Boston College
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Department of
Teaching, Curriculum, and Society

Program of
Curriculum and Instruction

# EXPLORING HOW MATHEMATICAL AUTHORIAL IDENTITY EMERGES: AN APPLIED CONVERSATION ANALYSIS STUDY OF STUDENTS' SMALL GROUP DISCUSSIONS

Dissertation by

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Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

May 2022

**Abstract**

Exploring how mathematical authorial identity emerges: An applied conversation analysis of students' small group discussions

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The recent mathematics curriculum reforms in the United States resulted in various classroom initiatives and research on cultivating students' mathematical identity. Among many dimensions of mathematical identity (Fellus, 2019), mathematical authorial identity is connected to how students leverage the interactional space and communicate their ideas about mathematical concepts while invoking authority, especially during students' peer discussion in mathematics classrooms (Povey & Burton, 2003; Schoenfeld & Sloan, 2016).

Despite the emerging importance of students' mathematical authorial identity, most research on authorship and authority in mathematics classrooms has focused on the relationship between teachers and students, and not on the relationships of students with one another in small groups (Amid & Fried, 2005; Cobb et al., 2009; Wagner & Herbel-Eisenmann, 2014). More attention is needed to understand how the notions of authorship and authority work in students' interactions with others, and what interactional patterns occur as students construct mathematical authorial identity through classroom discourses (Langer-Osuna, 2016, 2017, 2018; Langer-Osuna et al., 2020).

The current study used an applied conversation analysis to investigate students' interactional patterns of seven small group discussions. These students met virtually four

times over one school year to exchange feedback on each other's mathematical arguments. After transcribing students' small group discussions, I focused on the occurrences of accounts, which are statements "made by a social actor to explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46). They are typically used by interactants when they offer additional explanation or elaboration in situations when they are accomplishing a dispreferred action.

The results indicate that mathematical authorial identity was manifested in three different types of account turns. The first type of account turns was 'missing accounts,' which were expected to occur but were missing due to students accomplishing other interactional work. Students deployed this type of accounts as they accomplished various forms of disagreement. The second type of account turns invoked external authority. Students typically deployed this type of account turns towards the end of a sequence, and they were likely to use strong expressions of disagreement. The third type was account turns that invoked shared/internal authority. These account turns usually occurred at the beginning of a new sequence and when students expressed weaker disagreement.

The various types of account turns and interactional environments suggest that students actively conceptualize and manage interactional work, such as facework and preference organization, when navigating mathematics classroom discourse. Based on the findings, this dissertation offers pedagogical implications for mathematics educators to actively cultivate group norms that could occasion more interactional affordances for students and be aware of interactional features and sequences that foster students' construction of mathematical authorial identity.

## Acknowledgements

This dissertation would not have been possible without the constant support, guidance, and patience of many people. First and foremost, I would like to thank my dissertation committee members for their steadfast encouragement and thoughtful feedback. Dr. Dennis Shirley, I am beyond fortunate to have worked with and learned from you for the past six years through many research and conference trips. Your kind, inspiring, and insightful mentorship is what led me to complete this program. Dr. Kristen Bottema-Beutel, thank you for what you have taught me about conversation analysis. Because of you, I discovered the joy of playing with data. Dr. Osnat Fellus, you have expanded my understanding of identity, discourse, and mathematics education. Thank you for your thorough comments and passionate discussion on these topics. I also would like to thank Dr. Andy Hargreaves for many opportunities to learn from a vast network of scholars and practitioners in the field of educational change and leadership.

To the teachers who participated in this study through the NW RISE network, your relentless passion for students' engagement and collaborative learning has always been an inspiration for me. You have shown me what dedication to professional development looks like in real life. Thank you for welcoming me to your classrooms and working with me to create and implement a curriculum that heavily utilizes virtual meetings (before COVID!).

To friends and colleagues at the Lynch School of Education and the Center of Digital Innovation in Learning, thank you for your camaraderie and support. It did not matter whether we connected in person or virtually. I am incredibly grateful for the times and conversations we shared over cups of coffee or glasses of wine.

Most importantly, I would not be where I am without my family's relentless and unconditional love. To my husband, Hoon, thank you for believing in me and giving me the confidence to be myself. Thank you for listening to me, offering your perspectives, and keeping me grounded. Without your amazing cooking skills and thoughtful, sharp questions, this dissertation would not have been possible. I value and cherish every moment I share with you, Reykjavik, and Kona.

And finally, thank you to my parents, Hyungchul Kim and Eunyoung Bae, for never once losing faith in me and lovingly encouraging me for every step of my life. I also thank my brother, Minwoo Kim, for always supporting me. Living as an immigrant family has been challenging to say the least; however, overcoming those challenges and finding little joys along the way with you shaped who I am today, and I am proud of it. Thank you for being my everyday inspiration, courage, and strength.

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# Chapter 1: Introduction

Mathematics classrooms are increasingly becoming places where students are expected to verbalize their thoughts about mathematical concepts. The United States' Common Core Standards for Mathematical Practices (SMP) state that students need to be able to construct their own mathematical arguments and communicate those ideas that are backed up with evidence to their peers (Common Core State Standards Initiative, 2010). In addition, previous research (Chapin et al., 2009; Yackel et al., 1991) has shown that students' verbal interaction with peers enhances their mathematical understanding by providing more opportunities for them to make sense of and critique mathematical ideas. These interactions, in turn, have demonstrated a positive impact on students' mathematical identity construction (Bishop, 2012; Wood, 2013), an essential component of becoming a life-long learner of mathematics.

One critical, yet often overlooked factor that makes interactive and collaborative mathematical learning possible, is having enough students to form small groups. Among many challenges that rural schools face, such as geographic isolation and inadequate funding, small class sizes can be a hindrance, when rural students do not have an adequate number of same-grade level students with whom they can interact (Budge, 2006; Irvin et al., 2012). Therefore, the challenge of implementing collaborative activities must be addressed so that students in rural contexts can benefit from learning in groups, just as their urban and suburban counterparts are able to do.

To alleviate this challenge of rural schools and provide engaging, collaborative mathematical learning opportunities, three rural educators and I designed an innovative curriculum activity together. It virtually connected 4th- and 5th-grade rural students by using a videoconference platform, so the students could exchange feedback on mathematical argumentative writing with peers. In this learning environment, students interacted with their peers from different schools about mathematical arguments and problem-solving strategies. This study focuses on identifying their interactional patterns of students' small group discussions to explore how they use a specific interactional strategy called "accounts" (Scott & Lyman, 1968; Waring, 2007). It investigates how students' deployment of accounts and their mathematical authorial identity are related.

Accounts are statements "made by a social actor to explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46). They are typically used by interactants when they offer additional explanation or elaboration in situations when they are involved in a dispreferred action. When students produce accounts, due to how interactions are organized, students can access opportunities to communicate their personal interpretations and explanations about mathematical concepts. This study focuses on the instances when students use accounts and explores how accounts are deployed and related to students' mathematical authorial identity.

## Problem Statement

Research on students' mathematical identity has shown that identity is closely related to students' perseverance and motivation to study mathematics (Boaler & Greeno, 2000; Sherman & Fennema, 1977), academic achievement (Schoenfeld, 1989), and engagement with the subject (Nasir & Cobb, 2006). The National Council of Teachers of Mathematics (NCTM, 2000) explained that one of the goals of mathematics education is to encourage learners to be "confident in their ability to tackle difficult problems, eager to figure things out on their own, and willing to persevere" (p. 21). By empowering students to be certain of their mathematical abilities and seek mathematics problems willingly, the NCTM suggested that educators provide a learning environment that fosters students actively participating in discussions about mathematics and supports students in developing a positive mathematical identity.

To systematically operationalize the understanding of mathematical identity, Fellus (2019) presented an interconnected network of mathematical identity by adapting the theoretical framework developed by Ivanič (1998) in writing education. The network of identity includes aspects of autobiographical identity, discoursal identity, authorial identity, and socioculturally available selfhood. Fellus (2019) argued that these aspects are part of a "networked identity model" that represents the multifaceted and connected nature of mathematical identity (p. 445).

In Fellus' (2019) networked model of identity, she presented an overview of four mathematical identity dimensions – *autobiographical, discoursal, authorial, and socioculturally available selfhoods.* Among these identity dimensions, mathematical authorial identity is the focus of this dissertation. This is due to the potential of mathematical authorial identity being connected to developing students' sense of self as authors who communicate their ideas about mathematical concepts while exercising internal authority, especially during students' peer discussion in mathematics classrooms (Povey & Burton, 2003; Schoenfeld & Sloan, 2016).

The topics of authority in mathematics classrooms and students' mathematical authorial identity development are becoming more critical. This is because, in addition to the need to demonstrate proficiency in knowing key mathematical concepts, students also need to "justify *their* [emphasis added] conclusions, communicate them to others, and respond to the arguments of others" (SMP #3). In other words, students are expected to become authors by producing personal thoughts about mathematical concepts and communicating their ideas with a certain level of authority.

Despite the emerging importance of students' sense of self as authors, most research on authority in mathematics classrooms has focused on the relationship between teachers and students, and not on the relationships of students with one another in small groups (Amid & Fried, 2005; Cobb et al., 2009; Gresalfi and Cobb, 2006; Wagner & Herbel-Eisenmann, 2014). More attention is needed, then, to understand how the notion of authority works in students' interactions with others, and how students construct mathematical authorial identity through classroom discourses (Langer-Osuna, 2016, 2017, 2018; Langer-Osuna et al., 2020).

In addition, regarding mathematical authorial identity, Fellus' (2019) networked model of identity could benefit from an elaboration on the details of what each dimension constitutes and by producing new analytical frameworks to investigate those dimensions. Dissecting various aspects of mathematical identity and discovering specific elements of those aspects would bolster the current body of knowledge we have regarding mathematical identity. It would also serve to inform and to strengthen educators' pedagogical practices, to encourage the learning of mathematics, and the continual construction of students' positive mathematical identity.

Therefore, this study explores mathematical authorial identity, specifically for the 4th- and 5th-grade students who participated in small group discussions via a videoconferencing platform. I narrow down the data to focus on just one specific interactional strategy that became apparent in the students' classroom discourse. By analyzing students' interaction patterns during their learning activities, I am able to investigate how students use the interactional strategy called "accounts" (Scott & Lyman, 1968; Waring, 2007). I then show how the students' deployment of accounts is related to their mathematical authorial identity in ways that have not been evident in prior research.

## Purpose of the Study and Research Questions

In this dissertation, I explore how 4th- and 5th-grade rural students signal and develop mathematical authorial identity when they deploy accounts during their small group learning activities. Specifically, based on existing literature on the topics of authority, authorship, and authorial identity, I conduct an applied conversation analysis (CA) and an analysis of linguistic features that are embedded in students' account turns. Building onto Fellus' (2019) proposed conceptual framework on the networked model of identity, I aim to capture and understand the interactional patterns that students use when their mathematical authorial identity is manifested and developed. As such, the following research questions will be investigated:

- How are accounts implicated in students' signaling and development of mathematical authorial identity?
    - How are accounts occasioned?
    - What did students achieve when they deployed accounts in small group discussions?
    - How are different account types related to the emergence of mathematical authorial identity?

# Chapter 2: Theoretical Background

For the past few decades, studies of mathematical identity have investigated how students' social contexts impact learning (Heyd-Metzuyanim, 2017; Langer-Osuna, 2016, 2018) and how diverse student populations experience mathematical learning.  Studies have addressed those who have learning difficulties (Heyd-Metzuyanim, 2015); disability, (Heyd-Metzuyanim & Sfard, 2012); female students (Bhana, 2005; Lim, 2008); and Black male students (Stinson, 2013).

Mathematical identity also has been a valuable conceptual tool to explore teachers' professional understanding of selves and their pedagogical choices in teaching practices (Beijaard et al., 2000; Brown & McNamara, 2011; Skott, 2019). Furthermore, researchers have investigated pre-service and in-service teachers' growth into their professional identity (Losano et al., 2018; Lutovac & Kaasila, 2014; van Putten et al., 2014) and teachers' reception of reform efforts (Jong, 2016). However, it is still necessary to explore how identity is constructed in various contexts (Wortham, 2006).

The purpose of this chapter is to provide the theoretical backgrounds of this study's approach to identity, conversation analysis (CA), and extant research on mathematical identity. I begin by describing relevant identity theories and explaining this study's view of identity as a socially and interactionally emergent and dynamic phenomenon. Next, I elaborate on the fundamental principles of CA and explain why CA is relevant to this dissertation. Finally, I narrow my focus on the topic of mathematical identity and discuss why this study utilized Fellus' (2018, 2019) networked identity model to concentrate on the dimension of mathematical authorial identity.

## Theories of Identity

There are three significant perspectives of identity: psychological/developmental, sociocultural, and poststructural (Grootenboer et al., 2006). In general, these perspectives guide how identity can be investigated. First, a psychosocial/developmental perspective focuses on individuals and their inherent characteristics. A primary theory under this perspective is Erikson's (1968) identity theory. The theory posits that individuals acquire specific characteristics during the early years of identity formation phases and eventually construct a coherent sense of identity. Erikson (1968) further elaborated that individuals commit to particular values and beliefs that impact their social positions. Consequently, such commitments become an inseparable part of the individuals. This perspective highlights individuals' choices impacting identity formation. As a result, identity is perceived as a self-determined notion (Flum & Kaplan, 2006; Heffernan et al., 2020).

Second, a sociocultural perspective focuses on the interaction between individuals and their social contexts. Mead's (1913/2011) identity theory falls in this perspective and posits that identity is multifaceted, situative, and performative. Identity in a sociocultural perspective is often manifested as a form of participation. The sociocultural perspective of identity claims that it is socially and discursively constructed as individuals follow certain discourse norms practiced within the group (Cobb, 1994) and by going through a process of socialization into a "community of practice" (Wenger, 1998). When the sociocultural perspective is applied in the context of schools, students experience and construct classroom norms and obligations when interacting with others. As a result, in addition to learning the subject matter, students develop a sense of self based on ongoing interactions (Cobb et al., 2001; Cobb et al., 2009).

Third, Foucault's (1972) work heavily influenced the poststructuralist perspective. This perspective focuses on how power and institutional discourse impact identity, considered a continual and dynamic process (Goos, 2014; Grootenboer et al., 2006). Identity is an ongoing process shaped by existing institutional structures, such as schools and curricula.

Among these broad perspectives of identity, Darragh's (2016) systematic review of mathematical identity found that psychological/developmental and sociocultural are primarily utilized in mathematics education, as researchers have turned to Erikson (1968) and Mead (1913/2011) to frame the notion of identity theoretically. In a more recent publication providing an overview of research on mathematics identity, Graven and Heyd-Metzuyanim (2019) noted that most research on mathematics identity utilizes the sociocultural perspective.

Situating the extant research on mathematics education from a sociocultural perspective, Lerman (1994) argued that mathematical teaching and learning is based on "frameworks which build on the notion that the individual's cognition originates in social interaction…and therefore the role of culture, motives, values, and social and discursive practices are central" (p. 4).

The sociocultural perspective emphasizes that students learn by interacting with others (Vygotsky, 1978), including teachers, peers, books, and the internet. Based on the students' experience of participating in mathematical learning activities through multiple mediums of interaction, the sociocultural perspective offers the appropriate theoretical foundation for investigating the dynamic notion of students' mathematical identity through analyzing the students' classroom discourse data.

**Sociocultural Perspective of Identity**

This subsection focuses on the sociocultural perspective of identity, including three approaches to conceptualizing identity. These approaches include identity as *a form of situated participation* (Hand & Gresalfi, 2015; Lave & Wenger, 1991), as *stories or narratives* (Sfard & Prusak, 2005), and as *manifested in social interactions* (Bucholtz & Hall, 2005; Ochs, 1993).

First, *the situative approach* posits that identity formation occurs as individuals participate in communities (Lave & Wenger, 1991; Wenger, 1998). When an individual joins a community, multiple lines of communication are established with the community's members. Through these interactions, individuals contribute to group norms, adapt to new norms, and learn the ways of being in the community. Lave and Wenger (1991) explained the interconnectedness of learning and identity formation, stating that "learning…implies becoming a different person [and] involves the construction of identity" (p. 53). In this approach, individuals take on different forms of identity by going through social interactions over time. Accordingly, learning and identity development essentially occur simultaneously in communities of practice.

Second, Sfard and Prusak (2005) defined identity as "collections of stories about persons, or more specifically, as those narratives about individuals that are reifying, endorsable, and significant" (p. 16). The authors noted that a collection of stories shapes identity, instead of using stories as analytical tools. They elaborated that reifying refers to talking about what people are instead of what they do. A story is deemed endorsable when the individual who is being discussed "would say that it faithfully reflects the state of affairs in the world" (p. 16). Finally, the aspect of significance concerns how "an alteration or removal of any of the story's main elements would change how the author feels about the protagonist" (Heyd-Metzuyanim & Sfard, 2012, p. 132). The last part of the definition also includes an emotional aspect. By equating one's identity as an ongoing story, Sfard and Prusak (2005) shared how transforming an obscure identity concept could become tangible as a form of data.

The third approach to the sociocultural perspective of identity is based on Bucholtz and Hall's (2005) and Ochs' (1993) research. Bucholtz and Hall (2005) defined identity as "the social positioning of self and other" (p. 586). They also suggested five principles that serve as a framework for studying the nature of socioculturally dynamic and discursively emerging identity. This definition adds a linguistic dimension to a sociocultural perspective of identity since sociocultural linguistics is "concerned with the intersection of language, culture, and society" (Bucholtz & Hall, 2005, p. 586). The advantage of this approach is that researchers can draw conclusions about how the linguistic resources employed by individuals gain specific meanings that contribute to identity construction by analyzing interactions.

First, the *emergence principle* emphasizes that identity is not a pre-existing product but a dynamic "social and cultural phenomenon" (Bucholtz & Hall, 2005, p. 588). Second, the *positionality principle* broadens the view of identity to include macro-level social groups, local and context-specific positions, and temporal stance taking and roles. This principle argues that identity should be examined under various timescales, including the broad social discourses and moment-to-moment stance taking. Third, the *indexicality principle* highlights the discursive aspect of identity. Discourses impact identity through numerous methods, such as presuppositions, stances expressed as evaluative comments, and the use of particular language to identify a person with a group. Fourth, the *relationality principle* underscores the intersubjective nature of identity and describes how identity relies on other social relationships. Lastly, the *partialness principle* emphasizes that identity is flexible, dynamic, and context-dependent.

Bucholtz and Hall (2005) argued that identity is discursively constructed because of how one speaks and how one is spoken to shape a sense of self. Therefore, it is possible to “locate identity as an intersubjectively achieved social and cultural phenomenon” (p. 607). Similar to Bucholtz and Hall (2009), Ochs (1993) understood identity from a discursive approach, arguing that “speakers attempt to establish the social identities of themselves and others through verbally performing certain social acts and verbally displaying certain stances” (p. 288). The research argued that spoken words are mechanisms that construct identity by accomplishing specific actions and indicating speakers’ attitudes or commitment to ideas.

Among the three sociocultural perspectives that offer various ways to observe and analyze identity, this dissertation employs the third approach, which focuses on identity construction's social and discursive aspects. Within the institutional setting of schools, students spend most of their time engaging in verbal interaction with teachers and peers. In addition, these days, students are more likely to participate in collaborative and discursive learning activities because of mathematics curriculum standards that expect students to construct mathematical arguments and communicate their ideas. Employing this approach to identity offers the analytical advantage of exploring identity from classroom discourse data. Interviewing students to understand their perception of identity could be beneficial; however, interview data rely on individuals' recollections, which might be nebulous and ambiguous. Moreover, responses may be limited by the scope of interview questions. The analysis of classroom discourse data eliminates the additional layer of understanding how identity construction works because researchers do not have to rely on individuals' recollections of identity (Heyd-Metzuyanim, 2013).

**Roles of Discourse in Identity Construction**

In the previous section, I described the background information on how I choose to view identity through the social and discursive lens. I now elaborate on this study's approach to discourse and its relations to identity. As Stentoft and Valero (2009) claimed, discourse and identity are inseparable, and discourse is a mediating tool of identity construction. Especially for classroom discourse, students and teachers practice communication acts in a particular context bounded by institutions called "schools." Students and teachers create new meanings within that community while they are building social relationships and developing their identity (Planas & Gorgorió, 2004).

Essential theoretical foundations of how I understand the role of discourse are anchored in Bakhtin's (1981) work on dialogic theory. Bakhtin stated that the "word is born in a dialogue as a living rejoinder within it; the word is shaped in dialogic interaction with an alien word that is already in the object. A word forms a concept of its own object in a dialogic way" (1981, p. 279). All words that people speak come from words that have been spoken previously. What people say is essentially a response to discourses that have existed before, and this nature of discourse is why it can be described as a living, dynamic, and biased force. Language and its interpretations change based on who the speakers are, who the audience is, and how and when the language is spoken. Through this discursive process, an individual's subjectivity, or identity, is formed “through the dialogic struggle between contending voices or discourses. The phenomenon of ‘selfness’ is constructed through the operation of a dense and conflicting network of discourses, cultural and social practices and institutional structures, which are” part of the “complex interplay of the self-other relation” (Lim, 2008, p. 618).

In Bakhtin’s theory (1981), discourses are considered relational, meaning that the words we use and the discourse we participate in, do not exist in isolation. Historical and sociocultural meanings that have preceded us are embedded in discourse. When we speak, we ‘appropriate’ the particular words with how others have used the language previously. Bakhtin (1981) argued that “language is not a neutral medium that passes freely and easily into the private property of the speaker’s intentions; it is populated - overpopulated - with the intentions of others” (p. 294).

Gee (2000) also underscored the role of discourse in identity construction. He defined identity as “being recognized as a certain ‘kind of person,’ in a given context” (p. 99). He referred to the “little ‘d’” discourse as the “flow of language-in-use across time and the patterns and connections across this flow of language” (Gee, 2015, p. 2) and Discourse (with a capital “D”) as “the ways in which such socially-based group conventions allow people to enact specific identities and activities” (p. 2). In other words, Discourse is a broader societal norm about ways of being, and discourse is how people use language in particular ways based on their social contexts. These two types of d/Discourses influence each other. How we communicate in our daily lives, moment-to-moment local interactions all contribute to building the Discourse; on the other hand, Discourse, which refers to meta-narratives that exist in the society, impacts what we say and how we say certain language in our local interactions.

When the theoretical background mentioned above is applied to mathematics classroom contexts, it can be argued that students get involved in the "constant struggle," where the language they use is laden with personal, historical, and sociocultural meanings. In addition, students interact directly with their peers and with Discourse, the broad societal norms. Consequently, their interaction becomes a site where language functions as a mediating force that impacts identity for all involved interactants. This theoretical assumption is another justification that supports this dissertation's investigation of classroom discourse to explore mathematical identity.

**Identity as a Socially and Interactionally Emergent Phenomena**

The previous section acknowledged the role of discourse in the identity construction process. Now, I explain how I specifically view identity by relying on the emergence principle, which highlights the ontological status of identity that becomes emergent in discourse (Bucholtz & Hall, 2005). Identity can be viewed as an interactional achievement arising from the course of interaction instead of an end product preconceived from societal norms, biology, or culture (Antaki & Widdicomb, 1998; Sidnell, 2003).

To illustrate that identity is a notion accomplished through social interaction, Bucholtz and Hall (2005) provided an example of how hijras use gender pronouns as discoursal resources. Hijras are a group of Indians born biologically male but choose not to identify with that gender. Instead of explicitly talking about their gender choices, hijras make a discoursal choice of using the feminine gender pronouns when talking about themselves and letting their gender identity emerge from interactions.

Similarly, in mathematics education, the language used in classroom discourse also reflects interactants' identity construction. According to the emergence principle, students' mathematical authorial identity can become visible and emergent in classroom discourses. Previous researchers on classroom discourses in mathematics education have emphasized the relationship between students' mathematical identity and classroom discourse. In particular, Sfard (2008) argued that students learn mathematical concepts and become members of mathematical discourse communities through "participation in communicational activities of any collective that practices this discourse" (p. 91).

Through classroom discourse, students' identity is constructed through an iterative cycle of expressing themselves and being perceived from the expression. Students' learning opportunities also depend on their access to classroom discourses (Cobb & Yackel, 1998; Krummheuer, 2011). As more instructions and learning activities occur verbally, students are increasingly expected to meet the "interactional demands" of mathematics classroom discourses (Bucholtz & Hall, 2005, p. 591).

Another aspect highlighted by researchers concerns mathematical identity's interactional and emergent nature. Heyd-Metzuyanim (2013) taught one student who experienced learning difficulties. During class, the student repeatedly "devia[ted] from normative routines" (p. 341); eventually, the student's identity as a struggling mathematics student emerged.

