

Advanced Research on Biologically Inspired Cognitive Architectures

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Chapter 9

Mathematical Models of Desire, Need, Attention, and Will Effort

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ABSTRACT

According to Spinoza, “Love is nothing else but pleasure accompanied by the idea of an external cause”. Author proposes that desire is nothing else but a change of pleasure accompanied by the idea of its cause, that terms ‘desire’, ‘want’ and their cognates describe change of the pleasantness of the state of a subject (PSS in short) associated with X, that if change of PSS is positive/negative, then X is called desirable/undesirable correspondingly. Both positive and negative desires can be strong, so strength of desire characterizes its magnitude. Need of X is defined here as a cyclical desire of X that gets stronger/weaker with dissatisfaction/satisfaction of its need. Author also explores an idea that the stronger is desire of X by a subject, the more attention this subject pays to X. Distribution of attention and influence on it by the will effort are analyzed in this paper.

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INTRODUCTION

The main objective of this chapter is to present new, closely linked mathematical models of desire, need, and attention. According to Spinoza (1674/1955), “*Love is nothing else but pleasure accompanied by the idea of an external cause: Hate is nothing else but pain accompanied by the idea of an external cause*” (p. 140). The author posits that desire is nothing else, but a *change* of pleasure accompanied by the idea of its cause. These definitions are so close, because loving/liking and desiring are two facets of the same process that author calls “Hedonic Recognition”¹ – the terms ‘desire’, ‘want’ and their cognates are used to describe *change* of the pleasantness of the state of a subject² associated with X, while the terms ‘love’/‘like’ and their cognates are used to describe the hedonic *end result of this change*. If X causes a positive/negative change, then X is called “desirable”/“undesirable” correspondingly.

Some support for this view on desire can be found in the classical literature, for example, in the writings of Aristotle and Locke; it also has some experimental backing (Ovsich & Cabanac, 2012). The author finds verification of this idea in the analysis of the process of needs satisfaction that has a typical pattern: dissatisfaction of a need for X creates desire for X by *lowering* current pleasantness of the state of a subject (pangs of hunger, pain of the withdrawal from a drug, etc.) while, at the same time, usually *raising* pleasantness of perceiving or even imagining X. These two simultaneous processes make X to be a factor of maximization of pleasantness, make X desirable. In other words, this creates the positive hedonic gap between the pleasantness of a state of a subject *with* and *without* X and this gap is called “desire for X”. The magnitude of the hedonic gap of desire is the measure of X’s desirability or desire strength; it increases with growth of dissatisfaction of the need for X, that in the terminology of this theory means that *desire for X gets stronger*. The exact opposite happens with the satisfaction of a need.

Desires attract attention of a subject to their objects. For example, objects of a dissatisfied need come to the attention of a subject more and more persistently with the growth of this need’s dissatisfaction. If a need is grossly dissatisfied, then objects and activities of satisfaction of this need can dominate the center of attention of a subject, consume attention. In the first approximation, the stronger the desire is for X, the more attention X gets and this proportionality is explored in this chapter. Voluntary attention is driven by the will effort that can suppress or support competing desires. This mechanism is addressed here in the framework consistent with William James’s (1927) approach.

BACKGROUND

The author presents here closely linked mathematical models of desire, need, and attention. Need and attention models are built upon the model of desire. Hence, desire is the pivotal entity of this inquiry and its first topic. There are many ways (Marks, 1986) and faces (Schroeder, 2004) of desire, but, first of all, there is one fundamental question about the meaning and definition of desire.

There is not much of a consensus about the notion of desire. The quite common understanding of desire as a propositional attitude was highly criticized by Bence Nanay (2013). “A number of philosophers have drawn attention to an ambiguity in the word ‘desire’ ” (De Sousa, 2011, p. 227). Schueler (1995, p. 6), who “... focused on contemporary philosophers...”, noted that “... the views I am criticizing suffer from a deep ambiguity in terms such as ‘desire’, ‘want’ and their cognates”. Frankfurt (2004, p.10) called the notion of desire “rampantly ubiquitous” and wrote:

Moreover, its various meanings are rarely distinguished; nor is there much effort to clarify how they are related. These matters are generally left carelessly undefined in the blunt usages of common sense and ordinary speech.

The level of ambiguity in understanding desire is such that the validity of the notion of desire itself is sometimes questioned or even denied outright. For example, DeLancey (2002, p. ix) wrote:

*Since my concern in this book is with basic emotions and other motivational states, I will on several occasions discuss the inappropriateness of the philosopher’s notion of desire; it is hard to overestimate the harm that this notion has done to moral psychology, action theory, and other aspects of philosophy of mind.
... (for example, there are many kinds of motivational states, but no generic one corresponding to the philosophical notion of desire)....*

However, as Marks (1986, p. 10) carefully noted:

...it may well be the case, as I believe, that there remains a single, significant, psychological phenomenon appropriately named “desire.” If so, then it is this – desire proper – which, ultimately, constitutes the subject matter of the theory of desire.

His belief is shared by the author of this paper.

There is not much clarity in understanding need, attention, will effort, and their relation to desire. According to Douglas and Peters (1979), "...attention was a neglected topic within psychology for many years" and there was "Confusion Caused by Definitions of Attention" (p. 178). The history of philosophy presents a substantial collection of notions on will. Bourke (1964) thought there to be "... eight distinctive views" of these notions "taken by Western thinkers" (p. 8).

MATHEMATICAL MODEL OF DESIRE

Hedonistic Approach to Desire

Schroeder (2004, pp. 27-31) identified three types of the desire theories. Two of them are represented by motivational and hedonistic theories of desire³. Schroeder considered hedonistic theories of desire to be superior to the motivational. Indeed, the hedonistic approach to desire has a very long and impressive history. Aristotle (2004, I, 11, 1370b) directly defined desire through pleasure: "Everything, too, is pleasant for which we have the desire within us, since desire is the craving for pleasure". The same can be said about Spinoza (1674/1955, Proposition XXXVII) and Mill (1861/1957, p. 49), who wrote: "...desiring a thing and finding it pleasant, aversion to it and thinking of it as painful, are phenomena entirely inseparable or, rather, two parts of the same phenomenon". Schroeder (2004, p. 27), referring to this Mill's opinion, wrote "Mill is not the only distinguished historical figure to have considered such a view." Schroeder further elaborated: "Hobbes, Hume, and Kant apparently had similar thoughts, though interpretation of these thinkers is difficult" (2004, p. 185). Arpaly and Schroeder (2014) think that "Thus, intrinsic desires are not made desires by their relations to action or pleasure" (p. 125), though they see some connection between pleasure and desire satisfaction.

Aristotle (2004, I, 11, 1370b), in line with his clearly hedonistic definition of desire as "the craving for pleasure" quoted above, not only defines anger as a desire for revenge (Konstan, 2006, p. 41; Kenny, 1963, p. 193) or retaliation (Tailor, 1986, p. 231), but also provides rather detailed descriptions of what it means at the hedonic level (Aristotle, 2004, I, 11, 1371a): "Revenge, too, is pleasant; it is pleasant to get anything that it is painful to fail to get, and angry people suffer extreme pain when they fail to get their revenge; but they enjoy the prospect of getting it" (also see 2.2, 1378b). It is important here to note that Aristotle's desire for revenge (anger) involves a *positive hedonic change*, transition from the hedonically negative to the hedonically positive state experienced even while imagining "the prospect of getting it".

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Aristotle's hedonistic approach to desire was echoed by Locke who defined desire as follows: "The uneasiness a man finds in himself upon the absence of anything whose present enjoyment carries the idea of delight with it, is that we call desire" (1690/1824, Book II, Chapter 20, Section 6). Desire for Locke is also about the *hedonic gap* between the more negative hedonic level ("uneasiness") of the state of the desiring subject without an object of desire and the more positive hedonic level ("enjoyment") with it. As for Aristotle, Locke's interpretation of desire is also about the *positive hedonic change* associated with the desired phenomenon.

