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ESSAYS IN MARKET DESIGN

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Abstract

This dissertation consists of two chapters. Both are centered around the theory and design of markets, in which the use of money is prohibited and/or strongly undesirable. In my first chapter, I study multi-object assignment problems. Here, the assignment of graduate students to teaching assistant positions over the course of two semesters, serves as an illustrative application. In my second chapter, I propose an alternative way to distribute asylum seekers among European member states based on the preferences of both sides.

Chapter 1: Multi-Object Assignment: Booster Draft

In my first chapter, I ask the question of how to divide among a set of n individuals a set of $n \times m$ indivisible objects without using monetary transfers, in a way that is efficient, incentive compatible, and ex-post fair. A well known impossibility result shows that the only mechanisms that are both incentive compatible and efficient are dictatorship mechanisms. I fill a gap in the literature by describing a novel mechanism that is both incentive compatible and fair in the responsive preference domain. The mechanism is inspired by booster drafts used in competitive card game tournaments. The idea is to arbitrarily divide the set $n \times m$

objects into m "boosters" (sets) of size n and specify a priority order for each such booster. Afterwards the individuals pick objects from the boosters in order of priority. The outcome of the booster draft mechanism can be improved if additional knowledge about a particular market is incorporated into the creation of boosters. I point out a special case of multi-object assignment problems, motivated by the allocation of teaching assignments among graduate students. In this domain the creation of the boosters is straightforward. Indeed, at the Boston College economics department, graduate students are assigned exactly one fall and one spring semester task over the academic year. Here the optimal way of creating boosters is to group up all spring teaching assignments in one booster and all fall semester assignments in the other. In this case the balanced booster draft is not only strategy-proof and fair, but also weakly efficient (dominance efficient). Moreover, for this restricted assignment domain I characterize the set of all booster drafts as any (strongly) strategy-proof, neutral and non-bossy mechanism. In the final part of the paper I take a closer look at the teaching assistant assignment problem, using date on the submitted rankings over semester-tasks by graduate students. The simulation exercise provides additional evidence that the proposed mechanism is a sensible practical solution. In particular, I show that for a simple measure of welfare students prefer a balanced booster draft to a serial dictatorship mechanism if they are mildly risk averse.

Chapter 2: An Alternative Asylum Assignment

The 2015 refugee crisis has demonstrated the necessity of revising the current European asylum system. As an alternative, I propose to take into account preferences of asylum seekers as well as preferences of member states. Asylum seekers indicate how long they are willing to wait for their asylum application for any given member state, allowing them to avoid overburdened member states by opting for "less popular" member states. Within the market design literature, this is the first paper proposing to match asylum seekers as opposed to refugees. In other words, its stays much closer to the template of the Common European Asylum System.

From a theoretical perspective, it turns out that the asylum seeker framework can be formulated as an application of the well-known matching with contracts model by Hatfield and Milgrom (2005*a*). This simplifies the analysis a great deal, as matching with contracts is a well studied framework within the matching/market design literature. I show that the standard cumulative offer mechanism (Gale and Shapley, 1962*a*; Hatfield and Kojima, 2010*a*) is asylum seeker incentive compatible and leads to stable outcomes, using the fact that the proposed choice functions have a completion satisfying substitutability and the law of aggregate demand Hatfield and Kominers (2016). Moreover, stability implies two sided Pareto efficiency, giving consideration to both preferences of member states and asylum seekers.

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Chapter 1

Booster Draft Mechanism for Multi-Object Assignment

1.1 Introduction

In this paper, we study the multi-object assignment problem. That is, $m \times n$ indivisible objects have to be distributed among n individuals, without the use of monetary transfers. Examples include the assignment of shifts to interchangeable workers, players to sport teams, courses to students, and teaching assignments to graduate students. We introduce a new mechanism, which stands out in terms of simplicity and strategy-proofness, that is, it will be always optimal for individuals to reveal their true preferences. The mechanism is inspired by the following multi-object assignment process used in Magic: The Gathering, a competitive trading card game:[1]

> `In a booster draft, players each receive three booster packs of 15 cards. After being seated around a table, each player simultaneously opens one booster pack, selects a single card, and then passes the rest over to the next player. After all players have drafted fifteen cards, they each open their second pack, and drafting continues, sometimes in reverse order during the second pack (MTGWiki, 2019).`

We formalize an algorithm that captures the essential elements of the mechanism described above. As input, individuals report rankings over the available objects. In a first step, the objects are arbitrarily divided into m sets of size n, and a separate priority order for each set determines the sequence in which objects are picked. In line with the motivation, we refer to these sets as "boosters." Following his/her reported ranking, the individual whose turn it is to select from a given booster adds the best available object to his/her collection. Once all objects are distributed, a final allocation is reached. Fixing the objects within each booster as well ass the corresponding priority orders the algorithm induces a function from rankings to allocations, which we refer to as a booster draft (BD) mechanism. More precisely, we introduce a class of mechanisms, as every design of boosters and priorities

[1] Magic: The Gathering (MTG) was the first commercially successful trading card game, developed by Richard Garfield and published in 1993 by Wizards of the Coast (https://company.wizards.com/). Analogous to a sports drafts, MTG introduced a play mode in which players draft cards from a common pool. Afterwards every player builds a deck using a subset of the drafted cards. Only then do players compete against each other with their constructed decks. There are countless other online/offline trading card games featuring a booster draft-inspired play-mode.

yields a distinct BD mechanism.

We start by analyzing how well BD mechanism perform if used for a general multi-object assignment problem. Postponing the details to a later part of the introduction, we show that any BD mechanism is strategy-proof in the "responsive" preference domain.[2] To ensure that individuals cannot benefit from manipulation, we need them to pick at most once from every booster, therefore we restrict the sets to be of size n. Moreover, responsiveness ensures that an individual does not want to change its earlier picked objects, based on the objects received later on. Furthermore, we show that any "balanced" booster drafts satisfies a reasonable notion of fairness.[3] It is well known that in the multi-object environment, there does not exist any mechanism simultaneously satisfying strategy-proofness, efficiency, and fairness. Therefore, we cannot ensure that the outcomes of the BD mechanisms will be efficient.

Following these observations, in the second part of the paper, we take a market design perspective, exploiting additional restrictions on preferences and allocations for specific markets, in order to improve the outcome of the BD mechanism. Paying attention to detail, we ask how to design the underlying boosters and priority orders. We focus our attention on the assignment of teaching positions to graduate students, in which a restriction is placed on the set of feasible allocations. Specifically, at the economics department under consideration,

[2] Responsiveness requires that an individual will always prefer a higher ranked object to a lower ranked one regardless of any other items in his/her possession.

[3] Balanced BD mechanisms place the following restriction on the priority orders: For any pair of individuals i and j, i has lower priority than j in at most half of the available boosters, rounded down if there is an odd number of boosters, i.e., $\lceil \frac{m}{2} \rceil$.

graduate students are supposed to work as a teaching assistant for exactly one fall and one spring semester course.[4] Here, the optimal way of creating boosters is to group up all spring teaching assignments in one booster and all fall semester assignments in the other. In this case, the balanced booster draft is not only strategy-proof, but also (weakly) efficient and fair. More generally, we describe the *partition-restricted* assignment domain, i.e., any multi-object assignment problem for which there exists an exogenous partition of objects, such that any two objects within the same set cannot be obtained by the same individual. As before, the creation of boosters for running the BD algorithm is no longer arbitrary, but naturally follows the exogenously given partition. In theorem 1, we characterize the set of all "partition-consistent" BD mechanism for this domain. In particular, given the standard requirements of non-bossiness and neutrality any strategy-proof mechanism must fall into the category of booster draft mechanisms.[5] We conclude that in any partition-restricted multi-object assignment problem, the balanced BD mechanism arises as a natural candidate to be employed. How to create boosters for other multi-object assignment problems remains an open question.

In the third part of the paper, we take a closer look at the assigning of graduate students to teaching positions. We use 2018 data on the preferences of graduate students, at a particular economics department, to simulate assignments under the balanced BD mechanism. That

[4]One relevant rational for the requirement is the following: As many graduate students in the economics department are international students, their visa status (F1) allows them to work up to 20 hours per week, preventing them from fulfilling the work-requirement for two positions in the same semester, without violating their visa regulations.

[5]Non-Bossiness requires that no individual can influence the allocation of another individual without affecting its own allocation, while neutrality states that the mechanism should be immune to a relabeling of the object.

is, at the end of the academic year, students separately rank all the fall and spring semester tasks. Before the start of the new academic year, an assignment is created based on the submitted rankings. For the simulation, students are randomly ordered and pick their preferred fall semester assignments one after another. Then the initial random order is reversed and students pick their spring semester assignments in the same fashion. The BD mechanism outperforms the actual assignment in that year, both in terms of efficiency and fairness. Moreover, simulating the outcome of serial dictatorship as an alternative, we show that the BD mechanism reaches comparable outcomes in terms of efficiency, while achieving higher fairness.[6] This discussion concludes the main part of the paper. We belief that an additional strength of the booster draft lies in its simplicity. Therefore, in the remainder, we evaluate in which sense the BD rule's non-manipulability is simple to grasp, following the concept of obvious strategy-proofness.[7] We now will discuss some of the previously omitted definitions and ideas in more detail.

Running the BD mechanism, every individual reports a simple order/ranking over the available objects.[8] As strategy-proofness, efficiency, and fairness are all formulated in terms of individuals preferences, we first need to establish a link between the reported order over objects and the underlying preferences. In particular, consider the following partial order,

[6] Under a serial dictatorship, for any two students A and B, one of them is going to choose all his/her objects before the other. The mechanism is simple to implement, efficient, and strategy-proof and has therefore often been used in practice. An important shortcoming is that serial dicatorships lead to very unfair allocations, especially if individuals value similar objects.

[7] In practice strategy-proofness is not always strong enough. People sometimes will try to manipulate a mechanism, failing to recognize its strategy-proofness. Li (2017) introduces the strengthening of strategy-proofness called obvious strategy-proofness that addresses the issue.

[8] For any preference relation, we refer to the ranking of singleton sets as the "underlying simple order."

which we refer to as a dominance relation: Given a simple order over objects, for any two (same size) sets A and B, A dominates B if for every object in B there is a concomitant object in A that is (weakly) preferred to the one in B. In lemma 1, we show that if A is preferred to B by set-wise domination, then the same holds true for the actual preferences. This connection allows us to analyze which properties the BD mechanism satisfies.

A mechanism is strategy-poof if an individual cannot obtain a better outcome by submitting a untruthful ranking over objects. The BD mechanism is strategy-proof, as the final allocation is (weakly) set-wise dominating any other outcome obtainable by submitting an different ranking. The idea behind the fairness concept is as follows: Pick a final allocation and suppose individual i prefers individual j's bundle to his/hers. Let us sequentially remove the best object from j's and the worst object from i's assignment, following i's simple order. At some point, i (weakly) prefers her reduced bundle to j's reduced bundle. A mechanism is k *envy-free*, if the maximum number of pairs of objects that have to be removed to eliminate envy for any individual i over the bundle of any j, at any possible allocation, is equal to k. The larger k is, the higher the envy of an individual. The maximum envy under the balanced BD mechanism is equal to half of the obtained objects rounded up.

Efficiency requires that for any final allocation, no other allocation of objects makes everyone weakly and at least one individual strictly better off. We relax efficiency to dominance efficiency, ruling out that all individuals can be made better off in terms of set-wise dominance. In proposition 4 we show that an allocation is dominance efficient if and only if there does not exist any "exchange-cycles" between a subset of individuals, s.t. everyone gives and receives exactly a single object, and everyone is better off after the trade takes place. However, our weakened efficiency criteria does not rule out that swapping a combination of objects, some deemed better and some worse than the ones exchanged, can lead to a more desired allocation for all involved parties. The BD mechanism violates dominance efficiency in the standard responsive domain. We also show that in the standard responsive preference domain no mechanism can simultaneously satisfy envy-freeness for half of the objects, dominance strategy-proofness, and dominance efficiency.[9] As discussed previously, we can avoid the impossibility result by incorporating market specific restrictions. For instance, we show that in the partition-restricted assignment domain the BD mechanism satisfies dominance efficiency on top of strategy-proofness and envy freeness for half of the objects.

Finally, we ask whether the BD mechanism is implementable via an extensive form game in an obviously strategy-proof (OSP) way. A mechanism is OSP implementable if there exists an extensive form game that yields the same outcome as the proposed mechanism with the added restriction that, at any information set in which an individual is called to play, the best outcome under truthful play is weakly preferred to any possible history reachable from the same information set. The BD mechanism is not OSP implementable. We introduce a weakening of OSP called *dominance obvious strategy-proofness* (DOSP), that limits the attention to outcome pairs comparable by set-wise domination. Unlike the

[9] Dominance strategy-poofness is a weakening of strategy-proofness, requiring that no individual can manipulate the mechanism, s.t. his/her assignment (strictly) improves under the dominance relation. Hence, this notion allows for some manipulations to take place.

standard responsive domain, the BD mechanism is DOSP implementable in the partition-restricted domain, providing additional evidence that the BD mechanism is a strongly viable candidate for the restricted multi-object assignment problems. We continue by discussing the relevant literature.

1.2 Related Literature

A series of impossibility results have pointed out that any efficient and strategy-proof mechanism is a serial dictatorship (Pápai, 2001; Klaus and Miyagawa, 2002; Ehlers and Klaus, 2003). We add a new impossiblity result to the literature, showing that even weaker versions of fairness, strategy-proofness, and efficiency cannot be simultaneously satisfied by a mechanism.

Initially, we are interested in strategy-proof mechanisms that are also fair. Related to this, Moulin (2019) provides a comprehensive survey of the long-standing literature on fair division problems. Our definition of envy-freeness is adapted from Budish (2011), although we modify their definition to account for the possibility of distributing bads, as well extending it, allowing for the removal of an arbitrary number of objects.[10] For practicality, we let individuals report simple orders over objects. We then have to establish a link between the reports of individuals and their preferences across sets of objects. This

[10] Suppose for example that only non-disposable bads are distributed. Hence removing a bad from the bundle of individual j will only increase the envy of individual i. In this case one should remove the worst object from i's bundle instead. We take care of both cases by always removing the best object from j and the worst object from i simultaneously. Aziz et al. (2018) have a similar definition based on removing a single good on each side. Moreover, instead of considering 1 envy-freeness, we allow for an arbitrary k.

approach relates to Brams and Fishburn (2000), Brams, Edelman and Fishburn (2003), and Edelman and Fishburn (2001).

In the second part of the paper, we take a market design perspective. We exploit the additional structure specific markets impose on preferences and or final allocations, to adjust our mechanism to the problem at hand. This has been done before for multi-unit assignment problems, in the context of course allocation at business schools (Sönmez and Ünver, 2010; Budish, 2011; Budish and Cantillon, 2012).[11] Budish (2011) provides a solution to the more general combinatorial assignment problem, introducing the approximate competitive equilibrium from equal incomes (ACEEI) mechanism. Efficiency and strategy-proofness of the ACEEI mechanism rely on the market being large enough such that people become price takers. Unfortunately, ACEEI cannot be obtained in a constructive way (Othman, Sandholm and Budish, 2010; Budish et al., 2016). This might cause legitimacy issues (Bo and Li, 2019), as its not possible to publicly implement the outcome of ACEEI. Related to this, Li (2017) points out that if a mechanism is hard to understand in practice, some individuals will employ dominated strategies, even if the mechanism is strategy-proof. An additional appeal of the BD mechanism lies in its simplicity. The discussion of the dominance obviously strategy-proofness, relates to the small body of literature on obviously strategy-proofness (Li, 2017; Zhang and Levin, 2017; Ashlagi and Gonczarowski, 2018; Pycia and Troyan, 2018; Troyan, 2016).

[11] The multi-unit assignment problem is a slight variation to on the multi-object assignment problem, in which several units of the same object are available, e.g., representing the number of available seats for each course. None of the results we present depend on the absence of multiple copies of objects. We ignore it to reduce the notation, as it does not give any additional insight.

In general, our research relates to the larger field of matching theory started by Gale and Shapley (1962*a*). In particular, the characterization result draws from Svensson (1999) and Hatfield (2009), while the responsiveness preference assumption is based on Roth (1985). Finally, the assignment of graduate students to teaching assignments falls into the category of applied matching problems (Abdulkadiroglu and Sönmez, 2003*a*; Sönmez and Switzer, 2013*a*; Delacrétaz, Kominers and Teytelboym, 2019). We are not aware that this particular application has been discussed in any previous literature.

1.3 Model

A **multi-object assignment problem** is a triple $\langle I, O, \succsim \rangle$, where

1. I is a finite set of $|I| = n$ individuals,
2. O is a finite set of $|O| = m \times n$ objects with $m \geq 2$, and
3. $\succsim = (\succsim_i)_{i \in I}$ a list of preferences over sets of objects 2^O.

We want to distribute all the available objects among the individuals. An **allocation** $A = (A_i)_{i \in I}$ gives every individual $i \in I$ a subset of objects $A_i \in 2^O$. An allocation is **feasible** if for any two distinct individuals $i, j \in I$ with $i \neq j$, their assignments do not overlap $A_i \cap A_j = \varnothing$, and all objects are distributed $\bigcup_{i \in I} A_i = O$. Let $\mathcal{A}$ denote the set of all feasible allocations. Every individual has a preference relation $\succsim_i$ over sets of objects.[12]

[12] For any $O', O'' \in 2^O$ with $O' \succsim O''$ but $O'' \not\succsim O'$ we write $O' \succ_i O''$, similarly for any $O', O'' \in 2^O$ with $O' \succsim O''$ but $O'' \succsim O'$ I write $O' \sim_i O''$.

Slightly abusing notation $\succsim_i$ denotes preferences over sets of objects as well as allocations, such that $A \succsim_i A'$ if and only if $A_i \succsim_i A'_i$. The underlying assumption is that individuals only care about their own assignment.

From individual i's perspective, an object o is a **good** if receiving the object is preferred to not receiving it, i.e., $\{o\} \succ_i \varnothing$. The empty-set, $\varnothing$, represents an empty assignment. Conversely, an object is a **bad** if not receiving it is preferred to receiving it, i.e., $\varnothing \succ_i \{o\}$. For simplicity, we focus on the case where everyone agrees whether objects are good or bad. Formally, we have $\{o\} \succsim_i \varnothing$ if and only if $\{o\} \succsim_j \varnothing$ for all $j \in I$ and hence $\varnothing \succ_i \{o\}$ if and only if $\varnothing \succ_j \{o\}$ for all $j \in I$. Preferences over objects are **responsive**. The idea is that if an individual prefers object o to another object o' then we can infer, no matter what other objects $O' \subset O$ the individual possesses, that $\{o\} \cup O'$ is preferred to $\{o'\} \cup O'$. I.e., for any $o, o' \in O$ and $O' \subset O \setminus \{o, o'\}$ we have that $O' \cup \{o\} \succsim_i O' \cup \{o'\}$ if and only if $\{o\} \succsim_i \{o'\}$. Likewise, a good is always desirable, while a bad makes an individual always worse off, i.e., for any $o \in O$ and $O' \subset O \setminus \{o\}$ we have $O' \cup \{o\} \succ_i O'$ if and only if $\{o'\} \succ_i \varnothing$. Finally, we restrict our attention to preferences that strictly rank any pair of singletons. That is for any $o, o' \in O$ with $o \neq o'$ either $\{o\} \succ_i \{o'\}$ or $\{o'\} \succ_i \{o\}$.