Furthermore, Bishop (2012) analyzed the interactions between two students and concluded that students' mathematical identity could emerge from pair work. Through the classroom interactional contexts occasioning one student as “the dumb one,” that student eventually considered herself incompetent in mathematics. In this study, Bishop (2012) described mathematical identity as “the ideas, often tacit, one has about who he or she is concerning the subject of mathematics and its corresponding activities” (p. 39).

Bucholtz and Hall’s (2005) emergence principle of identity was evident in previous empirical research on students’ mathematical identity. These studies perceived identity as dynamic, socially emergent, and interactional. The studies analyzed participants’ spoken words to understand their sense of self and represent the relationship between students and mathematics. In sum, understanding identity as a socially and interactionally emergent phenomenon opens the analytical opportunity to examine mathematics classroom discourse data when exploring the notion of identity.

Conversation analysis (CA) is a theoretical and methodological approach (ten Have, 2007) that focuses on inductively examining interactional structures of conversations that occur naturally in various social contexts. The primary focus of this methodology is on what interactants say and when they say it in those contexts. Therefore, CA’s unit of analysis is the interaction, rather than the individuals. CA researchers examine the microscale details of these conversations to discover what the sequential interactional patterns are and how they shape social actions (Sacks et al., 1974). CA defines social actions as “what the participants are doing interactionally vis-à-vis one another” (Pomerantz & Fehr, 2011, p. 168).

Turns consist of one or more “linguistic units (i.e., words, phrases, clauses, etc.) that form a recognizably complete utterance in a given context” (Hoey & Kenrick 2017, p. 3). By focusing on how turns occur one after another and how “speakers orient to whatever has gone before and to what might come after" (Benwell & Stokoe, 2006, p. 60), CA researchers aim to discover interaction patterns, sequence organization of turns, and interactional strategies that interactants adhere to when they achieve social actions.

## Basic Principles of Conversation Analysis

There are three main guiding principles in CA. The first is that all talk is orderly. This principle describes a sense of orderliness created, maintained, and reinforced by interactants through naturally occurring talk in various settings (Sacks, 1992). In CA, a sense of orderliness does not refer to a strict regulation that must be followed. Instead, it involves interactants adhering to certain patterns of interactions that become normative. One example of this principle is the implicit rule that one person speaks at a time during conversations (Sacks et al., 1974). For instance, if there is the main speaker at an event and an individual sitting in the audience would like to communicate with the person sitting next to them, that individual would whisper to another person, trying not to interrupt the main speaker (Hoey & Kendrick, 2017). No one at the event ordered that all audiences whisper to others when the main speaker is talking; however, because of the implicit rule created as a norm, the individual whispers.  Each turn was socially organized in this particular context. CA uncovers implicit yet socially ingrained rules to which people orient themselves.

The second principle is that all talk shapes and creates context (Goodwin & Heritage, 1990, p. 287). CA emphasizes that conversations must be interpreted by considering who speakers are, where the conversation takes place, and under what contexts (Heritage, 1984). The words "I am here" depend on who the speaker is and where the speaker is located. If person A is speaking this phrase in front of a friend's house, the words “I” and “here” would point to different objects than person B saying the phrase in front of his parents’ house. In other words, this principle embodies the importance of speakers’ identity and social contexts.

The third principle is that all details of talk are essential for analysis (Seedhouse, 2005, p. 166). CA methods focus on the structure of local interactions to understand how actions are achieved. In other words, instead of bringing in external frameworks for analysis, CA observes the patterns that interactants display in various contexts. Because of this approach, the primary data for CA are usually detailed transcripts of conversation instead of participant interviews or observation notes (Seedhouse, 2005). Conversation data is then examined to “trace how participants analyze and interpret each other’s actions and develop a shared understanding of the progress of the interaction” (Seedhouse, 2005, p. 166).

From CA’s perspective, mathematics classrooms are a dynamic social setting where teachers and students use language as a mediating tool for teaching and learning mathematical concepts and co-constructing the notion of identity. In addition to employing Bucholtz and Hall’s (2005) emergence principle, applying the CA method in this context facilitates considering identity as a socially and interactionally accomplished phenomenon. As noted by Sfard (2007), “learning mathematics may now be defined as…the process of becoming able to have mathematical communication not only with others, but also with oneself” (p. 573).

For example, teachers talk to students when introducing new mathematical concepts or checking for understanding, while students speak to each other to demonstrate their knowledge or ask questions. Students proceed to co-construct social actions through interactions through this discursive process. They contribute to mathematical discourse practices and shape their mathematical identity (Barwell, 2005; Esmonde, 2009; Sfard, 2001). For this study, I employ CA to identify how students' turns are sequentially organized when they use accounts, how these turns accomplish social actions, and how mathematical identity emerges from the classroom discourse.

**Adjacency Pair**

Based on the established principles of CA, researchers have identified key interactional patterns. Among them, I highlight two of the well-known interactional patterns that support the description of how accounts, which are the key phenomenon of this study, occur in conversations. The first type of interactional pattern is adjacency pair. An adjacency pair is a set of two utterances that have the following characteristics: 1) expressed by different speakers, 2) spoken "one after the other," 3) have first pair parts (FPPs) and second pair parts (SPPs), and 4) belong to pair types (Schegloff, 2007, p. 13).

First pair parts (FPPs) are utterances that initiate talk. Examples of FPPs include speakers asking questions, making offers, requesting information, and deploying invitations. Then second pair parts (SPPs) are utterances that respond to FPPs, and some examples include speakers providing answers, accepting or declining offers, and agreeing or disagreeing with claims. Because certain pairs of FPPs and SPPs have regularly occurred together, they are put together as "pair types," such as "greeting-greeting, question-answer, offer-accept/decline" (p. 13).

Adjacency pairs operate based on "conditional relevance" (Seedhouse, 2005, 167). Relevant SPPs are expected to be produced when FPPs are expressed. For example, when a teacher asks a student to come to the board to write the answer, the student is expected to respond (and to comply with the teacher's request). This form of interaction pattern is a question-answer adjacency pair. Alternative responses happen in social interaction. However, suppose an alternative response is produced after FPPs. In that case, CA analysts look for an explanation of why the SPPs did not occur as expected, and what utterances were produced instead.

**Preference Organization**

The second type of interactional pattern I describe is preference organization, which is "the structural relationship between two adjacent turns" (Ingram et al., 2019, p. 56). In CA, the term preference does not represent an individual's feelings of partiality or liking. It refers to how responses are given to achieve "thoroughly institutionalized and largely normative, and that systematically promote certain interactional outcomes over others" (Robinson & Bolden, 2010, p. 502). Interactants are oriented to certain rules of interaction (as represented by adjacency pairs), and they also orient to achieve social harmony through interaction. So, based on the normative rule of interaction, the favored interactional outcome of SPPs is "the avoidance of conflict" (Heritage, 1984, p. 265). When this rule is violated, then it would require a warrant.

For example, when a teacher asks a question (example of an FPP), the expected response is an answer (example of a SPP) and is considered preferred. When the answer is not provided or when a student responds by saying, "I don't know," these SPPs are considered dispreferred. Preferred responses are usually given right away and short (Pomerantz & Heritage, 2014). On the other hand, a "dispreferred alternative is avoided or mitigated or delayed or, at least, accounted for" (Blimes, 2014, p. 53). Therefore, dispreferred responses are typically longer and include more information because the speaker feels obligated to explain why a dispreferred answer was given.

**Accounts**

In addition to the systematic, implicit, and context-based normative rules of conversations, CA also illuminates different interactional strategies employed in social interactions. This dissertation study focuses on students' deployment of accounts among many of them. Accounts are often referred to as a strategy representing speakers' orientation to preference organization and mitigating the consequences of dispreferred action. Interactants use accounts to "repair or mitigate the offensive or discrediting dimensions of the event" (Buttny, 1987, p. 67), "explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46), or elaborate on "the reasoning (i.e., cause and effect) …to bolster the viability of the advice" (Waring, 2007, p. 372).

Basic CA principles explain that accounts also operate under specific interactional contexts. In other words, accounts are usually deployed by interactants as SPPs, and there are certain types of FPPs that occasion accounts. This dissertation study identifies certain sequences of turns when interactants use accounts and describes how accounts are related to students' mathematical authorial identity. More detailed information about accounts and their relationship with mathematical authorial identity is provided in chapter 3.

**CA in Mathematics Education**

Although identity and classroom discourse have been topics of interest in mathematics education for a long time, there is still a need for "a more thorough approach to the analysis of classroom interaction" (Barwell, 2003, p. 206). CA offers a theoretical and analytical framework that results in a thorough examination of mathematics classroom interactions. Highlighting the potential value that CA brings to research in mathematics education, Ingram (2018) stated that CA is "particularly powerful when we want to examine the co-construction of mathematical knowledge and practices, such as explaining, arguing or convincing, but can also be used to explore identity work" (p. 1066). Analysis of mathematics classroom discourse using the CA lens reveals how particular interactional patterns result in social actions and how mathematical identity is shaped during the discursive process.

Previous empirical studies that employed CA in mathematics education research include Krummheuer's (2011) project. It examined sequential structure of students' turns and concluded that students picked up different positions. This study also highlighted the systematic nature of students' conversations that includes "formulations (syntax) and/or the content (semantics) of previously produced utterances" (Krummheuer, 2011, p. 86). Ingram and Elliott (2014) conducted CA on seventeen mathematics lessons and identified three instances that deviated from the ordinary patterns of turn taking suggested by McHoul (1978). The CA approach to interactional patterns in classrooms often described how teachers usually take control over who can speak and when. It also noted that there are implicit rules in classrooms to "minimize the possibility of overlap in classroom interactions" (Ingram & Elliott, 2014, p. 2). Therefore, students did not have opportunities to self-select when speaking or interrupt while the teacher was speaking.

This study discovered the interactional contexts that were different from what was considered typical classroom interaction. These instances were when students participated in a debate to argue for one's stance about mathematical ideas, asked questions, and spoke out of turn when their teacher made a mistake while writing on a whiteboard. The discovery of these exceptions demonstrated how mathematics classroom discourses are different from how individuals talk in natural settings.

In another CA study that utilizes mathematics classroom discourse data, Ingram, Andrews, and Pitt (2019) concluded that two interactional contexts were evident when students produced explanations without a prompt from the teacher. The first context involved students providing additional explanations to a question which had already been discussed. The second context pertained to students speaking out of turn. These interactional contexts became relevant to a conversation because students demonstrated orientation to preference organization, which refers to how interactants participate in conversations with a desire to either seek or avoid social harmony (Pillet-Shore, 2017). The study discovered that when students offered explanations without their teachers' request, the students realized that they were performing a dispreferred action and elaborated their positions.

Bishop (2021) conducted a combination of CA and multilevel modeling, concluding that "highly responsive teacher moves" (p. 500)—such as revoicing, asking probing questions, and establishing connections among students' ideas—were positively related to students' mastering of topics on ratio and proportions. By examining the adjacency pair of teacher requests and student responses, the author concluded that teachers' offering of discursive opportunities shaped students' responses. When teachers asked high-level questions, students provided high-level responses.

Examining the moment-to-moment sequential organization of mathematics classroom discourses holds the potential to discover which interactional contexts provide affordances for students to make their voices heard and participate in the mathematical meaning-making process. By conducting a similar CA study but in different contexts, this dissertation illuminates how students' talk is organized when they deploy accounts.

## Mathematical Identity

The previous two sections described the theoretical background of this dissertation. I discussed the sociocultural perspective to identity and explained why I decided to view it as a socially and interactionally emergent phenomenon by employing Bucholtz and Hall's (2005) principles. In addition, I covered basic principles of CA to justify why this theoretical and analytical approach was best suited for this study.

Fellus' (2019) networked identity model stands out as a holistic approach to investigating identity compared to previously proposed frameworks, limiting identity researchers to choose one dimension. This section focuses on mathematical identity and frameworks that better represent the concept. Next, I describe my rationale for exploring mathematical authorial identity based on Fellus' (2019) networked identity model.

Since the sociocultural shift in mathematics education (Lerman, 2004), researchers have investigated various factors impacting the identity construction process, including teachers' communication styles (Empson, 2003; Forster, 2000), peer interactions (Andersson & Wagner, 2019; Esmonde & Langer-Osuna, 2013; Langer-Osuna, 2018; Wagner & Herbel-Eisenmann, 2008), and curricular materials (Anderson, 2009). In addition to addressing learning experiences, the notion of identity has been utilized to discuss issues related to equity, power, and authority in mathematics classrooms (Bhana, 2005; Ben-Yehuda et al., 2005; Esmonde & Langer-Osuna, 2013; Gutierrez, 2013; Langer-Osuna, 2016). Some researchers have focused on the mathematical identity of different types of student populations, such as immigrants

(Planas & Gorgorió, 2004), English language learners (Lee et al., 2011; Takeuchi, 2016), and Black male students (Grant et al., 2015). Another body of research on mathematical identity focused on how an individual's sense of self shifted throughout the progression of classroom discourse (Bishop, 2012; Darragh, 2013; Wood, 2013).

A notable critique within mathematical identity research emerged as numerous studies were conducted. Sfard and Prusak (2005) criticized how the operationalization and definition of identity were unclear. Indeed, there were various ways of interpreting and studying mathematical identity. The notion has often been described in a fragmented manner.

Researchers have described their understandings of mathematical identity as one's relationship with mathematics (Boaler, 2002), "stories about persons" (Sfard & Prusak, 2005, p. 14), and perceptions of self that are related to mathematics (Bishop, 2012). In addition, some researchers have regarded identity as a malleable and context-specific notion (Andersson et al., 2015; Forster, 2000; Wood, 2013), while others have understood it as a more stable concept (Bishop, 2012) that transcends context. These varying representations indicate the challenging nature of capturing the dynamics of identity.

Given the dynamic characteristic of identity that could not be covered in a continuous study, a need to shift toward a broader and more holistic framework to approach identity became evident. Overall, because identity is a versatile, indispensable, and multidimensional construct, a theory of identity should reflect such characteristics. A few literature reviews and frameworks of mathematical identity demonstrate that when research on mathematical identity first received attention, much of the focus was oriented towards defining what different categories are, and isolating each proposed category from one another. First, Darragh (2016)'s systematic review initiated that process, and argued that researchers should not combine two different theoretical approaches from Erikson (1968) and Mead (1913/2011). Then, the second major review of students' mathematical identity argued that there are different dimensions to the concept of students' mathematical identity (Radovic et al., 2018). The researchers argued that there is a subjective/social dimension, a representational/enacted dimension, and a change/stability dimension to identity.

While there were more categories developed to offer a more granular level of research on students' mathematical identity, the framework was still missing a way to discuss how the different dimensions are related to each other. Hence, a third framework was created by Fellus (2019), who suggested a new, networked model of identity. She argued that identity constitutes multiple dimensions that are analytically distinct but nonetheless interrelated. This framework is different from others' systematic review papers because it offers a structured way of discussing how each dimension affects one another.

**Darragh's (2016) Review of Mathematics Identity**

Darragh's (2016) systematic review initiated the cartography of theoretical backgrounds of mathematical identity. The author suggested that socially constructed identity within mathematics education research can be categorized based on five traditions of research. These are: participative, narrative, discursive, psychoanalytic, and performative.

First, participative involves one's engagement in social groups. Studies in this category have mainly drawn from Wenger's (1998) theory of communities of practice or Holland et al.'s (1998) theory of figured worlds to investigate the social contexts of how students' and teachers' identities develop. Second, narrative identity uses stories as representative artifacts that serve as a lens to understand identity. Researchers who approach identity in this way use narrative inquiry or narrative analysis as analytic methods. Furthermore, Sfard and Prusak's (2005) seminal study in this category equated identity and stories that are "reifying, endorsable, and significant" (p. 16). Third, discursive identity views the concept of identity as shaped by discourse among individuals and by discourse derived from meta-narratives perpetuated by society. Studies in this category have often drawn from Gee's (2011) definition of 'discourse-identity,' which refers to a dimension of identity "that is produced and reproduced in the ways in which people…talk to and about others in discourse and dialogue" (p. 108). Another prevalent view of discourse common today considers how narratives impacted by popular media also affect one's discursive identity (Mendick, 2005). Fourth, psychoanalytic identity employs psychoanalysis as a conceptual tool and methodology. It proposes that identity can and should be related to social interactions and power structures in society. Studies in this category have often cited Foucault's work to discuss how "social organization of power" and discourse play a role in shaping identity (Darragh, 2016, p. 26). Within this category, identity is shaped through discourses that "imply forms of social organization and social practices that, at different times, structure institutions and constitute individuals as thinking, feeling, and acting subjects" (Walshaw, 2013, p. 102). In mathematics classrooms, participants are labeled with roles such as students, teachers, or researchers. This is done habitually through the power of social relationships that are manifest "through classroom's traditions" (p. 103). Finally, performative identity primarily draws from positioning theory (Harré & van Langenhove, 1999), Goffman's (1959) performance theory, and Butler's (1997) work on gender performativity. In this category, identity is embodied through repetitive acts or performance.

In addition to these categories, Darragh (2016) stated that researchers should explicitly orient their scholarship toward either sociological or psychological theories of identity. Sociological perspectives of identity, which primarily derives from the work of Mead (1913/2011), recognize that identity is constructed as a result of interaction with others. Conversely, psychological identity theories, impacted by Erikson (1968), view identity as a coherent characteristic that is acquired over time. With these two perspectives about identity, Darragh (2016) recommended that theoretical orientations be aligned with the conceptual and analytical approaches to understand identity. In other words, when identity is understood as a social construct, it should be investigated as such explicitly, and scholars should view identity as a social act rather than as an individualistic acquisition.

**Radovic et al.'s (2018) Review of Mathematics Learners' Identity**

Radovic et al.'s (2018) review of mathematical identity argued that three primary dimensions conceptually define mathematical identity. These are a subjective/social dimension, a representational/enacted dimension, and a change/stability dimension. Radovic et al. (2018) suggested that each dimension of identity constitutes a spectrum between the two opposing concepts and claimed how different theories of identity, such as Wenger (1998) and Holland et al. (1998), overlap in the dimensions to describe the complex nature of identity.

The authors argued that identity could be considered an individual's perception or social experience in relation to the first subjective/social dimension (Radovic et al., 2018). The second, representational/enacted dimension addresses how identity becomes evident. Some researchers have contended that narratives or expressions represent one's identity; others have reasoned that identity is enacted as action, engagement, or participation. The third, change/stability dimension pertains to whether identity is a malleable or stable notion. Additionally, Radovic et al. (2018) described the operational definitions of mathematical identity as follows: "identity as individual attributes; identity as narratives; identity as a relationship with specific practices; identity as ways of acting; and identities as afforded and constrained by local practices" (p. 21).

Radovic et al. (2018) found that within the representational/enacted dimension, the representational dimension was often utilized by researchers who focus on higher education students' mathematical identity, while the enacted dimension of mathematical identity was the most used by researchers whose concentration is in primary and secondary classrooms. The authors explained that these conceptual and operational definitions could serve as a resource for researchers to elaborate on mathematical identity's conceptual and operational approaches specific to the studies they conduct (Radovic et al., 2018). However, the framework lacks a description of the interrelationship between each dimension.

**Fellus' (2019) Networked Identity Model**

The third framework is based on Fellus' (2019) networked identity model, which derived from Ivanič's (1998) framework that investigates writers' identity construction. In contrast to Darragh (2016) and Radovic et al. (2018), Fellus' (2019) networked model followed a deductive approach and described the relationship between the dimensions of identity. Fellus (2019) argued for the need for "a unifying theory that highlights the compatibility among the different identity nodes" (p. 447).

The networked identity model consists of four dimensions: 1) autobiographical identity, 2) discoursal identity, 3) authorial identity, and 4) socioculturally available selfhoods. Fellus (2019) emphasized that students' past experiences impact their current understandings and perceptions of themselves as mathematicians with autobiographical identity. This dimension of identity points to "learners' experiences and their interpretation of these experiences [that] shape their behaviors, attitudes, and beliefs about mathematics" (Fellus, 2019, p. 448). Discoursal identity addresses "how students are talked to and about as learners of mathematics" and "how these storylines inform learners' mathematics-related choices and actions" (Fellus, 2019, p. 449). Discoursal identity is highlighted when individuals' identity is shaped by how others speak to them and how individuals are spoken to in mathematics learning. Authorial identity refers to individuals' stance, opinions, and beliefs (Ivanič, 1998) and in the context of mathematics as "instances when speakers populate mathematical concepts with their own accents and intentions" (Fellus, 2019, p. 449). Finally, socioculturally available selfhoods represent how individuals are influenced by multiple layers of discourse that exist in the societal level, popular media, or sociocultural and political contexts. Fellus (2019) noted that her networked identity model emphasizes the multidimensional aspect of identity. Furthermore, because these dimensions are considered distinct and interactive, each warrants in-depth research.

For example, when students participate in small group discussions and experience making meaning, they would be constructing *autobiographical* identity. Responses from peers and teachers would contribute to the *discoursal* dimension. The *authorial* dimension is built as students leverage the interactional space to express agency and ownership over their ideas. Finally, *socioculturally available selfhoods* represent how students are impacted by discourses that exist in the societal level. An example would be when students discuss how students who like mathematics are portrayed in media and compare others to such images.

While these dimensions are different and distinct, they are still interconnected and unified. How students respond to each other (discoursal dimension) impacts how individual students leverage interactional space to share their thoughts about mathematics (authorial dimension). These discursive experiences contribute to students' autobiographical identity. During students' conversations, what and how students talk to each other are influenced by discourses that exist on a broader societal level. Because of each dimension's profound impact on other dimensions, it would be nearly impossible to completely isolate one dimension from the others. Despite the differences that exist among dimensions, these dimensions all refer to the notion of identity.

For this dissertation study, I will be referring to the notion of identity in a singular form of noun to follow the terminology used in Fellus' (2019) networked model of identity. Even though I use a singular form to refer to identity, this study assumes that identity is multidimensional, intertwined, and co-constructed through social interactions.

**Mathematical Identity as Socially and Interactionally Emergent Phenomena**

Regarding the current status of identity research, Graven and Heyd-Metzuyanim (2018) criticize the potential "objectification of identity into a mental stable entity…[which] goes hand in hand with ontological collapses that hide the fluid, dynamic and situated nature of identity" (p. 370). They described a potential pitfall where researchers could use the notion of identity as an objective category to label students or pedagogical practices. They reiterated that research in mathematics education should continue to perceive identity as a dynamic and organic process.

Earlier in this chapter, I proposed that identity should be considered a socially and interactionally emergent phenomenon. Hence, this study's approach toward identity places discourse at the forefront of identity's definition and manifestation. As previous scholars (de Fina et al., 2006; Gee, 1996; Lerman, 2011) have shown, the relationship between discourse and identity is multifaceted and complicated. Therefore, limiting the analysis to a single dimension of how discourse impacts individuals would not be able to capture the innate nature of discourse and its impact on identity.

A more comprehensive theoretical framework such as Fellus' (2019) networked model of mathematical identity facilitates the discussion about identity and discourse from various perspectives (Figure 1).

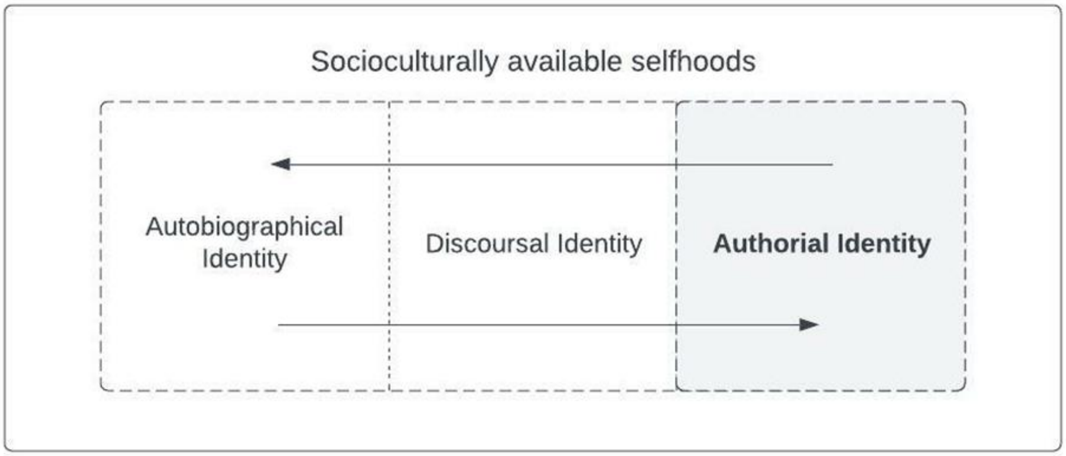


*Figure 1. Networked Identity Model*

The first three dimensions (autobiographical identity, discoursal identity, and authorial identity) refer to the individual's identity work through local discourse. The fourth dimension of socioculturally available selfhoods pertaining to mathematics identity presents the social and institutional possibilities available to individuals.