The vital fact of the matter here is that such a hedonic gap, which is a hedonic change associated with the object of desire, is a regular property of the subjective experience of desire. This is true for the "low" physiological desires as well as for the "high" psychological desires. This sameness allows one to express desire for an action, power or sex, metaphorically, as being "power hungry", "hungry for the loved one."

Definition and Model of Desire

The model of desire discussed here (and in Ovsich 1998a, 1998b, 2012; Ovsich & Cabanac, 2012) is based upon the dynamic interpretation of the Hedonistic Principle, declaring that animals and humans alike are motivated/driven to maximize pleasantness of their internal state (Pleasantness of the State of a Subject or PSS⁴ here). The direct inference from the Hedonistic Principle is that one of the most important characteristics of any phenomenon (X) for a subject (S) driven to maximize her PSS should be *how much* X maximizes (or minimizes) the PSS as measured by the PSS change by X ($\Delta PSS_{s,x}$). For the human subject it should also mean that words and expressions describing the PSS changes ought to be notable and widely used.

The author proposes that terms such as 'desire,' 'want,' and their cognates describe the PSS change (ΔPSS) associated with (caused by) a phenomenon (object, activity, etc.):

- X associated with positive $\Delta PSS_{s,x}$ is called "desirable", "wanted";
- X associated with negative $\Delta PSS_{s,x}$ is called "undesirable", "unwanted";
- X associated with zero $\Delta PSS_{s,x}$ is called "indifferent", though sometimes it is called "undesirable" in the sense of the *lack* of any desire.

The common feature in the last two cases above is a non-positive (zero or negative) change of the PSS ($\Delta PSS_{s,x} \leq 0$) or an absence of the positive change of the PSS by X. It indicates, that:

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- A subject reports a presence or absence of desire for a phenomenon depending upon the presence or absence of the positive change of the PSS associated with that phenomenon;
- What is usually called ‘desire’ of X is a positive change of the PSS associated with X;
- An object of desire is a factor of the PSS maximization.

From the hedonistic viewpoint, it is quite clear why a positive rather than a negative or zero change of the PSS is used as the bases for terms ‘desire’ and ‘want’ describing PSS alteration. According to the Hedonistic Principle, a subject is looking for *maximization* of the PSS that is represented by a *positive* PSS change, $\Delta\text{PSS}_{s,x} > 0$. The use of the negative prefix to describe something as undesirable, unwanted, usually points to the opposite of the positive PSS change that subjects are seeking or, sometimes, to the absence of the positive PSS change.

If we interpret desire as an algebraic variable that can be positive or negative (where the desirable X is an object of the positive desire and undesirable X is an object of the negative desire), then we can define the desire of X in general as a term describing a change of the PSS ($\Delta\text{PSS}_{s,x}$) associated with X. Here is the definition of a desire: *a subject’s (S) desire for X is a word to describe a change of the Pleasantness of the State of this Subject ($\Delta\text{PSS}_{s,x}$) associated with (‘caused’ by) X, where X can be an object or an activity, perceived, remembered, or imagined. Desirability of X for S is an ability of X to maximize/minimize the PSS.*

Below is the formula of desire that incorporates all three types of the ΔPSS , where S is a subject experiencing desire, X is an object of desire, ΔPSS is the change of the Pleasantness of the State of the Subject:

$$\text{DESIRE}_{s,x} = \Delta\text{PSS}_{s,x} \tag{1}$$

If $\Delta\text{PSS}_{s,x} > 0$, then x is called “desirable”.

If $\Delta\text{PSS}_{s,x} < 0$, then x is called “undesirable”.

The above definition and formula of desire are consistent both with hedonistic/utilitarian approach to desire and with the contemporary point of view, that “... the primary linkage of the notion of desire to a notion other than itself is to the notion of affect – pleasure or displeasure in the widest sense” (Strawson, 2010, p. 284). Experimental support of this model of desire is demonstrated in Ovsich & Cabanac, 2012.

Desire Strength

A desire is often characterized or measured by its *strength*. Both positive and negative desire can be experienced as strong or weak. This means that the strength of desire is a sign-independent characteristic of desire. Therefore, a mathematical sign of the magnitude or an absolute value ($|value|$) should be applied to express strength of the subject's (S) desire for X ($\Delta PSS_{s,x}$):

$$\text{Strength of S desire for X} = |\text{DESIRE}_{s,x}| = |\Delta PSS_{s,x}| \quad (2)$$

Desiring and Liking as Facets of the Hedonic Recognition

According to Spinoza (1674/1955), “Love is nothing else but pleasure accompanied by the idea of an external cause” (p. 140). The author proposes that desire is nothing else but a *change* of pleasure accompanied by the idea of its cause. These definitions are so close because loving/liking and desiring are two different aspects of the same process. Indeed, Spinoza’s level of “pleasure accompanied by” X is reached as a result of the PSS change triggered by X, i.e., as a consequence of desiring X. The author posits that the terms ‘desire’ and ‘want’ are used to describe *change of the PSS* generated by X while terms ‘love’/‘like’, etc. are used to describe the *hedonic end result of this change* - the end level of ΔPSS .

In the process of the Object Recognition an Object is not only recognized and categorized as a banana, a chair, John, etc., but also recognized hedonically as pleasant/unpleasant (P/U), liked/disliked, desirable/undesirable. The author calls the latter form of recognition “Hedonic Recognition”. The fact of the matter is that in the process of the Object Recognition that includes Hedonic Recognition, the PSS of the subject changes from the antecedent, preceding perception/imagination of the object level PSS_1 to the end result level PSS_2 . This hedonic transition creates a PSS change: $\Delta PSS = PSS_2 - PSS_1 = PSS_{end} - PSS_{bgn}$. The author posits that P/U, or liking/disliking of X, is determined by the end level PSS_2 , while desirability of X is determined by the PSS change (ΔPSS). In other words, liking is determined by the position PSS_{end} on the hedonic axes while desiring is determined by the hedonic change depending not only on PSS_{end} , but also on the preceding level PSS_{bgn} . This means that though X can be equally liked in two different situations by the same subject, it can be desirable in one of them and undesirable in another, depending on whether the end level PSS_2 was reached by:

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1. *Raising* the PSS from the *lower* preceding/beginning level PSS_1 , creating positive ΔPSS , making X desirable, or by
2. *Lowering* the PSS from the *higher* preceding level PSS_1 , creating negative ΔPSS , making X undesirable.

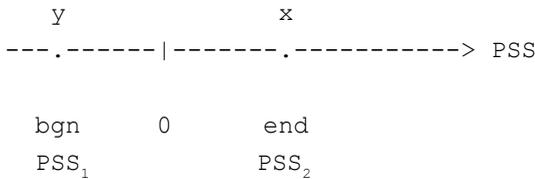
There are many possible combinations of liking and desiring, depending on where on the P/U continuum the hedonic change of desire begins and ends. Five cases of these combinations are analyzed below.

1. X is liked/pleasant ($PSS_{s,x} > 0$) and desirable ($\Delta PSS_{s,x} > 0$).

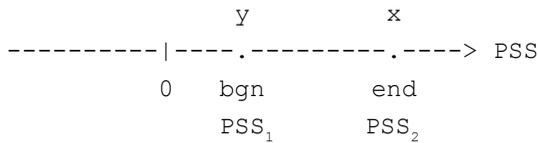
X can be represented in this case by food for a hungry S, entertainment for a bored one, etc. Two different types of this case are represented graphically below - one with negative PSS in the beginning (PSS_1 or PSS_{bgn}) and another one with positive PSS_1 . In both cases 1a and 1b,

$$DESIRE_{s,x} = PSS_{s,x} - PSS_{s,y} > 0.$$

Case 1a:



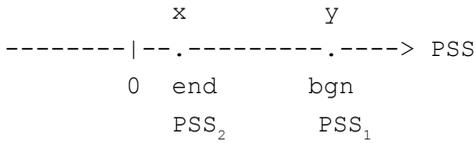
Case 1b:



There is nothing painful about the kind of desire that is represented by the last graph above (Case 1b) - its 'beginning' and 'end' are both in the area of positive P. This case supports Edwards (1979) critique of "... the false claim of Schopenhauer's pessimism that all pleasures result from the satisfaction of painful desire" (p. 95).