1.3.1 Submitted Rankings and the Booster Draft Mechanism

As it is impractical to ask individuals for their full preference relation over all sets of objects, throughout the analysis, we let each individual report a **strict simple order** P_i over the available objects O, with the associated simple order R_i.[13] The set of possible rankings for any $i \in I$ is denoted as $\mathcal{P}_i$, and represents all possible ways one can order the available objects O. $P = (P_i)_{i \in I}$ denotes a list of simple orders for every individual $i \in I$ with $\mathcal{P}$ representing the set of all possible lists. For a given preference relation $\succ_i$ over 2^O we say P_i is the **associated simple order** over O if for all $o, o' \in O$ we have that $o \; P_i \; o'$ if and only if $\{o\} \succ_i \{o'\}$. That is, the associate simple order ranks all the objects in the same way as the underlying preference relation.[14]

Even though it is not possible to infer the whole preference relation $\succsim_i$ over 2^O from the associated simple order P_i over O, the responsiveness assumption lets us compare some sets by element-wise dominance. To express this relationship, we define a partial order $\geq_i$ over 2^O based on a simple order P_i. We will refer to the partial order $\geq_i$ as the **dominance relation**. The idea is that two sets of objects are comparable if for every object in one set we can find a weakly preferred concomitant object in the other set. Formally, let $o'_{i,l} = \{o \in O' : |\{o' \in O' : o' \; R_i \; o\}| = l\}$ be the lth best object in subset $O' \in 2^O$ following simple order P_i. Then, for any two subsets $O', O'' \in 2^O$ of equal size $|O'| = |O''| = m'$, we have $O' \geq_i O''$ if and only if $o'_{i,l} \; R_i \; o''_{i,l}$ for all $l \in \{1, \dots, m'\}$.[15] In the following we show

[13]The strict simple order P_i is *transitive*, *asymmetric*, and *complete*. The associated simple order R_i is *transitive*, *antisymmetric* and *strongly complete*. Strong completeness implies reflexiveness and is therefore not listed under the properties of a simple order. Simply put, unlike P_i which is asymmetric, R_i also compares any object with itself. Otherwise both relations ranks every pair in the same way. See Roberts (1985) for an excellent overview on binary relations and their properties. Finally as every person agrees whether an object is a good or a bad, together with requiring every object to be assigned to someone, it is sufficient to let individuals rank O as opposed to $O \cup \{\varnothing\}$.

[14]Note that P_i is a strict simple order (*transitive*, *asymmetric*, and *complete*) as $\succ_i$ strictly ranks all pairs of singleton sets in a transitive way. Moreover, the associated simple order P_i is uniquely determined for each preference relation $\succ_i$, while multiple (responsive) preference relations $\succ_i$ are consistent with any given simple order P_i.

[15]A partial order is a *reflexive*, *antisymmetric*, and *transitive* binary relation (Roberts, 1985). We use

that if the dominance relation $\geq_i$ based on P_i rank two sets of objects, then it does so in the same way as the responsive preference relation with the same associated simple order P_i.

Lemma 1. *Let $\succsim_i$ be any responsive preference relation over 2^O with associated simple order P_i, and $\geq_i$ the corresponding dominance relation. For any $O', O'' \in 2^O$ if $O' \geq_i O''$ then $O' \succsim_i O''$.*

Proof. Suppose we have $O', O'' \in 2^O$ with $O' = \{o'_{i,1}, \ldots, o'_{i,m'}\} \geq_i O'' = \{o''_{i,1}, \ldots, o''_{i,m'}\}$. As $O' \geq_i O''$ we have $o'_{i,1}\ R_1\ o''_{i,1}$ as well as $\{o'_{i,1}\} \succsim_i \{o''_{i,1}\}$. Using responsiveness for $\{o''_{i,2}, \ldots, o''_{i,m'}\} \subseteq O \setminus \{o'_{i,1}, o''_{i,1}\}$ and $\{o'_{i,1}\} \succsim_i \{o''_{i,1}\}$ we get $\{o'_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} \succsim_i O'' = \{o''_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\}$. Replacing one-by-one $o''_{i,k}$ by $o'_{i,k}$ for all $k \in \{2, \ldots, m'\}$ and invoking responsiveness we get $O' = \{o'_{i,1}, o'_{i,2}, \ldots, o'_{i,m'}\} \succsim_i \ldots \succsim_i \{o'_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} \succsim_i \{o''_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} = O''$. By transitivity of $\succsim_i$ we reach the conclusion that $O' \succsim_i O''$. $\square$

We now go back to the question of how to distribute the available objects. That is, we are interested in finding a **simple mechanism** $\psi : \mathcal{P} \to \mathcal{A}$ that selects an allocation $A \in \mathcal{A}$ for any reported list of orderings $P \in \mathcal{P}$.

$O' >_i O''$ to denote that $O' \geq_i O''$ but $O'' \not\geq_i O''$. Similarly we use $O' =_i O''$ whenever $O' \geq_i O''$ and $O'' \geq_i O''$, where given our assumptions on preferences in this case the two sets O' and O'' must be identical. For any $i \in I$ and any ranking $P_i \in \mathcal{P}_i$ respectively $\hat{P}_i \in \mathcal{P}_i$ we will use $\succsim_i$ respectively $\hat{\succsim}_i$ to denote any responsive preference consistent with order P_i respectively $\hat{P}_i$ and $\geq_i$ respectively $\hat{\geq}_i$ for the dominance relation based on P_i respectively $\hat{P}_i$. To reduce notation, we only define the dominance relation for of equal size, which will be sufficient for our purpose.

Booster Draft (BD) Algorithm

Step 0.

Let the set of objects O be arbitrarily partitioned into m boosters of equal size $\{O^1, \ldots, O^m\}$ with $|O^k| = n$ for all $k \in \{1, \ldots, m\}$. Moreover construct m different priority for every booster $\{f^1, \ldots, f^m\}$.[16]

Step $1 \leq t \leq n+1$.

For $k \in \{1, \ldots, m\}$ following the priority orders let any person $i \in I$ *claim* her most preferred object according to P_i among remaining ones in any booster O^k where her priority is $f^k(i) = t$.

In each of the m buckets there are $(n - t)$ objects left. If there are no objects left the algorithm terminates and every person gets assigned her *claimed* objects.

For those interested we next discuss a short example, illustrating our mechanism.

Example 1. Family Heirloom Assignment Problem

Let the set of $n = 3$ individuals, respectively siblings, be $I = \{\text{i, j, k}\}$. The available objects are $O = \{\text{Armchair, Bagpipe, Clock, Diamond-ring, Earings, Fine wine}\}$ with $n \times m = 2 \times 3$. Moreover, individual i reports P_i : D - B - A - C - F - E, individual j P_j : B - A - C - E - F - D, while individual k reports P_k : B - D - E - F - A - C. Figure 1.1 illustrates the functioning of the described booster draft algorithm, with boosters $O^1 = \{A, B, C\}$, $O^2 = \{D, E, F\}$, and priority orders $f^1 : i - j - k$, $f^2 : k - j - i$. Even though we have not yet formally introduced the definition, this example portrays a balanced booster draft.

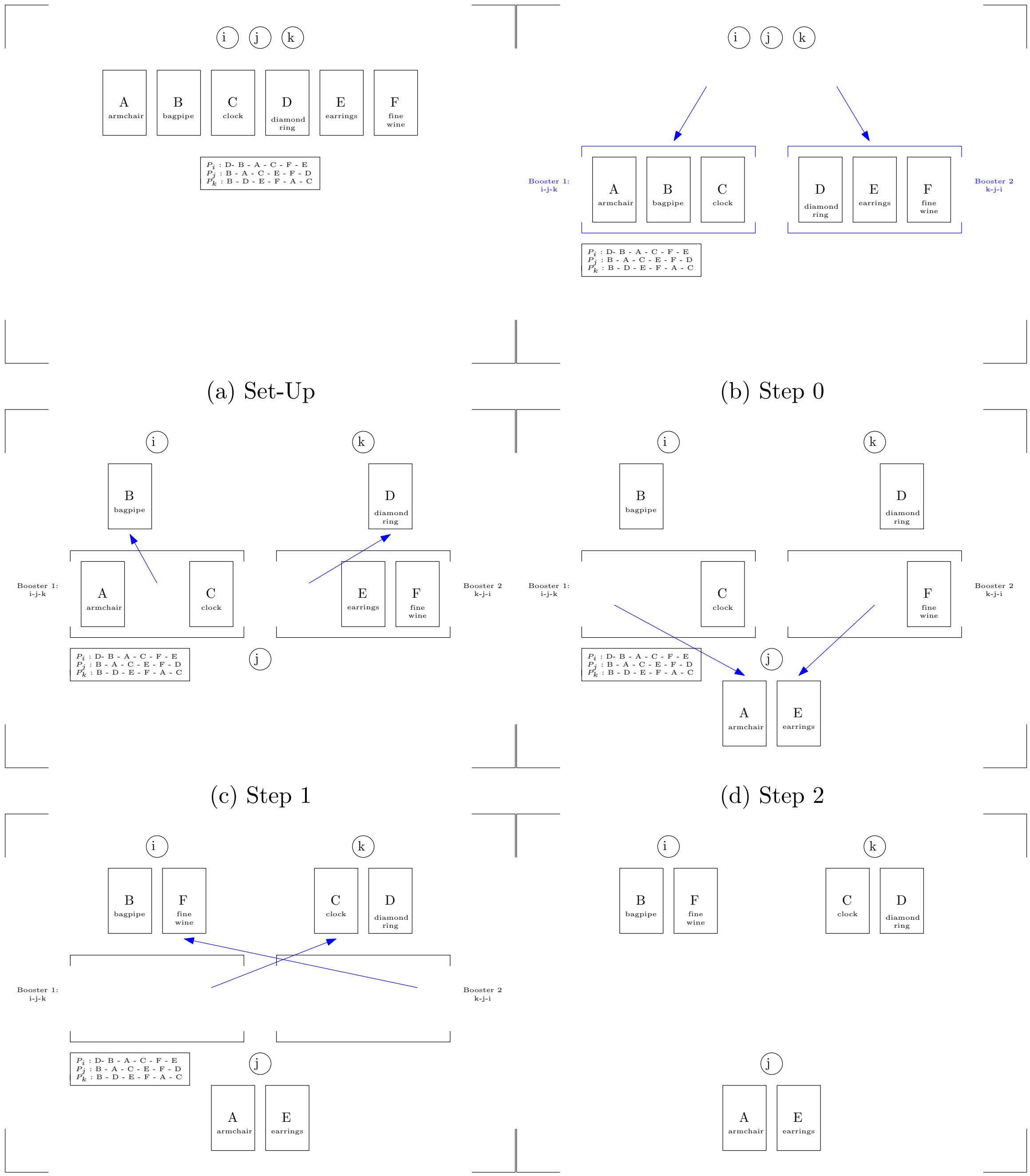
It can be easily verified that the final allocation in this case is $A_i = \{B, E\}$, $A_j = \{A, E\}$, and $A_k = \{C, D\}$.

1.3.2 Properties of the Booster Draft mechanism

We evaluate mechanisms along three dimensions, whether they are manipulable by submitting untruthful rankings, the efficiency of their outcome, and how fair their assignment is ex post. We start by defining the requirement that individuals should not be able to get a better outcome by misrepresenting their true preferences. A simple mechanism ψ is **strategy-proof** if for all $i \in I, P_i, \hat{P}_i \in P_i$, and $P_{-i} \in \mathcal{P}_{-i}$ we have $\psi_i(P) \succsim_i \psi_i(\hat{P}_i, P_{-i})$. Intuitively, we can think of P_i as the truthful report and $\hat{P}_i$ as a possible lie. Strategy-proofness requires that the outcome under a truthful report must be (weakly) preferred to any possible outcome associated with a lie. In order to show that the BD mechanism is strategy-proof we proof the following stronger property: A mechanism ψ is **strongly strategy proof** if for all $i \in I$, $P_i, \hat{P}_i \in \mathcal{P}_i$, and $P_{-i} \in \mathcal{P}_{-i}$ we have $\psi_i(P) \geq_i \psi_i(\hat{P}_i, P_{-i})$. The logic is the same as before, but we require that the outcome is (weakly) preferred under the associated dominance relation $\geq_i$. Together with lemma 1, strong strategy-proofness implies strategy-proofness.

Proposition 1. *The BD mechanism is strongly strategy proof.*

Proof. Suppose by contradiction that there exist $\psi_i(P) \ngeq_i \psi_i(P'_i, P_{-i})$. Then there exists at least one bucket $k \in \{1, \ldots, m\}$ such that $\psi_i^k(P'_i, P_{-1})\ P_i\ \psi_i^k(P)$. But as P_{-i} is fixed



(a) Set-Up

(b) Step 0

(c) Step 1

(d) Step 2

(e) Step 3

(f) Final Allocation

Figure 1.1: Booster Draft Mechanism

all individuals with higher priority will pick identical items in bucket k independent of i reporting P_i or P_i', so i gets to choose from the same set of remaining objects. Hence we have that the obtained item under P_i is weakly preferred to any item obtained by reporting another simple order, i.e. $\psi_i^k(P)\ R_i\ \psi_i^k(P_i', P_{-i})$ for all $k \in \{1, \dots, m\}$ contradicting the initial statement. $\square$

Corollary 1. *The BD mechanism is strategy proof.*

We move on, defining the (ex post) fairness of an outcome. For $j \neq i$ and an outcome of a mechanism $\psi(P) \in \mathcal{A}$ let $\psi(P)_{j,i}^k = \{o \in \psi(P)_j : |\{o' \in \psi(P)_j : o'\ R_i\ o\}| \leq k\}$ denote the set of objects obtained from $\psi(P)_j$ by removing the best k objects according to P_i. Similarly let $\psi(P)_{i,i}^k = \{o \in \psi(P)_i : |\{o' \in \psi(P)_i : o'\ R_i\ o\}| \geq m - k + 1\}$ denote the set obtained by removing the worst k objects from $\psi(P)_i$ following the ranking P_i. We say that mechanism ψ is ***k*-envy free** if for all $P \in \mathcal{P}$ and for all $i, j \in I$ we have $\psi(P)_{i,i}^k \succsim_i \psi(P)_{j,i}^k$. That is, for any individual i that prefers her bundle to another persons bundle j, we can always remove the best k objects from j's bundle and the k worst objects from i's bundle to eliminate i's envy. Note that if both bundles contain only goods, it would be sufficient to remove only k object from j's bundle to eliminate envy of i. Likewise if both bundles only contain bads we could only remove k objects from i's bundle. By removing both simultaneously we do not need to pay attention whether we remove goods or bads. Naturally, envy-freeness is more demanding the smaller the chosen k. We restrict our attention to a subset of BD mechanism that equalize the priorities across individual as much as possible across the available buckets.

That is, for any two individuals $i, j \in I$, we have that i has higher priority than j is at most half of the boosters - rounded up. Formally, a BD mechanism is a **balanced booster draft mechanism** if for all $i, j \in I$ we have $|k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. Let us next state the trivial observation that the set of balanced booster drafts is always non-empty, followed by proposition 3, stating that balanced booster drafts are $\lceil \frac{m}{2} \rceil$ envy-free.

Proposition 2. *The set of balanced BD rules is non-empty.*

Proof. We simply show this by construction for any m boosters. Fix any priority order f^1. For all $i \in I$ let $f^2(i) = n+1-f^1(i)$, i.e. f^2 reverses the order of priority of f^1. For all odd $k \in \{1, 3, \dots\}$ let $f^k = f^1$ and for all even $k \in \{2, 4, \dots\}$ let $f^k = f^2$. For any $i, j \in I$ if i has a lower priority in all odd (even) priorities than j, i has higher priority than j in all even (odd) priorities, hence it directly follows that $|k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. □

Proposition 3. *The balanced BD mechanism is* $\lceil \frac{m}{2} \rceil$ *envy-free.*

Proof. Consider the outcome of any balanced booster draft mechanism $\psi(P)$ where some i envies j. Let $K_i = \{k \in \ \{1, \dots, m\} : f^k(i) < f^k(j)\}$ denote the set of all bucket where i has higher priority than j. Note that every object obtained by i in these buckets must be weakly preferred to any object obtained by j, and hence we have:

$$\bigcup_{k \in K_i} O^k \cap \psi(P)_i \geq_i \bigcup_{k \in K_i} O^k \cap \psi(P)_j$$

Moreover, following P_i, the set obtained by removing the $m - |K_i|$ worst objects from $\psi(P)_i$, denoted by $\psi(P)_{i,i}^{m-|K_i|}$, must weakly dominate the set $\bigcup_{k\in K_i} O^k \cap \psi(P)_i$. Similarly, the set obtained from removing the best $m-|K_i|$, denoted by $\psi(P)_{j,i}^{m-|K_i|}$, objects form $\psi(P)_i$ must be weakly dominated by $\bigcup_{k\in K_i} O^k \cap \psi(P)_j$, and hence:

$$\psi(P)_{i,i}^{m-|K_i|} \geq_i \psi(P)_{j,i}^{m-|K_i|}$$

Note that balancedness implies that $|K_i| \geq \lfloor \frac{m}{2} \rfloor$, i.e. for all $i, j \in I$ we have $|k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. From this it follows directly that the lower bound on envy for each individual i is $m - |K_i| = m - \lfloor \frac{m}{2} \rfloor = \lceil \frac{m}{2} \rceil$. By lemma 1 we have $\psi(P)_{i,i}^{m-|K_i|} \geq_i \psi(P)_{j,i}^{m-|K_i|}$ implying $\psi(P)_{i,i}^{m-|K_i|} \succsim_i \psi(P)_{j,i}^{m-|K_i|}$, which concludes the proof. $\square$

The last criteria concerns efficiency. We say that a simple mechanism ψ is **Pareto efficient** if for each preference profile $P \in \mathcal{P}$ there does not exist a different allocation $A \in \mathcal{A}$ s.t. everyone prefers the allocation to the outcome under the mechanism, i.e., $A_i \succsim_i \psi(P)_i$ for all $i \in I$ and $A_i \succ_i \psi(P)_i$ for at least some $i \in I$. Following the same logic, we introduce a weaker notion of efficiency, requiring that no allocation can make everyone better off under the dominance relation. A mechanism rule ψ is **dominance efficient** if for each $P \in \mathcal{P}$ there does not exist an allocation $A \in \mathcal{A}$ s.t. $A_i \geq_i \psi(P)_i$ for all $i \in I$ and $A_i >_i \psi(P)_i$ for at least some $i \in I$.

We show that dominance efficiency rules out that any number of individuals can trade single objects with each other and all benefit from the exchange. A **(feasible) single object trade**, under allocation A, is a sequence of individual-object pair $(i_1, o_1), (i_2, o_2), \dots, (i_k, o_k)$

with $o_1 \in A_{i_1}, \ldots, o_k \in A_{i_k}$ such that i_2 receives o_1, i_3 receives o_2, so on and so forth, until i_1 receives o_k. An **efficient single object trade** requires that all individual are strictly better off after the trade takes place, i.e., $A_{i_1} \cup \{o_k\} \setminus \{o_1\} \succ_{i_1} A_{i_1}, \ldots, A_{i_k} \cup \{o_{k-1}\} \setminus \{o_k\} \succ_{i_k} A_{i_k}$. We show the following characterization result.

Proposition 4. *Under responsive preferences, an allocation A is dominance efficient if and only if there are no efficient single object trades at A.*

Unfortunately, the increased fairness of the BD rule comes at the cost of loosing efficiency, even in its weaker form.

Proposition 5. *The BD mechanism is not dominance efficient.*

Proof. We proceed by counterexample. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Let ψ be the draft mechanism with $O^1 = \{o_2, o_3\}$, $O^2 = \{o_1, o_4\}$ and priority order $f^1 : 1, 2$ and $f^2 : 2, 1$, where we list individuals in order of assigned priority. Suppose the reported ranking is $P_1 : o_1, o_2, o_3, o_4$ and for individual 1 and $P_2 : o_2, o_1, o_4, o_3$ for individual 2. It can easily be checked that the draft mechanism assigns $\psi(P)_1 = \{o_2, o_4\}$ for individual 1 respectively $\psi(P)_2 = \{o_1, o_3\}$ for individual 2. Consider the outcome A obtained by both individuals switching their assignments, i.e. $A_1 = \psi(P)_2$ and $A_2 = \psi(P)_1$. As $A_1 >_1 \psi(P)_1$ and $A_1 >_\psi (P)_2$ the draft mechanism is not dominance efficient. $\square$

As a final remark, we show an impossibility result via counterexample, illustrating that no mechanism can fulfill weak versions of efficiency, fairness and strategy-proofness. We define our weakening of strategy-proofness, allowing only manipulations in which an individual gets at least one object that is strictly better. A mechanism ψ is **dominance strategy proof** if there does not exist $P_{-i} \in \mathcal{P}_{-i}$ and $P_i, \hat{P}_i \in \mathcal{P}_i$ s.t. $\psi_i(\hat{P}_i, P_{-i}) \geq_i \psi_i(P)$.