This dimension is embedded in the discourse that is part of the metanarratives that construct sociocultural norms, similar to what Gee (2000) referred to as Discourse with a capital 'D.' Ideally, each dimension deserves an in-depth exploration of its own, and this should be undertaken by researchers into mathematical identity in the future. At the current moment, thoroughly researching each dimension of the networked identity model is an ambitious undertaking that is beyond the scope of this dissertation study. Therefore, at this point, I focus on only one of these—authorial identity—in the context of mathematics education.

## Focus on Authorial Identity from the Networked Identity Model

I concentrate on mathematical authorial identity, which is one of the four dimensions of identity suggested by Fellus' (2019) for two different reasons. In the original networked identity model, Ivanič (1998) argued that the first three dimensions (autobiographical identity, discoursal identity, and authorial identity) are “socially constructed and socially constructing in that they are shaped by and shape the more abstract ‘possibilities for self-hood’” (p. 24). Unlike the fourth dimension, these three dimensions are more closely related to an individual and have a reflexive relationship with sociocultural contexts. Then, among these three dimensions, authorial identity stands out because of its profound connection to the other two dimensions. Individuals’ previous experiences of interaction with others in particular settings—autobiographical identity—contributes to one’s projected sense of authority.

Students who have previous personal experience of their ideas being validated by their teacher are more likely to speak with authority due to their autobiographical experiences. Students who have opportunities to speak with a sense of authority in classrooms construct positive mathematical identity. Individuals construct identity through the interaction between discoursal identity and authorial presence (Ivanič, 1998, p. 26). Therefore, investigating authorial identity in the networked identity model explains how each dimension is enacted and how the dimensions interact.

Additionally, as students are increasingly asked to craft arguments and convince others in mathematics, the field of mathematics education could benefit from a more nuanced understanding of authorial identity, which considers students as authors of mathematical ideas. With these skills, students have to learn mathematics content and the specific social and interactional skills necessary for communicating mathematical ideas. Therefore, investigating what students say and how they use interactional features when developing mathematical authorial identity will inform researchers and educators to properly support students.

Thus far, I have presented various theoretical approaches to describe the rationale behind focusing on the relationship between identity and classroom discourse in this dissertation study. I take Mead's (1913/2011) sociological approach to view identity as a socially and interactionally emergent phenomenon. This understanding justifies the approach used in this study to explore students' mathematical identity in social contexts where students participate in mathematical learning activities with peers via a video-conferencing application. Moreover, I utilize CA to observe how mathematical authorial identity emerges from students' conversations and to describe interactional patterns when students deploy accounts, which is a type of interactional feature.

## Chapter 3: Toward a Deeper Understanding of Mathematical Authorial Identity

Fellus (2019) suggested a holistic and broad approach to understanding identity that includes four dimensions: (a) autobiographical identity, (b) discoursal identity, (c) authorial identity, and (d) socioculturally available selfhoods. Among these dimensions, this study concentrates on authorial identity because of its significant role within the overall identity network. In the original networked identity model, Ivanič (1998) argued that the first three dimensions (autobiographical, discoursal, and authorial identity) are "socially constructed and socially constructing in that they are shaped by and shape the more abstract 'possibilities for self-hood'" (p. 24). Socioculturally available selfhoods refer to the broader narrative of identity that exists in the historical and sociocultural contexts. Examples of this dimension include portrayals of mathematics in the media and "discussion of widespread beliefs about mathematics and mathematicians" (Fellus, 2019, p. 451). The other three dimensions are different from the socioculturally available selfhoods because they are more closely related to the individuals' local experience of interaction with others.

Authorial identity stands out among the three dimensions because of its profound connection to the first two dimensions. First, autobiographical identity refers to an individual's previous experiences of interaction with others in particular settings, contributing to a projected sense of authority. For example, suppose students have a prior experience where their teacher validated their ideas. In that case, they are more likely to speak with authority due to the confidence the autobiographical experience instilled. In addition, students who have opportunities to talk with a sense of authority in classrooms construct an autobiographical identity that contributes to a positive mathematical identity.

Second, discoursal identity pertains to individuals' sense of self constructed when they engage in interactional strategies to convey who they are to others, influencing their authorial presence (Ivanič, 1998, p. 26). Therefore, investigating authorial identity first within the networked identity model could offer further insight into each dimension and how the different dimensions interact.

This chapter aims to investigate mathematical authorial identity from two different perspectives. The first perspective offers a conceptual understanding of mathematical authorial identity by drawing from previous research on authorship and authority in mathematics education. Regarding the notion of authorship, I highlight three relevant aspects. These concern the meaning-making process, cultivating a sense of agency, and broadening individuals' repertoire to become a part of a community of practice. These three aspects represent how the notion of authorship can be detected from data. Then, I suggest that there are two broad categories of authority – external and shared authority – that are relevant to this study. These aspects and categories related to authorship and authority constitute mathematical authorial identity. They will be used to understand how accounts are related to how students' mathematical authorial identity emerges from discourses.

The second perspective establishes a connection between relevant CA theories and mathematical authorial identity. Based on the theoretical orientation of this study that considers identity as a socially and interactionally emergent phenomenon, I argue that CA provides a systematic analytic method to examine students' small group discussions. This study focuses on *accounts*, which are interactional resources that speakers use to mitigate dispreferred actions. More specifically, I suggest that identifying interactional patterns around *accounts* could reveal critical information regarding how accounts are implicated in students' signaling and developing mathematical authorial identity.

## Toward a Conceptual Understanding of Mathematical Authorial Identity

The fundamental concepts underpinning authorial identity are authority and authorship. Ivanič (1998) defined authorial identity as how "writers see themselves…as authors, and present themselves…as authors" (p. 26) with an authorial presence. Then, when Fellus (2019) introduced the networked identity model into the field of mathematics education, she contended that students in mathematics classrooms are expected to learn how to conduct "authoring-related actions" (p. 449) such as how to "reason, argue, provide explanations, and defend their respective stance" (p. 450) to construct authorial identity. In this study, Fellus (2019) defined authorial identity as evident in those "instances when speakers populate mathematical concepts with their own accents and intentions" (p. 449), while simultaneously emphasizing that students should be afforded "opportunities…to develop their authorial identity in mathematics through processes of taking ownership over mathematical ideas" (p. 449).

Both descriptions of authorial identity point to the idea of students expressing their opinions or stances with a sense of authority or creating meaning as authors. Based on these studies, I expand on the conceptual understanding of authorial identity and then discuss authorial identity in the context of mathematics education by drawing from the "interlinked concepts that…provide a comprehensive understanding of phenomena" (Jabareen, 2009, p. 50).

***Conceptualizing Authorial Identity***

Authorial identity is researched mostly in the contexts of academic writing and writing education (Pittam et al., 2009). This research has described authorial identity as how writers come to see themselves as authors (Singh & Daniel, 2018) and how students acquire images of themselves as authors by communicating their ideas through writing with a sense of authority (Matsuda, 2015; Olmos-Lopez, 2015; Pittam et al., 2009).

Prior to examining how the concept of authorial identity is understood, it is worthwhile to acknowledge the definition and etymology of the word ‘authorial.’ The *Oxford English Dictionary* (OED) defines the term ‘authorial’ as “of, belonging to, or characteristic of an author.” The term “author” has its origins in French and Latin. The Old French word “auctor” means a creator, originator, source, and a person or thing that gives rise to ideas. The Latin word “auctor” refers to a person with authority or power to act or decide. Donovan et al. (2015) found that the word “auctor” in ancient Rome represented a writer “who gives the meaning” or “he that brings about the existence of any object or promotes the increase or prosperity of it” (p. 2).

Previous research on authorial identity and the etymology of the word ‘authorial’ indicates that there are two distinct yet interrelated constituting factors of authorial identity. Authorship's first constitutive factor describes how individuals express ideas to negotiate meaning through social interaction. The second factor is related to authority, which generally describes individuals’ capability to influence the decision-making process. This section briefly outlines the basic characteristics of authorship and authority then contextualizes both notions in mathematics education (Figure 2).

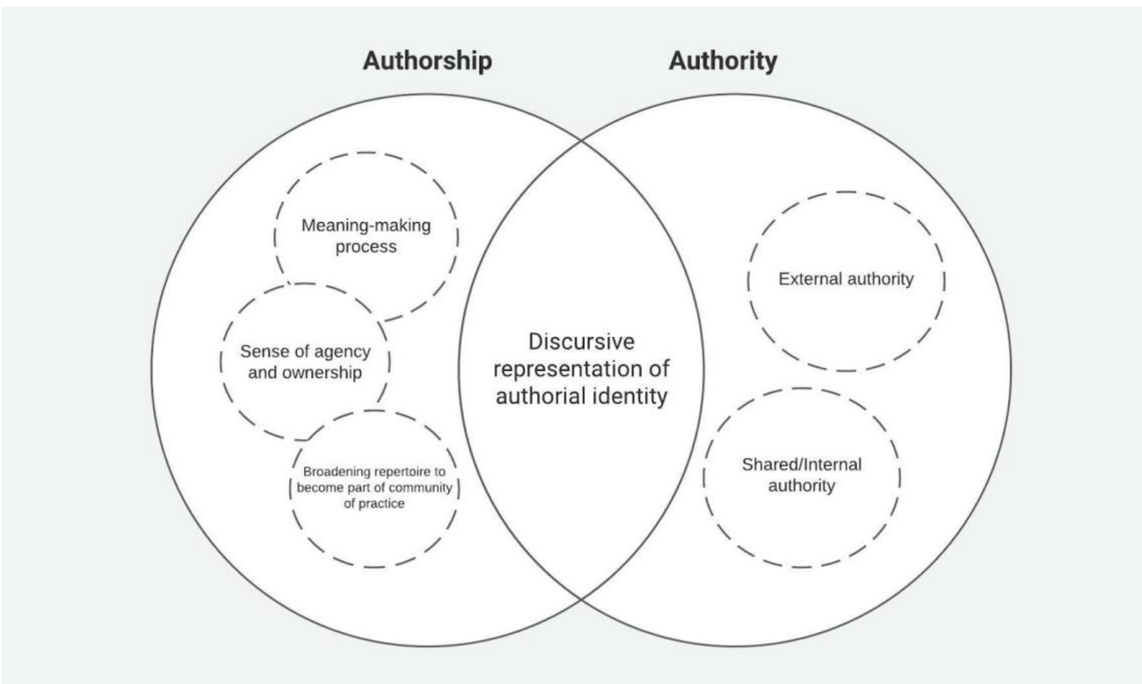


*Figure 2. Concept of Mathematical Authorial Identity*

First, authorship occurs “in the context of an ongoing dialogue between self and others” (Day & Tappan, 1995, p. 48). It is a concept that can be interpreted in various ways based on theoretical perspectives, such as constructivism, social constructionism, or dialogism (Warschauer & Grimes, 2007). Among the interpretations, the aspects of authorship relevant to this study explore the notion in three ways.  These are: (i) *making* *meaning through social interactions* (Holquist, 1983, 1990); (ii) *cultivating a sense of ownership and agency* (Burton, 1999; Schoenfeld, 2020); and (iii) *broadening individuals' repertoires as they become part of a community of practice* (Bizzel, 1992).

Second, authority can be defined as "a social relationship in which some people are granted the legitimacy to lead, and others agree to follow" (Pace & Hemming, 2007, p. 6). Interpreting authority in the context of education, Oyler (1996) proposed two dimensions of authority: process and content. The process dimension of authority represents "controlling the flow of traffic and talk in the classroom." The content dimension of authority refers to "what counts as knowledge and who is validated as a 'knower.' (Oyler, 1996, p. 149). The nature of teacher authority was used to describe the difference between these two dimensions. Referring to Peters (1966), Oyler (1996) described that when teachers are "*in* authority," they would direct students on what they should do and how to complete the process (p. 149). When teachers are considered "*an* authority" (Oyler, 1996, p. 149) by students, the teachers would be considered experts or validators of students' thoughts.

***Conceptualizing Mathematical Authorial Identity***

This dissertation study explores how accounts are related to students signaling and developing mathematical authorial identity by analyzing their small group discussions. The notion of mathematical authorial identity has not been extensively examined in mathematics education research yet; however, it is possible to draw from previous research on authorship and authority. In this section, I discuss what authorship and authority mean in this study and explain how these two concepts are relevant to students' mathematical authorial identity.

**Authorship in Mathematics Classrooms.** Authorship is a socially and discursively constructed notion (Burton, 1999; Povey, 1995). The word 'authorship' conveys "the sense of personal derivation *and* responsibility" (Burton, 1999, p. 22). Three interactive aspects contribute to the construction of authorship and are relevant to this dissertation study. The aspects are (i) *making meaning* through social interactions (Holquist, 1983, 1990), (ii) *cultivating a sense of agency and ownership* (Burton, 1999; Greeno, 2006; Schoenfeld, 2020), and (iii) *broadening an individual's repertoires to be part of a community of practice* (Bizzel, 1992).

The *meaning-making process* can be described as inherently discursively situative, diverse, and agentive (Barwell, 2018; Lave, 1991; Morgan, 2006; Planas, 2018; Seeger, 2011; Sfard, 2001). Barwell (2018) argues that "mathematical meaning-making happens through the dialogic relations between the diverse discourses, voices, and languages that arise, in written and spoken classroom interaction…language itself plays an active role in the meaning-making process, as each utterance is a response to preceding utterances" (p. 162).

Students and teachers co-construct the meaning of the words and concepts during classroom discussions through situative, local utterances. For example, Barwell (2018) shares how the meaning of the word 'curved' is co-constructed when a teacher and a student were discussing the concept of 'non-polygon.' The students explained that a shape is not a polygon and said: "because it's not a straight line." Then the teacher said, "It's not a straight line. The line is curved." By saying the word 'curved' after the phrase 'not a straight line,' the teacher indicated that the meaning of the word 'curved' was what the student said earlier. The meaning-making process occurred from how the student and teachers' turns were spoken consecutively.

Barwell (2018) also explains that the meaning that arises from the process is diverse. In the brief example conversation mentioned above, both the student and teacher's voices are present in this process. Also, the concept of 'non-polygon' was represented in two different ways. The student's description of "not a straight line" was an informal description, whereas the teacher used a formal term of 'non-polygon.'

Finally, the meaning-making process is also agentive. When students first use the word 'non-polygon,' the term is "populated with the intentions of others" since they heard the term from their teacher during classroom discussion (Bakhtin, 1981, p. 294). As students continue to use the word 'non-polygon,' they are populating *their* personal meaning in addition to the intentions of others who have spoken the word previously. As students' intentions are added to the "intentions of others" that already exist in the word, the meaning-making process occurs "each time anyone learns these ideas" (Sfard, 2003, p. 357).

Another aspect of authorship is students *cultivating a sense of agency and ownership*. Students' experience of creating the mathematical meaning of their own "repositions learners from being dependent on their teachers to adopting agency for their own learning" (Burton, 2004, p. 372). Agency in mathematics classrooms can be described as students having the interactional space to freely use mathematical concepts to identify and solve problems, evaluate, and modify what they have learned (Boaler & Greeno, 2000; Greeno, 2003, 2006). Furthermore, a sense of agency is about students "acting as an accountable author…[who] are positioned as contributors whose inputs are recognized and credited" (Lipponen & Kumpulainen, 2011, p. 813).

A sense of ownership is also a significant factor of authorship since students make linguistic choices to participate in classroom discussions. Ivanič (1998) argues that writers are engaged in "a process of self-attribution: forging their own allegiances to particular traditions and sets of values by their language choices" (p. 3). Additionally, drawing from Bakhtin (1981)'s theory of 'appropriation,' Fellus (2018, 2019) emphasizes that students develop a sense of ownership as they reproduce the words that others have spoken and add their personal interpretations during discussions. The interactional environment where students can initiate different ways to interpret and apply mathematical concepts and contribute their intentions allows students to construct authorship and a sense of agency and ownership.

The final aspect of authorial identity is *broadening individuals' repertoires to be part of a community of practice*. To reiterate, authorship is a notion that is constructed from social interactions. Fellus (2018) specifically refers to Wenger's (1998) argument of how speakers learn "a shared repertoire" (p. 73) to become members of a community. This idea resonates with Povey and Burton's (1999) argument that teaching students should be about preparing them to become a member of a "knowledge-making community" (p. 234) and Benne's (1970) description of the relationship between the bearer of authority and the subject within the environment of shared authority. The bearer of authority is responsible for guiding the subject through a new community until the subject can communicate independently by using the language repertoire specific to the group. In the conversation mentioned above between the student and teacher about 'non-polygon,’ the teacher introduced the word ‘curved’ that represents the meaning of ‘not a straight line,’ This example demonstrates how the teacher is broadening students’ repertoire of language and guiding them to be part of the mathematics community of practice.

While these three aspects of authorship discussed above are not an exhaustive list of what authorship means, they are essential for understanding how the notion of authorship operates in mathematics classrooms and contributes to the construction of mathematical authorial identity. The underlying and foundational concept that underpins all three aspects of authorship is the need for interactional space. Students should be able to leverage the interactional space during classroom discussions to contribute to the meaning-making process, cultivate a sense of agency and ownership, and broaden their repertoire. In other words, students’ participation in mathematics classroom discussions is more significant than a simple act of speaking. How their voices are heard and responded to impact students’ identity construction (Bishop, 2021; Langer-Osuna, 2016; Wood, 2013).

In sum, the construction of mathematical authorial identity is associated with students communicating their opinions and stances by participating in the meaning-making process. Students also demonstrate a sense of agency and ownership when they populate words with personal intentions and express their opinions and stances on understanding and applying mathematical concepts. As students engage in the discursive practice of authorship, they broaden their language repertoire related to mathematics and eventually become part of the community of practice that uses a similar repertoire.

These three aspects of authorship represent how mathematical authorial identity could emerge from interactions. This study considers identity a flexible, emerging, and interactive phenomenon instead of a stable and tangible end product. Based on this view of identity, how students' identity emerges from discourse also varies. However, these three aspects of authorship offer a framework for identifying the emergence of mathematical authorial identity. Students' turns related to the meaning-making process, sense of agency, and becoming a part of the community of practice would make the concept of mathematical authorial identity, especially the dimension of authorship, visible.

***Authority in Mathematics Classrooms***

Authorship and authority are interrelated and reciprocal. How interactants speak and construct authorship is influenced by the authority dynamics of a group. At the same time, the authority dynamics impact who takes over the interactional space during discussions, and how authorship is constructed by individuals in those environments. Gee (2015) specifically explains the reciprocal relationship between meaning-making and authority:

> Meanings are ultimately rooted in negotiations among people in different social practices with different interests, people who share or seek to share common ground. Power plays an important role in these negotiations. The negotiation can be settled for the time being, in which case meaning becomes conventional and routine (p. 27).

It is imperative to understand how authority works in mathematics classrooms to conceptualize mathematical authorial identity comprehensively. Authority is a social and interactional notion that plays a significant role in students' classroom interactions, mathematics learning, and mathematical identity construction in various settings. These include its manifestations in whole class discussions (Boaler, 2003; Cobb et al., 1992; Engle & Conant, 2002; Ng et al., 2021; Schoenfeld, 2014) and in small groups with peers (Amit & Fried, 2005; Engle, Langer-Osuna, & McKinney de Royston, 2014; Langer-Osuna, 2016, 2017, 2018; Langer-Osuna et al., 2020; Wagner & Herbel-Eisenmann, 2014; Wood & Kalinec, 2012).

Previous researchers have defined authority as "a social relationship in which some people are granted the legitimacy to lead, and others agree to follow" (Pace & Hemmings, 2007, p. 6). Amit and Fried (2005) emphasized the relational aspect in authority when they defined authority as "a relationship in which one person (or group of people) tends to *obey, act on, or accept without question* [emphasis added] the statements or commands of another person (or group of people or any other entity capable of producing statements or commands)" (p. 162). Within mathematics classrooms, having authority can be described as being manifested as a source of veracity when communicating opinions or stances about mathematical concepts. There are various ways that authority emerges from social interactions in these classrooms. For this study, I discuss two broad categories of authority: external and shared authority (Povey & Burton, 1997, p. 332).

**External Authority.** When students experience external authority, they assume that mathematical "meaning is taken as given and knowledge is assumed to be fixed and absolute rather than contextual and changeable. The knower is deeply dependent on others, especially authoritative others" (Povey & Burton, 1999, p. 233 - 234). For example, Cobb et al. (1992) shared a mathematics teacher's case of implementing a "discovery" activity. During this activity, the teacher was the only individual who knew the correct answer and was perceived by the students as the sole source of legitimacy. Therefore, it did not matter whether the activity was labeled 'discovery' or not. Students still relied on their teacher, who held the external authority, to seek a sense of validation, and this influenced their interactions and their learning.

The fundamental assumption behind this perspective is that students participate in the mathematics learning activity and classroom discussions not to make meaning or express their opinions about mathematical concepts, but rather "to match the teacher's intellectual expectations, in a sense to retell the teacher's story" (Povey & Burton, 1999, p. 234). Instead of engaging in a constructive discussion in which students express their opinions, evaluate others' stances, and modify their understanding of mathematical concepts, they are likely to focus on saying the correct answer. Even when students participate in classroom discussions, they would rely on external authority, such as the teacher or their peers, "for instructions, not, by contrast, for a discussion" (Amit & Fried, 2005, p. 148). Below, I describe two different types of external authority that often emerge in mathematics classrooms.

***Teachers' Authority.*** Students experience the notion of authority when they participate in whole-class discussions or listen to teachers' lectures. A typical example of how authority is enacted in mathematics classrooms is when students accept what their teachers or peers tell them without asking questions or expressing any doubt. This is commonly due to schools' institutional setting, which encourages students to accept what their teachers say. Often in mathematics classrooms, teachers are seen as the person who speaks the truth about mathematical concepts and has authority over subject matter knowledge (Gerson & Bateman, 2010; Pace, 2003). Students perceive their mathematics teachers as “an expert authority” who serves as “a source of information and guidance; one turns to an expert authority for instructions, not, by contrast, for a discussion” (Amit & Fried, 2005, p. 148).

The issue at stake is not that considering a teacher as “an expert authority” or listening to what their teachers say is harmful to students’ learning; instead, it is the nature of external authority that results in students *blindly* following others’ claims. For example, Yackel and Cobb (1996) stated that students are “accustomed to relying on authority and status to develop rationales” (p. 467). The authors noted that lesson structures and classroom discourse norms resulted in expectations placed on students to listen to their teachers. Students are familiar with interactional patterns such as regurgitating key information their teachers relayed, or accepting the teachers’ presentation of mathematics. Amit and Fried (2005) interviewed middle school students to explore who is considered an authority in mathematics classrooms. All of the sources of authority that students identified were all external authorities. The authors argued that when students turn “always from one figure to another, and never to themselves, [they] not only fail to develop their own mathematical thinking but they also perpetuate this failure by always defining themselves as *outsiders* with respect to mathematical discourse” (Amit & Fried, 2005, p. 165).

***Authority among Peers.*** Teachers are not the only kind of external authority available in mathematics classrooms. Given that authority is a social and interactional concept, social status and power also contribute to the group dynamics of authority among peers. Students often determine who has authority by their social status and using certain discursive moves instead of their mathematical merits (Engle et al., 2014; Langer-Osuna, 2016, 2017). For example, Bishop's (2012) analysis of two seventh-grade students' talk during peer activities revealed that the student who used discursive moves, such as using "authoritarian voice" and "statements of superiority" (p. 53-54), controlled whose ideas were considered valid and legitimate.

In another study, Wood (2013) observed how one student's opportunities of participating in small group discussions varied depending on how the group members referred to the student. He had greater access to the discussion when considered a "mathematical explainer" or "mathematical student." However, Wood (2013) noticed how the student was perceived by their peers shifted when his white female peer called the student, who was a black male, "boy." Since then, the student was positioned as a "menial worker," and his opportunities to contribute to the discussion became restricted.

How classroom talk impacts the notion of authority in peer discussion is also evident from Langer-Osuna's (2016) study. The researcher reported that intellectual authority was afforded to fifth-grade students who successfully utilized social directives, despite a lack of content knowledge in mathematics. Students were more likely to listen to a peer who told what others should do by using directives. In other words, the mathematical accuracy of the statements did not matter as much as their status. When certain students spoke using directives, other students took that interaction pattern as conveying a sense of authority that they needed to follow. These studies demonstrate *what* and *how* students say particular words to others influence mathematical identity construction and learning.