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2. X is liked/pleasant ($PSS_{s,x} > 0$) and undesirable ($\Delta PSS_{s,x} < 0$).



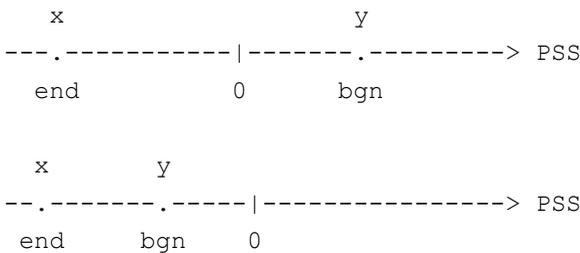
In this case, X may be offered/available when S is in possession of Y, or doing Y that is more pleasant than X. X minimizes the PSS here, i.e. X is undesirable, though X is pleasant in and of itself, and is liked by S.

X does not have to be unpleasant/disliked/aversive/associated with a negative PSS in order to be categorized as undesirable - it just has to be *less pleasant* than Y available at the same time. This point of view is in disagreement with Arpaly & Schroeder’s (2014) description of the negative desirability (“intrinsic aversion” in their terminology) when they equate it with “...desiring that things not be the case because we have an aversion to them in themselves” (p. 128).

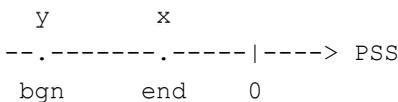
Here is an example for this case (2): S is eating S’s favorite dish Y. When dish X is offered S declines it and says that s/he doesn’t want it. This does not necessarily mean that S has an aversion to X in itself or does not like it, but only that S likes Y *more* than X. When X is offered, S experiences negative $\Delta PSS_{s,x}$ and, therefore, categorizes X as undesirable to S at the moment.

3. X is disliked/unpleasant ($PSS_{s,x} < 0$) and undesirable ($\Delta PSS_{s,x} < 0$).

In this case both desire and attitude toward their object (x) are negative: a subject does not want something unpleasant in and of itself - bad smell, view, event, news, etc.



4. X is disliked/unpleasant ($PSS_{s,x} < 0$) and desirable ($\Delta PSS_{s,x} > 0$):



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What is common and notable about cases 4 and 5 is that they describe desires with no end pleasure - the hedonic end result of desire is not positive (it's negative or zero) while the PSS change experienced by the subject is positive. An object of such desire is still a factor of the PSS maximization, but satisfaction of such desire doesn't bring the PSS into the area of positive PSS, or into the domain of happiness. In the author's opinion, this 'hedonic incompleteness' of desire is the source of that reluctance to use the terms 'desire', 'want' and their cognates in these cases.

It seems that Aristotle's understanding of desire as "craving for pleasure", as well as Mill's one (see quotes in the paragraph "Hedonistic Approach to Desire") are too narrow, because they do not cover cases of desire with no end pleasure. On the contrary, a proposed understanding of desire as craving for pleasure *increase* (or displeasure *decrease*) covers a lot more, if not all, of the ground - in all the above cases, there is a PSS *change* associated with the object of positive or negative desire.

Desire, Motivation, and Pleasure

There is a question about the "disconnection between desire, motivation and pleasure" (De Sousa, 2011, p. 242; Corns, 2014) associated with the above discussion. It seems to be based upon the narrow understanding of desire as craving for pleasure⁵, and/or the static, fragmented interpretation of the Hedonistic Principle, where pleasure is the source of a positive motivation and displeasure/pain is the source of a negative one. These views on desire present a contradiction in understanding of cases 4 and 5 above as having positive motivation of desire without end pleasure. The dynamic interpretation of the Hedonistic Principle exercised here, where the source of positive motivation is not pleasure, but *pleasure maximization*, eliminates this contradiction. The point here is that it is not pleasure *per se*, but a *change* of pleasure that is a necessary and often sufficient condition of the hedonic experience called desire.

It is especially clear from the point of view of the strong form of the Hedonistic Principle, declaring that a subject is motivated by pleasure maximization *and only by it*. If only pleasure maximization motivates, then there is no motivation without it and only the PSS *change* of desire carries what was called a "motivational oomph" in Robinson & Berridge (2015), Corns (2014). According to Berridge (2004, p. 194):

'Liking' is essentially hedonic impact - the brain reaction underlying sensory pleasure-triggered by immediate receipt of reward such as a sweet taste (unconditioned 'liking'). ... 'Wanting', or incentive salience, is the motivational incentive value of the same reward. ... But incentive 'wanting' is not a sensory pleasure. 'Wanting' is purely the incentive motivational value of a stimulus, not its hedonic impact.

A good illustrative analogy with liking and wanting, subscribing motivational power to the latter rather than to the former, is through the comparison of them with an electrical potential and voltage respectively. Any point of the conductor can have an electric potential, but it takes *difference* of potentials (voltage) between two points to move electrons between them, to “motivate” them into action. Similarly, every phenomenon has its pleasantness/unpleasantness for a subject (including zero level), but it takes a *change* of pleasantness, a *difference* in the pleasantness level (desire in the author’s theory) to motivate and move a subject. In real life, your answer to the question “Do you *like* tuna sandwiches?” does not describe your current motivation toward them, while your answer to the question “Do you *want* tuna sandwich?” does; there is no principal inconsistency or contradiction if the former answer is “yes” and the latter is “no”, or *vice versa*.

The best proof and illustration of the presented view on desire is provided by the following analysis of the process of strengthening and weakening of desires generated by needs during the process of their satisfaction.

NEED AS A PERIODIC DESIRE

Definition of a Need

As Audi (1993, p. 29) wrote, “Human needs are innate and quickly give rise to desires”. Rubinshtein (1957) has declared that “*desire is a concrete form of the need’s existence*”⁶. Experiencing a need means feeling the corresponding desire. If a subject experiences a desire for X repeatedly or regularly it is usually said that the subject *needs* X. This is clearly demonstrated by the needs that emerge and cease to exist with age or during changing circumstances, for example, the needs for sex, smoking, or drugs. The origination/disappearance of such needs is acknowledged when the corresponding desire begins/stops being recurring, cyclical, periodic. One can properly say that s/he occasionally wants X, but does not need it (anymore).

Need is defined here as a term used for a periodic or cyclical desire. This is true for all kinds of needs. A need is characterized by the strength and frequency of its desire. Need, being a cyclical process is like a ‘wave’ of desire.

Satisfaction and Dissatisfaction of a Need and its Desire

According to the Hedonistic Principle, animals and humans alike are motivated/driven by the hedonic strive to maximize their PSS. Therefore, a major tool of their orientation is hedonic ‘pricing’ through attaching a factor of pleasantness/unpleasantness (P/U) to a phenomenon in order to establish it as a positive or negative

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factor of the PSS maximization, and determine its likability and desirability. By using variants of reward and punishment, like the carrot and stick scenario, both nature and society affix hedonic sticker-prices of P/U and set values of good and bad. Alteration of this P/U or hedonic ‘pricing’ is the most significant instrument of adjustment of animal and human orientation and choice. This process has been experimentally studied by Cabanac (1971, p. 1105) who called it “alliesthesia”:

In order to avoid using a whole sentence saying that a given external stimulus can be perceived either as pleasant or unpleasant depending upon signals coming from inside the body, it may be useful to use a single word to describe this phenomenon. I hereby propose the word alliesthesia coming from esthesia (meaning sensation) and allios (meaning changed).

Modification of the hedonic values of objects and activities by alliesthesia, which depends on the level of satisfaction of a subject’s needs, is very important for understanding the mechanism of needs satisfaction through desire generation. Most needs have two definable features:

- The dissatisfaction of any need of a subject negatively affects the PSS, the PSS level *goes down* with time;
- At the same time, the pleasantness of perceived or imagined objects/activities related to this need’s satisfaction *goes up* for the subject.