Proposition 6. *In the responsive preference domain, there does not exist a simple mechanism that is dominance strategy-proof, dominance efficient, and* $\lceil \frac{m}{2} \rceil$ *envy-free.*

In the appendix, we point out two mechanisms from the literature, one 1-envy free and dominance efficient but manipulable (Harvard business school mechanism) and the other efficient and strategy-proof but k envy-free (serial dictatorship).

1.4 Partition-Restricted Assignment Domain - Characterization

Intuitively, the arbitrary creation of boosters for the booster draft, leads to a lack of efficiency. Likewise, ex-post fairness suffers from the same problem to a lesser degree. But, if additional information about the specific features of the underlying multi-object assignment problem, is incorporated into the construction of boosters, these issues can be mitigated or even avoided. Of course that only works if there is additional structure to be exploited. Going back to our illustrative family heirloom example, suppose that we have n siblings and $3 \times n$ family heirlooms, consisting of n expensive, n medium priced, and n cheap objects. Moreover, everyone prefers the expensive objects to medium priced, and these to cheap ones, but individuals potentially have different valuations within the three categories. In that case, we get a dominance efficient outcome if the objects are grouped together according to their value, and the outcome of the booster draft is 1-envy free. In this simple example we use additional structure on preferences to build boosters.

For this section, our motivation is based on the assignment of graduate students to teaching positions. Here, students are required to work for exactly one course in each semester, placing a restriction on the allowed allocations. More general, we are given an exogenous partition of objects, in which every individual can be assigned at most one object from every set of the partition.

The **partition-restricted multi-object assignment domain** is a multi object assignment problem $\langle I, O, \succ \rangle$ subject to the constraint that every person can be assigned at most one object from every set O^k for a exogenous give partition $(O^k)_{k\in\{1,\dots,m\}}$. For example, we can think of $\{1,\dots,m\}$ as different time periods for m sets of tasks that have to be carried out, but individuals are not able to work on simultaneously on tasks within the same period. Here, $\succ = (\succ_i)_{i\in I}$ is a list of preferences over **schedules** $S = \{O' \in 2^O : |O^k \cap O'| \leq 1 \text{ for every } k \in \{1,\dots,m\}\}$. A feasible, **restricted allocation** $A = (A_i)_{i\in I}$ is a feasible allocation $A \in \mathcal{A}$ s.t. $A_i \in S$ for all $i \in I$. Let $\mathcal{B}$ denote the set of restricted allocations, clearly $\mathcal{B} \subset \mathcal{A}$. Hence, we refer to this as the partition-restricted assignment domain. For the **partition consistent booster draft**, the m booster are simply $(O^k)_{k\in\{1,\dots,m\}}$. Similarly to before, preferences are responsive if for all $k \in \{1,\dots,m\}$, and $o, o' \in O^k$, as well as $O' \in S$ such that $O' \cap O^k = \varnothing$ we have $\{o\} \cup O' \succ_i \{o'\} \cup O'$ if and only if $\{o\} \succ_i \{o'\}$.

Given the restriction, individuals no longer need to indicate their preferences across different sets. Therefore, we require every individual $i \in I$ to submit a list of m rankings $P_i = (P_i^1, \dots, P_i^m)$ where P_i^k is a simple order over O^k. For individual $i \in I$, the set of possible messages is $\mathcal{P}_i$. $P = (P_i)_{i \in I}$ is a list of orders for every individual, with the set of all such message profiles being $\mathcal{P}$. We slightly adjust the definition of the dominance relation, i.e. the partial order connecting the reported rankings with the preferences. We say P_i^k is the underlying ranking over O^k if for all $o, o' \in O^k$ we have that $o \; P_i^k \; o'$ if and only if $\{o\} \succ_i \{o'\}$. Let $o'_k = O' \cap O^k$ be the best object simultaneously in subset $O' \in S$ and O^k. For any two subsets $O', O'' \in S$ with $|O'| = |O''| = m$, we have $O' \geq_i O''$ if and only if $o'_k \; R_i \; o''_k$ for all $k \in \{1, \dots, m\}$. The relation between the original preferences $\succsim_i$ of an individual $i \in I$ and the dominance relation $\geq_i$ based on the submitted order P_i remains unchanged. For those interested we moved the exact statement to the appendix.

In the restricted multi-object assignment domain the draft mechanism is dominance efficient. Moreover, we characterize the set of booster draft mechanism as the set of strongly strategy-proof, non-bossy and neutral mechanisms. Formally non-bossiness and neutrality are defined as follows. First, a simple mechanism ψ is **non-bossy** if $\psi_i(P) = \psi_i(\hat{P}_i, P_{-i})$ then $\psi(P) = \psi(\hat{P}_i, P_{-i})$. Secondly, let $\pi : O \to O$ be a permutations s.t. for all $o \in O^k$ we have $\pi(o) \in O^k$. We permute a list of simple orders P, denoted by πP, as follows: For all $k \in \{1, \dots, m\}$ and $o, o \in O^k$ we have $o \; \pi[P_i^k] \; o'$ if and only if $\pi^{-1}[o] \; P_i^k \; \pi^{-1}[o']$. We say a choice rules ψ is **neutral** if for all $k \in \{1, \ldots m\}$, for all $i \in I$, and for all possible permutations we have $\pi[\psi(P)_i^k] = \psi(\pi P)_i^k$. Lemma 2 is adapted from Svensson (1999), but we need strong strategy-proofness for the result to go through.

Lemma 2. *Let ψ be a non-bossy and strongly strategy-proof mechanism. Consider $P_i, \hat{P}_i \in \mathcal{P}_i$ and $P_{-i} \in \mathcal{P}_i$. Suppose for all $A_i \in \mathcal{A}_i$ s.t. $\psi_i(P) \geq_i A_i$ we have $\psi_i(P) \hat{\geq}_i A_i$. Then $\psi(P) = \psi(\hat{P}_i, P_{-i})$.*

Proof. By strong strategy-proofness we have $\psi_i(P) \geq_i \psi_i(P_i', P_i)$.
By the assumption of the lemma we have $\psi_i(P) \hat{\geq}_i \psi_i(P_i', P_i)$
Using strong strategy-proofness again we get $\psi_i(\hat{P}_i, P_i) \hat{\geq}_i \psi_i(P)$.
Combining the second and third line we get $\psi_i(\hat{P}_i, P_i) = \psi_i(P)$ as this is the only indifference case under the dominance relation $\psi_i(\hat{P}_i, P_i) \hat{=}_i \psi_i(P)$.
By non-bossiness it directly follows that $\psi(P) = \psi(\hat{P}_i, P_{-i})$. □

The set of all **identical preference profiles** is defined as $\mathcal{I} = \{P \in \mathcal{P} : P_j^k = P_i^k \text{ for all } i, j \in I \text{ and for all } k \in \{1, \ldots, m\}\}$. Considering only identical preference profiles, we show that neutrality strongly restricts the way in which individuals can be assigned objects.

Lemma 3. *Let ψ be a neutral mechanism. For every identical preference profile $P \in \mathcal{I}$, $k \in \{1, \ldots, m\}$, and $l \in \{1, \ldots, n\}$ the same individual $i_l^k \in I$ is assigned the lth best choice in O^k according to preference P.*

Proof. Consider the outcome of a neutral mechanism ψ for any two identical preference profile $P \in \mathcal{I}$ and $\hat{P} \in \mathcal{I}$. Let us define the lth best choice in O^k under the identical preference profile P as well as $\hat{P}$: For all $l \in \{1, \dots, n\}$ and $k \in \{1, \dots, m\}$, let o_l^k denote $o \in O^k$ s.t. $|\{o' \in O^k : o' \; R^k \; o\}| = l$ respectively $\hat{o}_l^k$ denote $o \in O^k$ s.t. $|\{o' \in O^k : o' \; \hat{R}^k \; o\}| = l$. Consider the individual i_l^k that is assigned o_l^k under P, i.e. $\psi(P)_{i_l^k}^k = o_l^k$. We want to show that the same individual gets the lth best choice in O^k under any other identical preference profile $\psi(\hat{P})_{i_t^k}^k = \hat{o}_l^k$. Consider the following permutation $\hat{\pi}$ defined for all $k \in \{1, \dots, m\}$ and $l \in \{1, \dots, n\}$ as $\hat{\pi}(o_l^k) = \hat{o}_t^k$. For this particular permutation the following holds true:

Claim 1. We have that $\hat{\pi}(P^k) = \hat{P}^k$ for all $k \in \{1, \dots, m\}$.

Suppose not, then for some $l' < l$ there exists $\hat{o}_{l'}^k \; \hat{P}^k \; \hat{o}_l^k$ such that $\hat{o}_l^k \; \pi[P^k] \; \hat{o}_{l'}^k$. Note that the permuted preference $\hat{o}_l^k \; \pi[P^k] \; \hat{o}_{l'}^k$ is equivalent to the original preference over permuted outcomes $\hat{\pi}^{-1}[\hat{o}_l^k] \; P^k \; \hat{\pi}^{-1}[\hat{o}_{l'}^k]$. But using our defined permutation, this implies $o_l^k \; P^k \; o_{l'}^k$ for $l' < l$ leading to a contradiction.

By neutrality and claim 1 we get $\hat{\pi}[\psi(P)_{i_l^k}^k] = \psi((\hat{\pi}[P]))_{i_t^k}^k = \psi(\hat{P})_{i_t^k}^k$. Moreover by the definition of the permutation $\hat{\pi}$ we have $\hat{\pi}[\psi(P)_{i_l^k}^k] = \hat{\pi}[o_l^k] = \hat{o}_l^k$. Combining both leads the desired conclusion that the same individual gets the lth best object in set O^k for any two identical preference profiles $\psi(\hat{P})_{i_t^k}^k = \hat{o}_l^k$. □

Lemma 3 shows that for identical preference profiles any neutral mechanism can be obtained through a BD mechanism. Note that, in the partition-restricted domain the standard serial dictatorship mechanism is a booster draft where the same individual has the highest priority everywhere, followed by an individual having the second highest priority everywhere and so on and so forth. It remains to be shown, what happens for arbitrary preference profile. We will invoke lemma 2 to show that for any $P \in \mathcal{P} \setminus \mathcal{I}$ there exists an identical preference profile $P \in \mathcal{I}$ leading the same outcome.

Theorem 1. *In the partition-restricted assignment domain with responsive preferences a simple mechanism ψ is strongly strategy proof, non-bossy, and neutral if an only if ψ is a BD choice rule. The outcome of the BD choice rule is dominance efficient.*

Proof. It is obvious that the BD mechanism is strongly-strategy proof (see proposition 1), neutral and non-bossy.

We now show that any strongly strategy proof, non-bossy, and neutral simple mechanism ψ is booster draft. Apply ψ to the subset of identical preference profiles $\mathcal{I}$. By lemma 3 for each $k \in \{1, \dots, m\}$ and $l \in \{1, \dots, n\}$ we can uniquely identify an individual $i_l^k \in I$ that is assigned her t-th choice in O^k according to preference P. Formally the outcome of the BD mechanism is defined for all $k \in \{1, \dots, m\}$ recursively from highest to lowest priority individuals as $\psi_i^k(P) = \{o \in O^k : o \ R_i \ o' \text{ for all } o' \in O^k \setminus \bigcup_{j \in \{j \in I : f^k(j) < f^k(i)\}} \psi_j^k(P)\}$. Indeed assigning the individuals priorities in the same order they obtain the objects from each booster we get that $\psi_i^k(P) = o_t^k$ for all $l \in \{1, \dots, n\}$ and $k \in \{1, \dots, m\}$. Therefore for each identical preference profile $\mathcal{I}$ the outcome of any neutral mechanism ψ is obtained by a booster draft mechanism.

It remains to be shown for any other preference profile that is not identical across agents. Consider a preference profile $\hat{P} \in \mathcal{P} \backslash \mathcal{I}$ and construct an identical preference profile $P(\hat{(P)}) \in \mathcal{I}$ from it as follows: For any O^k with $k \in \{1, \ldots m\}$ the preference P^k order ranks individual i_1^1's first choice highest, and individuals i_1^t first choice among the remaining objects in O^k as t-th highest for $t \in \{2, \ldots, n\}$.

By lemma 2 we can move all agents preferences one-by-one from the constructed identical preference profile $P(\hat{(P)})$ back to the initial preference profile $\hat{P}$ without changing the outcome of the mechanism. Hence for any preference profile the outcome of any neutral, strongly strategy-poof and non-bossy mechanism is a booster draft mechanism.

Finally we show that the outcome of the booster draft mechanism in this domain dominance efficient. Suppose by contradiction there exists $A_i \geq_i \psi(P)_i$ for all $i \in I$ holding strictly for at least one individual. $A_i \geq_i \psi(P)_i$ means that $A_i^k \ R_i^k \ \psi(P)_i$ for all $k \in \{1, \ldots, m\}$ and for all $i \in I$ holding stickily for at least some k and i. If there are multiple, pick the first basket k and the first agents that gets a strictly better object in k. As all agents with higher priority in O^k get the same items as before A_i^k is still available and therefore we have $\psi(P)_i^k \ R_i^k \ A_i^k$ contradicting $A_i^k \ P_i^k \ \psi(P)_i$.

□

The subset of balanced booster draft mechanism is $\frac{m}{2}$ envy-free. As a robustness check for the theorem, note that the result no longer holds in the unrestricted multi-object assignment problem.

As mentioned, in the partition-restricted assignment domain the serial dictatorship mechanism is a special case of the (non-balanced) BD mechanism, which is not true for the responsive domain. So the result doesn't go through for the unrestricted problem, as there exists a mechanism that is strongly strategy-proof, non-bossy and neutral outside the set of BD mechanism.

1.5 Teaching Assignments for Graduate Students

In this section, we take a closer look at the teaching assignment problem, which is an example of the previously described partition-restricted assignment domain. The data consists of the rankings submitted by graduate students in economics at Boston College for the academic year 2018, as well as the final assignment made for that year. Analogous to our theoretical part, everyone separately ranked the available positions for each semester and was assigned a single teaching position for both the fall and spring semester.[17] We simulate the outcome of the balanced booster draft (**BD**) as well as the serial dictatorship (**SD**) for 10000 different priority orders, while the actual assignment (**AA**) remains unchanged. As in the theoretical

[17]For each semester, graduate students give their preferences over ta (teaching assistant) principles, ta statistics, ta econometrics, lab (laboratory) stats, lab econometrics, tf (teaching fellow) principles, and tf statistics. In our data 5 out of 37 students made special arrangements with a specific professor or got a fellowship that freed them of work for one semester. In those cases, we always assigned them their pre-arranged positions before assigning positions to the remaining students based on their reported rankings. As the was a new person in charge of the assignment for 2018 and it was unknown how reported rankings would translate into the final assignment, it is reasonable to expect that graduate students reported their rankings truthfully.

part we want to analyze the different assignments in terms of efficiency and fairness.

For fairness, we care about the percentage of students envying at least one other student, who was given a strictly better assignment in both semesters (2 Envy). For completeness, we also show the percentage of remaining students envying at least on student for his or her assignment in one semester (1 Envy), and the percentage of students getting their two top choices (Envy 0). Figure 2 shows that the balanced booster draft mechanism avoids "2 Envy" altogether, and therefore outperforms serial dictatorship where "2 Envy" is roughly 15%, as well as the actual assignment where "2 Envy" reaches almost 30%.

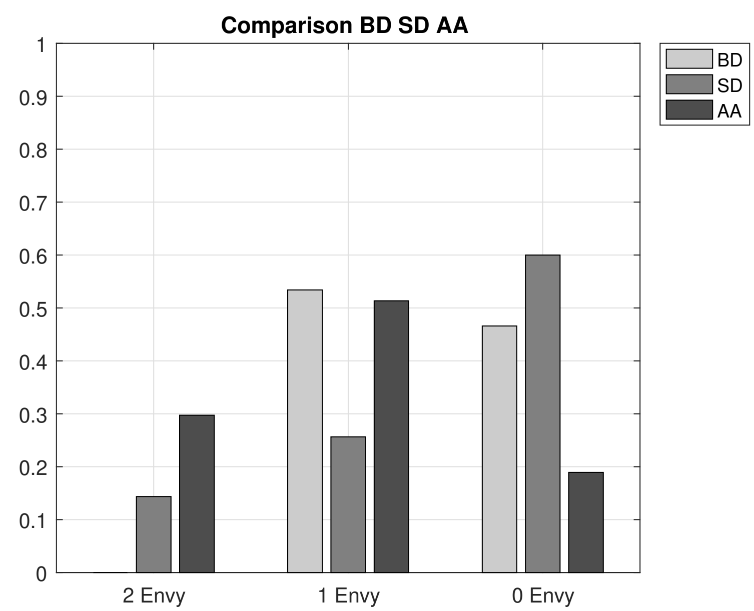


Figure 1.2: Envy

For efficiency figure 3 depict the percentage of individuals that get at least one strictly better and one weakly better assignment in both semesters under one outcome relative to another.

The actual assignment in that year is unsatisfactory in terms of efficiency compared to the two alternatives. On the other hand there is no noticeable difference between serial dictatorship and the balanced booster draft mechanism.

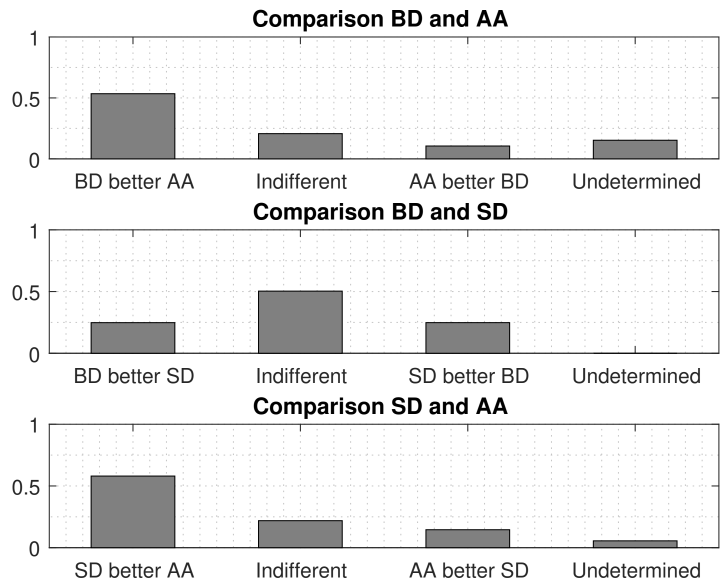


Figure 1.3: Efficiency

Finally, following Budish and Cantillon (2012), we consider the average rank, i.e., a simple measure of welfare to compare the tree alternatives. For example, if a graduate student is assigned her first choice in one semester and her third choice in the other, her rank is four. We then simply average across all students for a given final allocation. Again the results are consistent with the previous analysis in that both serial dictatorship and the booster draft have an average rank of 3.21, while the average rank for the actual assignment is 4.72. Moreover, the balanced booster draft leads to a lower dispersion in terms of rank compared the serial dictatorship.

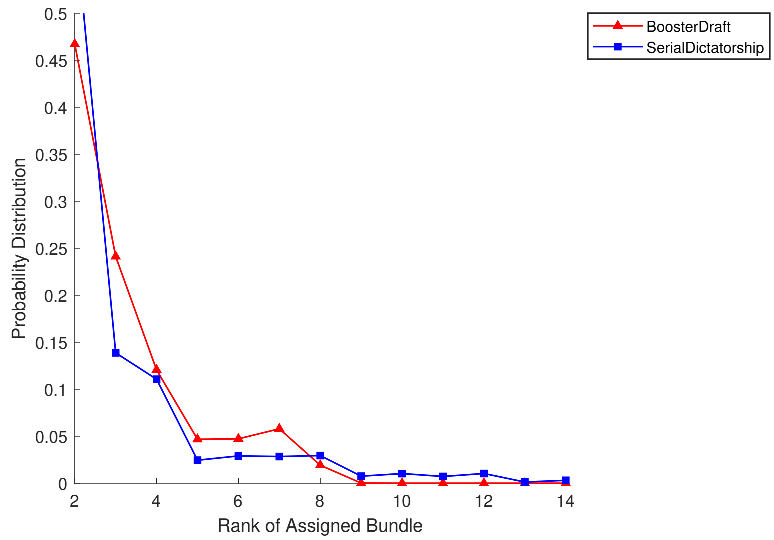


Figure 1.4: Distribution Average Rank

We conclude that if individuals are mildly risk-avers their will prefer the (random) balanced booster draft to the (random) serial dictatorship.