In addition to these studies, researchers have noticed how complicated it is for students to navigate peer discussion in mathematics classrooms. Students relying on other peers who were perceived as mathematically competent were not able to engage in a "true dialogue…between [students], and [therefore]…no true collaborative learning" (Amit & Fried, 2005, p. 161). Most of the conversations among the students involved figuring out the accurate answer from particular students who are considered 'good at mathematics.' Additionally, Cobb and Bauersfeld (1995) asserted that students *who dominate* social interaction and *who are dominated* face challenges engaging in meaningful mathematical collaboration. Students who dominate do not have access to critically review their reasoning through peers' questions and ideas. In contrast, those who are dominated often do not have interactional affordances to express their thoughts about mathematics.

**Shared/Internal Authority.** While authority can be a restricting factor in students' learning of mathematics concepts, it can also be an empowering tool. To tap into the potential of authority, the critical question should not be about *who* has authority but about *how* authority is *distributed* in mathematics classrooms. Shared authority is "a kind of authority which is non-localized, that is, in which there is no immovable division between the subject and agent of authority" (Amit & Fried, 2005, p. 58). When students experience shared authority, they can express personal thoughts about mathematical concepts. Also, they can be more susceptible to the idea that authority exists in a non-rigid hierarchy, and anyone in the classroom community can practice it.

A foundational work on shared authority comes from Benne's (1970) theory of "anthropogogy authority," which refers to a combination of "anthropology" and "pedagogy" (p. 399). Benne (1970) described how authority works as the following:

> [Authority] operates in situations in which a person or group, fulfilling some purpose, project, or need, requires guidance or direction from a source outside [them]. The bearer of authority received willing obedience from the subjects of [their] authority as the bearer exercises [their] claim to help mediate the field of conduct or belief in which the subjects are in need of advice, leadership, guidance, or direction (pp. 392-393).

Applying the above description of authority to the context of education, Benne (1970) underscored two critical concepts: (a) the importance of community and (b) individual's growth to achieve autonomy within the community - through an example of a doctor and a medical student. First, in the context of medical education, a successful authority relationship between a doctor and a medical student should be marked with "a movement toward collegiality between teacher and student [since] the task of medical education is to help induct aspirants into membership in the medical community" (Benne, 1970, p. 400). Second, Benne (1970) also emphasized that through the authority relationship, "the student must make the role of doctor his own, must integrate the role into his own personality…to become an autonomous and independent member of the medical community" (p. 401).

A key characteristic of shared authority described by Benne (1970) is that all participants in the community have a significant and active role to play at different stages of development. The person who first bears authority from the perspectives of the students provides guidance or directions that the subject of authority requests. In this relationship, the goal is to foster the subject's personal and professional growth. Eventually, the individual who was once the subject of authority becomes the bearer of authority for other subjects, who in turn take on that role as well with the passage of time.

Previous research on shared authority emphasizes similar aspects Benne (1970) described. The first aspect is providing students with ample interactional opportunities to participate in mathematics classroom discussions as a source of authority (Amit & Fried, 2005; Gerson & Bateman, 2010; Povey & Burton, 1990; Schoenfeld, 2014). The second aspect concerns legitimizing and granting agency to students so that their contribution becomes valuable to the classroom community's discourse and meaning-making process about mathematics (Amit & Fried, 2005; Bell & Pape, 2012; Engle & Conant, 2002; Fried & Amit, 2008; Kinser-Traut & Turner, 2020; Langer-Osuna, 2018).

The aspect of shared authority related to student's agency can also be described as internal authority. For the purpose of this dissertation study, I am referring to internal authority as what Povey and Burton (2003) described as "author/ity" (p. 332). Students who experience internal authority would consider knowledge and mathematical meaning as a notion that is "contingent, contextual, and personal" that is co-constructed through an interaction of external sources of authority and individual's interpretation (Povey, 1997, p. 338). Reinholz (2012) elaborated on how students would invoke internal authority in three different actions: (i) explain their reasoning, (ii) justify conjectures, and (iii) evaluate their work once they reach a solution (p. 242). These actions describe what students would do when they interpret mathematical concepts with their own lens and communicate those ideas with peers to co-construct meaning.

Addressing these two related aspects, Schoenfeld (2014) elaborated that the students experiencing shared authority should have "opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency…and authority…resulting in positive identities as doers of mathematics" (p. 9). In other words, for students to become "bearer[s] of authority" (Benne, 1970, p. 392), they should be *guided* by other bearers of authority in mathematics classrooms. These include classroom teachers, their peers, and even their textbooks, or other curriculum materials. Within this community that practices shared authority, students should be able to experience a sense of agency when communicating opinions and stances about mathematical concepts with their "own personality" (Benne, 1970, p. 393).

Shared/internal authority, for the context of this dissertation study, would mean that all members of mathematics classrooms practice authority when understanding and applying the mathematical concepts they are learning. In other words, students act with a sense of agency and actively decide how certain mathematical concepts should be interpreted and applied when solving problems. Authority is not skewed or limited to one person in this learning environment. Students acknowledge that all class participants can contribute to each other's meaning-making process and knowledge building. Everyone is positioned as a valued stakeholder in deciding which mathematical concepts are relevant and how they should be applied. Claims about mathematical concepts are produced with students' personal intentions, and their words are perceived as legitimate contributions.

In this section on authority, I described two broad categories of authority and how they work in mathematics classrooms. Previous research argued that authority could hinder learning when it is only placed on the teacher or specific students. When authority is located external to the students or limited to a few individuals in the classrooms, community members eventually become passive *receivers* of knowledge. They would not be able to leverage the interactional space to raise questions on the rationale behind understanding mathematical concepts or even to express their voices at all in some instances (Empson, 2003).

The notion of shared/internal authority serves as a foundation for promoting students' agency in the meaning-making process of learning mathematical concepts, having ample opportunities to express their opinions and stances about mathematics, and being perceived by others in the classroom community as valuable contributors. The conceptual perspective of shared authority is closely related to authorship since when students practice shared authority during classroom discussions, they engage in "talk, discussion, suggestions and conjectures, refutations, or shifts of thought through resonance" (Lerman, 1994, p. 196) to produce knowledge and become authors with their meanings of mathematical concepts.

### *Operationalizing Mathematical Authorial Identity*

So far in this chapter, I have presented this dissertation study's conceptual orientation of mathematical authorial identity. Authorial identity is a concept that has been researched in writing education; however, it has critical and promising implications for mathematics education as well. I proposed that authorship and authority are contributing factors to authorial identity and discussed previous research on the two notions in mathematics education.

The specific aspects of authorship that I highlighted include students (i) *making meaning through social interactions* (Holquist, 1983, 1990), (ii) *cultivating a sense of agency and ownership* (Burton, 1999; Greeno, 2006; Schoenfeld, 2020), and (iii) *broadening an individual's repertoires to be part of a community of practice* (Bizzell, 1992). These aspects represent how mathematical authorial identity could emerge from students' conversations. These are not meant to be mutually exclusive nor exhaustive, however. If researchers can specify various ways that authorship can be manifested, these would facilitate the task of identifying the precise expression of students' turns that contribute to their growth into a confident mathematical authorial identity.

A related notion that I also discussed was authority. I suggested that authority can be divided into two broad categories of external and shared authority. The most significant difference between the two categories entails ascertaining just *where* authority is located. Students experiencing external authority would refer only to types of authority such as teachers, textbooks, or other experts. Their own participation would be muted or discounted altogether. On the other hand, students who experience shared authority would leverage the interactional space to voice their opinions and participate in the meaning-making process. The two constitutive factors of mathematical authorial identity, authorship and authority, are negotiated and constructed through social interactions. Then how is identity constructed in this process?

Let's refer to the brief exchange between the student and teacher about the word 'curved' (Barwell, 2018). From the lens of Bucholtz and Hall's (2005) emergence principle of identity, it can be argued that mathematical authorial identity was constructed from the sequences of the turns between the student and the teacher. In that conversation, the student was answering a question about whether a shape is a polygon or not. The student said that the shape is not a polygon "because it's not a straight line." Then the teacher said, "It's not a straight line. The line is curved."

First, regarding authorship, the question that occasioned the student's answer provided the interactional space to produce and express meaning about the concept of non-polygon. The student's discursive action created a site where the student's authorship emerged, since the student 'appropriated' the words to express an opinion and adapt a stance regarding a mathematical concept. Second, regarding authority, the teacher's following turn, which revoiced what the student said, authorized the student's voice as a legitimate contribution to the classroom discussion. This conversation is an example of shared authority, as the student contributed to the meaning-making process of the word 'curved.' Overall, the student's mathematical authorial identity became audible through the interactional space provided by the question. Critical also here is the teacher's revoicing that shared authority with the student, while also further elaborating by stating that the line was curved.

These two turns captured here also illustrate how the four dimensions of the networked model of identity proposed by Fellus (2019) are related to each other. The dimensions were (i) *autobiographical identity*, (ii) *discoursal identity*, (iii) *authorial identity*, and (iv) *socioculturally available selfhoods*. Because the first three dimensions refer to how identity is constructed locally, the interaction mentioned above can be interpreted through the lens of autobiographical, discoursal, and authorial identity.

The interaction mentioned above documents the student's experience of making meaning, exercising a sense of agency, and learning what vocabularies the community of practice uses (i.e., curved). This experience would contribute to the student's *autobiographical* identity in presenting independent claims regarding the task at hand. The *discoursal* dimension refers to how the teacher responded to the student's turn. The teacher's revoicing indicates that the student's voice is valued as a doer of mathematics that can be further elaborated upon with a contrast between a line and a curve. The *authorial* dimension highlights the interactional space that the student was able to leverage to indicate agency and ownership over the opinion on a mathematical concept. This dimension also facilitates a discussion around how the teacher validated the student as an author of mathematical meaning.

As described above, the sequence of turns and what interactants say can be analyzed to draw insights into how mathematical authorial identity emerges from interactions. This kind of interactional analysis can be done not only between a student and teacher, but also among students. Then what would be a systematic method of analyzing the patterns of conversations? The following section discusses the CA aspect of mathematical authorial identity.

## Toward a CA Understanding of Mathematics Authorial Identity

The purpose of this dissertation is to explore the interactional patterns of students' deployment of accounts and investigate how accounts are implicated in students' signaling and development of mathematical authorial identity. There are an infinite number of interactional resources that we use in our everyday lives. Among many interactional features, accounts are closely aligned with the conceptual understanding of mathematical authorial identity. As explained in Chapter 2, accounts refer to statements "made by a social actor to explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46). Account's interactional feature that provides an additional explanation to mitigate dispreferred actions could serve as a window into how mathematical authorial identity emerges.

### *Indexicality Principle of Identity*

Before providing the details of accounts, it is essential to recall the relevant identity principle that describes the relationship between identity and discourse. Bucholtz and Hall's (2005) *indexicality principle* connects identity theories and empirical evidence from classroom discourse. I rely on this principle to explain the relationship between conversational features and students' identity throughout my discussion of accounts and epistemic stance markers.

The indexicality principle highlights the mechanical process of how linguistic forms implicate identity. Indexicality refers to "the creation of semiotic links between linguistic forms and social meanings" (Bucholtz & Hall, 2005, p. 594). An index, which is the core idea of the indexicality principle, is a connection between objects or ideas and their meanings from social contexts. Hanks (1999) provided an example of a situation where an individual hands over a book to another person and says, "I want you to have this." In this sentence, the words 'I,' 'you,' and 'this' are examples of *indexical* linguistic forms because the meaning of these words is only significant when interpreted within the interactional context. This sentence would not carry the exact same meaning if spoken in a different setting and with different interactants. This example demonstrates how indexes work to attribute meanings to linguistic forms.

Here, I refer back to this study's approach that identity is a social and interactional achievement. It is not a preordained, objective concept but rather an intersubjective accomplishment that obtains meaning from social contexts. In other words, linguistic forms are given social meanings related to identity through indexical processes. Based on this logic, the indexicality principle enables an examination of linguistic structures to understand identity further as they emerge from interactions. Bucholtz and Hall (2005) detailed four processes as to how various linguistic forms discursively produce identity:

Identity relations emerge in interaction through several related indexical processes. These include (a) *overt mention of identity categories and labels*; (b) *implicatures and presuppositions regarding one's own or others' identity position*; (c) *displayed evaluative and epistemic orientations to ongoing talk, as well as interactional footings and participant roles*; and (d) *the use of linguistic structures and systems that are ideologically associated with specific personas and groups* (p. 594).

The first indexicality process is when specific identity categories are explicitly used during discourses. A minimum amount of inferential work is needed by interlocuters to understand what a particular word means in terms of identity. To go back to the example of hijra, the word *hijra* means 'impotent' and denotes a derogatory force in Indian society. When an individual is referred to as a *hijra* in interactions, it is associated with impotence and is an insult to that individual. Therefore, the word *hijra* is used to invoke one's identity, which is ridiculed and disparaging. Because it has been attributed to the meanings of specific identity categories and labels through metanarratives, individuals familiar with the sociocultural norms that use this word as an insult would invariably understand its meaning in terms of one's identity.

The second indexicality process is less direct and needs more inferential work. Liang (1999) detailed the use of gender-related references among lesbian and gay individuals. For example, in everyday conversations where lesbians talk about relationships, they might use the 'she' pronoun to refer to their spouses. Specific pronoun linguistic forms implicate the speaker's sexual identity without explicitly declaring that she is a lesbian.

The third indexicality process is well represented by *stance*, or "the display of evaluative, affective, and epistemic orientations in discourse" (Bucholtz & Hall, 2009, p. 595). The stance marks speakers' attitudes and positions. Stances are meaningful when understanding identity because they are formed from the interaction between personal experience or standards and objects of evaluation. Expanding on the indexicality principle, Ochs (1993) argued that stance is a mediator between linguistic forms and social identity, as "social identity is a complex social meaning that can be distilled into the act and stance meanings that bring it into being" (p. 289).

Beyond linguistic forms, including specific vocabularies, pronouns, or stance, norms regarding language systems can index identity. An example of this fourth indexicality process is based on how globalization impacts language use and identity. Besnier (2004) examined how a New Zealand seller used English more often than Tongan when communicating with buyers at a tourist site to display the seller's cosmopolitan identity.

These processes describe how various linguistic forms index identity. I argue that specific interactional features like accounts can index students' mathematical authorial identity. The indexicality principle offers the theoretical justification for examining students' conversations to infer how their mathematical authorial identity interactionally emerges. The following section describes how accounts could contribute to a deeper understanding of mathematical authorial identity from a CA perspective.

***Accounts***

As with other interactional features, accounts are intentionally produced to achieve specific social actions and carefully placed in conversation sequences (Firth, 1995). Accounts refer to statements "made by a social actor to explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46). Accounts can be used to achieve the following social actions (Buttny, 1987, p. 67): (a) managing problematic situations (Scott & Lyman, 1968); (b) restoring social equilibrium (Goffman, 1967; Semin & Manstad, 1983); (c) responding to social embarrassment (Petronio, 1984); and (d) representing a form of impression management (Goffman, 1959). Accounts are usually associated with dispreferred actions because researchers have shown that accounts are expressed along with them while missing from preferred actions (Heritage, 1984; Pomerantz, 1984; Sacks, 1987; Schegloff, 2007).

In contrast, CA researchers have discovered that dispreferred actions that could threaten hearers' faces are usually "avoided or mitigated or delayed or, at least accounted for" (Blimes, 2014, p. 53). When someone has to decline a dinner invitation, that person is expected to provide accounts to mitigate the face-threatening act of rejection.

Suppose that person A extends person B the following invitation: “Would you like to come by for dinner next week?” Person B’s response, which provides accounts, might be as follows: “Unfortunately, I can’t make it next week because I need to work on my final papers.” Person B rejects the invitation, which threatens person A’s face (Brown & Levinson, 1987) but explains the rejection. The response is specifically used to indicate that the invitation was rejected *not* because their relationship is at risk but rather due to an external circumstance (Heritage, 1984, p. 271). Person B’s accounts were used to manage the face-threatening act and maintain their social relationship.

Person A might question their social relationship because the face-threatening act was not mitigated or qualified. Person A could think that person B is rude and would never be invited to other events. If person B had said “No” without additional explanation, person A would notice that accounts were absent from the talk (Goodwin & Heritage, 1990). A dispreferred action not accompanied by accounts would be considered disaligning or improper (Heritage, 1984, 1988).

Accounts have three characteristics. First, as evident in the above example, accounts index untoward or dispreferred behaviors (Firth, 1995). The distinction between accounts and explanations highlights accounts’ specific role in marking dispreferred actions. Although both provide additional contextual information during interactions, with explanations, “untoward action is not an issue and [they do] not have critical implications for a relationship” (Scott & Lyman, 1968, p. 47).

Second, accounts are linguistic devices that also work as an “important problem-solving resource…that…provide[s] the negotiating parties with ‘negotiable materials’” (Firth, 1995, p. 205). When accounts are produced to mitigate a dispreferred action, interactants can use what has been said to find solutions or move on to the following conversation sequence (Buttny, 1985). Following the previous example, after person B says, “Unfortunately, I can’t make it next week because I need to work on my final papers next week,” person A can suggest another time for dinner or inquire further about the final papers. Therefore, accounts are used to smooth out the dispreferred action and connect the subsequent “phases” of conversation sequences (Scott & Lyman, 1968, p. 60). In other words, accounts are typically an interactional site where one’s positions, reasoning, or beliefs are elaborated to preserve social harmony when interactants’ faces are interrupted (Goodwin & Heritage, 1990; Heritage, 1988).

Third, although accounts are expressed to mitigate the impact of untoward actions, the hearers of accounts are responsible for restoring social relationships and can choose whether to honor or dishonor an attempt to mitigate the impact of dispreferred action (Scott & Lyman, 1968). Speakers typically deploy accounts about a problematic event or a dispreferred action (Buttny, 1987). After accounts are deployed, hearers of accounts determine whether the explanation provided by the speakers was enough to mitigate the dispreferred action. In other words, the hearers of accounts typically let the speakers of accounts know whether the speaker has been “re-establish(ed)…as a person” (Goffman, 1971, p. 119). If the accounts were insufficient and did not restore social cohesion, the hearers would let the speakers know in the conversation.

To understand mathematical authorial identity from the lens of CA and students’ deployment of *accounts*, two assumptions should be established: (a) framing the activity of mathematical argumentation as a type of accounting practice that involves facework; and (b) considering accounts as a site of identity work.

**Facework in Mathematics Classrooms.** When students participate in mathematical argumentation, they perform "authoring-related actions," including how to "reason, argue, provide explanations, and defend their respective stance" to construct authorial identity (Fellus, 2019, p. 450). These actions are considered face-threatening acts from a CA perspective (Goffman, 1974; Brown & Levinson, 1987).

Connected to one's identity and emotion, *face* is defined as "the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact" (Goffman, 1955, p. 213). Face can be further categorized into *positive face* and *negative face* (Brown & Levinson, 1987). A positive face is an individual's desire for social approval, while a negative face is an individual's need for autonomy and desire to act with minimum interruption by others.

Following the Politeness Theory, the assumption is that "the mutual knowledge of members' public self-image or face, the social necessity to orient oneself to it in interaction are universal" (Brown & Levinson, 1987, p. 312). In other words, when engaging in social interaction, people are aware that what they say impacts other peoples' faces. It also can be assumed that individuals try to protect each others' faces during conversations. Given that face is an intersubjective concept, *facework* refers to the negotiation of face that aims "to preserve one's own and others' face or positive social value" (Polo et al., 2017, p. 127).

The same principle of facework applies to classroom discourse—teachers' and students' needs to save face are constantly negotiated through their interactions. To describe the different dynamics of facework, Tatsis et al. (2018) proposed five facework acts (see Table 1).

***Table 1: Facework Acts***

| Type of Act | Description | General Indicators of Act |
|---|---|---|
| Face-threatening act | Explicitly threatens the other's face | Requests, orders, rejection of the other's suggestion, expressions of sarcasm and irony |
| Face-empowering act | Explicitly or implicitly empowers the other's face | Acceptance of the other's suggestion, expressions of appraisal |
| Face-weakening act | Implicitly weakens one's own face | Expressions of uncertainty, withdrawal of one's own suggestion, admittance of being mistaken |
| Face-maintaining act | Implicitly aims at maintaining one's face, even when it is not being explicitly threatened | Initiation of talk, expression of one's ideas |
| Face-saving act | Aims at 'repairing' one's face after having received a face-threatening act | Argumentation, justification of one's own acts, repetition or elaboration of a suggestion, expression of face-threatening acts against the other |

*Note*. Adapted from Tatsis et al. (2018), p. 1033.

Returning to the discussion of students' "authoring-related actions" (Fellus, 2019, p. 450), suppose that two students have different approaches to solving a problem. Student A initiates the conversation by making a claim. Then, student B is expected to respond to the claim by either agreeing or disagreeing with student A. When student B expresses disagreement, this expression is considered a *face-threatening act* because it impedes student A's need to obtain social approval. If student A produces a counterclaim, this expression is regarded as a *face-saving act* because student A responded to a basic need of restoring face after the impact of a face-threatening act. As such, when students make claims and counterclaims about mathematical concepts, they are engaging in facework.

**Accounts as a Site of Identity Work.** I elaborated on the conceptual understanding of mathematical authorial identity in the previous section of this chapter. In particular, I built on the theoretical approach that considers identity as a socially and interactionally emergent achievement (Bucholtz & Hall, 2005) and argued that aspects of mathematical authorial identity include the notion of authorship and authority. This dissertation study operates under the assumption that identity emerges from interactions. As an interactional feature, accounts can be investigated to make inferences about speakers' identity. As Scott and Lyman (1968) argued, "every account is a manifestation of the underlying negotiation of identities" (p. 59).

Accounting practices have been investigated in various settings to uncover how identity is constructed during accounts. Previous studies had focused on how "normative gender behaviors" were embedded in teenage girls' accounting practices when they identified 'good versus bad friends' and formed cliques (Evaldsson, 2007, p. 379). Scholars have also shown how accounting practices shaped interactants' identity as "ordinary users" of technologies (Robles et al., 2018, p. 150). Sterponi (2004) demonstrated that by participating in account episodes, high-functioning children with autism achieved "practical skills for constructing a satisfactory moral identity" (p. 223).

As evident in these examples, accounts are a valuable conceptual and methodological tool that offers insights into how discourse constructs identity. For the context of this dissertation study, I am employing accounts to explore how mathematical authorial identity emerges from students' small group discussions in mathematics classrooms.

## Chapter 4: Methodology

This study was part of a larger project called Northwest Rural Innovation and Student Engagement (NW RISE), which offered professional development and networking opportunities for K-12 educators, administrators, and state education agency staffs in schools in the Pacific Northwest states. NW RISE network aims to enhance students' engagement and facilitate collaboration and innovation among rural educators. The network functions in a blended format that includes both in-person convenings and online meetings. More information about the organization is provided in the next section.

Within the NW RISE network, Katie[1] (an instructional coach from Alaska), Heather (a fourth-grade classroom teacher from Alaska), and Emily (a fourth- and fifth-grade classroom teacher from Idaho) collaborated to develop a curriculum that would virtually connect their students and teach mathematical argumentative writing skills. The teachers spent summer 2018 developing the curriculum, implementing learning activities during the 2018-2019 school year.

The teachers placed students in small groups that constitutes of one student from Alaska and two to three students from Idaho. First, students who did not have a consent were placed in one group to ensure that their conversations were not recorded. Second, the remaining students were placed so that their grade levels and mathematics proficiency levels varied. After the groups were formed, they stayed in the same group and met once a quarter (four times in one school year) via Zoom.

During the videoconference meetings, the teachers observed the students in person while walking around to see different small groups. I observed various small

[1] All names mentioned in this dissertation are pseudonyms.

groups virtually. After each meeting, Katie compiled the recordings and shared the files with me. Only audio data was utilized from the recordings; any other than audio was considered beyond the scope of this study. Because I did not recognize the students who participated in this study, I used the recordings to determine who spoke and when. Students' audio data were then transcribed and analyzed.

I focused on students' interaction sequences using CA methods that explored how students' mathematical authorial identity emerged in their use of accounts. The research questions guiding this study were as follows:

- How are accounts implicated in students' signaling and development of mathematical authorial identity?
    - How are account occasioned?
    - What did students achieve when they deployed accounts in small group discussions?
    - How are different account types related to the emergence of mathematical authorial identity?