These two aspects are easily recognizable in the following description of Bertrand Russell (1921, p. 67):

...it seems clear that what, with us, sets a behavior-cycle in motion is some sensation of the sort which we call disagreeable. Take the case of hunger: we have first an uncomfortable feeling inside, producing a disinclination to sit still, a sensitiveness to savory smells, and an attraction towards any food that there may be in our neighborhood.

This means that the hedonic gap between the PSS *without* the object(s) of a need satisfaction and the PSS *with it* grows. *This gap is a desire and its magnitude is its strength that grows.*

Satisfaction of any need of a subject produces exactly opposite effects:

- The PSS becomes more positive, goes up as a result of satisfaction of a need;
- The P (pleasantness) of the objects of this need’s satisfaction goes down.

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As the hedonic gap of desire gets smaller, desire gets weaker, all the way down to the satiation point when ΔPSS of desire becomes equal to zero – desire is satisfied, it “disappears”. At this time, the opposite side of the desire cycle starts again.

The needs satisfaction process clearly shows that motivation, or motivational ‘oomph’, grows and diminishes together with desire, and not with liking. Indeed, though the pleasantness-likability of food increases together with dissatisfaction of the need for food due to alliesthesia, one still generally likes food after one has eaten, though one does not want to eat anymore, and is not motivated to eat. One likes what s/he put in their refrigerator, but is motivated to open it only when hungry. On the other hand, one who is extremely hungry ($\Delta\text{PSS}_{s,\text{food}} \gg 0$) may be motivated to eat things that s/he dislikes ($\text{PSS}_{s,\text{this food}} < 0$). Aren’t these patterns tried-and-true with regard to the entire smörgåsbord of life?

MATHEMATICAL MODEL OF ATTENTION

Attention and Hedonistic Principle

Desires catch attention, and the stronger the desires are, the more attention they and their objects get. This statement can be presented as an empirical observation. It can be also deduced from the Hedonistic Principle: if it is true that subjects are motivated/driven to maximize the pleasantness of their internal state (PSS), then the more a phenomenon influences the process of the PSS maximization, the more attention should be paid to it. The magnitude of the influence of X on the process of the PSS maximization is measured by the magnitude of the PSS change ($|\Delta\text{PSS}_{s,x}|$) associated with X, or, said another way, the magnitude of the influence of X is determined by the strength of the desire of S for X. In the first approximation, attention of a subject S toward a phenomenon X can be considered to be proportional to the strength of desire for it:

$$\text{ATT}_{s,x} = k|\Delta\text{PSS}_{s,x}| = k|\text{DESIRE}_{s,x}|, \quad (3)$$

where k is a positive coefficient of proportionality.

Attention to a Single Phenomenon

Let’s analyze the formula of attention to a single phenomenon (3) to see if it describes the reality of attention correctly.

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Case 1: $\Delta PSS_{s,x} > 0$ or $DESIRE_{s,x} > 0$.

In this case X is a factor of PSS *maximization*, meaning that the subject wants X.

If $\Delta PSS_{s,x} > 0$, $DESIRE_{s,x} > 0$ then $ATT_{s,x} > 0$.

According to Equation (Eq.) 3, $ATT_{s,x}$ increases/decreases if the positive $DESIRE_{s,x}$ increases/decreases. The greater the desire for X by a subject S, the more attention is paid by S to X.

Case 2: $\Delta PSS_{s,x} < 0$ or $DESIRE_{s,x} < 0$.

In this case, X is a factor of PSS *minimization*, meaning that the subject does not want X.

If $\Delta PSS_{s,x} < 0$, $DESIRE_{s,x} < 0$ then $ATT_{s,x} > 0$.

The formula $ATT_{s,x} = k|DESIRE_{s,x}|$ illustrates that the stronger the negative desire for X - the more bothersome, painful, and undesirable X is – the more attention is paid to it.

Cases #1 and #2 show that, according to Eq. 3, a subject pays attention to both desirable and undesirable phenomena; the stronger (more desirable or undesirable) it is, the more attention will be paid to it. The substance of this matter is that eliminating the sources of the PSS minimization is just as important for the hedonistic process as acquiring the sources of the PSS maximization because of the integrative character of the PSS. A subject's concentration on the sources of a positive ΔPSS for their hedonic *exploitation* as well as concentration on the sources of a negative ΔPSS for their *elimination* is equally important for this process of the PSS maximization. Attention paid to X doesn't depend on the *sign* of ΔPSS_x or a desire for X, but only on the *magnitude* of the PSS change that is the strength of desire for X. In summary, attention paid to X is *sign-independent* of whether X is desirable or undesirable, but depends only on the *strength* of desirability/undesirability of X. Attention is "blind" to the sign of desire; attention depends only on its strength.

Case 3: $\Delta PSS_{s,x} = 0$ or $DESIRE_{s,x} = 0$.

If $\Delta PSS_{s,x} = 0$, $DESIRE_{s,x} = 0$ then $ATT_{s,x} = 0$.

If X doesn't affect the PSS maximization, if X is neither desirable nor undesirable, then a subject does not pay attention to X. It does not mean that X is hedonically indifferent, or is not liked or disliked. One likes what s/he put in the refrigerator, but does not pay attention to it, is not motivated to eat until getting hungry, experiencing the desire to eat.

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creature facing an infinite Universe. This is the essence of "... the human talent for *ignoring* what should be ignored, while staying alert to relevant recalcitrance when it occurs" (Dennett, 1990, p. 162).

The author would suggest that imitation of this hedonic mechanism is the key to resolution of one of the fundamental problems of Artificial Intelligence, called "the qualification problem" by McCarthy (1968), though usually called a "frame problem", and described by Dennett (1990, p. 161) as follows:

What is needed is a system that genuinely ignores most of what it knows, and operates with a well-chosen portion of its knowledge at any moment. Well chosen, but not chosen by exhaustive consideration. How, though, can you give a system rules for ignoring - or better, since explicit rule-following is not the problem, how can you design a system that reliably ignores what it ought to ignore under a wide variety of different circumstances in a complex action environment?

The author agrees with McFarland's (2008, p. 156) point of view:

It is worth noting that animals do not suffer from the frame problem, and this may be because they have a value system (see Chapter 8), the cost and risks involved in their decision-making acting as constraints on their behavior.

Attention to Multiple Phenomena

The model of attention to a 'single' phenomenon above is very much an abstraction, because in reality a subject always perceives multiple phenomena. This model, however, represents an approximation of a real situation, where the subject concentrates mainly on one phenomenon in the center of attention. The higher the percentage of total attention paid to the phenomenon in the center of attention, the closer this model comes to reality.

There are certain situations when a phenomenon is singled out and placed in the center of attention. This occurs in a process of choice making when the elements of choice are appraised by a subject one by one: when a 'new' phenomenon catches the attention of a subject and is appraised, or perhaps an 'old' phenomenon is re-appraised. This also happens when a phenomenon becomes 'the chosen one', catching the center of attention while competing phenomena are pushed to the periphery of attention.

The fact of the matter is that at any given moment the attention of a subject is *distributed* between multitudes of simultaneously perceived phenomena⁷. The author proposes that the total volume of attention of a subject perceiving n phenomena at the moment t ($ATT_{total, s, t}$) can be described as the sum of attention paid to each of them:

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$$ATT_{total\ s,t} = ATT_{s,t,1} + ATT_{s,t,2} + \dots + ATT_{s,t,n} \quad (4)$$

Now, let's merge two last formulas by replacing every component of the right part of Eq. 4, representing attention to one of the n phenomena, with its expression from Eq. 3:

$$\begin{aligned} ATT_{total\ s,t} &= k|\Delta PSS_{s,t,1}| + \dots + k|\Delta PSS_{s,t,n}| = \\ &= k|DESIRE_{s,t,1}| + \dots + k|DESIRE_{s,t,n}| \end{aligned} \quad (5)$$

This formula of attention describes distribution of attention between n simultaneously perceived phenomena in proportion with their desire's strength⁸.