1.6 Dominance Obvious S-P

Following the idea of obviously strategy-proofness introduced by Li (2017), we give some insight in which sense the BD mechanism can be implemented as a extensive form game that is easy to understand. In other words, we think an additional strength of the booster draft lies the mechanisms simplicity. This turns out to be important in practice, as mentioned in the literature review.

As in the previously, I is the set of individuals, $\mathcal{A}$ is the set feasible allocations, and each individual i has a preference relation $\succsim_i$ over the outcomes, which we will sometimes refer to as the **type** of an agent. Preferences are responsive, and assignments are either made in the partition-restricted or the unrestricted domain. A type profile $\succsim = (\succsim_i)_{i \in I}$ specifies a preference relation for each person, and the set of all type profiles is denoted by $\succsim^I$.

Consider an extensive game form where each terminal history z results in some outcome $g(z) \in \mathcal{A}$. For ease of presentation we focus on the special case of deterministic games with finite preference, finite outcomes sets, and complete information.[18] $\mathcal{G}$ denotes the set of all such game forms, with representative element G. Table 1 depicts useful notation.

Name	Notation	Representative Element
histories	H	h
initial history	$h_\varnothing$	
terminal histories	Z	z
outcome resulting from z	$g(z)$	
individual called to play at h	$i(h)$	
information sets for agent i	$\mathcal{I}_i$	I_i
actions available at I_i	$A(I_i)$	$a(I_i)$

Table 1.1: Notation Extensive Form Games

A **strategy** S_i for agent i chooses an action $S_i(I_i) \in A(I_i)$ at every information set. A strategy profile $S = (S_i)_{i \in I}$ specifies a strategy for each agent, and $\mathcal{S}$ denotes the set of all strategy profiles. A **type-strategy profile function** $T : \succsim^I \to \mathcal{S}$ specifies a strategy profile for every type profile. Any persons type-strategy depends only on her own type. $T(\succsim_i) \in \mathcal{S}_i$ refers to the strategy assigned to type $\succsim_i$. Let $z^G(h, S)$ be the terminal histories

[18] In a complete information games every information set is a singleton.

that results in game form G when starting from h and play proceeding according to S.

For a given extensive form game G and a particular type $\succsim_i$, strategy S_i is **weakly dominant** if $\forall S_i'$ and $\forall S_{-i}$ we have $g(z^G(h_\varnothing, S_i, S_{-i})) \succsim_i g(z^G(h_\varnothing, S_i, S_{-i}))$. Similarly we can define the stronger requirement of an obviously strategy-proof strategy profile. For this we first need to introduce some additional notation. For two distinct strategies S_i and S_i', an information set is in the set of **earliest points of departure** $I_i \in \alpha(S_i, S_i')$ if it is on the path of play under both S_i and S_i', and both strategies choose the same action at all earlier information sets but select a different action at I_i. Furthermore let $Z^G(I_i, S_i)$ denote the set of **reachable terminal histories** by playing strategy S_i when starting from information set I_i in game G. Given G and $\succsim_i$, S_i is **obviously dominant** if $\forall S_i'$ and $\forall I_i \in \alpha(S_i, S_i')$ there does not exist $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ such that $g(z') \succsim_i g(z)$.[19]

A **mechanism** is a function $\psi : \succsim^I \to \mathcal{A}$ from type profiles to assignments. A **solution concept** $C(\cdot)$ maps any game G into a subset of strategy profiles $C(G) \subseteq \mathcal{S}$ satisfying the solution concept C. An extensive form game together with a type strategy profile function (G, T) is said to **C-implement** a mechanism ψ if $\forall \succsim \in \succsim^I$ we have $T(\succsim) \in C(G)$ as well as $\psi(\succsim) = g(z^G(h_\varnothing, T(\succsim)))$. Similarly ψ is C-implementable if there exist (G, T) that satisfy the above requirements. In particular a type-strategy profile $T(\succsim) \in SP(G)$ is in the set of **strategy proof (SP)** profiles if for all $i \in I$, $T(\succsim_i)$ is weakly dominant. A type-strategy profile $T(\succsim) \in OSP(G)$ is in the set of **obviously**

[19] Unlike the original definition, our version of obvious dominance is slightly modified, allowing to consider preferences that do not compare all available options in $\mathcal{A}$.

strategy-proof (OSP) profiles if for all $i \in I$, $T(\succsim_i)$ is obviously dominant. Unfortunately the booster draft mechanism is not implementable in a obvious strategy-proof way.

Proposition 7. *In the restricted and unrestricted assignment domain with $m \geq 2$, $n \geq 2$ the balanced BD mechanism is not OSP implementable.*

We weaken the concept of obvious strategy-proofness, by specifying a subset of pairwise comparisons between outcomes, an individual "pays attention to". The more comparisons can be made, the closer the definition is to standard OSP. Here we focus on our dominance relation, though we provide a more general definition, for an arbitrary partial order, in the appendix. Given G and $(\succsim_i, \geq)$, S_i is **dominance obviously dominant** if $\forall S_i'$ and $\forall I_i \in \alpha(S_i, S_i')$ there does not exist $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ such that $g(z') >_i g(z)$. A type-strategy profile $T(\succsim) \in DOSP(G)$ is in the set of **dominance obviously strategy-proof (DOSP)** profiles if for all $i \in I$, $T(\succsim_i)$ is dominance obviously dominant.

Proposition 8. *In the unrestricted domain BD is not DOSP implementable. In the partition-restricted assignment domian the balanced BD mechanism is DOSP implementable.*

Returning to our original motivation, we note that the extensive form game specified by the card version of the booster draft mechanism actually obviously dominance strategy-proof implements the BD mechanism. In other words it provides us with an additional explanation why drafting rules might be easy to understand pointing to their prevalence in practice.

1.7 Conclusion

We have introduced the booster draft mechanism, a new allocation scheme for the multi-object assignment problem, inspired by existing drafting procedures in competitive card games. In the responsive preference domain, the BD mechanism is strategy-proof and envy-free equal to half of the objects, but it is neither dominant efficient nor dominance obvious strategy-proof implementable. In the partition-restricted assignment domain, any neutral, non-bossy, and strongly strategy-proof simple mechanism is a BD mechanism. Moreover, the subset of balanced BD is dominance efficient, strategy-proof, envy-free equal to half of the object, and dominance obvious strategy-proof implementable. We discuss a practical application in the partition-restricted assignment domain, the assignment of graduate students to teaching assistant positions. The simulated assignments support the claims made in the theoretical argument of the paper.

Chapter 2

An Alternative Approach to Asylum Assignment

2.1 Introduction

When arriving in the European Union, an asylum seeker has to launch an application for protection in a single member state. If successful, the person will be granted refugee status or subsidiary protection by the country that examined the asylum claim. The responsible member state cannot be chosen freely. Under the Common European Asylum System (CEAS), most asylum seekers are required to lodge their application for protection in the country in which they initially set foot, following the principle of first entry outlined in the Dublin Regulation (European Commission, 2016*b*).

The Dublin Regulation has been heavily criticized for many shortcomings. It places a disproportionate burden on countries located at the border of the European Union. Its complex bureaucratic approach leads to delays and disputes over responsibility. Its strict no-choice approach incentivizes asylum seekers to engage in illegal secondary movements to reach preferred member states (European Commission, 2016*b*).

We propose an alternative way of determining responsibility for asylum claims, based on the preferences of asylum seekers and priorities of member states with the intent of improving the outcome for both sides. In line with current practices, we allow member states to keep control over their eligibility determination process, and restrict protection seekers to a single application for asylum.[1] Upon initial registration, asylum seekers have to submit their preferences to the local authorities, and a centralized clearing house determines a responsible member state, based on the available information. Compared to the current system, this gives more autonomy to asylum seekers as well as more control to member states over whom to accept.[2] Our proposal is conceptually different from the previous literature as the responsible member state is identified prior to any decision on the asylum application. An obvious advantage of our approach is that its implementation requires relatively few adjustments in the underlying Common European Asylum System.

[1]As pointed out by Jones and Teytelboym (2017*a*), implementing a centralized system will lead to a harmonization of eligibility procedures and refugee status determination decisions.

[2]Member states would be required to rank protection seekers, and criteria could be chosen freely as long as they don't interfere with article 14 of the European Convention on Human Rights (Jones and Teytelboym, 2017*a*), which prohibits discrimination on any ground such as religion or race (European Council of Human Rights, 2013). For example, in 2015, the former premier minister of Britain, David Cameron, announced the acceptance of up to 20'000 refugees from Syria, prioritizing vulnerable children and orphans (BBC, 2015*b*).

In practice, member states often fail to guarantee a person's right to have her asylum claim handled within a reasonable time frame, as required by the Charter of Fundamental Rights of the European Union (Beck, Mole and Reneman, 2014).[3,4] We partially circumvent this problem by asking asylum seekers to express their preferences over potential time frames for the completion of their asylum applications, allowing them ceteris paribus to avoid overburdened member states with significant higher wait times. Moreover, explicitly scheduling a time period opens up the possibility of assigning member states to asylum seekers prior to their arrival on European territory. Contrary to the criteria of first entry, this provision makes our proposal compatible with the idea of humanitarian visas, enabling asylum seekers to reach the state in which they will apply for asylum safely and legally (Neville and Rigon, 2016).[5]

In our framework, member states commit to processing a minimum number of asylum applications during each time period. This burden-sharing quota ensures that every asylum seeker is accommodated by some member state, as guaranteed by the Charter of Fundamen-

[3] We assume that the limited capabilities of member states to process asylum applications can be reasonably approximated by bureaucratic capacity constraints representing the total number of asylum applications that a European country is responsible for handling within a given time frame. As an example, over 11,000 people per month applied for asylum in Italy in 2016, and on average between 6,000 and 8,000 were processed every month. In Greece, around 62,000 people were waiting to have their asylum applications processed, while each month, fewer than 1,000 asylum decisions were being made, with more than that number of asylum seekers arriving (Open Society Initiative, 2016).

[4] The guideline for the time to handle a standard asylum claim is six months, and member states are required to inform asylum seekers when the process takes longer. A statement of the approximate date of completion of an asylum claim does not imply an obligation to handle the claim in the given time frame (Beck, Mole and Reneman, 2014). In the theoretical part we implicitly assume that member states will be able to handle asylum applications on time. Moreover, within reasonable bounds, the bureaucratic constraints should be dictated by the European Union as opposed to be freely chosen by member states.

[5] Humanitarian visas would allow asylum seekers to avoid the dangerous journey across the Mediterranean sea, which has caused more than 15'000 deaths since 2014 (Jilani, 2018).

tal Rights of the European Union (European Commission, 2016*b*). We do not take a stance on how quotas are determined, keeping our proposal compatible with various policies, such as tradable refugee quotas (Moraga and Rapoport, 2014), or the burden sharing rule based on a country's size and GDP outlined in the newest iteration of the Dublin (IV) Regulation (European Commission, 2016*b*). Naturally member states can accept more asylum seekers than they have committed to. Therefore countries unwilling to enter an agreement can nonetheless take part in the centralized matching system.[6]

We present a dynamic model, where at any point in time the asylum seekers present in the system are composed of some exogenously newly arrived individuals and existing asylum seekers waiting for their asylum claim to be processed. We require member states to commit to previously made assignments, which is in line with the "one-time determination" policy proposed by the European Commission.[7] In our case this leads to asylum seekers being assigned exactly once upon initial participation in the matching algorithm. It turns out that even if one develops independently a model to fit in the asylum seeker framework, it can be formulated as an application of the well-known matching with contracts model by Hatfield and Milgrom (2005*b*). This simplifies the analysis a great deal, as matching with contracts is a well studied framework within the matching literature. We show that the

[6]For example, in 2015 the Hungarian, Czech Republic, Slovakian and Polish prime ministers have rejected the idea of quotas for EU nations (BBC, 2015*a*). At the same time Slovakia, Poland, and the Czech Republic announced that if they had to accept refugees they would allow Christian refugees (Wasik and Foy, 2015).

[7]"The Regulation introduces a rule that once a Member State has examined the application as Member State responsible, it remains responsible also for examining future representations and applications of the given applicant. This strengthens the new rule that only one Member State is and shall remain responsible for examining an application and that the criteria of responsibility shall be applied only once." European Commission (2016*a*)

standard cumulative offer mechanism is strategy-proof and leads to stable outcomes, using the fact that our proposed choice functions have a completion (Hatfield and Kominers, 2016) satisfying substitutability and the law of aggregate demand. We implicitly assume that priorities reflect member states preferences, and interpret stability as an equilibrium concept taking into account preferences of asylum seekers, while giving more control to member states over who to accept. In particular, stability avoids situations where an asylum seeker prefers her claim to be handled by a member state other than the one to which she is assigned under the asylum system, and the more desirable member state is willing to accept her. This requirement is clearly violated under the Dublin Regulation, as assignments are made without any consideration of preferences from either side.[8] Strategy proofness implies that it is in every asylum seeker's best interest to state their preferences truthfully, as there are no gains from manipulation. For this result the commitment of member states to previous made arrangements is crucial. On the other side, the outcome is manipulable only by over-demanded member states. We do not consider this to be an issue. In particular, member states preferences reflect the will of the political parties, which are in charge of formulating a priority system. In any European member state political parties belong to the legislative, while the bureaucrats mandated with the implementation of the asylum process are part of the executive, and hence are thought of as being neutral and independent of any influence by the legislative.

[8]In 2015, the Germany government allowed all Syrian refugees that made it to German soil to apply for asylum in Germany. This suspended the Dublin Regulation, as most of the Syrian refugees were assigned a different responsible member state following the principle of first entry (Hayden, 2015). It should also be noted that several months later Germany stopped its initial proposal after being overwhelmed by the sheer number of arriving asylum seekers (Harding, 2015).

2.2 Literature

To the best of our knowledge Schuck (1997) was the first to focus directly on the management of refugee flows, taking the suffering of refugees and the root causes of their flights as tragically given. He proposed that each state bears a share of responsibility for temporary protection and permanent resettlement of refugees based on a quota, and all governments can pay another to fulfill their obligations.[9]

Moraga and Rapoport (2014) developed this idea further, proposing a system whereby states trade quotas multilaterally without exchange of money. They envisioned this approach for the resettlement of longstanding refugees, as well as for refugees and asylum seekers within the European union (Fernández-Huertas Moraga and Rapoport, 2015). Furthermore, they were the first to explicitly mention the possibility of combining a quota system with a matching mechanism.

Jones and Teytelboym (2016, 2017*a*,*b*) informally discussed in great detail how the methods of matching could be applied to improve or replace practices currently in place. They distinguish between the global refugee match, on an international level, and the local refugee match, on a community level within a country. The latter was rigorously analyzed by David Delacretaz, Alexander Teytelboym and Scott Kominers (2016), leading to the introduction of a new framework for matching with multidimensional constraints. Moreover, Trapp et al. (2018) develop a software tool that assists a resettlement agency in the United States with

[9] More recently burden sharing quotas have also found their way into the current 4th iteration of the Dublin Regulation (European Commission, 2016*b*).

matching refugees to their initial placements. Similarly to David Delacretaz, Alexander Teytelboym and Scott Kominers (2016), Andersson et al. (2018) proposed a dynamic model of assigning refugees to localities based on types. To the best of our knowledge, we are the first to provide a theoretical model for the international refugee match and to introduce wait times. Furthermore, our proposed matching is conceptually different from those of previous studies in that it requires any assignment to be made prior to the determination of refugee status.

As pointed out by Jones and Teytelboym (2017*a*,*b*) in the international context, due to the "thickness" of the market, the refugee problem can be reasonably modeled as a standard school choice problem (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003*b*). In other words, we do not need to be concerned whether an asylum seeker in our model represents an individual or a family. In general the asylum seeker problem falls into the category of application-focused matching problems, like the matching of residents with hospitals (Roth, 1984), or of cadets with military branches (Sönmez and Switzer, 2013*b*). The latter relates to our paper in that the cadet branch assignment problem turns out to fit the matching with contract model (Hatfield and Milgrom, 2005*b*), and illustrates the practical importance of the unilateral substitutability condition (Hatfield and Kojima, 2010*b*). Showing the relevance for hidden substitutability (Hatfield and Kominers, 2016) in a practical problem is an important contribution of our paper. Our set-up bears some similarities to the German entry-Level labor market for lawyers application proposed by Dimakopoulos, Heller et al. (2015), as both make use of designating wait times. But while our model is dynamic, they look at a single period. Moreover, one can check that hidden substitutability (Hatfield and Kominers, 2016) will suffice for their set-up, without imposing any monotonicity assumption on preferences over waiting times.

In our model the choice functions of member states violate the unilateral substitute condition (Hatfield and Kojima, 2010*b*) and law of aggregate demand (Hatfield and Milgrom, 2005*b*), but there exists a completion of the choice function that satisfies substitutability (Hatfield and Milgrom, 2005*b*; Kelso Jr and Crawford, 1982) and the law of aggregate demand. Therefore, this paper is part of a small body of literature using or directly applying this result (Aygun and Turhan, 2016; Hassidim, Romm and Shorrer, 2017; Yenmez, 2018). We utilize the irrelevance of rejected contracts conditions (Aygün and Sönmez, 2013) to show that stability implies our version of two-sided efficiency and employ the widely used cumulative offer mechanism (Hatfield and Kojima, 2010*b*), originally introduced as deferred acceptance algorithm by Gale and Shapley (1962*b*).

Finally, our work is related to a large literature on assigning people to tasks in operation research (Pentico, 2007), as well as dynamic matching problems (Kotowski et al., 2015; Kurino, 2014; Ünver, 2010), where our dynamic set-up is most similar to Pereyra (2013), who discusses the placement of teachers with tenure.

In section 3, the model is introduced, followed by a discussion of the choice functions of member states, as well as the solution concepts used and the cumulative offer mechanism to find a solution. Section 4 provides conclusions of the study. All proofs together with an illustrative two period example can be found in the appendix.

2.3 The Asylum Seeker Matching Problem

In the following we outline the dynamic asylum seeker problem. In regular time intervals member states rank all newly arrived individuals, while the asylum seekers submit their preferences over combinations of processing times and member states. Afterwards asylum seekers are assigned to member states and wait times based on the information provided by both sides. In accordance with the CEAS, asylum seekers are restricted to a single application for protection, making it a many-to-one matching problem. European countries keep control over their asylum application process but are required to make an asylum decision in the agreed upon time period. When considering how many applications to accept they are constrained by their bureaucratic capacity, representing the maximum number of asylum claims that can be handled in a given time period. We think of these capacities as not being chosen freely by the countries, but rather dictated to them by the relevant authorities. To ensure individuals rights to have their asylum application examined, member states take part in a burden sharing agreement sufficiently large to encompass all incoming asylum seekers. With these considerations in mind, we are now ready to formally introduce the model.

2.3.1 The Model

A **dynamic asylum seeker matching problem** goes on for some finite time periods T, where

1. $N = (N_t)_{t \in T}$ are the sets of newly arriving asylum seekers each period, with $A = \bigcup_{t \in T} N_t$,
2. M is the finite set of member states,
3. $W = T \cup \{\bar{w}\}$ is the set of period in which asylum decisions can be concluded, with $\bar{w}$ representing any period after $|T|$ and $W_t = \{s \in T : s \geq t\} \cup \{\bar{w}\}$,
4. $q = (q_{t,m})_{t \in T, m \in M}$ is a list of burden sharing quotas with $\sum_{m \in M} q_{t,m} \geq |N_t|$ for all $t \in T$,
5. $r = (r_{m,w})_{m \in M, w \in W}$ represents the number of asylum claims a member state is required/able to process in a given period with $r_{m,\bar{w}} \geq |A|$ for all $m \in M$,
6. $P = (P_a)_{a \in A}$ is a list of preference rankings over $(M \times W) \cup \{\varnothing\}$,
7. $> = (>_{t,m})_{t \in T, m \in M}$ is a list of priorities for every period, where $>_{t,m}$ orders $N_t \cup \{\varnothing\}$.