Adopting a CA approach to explore how mathematical authorial identity emerges in social interactions, specifically in accounts, offers a unique opportunity to discover *how* students' mathematical authorial identity are constructed and negotiated in moment-to-moment classroom interactions. In this chapter, I detail (a) the background information and study design of this study; (b) a description of learning activities that students completed; and (c) the CA analytic processes I took.

## Background of the Study

### Larger Project Background

Study data were gathered as part of a larger curricular project from a professional learning network of rural teachers in the Pacific Northwest states in the United States. The most recent U.S. Census data on ‘Urban and Rural Classification and Urban Area Criteria’ defined ‘rural’ areas as those with populations of less than 2,500 (U.S. Census Bureau, 2021). Rural schools make up about 50% of all school districts in the U.S. (Cicchinelli & Beesley, 2017). The number of students who attend rural schools exceeds “the enrollments of New York City, Los Angeles, Chicago, and…the next 75 largest school districts combined” (Showalter et al., 2017, p. 1).

However, more attention and research are still needed compared to the research focused on urban schools. In rural communities, schools are often the only institution that support youths (Biddle & Azano, 2016). Therefore, schools are more than educational institutions for children—they are where communities gather and build relationships.

Rural schools often struggle with limited funding, as rural communities collect comparatively less revenue from property taxes (Hansen-Thomas, 2018; Reynolds, 2017) and have low school enrollment, both of which lead to lower funding amounts (Gutierrez, 2016). Teachers in rural schools are paid less, on average making 88% of the salary of colleagues who teach in nonrural areas (Beeson & Strange, 2003), and have limited access to professional development opportunities (Coady, 2020). Teacher shortage issues are critical in rural schools and lead to complications such as limited course offerings (Monk & Haller, 1993). In many instances, one teacher teaches multiple subjects without in-depth subject matter knowledge (Moskal & Skokan, 2011).

To combat the challenges of rural schools, the Northwest Comprehensive Center (NWCC) at Education Northwest started a professional learning network of rural educators called NW RISE in 2012 (Hargreaves et al., 2015; Shirley & Hargreaves, 2021). At its inception stage from 2012 to 2013, NW RISE network partnered with Drs. Andy Hargreaves and Dennis Shirley at the Lynch School of Education at Boston College to design and implement the education network with an aim to connect rural educators who are teach similar subjects or grade-levels and enhance student engagement. From 2014, NW RISE began hosting in-person meetings twice a year in June and November, in addition to sponsoring online meetings in between. The educators who joined the NW RISE network communicated with one another frequently via email and a web platform called "Schoology."

The highlight of the in-person meetings is the "job-alike groups." Due to teacher shortages and limited curriculum offerings, rural teachers are often the only subject- or grade-level teachers. For example, there would be one fourth- and fifth-grade teacher or one high school mathematics teacher in the entire staff. The rural teachers often would not have grade-level teams or subject-matter department teams to discuss lesson plans, ask questions, or request feedback. Also, teachers are often asked to teach multiple grade-levels or subjects at the same time (i.e., teaching a combined class of 4$^{th}$ and 5$^{th}$ grade students or teaching both mathematics and art) or teach subject areas they are not familiar with. Through the network, and especially through "job-alike groups," the teachers who were part of NW RISE were able to share similar challenges and seek solutions together, especially through collaboration related to developing and implementing curricular activities.

Examples of job-alike groups include kindergarten teachers, third- through fifth-grade teachers, secondary social studies, and special educators. In these groups, teachers discussed common challenges experienced by their students, drafted collaborative curriculum solutions, and analyzed student data by examining students' classwork. Between in-person meetings, the members communicated virtually to share lesson plans, innovated, and held each other accountable for the sustainability of the NW RISE network. Teachers were encouraged to choose one job-alike group to maintain consistency and streamlined collaboration.

There were about 10 to 20 teachers in job-alike groups. In each job-alike group, the participants were divided into small groups of three to four who shared similar interests. For example, in the third- and fifth-grade teachers' job-alike groups, the teachers were divided into three to four smaller groups to pursue different interests related to curriculum development. The teachers decided what they wanted to work on, how they wanted to design and implement the curriculum activities, and how to measure students' progress.

I joined the NW RISE convenings from November 2016 until November 2018 as part of the Boston College team with Drs. Hargreaves and Shirley. We supported the planning and facilitating in-person convenings and virtual meetings such as webinars and online steering committee meetings. In addition, I worked as a co-facilitator of secondary mathematics teachers' job-alike group along with another NWCC staff member. After one year of working with secondary mathematics teachers, I wanted to explore what other groups were like. So, I joined the third- and fifth-grade teachers' job-alike group and began working closely with Katie, Heather, and Emily.

I first began working with the teachers as a co-facilitator who supported them as they thought about how to design and implement their curricular project. Within job-alike group, the teachers were asked to complete a worksheet that shares how their curricular project is going. I worked with the teachers to think about questions such as 'what specific activities will students complete,' 'how would the activity enhance students' engagement in learning,' and 'how do you plan to understand and measure students' level of engagement.' After working with Katie, Heather, and Emily for about six months, I became more involved since I actively participated in designing the curricular activities. Then, as the teachers implemented the learning activities, my position shifted to being a researcher since I focused on collecting and analyzing data.

**NW RISE Third- and Fifth-Grade Teachers' Job-Alike Group**

The teachers who participated in this study met through NW RISE's job-alike groups. Among third- through fifth-grade teachers who participated in the same job-alike group, Katie, Heather, and Emily collaborated because they shared a similar goal of developing an engaging and collaborative mathematics curriculum for their students.

Because Katie and Heather worked at the same school, two schools (one from Alaska and another from Idaho) were involved in this study. These two schools represent typical rural schools in the Pacific Northwest. Both schools are in geographically isolated rural areas, with the total student population in these schools at about 150-200 students ranging from kindergarten to 12th grade. Because of their small size, these schools belong to "one school" districts. The number of students from economically disadvantaged backgrounds ranges from 20% to 70%; approximately 30% come from minority backgrounds (Kim & Martin, 2020).

The three teachers first began developing a collaborative mathematics curriculum for their students during the summer of 2017. After solving the same mathematics problems, the teachers used Zoom to connect their classes. During the shared instructional time via Zoom, only a few students from Heather's classroom and Emily's classroom were able to share their mathematical thinking and their answers with peers from another school. The teachers met again in the summer of 2018 to discuss improving the collaborative curriculum and engaging more students in the learning activity. During this time, I attended the NW RISE meetings as an observer and decided to join Katie, Heather, and Emily's curriculum project.

Based on their experiences running Zoom meetings with the two classes, the teachers discussed some of their ideas concerning ways to facilitate more meaningful conversations about mathematics among their students. The teachers also wanted more students to communicate with peers from another school. In addition, the teachers had specific goals regarding mathematics argumentation, as they were aware of the new mathematics standards.

**Preparing Mathematics Argumentation Learning Activities**

After meeting at the NW RISE in-person conference in June 2018, we began creating a curriculum for students with three specific goals. The first goal was to develop learning activities and put students in small groups to afford opportunities to talk to their peers about ideas on mathematics. The second goal was to facilitate students' discussions on mathematics argumentation. Finally, the third goal was to develop students' positive mathematical identity and provide opportunities to perceive themselves as competent mathematicians during class discussions.

After identifying these goals, we met on three occasions over summer 2018 (July 9, July 23, and August 2) to learn about mathematical argumentations and plan learning activities for 2018-2019. These planning sessions lasted 1 hour, and we reviewed recent developments in the Common Core Standards for Mathematics Practices (SMP, 2010) that emphasize students' competency in crafting mathematical arguments. We also discussed managing groups with different grade levels so that all students could talk to peers from a different school to share mathematical argument drafts and exchange feedback on each other's arguments. The learning activities were primarily divided into four parts: (a) students individually solved the same mathematics problem and drafted mathematical arguments; (b) students then met in small groups via Zoom to share their drafts of arguments; (c) also via Zoom, students provided feedback to each other on how to improve their arguments; and (d) after receiving feedback, students then took the time to reflect on their peers' comments and individually write the final draft of mathematical arguments.

To implement these learning activities, we created three sets of curriculum materials during summer 2018. The first set of documents were PowerPoint slides and worksheets that introduced students to components of mathematical arguments and shared examples of effective and ineffective arguments. The second was a set of worksheets that facilitated students' small group discussion over Zoom. The third was a self-grading rubric for students to assess the quality of their mathematical arguments. After finalizing the details of the learning activities, the teachers chose the dates to hold the Zoom meetings in advance (October 18, 2018; December 13, 2018; March 21, 2019; and April 30, 2019). The teachers also selected the mathematics problems that would allow students with various grade levels to solve because Emily had a group of fourth- and fifth-grade students.

The math problems were selected from Stanford's research-based mathematics learning resource website called Youcubed.org and resources from the University of Cambridge's NRICH program. The mathematical learning activities on Youcubed.org were created to encourage growth mindsets in mathematics, reduce anxiety related to mathematics, and foster positive mathematical identity (Boaler et al., 2016). Initially, we selected four problems from the website; however, we decided that the problems from Youcubed.org were too demanding for students based on reflections after each Zoom meeting. Therefore, we instead implemented another problem from the NRICH's website based on Emily's suggestion.

In addition to the planning sessions, Katie and I met via Zoom on September 14 and 16 to become familiar with the platform. We needed to ensure that multiple breakout sessions were opened with the preassigned students to small groups, including two from Idaho and one from Alaska. In addition, we invited the technology teacher from Katie's school to determine how to record the small group breakout sessions. I visited Katie's and Heather's school from October 15 to 19, 2018. At the school, I oversaw the implementation of the Zoom meeting so that students were able to complete all four parts of the learning activities discussed above.

**Description of Mathematics Argumentation Learning Activity**

To make the most of this innovative approach to small group learning online, we developed activities based on the aforementioned goals. First, ensuring participation in small group discussions with peers from another school required advanced planning to ensure productive conversations during the Zoom sessions. Second, teaching skills on writing mathematical arguments and exchanging feedback required specific lessons on how to write in mathematics classrooms. Finally, to promote positive mathematical identity, we crafted the language used in worksheets and teachers' instructions. Therefore, we planned the learning activities in advance, especially the mathematics problems and small group discussion guide.

The teachers chose mathematics problems from Youcubed.org and NRICH. Since the teachers had already taught most of the students in the previous school year, based on their knowledge of students, they picked the problems that could accommodate varying levels of mathematics understanding. Below are the problems which were given to students for each meeting (Table 2):

***Table 2: Mathematics Problems Assigned to Students***

| **Meeting Date** | **Problem Assigned to Students** |
|---|---|
| First Meeting Task (October 18, 2018) | School Fair Necklaces: Rob and Jennie were making necklaces to sell at the school fair. They decided to make them mathematically. Each necklace was to have eight beads: four of one color and four of another.<br><br>If there were eight beads with four of each color, what would the different necklaces look like? What if they had nine beads with five of one color and four of another? What if they had ten beads? Can you find all of the different patterns? How do you know there are no other patterns? |
| Second Meeting Task (December 13, 2018) | Leo the Rabbit: Leo the Rabbit is climbing up a flight of 10 steps. Leo can only hop up one or two steps each time he hops. He never hops down, only up.<br><br>How many different ways can Leo hop up the flight of 10 steps? Provide evidence to justify your thinking. |

| | |
|---|---|
| Fourth Meeting Task (April 30, 2019) | Fruity Total: In the 4x4 table below, each fruit has a value between 1 and 15 inclusive. The sum of the values of the fruit in each row and column is shown.<br><br>How do you find the value of each individual fruit? How do you know each value is the correct one? |

Students solved one task during each Zoom meeting and had about 1 hour of independent work time to develop their responses before sharing the mathematical argumentation. During the first independent work time allotment, they answered the following four questions (Table 3):

***Table 3: Questions for Independent Work***

| | **Questions for Independent Work** |
|---|---|
| Question #1 | Identify the problem and make a conjecture.<br>Problem: What is the problem asking? What do you need to know to solve this problem?<br>Conjecture: How do you think you can solve this problem? |
| Question #2 | Develop mathematical claims based on evidence.<br>Claim: What is your response?<br>Evidence: What mathematical work supports your response? You can use pictures, symbols, graphs, or words. |
| Question #3 | Develop a justification. How does your evidence support your claim? How do you know this is true? |
| Question #4 | Review your argument. |

Responses to these questions were shared during the Zoom meetings. Students took turns sharing their responses, and those who were giving feedback answered the following questions (Table 4):

*Table 4: Questions for Feedback*

| | Questions for Feedback |
|---|---|
| Question #1 | Do you agree or disagree with the argument, and why? |
| Question #2 | Did your friend identify the problem? |
| Question #3 | Did your friend develop mathematical claims that are based on mathematical evidence? |
| Question #4 | Did your friend describe the relationship between the claim and evidence? |

After receiving feedback, students could express whether they agreed with the given feedback. Finally, after each Zoom meeting, students were given a second independent work time allotment to write their final draft of the mathematical argument. Students again answered the four questions stated above and improved their arguments. While their written responses provided a wealth of information, this study only focused on their verbal interaction during small group discussions. Exploring how students' mathematical authorial identity emerges from small group talk could lead to a more in-depth study of the relationship between students' mathematical authorial identity and writings in the future. I will discuss additional research topics to be explored like this in the conclusion of this dissertation.

**Small Group Discussion via a Videoconferencing App**

Thirty-four students participated in virtual meetings: 24 were from Idaho and 10 were from Alaska. While students from Alaska were all from the fourth grade, students from Idaho included the fourth and fifth grades. There were 10 small groups of three to four students, and seven groups participated in this study.

We decided to keep students in the same groups for the 2018-2019 school year. To ensure students would feel comfortable talking about mathematics, the teachers hosted a “meet and greet” session on October 9, 2018. Before the first meeting on October 18, 2018, the teachers implemented two lessons on mathematics argumentation based on Toulmin’s (2003) method for argumentation, including claims, data, warrants, rebuttals, and qualifiers. After learning about mathematical argumentation, students had a chance to individually work on mathematical arguments based on the first problem assigned by the teachers.

Students were given the same set of worksheets to facilitate conversation during the virtual meetings, each of which lasted between 24 and 67 minutes. Students then took turns reading the arguments they came up with before the meeting. After students shared their arguments, the other students provided feedback and, if applicable, were asked to convince others to agree with their answers.

Students were expected to write down their peers’ feedback to work on another draft of the argumentation after the virtual meetings. No specific instructions were given about having the same or correct answers. Instead, students were encouraged to think about ways to arrive at the solution they found to be most credible and to support their claims with mathematics-related evidence.

After receiving feedback from the virtual meetings, students worked on the final draft of the mathematics argumentation and were asked to write a paragraph on their answer to the problem and demonstrate the validity of their claims using evidence. Students were expected to reflect on the comments received from peers. They then submitted the final draft and a self-graded rubric for a completion grade. In addition, they answered a three-question survey about (a) three things they learned from the Zoom meeting and writing mathematical arguments; (b) two things they liked about the activity; and (c) one thing they would change about it.

**Data Source and Collection**

Video data for the study were gathered from students' small group discussions via a videoconferencing app. The teachers were aware of my research project but were not given specific questions when they implemented the curriculum. Most of my conversations with the teachers focused on improving the curriculum content, reflecting on students' comments about the activity, and providing more opportunities for students to practice constructing positive mathematical identity.

Before collecting the data, I obtained Boston College Institutional Review Board approval. Next, I met with the administrators from both schools to provide an overview of the dissertation project, the types of data that would be collected, and the potential risks involved. At this time, they were given an option to sign the consent form. I subsequently met with the teachers to discuss the same issues. The teachers explained the research to students and families and sent physical copies of consent forms home. Students' participation in this research was completely voluntary and did not impact their evaluation in class. On the consent forms, I explained that the small group conversations would be recorded via Zoom and that their classwork will be collected and used only for research purposes. Only the recordings of students' conversations were utilized to study students' interaction patterns.

In addition to the small group discussion recordings, I created and utilized transcripts of the recordings during the data analysis phase. In the transcriptions, I only documented information regarding students' verbalization. Video data offer a rich source of information, including "gaze, posture, gesture, and other visual and verbal cues" (Silva et al., 2010, p. 905). However, I only focused on audio data extracted from the videos to deepen my understanding of how mathematical authorial identity emerged from students' talk and choices of interactional features. Had I used an audio recording device to collect students' data, I would have risked not being able to recognize different students' voices, thus leading to potential errors in transcription. In other words, the video recordings helped me identify who was speaking and when. More information on how the transcripts were produced will be detailed in the following section.

For the video recordings, a total of three meetings' small group activities were recorded except for the meeting held on March 21, 2019. For this specific meeting, students were supposed to attend the virtual meetings prepared with a draft of mathematical arguments, but there was a miscommunication between the teachers about this meeting. After the meetings had started, the teachers realized that the students in Alaska solved a different mathematics problem from those in Idaho. There was not enough time to let students solve the same question and hold another virtual meeting again. As a result, the teachers decided to end the third meeting early, and the students did not complete the learning activities.

Overall, 21 video recordings of small group discussions were used for data transcription and analysis (the dates and number of participants in each meeting from each site are listed in Table 5). Among the students who were part of these seven groups, 17 were from Idaho and seven were from Alaska (see Table 6). These students remained in the same small groups throughout the school year.

*Table 5: Total Number of Video Recordings*

| Item | 1st Meeting: 10/18/18 | 2nd Meeting: 12/13/18 | 4th Meeting: 4/30/19 |
|---|---|---|---|
| Number of Groups | 7 | 7 | 7 |

*Table 6: Participant Information*

| Characteristic | | *n* | % |
|---|---|---|---|
| Location | Alaska | 7 | 29 |
| | Idaho | 17 | 71 |
| Gender | Female | 17 | 29 |
| | Male | 7 | 71 |

*Note. n* = 24.

## Applied Conversation Analysis Process

CA involves an investigation of sequential patterns and structures of everyday, ordinary conversations (Sacks, 1984; Sacks et al., 1974; Schegloff, 2007). One general distinction should be highlighted between 'pure' and 'applied' CA due to the nature of my data, which takes place in an institutional school setting. Because this setting may impact what is spoken and how participants speak, one may contend that ensuing conversations are not naturally occurring talk. However, there is also an argument that CA may also be used to study "any kind of talk-in-interaction, whatever its context or purpose" (ten Have, 2007, p. 174). To prevent confusion or questions regarding how CA was applied with my data, I provide a brief review of the difference between pure and applied CA below.

Pure CA was originally developed as a technical approach to studying "systematic organization" (Sacks et al., 1974) of everyday talk. The everyday, ordinary talk in CA terms represents a type of conversation where "participants in conversation generally share equal rights of speakership" (Drew, 1991, p. 22). Talk between friends, families, or neighbors is an example of everyday talk. Another characteristic of pure CA is its analytic focus on identifying and analyzing the sequential organization of talk (Heap, 1990).

In contrast, applied CA provides justification for examining 'institutional talk,' which refers to conversations occurring in social contexts where "there might be quite striking inequalities in the distribution of communicative resources" (Drew, 1991, p. 22). Conversations in schools are an example of 'institutional talk' because in most classrooms students do not have equal access to speaking rights as teachers. Usually, teachers have the right to create speaking opportunities for students by permitting them to speak or having them participate in small group activities. Applied CA has a comparatively broader focus, as this approach focuses on "the structures of phenomena, and especially…the consequences of those structures for realizing ends and objectives regarded as important outside of [CA]'s analytic interests" (Heap, 1990, p. 44). Applied CA still adheres to CA principles, but the main differences are the data type and analytical approach used to explore research questions (Antaki, 2011; Drew, 2005).

In this dissertation, I adopt the applied CA approach based on the data type and aim of the study. First, my data consisted of students' virtual small group conversations, in institutional settings where students and teachers operate under implicit interaction rules. To examine the interactional patterns of students' talk in this setting, the applied CA approach was used. Second, this study draws conclusions about how students' mathematical authorial identity emerges through the linguistic references to various authority structures (Wagner & Herbel-Eisenmann, 2014).

**Transcription**

As an iterative, reflexive, and data-driven process (Lester & O'Reilly, 2019; Ochs, 1979), CA involves two major steps: (a) creating transcriptions that document conversation data; and (b) going back and forth between different stages of analysis to identify and analyze conversation patterns. Transcribing is a significant component of CA analysis. At first, data transcription may seem like a simple task of writing down what is said in conversations. However, CA scholars have argued that transcripts are "a representation of a recording that is shaped by the researcher's theoretical position" (Lester & O'Reilly, 2019, p. 125). Transcribing what is said in video data in a neutral manner is impractical because the transcriber must complete the task and make choices regarding which details to include and exclude in conversations and how to present the data. Therefore, transcription reflects researchers' positionalities and stances (Ochs, 1979).

To begin the analysis, I first watched all 21 recordings of the small group discussions. The videos from each Zoom meeting totaled 314 minutes (October 18, 2018), 199 minutes (December 13, 2018), and 272 minutes (April 30, 2019), respectively. While watching the videos, I created a rough transcript to document what was spoken and who was speaking and excluded students' talk that was not related to the learning activities. However, when talk unrelated to the activities took place in between relevant turns, I documented what students said to understand the interaction flow. Students' talk referring to the learning activities was transcribed whether the talk was about logistics or content (Table 7).

***Table 7: Included and Excluded Turns***

| Examples of Included Turns | Examples of Excluded Turns |
|---|---|
| • The second one's not symmetrical.<br>• The first pattern I did was…<br>• Should I give you feedback now?<br>• It's your turn to give feedback. | • Where do you live?<br>• I live in Alaska.<br>• I can't hear you.<br>• I'll be right back. |

After writing down students' talk, I indicated who the speaker was, made note pauses and silences took place, added time codes, and briefly described what was happening in the videos to create 'a content log' (Goodwin, 1994; ten Have, 2007).

**Clarifying Analytic Focus**

The next process involved a combination of data analysis and transcription. I first transported the rough transcripts to MAXQDA 2020. At this point in the process, there were 21 separate transcripts documenting seven small groups meetings over three separate sessions. By listening to the videos again, I read the transcripts to highlight the most relevant interaction parts (Pomerantz & Fehr, 1997). I also took notes of social actions that emerged from the data by identifying what the speaker was trying to achieve through their talk (Lester & O'Reilly, 2019). Some of the most frequently occurring social actions in the data set were as follows (Table 8):

***Table 8: Emerged Social Actions from the Data***

| Emerged Social Actions | Example Turns |
|---|---|
| Disagreement | • So, I basically wrote that I disagree because I think you have to switch the colors.<br>• I don't think that would be symmetrical. |

| | |
|---|---|
| Agreement | • I agree with you because you have all the mathematical phrases that you need.<br>• I did it like that. |
| Elaboration | • Because if it's nine, it would kind of have to be a different color bead.<br>• It could be more than 79 because the communicative property might give you a lot. |
| Request | • Can I ask you a question about how you have 11 beads?<br>• How did you get a nine? |

After identifying the social actions that emerged across the small group discussions, I made two main observations to narrow my analytic focus on a specific interactional feature. The first observation was that students' talk about mathematical ideas occurred most frequently when they were doing *elaborations*. Second, elaborations typically occurred before or after disagreements. These two observations were valuable because they aligned with my focus on mathematical authorial identity.

Students who practice *author/ity* (Povey & Burton, 1999) have opportunities to express their thoughts about mathematical ideas with personal intentions. When students elaborate on their thought processes about mathematical ideas, they use the interactional feature called *accounts*. I noticed that when students were doing accounts, they talked for a more extended time and about ideas related to mathematics. Based on this initial observation, I decided to further examine the sequence organizations of students doing accounts.

***Identifying Accounts Episodes***

ten Have (2007) suggested detailed transcriptions that include information about *how* interactants speak should only be completed for parts of data that display the interactional feature of interest. Therefore, based on 21 rough transcripts documenting what students said while engaged in the learning activities and who said it, I began identifying episodes of interaction sequences where accounts occurred or where accounts were expected to occur based on discourse markers students used to indicate dispreferred actions.

To identify the accounts episodes, I first located where accounts took place. As mentioned in Chapter 3, accounts are usually produced to mitigate the consequences of dispreferred actions such as disagreements or rejection (Buttny, 1987). Therefore, I located the interactional features signaling dispreference, such as pauses, silence, turn-initial particles including *well* or *uh*, and elaborations (Heritage, 1984; Kendrick & Torreira, 2015; Pomerantz, 1975). Next, I examined the preceding and following turns to see where and how accounts took place.