Center of Attention

Attention has its periphery, and most focused, or 'brightest' area, which is called the 'center of attention'. Let's assign numbers in Eq. 5 to perceived phenomena from 1 to n *in descending order*, in accordance with the volume of attention paid by a subject to each of them, so that:

$$ATT_{s,t,1} > ATT_{s,t,2} > \dots > ATT_{s,t,n} \quad (6)$$

Thus, the number one ($ATT_{s,t,1}$) will be assigned from now on to the phenomenon having the most attention, or being at the center of attention - the phenomenon associated with the largest positive or negative PSS change, the one that is most desirable or undesirable, i.e., corresponding to the *strongest desire*:

$$|\Delta PSS_{s,t,1}| > \dots > |\Delta PSS_{s,t,n}| \quad (7)$$

or

$$|DESIRE_{s,t,1}| > \dots > |DESIRE_{s,t,n}| \quad (8)$$

General Formula of Attention

There is another general feature of attention that has to be taken in consideration: attention has an upper limit. In the words of Csikszentmihalyi (1978, p. 337): "The main assumption I shall be making is that attention is a form of a psychic energy needed to control the stream of consciousness, and that attention is a limited psychic resource". This means that at any moment (t) there is a maximum or an upper

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limit ($ATT_{s,t}^{max}$) for the attention of a subject (S) and that at any moment (t) this maximum is not greater than the total attention of a subject:

$$\begin{aligned}
 ATT_{s,t}^{max} &\geq ATT_{s,t}^{total} = \\
 &= k|\Delta PSS_{s,t,1}| + \dots + k|\Delta PSS_{s,t,n}| = \\
 &= |DESIRE_{s,t,1}| + \dots + k|DESIRE_{s,t,n}| \quad (9)
 \end{aligned}$$

It is important that this *general formula of attention* includes within itself the formula for attention to a single phenomenon (Eq. 3) as a particular case, corresponding to the situation when $n = 1$, when one phenomenon consumes all attention (see Eq. 13).

$$ATT_{s,t}^{max} = ATT_{s,t}^{total} = k|DESIRE_{s,t,1}| \quad (10)$$

Let's find out how the general formula of attention works with some combinations of values of its variables/parameters, and how the formula's implications reflect reality.

Change of Desirability and Redistribution of Attention

Here we will consider what happens with distribution of attention if desirability of one of the n simultaneously perceived phenomena changes.

Case 1: Change of *positive* desirability of the phenomenon X; $DESIRE_{s,x} > 0$.

If positive desire grows, then its magnitude or strength ($|DESIRE_{s,x}|$) gets larger. According to Eq. 3 ($ATT_{s,x} = k|DESIRE_{s,x}|$), attention towards the phenomenon grows together with the *strength of the desire* for it.

With the additional attention paid to one of the n phenomena, this particular one will move up in the 'attention hierarchy'; it will get an *attention 'promotion'*. This phenomenon would rise in the hierarchy of the attention levels corresponding to n different phenomena perceived at the same time (t).

$$ATT_{s,t,1} > ATT_{s,t,2} > \dots > ATT_{s,t,n} \quad (11)$$

Its position will move from right to left in Eq. 11 and its number (n) will *decrease* until it becomes the *number one* ($n=1$) phenomenon – one in the center of attention. The reverse process of an *attention 'demotion'* occurs when the strength of desirability of the phenomenon decreases.

Attention ‘promotion’ and ‘demotion’, as prescribed by Eq. 9, do take place in reality. A good illustration of such a promotion is provided by taking note of a growing desire corresponding to an ongoing unsatisfied need. Such a desire strengthens until it gets into the center of attention of a subject, together with the objects and ways of its satisfaction. In the course of satisfaction of a need, the reverse process takes place. Desire gets weaker, and the attention paid to the objects and actions of satisfaction for this desire decreases, and as such, these objects and acts move out from the center of the subject’s attention to its periphery, and finally completely out of the attention’s range. The center of attention gets overtaken by the phenomenon that is next in the desire strength hierarchy.

Case 2: Change of *negative* desirability (undesirability) of X: $DESIRE_{s,x} < 0$.

If negative desire grows, the magnitude of its undesirability ($|DESIRE_x|$) gets larger. Eq. 3 shows that attention towards the phenomenon grows together with the strength or magnitude of its *undesirability*. As in Case #1, with the additional attention paid to one of the n phenomena, that particular one will move up in the attention hierarchy, and will earn an attention promotion. A good illustration of the cases where attention grows toward *undesirables* is provided by any kind of the increase of unpleasantness, discomfort or pain. The more unpleasant and undesirable something becomes to a subject, the more attention is drawn thereto.

Comment about Cases 1 and 2: The similarity in changes of attention in the above Cases 1 and 2 illustrates the independence of attention paid to a phenomenon from the positive or negative sign of its desirability. Attention to X is “blind” to the sign of X’s desirability, and it is so for a good reason - animals and humans alike should pay attention to both pleasures of life and its pitfalls in order to act upon them.

Hedonic ‘Pricing’ and Redistribution of Attention

Let us suppose that a subject perceives the same n phenomena for a given time when $ATTmax_{s,t} = ATTtotal_{s,t}$, but the attention that is required for one of n phenomena grows.

$$\begin{aligned}
 ATTmax_{s,t} &= ATTtotal_{s,t} \\
 &= ATT_{s,t,1} + \dots + ATT_{s,t,n} \\
 &= k|DESIRE_{s,t,1}| + \dots + k|DESIRE_{s,t,n}| \tag{12}
 \end{aligned}$$

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This formula shows that as one of the n phenomena (number $x \leq n$) gathers more attention, then the other $(n-1)$ phenomena will have less attention left to them. If the maximum of available attention ($ATT_{s,t}^{\max}$) is not used up ($ATT_{s,t}^{\max} < ATT_{s,t}^{\text{total}}$), then the total disbursed attention ($ATT_{s,t}^{\text{total}}$) can be increased up to the level of $ATT_{s,t}^{\max}$. Conversely, if the maximum of available attention is already used up ($ATT_{s,t}^{\max} = ATT_{s,t}^{\text{total}}$), then the total of available attention ($ATT_{s,t}^{\text{total}}$) must be redistributed. If the remainder of attention is not enough for the rest $(n-1)$ of the evident phenomena, then some of them will receive no attention at all. Hence, a reduction of the number (n) of the perceived phenomena takes place. At this point, attention becomes more focused or narrowed. If attention to X grows so much that it requires *all* of the available attention of a subject, then all of it has to be spent on X only:

$$ATT_{s,t}^{\max} = ATT_{s,t}^{\text{total}} = ATT_{s,t,x} \quad (13)$$

It may be that an adult deeply concentrated on inner thoughts, or a child running after a ball, may not pay enough attention to that oncoming car. The more concentrated a subject is on something, the more difficult it will be for anything else to catch one's attention. And conversely, if the concentration of attention for a subject is low, then any new phenomena can easily get to the center of attention. For example, a bored child in the classroom is just looking for anything new to switch attention to.

A good example of the narrowing down of attention is the case when a need of a subject has not been satisfied for a long period of time. (A 'long' period of time can be probably defined as a multiple of the regular or average period of time between satisfactions of this need). In this case, objects and activities associated with satisfaction of the subject's need usually become both more pleasant and desirable (due to alliesthesia), and demand more and more attention. They gradually push everything out of the center of the subject's attention to the periphery. Eventually the objects and activities of this need's satisfaction become 'super-values' for that moment.

This converges with one of the basic postulates of Ethology, as described by Cabanac (2000, p. 1): "One basic postulate of Ethology is that behavior tends to satisfy the most urgent need of the behaving subject (Tinbergen, 1950; Baerends, 1956)", because the strongest desire corresponds to the 'most urgent need' of this postulate. As James (1927) said: "What holds attention determines action" (p. 448).