To encompass waiting times we use a matching with contracts framework. A **contract** $x = (a, m, w) \in X = (A \times M \times W) \cup \{\varnothing\}$ specifies an asylum seeker $a \in A$, a member state $m \in M$, and a time by which an asylum decision will be made $w \in W$. Given a contract x, let $a(x)$ represent the asylum seeker, $m(x)$ the member state, and $w(x)$ the wait time specified in contract x. For every subset of contracts $X' \subseteq X$ we use $X'_a = \{x \in X' : a(x) = a\}$ to denote the set of contracts asylum seeker a is part of, where equivalent notation is used for member states as well as wait times. $A(X') = \{a \in A : a(x) = a \text{ for some } x \in X'\}$ denotes the asylum seekers specified in set $X' \subseteq X$ again equivalently defined for member states and wait times.

As this is a dynamic model we need to keep track of the current asylum seeker population, as well as any assignment made in a previous period. For all $t \in T$ we let $A_t = E_t + N_t$ denote the set of all **asylum seekers present** at the beginning of period t consisting of **existing asylum seekers** E_t awaiting a decision and the **newly arrived asylum seekers** N_t not currently assigned to any member state. A **period *t* allocation** $Y_t \subseteq X_t = A_t \times M \times W_t$ is a set of contract with $|Y_{t,a}| \leq 1$ for all $a \in A_t$, and $|Y_{t,m,w}| \leq r_{m,w}$ for all $m \in M$ and $w \in W_t$.[10] Let $\mathcal{Y}_t$ denote the set of all period t allocations. In words, a period t allocation specifies for every current asylum seekers a responsible member states and a wait time, while ensuring that no bureaucratic capacity is violated. We set $Y_0 = \varnothing$, respectively $\mathcal{Y}_0 = \{\varnothing\}$, while the period 1 the set of existing asylum seekers is empty $E_1 = \varnothing$ and for all $t \in T$ with $t \geq 2$ and any $Y_{t-1} \in \mathcal{Y}_{t-1}$ we have $E_t = \{a \in A : w(Y_{t-1,a}) \geq t\}$. Therefore looking at the previous period allocation any agent that is still awaiting her decision is an existing asylum seeker. Note that they will use up some bureaucratic capacity $r_{m,w}$ reducing the new asylum seekers that can be processed in the same time. For completeness we say that a **dynamic allocation** is a sequence of period t allocations $Y = (Y_t)_{t \in T}$, where $\mathcal{Y} \equiv \mathcal{Y}_1 \times \ldots \times \mathcal{Y}_{|T|}$.

[10] The set of contracts available at a particular point in time t is $X_t = A_t \times M \times W_t \subseteq X$, where any subset of these contracts is denoted by $X'_t \subseteq X_t$.

Every asylum seeker has a strict preference relation P_a over $M \times W$, with weakly preferred written as R_a. It is implicitly assumed that asylum seekers preferences do not change over time. Given an asylum seeker $a \in A$ and an allocation Y_t with $Y_{t,a} = \{(a, m, w)\}$ we refer to the pair (m, w) as the *assignment* of applicant a under Y_t. Slightly abusing notation we will use P_a for preferences over contracts as well as assignments.[11]

2.3.2 Choice Function

We define member states behavior in each period using a choice function based on the corresponding priorities. A choice function $C_{m,t}(X'_t|Y_{t-1}) \subseteq X'_t$ simply gives a subset of accepted contracts for any possible selection of options $X'_t \subseteq X_t$. We requires member states to commit to any previously accepted contract, and to not accept asylum seekers they are not responsible for, introducing a dependence on the allocation of the previous period. We next outline the choice functions, while a more rigorous definition can be found in the appendix. A **choice function** $C_{t,m}(X'_t|Y_{t-1}) \subseteq X'_t$ determines the set of selected contracts as follows:

[11] For any $y, z \in X_t$ we have $y \; R_a \; z$ if and only if $(m(y), w(y)) \; R_a \; (m(z), w(z))$ respectively for any $Y_t, Y'_t \in \mathcal{Y}_t$ we say $Y_t \; R_a \; Y'_t$ if and only if $Y_{t,a} \; R_a \; Y'_{t,a}$.

Step 0.

- Accept all contracts accepted in the previous period $X'_{t,m} \cap Y_{t-1}$.
- Reject all other contracts available previously $X'_{t,m} \cap X_{t-1} \setminus Y_{t-1}$, and all contracts specifying different member states $X'_t \setminus X'_{t,m}$.

Step $j \geq 1$. Consider all contracts of the highest priority applicant a_j among the remaining. If there is no such asylum seeker the algorithm stops.

a) If $\varnothing >_{t,m} a_j$ and there are already at least $q_{t,m}$ contracts specifying newly arrived asylum seekers accepted the algorithm stops.

b) Otherwise (if possible) accept exactly one contract specifying a_j for which the maximum bureaucratic capacity $r_{m,w}$ has not yet been reached, and reject all other contract involving a_j. (Assume the lowest feasible waiting time contract is chosen.)

If the member state has multiple contracts with the same individual at most one contract for which the bureaucratic capacity has not been filled yet is picked. For convenience we assume that the contract with the lowest waiting time available is accepted. We note that this assumption is not relevant for the obtained results. In fact any other order in which a contract for the same asylum seeker is chosen will suffice, and the order could also depend on previously accepted contracts during the algorithm describing the choice function. We let $C = (C_t)_{t \in T}$ denote the list of all choice functions, where $C_t = (C_{t,m})_{m \in M}$ is a list of period t choice function.

2.3.3 Cumulative Offer Mechanism, Stability, Strategy-Proofness, Efficiency

We will present some standard properties adjusted to our dynamic framework. A **direct mechanism** φ is a function $\varphi : \mathcal{P} \times \mathcal{C} \to \mathcal{Y}$ selecting a sequence of allocations for each profile of preferences and choice functions. We use $\varphi_t(P, C) \in \mathcal{Y}_t$ to denote the assignment Y_t induced by mechanism φ, when $P \in \mathcal{P}$ is the reported preference profile and C are the choice function based on the priorities $>$. A direct mechanism is **dynamically (asylum seeker) strategy proof** (for asylum seekers) if there does not exist $a \in A$, $P_a, \hat{P}_a \in \mathcal{P}_a$, $P_{-a} \in \mathcal{P}_{-a}$ and $C \in \mathcal{C}$ such that $\varphi_t(\hat{P}_a, P_{-a}, C)\ P_a\ \varphi_t(P, C)$. This is a standard requirement making truth-telling a dominant strategy, an no asylum seeker can get a better outcome by submitting an incorrect preference relation. Its worth noting that we have defined strategy proofness only for a subclass of mechanisms where asylum seekers submit a preference once. Theoretically new preferences could be elicited every period even though this would hardly be practical.

An allocation $Y = (Y_t)_{t \in T}$ is **dynamically stable** if for all $t \in T$ we have

i) $Y_{t,a}\ P_a\ \varnothing$ for all $a \in A_t$,

ii) $C_{t,m}(Y_t | Y_{t-1}) = Y_{t,m}$ for all $m \in M$,

iii) and there does not exist a pair $a \in A$, $m \in M$, and a contract $x \in X_t \setminus Y_t$ such that $x\ P_a\ Y_{t,a}$ and $x \in C_{t,m}(Y_t \cup \{x\} | Y_{t-1})$.

As previously discussed our main aim it to have an strategy-proof and stable assignment process, as stability takes into account asylums seekers preference as well as member states priorities. In particular stability (almost) implies the following notion of efficiency. An allocation $Y = (Y_t)_{t \in T}$ is **two sided efficient** if there does not exist an allocation $Y' = (Y'_t)_{t \in T}$ with $Y' \neq Y$ s.t. $Y'_{t,a} \; R_a \; Y_{t,a}$ for all $a \in A$ and $t \in T$ and $C_{t,m}(Y'_t \cup Y_t | Y'_{t-1}) = Y'_{t-1}$.

We going to apply the asylum seeker optimal stable algorithm, which was first introduced by Gale and Shapley (1962*b*) and adapted to the matching with contracts by Hatfield and Kojima (2010*b*).

For $t = T$ the outcome $\psi^c_t(P, C)$ of the asylum seeker proposing **cumulative offer** algorithm ψ^c is defined as follows:

Step $k \geq 1$.

- If there exists at least a single applicant $a \in A_t$ currently not being assigned a member state, let one of them propose her preferred contract according to P_a that has not yet been rejected, $x^k \in X_t \setminus X^k_t$. Set $X^{k+1}_t = X^k \cup \{x^k\}$ where $X^0 = \varnothing$. Member state $m = m(x^k)$ *holds* all contracts in $C_{t,m}(X^k_t | Y^{t-1})$ and *rejects* all other contracts in $X^k_t \setminus C_{t,m}(X^k_t | Y^{t-1})$.
- Otherwise the process terminates with $\psi^c_t(P, C) = \bigcup_{m \in M} C_{t,m}(X^K_t | Y^{t-1})$.

Our main result of this paper is as follows:

Theorem 2. *The outcome of the asylum seeker proposing cumulative offer mechanism ψ^c is dynamically stable and dynamically (asylum seeker but not member state) strategy-proof and two-sided efficient.*

As mentioned, we do not consider the lack of strategy proofness for member states a deal breaker, as the division of power between legislative and executive is designed to naturally address the issue at hand. In the next subsection we will discuss our main result. For those interested we provide a simple 2 period example of our model in the appendix, illustrating the deferred acceptance algorithm.

2.3.4 Conditions on Choice Functions

We introduce a condition on choice functions for the dynamic model, ensuring that asylum seekers can not switch contracts over time as they are awaiting their status determination, requiring member states to commit to any previously made allocation. A choice function $C_{t,m}(\cdot|Y_{t-1})$ satisfies **one-time determination** if $x \in C_{t,m}(X'_t|Y_{t-1})$ for all $x \in X'_{t,m} \cap Y_{t-1}$, and $x \notin C_{t,m}(X'_t|Y_{t-1})$ for all $x \in X'_{t,m} \cap X_{t-1} \setminus Y_{t-1}$. In our results this requirement is only used for strategy proofness as it is not necessary for stability. Intuitively without commitment asylum seekers can be accepted by member states that previously rejected them, creating an incentive to rank long wait time contracts higher. Similarly asylum seekers could be excluded from a contract to which they were assigned in a previous period, due to newly arriving asylum seekers, which incentives them to rank short wait time contracts higher than under their true preferences. Both reasons make one-time determination a crucial condition for strategy proofness in our dynamic set-up.

We now move on to standard conditions for showing that the outcome of the cumulative offer mechanism is stable and strategy-proof. Hence we refrain from discussing them detail. For all $t \in T$ we say a choice function $C_{t,m}(\cdot|Y_{t-1})$ satisfies the **law of aggregate demand** if for $X_t' \subseteq X_t$ and $x \in X_t \setminus X_t'$ we have $|C_{t,m}(X_t' \cup \{x\}|Y_{t-1})| \geq |C_{t,m}(X_t'|Y_{t-1})|$. A choice function $C_{t,m}(\cdot|Y_{t-1})$ is **substitutable** if there does not exist $X_t' \subseteq X_t$ and $x, z \in X_t \setminus X_t'$ such that $x \notin C_{t,m}(X_t' \cup \{x\}|Y_{t-1})$ and $x \in C_{t,m}(X_t' \cup \{x, z\}|Y_{t-1})$. A choice function $C_{t,m}(\cdot|Y_{t-1})$ satisfies the **irrelevance of rejected contracts** condition if for all $X_t' \subseteq X_t$ and $x \in X_t \setminus X_t'$ we have $x \notin C_{t,m}(X_t' \cup \{x\}|Y_{t-1})$ implies $C_{t,m}(X_t' \cup \{x\}|Y_{t-1}) = C_{t,m}(X_t'|Y_{t-1})$.

Lemma 4. *Consider any direct mechanism $\psi(P, C)$ producing a stable allocation. Whenever the choice functions $C_{t,m} \in C$ satisfy irrelevance of rejected contracts the outcome satisfies two sides efficiency.*

Our choice functions do not satisfy substitutability nor the law of aggregate demand. In the following counterexample we again assume that the lowest available waiting time contract is chosen if there are multiple contracts available specifying the same asylum seeker. We note that it is straightforward to construct an equivalent violation for any other order in which a member state might select a contract when multiple are available for the same individual.

Example 2. Consider the set of contracts $X_t = \{(a_1, m_1, t), (a_1, m_1, t + 1), (a_2, m_1, t), (a_2, m_1, t + 1), \dots\}$, where $a_1, a_2 \in N_t$ and $m_1 \in M$. For the choice function $C_{t,m_1}(\cdot|Y_{t-1})$ assume $q_{t,m_1} = 2$, $r_{m_1,t} = r_{m_1,t+1} = 1$, and $a_1 >_{t,m_1} a_2$.

To see that the choice function violates substitutability consider $X_t' = \{(a_1, m_1, t + 1)\}$, $x = (a_2, m_1, t + 1)$, and $z = (a_1, m_1, t)$. We have that $x \notin C_{t,m_1}(X' \cup \{x\}|Y_{t-1})$ but

$x \in C_{t,m_1}(X'_t \cup \{x,z\}|Y_{t-1})$.[12]

Similarly to find a violation of the law of aggregate demand consider $X'_t = \{(a_1, m_1, 2), (a_2, m_1, 1)\}$ and $x = (a_1, m_1, 1)$. Then we have $|C_{t,m_1}(X'_t)| = |X'| > |C_{t,m_1}(X'_t \cup \{x\})| = |\{x\}|$. [13]

We show that there exists a completion (Hatfield and Kominers, 2016) of our proposed choice function that satisfies the desired properties. We formally state the concept of completion, before establishing the lemma which we use to prove theorem 2. A **completion** of a choice function $C_{t,m}(\cdot|Y_{t-1})$ is a choice function $C'_{t,m}(\cdot|Y_{t-1})$ such that for all $X'_t \subseteq X_t$, either $C'_{t,m}(X'_t|Y_{t-1}) = C_{t,m}(X'_t|Y_{t-1})$, or there exist distinct $x, x' \in C'_{t,m}(X'_t|Y_{t-1})$ that are associated with the same asylum seekers, i.e. $a(x) = a(x')$.

Lemma 5. *The choice function $C_{t,m}(\cdot|Y_{t-1})$ satisfies irrelevance of rejected contracts and has a completion $C'_{t,m}(\cdot|Y_{t-1})$ satisfying substitutability and the law of aggregate demand (and irrelevance of rejected contracts).*

Both lemmas are used to establish theorem 2. Lemma 5 establishes that there exists a completion satisfying substitutability and the law of aggregate demand used to establish strategy-proofness and stability. Moreover, using lemma 4, the outcome satisfies our definition of two sided efficiency.

[12] The choice function also violates the weaker **unilateral substitutes** condition Hatfield and Kojima (2010*b*), which toghether with the law of aggregate demand is sufficient for stability and agent strategy proofness. A choice function $C_{t,m_1}(\cdot|Y_{t-1})$ is *unilateral substitutable* if there does not exist $X'_t \subseteq X_t$ and $x, z \in X_t \setminus X'_t$ such that $a(x) \notin A(X'_t)$ and $x \notin C_{t,m_1}(X'_t \cup \{x\}|Y_{t-1})$ but $x \in C_{t,m_1}(X'_t \cup \{x,z\}|Y_{t-1})$. This example is also a violation of the unilateral substitutability condition as $a(x) \notin A(X'_t)$.

[13] Dimakopoulos, Heller et al. (2015) use a similar example for the lawyer match.

2.4 Conclusion

The Dublin Regulation leads to an inefficient European Asylum System. As an alternative, we propose to match asylum seekers to member states based on their preferences and member states priorities improving the outcome on both sides, while not interfering with member states control over their eligibility determination process. An important observation is that the independently developed asylum seeker framework can be modeled as an application of the matching with contracts model. We set up a dynamic model and show that the cumulative offer mechanism is stable, (asylum seeker) strategy proof, and two sided efficient. The key is to recognize that there exists a completion of the proposed member state choice function that satisfies the law of aggregate demand and the substitutability condition. This illustrates the practical importance of the hidden subsitutability framework. Moreover, for dynamic stategy-proofness we need member states to commit to all contracts signed in earlier periods. This aligns with the European Unions demand that the responsible member state is determined only once upon initial arrival of an asylum seeker. We suggest that the asylum seeker problem might be a fruitful area for future research, as we abstracted away from many complexities that are not present in the standard school choice framework.

Appendix A

Multi-Object Assignment: Booster Draft

A.1 Mathematical Appendix

A.1.1 Section 3

Proposition 4: Characterization Dominance Efficiency

Proof. We start with the if-statement. Suppose that it does not hold, then the allocation A is dominance efficient, but there exists a efficient single object trade. One can easily confirm that, under responsive preferences a single object trade $(i_1, o_1), (i_2, o_2), \dots, (i_k, o_k)$ makes every individual involved $\{i_i, \dots, i_k\}$ better off under the dominance relation, i.e., $A_{i_1} \cup \{o_k\} \setminus \{o_1\} >_{i_1} A_{i_1}, \dots, A_{i_k} \cup \{o_{k-1}\} \setminus \{o_k\} >_{i_k} A_{i_k}$. Hence carrying out the trade makes all individual in the trade strictly better of under the dominance relation while everyone else is indifferent, contradicting that A is dominance efficient.

Next consider the only-if-statement. Suppose it is does not hold, then there exists no efficient single object trade at A, but A is not dominance efficient. Let A' be an allocation that is dominant efficient relative to A and consider the following argument.

Step 0. Pick any individual i for which $A' >_i A$ and call it i_k. Pick the best object $o \in A'_{i_k} \setminus A_{i_k}$ and call it o_{k-1}. This object must have been assigned to a different person under A. Call that person i_{k-1} and go to the next step.

Step t. Consider individual i_{k-t}. Pick the best object $o \in A'_{i_{k-t}} \setminus A_{i_{k-t}}$ and call it o_{k-t-1}. This object must have been assigned to a different person under A. If the individual is in $\{i_{k-t+1}, \ldots, i_k\}$ we found a efficient single object trade, starting from the original distribution A, and hence reach a contradiction. Otherwise call that person i_{k-t-1} and go to the next step.

As the set of individuals is finite we reach a contradiction after a finite number of steps. Every person in the circle get his/her best object among new ones. □

Proposition 6: Impossibility Result

Counterexample. Let $I = \{i, j\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose ψ is a dominance strategy-proof, dominance efficient and $\lceil \frac{m}{2} \rceil = 1$ envy-free mechanism. We abbreviate rankings P_i as 1234_i to represent $o_1\ P_i\ o_2\ P_i\ o_3\ P_i\ o_4$. Similarly for an allocation A with $A_i = \{1, 2\}$ and $A_j = \{3, 4\}$ we simply write $(12, 34)$.

Case 1: $\psi(1234_i, 1234_j) = (12, 34)$

This outcome violates $\lceil \frac{m}{2} \rceil = 1$ pick envy freeness. We have that j envies i's assignment as $\{12\} >_j \{34\}$ and even after removing the best object from i and the worst form j the envy prevails as $\{1,2\} \setminus \{1\} >_j \{3,4\} \setminus \{4\}$, respectively $\{2\} >_j \{3\}$.

Case 2: $\psi(1234_i, 1234_j) = (34, 12)$

The reasoning is symmetric to the one in case 1.

Case 3: $\psi(1234_i, 1234_j) = (24, 13)$

Observation 1: $\psi(1234_i, 3214_j) = (14, 23)$. Individual j can always change her preference from 3214_j to 1234_j and get $\{1, 3\}$. This leaves us already with only two possible outcomes of the mechanism either $(24, 13)$ or $(14, 23)$. Given dominance efficiency we must have $\psi(1234_i, 3214_j) = (14, 23)$ as $\{1, 4\} \geq_i \{2, 4\}$ and $\{2, 3\} \geq_j \{1, 3\}$.

Observation 2: $\psi(1234_i, 2341_j) = (14, 23)$. By observation 1, any other outcome would violate dominance strategy proofness as j can switch back to 3214_j and enforce her best outcome $\{2, 3\}$.

Observation 3: $\psi(2314_i, 1234_j) \neq (34, 12)$. This follows by dominance strategy profness at otherwise i would be better of under preferences consistent with 2314_i to report 1234_i instead and get $\{2, 4\}$ instead of $\{3, 4\}$.

Observation 4: $\psi(2314_i, 2341_j)$ has no outcome is consistent with the required criteria.

- Naturally $(14, 23)$ and $(23, 14)$ are both violating 1 envy freeness, as i respectively j

get their worst two objects.