In addition, conversation structure also informed this process, which proposed that the social action of arguing consists of a three-turn conversation structure (Muntigl & Turnbull, 1998). In this format, Speaker A makes a claim in turn 1, and speaker B responds by disagreeing with the claim in turn 2. Then, speaker A either supports the claim expressed in turn 1 or disputes speaker B's disagreement. By definition, disagreement refers to "the expression of a view that differs from that expressed by another speaker" (Sifanou, 2012, p. 1). This basic conversation structure implies that dispreferred actions such as disagreements require a prior turn. For disagreements to occur, there has to be a claim or statement expressed in prior turns to which interactants can respond (Jenks et al., 2012).

Following this structure, I traced the preceding turns to identify the beginning of an episode, which was identified by “the turn whereby one of the speakers initiated an action or topic and was responded to by the other interlocutors” (Lester & O’Reilly, 2019, p. 156) For the end of sequences, I followed the flow of interaction until “speakers [were] no longer specifically responding to the prior action or topic” (Lester & O’Reilly, 2019, p. 156). I implemented this process for all transcript documents; as a result, I came up with 46 accounts episodes.

***The Jefferson Transcription System***

Finally, I applied Jefferson’s (1984) Transcription System to document the interaction details in the 46 accounts episodes. This method provides a systematic framework to document the “vocal, verbal, and multimodal detail of the interaction” and the “characteristics of speech delivery” (Lester & O’Reilly, 2019, p. 128). In other words, the transcriptions created through the Jeffersonian method include participants’ actions during turns and what takes place in between turns, including whether there is a gap or an overlap (see Table 9). I examined these details to identify patterns across accounts episodes and draw conclusions on how social actions are constructed through interactions.

***Table 9: Transcription Conventions***

| **Convention** | **Meaning** |
|---|---|
| [ text ] | Indicates the start and end points of overlapping speech |
| = | Indicates the break and subsequent continuation of a single interrupted utterance |
| (# of seconds) | A number in parentheses indicates the time, in seconds, of a pause in speech |
| (.) | A brief pause, usually less than 0.2 seconds |
| ::: | Indicates prolongation of an utterance |

| | |
|---|---|
| ( text ) | Speech which is unclear or in doubt in the transcript |
| ? | Raised intonation |
| . | Falling intonation at the end of a turn |
| Underlined | Represents emphasis on the word |
| <word> | Talk is slowed down |
| >word< | Talk is speed up |
| ((comment)) | Transcriber comments |

*Note. From "Transcription Notation," by G. Jefferson, in J. Atkinson and J. Heritage (Eds.), Structures of Social Interaction, 1984, Cambridge University Press.*

**Identifying Patterns in Account Episodes**

After transcribing the 46 accounts episodes based on the Jeffersonian method, I examined the episodes to identify patterns in interactional features (Drew, 2015). For each episode, I searched for recurring social actions, turn-taking processes, and sequence organizations such as adjacency pairs (ten Have, 2007). Then, I reviewed the patterns that emerged across different accounts episodes and refined the categories of interactional patterns representing the characteristics of how accounts worked in my data.

Based on these observations, I cataloged similarities and differences between accounts episodes to find interaction rules that became visible in the episodes. During this process, I also referred to the CA literature to compare my findings with previous research. Finally, I identified excerpts that could be used to demonstrate interactional patterns of accounts that occurred in my data.

**Identifying Authority Structures Invoked in Account Turns**

Wagner and Herbel-Eisenmann's (2014) framework offers linguistic features typically used to invoke four different types of authority in mathematics classrooms. The categories are *personal authority*, *discourse as authority*, *implicit discursive authority*, and *personal latitude* (Table 10).

***Table 10: Linguistic Features of Authority Structure***

| Authority Structure | Linguistic Clues | General Indicators of the Structure |
|---|---|---|
| Personal authority | ● *I* and *you* in the same sentence<br>● Exclusive imperatives<br>● Closed questions<br>● Choral response | Look for other evidence that someone is following the wishes of another for no explicitly given reason |
| Discourse as authority | ● Modal verbs suggesting necessity (e.g., *have to, need to, must*) | Look for other evidence that certain actions must be done where no person/people are identified as demanding this |
| Discursive inevitability | ● Going to | Look for other evidence that people speak as though they know what will happen without giving reasons why they know |
| Personal latitude | ● Open questions<br>● Inclusive imperatives<br>● Verbs that indicate a changed mind (e.g., *was going to, could have*)<br>● Constructions that suggest alternative choice (e.g., *if you want, you might want to*) | Look for other evidence that people are aware they or others are making choices. |

*Note.* Adapted from Wagner and Herbel-Eisenmann (2014), p.879

Considering how this study approaches identity and authority as socially and discursively emerging notions, I decided to use Wagner and Herbel-Eisenmann (2014)'s framework that identified various types of authority structures by specific language clues that speakers use. This framework was applied after identifying account turns within episodes.

The first category is *personal authority*. Within this structure, students "[follow] the wishes of another for no explicitly given reason" (Wagner & Herbel-Eisenmann, 2014, p. 875). Teachers give instructions to students without providing reasons, and students follow the instructions without asking questions. Some of the linguistic features that indicate personal authority are saying both personal pronouns ("I" and "you") to communicate what the person in charge wants the listener to do.

The second category is *discourse as authority*, evidenced by modal verbs such as "have to" or "need to" that indicate an obligation must be followed. Even though students' conversation does not allude to the specific person demanding actions, students talk as if they need to follow specific rules. Within this authority structure, "the sense of who is in real authority is therefore obscure" (Ng et al., 2020, p. 590).

The third category is *discursive inevitability*. This authority structure is similar to the second category but does not directly reference a specific authority source. However, students still talk as if they know what will happen. Within this authority structure, "what matters is not the actual probability of an event but rather the language that suggests inevitability" (Wagner & Herbel-Eisenmann, 2014, p. 873). The phrases typically used to indicate this source of authority are "you are going to" and "it is going to," as they suggest that students do not have the agency to make decisions.

The final category is *personal latitude,* which indicates that students are aware that they are making choices among various methods of understanding and applying mathematical concepts. Linguistic features representing this authority include open questions, conditional statements, and inclusive imperatives (Wagner & Herbel-Eisenmann, 2014).

Even though there were four categories mentioned in this framework, I decided to organize them into external authority and shared/internal authority to simplify the process. The first three authority structures (i.e., personal authority, discourse as authority, and discursive inevitability) were considered as external authority. The linguistic clues that indicate personal latitude were classified as shared/internal authority. The primary purpose of this dissertation study is not to examine what authority structures students invoke but to explore how the notion of mathematical authorial identity is related to account turns. Therefore, identifying broad types of authority structures that emerge from students' classroom discourse was sufficient for answering the research question of this dissertation study.

**Accounting for Patterns Through Writing**

During the last stage of the data analysis process, I elaborated on the findings to provide an "analytically inspired description" of the interaction features used in the accounts episodes (ten Have, 2007, p. 146). I referred to specific excerpts from accounts episodes and elaborated on different social actions that were accomplished through accounts. During this phase of analysis, I started to focus on episodes that capture students' accomplishment of disagreement and their deployment of accounts. Through writing, I was able to identify key interactional patterns that are relevant to the research question and the notion of mathematical authorial identity. In addition, to analyze the content of the account turns, I also referred to Wagner and Herbel-Eisenmann's (2014) authority structure to describe the connections between interactional features used in students' talk and their mathematical authorial identity that emerged.

# Chapter 5: Findings and Discussion

This study aims to explore the concept of mathematical authorial identity from a CA perspective.  It seeks to identify interactional patterns that emerge when students deploy accounts in small group discussions, and to describe the relationship between account turns and mathematical authorial identity. The study's primary research question is "How are accounts implicated in students' signaling and developing mathematical authorial identity?" To answer this question, I explored the following three sub-questions:

(i) How are accounts occasioned?

(ii) What did students achieve when they deployed accounts in small group discussions?

(iii) How are different account types related to the emergence of mathematical authorial identity?

The first sub-question describes the three specific types of first pair parts (FPPs) that occasioned accounts in the data set. The second sub-question focuses on the two primary functions that were accomplished by students' deployment of accounts. The third sub-question suggests three types of account turns that occurred in the data set. For each type of account turns, I describe the interactional environments that became apparent when students deployed the different types of accounts. Finally, I discuss the implications of the three account types and their interactional environments in the discursive emergence of students' mathematical authorial identity, thereby setting up the stage for the ensuing discussion and interpretation that will be advanced in the remainder of the dissertation.

## Research Question #1: How Are Accounts Occasioned?

One of the basic principles of CA is that all talks are organized. Adjacency pair refers to a set of turns that typically occur sequentially. First pair parts (FPP) are turns that “initiate some exchange,” and second pair parts (SPP) are turns that are “responsive to the action of a prior turn” (Schegloff, 2007, p. 13). In the example below, speaker A’s FPP opens the conversation sequence by occasioning the interactional space for speaker B to answer. A relevant SPP in this example is a type-fitted answer, such as “It’s tomorrow at 4 pm.”

***Table 11: Extract 1***

| | | |
|---|---|---|
| 1 | A: | “What time is your appointment?” |
| 2 | B: | “It’s tomorrow at 4 pm.” |

Without the FPP that asks a question, what speaker B says would violate “relevance rules” because turn 2 would not make sense without being preceded by a question. In other words, FPPs create both relevancies and restrictions so that only certain forms of SPPs can follow to ensure the talk’s efficiency (Schegloff, 2007, p. 252). Accounts are also systematically deployed to explain or offer additional information to minimize the impact of dispreferred actions. For accounts to be relevant and meaningful in conversations, “unanticipated or untoward behaviors” (Scott & Lyman, 1968) are usually detected in prior or following turns. Accounts are also typically produced as SPP turns.

Prior CA research on mathematics classroom discourse elaborates on how specific sequences of turns resulted in students providing explanations during whole-class discussions without teachers’ directions (Ingram et al., 2019) and how teachers’ questioning moves afforded the interactional opportunity for students to contribute to the discussion with an enhanced sense of agency (Ng et al., 2020). These studies describe how the sequential organization of turns accomplished interactional phenomena (i.e., explanation, disagreement, questioning). As a result, CA research contributes to a deeper understanding of underlying interactional structures of mathematics classroom discourse. The first sub-question describes the three different interactional contexts that occasioned accounts in this study's data in a similar vein.

Speakers' deployment of accounts is worth investigating, especially in the context of mathematics classroom discourse, because when accounts are deployed, they usually create space in students' conversations to offer additional explanations or describe the reasoning behind their claims. Account turns' potential that could offer interactional space for students to explain their claims warrants a deeper understanding of when and how accounts occur in students' small group discussions.

Students deployed accounts as a response to the following FPP turns that offered the interactional space for accounts to become relevant to the conversation. The three specific FPP turns were: (a) when 'how' questions were asked, (b) when claims for non-understanding were produced, and (c) when speakers expressed disagreement.

### *'How' Questions*

Extract 2 is from Group A's first virtual meeting. There were four students in this group - three from Idaho (Amy, Kayla, Miles) and one from Alaska (Riley). Students were discussing a question that asked them to count how many different symmetrical necklaces can be made with eight beads, nine beads, and ten beads of two different colors.

At the beginning of the virtual meeting, the students took turns sharing their answers. This is what Amy is doing in line 1. After all students shared their responses, it was Kayla's turn to give feedback. Kayla asks, "how did you get twelve rows though?" (line 6). This question prompts Amy to deploy accounts as she describes what she did to get the answer twelve in line 7. So Kayla's 'how' question invited Amy's account to be relevant to the conversation. As a form of an open-ended question, the 'how' questions typically offer recipients the interactional space to express and elaborate their ideas (Blosser, 1973; McNeil & Pimentel, 2010).

***Table 12: Extract 2***

| | | |
|---|---|---|
| 1 | Amy | Okay. So. Um. My cla::im↑ is for number eight, I can make |
| 2 | | twelve patterns? number ni::ne, I can make eight patterns?and |
| 3 | | number ten, I can make ten patterns (.) And then evidence:: |
| 4 | | that I wrote do::wn is::: this? (.) >I don't know if you can |
| 5 | | see it< |
| (18:00) | | |
| 6 | Kayla | **Ho::w↑ did you get twelve rows though?** |
| 7 | Amy | (1) cause (.) um (.) because six times two equals twelve |
| 8 | Kayla | (1) uh (.) you can't do that (.) that way= |

Extract 3 is another example of the 'how' question preceding an account turn. Group E's members were Pablo, Devyn, Isabel from Idaho, and Allison from Alaska. Before this extract, Allison claimed that it is impossible to create a symmetrical necklace with two different colored beads when there are nine beads. After Allison's claim, Pablo disagreed, but the rest did not pay attention. Hence, he did not offer more details.

At the beginning of extract 3, Pablo brings up the topic again in lines 1-3. He expresses disagreement with Allison and provides an account, specifically in lines 2-3.

This account turn describes why Pablo thinks Allison is wrong. Responding to Pablo, Isabel asks the ‘how’ question in line 4. Also notable is Devyn’s turn in line 5 that prompts Pablo to explain what he did to get nine as the answer.

Isabel’s question in line 4 provides the interactional space for Pablo to account for his answer. As a form of an adjacency pair, the ‘how’ question initiates a conversation sequence in which an answer is expected. Therefore, we can see that Isabel’s question made Pablo’s accounts (lines 6-10) relevant to the conversation.

***Table 13: Extract 3***

| | | |
|---|---|---|
| 1 | Pablo | [I kinda] >I actually kinda< think Allison was wro::ng? cause I |
| 2 | | got (.) cause she says you can’t really do ni:::nes? (.) like |
| 3 | | (.) But I:: like (.) I got a few (.) of ni::ne?= |
| 4 | Isabel | **=Wait how did you get a nine?** |
| 5 | Devyn | like (.) You can (.) you can (.) you can explai::n |
| 6 | Pablo | Uh:::m(.) so like (.5) so I like (.) I put four uh::m (.) two |
| 7 | | on one side two on another? and then (.) on the bottom? in the |
| 8 | | middle I put one? and on the other two sides? on the bottom? |
| 9 | | ugh (hhh) I’ll just show you. like (.) This is how I came up |
| 10 | | with it. it's like (3) you see:: (.) |

***Claims of non-understanding***

Students’ claims of non-understanding were another type of FPP turns that afforded the interactional space for accounts to occur. CA considers understanding and non-understanding interactional achievements (Schegloff, 1984). When understanding is not achieved, speakers use claims of non-understanding to indicate that a repair practice is needed to move the conversation forward (Schegloff, 2007). Repair is a “self-righting mechanism” (Schegloff et al., 1977, p. 381) embedded in conversations that addresses and resolves the trouble sources, such as factors that hinder understanding or hearing. In classroom discourses, students often indicate non-understanding by saying, "I don't understand," or requesting clarification (Aldrup, 2019; Somuncu & Sert, 2019). When speakers initiate repair, accounts are one of the strategies for resolving the trouble.

Extract 4 illustrates two occasions of Amy's requests for clarification preceding Riley's accounts. The first occasion begins from lines 1-2 when Riley disagrees with Amy's answer. Then, as a response, Amy expresses non-understanding (line 3) by repeating what Riley said in the previous turn. Additionally, using rising intonation signals that Amy does not understand Riley's claim and needs more information to achieve mutual understanding (Cogo & Pitzl, 2016). Amy's turn in line 3 makes Riley's accounts in lines 4-5 relevant to the conversation.

The second occasion begins from lines 14-15. Amy deploys another, more explicit claim of non-understanding. She specifically mentions that she does not understand Amy's explanation this time. Again, Riley's accounts (lines 16-21) sound relevant in this interactional context because her turns respond to Amy's repair needs (lines 14-15). As demonstrated in these two examples, students occasion the interactional space for accounts by using claims of non-understanding (Goodwin, 2007).

***Table 14: Extract 4***

| | | |
|---|---|---|
| 1 | Riley | um (1) so (.) okay (.) I disagree with th(.)er (.) your (.) |
| 2 | | argument because your justification↑ doesn't stand alone↑ (5) |
| 3 | Amy | **doesn't stand alone?** |
| 4 | Riley | (3) yeah so (.) it doesn't stand alone↑ without your |
| 5 | | evidence↑ (1) |
| ((lines omitted where Amy was writing down Reina's comment)) | | |
| 6 | Amy | (19:40) oka::;y (.) uhm (.) is that all of your feedback? |

Riley one second (.) I have a little more (5) ((writing)) also uhm
(.) u::se (1) some mathematical ideas↑ in your justification↑
(2) like (.) that actually support your evidence↑ (1) like
say how you use them↑ (1)
Amy Okay (.) what [okay]
Riley [I can] repeat that (.) do you want me to
repea::t that?
Amy **uh::m (.) but the (.) main thing is that I do::n't understand**
**much about what it means by mathe(.) mathematical idea:::s↑**
Riley yeah (.) I think it mean (.) like (.) addition (.) and like
(.) subtraction↑ the (.) um (.) like use the ones like (.)
say in your justification↑ what like (.) like (.) what
strategies↑ you used↑ like (.) um (.) did you use
subtra::ction or addition↑ or multiplication↑ like say (.)
kind of some of those things (.) you know?

### *Disagreement*

The third type of FPP turns that occasioned accounts was disagreement statements. Accounts are closely related to preference organization and are considered a "way of doing dispreference" (Buttny, 1993, p. 44). Dispreferred actions like disagreement or rejection are usually accompanied by 'dispreferred markers,' such as elaborations or delays that mitigate the threat to hearer's face (Jackson & Jacobs, 1980; Heritage, 1984; Pomerantz, 1984). Doing disagreement in a classroom setting is complicated because speakers have to manage the content knowledge and simultaneously manage the social interactions (Muntigl & Turnbull, 1998). Accounts usually serve as an interactional resource supporting speakers' navigation of disagreement talk, which students demonstrated in this study's data.

In extract 5, Kathy's (lines 2-3) and Sierra's (lines 4-6) turn sequence resonates with what Muntigl and Turnbull (1998) identified as a conversation pattern in arguments. They found that when claims are deployed as an FPP, it is common for speakers to express contradiction as an SPP. In this context, contradiction is defined as a "negated proposition expressed" to the previous claim (p. 231).

In line 2, Kathy claims that nine beads in two different colors can make a symmetrical necklace. Then, in line 4, Sierra indicates contradiction to Kathy's claim and complements the disagreement by saying, "because it ends with blue and then it starts back up with yellow." Sierra is doing a dispreferred action of threatening Kathy's face by disagreeing with her. To mitigate the impact of face threat, Sierra deployed an account turn that explains why she thinks Kathy's answer is incorrect.

***Table 15: Extract 5***

| | | |
|---|---|---|
| 1 | Sierra | (8) Wa::::it |
| 2 | Kathy | (5) >Basically< it's right here (3) this is the four |
| 3 | | necklaces (.) and then it splits in the middle = |
| 4 | Sierra | **=I don't think that one is symmetrical?** because um |
| 5 | | (2) it ends with blue? and then it starts back up |
| 6 | | with yellow = |

So far, I have described three distinct FPPs that occasioned accounts in students' small group discussions as they shared stances on solving a mathematics problem. The three interactional contexts that resulted in students' deployment of accounts were: when students (a) asked 'how' questions, (b) claimed non-understanding, and (c) expressed disagreement. In these cases, students oriented themselves to the patterns of adjacency pair, facework, and preference organization that made accounts relevant. These extracts demonstrate that accounts do not occur randomly. Accounts usually occur as an SPP turns as they function as an interactional strategy to mitigate face-threats and suggest solutions to problems (Waring, 2007). In other words, for accounts to be relevant to the conversation, there should be certain FPPs that occasion the interactional space to make accounts relevant as the following turn.

In the context of mathematics classroom discourse, students may deploy account turns to explain their stances or provide additional information to validate their suggestions. Students' opportunities to leverage the interactional space through account turns are deeply related to how students co-construct authorship, and eventually construct mathematical authorial identity. The conceptual dimension of authorship, such as making meaning through social interactions, cultivating a sense of agency and ownership, and broadening repertoires to become part of a community of practice, requires that individuals make their voices heard in social interactions. For students to make their voices heard, they should be able to leverage the interactional space.

The extracts shared above illustrate that for students to leverage the interactional space and potential benefits that account turns could offer, certain FPP turns should be deployed first to make account turns relevant to the conversation. Understanding specific interactional patterns that result in the occasion of accounts would illuminate how educators can create interactional contexts that encourage students' deployment of accounts. So far, I discussed how account turns are occasioned, the next sub-question explores what account turns achieved in students' small group discussions.

## Research Question #2: What Did Students Achieve When They Deployed Accounts in Small Group Discussions?

There were primarily two distinct cases where accounts were part of the action trajectory that students accomplished. The first case was students elaborating on their opinions or stances through the deployment of accounts. The second case was students invoking various authority structures within the account turns they deploy.

***Accounts as a Site of Elaborating on Students' Opinions or Stances***

When students deployed accounts, they leveraged the interactional space to elaborate on their opinions about mathematical concepts and construct a sense of authorship. The bolded texts in extracts 6 and 7 serve as examples of this argument. In these extracts, when students engaged in the phase of disagreement in which they were trying to convince each other, accounts offered the "discourse materials of change and for change" (Firth, 1995, p. 221). The interactional space created by accounts became a site where students explained their reasoning behind their stances. The act of explanation constituted "discourse materials" that other interactants could respond to.

In extract 6, Allison and Pablo leveraged the interactional space created by the deployment of accounts. Through the account turns that became relevant to the conversation, both students expressed their stances on applying the concept of 'symmetry' when designing a necklace with nine beads of two different colors. In lines 3-4, Allison explains that her understanding of symmetry does not include changing the shape of the beads by splitting them in half. Pablo responds in lines 5-8 by referring to the example he created, and he elaborates that the nine beads can be divided into four and five beads on each side.

Allison and Pablo engage in the meaning-making process and exercise a sense of authorship in this extract. Both heard the word ‘symmetry’ when their teachers explained the problem the students had to solve in order to participate in this learning activity. The word ‘symmetrical’ is populated with Allison and Pablo’s personal intentions here because both students interpret what ‘symmetry’ means. The account also captures the process of Allison and Pablo becoming part of the mathematics community of practice. Instead of using a description like “same on both sides,” Allison and Pablo used the word ‘symmetry,’ which is part of the language repertoire that a community of mathematics learners would use.

This extract demonstrates a conversation in which accounts were relevant. The students took advantage of the interactional space made available by accounts. Within that space, the students were able to actively and agentively participate in the meaning-making process and broaden their interactive repertoire related to the concept of ‘symmetry.’

***Table 16: Extract 6***

| | | |
|---|---|---|
| 3 | Allison | So I think I can't do the nine? **becau:::se if you split em** |
| 4 | | **in ha::lf like(.) it won't be symmetrical.** |
| 5 | Pablo | Oh I found the one that can be symmetrical. **It's like this** |
| 6 | | **(.2) uh::m (.) It's like one, two, three, four, fi::ve, and** |
| 7 | | **then you have four right there (.) it's symmetrical on** |
| 8 | | **ea::ch side=** |

Similarly, in extract 7, Amy’s question in lines 33-34 occasioned Riley’s deployment of accounts. The interactional space became available where Riley further explained her understanding of using mathematical ideas to support justification. Riley’s turns (lines 35-40) that follow Amy's question indicate Riley's understanding of 'using mathematical ideas' and include her description of mathematical operations such as addition, subtraction, or multiplication. Furthermore, her understanding of 'using mathematical ideas' constituted of saying 'those things,' which refers to a general category of mathematics-related terms. This extract exhibits how Riley leveraged the interactional space and contributed to the meaning-making process about 'using mathematical ideas' when justifying her stances.

***Table 17: Extract 7***

| | | |
|---|---|---|
| 33 | Amy | uh::m (.) but the (.) main thing is that I do::n't understand |
| 34 | | much about what it means by mathe(.) mathematical idea:::s? |
| 35 | Riley | yeah (.) I think it mean (.) **like (.) addition (.) and like** |
| 36 | | **(.) subtraction? the (.) um (.) like use the ones like (.)** |
| 37 | | **say in your justification? what like (.) like (.) what** |
| 38 | | **strategies? you used? like (.) um (.) did you use** |
| 39 | | **subtra::ction or addition? or multiplication? like say (.)** |
| 40 | | **kind of some of those things (.) you know?** |

Both extracts shared above provide an example of how students leveraged the interactional space occasioned by account deployment. In extract 6, accounts became relevant when both students were accomplishing the act of disagreement, and in extract 7, Amy's claim of non-understanding prompted Riley to deploy accounts. In other words, how the turns were sequenced resulted in accounts being relevant. Consequently, when accounts became relevant, the students were able to leverage the interactional space and elaborate on their opinions and stances. This overall discursive process is related to how students construct a sense of authorship and mathematical authorial identity by participating in a meaning-making process and agentively using the language that a community of practice utilizes. When students talk about their stances with vocabularies they learned in classes, they are reproducing the meaning of the words that are laden with their personal interpretations. As students experience this process, their authorship is being co-constructed through social interactions.