If James is right, then the 'division of labor' between desiring and liking in regard to choice and action seems to be as follows: desire (strength) determines *what* gets in the center of attention, holds it and is acted upon, while liking/disliking defines the *type of an action* as positive/negative, accepting/rejecting, consuming/destroying, assimilation/fight-or-flight correspondingly.

Change of the Objects of Attention

At any moment, new phenomenon can appear and make demands on a subject's attention. The following redistribution of attention can result in possible promotion of a hedonically important new phenomenon to the center of attention. This can be as sudden and unpredictable as its appearance.

The stage of the attention distribution described by Eq. 13 can be reached at once in the case of a hedonically significant new phenomenon, like an unforeseen extreme danger or excitement. For example, while walking down the street one perceives numerous objects but pays little attention to most of them. A subject can see many cars on the street and pay little or no attention to them. However, the distribution of a subject's attention changes right away with the recognition of a friend inside a car, or when it seems that one of these cars is going to hit the subject.

Maslow's Pyramid of Needs in Real Time as a Table of the Current Desires

It is generally accepted that animals and humans alike are driven by satisfaction of their needs and that the composition of the human needs is represented by Maslow's pyramid of needs. Let's transform Maslow's pyramid of needs into a table of desires generated by these needs, and sort them by their current strength in descending order. This way the strongest desire will be represented by the element number one on the top of this table. Let's also include in this table all other desires experienced by a subject at the moment. This would include unique, occasional, discrete desires of a subject (versus repetitious, cyclical desires of needs).

Such a table can be very informative in describing the motivational state of a subject, her/his preferences at the moment, especially if this table is constantly re-sorted by the desire's strength. It is important because the stronger desire is the more attention it gets. The strongest desire, its objects and associated with it activities have the highest probability to get in the center of attention of a subject, to stay in it, and to be acted upon.

Voluntary Attention

By a certain age humans become able to somewhat control their attention. This ability is called voluntary attention and is considered to be attention driven by will or will effort. The question is - how do attention and will connect and interact?

Voluntary Attention and Will Effort

The history of philosophy presents a substantial collection of notions on will. According to Bourke (1964), there are "... eight distinctive views" of these notions "taken by Western thinkers" (p. 8). The point of view on will effort discussed here has a lot in common with some of these meanings, i.e., understandings of will as an "intelligent preference", a "rational appetite", a "dynamic power" (Bourke, 1964, pp. 9-13).

Locke (1690/1824) asserted, that "...the will in truth signifies nothing but a power, or ability, to prefer or choose..." (p. 229, Book 2, § 17, Edit 12). It would be, probably, better to say that will signifies nothing but a power, or ability of a subject, to *influence* one's own preference or choice. The difference is significant because choice does not employ will at all times. A lot of common choices are 'will-effortless', do not involve will. Desires are often quite sufficient for determining choice. Animals and human infants can be said *not* to possess will, but they certainly have needs and desires, display preferences and choose. As Vilunas (1976) has noted⁹, "Will effort is necessary while one has to act either without an immediate emotional impulse or against it, when, for example, one has to restrain anger, interrupt an exciting activity, or ignore physical pain, etc." (p. 51).

There is an important difference between choice made with and without the involvement of will. Without the will's participation, elements of choice are usually (1) first evaluated by a subject and then (2) a choice of one of these elements is made based upon the balance of their desirabilities. This algorithm is *reversed* in the case when a choice is made with participation of the will: choice is made first (1), often by a rational decision and then (2) is enforced by the will effort that re-estimates and outright adjusts desirabilities of the elements of choice in order to support the predetermined preference. Such an 'unnatural' algorithm of choice made by will is the main reason why this voluntary type of choice needs the reinforcement of the will effort. Will always fights off other choices that might have otherwise prevailed.

Hedonistic Choice and Will Effort

The above exercised approach to attention suggests that for a phenomenon to be in the center of attention of a subject, to be chosen, it must have the dominant hedonic importance for that subject at that time, i.e., it must be 'the number one phenomenon' - the one with the greatest strength/magnitude of desirability (see Eq. 8). The essence of the will effort is in shifting desirability balance in favor of a predetermined phenomenon. According to Eq. 8, this shift can be implemented by one of these two following ways or by both of them applied together:

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1. By maximizing the hedonic importance or magnitude of desirability for the pre-chosen phenomenon (will's end) by stimulating a corresponding hedonic association with it, such as thinking of or imagining an associated P/U, hedonic reward/punishment;
2. By minimizing the magnitude of desirability of phenomena competing with a pre-chosen one.

Let's consider a simple case of a choice between just two phenomena, A and B, with corresponding desires DES_a and DES_b .

Phenomenon A is chosen, is in the center of attention of a subject if,

$$|DES_a| > |DES_b|. \quad (14)$$

Buridan's ass case corresponds to:

$$|DES_a| = |DES_b|. \quad (15)$$

Phenomenon A is *not* chosen if,

$$|DES_a| < |DES_b|. \quad (16)$$

Let's consider the last case (Eq. 16) when phenomenon A is not chosen at the moment, but the subject has made a conscious, rational decision to choose A over B that requires what is described by Eq. 14. To accomplish this s/he has to apply will effort (WE in short) strong enough to shift the balance of desirabilities from the situation described by Eq. 16 to Eq. 14. Mathematically speaking this means that:

$$|DES_a + WE| > |DES_b| \quad (17)$$

or

$$|DES_a| > |DES_b - WE| \quad (18)$$

Equations 14 - 18 are similar to the formulas offered by William James (1927, p. 444) who spoke about a subject conquering and overcoming impulses and temptations in the chapter 'Will' of his book:

The facts can be most briefly symbolized thus, P standing for propensity, I for ideal impulse, and E for the effort:

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I per se < P.

I + E > P.

In other words, if E adds itself to I, P immediately offers the least resistance, and motion occurs in spite of it.

What is presented as will effort (“WE”) in Equations 17 and 18 has to be a variable of the same nature as other members of the equation in order to be added to them. The other two elements are desires or changes of the PSS. Therefore, what is coded as WE must also be a change of the PSS (ΔPSS) or desire created by the will effort, desire being a function of will: $\text{WE} = \Delta\text{PSS}_{\text{we}}$. As Kant (1785/1983, p. 11) wrote, “The faculty of desire whose internal ground of determination and, consequently, even whose liking [das Belieben] is found in reason of the subject is called the will”.

Accordingly, Eq. 17 can be rewritten as follows:

$$|\text{DES}_a + \Delta\text{PSS}_{\text{we}}| > |\text{DES}_b| \quad (19a)$$

or

$$|\text{DES}_a + \text{DES}_{\text{we}}| > |\text{DES}_b|. \quad (19b)$$

Will effort can be also applied to a *competing* element of choice in order to reduce its desirability:

$$|\text{DES}_a| > |\text{DES}_b - \text{DES}_{\text{we}}|, \quad (19c)$$

Of course, will effort ($\Delta\text{PSS}_{\text{we}}$) may not be strong enough to change the choice, to suppress the dominating desire. This case seems to correspond to the case of *akrasia*, or weakness of will.

Will Effort as Effort of Attention

There is another question about will that needs to be answered: how can a subject voluntarily, by using will effort, change a hedonic value of an element of choice, an attitude or desire toward it? William James (1927) wrote: “Volitional effort is effort of attention” (p. 450), “Effort of attention is thus the essential phenomenon of the will” (p. 452). But what does an “effort of attention” get done and how? According to James (1927, p. 452):

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The strong-willed man, however, is the man who hears the still small voice unflinchingly, and who, when the death-bringing consideration comes, looks at its face, consents to its presence, clings to it, affirms it, and holds it fast, in spite of the host of exciting mental images which rise in revolt against it and would expel it from the mind. Sustained in this way by a resolute effort of attention, the difficult object ere long begins to call its own congeners and associates and ends by changing the disposition of the man's consciousness altogether.

The dialectical approach, as well as common sense, tells us that any phenomenon has its good and bad aspects for a subject, with positive and negative sides. Evaluating a phenomenon involves estimating these sides and coming up with their total in a way that helps determine an attitude toward it. Hedonic integration¹⁰ facilitates the summation of this total. The more attention is paid to one side of a phenomenon the more influence it has on the total hedonic value of the phenomenon as a whole.