- By dominance strategy proofness we can rule out $(34, 12)$. For this we need to note that by observation 3 we have $\psi(2314_i, 1234_j) \neq (34, 12)$, and therefore fixing i's ranking no other preference of j can ever give her the outcome $\{1, 2\}$.
- Using observation 2 and the same logic we can rule out $(12, 34)$ and $(13, 24)$ as $\psi(1234_i, 2341_j) = (14, 23)$, fixing j's ranking we can never have that i gets $\{1, 3\}$ or $\{1, 2\}$.
- This leaves us with $\psi(2314_a, 2341_b) = (24, 13)$. Consider $(12, 34)$ which can be reached by letting i and j trade o_1 and o_4 making both strictly better off and hence violating dominance efficiency.

This leads to the conclusion that no dominance strategy-proof, dominance efficient, and 1 pick envy free can every assign $\psi(1234_a, 1234_b) = (24, 13)$.

Case 4: $\psi(1234_i, 1234_j) = (13, 24)$

We can symmetrically follow the previous reasoning of case 3.

Case 5: $\psi(1234_a, 1234_b) = (23, 14)$

Observation 1: $\psi(2341_a, 1234_b) = (23, 14)$. This simply follows from dominance strategy proofness as under 2341_a we have $(23, 14) = \psi(1234_a, 1234_b) >_a \psi(2341_a, 1234_b) \neq (23, 14)$.

Observation 2: $\psi(2341_a, 2314_b)$ has no outcome is consistent with the required criteria.

- Outcomes $(14, 23)$ and $(23, 14)$ are both violating 1 envy freeness.

- Outcome $(12, 34)$ is dominated by $(24, 13)$ similarly $(13, 24)$ is dominated by $(34, 12)$, hence both would violate dominance efficiency.

- The last two possible outcomes violate dominance strategy proofness as under 1234_j we have $(34, 12)$ or $(24, 13) = \psi(2341_a, 2314_b) >_j \psi(2341_a, 1234_b) = (23, 14)$.

This leads to the conclusion that no dominance strategy-proof, dominance efficient, and $\lceil \frac{m}{2} \rceil = 1$ envy free mechanism can every assign $\psi(1234_a, 1234_b) = (23, 14)$.

Case 6: $\psi(1234_a, 1234_b) = (14, 23)$

We can symmetrically follow the previous reasoning of case 5.

Case 1-6 together conclude the proof as regardless of what we assign $\psi(1234_a, 1234_b)$ we find a contradiction with at least one required property.

$\square$

Serial Dictatorship and Harvard Business School Mechanism

Fixing a single priority order serial dictatorship algorithm is formally defined by the following algorithm:

SD Algorithm

Step $1 \geq t \leq n$.

There are $m \times n - m \times (t - 1)$ objects left. Following the priority orders let person $i \in I$ with priority $f^{-1}(i) = t$ pick her k most preferred objects among the remaining objects.

Similarly we can define the Harvard Business School mechanism via the following algorithm.

This algorithm is based on two priority orders f^{odd} and f^{even} that have reverse priority, i.e. $f^{odd}(i) = n - f^{even}(i)$.

HBS Algorithm

Step $1 \geq t \leq n \times m$.

There are $m \times n - (t-1)$ objects left. The priority order used is changed all n steps from f^{odd} to f^{even} and back. Following the appropriate priority order $f \in \{f^{odd}, f^{even}\}$ let person $i \in I$ with priority $f^{-1}(i) = t$ pick her most preferred object among the remaining objects.

It is well know that serial dictatorship is efficient and strategy-proof, but is unsatisfactory in terms of ex-post fairness. The HBS algorithm on the other hand is 1 envy-freeness and dominance efficiency but fails even the weaker notion of dominance strategy proofness. We summarize these results for the responsive preference domain in the following propositions.

Efficiency

Proposition 9. *HBS is dominance efficient. SD is pareto efficient (among all outcomes giving each individual exactly m objects) and hence dominance efficient. BD is not dominance efficient.*

Proof. HBS is dominance efficient. Let $\psi(P)$ be the outcome of the HBS mechanism. Suppose to the contrary that there exists an assignment A s.t. $A_i \geq_i \psi_i(P)$ for all $i \in I$ and $A_i >_i \psi_i(P)$ for at least some $i \in I$, It is obvious that when ordering the objects in $\psi_i(P)$ following P_i we get that the object in lth place $\psi_i^l = \{o \in \psi_i(P) : |\{o' \in \psi_i(P) : o' \; R_i \; o\}| = l\}$

for $l \in \{1, \ldots, k\}$ is the lth object picked under the HBS mechanism. As $A_i \geq_i \psi_i(P)$ for every object ψ_i^l assigned at each step of the HBS mechanism there exists an object $o_i^l \; R_i \; \psi_i^l$ with $o_i^l = \{o \in A_i : |\{o' \in A_i : o' \; R_i \; o\}| = l\}$. Consider the first step of the HBS mechanism where $o_i^l \; P_i \; \psi_i^l$. Note that as P is a simple order, all previous objects must have been identical $o_i^l = \psi_i^l$. This leads to a contradiction as the object ψ_i^l assigned under the HBS mechanism must be the best available object following P_i but there exists $o_i^l \; P_i \; \psi_i^l$.

HBS is not pareto efficient. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose the reported ranking is $P_i : o_1, o_2, o_3, o_4$ for $i = 1, 2$. Under $f^{odd} : 1, 2$ and $f^{even} : 2, 1$, the outcome under the HBS mechanism is $\psi_1(P) = \{o_1, o_4\}$ and $\psi_2(P) = \{o_2, o_3\}$. Note that preferences $\succsim_1$: $\{o_2, o_3\}, \{o_1, o_4\}$ and $\succsim_2$: $\{o_1, o_4\}, \{o_2, o_3\}$ are both consistent with the reported order P_1 respectively P_2 as the relative ranking between the two bundles cannot be inferred from the reported simple order under responsive preferences. Therefore assignment A with $A_1 = \{o_2, o_3\}$ and $A_2 = \{o_1, o_4\}$ pareto dominates the outcome $\psi(P)$.

SD is pareto efficient. It is well known that serial dictatorship is efficient and therefore dominance efficient. Note that the highest priority person i_1 with $f^{-1}(i_1) = 1$ gets her m best objects. Under responsiveness of P_{i_1} the bundle containing the best m objects is the best set in $\{O' \in 2^O : |O'| = m\}$. Conditional on this the second highest priority person i_2 with $f^{-1}(i_2) = 2$ gets her best m objects among the remaining object. As we can never change i_1's assignment to another assignment containing m objects without making her worse off we can never changes i_2's assignment as well. Following this argument for the remaining individual we can conclude that there cannot exist an allocation assigning every person weakly better bundle of size m.

SD is not pareto efficient under any assignment. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose the reported ranking is $P_i : o_1, o_2, o_3, o_4$ for $i = 1, 2$. Suppose that for 1 we have that $\{o_2, o_3, o_4\} \succ_1 \{o_1, o_2\}$ while for 2 we have $\{o_1\} \succ_2 \{o_2, o_3, o_4\}$.

For example u_1 and u_2 are additive utility functions of individual 1 and 2 where u_1 : $100, 99, 98, 88$ and $u_2 : 100, 3, 2, 1$. Where $u_1(\{o_2, o_3, o_4\}) = 285 > u_1(\{o_1, o_2\}) = 199$ and $u_2(\{o_1\}) = 100 > u_1(\{o_2, o_3, o_4\}) = 6$.

Under priority order $f : 1, 2$ the outcome of m-serial dictatorship is $\psi(P)$ with $\psi(P)_1 = \{o_1, o_2\}$ with $\psi(P)_2 = \{o_3, o_4\}$ which is Pareto dominated by A with $A_1 = \{o_2, o_3, o_4\}$ with $A_2 = \{o_1\}$. $\square$

Envy Freeness

Proposition 10. *HBS is* 1 *envy-free. BD is* $\lceil \frac{m}{2} \rceil$ *envy-free. And SD is* m*-pick envy free.*

Proof. HBS. 1 envy freeness follows directly from the algorithm. Consider any pair of individuals $i, j \in I$. Under the reported preference profile P_i person i always prefers her first picked item to person j's second picked item, her second picked item to person j's third picked item, and so on and so forth. Hence under any allocation of the draft mechanism we get that that $o_i^k \; P_i \; o_j^{k+1}$ for $l \in \{1, \dots, m-1\}$ which implies 1 envy-free.

SD. Let ψ denote the serial dictatorship mechanism. We have that $|\psi_i(P)| = |\psi(P)_j| = m$

for any two assignments. Consider two individual with identical simple orders. It follows that the one with lower priority will have m objects all worse that the higher priority individual. Hence by removing m items from any two sets we end up with $\varnothing \geq_i \varnothing$. $\square$

Strategy Proofness

Proposition 11. *BD and SD are both strongly strategy-proof. The HBS mechanism is not dominance strategy proof.*

Proof. HBS. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose true preference are $P_1 : o_1, o_2, o_3, o_4$ for individual 1 respectively $P_2 : o_2, o_3, o_4, o_1$ for individual 2. Take priority orders $f^{odd} : 1, 2$ and $f^{even} : 2, 1$, then under the HBS mechanism ψ and true rankings we get $\psi_1(P_1, P_2) = \{o_2, o_3\}$ and $\psi_2(P_1, P_2) = \{o_1, o_4\}$. But there is a profitable manipulation for 1 by picking the more popular object first $\hat{P}_1 : o_2, o_1, o_3, o_4$ leading $\psi_1(\hat{P}_1, P_2) = \{o_1, o_2\}$ and $\psi_2(\hat{P}_1, P_2) = \{o_3, o_4\}$. Its easy to check that $\psi_1(\hat{P}_1, P_2) >_1 \psi_1(\hat{P}_1, P_2)$ and hence violating even dominance strategy proofness.

SD is strategy-proof. Let ϕ denote the serial dictatorship mechanism. Is is well known that the serial dictatorship mechanism is strategy proof. The highest priority agent $i_1 \in I$ with $f^{-1}(i_1) = 1$ obtains the best m objects. Under responsiveness of P_{i_1} the bundle containing the best m objects is the best set in $\{O' \in 2^O : |O'| = m\}$. As any outcome for i_1 under the serial dictatorship is in the set $\{O' \in 2^O : |O'| = m\}$ we have $\psi(P)_{i_1} \succsim_i \psi(P_{i_1}, P'_{i_1})_{i_1}$ for all $P_{i_1}, P'_{i_1} \in \mathcal{P}_{i_1}$. The second highest priority agent $i_2 \in I$ with $f^{-1}(i_2) = 2$ obtains the best

m object among the remaining ones and hence can never be better off by misrepresenting using an analogous argument Following this argument step by step for every individual leads us to the desired conclusion.

□

A.1.2 Section 4

The following is a restatement of lemma 1, adjusted to the partition-responsive domain.

Lemma 6. *Let $\succsim_i$ be any responsive preference relation over 2^O with underlying ranking P_i, and $\geq_i$ the corresponding dominance relation. For any $O', O'' \in S$ if $O' \geq_i O''$ then $O' \succsim_i O''$.*

Proof. Suppose we have $O', O'' \in S$ with $O' = \{o'_1, \dots, o'_m\} \geq_i O'' = \{o''_1, \dots, o''_m\}$. As $O' \geq_i O''$ we have $o'_1 \; R_1 \; o''_1$ as well as $\{o'_1\} \succsim_i \{o''_1\}$. Using responsiveness for $\{o''_2, \dots, o''_m\} \cap \{o'_1, o''_1\} = \varnothing$ and $\{o'_1\} \succsim_i \{o''_1\}$ we get $\{o'_1, o''_2, \dots, o''_m\} \geq_i O'' = \{o''_1, o''_2 \dots, o''_{m'}\}$. Replacing one-by-one o''_k by o'_k for all $k \in \{2, \dots, m\}$ and invoking responsiveness we get $O' = \{o'_1, o'_2, \dots, o'_m\} \succsim_i \dots \succsim_i \{o'_1, o''_2, \dots, o''_{m'}\} \succsim_i \{o''_1, o''_2, \dots, o''_{m'}\} = O''$. By transitivity of $\succsim_i$ we reach the conclusion that $O' \succsim_i O''$. □

A.1.3 Section 7

Proposition 7: Not OSP

Proof. Following proposition 2 (pruning principle) in Li (2017), we can restrict attention to "minimal" extensive form games, where no histories are off the path of play. It is also sufficient to show that a sub-function ψ is not OSP-implementable. Take agents $\{1,2\} \subseteq I$ and objects $\{a,b,c,d\} \subseteq O$ with $\{a,b\} \subseteq O^1$ and $\{c,d\} \subseteq O^2$ with priority orders $f^1(1) < f^1(2)$ and $f^2(2) < f^2(1)$. Consider the following subset $\succsim^{\{1,2\}} \subset \succsim^N$ for the partition-restricted assignment domain.

$$\succsim_1 : \{a,c\} \succ_1 \{b,c\} \succ_1 \{a,d\} \succ_1 \{b,d\} \tag{A.1}$$

$$\succsim_2 : \{a,c\} \succ_2 \{a,d\} \succ_2 \{b,c\} \succ_2 \{b,d\} \tag{A.2}$$

Take any G pruned with respect to the truthful strategy profiles, such that G OSP-implements ψ for domain $\succsim^{\{1,2\}}$. Consider some history h at which $i(h) = 1$ with a non-singleton action set. If OSP holds this cannot come before any non-singleton action history with $i(h) = 2$. Suppose not, then if 1 chooses an action corresponding to $a \; P_a \; b$ her worst payoff is $\{a,d\}$ while her best payoff under an action corresponding to $b \; P_a \; a$ is $\{b,c\} \succ_1 \{a,d\}$. Similarly consider some history h at which $i(h) = 2$ with a non-singleton action set. This cannot come before any such history with $i(h) = 1$. Suppose not, then if 2 chooses an action corresponding to $c \; P_a \; d$ her worst payoff is $\{b,c\}$ while her best payoff under an action corresponding to $d\ P_a\ d$ is $\{a,d\} \succ_2 \{b,c\}$. So all action sets of 1 and 2 must be singletons and G does not OSP-implement the BD rule. $\square$

Partial Order SP, OSP, WGSP

Slightly abusing notation, a **partial order function** $\trianglerighteq : \succsim^I \to \trianglerighteq^I$ is a consistent way to assign a subset of pairwise comparisons $\trianglerighteq(\succsim_i) \subseteq \succsim_i$ to each type. Given G and $(\succsim_i, \trianglerighteq)$, S_i is **partial order obviously dominant with respect to** $\trianglerighteq$ if $\forall S_i'$ and $\forall I_i \in \alpha(S_i, S_i')$ there does not exist $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ such that $g(z') \triangleright(\succsim_i)\ g(z)$. A type-strategy profile $T(\succsim) \in OSP_{\trianglerighteq}(G)$ is in the set of **partial order obviously strategy-proof (PoOSP)** profiles if for all $i \in I$, $T(\succsim_i)$ is partial order obviously dominant with respect to $\trianglerighteq$.

Similarly we can define the standard concepts of strategy proofness and weak group strategy proofness for a particular partial order. For the partial order profile $\geq = (\geq_i)_{i \in I}$ a type-strategy profile $T(\succsim) \in SP_{\trianglerighteq}(G)$ is in the set of **partial order strategy-proof (PoSP)** profiles if there does not exists an $\succsim \in \succsim^I$, individual $i \in I$ with deviation strategy $\hat{S}_i \neq T(\succsim_i)$ such that $g(z^G(h_\varnothing, \hat{S}_i, T(\succsim) \setminus T(\succsim_i))) \triangleright(\succsim_i)\ g(z^G(h_\varnothing, T(\succsim)))$. For the partial order profile $\geq = (\geq_i)_{i \in I}$ a type-strategy profile $T(\succsim) \in WGSP_{\trianglerighteq}(G)$ is in the set of **partial order weakly group-strategy-proof (PoWGSP)** profiles if there does not exists a coalition $I' \subseteq I$, type profile $\succsim \in \succsim^I$, deviating strategies $\hat{S} = (\hat{S}_i)_{i \in I'}$, non coalition strategies $T(\succsim) \setminus \hat{S}$ such that for all $i \in I'$ we have $g(z^G(h_\varnothing, \hat{S}, T(\succsim) \setminus \hat{S})) \triangleright(\succsim_i)\ g(z^G(h_\varnothing, T(\succsim)))$.

Proposition 12. *If $T(\succsim) \in OSP_{\trianglerighteq}(G)$ with respect to $\trianglerighteq$ then $T(\succsim) \in WGSP_{\trianglerighteq'}(G)$ with respect to the same partial order $\trianglerighteq' = \trianglerighteq$.*

Proof. Suppose $T(\succsim) \notin WGSP_{\trianglerighteq}(G)$. Then there exists coalition I' with types $(\succsim_i)_{i \in I'}$ that can deviate to strategy $\hat{S} = (\hat{S}_i)_{i \in I'}$ where all $i \in I$ are strictly better off following the partial order $\geq$. Along the resulting terminal history, there is a first agent $i \in I'$ in the coalition to deviate from $T(\succsim_i)$ to $\hat{S}_i$. That first deviation happens at some information set $I_i \in \alpha(T(\succsim_i), \hat{S}_i)$. We have for $h \in I_i$ and $S'_{-i} = (T(\succsim) \setminus \hat{S}) \cup (\hat{S} \setminus \{\hat{S}_i\})$ that

$$g(z^G(h, \hat{S}_i, S'_{-i})) \triangleright(\succsim_i)\ g(z^G(h, T(\succsim)))$$

We reach a contradiction as $T(\succsim) \notin OSP_{\trianglerighteq}(G)$ as there exists an earliest point of departure at which a preferred history following the partial order is reachable. $\square$

Proposition 8: DOSP

We fist show that in the standard domain the balanced BD mechanism is not DOSP implementable. We show that the mechanism is not DWGSP(G) implementable, and hence by proposition 12, in the previous appendix subsection, not DOSP implementable.

Proposition 13. *In the responsive preference domian for $m \geq 2$ the balanced BD mechanism is not DWGSP(G) implementable.*

Proof. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$ without loss of generality let the partition of objects into packages be $O^1 = \{o_1, o_2\}$ and $O^2 = \{o_3, o_4\}$ as well as $f^1(1) > f^1(2)$ and $f^1(2) > f^1(1)$. Suppose agent 1's type $\succsim_1$ produces the following simple order preference $P_1 : o_3, o_1, o_2, o_4$ respectively agent 2's type the simple order $P_2 : o_1, o_3, o_4, o_2$.

The BD mechanism leads $A_1 = \{o_1, o_4\}$ for individual 1 and $A_2 = \{o_2, o_3\}$ for individual 2. Now consider the manipulation corresponding to $\hat{P}_1 :: o_3, o_2, o_1, o_4$ and $\hat{P}_3 :: o_4, o_1, o_2, o_3$. The BD mechanism leads $A_1 = \{o_2, o_3\}$ for individual 1 and $A_2 = \{o_1, o_4\}$ for individual 2. This is strictly better for both 1 and 2 following the dominance relation as for individual 1 we have $o_3 \ P_1 \ o_1$, $o_2 \ P_1 \ o_4$ and while for individual 2 we have $o_1 \ P_2 \ o_3$, $o_4 \ P_2 \ o_2$. $\square$

Corollary 2. *In the responsive preference domain, if ψ is a balanced BD mechanism and $m \geq 2$, then there does not exist $G \in \mathcal{G}$ that DOSP-implements ψ.*

We now show the second part, i.e. that in the restricted domain the Balanced BD is DOSP implementable.

Note that the BD rule is a simple mechanism that treats all types with the same underlying simple order as identical. We will think of the problem as having a mechanism from $\psi : \mathcal{P} \to \mathcal{A}$ and specifying a type-strategy profile function $T : \mathcal{P} \to \mathcal{S}$.

Proof. Define the *extensive form game G* as follows. Take the buckets $\{O^1, \dots, O^m\}$ and the corresponding priority orders $\{f_1, \dots, f_m\}$. The set of players is I. The set of actions at each information set $A(I_i)$ is to claim a single available object. For histories of length $|h| \in [1, n-1]$ the player at each node is identified by the priority order f_1. Similarly for any history of length $|h| \in [kn, (k+1)n-1]$ the player $P(h)$ is defined by the priority order f_k. The terminal histories give every agents the set of object the person claimed at each step of the path leading to z.