How students could leverage the account turns' interactional space and elaborate their opinions becomes more explicit when discussing deviant cases. Deviant cases refer to "the ways in which the participants, through their actions, orient to these departures" (Heritage, 1988, p. 131). Heritage (1988) argues that discussing both certain interactional patterns and deviant cases contributes to "showing that a particular normative organization is operative in interaction (that is, underlying both the production of and reasoning about a particular social action or sequence of actions)" (p. 131). The deviant cases illuminate how the interactional patterns are actually practiced. The extracts that are shared below represent the interactional sequences where account turns could have been relevant based on the definition of accounts; however, students focused on other interactional work that did not make accounts relevant to the conversation.

Extract 8 exhibits Kayla and Riley accomplishing the action of 'doing disagreement' by indicating opposing opinions (Muntigl & Turnbull, 1998; Hüttner, 2014). Because disagreements can be "socially disruptive" (Geogakopoulou & Patrona, 2000, p. 323) and dispreferred, disagreements are usually marked with discourse markers that mitigate the impact of damaging interactants' faces (Pomerantz, 1984). It has been discovered that accounts are typically deployed in a situation where interactants accomplish dispreferred actions. However, there are also deviant cases where disagreements occur without accounts. Accounts may not be relevant to the conversation when both speakers are oriented to further disagreements or engage in aggravated disagreement (Goodwin, 1983; Kotthoff, 1993).

Extract 8 is an example of Kayla and Riley engaging in disagreement without account turns. This episode demonstrates how Kayla and Riley oriented themselves to disagreement through group norms. Before Kayla and Riley participated in the virtual small group discussion, their teachers instructed that students could engage in disagreements. Students were instructed to argue and justify when their peers disagreed with them. As a result, group norms were established when the teachers authorized students expressing disagreement, group norms oriented to disagreement were established. These interactional conditions diminished Kayla and Riley's needs to deploy accounts to manage the face-threatening act of disagreement. In other words, the students did not have to orient themselves to facework and preference organization principle that usually results in the deployment of accounts that mitigate the consequences of disagreement. The impact of a face-threatening act was already mitigated by their teachers' authorization of disagreements.

The implication of this type of disagreement without account turns is that there was no interactional space available for Kayla to elaborate on her stances. The students oriented themselves to other interactional features that did not make accounts relevant to the conversation and resulted in an interactional sequence without the interactional space to explain or provide additional information.

***Table 18: Extract 8***

| | | |
|---|---|---|
| 5 | Kayla | **(2) The second one::s not symmetrica::l?=** |
| 6 | Riley | =This one? (.) Or that one? = |

| | | |
|---|---|---|
| 7 | Kayla | **= That one. So:: the two >that are< (.) um (.) [so on]** |

Extract 9 is an example of students' interaction that accomplished aggravated disagreement and accounts did not become relevant. At the moment, students were occupied with expressing disagreement and saving their faces by trying to occupy the floor to make their claims heard. Kathy and Marlee were not interested in reaching an agreement (Kotthoff, 1993) and did not deploy turns that could have made accounts relevant to the conversation. Because account turns typically occur as an SPP or a response to a prior turn, it would have been difficult for students to leverage the interactional space without certain types of FPP that could prompt accounts. As a result, there was no interactional space for Kathy and Marlee to elaborate on their stances about how to solve the problem.

***Table 19: Extract 9***

| | | |
|---|---|---|
| 7 | Kathy | **I still [disagree].** |
| 8 | Marlee | **[you can't]=** |
| 9 | Kathy | **=No matter what (.) I still disagree (hhh)** |

Overall, it was observed that when account turns were relevant in the interactional sequences, the students were provided with the opportunity to leverage the interactional space, elaborate on their opinions or stance about mathematical concepts, and construct a sense of authorship through the process.

I am not arguing that there is a positive correlation between students' deployment of accounts and their construction of authorship. Not all account turns warrant the interactional space for students to elaborate on their opinions or stances, and not all students leverage the interactional space provided by account deployment. Nonetheless, the finding from this sub-question demonstrates that when account turns were relevant to the conversation, students did have the interactional opportunity to explain their opinions or stances, and by taking advantage of the opportunity, they were able to co-construct authorship.

### *Accounts as a Site of Invoking Authority Structure*

Another function of accounts observed in the data was apparent when students invoked various authority structures within the account turns. During mathematics classroom discourse, students may invoke various authority figures and structures, including teachers, textbooks, peers, or students' previous experiences (Wagner & Herbel-Eisenmann, 2014). These authority figures and structures are co-constructed discursively and emerge from interactional patterns and classroom interactions (Amit & Fried, 2005; Gerson & Bateman, 2010; Langer-Osuna, 2016; Ng et al., 2020; Pace & Hemmings, 2007; Wagner & Herbel-Eisenmann, 2014).

An interesting observation from this study, which aligned with previous research on the discursive nature of authority (Wagner & Herbel-Eisenmann, 2014), was that students' account turns were used to invoke authority structures. The different types of authority structures that emerged from the data can be divided into external and shared/internal authority.

For example, Extract 10 illustrates the interaction of students who referred to *discursive inevitability* and *personal authority*, thereby representing a form of external authority (Povey & Burton, 1999). Sierra's turn in line 1 is an example of *discursive inevitability* when she says, "nine is an odd number. You can't split up evenly." She spoke as if she knew that it would be impossible to split beads to make a symmetrical necklace with nine beads. However, she did not provide examples or give evidence of any reasoning that supported her claim. Another example of external authority comes from Kathy's turn in line 3. She refers to what "Mrs. Fisher said" to bolster her argument.

***Table 20: Extract 10***

| | | |
|---|---|---|
| 1 | Sierra | (2) Nine is an odd number. You can't split up evenly = |
| 2 | Marlee | =Yeah like you can't split up [eleven evenly] |
| 3 | Kathy | **[Mrs. Fisher said] that if we belie:::ve that you can split nine up, then you can say (.) say::: that (1) [But if you don't believe] it, it's fi::ne.** |

It can be implied that both Sierra and Kathy's account turns that invoked external authority indicate their diminished sense of agency in deciding how to apply mathematical concepts to solve this problem. The linguistic features used in Sierra and Kathy's account turns, such as "can't," and "Mrs. Fisher said," signal that the students rely on the rules established by external authority figures. Those rules would limit their ability to decide how to solve this problem. Another observation is that Kathy's reasoning for her disagreement derives from the external authority structure. There was no additional explanation of the disagreement other than the fact that her teacher said so. As such, these linguistic features represent students experiencing mathematical authority "as external to the self and located in experts" (Depaepe et al., 2012, p. 224). Furthermore, students who experience external authority often consider mathematical meaning "as given…and knowledge is assumed to be fixed and absolute” (Povey & Burton, 1999, p. 234).

Another example of accounts being a site of invoking authority structure is captured in extract 11. This interaction is different from extract 10 because it depicts an example of shared/internal authority. Wagner and Herbel-Eisenmann (2014) suggested that *personal latitude* is signaled by linguistic clues such as ‘could’ or ‘would’ and indicate that students have a sense of agency to make decisions on applying mathematical concepts to solve problems proactively.

***Table 21: Extract 11***

| | | |
|---|---|---|
| 2 | Morgan | = **But you could go like (.) >you could have< (.) like blue,** |
| 3 | | **purple, blue, purple(.)and then blue in the middle and then** |
| 4 | | **purple, blue, purple, blue, on the other side and then if** |
| 5 | | **you split that in half, it would be the same on both sides.** |
| | | |
| 14 | Morgan | Well(.) you wouldn't have to chop it in half. It'd be |
| 15 | | (inaudible). **>If it had nine beads< See? cause (.)** |
| 16 | | **If that was a purple and that was a blue, then** |
| 17 | | **you'd split that in ha::lf. (.)** Wait a second. |

Morgan’s turns in lines 2-5 and 14-17 illustrated *personal latitude* through her use of linguistic features such as “could” and “if.” These linguistic features demonstrate an open and flexible approach to mathematical problem-solving less restricted by an external authority. Wagner and Herbel-Eisenmann (2014) emphasized that students invoking *personal latitude* would indicate that what they are suggesting is one of the choices to solve a problem. Students would not indicate that they are obligated to follow certain rules to find an answer. Instead, they would be practicing a sense of agency to experiment with different approaches to apply mathematical concepts. Therefore, students would use linguistic clues that signal their openness to consider various possibilities of understanding and applying mathematical concepts.

Morgan first suggests that their peers ‘could’ arrange the beads in a certain way to create a necklace that has a symmetrical pattern (lines 2-5). She does not indicate a sense of obligation or that she is following immutable rules of mathematics in her account turns. Also, in lines 14-17, Morgan uses the word ‘if’ to suggest that their peers consider a conditional situation to think about whether splitting a bead in half would result in a symmetrical necklace. She does not use linguistic features like ‘have to’ or ‘must’ that could indicate the influence of external authority. She relies on *personal latitude*, which can be described as an example of shared/internal authority. Personal latitude indicates that the speaker can decide how to understand and apply mathematical concepts. This notion is closely related to shared and internal authority because personal latitude represents a sense of agency. Students’ expression of their opinions is considered legitimate ideas that have the authority to create meaning. Overall, Morgan leveraged the interactional space via account turns to signal how she understood the concept of 'symmetry' and applied it to solve the problem.

The second sub-research question of this study explored the account turns’ two primary functions. Findings from this question suggest the potential of account turns. When accounts became relevant to the conversation, the interactional space for students to construct a sense of authorship and invoke authority structures became available. First, the account turns occasioned the interactional space where students elaborated on their opinions or stances. Students’ sense of authorship was constructed in this interactional space as they participated in the meaning-making process, practiced a sense of ownership, and broadened their repertoire. Second, students leveraged the account turns to invoke different authority structures. Both categories – external and shared/internal authority – were represented in the data.

## Research Question #3: How Are Different Account Types Related to the Emergence of Mathematical Authorial Identity?

The previous two sub-questions focused on how accounts were occasioned and what they achieved when deployed. For the third sub-question, I provide an overview of three account types: (i) missing accounts, (ii) account turns where students invoked external authority, and (iii) account turns where students invoked shared/internal authority. For each account type, I describe the interaction environment when these account types were deployed. Finally, I conclude the section by discussing the relationship between the three different account types, their interactional sequences, and the discursive emergence of mathematical authorial identity.

### *Types of Accounts*

There were three types of accounts observed from the data (see Table 22). The different types were determined by whether accounts were relevant to the conversation and the authority structures invoked within the turns.

***Table 22: Different Types of Accounts***

| | Type A: Missing Accounts | Type B: Accounts that included external authority | Type C: Accounts that included shared/internal authority |
|---|---|---|---|

| Account Relevance | Accounts were **not** relevant to the conversation | Accounts were relevant to the conversation | Accounts were relevant to the conversation |
|---|---|---|---|
| **Invoked Authority Structures** | N/A | External authority (e.g., relying on teachers or peer's personal authority, discourse as authority, discursive inevitability) | Shared/Internal authority (e.g., indicating possibility of making individual decisions when solving mathematical problems) |
| **Interactional Environment** | • Disagreement as a response to an invitation<br>• Disagreement with a claim that had been already disputed<br>• Aggravated disagreement | • Towards the end of a sequence<br>• After Type C account turns have occurred<br>• Asserting speaker's opinion | • Beginning of a new sequence<br>• Building consensus |

**Type A: Missing Accounts.** The first type of account turns and the interactional sequence I discuss is when accounts did not become relevant to the conversation. There were episodes where accounts would have occurred based on the facework and preference organization but were missing. The deviant cases are important Those episodes occurred as students accomplished disagreement. More specifically, the students were (i) doing disagreement as a response to an invitation, (ii) doing disagreement with a claim that had been disputed already, and (iii) doing an aggravated disagreement. This section discusses how turns were organized for 'Type A' sequences.

***Disagreeing as a Response to an Invitation.*** Part of extract 8 was shared earlier when I described the function of accounts as a site of students elaborating on their opinions or stance. The rest of extract 8 begins with Riley downgrading her claims ("they might not all be symmetrical"). This sentence prefaces Riley's invitation for Kayla to review the answer. Riley uses the modal verb, “you can,” and implies that Kayla can implement the action of reviewing (Hill, 2011). Riley mitigates the face-threatening act of requesting by deploying an imprecision bundle “or something” (Conrad & Biber, 2004).

In line 3, Kayla readily accepts Riley’s request and evaluates the answer. Kayla then expresses her disagreement in line 5. She disagrees by asserting that Riley’s answer is “untrue” (Rees-Miller, 2000, p. 1554). However, Kayla does not provide an account turn to mitigate her dispreferred action. After Kayla’s disagreement, Riley immediately requests clarification on which answer Kayla disagrees with but does not ask why Kayla disagrees. In lines 7, 9, 10, and 11, Kayla explicitly disagrees with Riley’s answer. However, Kayla does not explain why she disagrees even in her follow-up answer.

As mentioned earlier, both students’ understanding of group norms could be a contributing factor to the absence of accounts. In addition, unmitigated disagreements may indicate a sense of intimacy between close friends or family (Georgakopoulou, 2001); however, Riley is from Alaska, while Kayla is from Idaho. So, this extract shows their second meeting via a video-conferencing application after the initial introductory virtual meeting. Another argument is that shared classroom discourse norms could result in students deploying unmitigated disagreements (Hellermann, 2009). If students share the understanding that they are expected to perform disagreements as a community of practice (Lave & Wenger, 1991), they may not feel the need to mark disagreements.

***Table 23: Extract 8***

| | | |
|---|---|---|
| 1 | Riley | But they might >not all be symmetrical<? (.) So you can look |
| 2 | | at it again? And see (.) check it or something? = |

| | |
|---|---|
| Kayla | = Okay. Hold on. |
| Riley | (2) I can just >hold it up< |
| Kayla | **(2) The second one::s not symmetrica::l?=** |
| Riley | =This one? (.) Or that one? = |
| Kayla | **= That one. So:: the two >that are< (.) um (.) [so on]** |
| Riley | [That] one? |
| Kayla | **the left si::de. The two on the bottom (.) between number** |
| | **one?(.) number one? I mean >number two and number three< are** |
| | **not symmetrical.** |

***Disagreeing with an Already Disputed Claim.*** The second interactional context where students accomplished disagreement without deploying accounts was when the disagreement turn occurred after another student had already expressed disagreement.

Extract 12 is an example of Kate and Heath disagreeing with Kiersten's claim. Kiersten shares her claim by describing the pattern of the symmetrical necklace she designed (lines 1-4). Kate acknowledges that she heard Kiersten's answer and then expresses disagreement through a negative evaluation (line 5). She mitigates her action by using the lexical bundle, "I don't think," to indicate her uncertain epistemic stance (Biber et al., 2004, p. 389).

Immediately after Kate's disagreement, Heath also indicates disagreement (line 6). Heath's disagreement is more straightforward than Kate's because it does not have discourse markers that usually indicate dispreferred actions. In line 7, Heath provides an account for why they should move onto the next part of the activity when he says, "she's (referring to Kiersten) trying to figure out which one." However, an account for disagreement is missing from both Kate and Heath.

This interactional pattern seems similar to the context that Ingram et al. (2019) described. In the study, students deployed explanations without teachers' solicitation when another student had already spoken the answer in classroom discussions. Similarly, Heath did not hesitate to align with Kate in disagreeing with Kiersten. Heath's turn in line 7 is an example of him agreeing with Kate and indicating a preferred action in response to Kate's claim in lines 5-6. Heath's orientation to preference organization could explain why his turn is simple and straightforward. Within this specific interactional pattern, Kate and Heath did not have access to the interactional space to deploy accounts to elaborate on why they disagreed with Kiersten within this specific interactional pattern. Furthermore, in the latter part of his turn, Heath prioritizes his obligation as a student to complete the class activity instead of explaining to Kiersten why her answer is not symmetrical. As a result, the conversation sequence moves on without providing the affordances for students to elaborate on their stances.

***Table 24: Extract 12***

| | | |
|---|---|---|
| 1 | Kiersten | For the thi::rd one? I did (.) I did green, green, yello |
| 2 | | (.) no (.) |
| 3 | | green, blue (.) green, >blue, blue, green< green. >Oh wait |
| 4 | | no< green, blue, green, blue, green, blue, green= |
| 5 | Kate | =okay (.) That one? **Actually >I don't think it's** |
| 6 | | **symmetrical< the last one you said=** |
| 7 | Heath | **=Yeah it's not.** She (.) She's trying to >figure out which |
| 8 | | one< but we'll jus(.) can we just move [on? |
| 9 | Kiersten | [(inaudible) that one? |

***Aggravated Disagreement.*** Sierra and Marlee disagree with Kathy in this extract. The students have disagreed since the virtual meeting began, and there was no sign of one side being persuaded. Their conversations lasted 38 minutes and 30 seconds, and these turns took place towards the end of the activity between 29:43 and 32:05.

Sierra, in line 1, suggests they decide on a constraint that they "can't cut a bead in half" to make a symmetrical necklace with nine beads in two different colors. However, in line 7, Kathy emphasizes saying "still" to indicate that she is not going to consider the constraint proposed by Sierra. Then, she expresses disagreement without stating the reason. Part of Kathy's turn overlaps with Marlee's in line 8 since Marlee begins talking when Kathy says, "I still." They are competing for the floor to promote their agenda and not let the other side speak (Scott, 2002).

***Table 25: Extract 13***

| | | |
|---|---|---|
| 1 | Sierra | Mkay how about we just can't >cut a bead in half<? |
| 2 | | be[cau::se] |
| 3 | Marlee | [Yes(.)] yes. |
| 4 | | That's very smart (.) Yea:::h, you can't cut a bead in |
| 5 | | ha::lf. |
| 6 | Sierra | Well [(inaudible)] |
| 7 | Kathy | **I still [disagree].** |
| 8 | Marlee | **[you can't]=** |
| 9 | Kathy | **=No matter what (.) I still disagree (hhh)** |
| 10 | | [Ohmygod] |
| 11 | Sierra | [Ho:::w] do you disagree:: (2) because (.) cause there's |
| 12 | | little (.) there's (.) there's problems. |
| 13 | Marlee | (3) Kathy (.) I hat:::e having to be mea::n (.) but you |
| 14 | | (.) you (.) **you just can't. Oka:::y? You just can't.** |

| | | |
|---|---|---|
| 15 | | ((Turns omitted where students talk about making a necklace for a friend's birthday party)) |
| 16 | Sierra | Okayokay. Let's (.) Let's stop talking about this for now = |
| 17 | Marlee | =Okay(.)okay(.) That's a very good idea, Sierra. |

These turns indicate that students' claims are beginning to move towards polarity and an aggravated disagreement (Goodwin, 1983). Kotthoff (1993) argued that the turns become more straightforward and terse once disagreement begins. In line 9, Kathy intensifies her disagreement by saying, "no matter what," she will maintain her position.

As the next turn, Sierra interjects and indicates her disagreement by asking a question (lines 11-12). Sierra slightly softens the aggravation by beginning her turn with "how." Kathy could have taken the opportunity to respond to Sierra. However, in line 13, Marlee pauses for three seconds and prefaces her disagreement when she says, "I hate having to be mean." Later in that turn, she returns to the aggravated form of disagreement by saying, "you just can't." Accounts are still missing from this turn.

Goodwin (1983) observed that aggravated disagreements lack mitigations and accounts. Especially when children accomplish aggravated disagreements, they do not usually adhere to the typical interactional patterns of preference organization and often do not seek consensus. In other words, they conclude the talk by agreeing to disagree. A similar pattern is observed when Sierra suggests that the group "stop talking about this for now" in line 16, and Marlee agrees with Sierra in line 17.

**Types B and C.** I discuss types B and C together in one section because they often occur within the same interactional sequence. Therefore, I will be using one extract to highlight the differences between the two types. Extract 6 was chosen because it best represented account types B and C's characteristics. Parts of extract 6 were shared in the previous section, and for the third research question, I am sharing the entire episode.

Allison, Devyn, Pablo, and Isabel were accomplishing the action of disagreement. Their interactional accomplishment can be divided into four parts: (i) statement, (ii) counterstatement, (iii) counterstatement to (ii), and (iv) aggravated disagreement (Antaki, 1994; Goodwin, 1983; Gruber, 1998; Muntigl & Turnbull, 1998). For each part, I highlight the account types and describe the interactional environment for each type of accounts.

***Part I: Statement (Lines 1- 4).*** Before line 1 of the extract, students in Group E took turns sharing their initial draft of mathematical arguments. At that time, Isabel first shared that she could only arrange a symmetrical necklace with eight beads of two different colors (02:56), and Devyn and Pablo shared that they figured out how to arrange a symmetrical necklace with eight, nine, ten, eleven, and twelve beads (04:10). Isabel requested that Devyn and Pablo examine her worksheet to see if her answer to the eight beads was correct.

During a brief pause in conversation as Isabel was gathering her paper, Allison launches a new sequence (line 1) with a discourse marker, "oh," and a pre-announcement by asking a question, "did I share my nine?" (Sacks et al., 1974; Schegloff, 2007). Allison is announcing that she is about to share her thoughts about the mathematical concepts around using nine beads of two colors to make a symmetrical necklace.

Then in line 3, Allison claims that it is impossible to make a symmetrical necklace with nine beads. She deploys an account (line 3). She says, "because if you split them in half, it won't be symmetrical." Her account serves as an interactional strategy that mitigates the potential face threat that her statement might cause. In her account, Allison also uses a conditional statement," if you split them in half," which weakens Allison's disagreement with Devyn and Pablo's answers (Goodwin, 1983).

The account turns deployed by Allison is ***Type C***, which invoked shared/internal authority. It is worth noting that Allison initiated the sequence and deployed accounts in line 3 by introducing a conditional statement using the word "if." Her linguistic choice indicates that she considers her opinion one of the possible answers to the problem. She did not invoke external authority as the reasoning of the claim. Instead, Allison used her understanding of what 'symmetry' means and referred to a scenario in which the beads were split in half as her justification.

***Table 26: Extract 6***

| | | |
|---|---|---|
| 1 | Allison | Oh (.) did I share my ni::ne? |
| 2 | Devyn | Yea= |
| 3 | Allison | So I think I can't do the nine? **becau:::se if you split em** |
| 4 | | **in ha::lf like(.) it won't be symmetrical.** |
| 5 | Pablo | Oh I found the one that can be symmetrical. **It's like this** |
| 6 | | **(.2) uh::m (.) It's like one, two, three, four, fi::ve, and** |
| 7 | | **then you have four right there (.) it's symmetrical on** |
| 8 | | **ea::ch side=** |
| 9 | Allison | =if= |
| 10 | Pablo | **=like right on the nines thou::gh (.) if you had a fi::ve(.)** |
| 11 | | **you can just put it on the bottom and then [like]** |
| 12 | Isabel | [if] |
| 13 | Pablo | **if you have it on the si::de=** |
| 14 | Devyn | =Yeah >that's what I meant< |
| 15 | Isabel | **(.) If you split ni::ne? if you split one? (.5) like If (.)** |
| 16 | | **you're doing ni::ne? uhm (.) you have to split one? to make** |

| | | |
|---|---|---|
| 17 | | **symmetrical** but that's not really symmetrical?= |
| 18 | Pablo | =well= |
| 19 | Allison | =That's what I said. |
| 20 | Isabel | You can't really >split beads in half< **cause you are making** |
| 21 | | **necklaces.** Unless (.) yeah (.) you just can't. So:::= |
| 22 | Pablo | =But you can do nine. |
| 23 | Allison | (.) So you did eight? ni::ne? Did you do ten?= |
| 24 | Allison<br>Devyn | =[Yea] |

***Part II: Counterstatement (Lines 5-13)***

Allison's deployment of the ***Type C*** account turns occasioned Pablo's counterstatement in line 5. He deploys a counterstatement that he "found the one that can be symmetrical" and asserts that it is possible to arrange nine beads to make a necklace with a symmetrical pattern. This assertion contradicts what Allison said in line 3. Pablo's counterstatement expressed through his account turns can be described as a ***Type C*** account. Pablo invoked shared/internal authority within this account turn and managed facework.

Pablo talks about a potential scenario of having five beads and argues that he can make a symmetrical necklace by placing five beads on one side. Using the conditional statement in lines 10 and 13, Pablo indicates that he is aware of making a choice when it comes to solving the problem. He also refers to the example that he came up with as a legitimate justification for his claim. Instead of relying on external authority to argue his stance, Pablo used his example and indicated a sense of shared/internal authority.