Just by controlling one's attention, by applying voluntary attention, one can concentrate more on the positive sides of a phenomenon and turn a blind eye to negative sides, thus shifting the balance of positivity or negativity of this phenomenon by one's own volition. Concentrating more on the fullness or emptiness of that 50% measured bottle, one can adjust one's perception of it, to be half full or half empty. The downside of this is that our attitudes can be somewhat off the mark or one-sided. The upside of this process is that it gives humans their summary attitudes and some measure of will power.

The control of attention or the usage of voluntary attention is the essence of will, as James has asserted, because attention is the will's executor. A level of mastery of this control determines the scope of freedom for a subject's will. Damasio wrote (1994, p. 175):

Willpower draws on the evaluation of a prospect, and that evaluation may not take place if attention is not properly driven to both the immediate trouble and the future payoff, to both the suffering now and the future gratification. Remove the latter and you remove the lift from under your willpower's wings. Willpower is just another name for the idea of choosing according to long-term outcomes rather than short-term ones.

Affective (Arnold, 1970) or hedonic memories can be retrieved by remembering or imagining something emotionally/hedonically charged, and this is exactly what will effort accomplishes. Will effort deliberately activates emotional/hedonic memories of a subject by insistently reminding a subject about certain features or consequences of choice and/or making a subject imagine these consequences. Will effort directs the attention of a subject toward those features of the elements of choice

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that change their desirabilities, and, therefore, their balance, in ways advantageous for the will's goal. Suppression of the leading desire can, of course, be produced by weakening it by closure of the hedonic gap of this desire, i.e., by lowering the anticipated or experienced pleasantness of the desired object ("poisoning" it for a subject), or by minimizing unpleasantness of the undesired one.

Will influences the subject's choice from the inside in the same way that other people, and society at large, do it from the outside - by reminding a subject about the consequences of these choices, and the external and internal rewards and punishments associated therewith. When imposed by will, however, these rewards and punishments are self-inflicted and self-determined which apparently qualifies such a will effort to be categorized as "free", and will's guidance of attention as being voluntary.

While willing oneself, a subject often talks to him- or herself internally or even externally - s/he recalls propositional attitudes of beliefs, deliberates, argues, shames him- or herself, etc. This verbalization may hold the key to understanding mechanism of the will effort. It is proven that words and propositions are emotionally, and therefore, hedonically charged. Humans, and only humans, possess this way to influence their own and others' PSS through language. Only normal and mature enough humans with the command of language possess will power, because only they can discuss with themselves the pros and cons of X and Y, and talk themselves into one behavior and against another.

FUTURE RESEARCH DIRECTIONS

Formalization of any process in mathematical terms paves the way for creation of its computer model. The author believes that proposed models of desire, need, attention, and will effort can be computerized and implemented by Artificial Intelligence. These models need to be developed further in greater detail. They also need experimental verification, though the model of desire presented here has experimental support (Ovsich & Cabanac, 2012).

One of the questions to be pursued further is about the relationship between desire and attitude. Desire is understood here as a kind of a relative attitude that depends on the relative positions of two points on the hedonic continuum, versus, so to speak, an "absolute" attitude of liking that is determined by only one such point. This view does not directly contradict the currently common, though contested (Nanay, 2013), categorization of desire as a propositional attitude. However, it should be taken into consideration that, "Despite its central place in explaining human behavior (Allport, 1935), there is no universally agreed upon definition of what precisely is represented by an attitude" (O'Reilly, Roche & Cartwright, 2015, p. 162).

CONCLUSION

The author has presented here new, closely linked mathematical models of desire, need and attention (including voluntary attention driven by the will effort) based upon the Hedonistic Principle. According to the Hedonistic Principle, subjects are motivated/driven to maximize pleasantness of their internal state (Pleasantness of the State of the Subject (PSS)). Hence:

1. The (most) important characteristic of any phenomenon (X) for such a subject (S) should be how much X influences the process of the PSS maximization that is measured by the PSS change associated with X ($\Delta PSS_{s,x}$). The author proposes that the terms ‘desire’, ‘want’ and their cognates describe the PSS change associated with (caused by) X:

$$\text{DESIRE}_{s,x} = \Delta PSS_{s,x}$$

If $\Delta PSS_{s,x} > 0$, then x is called “desirable”.

If $\Delta PSS_{s,x} < 0$, then x is called “undesirable”.

The magnitude of the PSS change is what is called the “strength of desire”:

$$\begin{aligned} \text{Strength of S desire for X} &= |\text{DESIRE}_{s,x}| = \\ &= |\Delta PSS_{s,x}| \end{aligned}$$

The author posits that the desirability of X is determined by the PSS change (ΔPSS) while the liking/disliking of X is determined by the end level of this PSS change.

Experiencing a need means feeling the corresponding desire. If a subject experiences a desire for X repeatedly or regularly, it is usually said that the subject needs X. Need for X is defined here as a recurring, periodic desire of X.

2. The more X affects the process of the PSS maximization, the more attention a subject should pay to it:

$$\text{ATTENTION}_{s,x} \sim |\Delta PSS_{s,x}| \sim |\text{DESIRE}_{s,x}| = k|\text{DESIRE}_{s,x}|$$

According to this formula, attention is “blind” to the sign of desire, depends only on its strength. Zero desire gets zero attention, and this works as a powerful hedonic filter facilitating efficient utilization of the finite resources of animals and humans alike.

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Considering that attention has an upper limit ($ATTmax_{s,t}$), and that the overall attention ($ATTtotal_{s,t}$) of a subject (S) at any moment (t) is distributed between a number (n) of phenomena in proportion with their desirability strength, the following formula is proposed:

$$\begin{aligned} ATTmax_{s,t} &\geq ATTtotal_{s,t} = \\ &= k|\Delta PSS_{s,t,1}| + \dots + k|\Delta PSS_{s,t,n}| = \\ &= |DESIRE_{s,t,1}| + \dots + k|DESIRE_{s,t,n}| \end{aligned}$$

The phenomenon with strongest desirability/undesirability gets to the center of attention.

It is generally accepted that animals and humans alike are driven by satisfaction of their needs, and that the basic composition of human needs is represented by Maslow's pyramid. The author's definition of a need as a periodic desire, combined with a mathematical model of desire, allows for the creation of the real time representation of Maslow's pyramid of needs in action. The author suggests transforming Maslow's pyramid of needs into a table of desires generated by these needs. Elements of this table should be sorted by the current strength of desires in descending order. This way the strongest desire will be represented by the number one element on the "top" of this table. This table should also incorporate occasional, discrete desires of a subject in addition to repetitious, cyclical desires of needs.

Such a table can be informative in describing the motivational state of a subject, and the subject's preferences at the moment.¹¹ The strongest desire, along with its associated objects and activities, have the highest probability of getting to the center of attention of a subject, and of being acted upon. For humans, possible influence of the will effort in suppressing or supporting desires should be taken into consideration. The essence of the will effort is in shifting desirability balance and, therefore, distribution of attention in favor of a predetermined phenomenon.

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KEY TERMS AND DEFINITIONS

Attention: At any given moment, the activity of a subject's perception, called 'attention', is distributed between multitudes of simultaneously perceived phenomena. The author proposes that the total volume of attention ($ATT_{total_{s,t}}$) for a subject (S) perceiving n phenomena at the moment (t) has an upper limit ($ATT_{max_{s,t}}$) and can be described as the sum of attention paid to each phenomenon: $ATT_{max_{s,t}} \geq ATT_{total_{s,t}} = k|\Delta PSS_{s,t,1}| + \dots + k|\Delta PSS_{s,t,n}| = |DESIRE_{s,t,1}| + \dots + |DESIRE_{s,t,n}|$ This formula describes the distribution of attention between n simultaneously perceived phenomena in proportion with their desire's strength. According to this formula, the largest portion of attention is allocated to the phenomenon causing (associated with) the largest positive or negative PSS change - one that is most desirable or undesirable, and corresponds to the *strongest desire*. This situation is usually described as the phenomenon being in the center of attention. Distribution of attention can be influenced by the will effort.