Define the *type strategy* $T(P_i)$ for each type $\succ_i$ corresponds to a simple order P_i over individual objects. In particular $T(P_i)$ is simply to take the best object available at each information set where a agent is called to play following the simple order P_i. Its straight forward to see that $\psi(P) = g(z^G(h_\varnothing, T(P)))$ where ψ is the outcome of the BD mechanism.

We want to show that $T \in DOSP(G)$ for all i and for all $\succsim_i$, $T_i(\succsim_i)$ is obviously dominance relation dominant. Suppose to the contrary that there exists $i \in I$ such that for some S_i' and $I_i \in \alpha(S_i, S_i')$ there exists $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ with $g(z') \geq (P_i)\ g(z)$. So for all $k \in \{1, \dots m\}$ we have that $g(z')_i \cap O^k\ R_i\ g(z)_i \cap O^k$. This implies that at I_i the object obtained under the two strategies differs $S_i(I_i) \neq S_i'(I_i)$ where $S_i'(I_i), S_i'(I_i) \in O^{k\prime} \subseteq O^k$ for some $k \in \{1, \dots m\}$. But we know that under the type strategy $S_i(I_i)\ R_i\ S_i'(I_i)$ holding strictly as $S_i(I_i) \neq S_i'(I_i)$, i.e. $S_i(I_i)\ P_i\ S_i'(I_i)$ contradicting that the assignment under S_i' dominates the assignment under S_i as $g(z)_i \cap O^k\ P_i\ g(z)_i \cap O^k$. $\square$

A.2 Illustration Algorithms

We schematically depict the described algorithms for eight object and four individual. In line with the original inspiration, the objects are represented by cards. Moreover, we invite the observer to interpret the depicted square as a tabletop with the individual 1,2,3, and 4 sitting around that table.

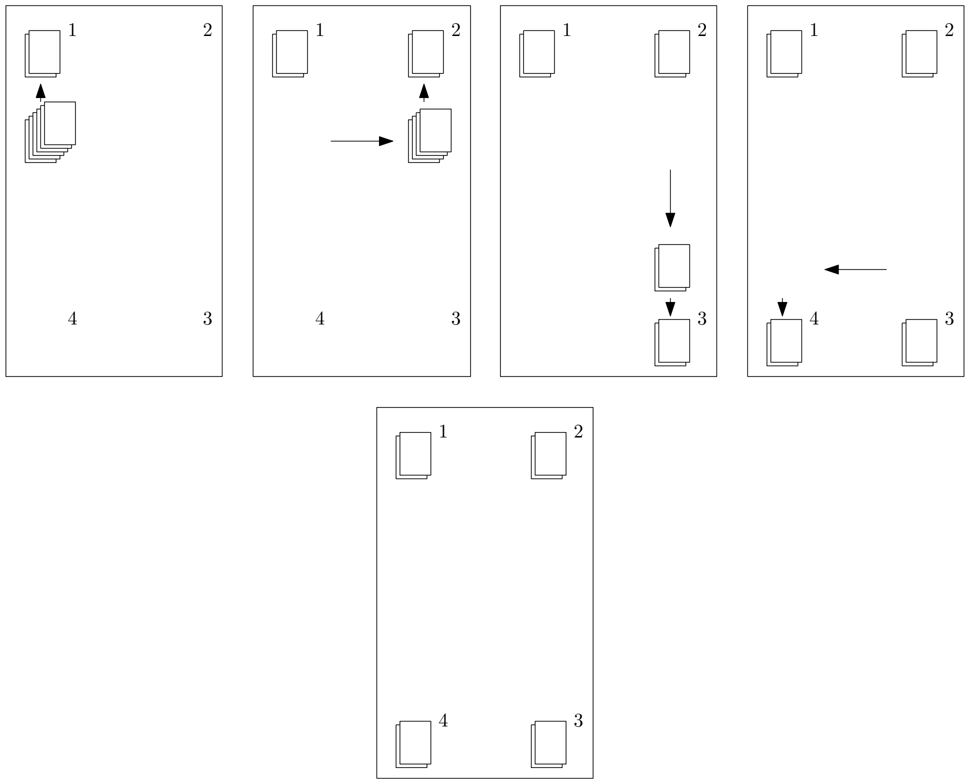


Figure A.1: Schematic Depiction Serial Dictatorship

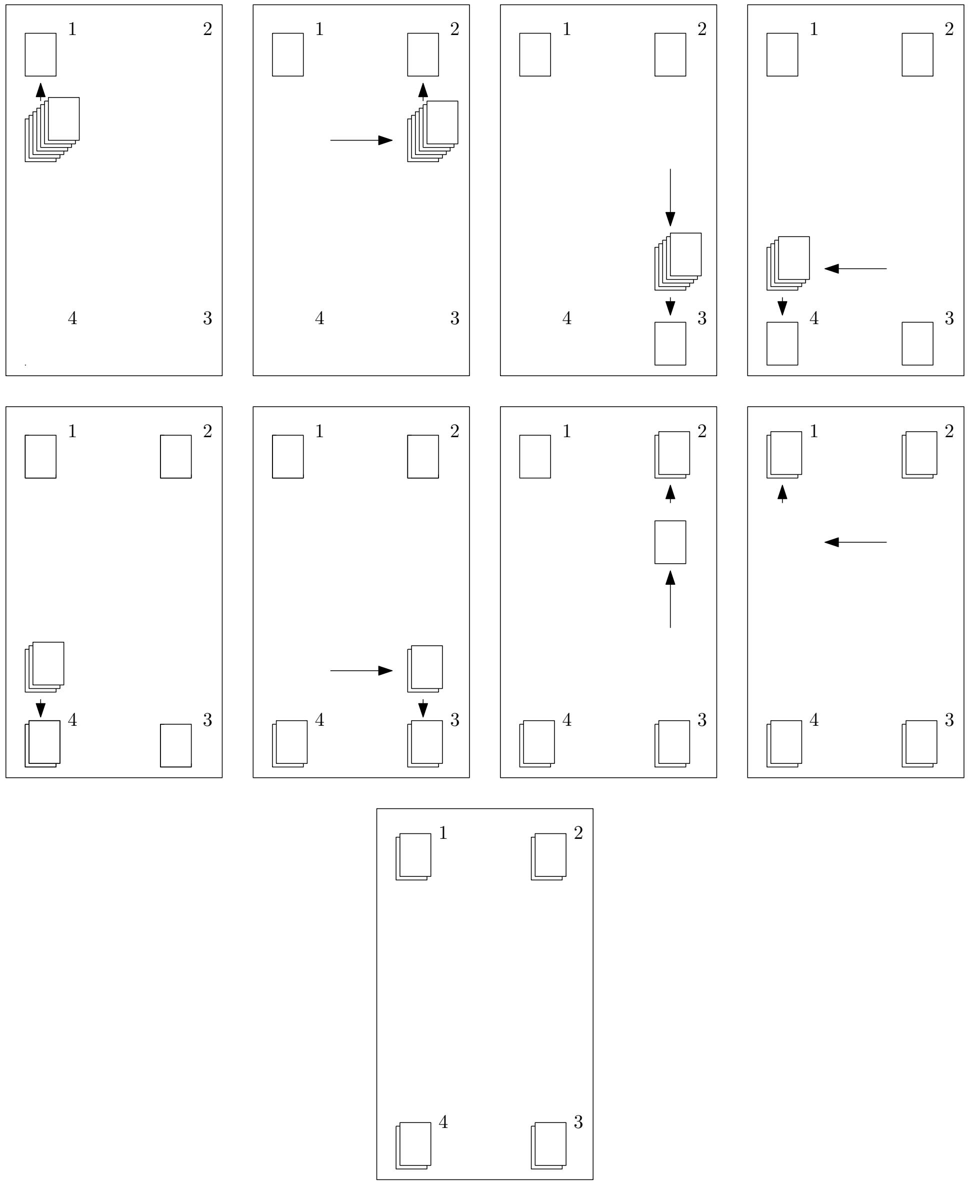


Figure A.2: Schematic Depiction Harvard Business School Mechanism

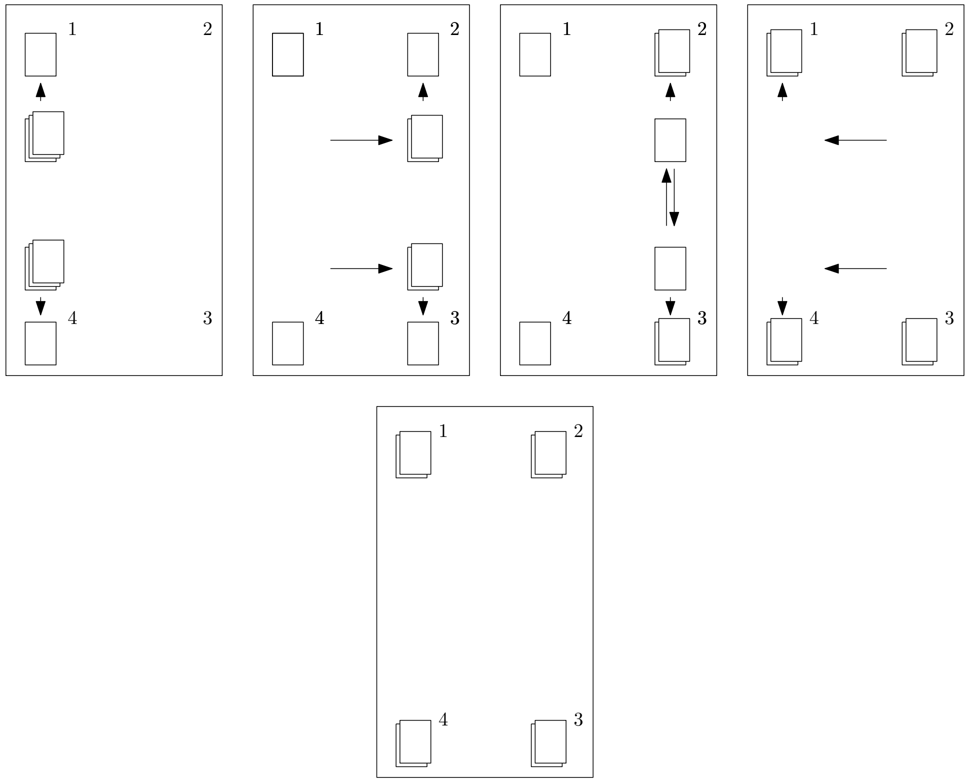


Figure A.3: Schematic Depiction Booster Draft Mechanism

Appendix B

An Alternative Approach to Asylum Assignment

B.1 Proofs

The **original choice function** $C_{t,m}(X_t'|Y_{t-1}) \subseteq X_t'$ is defined as follows:

Step 0.

- Accept all contracts specified in the previous period allocation $\overline{X}_t^1 = X_{t,m}' \cap Y_{t-1}$.
- Reject all contracts specifying different member states and all contracts available but not specified in the previous period allocation $\underline{X}_t^1 = (X_t' \setminus X_{t,m}') \cup (X_{t,m}' \cap X_{t-1} \setminus Y_{t-1})$.
- This leaves us with $X_t^1 \equiv X_t' \setminus (\underline{X}_t^1 \cup \overline{X}_t^1)$. Go to step 1.

Step $j \geq 1$. Consider all contracts X_{t,a_j}^j of the highest priority applicant among the remaining $|\{a \in A(X_t^j) : a \geq_{t,m} a_j\}| = 1$. If $X_{t,a_j}^j = \varnothing$ the algorithm stops and $C_{t,m} = (X_t'|Y_{t-1}) = \overline{X}_t^j$.

a) If $\varnothing >_{t,m} a_j$ and there are already at least $|\overline{X}_t^j \setminus \overline{X}_t^1| \geq q_{t,m}$ contracts specifying newly arrived asylum seekers accepted the algorithm stops.

b) Otherwise consider $\arg\min_{x \in X_{t,a_j}} = w(x) \in W(X_{t,a_j})$ such that $r_{m,w(x)} > |\overline{X}_t^j|$. If it exists then $\overline{X}_t^{j+1} = \overline{X}_t^j \cup \{x\}$ and $\underline{X}_t^{j+1} = \underline{X}_t^j \cup X_{t,a_j} \setminus \{x\}$, otherwise $\overline{X}_t^{j+1} = \overline{X}_t^j$ and $\underline{X}_t^{j+1} = \underline{X}_t^j \cup X_{t,a_j}$. Go to the next step with $X_t^{j+1} \equiv X_t' \setminus (\underline{X}_t^{j+1} \cup \overline{X}_{j+1}^1)$.

The **modified choice function** $C'_{t,m}(X'_t|Y_{t-1}) \subseteq X'_t$ is defined as follows:

Step 0.

- Accept all contracts specified in the previous period allocation $\overline{X}^1_t = X'_{t,m} \cap Y_{t-1}$.
- Reject all contracts specifying different member states and all contracts available but not specified in the previous period allocation $\underline{X}^1_t = (X'_t \setminus X'_{t,m}) \cup (X'_{t,m} \cap X_{t-1} \setminus Y_{t-1})$.
- This leaves us with $X^1_t \equiv X'_t \setminus (\underline{X}^1_t \cup \overline{X}^1_t)$. Go to step 1.

Step $j \geq 1$. Consider all contracts X^j_{t,a_j} of the highest priority applicant among the remaining $|\{a \in A(X^j_t) : a \geq_{t,m} a_j\}| = 1$. If $X^j_{t,a_j} = \varnothing$ the algorithm stops and $C'_{t,m} = (X'_t|Y_{t-1}) = \overline{X}^j_t$.

a) If $\varnothing >_{t,m} a_j$ and there are already at least $|\overline{X}^j_t \setminus \overline{X}^1_t| \geq q_{t,m}$ contracts specifying newly arrived asylum seekers accepted the algorithm stops. If $X^j_{t,a_j} = \varnothing$ the algorithm stops.

b) Otherwise consider $\arg\min_{x \in X_{t,a_j}} = w(x) \in W(X_{t,a_j})$ such that $r_{m,w(x)} > |\overline{X}^j_t|$. If it exists then $\overline{X}^{j+1}_t = \overline{X}^j_t \cup \{x\}$ and $\underline{X}^{j+1}_t = \underline{X}^j_t$, otherwise $\overline{X}^{j+1}_t = \overline{X}^j_t$ and $\underline{X}^{j+1}_t = \underline{X}^j_t \cup X_{t,a_j}$. Go to the next step with $X^{j+1}_t \equiv X'_t \setminus (\underline{X}^{j+1}_t \cup \overline{X}^1_{j+1})$.

Lemma 5

Claim. $C'_{t,m}(\cdot|Y_{t-1})$ is a **completion** of $C_{t,m}(\cdot|Y_{t-1})$.

Proof. We have to show that for any $X'_t \subseteq X_t$, whenever there do not exist two contracts $x, x' \in C'_{t,m}(X'_t|Y_{t-1})$ specifying the same asylums seeker $a(x) = a(x')$ we must have

$C'_{t,m}(X'_t|Y_{t-1}) = C_{t,m}(X'_t|Y_{t-1})$. We proceed by induction. Slightly abusing notation we refer to a step of $C'_{t,m}(X'_t|Y_{t-1})$ as all the steps where contracts X^j_{t,a_j} of the same agent are considered, while $C_{t,m}(X'_t|Y_{t-1})$ the original choice function does this in a single step.

Base Step: We have $\overline{X}^{1\prime}_t = \overline{X}^1_t$ and $\underline{X}^{1\prime}_t = \underline{X}^1_t$ at the end of step 0.

By definition $\overline{X}^{1\prime}_t = X'_{m,t} \cap Y_{t-1} = \overline{X}^1_t$, and $\underline{X}^{1\prime}_t = (X'_t \setminus X'_{m,t}) \cup (X'_{m,t} \cap X_{t-1} \setminus Y_{t-1}) = \underline{X}^1_t$.

Inductive Step. Suppose $\overline{X}^{j\prime}_t = \overline{X}^j_t$ and $\underline{X}^{j\prime}_t = \underline{X}^j_t$ at the end of step $j-1$. If no two contracts of the same applicant are accepted under $C'_{t,m}(X'_t|Y_{t-1})$, then $\overline{X}^{j+1\prime}_t = \overline{X}^{j+1}_t$ as well as $\underline{X}^{j+1\prime}_t = \underline{X}^{j+1}_t$ at the end of step j.

We have $X^{j\prime}_t = X'_t \setminus (\overline{X}^j_t \cup \underline{X}^j_t) = X^j_t$. It directly follows that the same contract must be chosen at j under both choice functions as $\{a \in A(X^{j\prime}_t) : a \geq_{t,m} a_j \text{ for all } a \in A(X^{j\prime}_t)\} = \{a \in A(X^j_t) : a \geq_{t,m} a_j \text{ for all } a \in A(X^j_t)\}$ and hence identifies the same agent, while $\arg\min_{x \in X'_{t,a_j}} = w(x) \in W(X'_{t,a_j})$ such that $r_{m,w(x)} > |\overline{X}^{j\prime}_t|$ identifies the same waiting time as $\arg\min_{x \in X_{t,a_j}} = w(x) \in W(X_{t,a_j})$ such that $r_{m,w(x)} > |\overline{X}^j_t|$. Therefore $\overline{X}^{j+1}_t = \overline{X}^j_t \cup \{x\} = \overline{X}^{j+1}_t$.

As no two contracts of the same applicant are accepted under $C'_{t,m}(X'_t|Y_{t-1})$ all contracts $X_{t,a_j} \setminus \{x\}$ must be rejected. Hence also the second part of the inductive claim holds $\underline{X}^{j+1\prime}_t = \underline{X}^{j\prime}_t = \underline{X}^j_t \cup X_{t,a_j} \setminus \{x\} = \underline{X}^{j+1}_t$. This concludes the proof. Let J be the final step of the algorithm, then $C'_{t,m}(X'_t|Y_{t-1}) = \overline{X}^J_t = C_{t,m}(X'_t|Y_{t-1})$.

□

Note that it is not necessary that the lowest available waiting time contract is chosen for the results to go through. It is easy to see that our choice functions can be modified, allowing for any rule that chooses a particular contract for the current asylum seeker under consideration. All we need for the results to go through is that the modified choice function has the exact same decision rule. In particular let $R_{t,m}$ be any order of contracts $X_t \setminus X_{t-1}$ such that if $a(x) >_{t,m} a(y)$ then $xR_{t,m}y$. Let the contract of any asylum seeker chosen at any step be the highest ranked contract according to $R_{t,m}$ under both the original and the modified choice function. It is straightforward to see that with slight modifications the same proofs works for these more general choice functions.

Claim. $C_{t,m}(\cdot|Y_{t-1})$ and $C'_{t,m}(\cdot|Y_{t-1})$ both satisfy **irrelevance of rejected contracts**.

Proof. We want to show if $x \notin C_{t,m}(X'_t \cup \{x'\}|Y_{t-1})$ then that $C_{t,m}(X'_t \cup \{x'\}|Y_{t-1}) = C_{t,m}(X'_t|Y_{t-1})$. We use induction, while the argument is similar to the previous proof. Consider the algorithm for $C_{t,m}(X'_t|Y_{t-1})$ and $C_{t,m}(X'_t \cup \{x\}|Y_{t-1})$. We use X^1_t to denote the initial set of contract for the former and $X^{1'}_t$ for the latter when also x' is in the choice set, and similarly for all other variable used during the algorithm.

Base Step: We have $\overline{X}^{1\prime}_t = \overline{X}^1_t$ and $\underline{X}^{1\prime}_t \setminus \{x\} = \underline{X}^1_t$ at the end of step 0.

By definition $\overline{X}^{1\prime}_t = (X'_{t,m} \cup \{x\}) \cap Y_{t-1} = X'_{t,m} \cap Y_{t-1} = \overline{X}^1_t$, where the second equality follows from the fact that x is not accepted by definition. Similarly $\underline{X}^{1\prime}_t \setminus \{x\} = [((X'_t \cup \{x\}) \setminus (X'_t \cup \{x\})_{t,m}) \cup ((X'_{t,m} \cup \{x\}) \cap X_{t-1} \setminus Y_{t-1})] \setminus \{x\} = (X'_t \setminus X'_{t,m}) \cup ((X'_{t,m}) \cap X_{t-1} \setminus Y_{t-1}) = \underline{X}^1_t$.