Pablo also managed facework in the account turns. To mitigate the consequences of doing a face-threatening act, Pablo uses discourse markers, such as “Uhm” and brief pauses that indicate a dispreferred action will be forthcoming (Schegloff, 2007, p. 68). He also mitigates the disagreement by saying, “if you had a five….” This is an example of a conditional statement used as a face-saving strategy (Brown & Levinson, 1987; Ferguson, 2001). He uses the conditional statement as an interactional strategy to soften the potential threat posed by his disagreement. Also, he downplays the significance of his example when he says, “you can *just* [emphasis added] put it on the bottom.” Furthermore, he repeats the phrase, “if you,” to “hold the floor, gain planning time…and promote understanding” (Kaur, 2012, p. 595).

In line 14, Devyn deploys an acknowledgment token, “yeah,” which continues the conversation (Schegloff, 1982; Bolden, 2015). She then aligns with Pablo’s assertion. Her agreement is considered a preferred action that does not require accounts or other noticeable discourse markers.

***Part III: Counterstatement to Previous Claim (Lines 15 - 19)***

In lines 15, 16, and 17, Isabel uses four discourse markers to accomplish a dispreferred action. She uses (i) micro pauses, (ii) repetition of the phrase, “if you split,” (iii) saying “*Uhm*,” (iv) a slight elongation on “nine,” and (v) a rising intonation after “one,” “nine,” and “symmetrical” (Goodwin, 1983; Nevil & Rendle-Short, 2009). These discourse markers delay the speaker’s expression of disagreement. Therefore, Isabel’s use of micro pauses and repetition of certain phrases at the beginning of the sentence indicates that she is doing a dispreferred action.

Similar to what Pablo did earlier, Isabel prefaces and delays expressing disagreement using the conditional phrase, “if you.” This phrase refers to a potential event instead of an actual situation. Isabel can refer to this scenario using the conditional statement, which weakens her stance (Goodwin, 1983).

Following the repetition, Isabel deploys an account in line 15. Her account would be categorized as a ***Type B*** account turn because she invokes external authority to justify her stance. Although she is using the conditional form, “if,” that was used to introduce her understanding of the concept of ‘symmetry.’ She explains that “you have to split one [the bead] to make it symmetrical.” It is noteworthy that Isabel uses the phrase “you have to” in the account (line 16), indicating “personal obligation rather than logical necessity” (Biber et al., 2002, p. 180). Isabel justifies her stance by referring to an obligation she thinks she must follow.

Pablo attempts to respond in line 18 by saying, “well.” Previous research has shown that “well” as a discourse marker can mitigate a potential face threat (Owen, 1983; Jucker, 1993). Considering that Pablo and Isabel expressed opposing perspectives already, it can be assumed that Pablo was about to deploy another counterstatement to Isabel’s claim in lines 15, 16, and 17.

However, instead of Pablo continuing his disagreement, Allison aligns with Isabel by equating what Isabel said to her perspective (line 19). At this point, Devyn and Pablo are on one side of the argument. Allison and Isabel are on the other side. Also, after Allison’s agreement with Isabel, Pablo could not leverage the interactional space to provide another account or express his idea about the claim.

***Part IV: Aggravated Disagreement and Closing (Lines 20 - 24)***

When Isabel begins her turn in line 20, she performs an "overt disagreement," which refers to an unmitigated and unqualified form of disagreement (Gruber, 1998, p. 482). Compared to her claim in lines 15, 16, and 17, she does not use discourse markers when she says, "you can't really split beads in half." But she does deploy a brief account by referring to a real-life situation of making a necklace.

In line 21, Isabel repeatedly expresses strong disagreement by saying, "you just can't." In this sentence, the word 'can't' is used as a negated modality representing a lack of possibility or ability (Turnbull & Saxton, 1997). According to the interactional pattern of facework or preference organization, accounts should accompany Isabel's disagreement; however, they are absent in line 21 and indicate a ***Type A*** account turn. Then Pablo immediately responds to Isabel without any noticeable disagreement discourse markers or accounts either (line 22).

The talk sequence that constitutes lines 20 - 22 is a form of aggravated disagreement, often seen in children's disagreement episodes. As opposing viewpoints continue without a resolution, disagreement turns progressively become terse, immediate, and direct without discourse markers (Goodwin, 1983). Students prioritize their claims instead of using the turns they have to deploy accounts in these instances. They may feel like it is more important to strongly express their stances instead of deploying accounts at this point of the conversation. Lines 20 - 24 serve as an example in which students directly express their positions instead of providing mitigation through accounts.

After the exchange of aggravated disagreement turns, Allison closes the conversation by introducing a new topic in line 23. As Goodwin (1990) described, children's argumentation often does not result in an explicit persuasion of other students that indicates how one side "won" or "lost" the argument. In this talk sequence, Allison's shifting to a new topic serves as an interactional strategy to close arguments (Greatbatch & Dingwall, 1997).

### *A Continuum of Mathematical Authorial Identity*

To address the third sub-research question, I presented three different account turns observed in this study. They were (i) missing accounts, (ii) accounts that included external authority, and (iii) accounts that included personal latitude. I also described the interaction environment for each account type when the turns were deployed. Now, I turn to the three account types' implications in students' signaling and developing mathematical authorial identity.

Chapter 3 discussed the conceptual dimensions of mathematical authorial identity where I specifically presented the notions of authorship and authority. Specifically, I highlighted three aspects of authorship (students participating in the meaning-making process, cultivating a sense of agency and ownership, and broadening their repertoire to become part of a community of practice) and suggested two types of authority (external and shared/internal authority) relevant to mathematical authorial identity. I employed the framework of authority structure's linguistic clues from Wagner and Herbel-Eisenmann's (2014) work to categorize *personal authority, discourse as authority,* and *discursive inevitability* as an external authority. I presented previous research on shared authority and employed Wagner and Herbel-Eisenmann's concept of *personal latitude* to categorize shared/internal authority.

Referring back to the conceptual understanding of mathematical authorial identity, I now discuss how the three types of account turns are related to the emergence of students' mathematical authorial identity. For each account type, I explain how the notions of authorship and authority could be related and, overall, the account type's implications in mathematical authorial identity.

For ***Type A***, which includes missing accounts, I shared examples of when students accomplished (i) disagreement as a response to an invitation, (ii) disagreement with a claim that had been already disputed, and (iii) aggravated disagreement. These interactional environments were accomplished due to students prioritizing other interactional work, such as answering questions as part of an adjacency pair and indicating agreement with another disagreement. During aggravated disagreements, students prioritized taking the floor and expressing their unwillingness to change stances.

Additionally, the interactional sequences that include Type A accounts did not offer the interactional space for students to elaborate on their opinions or stances. Therefore, students also did not have the interactional space to invoke authority. The lack of interactional space implies that students could not leverage an opportunity to construct a sense of authorship or invoke authority. As such, the interactional sequence that included Type A account turns could be described as a missed opportunity for students to signal and develop mathematical authorial identity *through* accounts.

When ***Type B*** account turns were deployed, students invoked external authority. In general, the episodes that included Type B account turns were usually occasioned (i) towards the end of a sequence, (ii) after Type C account turns had already occurred, and

(iii) when students were asserting their opinions instead of seeking consensus through reasoning.

In extract 6, Allison and Pablo had already expressed their opinions by deploying Type C accounts, which invoked shared/internal authority. Isabel later joined the discussion and aligned her stance with Allison by deploying Type B accounts that invoked external authority. Compared to Allison and Pablo's account turns, Isabel's turns can also be described as a stronger disagreement that does not include as many discourse markers indicating a dispreferred action. These characteristics of Type B accounts can be attributed to the delicate interactional work required for students to manage disagreement.

Isabel invoked *discourse as authority* when she mentioned the words 'have to,' which indicated a sense of obligation and a rule limiting her decision-making process when solving this problem. Especially in a conversation with other peers, disagreement is a complicated activity involving facework to mitigate the consequences of accomplishing a dispreferred action (Pomerantz, 1984). Isabel's reference to the external authority is an interactional strategy to manage disagreement (Hüttner, 2014; McQuade et al., 2018; Sharma, 2013). When an external authority structure is invoked, speakers can avoid responsibility for being the source of trouble in the interaction.

In addition, referring to an external authority can also occur when students think that invoking internal authority is ineffective in pursuing their peers' stances. So they seek support from what would be considered a more legitimate source of authority (McQuade et al., 2018; Sharma, 2013). Isabel's invocation of external authority occurred after her peers had already expressed their reasoning by invoking shared/internal authority. When a consensus was not reached, Isabel could have been seeking closure by invoking a different type of authority structure. Shortly after Isabel invoked external authority (lines 15-17 and lines 20-21), the conversation does reach closure by Allison switching the topic in line 23.

Another pattern observed about Type B account turns and the interactional environments that include Type B account turns often include expressions of strong disagreement (Maíz-Arévalo, 2014; Pomerantz 1984; Rees-Miller, 2000). Strong disagreement expressions usually are usually terse, blunt, and evaluative, whereas weaker disagreement expressions include speakers offering additional explanations, using hedges (e.g., “I guess,” “I think”), and offering suggestions (Maíz-Arévalo, 2014; Kreutel, 2007). Type C account turns deployed by Allison and Pablo include phrases such as “I think” and additional explanations and suggestions made by Pablo in lines 5-8. However, Isabel’s turns are more straightforward as she is more evaluative when she says “that’s not really symmetrical” (line 17) and “you just can’t” (line 21). When strong agreement or disagreement are expressed, they usually signal finalizing the topic on the table (Johnson, 2006; McQuade et al., 2018). As such, the episode reaches a closure soon after Isabel’s turn.

Then how are Type B account turns related to mathematical authorial identity? The interactional space became available for Isabel to participate in the meaning-making process when she deployed Type B account turns to indicate that her understanding of the concept of ‘symmetry’ is determined by whether a bead is divided. Although this may not be mathematically accurate, Isabel still had the interactional opportunity to express her stances. The action of voicing her stance would still enhance Isabel's sense of authorship because she was still able to make meaning.

Isabel's account turn, which reflected her understanding of 'symmetry,' is related to what Benne (1970) described as a shared authority when the bearer of authority guides the subject of authority to grow to achieve autonomy within a community of practice. Therefore, although Isabel may have reflected an inaccurate understanding of 'symmetry,' her expression is still part of the meaning-making process. Isabel's voicing her opinions is significant because it opens the opportunity for the bearer of authority to guide Isabel to use accurate terminology and continue to make meaning about 'symmetry.'

Because mathematical authorial identity is a notion that is co-constructed in social and discursive contexts, how Isabel's turn affects her peers' construction of mathematical authorial identity should be considered. Characteristics of Type B account turns contribute to the interactional sequence to reach closing or speakers to shift topics due to how speakers manage facework and disagreement. Therefore, students typically find it challenging to leverage the interactional space to elaborate on their stances further.

So once turns like Type B accounts are deployed, speakers begin to orient themselves to invoking external authority or asserting their stances using strong disagreement expressions to demonstrate opposition (Maíz-Arévalo, 2014; Goodwin, 1983). Based on how interactional sequences typically operate, opportunities for students to elaborate on their opinions or stances are likely to decrease after one student expresses a stance using external authority or strong disagreement. It becomes difficult for students to leverage interactional space to enhance their mathematical authorial identity.

***Type C*** account turns included when students deployed shared/internal authority. These account turns usually occurred (i) at the beginning of a new interactional sequence and (ii) when students were seeking to build consensus. For example, Allison and Pablo's account turns occurred at the beginning of a new sequence, after Allison made an announcement with an opening, "Oh" in line 1. Then, both students shared their interpretations of what the concept of 'symmetry' means in the context of this problem.

Account turns accompanying a dispreferred action like disagreement serve as "a 'bridge' between 'conflict' and its potential 'resolution" (Firth, 1995, p. 221). In other words, by providing additional explanations derived from internal authority, both students offered "discourse materials" for their peers to respond and interact with. The details that Allison and Pablo provide within their account could enable finding a solution together and building consensus.

The interactional environments that included Type C account turns were more inviting and made more interactional space available for students. The first factor contributing to the increased availability of interactional space relates to how speakers expressed weaker disagreement in the interactional environment with Type C accounts. As mentioned earlier, Allison and Pablo deployed weaker disagreement by using hedges and suggesting solutions. In other episodes with Type C accounts, the speakers were more likely to express weaker disagreement in various ways. Weaker agreement manifested in students' actions, such as requesting clarification.  For example, one account entailed the following queries: "Can I ask you a question about how…How could you have eleven? Since you can't get eleven into half? How could you have two sides that are symmetrical?" (from Episode 29 lines 1-3).  Students also used hedges to express uncertainty by saying, “I don’t think that one is symmetrical because it ends with blue and then it starts back up with yellow” (from Episode 37 line 8). The use of mitigated disagreement strategies signals minimized face-threat of doing disagreement. As a result, other interactants are more likely to respond and continue interacting with the topic.

The interactional environments with Type C accounts occasion even more interactional space when students invoke shared/internal authority. Type C account turns include linguistic clues that speakers are making conscientious decisions when understanding and applying mathematical concepts. In addition, students invoking shared/personal authority experience a more enhanced sense of agency and ownership. They refer to their interpretations of the mathematical concepts as legitimate justifications of their stances instead of having to invoke external authority. This experience would contribute to students’ sense of agency as their ideas are considered a legitimate and valid elaboration of their stances.

Overall, the types of account turn explained above represent how students’ mathematical authorial identity manifests in various ways during small group discussions. It would be ideal to encourage students to deploy more Type C accounts because these they occasion interactional opportunities for students to exercise agency and elaborate on their stances by referring to their personal latitude. However, it does not mean that mathematical authorial identity can only be constructed when students deploy Type C account turns. Even when students deploy accounts and invoke external authority, the experience of voicing their opinions and elaborating on their stances contribute to the meaning-making process.

The construction of mathematical authorial identity and disagreement are both interactional achievements constructed through social and discursive processes. What individuals do and say has a ripple effect on how others develop. Due to the nature of interactions, students and teachers would not be able to control how these different account types are occasioned. However, it would be worth striving to cultivate an interactional environment where Type C account turns are relevant to the conversation and offer more interactional space for students.

# Chapter 6: Conclusion

This dissertation explored the following research question: *How are accounts implicated in students' signaling and developing mathematical authorial identity?* I recorded and transcribed 21 different small group discussions of rural elementary school students who participated in this study to answer this question. The students were from schools located in two different states in the U.S. – Alaska and Idaho—that participated in a professional learning network designed to support rural teachers. They were placed in a small group to communicate with peers from another school. During the series of mathematical learning activities that occurred four times throughout the school year of 2018 - 2019, the students crafted mathematical arguments after solving a problem. They also exchanged feedback on each other's arguments using a video-conferencing application.

Based on the transcripts, I focused on instances when students deployed or were expected to deploy accounts. Accounts were defined as statements "made by a social actor to explain unanticipated or untoward behavior" (Scott & Lyman, 1968, p. 46). Using the episodes of students' deployment of accounts, I conducted a conversation analysis (CA) to identify and describe the interactional patterns of when students deployed accounts and explore how accounts are implicated in the signaling and development of students' mathematical authorial identity.

Overall, this study approached mathematical authorial identity from a sociocultural and discursive perspective. I focused on how students' mathematical authorial identity emerged in their discursive practices of deploying (or not deploying) accounts during conversations. I concluded that accounts are implicated in students' signaling and developing mathematical authorial identity by providing the interactional space where students' elaboration of their claims about mathematics and invocation of various authority structures become relevant to the conversation.

## Summary of the Findings

I identified interactional patterns of when students deployed accounts. These indicated that specific interactional contexts resulted in students' deployment of accounts. The findings on how specific FPPs led to the occasion of accounts indicated that accounts do not occur randomly. The FPPs that resulted in students' deployment of accounts were (i) when students asked open-ended 'how' questions, (ii) claimed non-understanding, and (iii) expressed disagreements.

The second significant finding was that when students deployed accounts, they served as a site of elaborating on their opinions or stances and invoking external or shared/internal authority structures.

Then, I categorized the three different types of account turns and described the interactional environments of those account turns. Type A account turns were 'missing accounts,' which refer to the interactional sequences that were supposed to include accounts but were missing due to speakers accomplishing different interactional work. The turns that resulted in the first type were when students accomplished (i) disagreement as a response to an invitation, (ii) disagreement with a claim that had been disputed already, and (iii) aggravated disagreement. Type B accounts were when speakers invoked external authority when elaborating on their stances. Students deployed this type of accounts towards the end of an interactional sequence and after other students had deployed account turns that invoke shared/internal authority. These turns also included expressions of strong disagreement that typically resulted in the closing of a sequence. Type C account turns were when speakers invoked shared/internal authority. These account turns were usually deployed at the beginning of a new sequence and included expressions of weaker disagreement that invited students to continue the interactional sequence.

The concept of mathematical authorial identity can be described as a continuum of two phenomena: authorship and authority. The three account types and the interactional environments represent various manifestations of mathematical authorial identity. Students would leverage the interactional space to deploy Type C account turns in an ideal world. Given the limitations of Type A and Type B account turns, it ensues that Type C account turns provide the optimal opportunities for students to leverage the interactional space and invoke shared/internal authority that could enhance mathematical authorial identity. Students would gain a sense of agency and ownership when they elaborate on their stances using their own justifications. In return, their peers would also consider their expressions based on shared/internal authority as a legitimate contribution to the meaning-making process. The overall discursive and interactive experience would support students' abilities to express themselves as authors with full authority over their stances on mathematical concepts.

However, students' mathematical authorial identity is also constructed when students deploy type B account turns. These account turns represent the process of students learning how to be independent thinkers first by invoking external authority. Navigating the discourse of disagreement is extremely challenging and complicated. There are instances when students have to invoke external authority to move the conversation forward. Even when students invoke external authority, they still contribute to the meaning-making process and construct mathematical authorial identity. One disadvantage would be that when Type B account turns are deployed, they are likely to bring an interactional sequence to a close or limit the interactional space offered to other students.

Finally, Type A account turns occur when speakers orient themselves to other interactional work, such as answering questions or engaging in an aggravated disagreement. These are instances when accounts were supposed to occur but were missing. Therefore, there is no interactional space for students to use to elaborate on their thoughts and invoke authority.

A critical underlying principle to note when discussing these three account types' implications in mathematical authorial identity is that accounts are a "collective activity" (Sterponi, 2003, p. 95). Accounts are co-constructed through social interactions, and students do not entirely control when these account turns occur. Students' deployment of different account turns, and their consequential development of mathematical authorial identity depend on the interactional environments and the trajectories of interactional sequences. Based on this understanding, I address three pedagogical implications for mathematics educators.

The three pedagogical implications are the following. First, the discussion on fostering students' mathematical authorial identity should be more detailed because simply having opportunities to interact is not enough to construct and develop mathematical authorial identity. Second, educators should be aware of the interactional work required for students to disagree. Finally, educators should also acknowledge that deploying accounts is a "collective activity" (Sterponi, 2003, p. 95). To effectively distribute how different account types occur, there should be group norms regarding classroom discussion participation to distribute how different account types occur effectively.

## Pedagogical Implications

Since the introduction of the reform movement in mathematics curriculum and the crystallization of eight Standards of Mathematical Practice (NCTM, 2000) in state and federal educational policies, researchers have emphasized the importance of creating a learning environment where students learn from participating in collaborative and discursive activities (Cobb et al., 1992; Engle & Conant, 2002; Langer-Osuna, 2017). More specifically, students are expected to learn how to "justify their conclusions, communicate them to others, and respond to the arguments of others" (NCTM, 2000) as essential parts of mathematics. These skills invite students to exercise agency over the meaning-making process in mathematics, display ownership and authority of their ideas about mathematical concepts, and ultimately develop mathematical authorial identity (Fellus, 2019; Langer-Osuna, 2016, 2017, 2018, 2020; Yackel & Cobb, 1996).

Shifting the interactional contexts of mathematics classrooms is a challenging process, as it has traditionally centered around teachers' authority (Wagner & Herbel-Eisenmann, 2014). On the surface level, implementing learning activities such as constructing mathematical arguments and providing feedback on peers' arguments might enhance students' sense of authorship in mathematics. However, as the findings from my study suggest, students can accomplish some actions, such as disagreement, without leveraging the interactional space. In other words, one of the major findings from my research is that interaction alone is not enough for students to develop mathematical authorial identity.

For example, I listed three instances when students accomplished the action of disagreement, but their account turns were not relevant to the conversation. The interactional environment around Type A accounts did not offer students the interactional space to elaborate on their opinions or stances. Because the action of disagreement is complicated, students can be involved in interactional work, such as aggravated disagreement when specific instructions are missing.

Therefore, what matters the most is the affordances of interactional space where students can accomplish two critical activities to develop a sense of mathematical authorial identity. The two activities leverage the interactional space to elaborate on their stances and invoke various authority structures. When students can engage in these two activities, actions like justifying, explaining, and arguing all can meaningfully contribute to the construction of mathematical authorial identity, thereby catalyzing students' learning forward.

The second pedagogical implication is for teachers to be aware that the interactional work required for students to justify and argue with each other is more complicated than commonly understood. For example, the deployment of type B account turns involves the interactional work of managing disagreement and facework. Students manage disagreement by using different mitigation strategies. They offer explanations, use hedges, or ask clarifying questions when expressing weaker disagreements. These kinds of mitigation strategies should be expected as part of their deliberation.

However, students can express strong disagreement by being terse, straightforward, and evaluative. We have seen from the data that once strong disagreement expressions are deployed, it is challenging for students to step back and deploy weaker disagreements. Students orient themselves to facework, and when strong disagreement expressions are used, they are likely to conclude the current interactional sequence.  Students switch the topic to avoid the heightened face-threats and the process of learning and developing authorial identity is terminated.

Teachers would benefit from being aware of how students have to navigate various layers of social relationships when they engage in disagreement. Therefore, when teachers monitor students' small group discussions, they can redirect students to avoid the perils of only accomplishing the kinds of aggravated disagreement that block further learning. Teachers would also benefit from learning what turns prompt students to elaborate more on their stances. The turns that occasion more interactional space for students to elaborate on their thoughts can be used as a sentence stem or questions that students can ask peers.

Finally, related to what students can do to maximize opportunities for leveraging interactional space and invoking authority, the third pedagogical implication emphasizes the importance of group norms. The findings show that the interactional environments for different account types are co-constructed. While students' deployment of Type C account turns might be beneficial, the turns cannot be occasioned every time. There are specific turns that occasion accounts. Therefore, students should get in the habit of figuring out what those stances are and how to use them effectively.

Another group norm that students can benefit from is redirecting the conversation when they are doing aggravated disagreement. Students do not have many opportunities to construct mathematical authorial identity when this type of disagreement continuously occurs. It is challenging to leverage the interactional space and invoke authority structures to justify their opinions. Group norms that instruct students on what to do when their conversation trajectories move towards aggravated disagreement would benefit students immensely.

## Limitations and Directions for Future Research

I analyzed the transcribed data of students' small group discussions via a video-conferencing application. This study aimed to explore the concept of mathematical authorial identity from a CA perspective. This study aimed to describe interactional patterns of accounts that emerged in my data and elaborate on the relationship between accounts and students' mathematical authorial identity. A micro-level analysis of students' conversations was the most viable research method to achieve the research objective.

While conducting CA to identify students' interactional patterns around accounts led to insights about mathematical authorial identity, there were also some limitations of the study. First, this study primarily considered one type of students' artifact: audio data extracted from small group discussion recordings. I acknowledge that using video data that include information about how students utilize interactional resources such as body language, gaze, and facial expressions may yield more information about the communication process (Pearce, Arnold, Phillips, & Dwan, 2010). However, given the scope of this dissertation study, it was appropriate to focus on the verbal interaction of students' conversations.

Additionally, students were aware that only their teachers and I had access to the recordings. This knowledge could have impacted their behavior. Due to implementation errors, I excluded the third set of recordings from the four small group learning activities. Including seven more recordings from the third video conferencing session could have resulted in identifying different patterns or selecting a more explicit extract to share in my findings chapter. Lastly, even though I participated in data sessions (ten Have, 2007) to discuss the CA analysis process, I did not have a second-rater who reviewed the dissertation data to check the accuracy of my analysis.

Identifying students' interactional patterns in various classroom contexts could contribute to the planning of specific collaborative and discursive learning activities (Ingram et al., 2019). Future research on the implications of accounts in students' mathematical authorial identity could include data from diverse student populations. In addition, the research agenda could benefit from different data types, such as written work or video data of students' small group discussions. As mentioned above, including a variety of data could enhance understanding of how accounts are related to mathematical authorial identity. Finally, because mathematical authorial identity is a dimension of the networked model of mathematical identity (Fellus, 2019), future researchers could examine the specific relationship between students' deployment of accounts and other identity dimensions.

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