Desire: The author posits that the subject's (S) desire for X is a word used to describe a change of the Pleasantness of the State of this Subject ($\Delta PSS_{s,x}$) associated with X, where X can be an object or an activity, perceived, remembered, or imagined. Desirability/undesirability of X for S is an ability of X to maximize/minimize the PSS. $DESIRE_{s,x} = \Delta PSS_{s,x}$. If $\Delta PSS_{s,x} > 0$, then x is called "desirable". If $\Delta PSS_{s,x} < 0$, then x is called "undesirable". Strength of S desire for X = $|DESIRE_{s,x}| = |\Delta PSS_{s,x}|$.

Happiness: Maximization of the PSS has an upper limit; this limit, representing the most pleasant internal state possible, is called "happiness". In other words, the state of happiness is an upper limit of the PSS maximization.

Hedonic Recognition: In the process of the Object Recognition, an Object is not only recognized and categorized as a banana, a chair, John, etc., but also recognized hedonically as pleasant/unpleasant (P/U), liked/disliked, desirable/undesirable. The author calls the latter form of recognition "Hedonic Recognition", that is, recognition

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of the hedonic properties of a phenomenon. In the process of the Object Recognition, that includes Hedonic Recognition, the PSS of the subject changes from the antecedent level PSS_1 to the end result level PSS_2 . This hedonic transition creates a PSS change: $\Delta PSS = PSS_2 - PSS_1$. The author posits that the desirability of X is determined by the PSS change (ΔPSS), while the liking/disliking of X is defined by the end level of this change (PSS_2).

Hedonistic Principle: Animals and humans alike are motivated/driven to maximize the pleasantness of their internal state called here the Pleasantness of the State of the Subject (PSS).

Need: Experiencing a need means feeling the corresponding desire. A need is characterized by the strength and frequency of its desire. If a subject experiences a desire for X repeatedly, periodically, it is usually said that the subject *needs* X. This is clearly demonstrated by the needs that emerge and cease to exist with age or during changing circumstances, for example, the needs for sex, smoking, or drugs. The origination/disappearance of such needs is acknowledged when the corresponding desire begins/stops being recurrent. *Need is defined here as a term used for a periodic or cyclical desire.* This is true for all kinds of needs - physiological, psychological, pathological needs of addictions, etc.

Pleasantness/Unpleasantness (P/U): P/U is one of many antagonistic pairs of expressions commonly used to describe positive/negative orientation to a phenomenon. A subject experiences orientation to the perceived phenomena - positive, negative or indifferent that is categorized by that human subject as P, U or indifferent. The only attribute that is common to all the numerous P/U phenomena is the +/- orientation of the subject toward it. This attribute, or quality of phenomena, is usually labeled as 'pleasantness' and 'unpleasantness'. This is the *meaning of P/U*. As Kant (1798, p. 99) has asserted: "Enjoyment is pleasure through the senses, and what delights the senses is called agreeable. Pain is displeasure through the senses, and what produces it is disagreeable. . . . We can also describe these feelings in terms of the effect that the sensation of our state produces on our mind. What directly (by the senses) prompts me to leave my state (to go out of it) is disagreeable to me - it pains me. What directly prompts me to maintain this state (to remain in it) is agreeable to me - it delights me." Referring to pleasure, Edwards (1979, p. 95) wrote: "Rather it means "the set of all feelings for which we have a psychic tension or attraction," and no circularity is involved. Similarly, "pain" in the generic sense means "the set of all feelings against which we have a psychic tension or aversion." The disposition to continue or to interrupt the current experience, to approach or avoid an action, or the tendency to go on or to stop, are the common features of pleasant or unpleasant experiences. A similar position is taken by Kahneman (1999, pp. 5, 8) in his definition of pleasantness/utility/Good-Bad dimension. In some primal form, the mechanism of a positive/negative orientation exists in the most primitive

of organisms, because any life form, regardless of its level of complexity, has to be involved in a selective exchange with its environment, i.e. choice or orientation (see Rado, 1964, p. 260; Johnston, 1999, pp. 66 - 67). “Pleasantness” and “unpleasantness” are considered in this work to be the most common terms used to describe an intentional, orientational quality of attractiveness/repulsiveness, which are the two existing types of orientation. *The acceptability or P/U of a phenomenon to a subject are terms indicating a +/- orientation of a subject toward the phenomenon.* Other feasible terms would be “agreeableness” (Kant, 1929, p. 290), or “agreeability” (Edwards, 1979, p. 43), because *the meaning of P/U of x is the positive/negative acceptability, agreeableness, or agreeability of x.* P/U are labels for the tendencies of the attraction/repulsion felt by a subject toward the phenomenon. They describe a vector of orientation so they are in turn vectors themselves. Their magnitude describes the degree or ‘strength’ of orientation. Their positive or negative designation describes the direction of orientation - toward or away from the object.

Pleasantness of the State of a Subject (PSS): There are at least seven components of the stream of consciousness which possess their own, intrinsic P/U that are usually experienced in the organ of the sensation: algic (pain), emotional¹², gustatory, olfactory, spatial, tactile, thermal sensations. A subject can experience all of them at once, or any number of them at the same time. This is one good reason to look for more precision and clarity while referring to pleasure and pain in order to clarify which one or ones are referred to. This is why it makes sense to utilize an inclusive characteristic, or a measurement, of the combined or total P of the state of a subject that includes *all* P/U components of a subject’s state. The author calls their combination or aggregate a ‘Pleasantness of the State of a Subject’ (PSS in short). The PSS at the moment (t) is a sum of all P/U components of the stream of consciousness experienced at t: $PSS_t = P_{algic} + P_{emotion} + P_{gustatory} + P_{olfactory} + P_{spatial} + P_{tactile} + P_{thermal}$. Different types of P can be totaled in the formula above, because the same quality of pleasantness is added up and because P “summate algebraically” (Plutchick, 1980, p. 115; Kahneman 1999, p. 5).¹³ The PSS is a sum of the P/U of all components of the stream of consciousness. In accordance with this definition, the PSS combines all the P/U, orientational or vectorial components of the stream of consciousness. Therefore, any influence on orientation ought to work through the PSS.

Will Effort: The essence of the will effort is to adjust the desirability balance in favor of a predetermined phenomenon. This can be done by strengthening the “favorite” desire or/and by weakening its competition. Will effort directs the attention of a subject toward those features of the elements of choice that change their desirabilities and, therefore, their balance in ways advantageous for the will’s goal. Will effort activates emotional/hedonic memories of a subject by insistently reminding a subject about certain features or consequences of the choice and/or making a subject imagine these consequences.

ENDNOTES

- ¹ See “Key Terms and Definitions”.
- ² See “Key Terms and Definitions”.
- ³ Schroeder presented in his book “the third face of desire”, his own reward and punishment theory of desire. For review of the theory see Katz (2005); for references to objections to it see Arpaly & Schroeder (2014, p. 127).
- ⁴ See “Key Terms and Definitions”.
- ⁵ See Aristotle’s quote in the paragraph “Hedonistic Approach to Desire”.
- ⁶ Translated by Ovsich.
- ⁷ See, for example, Damasio, 1994, p. 199.
- ⁸ The author suggests that in the first approximation, k is the same for all the simultaneous objects of attention.
- ⁹ Translated by Ovsich.
- ¹⁰ According to Plutchick (1980), “hedonic process summates algebraically” (p. 115). Also, see Cabanac (1992, p.182), Kahneman (1999, p. 5).
- ¹¹ Such a table can serve as a core of the hedonistic choice engine of an autonomous system, of a robot.
- ¹² Young (1961) describes experienced location of emotions as “thoracical” (p. 149).
- ¹³ A PSS is quite close to what is called a Valence of the Core Affect in Russell (2003), Barrett (2006).