Note that unlike the previous proof we now use the original choice function definition twice, once for the set X'_t and once for $X'_t \cup \{x\}$ where x' will be rejected.

Inductive Step. Suppose $\overline{X}_t^{j\prime} = \overline{X}_t^j$ and $\underline{X}_t^{j\prime} \setminus \{x\} = \underline{X}_t^j$ at the end of step $j-1$. Then we have $\overline{X}_t^{j+1\prime} = \overline{X}_t^{j+1}$ as well as $\underline{X}_t^{j+1\prime} \setminus \{x\} = \underline{X}_t^{j+1}$ at the end of step j.

As x' is never chosen, it directly follows that the same contract must be chosen at j under both choice functions as $\{a \in A(X_t^{j\prime}) : a \geq_{m,t} a_j \text{ for all } a \in A(X_t^{j\prime})\} = \{a \in A(X_t^j) : a \geq_{m,t} a_j \text{ for all } a \in A(X_t^j)\}$ and hence identifies the same agent, while $\arg\min_{z \in X'_{t,a_j}} = w(z) \in W(X'_{t,a_j})$ such that $r_{m,w(z)} > |\overline{X}_t^{j\prime}|$ identifies the same waiting time as $\arg\min_{z \in X_{t,a_j}} = w(z) \in W(X_{t,a_j})$ such that $r_{m,w(z)} > |\overline{X}_t^j|$. Therefore $\overline{X}_t^{j+1} = \overline{X}_t^j \cup \{x\} = \overline{X}_t^{j+1}$.

We have $\underline{X}_t^{j+1\prime} \setminus \{x'\} = \underline{X}_t^{j\prime} \cup X'_{t,a_j} \setminus \{x, x'\} = \underline{X}_t^j \cup X_{t,a_j} \setminus \{x\} = \underline{X}_t^{j+1\prime}$.

This concludes the proof, since the exact same contracts are chosen under both choice functions irrelevance of rejected contracts is satisfied.This concludes the proof. Let J be the final step of the algorithm, then $C_{t,m}(X'_t \cup \{x'\}|Y_{t-1}) = \overline{X}_t^J = C_{t,m}(X'_t|Y_{t-1})$. An analogous argument can be made for the modified choice function under $C'_{t,m}(X'_t \cup \{x\}|Y_{t-1})$ and $C'_{t,m}(X'_t|Y_{t-1})$ when x' is rejected. □

Claim. $C'_{t,m}(\cdot|Y_{t-1})$ satisfy both **the law of aggregate demand** and **subsitutability**.

Proof. Law of aggregate demand.

By construction of $C'_{t,m}(\cdot|Y_{t-1})$ a contract x can only be rejected if overall $q_{t,m}$ contracts are accepted or if $r_{m,w}$ higher priority contracts with the same wait time are accepted. Hence the chosen set can never shrink as the set of available contracts grows.

Subsitutability.

Suppose to the contrary that there exists exist $X'_t \subseteq X_t$ and $x, z \in X_t \setminus X'_t$ such that

$x \notin C'_{t,m}(X'_t \cup \{x\}|Y_{t-1})$ and $x \in C'_{t,m}(X'_t \cup \{x,y\}|Y_{t-1})$. As we already showed z has to be accepted as $C'_{t,m}(\cdot|Y_{t-1})$ satisfies the irrelevance of rejected contracts condition. If x was originally rejected because q_m higher contracts were accepted, then this still holds when z is accepted. Similarly if x was rejected because there were $r_{m,w(x)}$ higher priority contracts this still holds true when z gets accepted. Hence once a contract is rejected from a set X'_t the contract will be rejected from any larger set. □

Lemma 4

Proof. Recall that an allocation $Y = (Y_t)_{t\in T}$ is two sided efficient if there does not exist an allocation $Y' = (Y'_t)_{t\in T}$ with $Y' \neq Y$ s.t. $Y'_{t,a}\ R_a\ Y_{t,a}$ for all $a \in A$ and $t \in T$ and $C_{t,m}(Y'_t \cup Y_t|Y'_{t-1}) = Y'_{t-1}$. We will show that any such dynamic allocation will be the same as the original stable dynamic allocation whenever the choice functions satisfy irrelevance of rejected contracts.

Base step. We show that $Y'_1 = Y_1$.

Suppose by contradiction that $Y'_1 \neq Y_1$ then at least one asylum seeker gets assigned a strictly preferred contract $Y'_{1,a}\ P_a\ Y_{1,a}$. In particular consider $m \in M$ and all contracts that are different $\{x_1, \dots, x_k\} = Y'_{1,m} \setminus Y_{1,m}$, by efficiency we have $x\ P_a\ Y_{1,a}$ for all $x \in \{x_1, \dots, x_k\}$. Pick any $x \in \{x_1, \dots, x_k\}$, note that $Y_0 = Y'_0 = \varnothing$ and hence by stability and two-sided efficiency we have $x\ P_a\ Y_{1,a}$ and $x \in X_1 \setminus Y_1$ implying $x \notin C_{1,m}(Y_1 \cup \{x\}|\varnothing)$. By irrelevance of rejected contracts we have $C_{1,m}(Y_1 \cup \{x_1\}|\varnothing) = C_{1,m}(Y_1|\varnothing) = Y_1$ and similarly $C_{1,m}(Y_1 \cup \{x_1, x_2\}|\varnothing) = C_{1,m}(Y_1 \cup \{x_2\}|\varnothing) = C_{1,m}(Y_1|\varnothing) = Y_1$. Following this reasoning for the remain $\{x_3, \ldots, x_k\}$ we get $C_{1,m}(Y_1 \cup \{x_1, \ldots x_k\}|\varnothing) = C_{1,m}(Y_1) = Y_1$, and therefore $C_{1,m}(Y_1 \cup \{x_1, \ldots x_k\}|\varnothing) = C_{1,m}(Y_1 \cup (Y_1' \setminus Y_1)|\varnothing) = C_{1,m}(Y_1' \cup Y_1)|Y_0') = Y_1$ a contradiction except if $Y_1 = Y_1'$.

Induction step. If $Y_{t-1}' = Y_{t-1}$ then $Y_t' = Y_t$.

The induction argument is identical to the base step argument. Concluding the proof as we reach a contradiction with $Y' \neq Y$.

$\square$

Theorem 2

Proof. Dynamically stable.

By lemma 5 there exists a completion $C_{t,m}'(\cdot|Y_{t-1})$ of the choice function $C_{t,m}(\cdot|Y_{t-1})$ that is substitutable, and satisfies the law of aggregate demand. Therefore, together with lemma 5 we have that theorem 2 is a corrollary of theorem 1-3 of Hatfield and Kominers (2016). These three theorems show that the outcome of the cumulative offer process is stable and strategyproof for asylum seekers under the original choice function profile $(C_{t,m})_{m \in M}$.

Dynamically strategy-proof.

Consider any $a \in N_{t'}$ $P_a, \hat{P}_a \in \mathcal{P}_a$, $P_{-a} \in \mathcal{P}_{-a}$ and $C \in \mathcal{C}$ s.t. $\varphi_t(\hat{P}_a, P_{-a}, C)\ P_a\ \varphi_t(P, C)$, where $t' \leq t$. As previously for $t = t'$ the proof is a corollary of Theorem 1-3 of Hatfield and Kominers (2016). For $t > t'$ by one-time determination of the choice functions we have $\varphi_{t,a}(\hat{P}_a, P_{-a}, C) = \varphi_{t,a}(P, C)$ and therefore $\varphi_t(P, C)\ R_a\ \varphi_t(\hat{P}_a, P_{-a}, C)$ contradicting

$\varphi_t(\hat{P}_a, P_{-a}, C)\ P_a\ \varphi_t(P, C)$. As this holds for an arbitrary $a \in N_{t'}$ it holds for all $a \in A = \bigcup_{t \in t} N_t$ concluding the proof.

Two sided efficiency directly follows from lemma 4 as the outcome is dynamically stable and by lemma 5 the original choice function $C_{t,m}(X'_t|Y_{t-1})$ satisfies irrelevance of rejected contracts.

Finally, we provide a counter-example, showing the outcome would be manipulable by over-demanded member states. Consider the following (static) problem where bureaucratic capacities are not binding: $M = \{m_1, m_2\}$, $A = \{a_1, a_2, a_3\}$, $P_{a_1} : m_1 - m_2$, $P_{a_2} : m_2 - m_1$, $P_{a_3} : m_1 - m_2$, $>_{m_1}: a_1 - a_2 - a_3 - \varnothing$, $>_{m_2}: \varnothing a_3 - a_2 - a_1$, and $q_{m_1} = q_{m_2} = 1$. Its easy to check that the outcome of the cumulative offer mechanism is $Y_{m_1} = \{(a_1, m_1, w), (a_3, m_1, w)\}$ and $Y_{m_2} = \{(a_2, m_1, w)\}$. The report $\hat{>}_{m_1} : a_1 - a_2 - \varnothing - a_3$ gives member state m_1 a better outcome, as $Y_{m_1} = \{(a_1, m_1, w), (a_2, m_1, w)\}$ and $Y_{m_2} = \{(a_3, m_1, w)\}$. $\square$

B.2 Illustrating Example

Example 3 (Asylum Seeker Problem)**.** The matching algorithm is run in two periods $T = \{1, 2\}$. Initially three asylum seekers arrive $N_1 = \{a_1, a_2, a_3\}$, and two more in the next period $N_2 = \{a_4, a_5\}$. There are only two member states $M = \{m_1, m_2\}$, and the relevant wait time follows the calendar time $W = \{1, 2, \bar{w}\}$. We present burden sharing and bureaucratic capacities as follows:

$$q = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} m_1 \\ m_2 \end{array} & \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \end{array} \qquad r = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & \bar{w} \end{array} \\ \begin{array}{c} m_1 \\ m_2 \end{array} & \begin{bmatrix} 1 & 1 & 5 \\ 1 & 2 & 5 \end{bmatrix} \end{array}$$

Finally preferences for asylum seekers, and priorities for member states are listed next:

$P_{a_1} : (m_1, 2) - (m_2, 2) - \varnothing$

$P_{a_2} : (m_2, 1) - (m_1, 1) - \varnothing$

$P_{a_3} : (m_1, 1) - (m_2, 1) - \varnothing$

$P_{a_4} : (m_1, 2) - (m_2, 2) - \varnothing$

$P_{a_5} : (m_1, \bar{w}) - \varnothing$

$>_{1,m_1} : a_1 - a_2 - \varnothing - a_3$

$>_{1,m_2} : \varnothing - a_3 - a_2 - a_1$

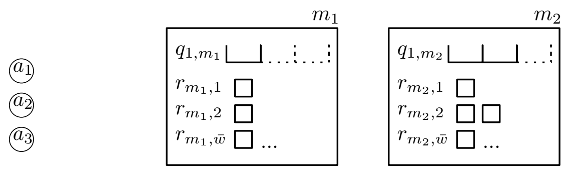
$>_{2,m_1} : a_4 - \varnothing - a_5$

$>_{2,m_2} : a_5 - a_4 - \varnothing$

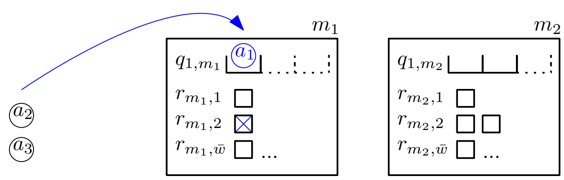
Period 1

The set of contracts $X_1 = A_1 \times M \times W_1$, where $A_1 = \{a_1, a_2, a_3\}$ and $W_1 = \{1, 2, 3\}$. We informally describe how the cumulative offer mechanism reaches an outcome, letting the lowest subscript applicant, currently not holding a contract, propose at the beginning or each step.

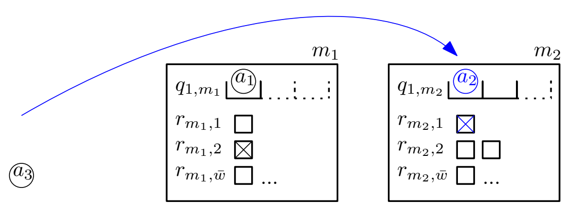
Start. The initial set of proposed contracts is empty, $X_1^0 = \varnothing$.



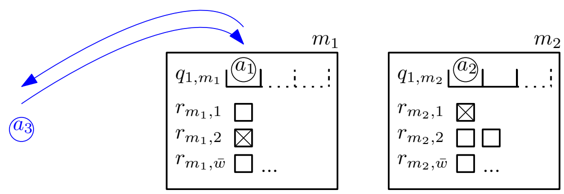
Step 1. Asylum seeker a_1 proposes her first choice $x_1 = (a_1, m_1, 2)$. $X_1^1 = \{x^1\}$ and the contract is tentatively accepted by member state m_1, i.e. $x_1 \in C_{1,m_1}(X_1^1|\varnothing)$ as the member state has committed to take at least one asylum seeker and the bureaucratic capacity for wait time 2 is sufficiently large.



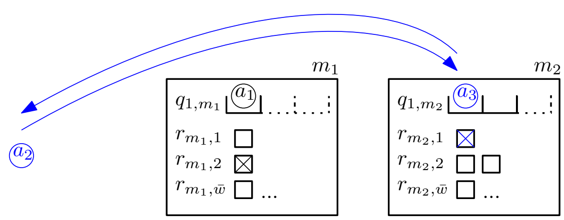
Step 2. Asylum seeker a_2 proposes her first choice $x_2 = (a_2, m_2, 1)$. $X_1^2 = \{x^1, x^2\}$ and the contract is tentatively accepted by member state m_2, for the same reasons as previously, hence $x_2 \in C_{1,m_2}(X_1^2|\varnothing)$.



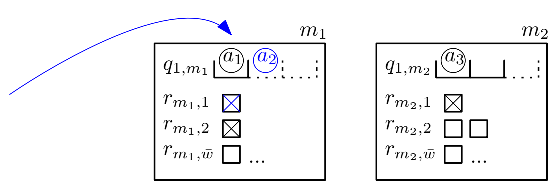
Step 3. Asylum seeker a_3 proposes her first choice $x_3 = (a_3, m_1, 1)$. $X_1^3 = \{x^1, x^2, x^3\}$ and the contract is rejected as $x_3 \notin C_{1,m_1}(X_1^2|\varnothing)$ as $\varnothing >_{1,m_1} a_3$ and the member state m_1 has already tentatively accepted $q_{1,m_1} = 1$ applications.



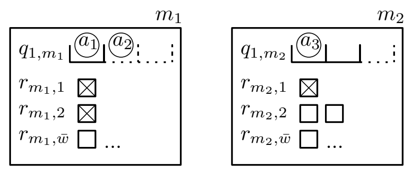
Step 4. Asylum seeker a_3 proposes her second choice $x_4 = (a_3, m_2, 1)$. $X_1^4 = \{x^1, \ldots, x^4\}$ and the contract is tentatively accepted by member state m_2. At the same time it triggers a rejection of contract x_2 as $x_2 \notin C_{1,m_2}(X_1^4|\varnothing)$ as the available bureaucratic quota $r_{1,m_2} = 1$ and therefore insufficient for handling both applications in the desired time.



Step 5. Asylum seeker a_2 proposes her second choice $x_5 = (a_2, m_1, 1)$. $X_1^5 = \{x^1, \ldots, x^5\}$ and the contract is accepted by member state m_1 even though its fair burden is already fulfilled, i.e. $x_5 \in C_{1,m_1}(X_1^5|\varnothing)$ as $a_2 >_{1,m_1} \varnothing$ and the asylum seekers can be processed within the specified time.



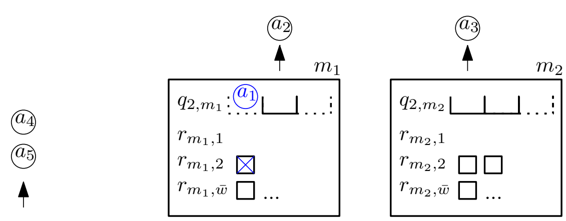
End. As everyone is tentatively assigned a contract the algorithm stops and every tentative assigned contract becomes permanently assigned, producing the period 1 allocation $\psi_1^c(P, C) = \{(a_1, m_1, 2), (a_2, m_1, 1), (a_3, m_2, 1)\}$.



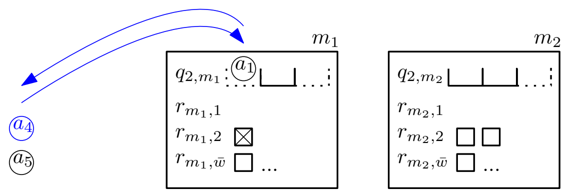
Period 2

The set of contracts $X_2 = A_2 \times M \times W_2$, where $E_2 = \{a_1\}$, $A_2 = N_2 \cup E_2 = \{a_1, a_4, a_5\}$ and $W_1 = \{2, 3\}$.

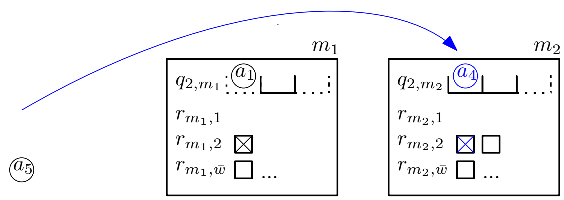
Step 1. All member state decision now depend on the previous period assignment $\psi_1^c(P, C) = Y_1 = \{(a_1, m_1, 2), (a_2, m_1, 1), (a_3, m_2, 1)\}$. To be consistent we require asylum seeker a_2 to proposes her (relabeled) contract $x^1 = (a_2, m_1, 1)$ again, which is now the only contract that will be accepted by any member state. Set $X_2^1 = \{x^1\}$.



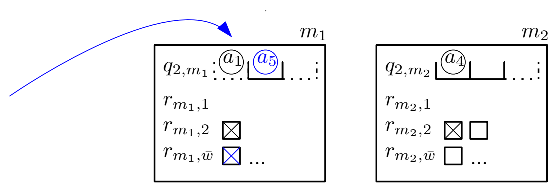
Step 2. Asylum seeker a_4 proposes her first choice $x_2^2 = (a_4, m_1, 2)$, and $X_2^2 = \{x^1, x^2\}$ and the contract is rejected by member state m_1, i.e. $x^2 \notin C_{2,m_1}(X_2^2|Y_1)$ as m_1 has already committed to process a_1, and is unable to process both in the given time frame.



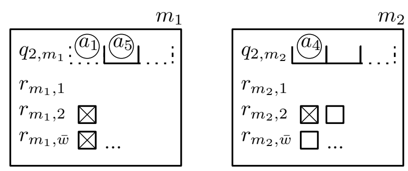
Step 3. Asylum seeker a_4 proposes her second choice $x^3 = (a_4, m_2, 2)$. $X_2^3 = \{x^1, x^2, x^3\}$ and the contract is accepted by m_2, i.e. $x^3 \in C_{2,m_2}(X_2^3|Y_1)$ as m_2 has committed to take at least one asylum seeker, and the bureaucratic capacity for the wait time 2 is sufficient.



Step 4. Asylum seeker a_5 proposes her second choice $x^4 = (a_5, m_1, 3)$. $X_2^3 = \{x^1, \ldots, x^4\}$ and the contract is accepted, i.e. $x^4 \in C_{2,m_1}(X_2^3|Y_1)$ following the same reasoning as the previous step. Note that as $q_{2,m_1} = 1$ is defined in terms of newly arrived asylum seekers, a_1 does not count towards the member states current burden share.



End. As everyone is tentatively assigned a contract the algorithm stops and produces the allocation $\psi_2^c(P, C) = \{(a_1, m_1, 2), (a_4, m_2, 2), (a_5, m_1, \bar{w})\}$.



Overall the asylum seeker proposing mechanism produced $\psi^c : (P, C) \to (Y_1, Y_2)$ with $Y_1 = \{(a_1, m_1, 2), (a_2, m_1, 1), (a_3, m_2, 1)\}$ and $Y_2 = \{(a_1, m_1, 2), (a_4, m_2, 2), (a_5, m_1, 3)\}$. This finally dynamic allocation is dynamically (asylum seeker) strategy proof, stable, and two sided efficient by theorem 2.

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