Mathematics Teacher Educators' Visions for Mathematical Inquiry in Equitable Mathematics Spaces:

Author: Miriam Rebecca Galpin Gates

Boston College Electronic Thesis or Dissertation, 2020

BOSTON COLLEGE

Lynch School of Education and Human Development

Department of
Teaching, Curriculum, and Society

Curriculum and Instruction

MATHEMATICS TEACHER EDUCATORS' VISIONS FOR MATHEMATICAL INQUIRY IN EQUITABLE MATHEMATICS SPACES

Dissertation
by

MIRIAM REBECCA GALPIN GATES

submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

May 2020

MATHEMATICS TEACHER EDUCATORS' VISIONS FOR MATHEMATICAL INQUIRY IN EQUITABLE MATHEMATICS SPACES

By Miriam Rebecca Galpin Gates

Lillie Richardson Albert, Ph.D., Chair

ABSTRACT

In mathematics education, there is an imperative for more just and equitable experiences in mathematics spaces, as well as ongoing efforts to move classroom instruction toward mathematical inquiry. While Mathematics Teacher Educators (MTEs) are expected to support multiple initiatives in mathematics education, they are particularly responsible for the professional learning of teachers and teacher candidates. MTEs must therefore prepare and support the professional learning of teachers to achieve twin goals. This study was designed to understand how MTEs envision their roles in supporting development of teachers across MTEs' many professional functions in their work toward the twin goals of equity and inquiry. The findings suggest that identifying the forms mathematical knowledge takes is important for mathematical inquiry and that interrogating these forms can be used to counter pervasive social myths about who can do mathematics. Further, MTEs articulated three interrelated values for application of mathematics inquiry teaching for justice and equity: creating space, supporting sense-making, and naming how power and privilege have operated and continue to operate in mathematics spaces. Finally, MTEs described how mathematics inquiry practices are a mode for understanding the world and can be used to promote equity by uncovering biases and assumptions. These findings suggest a promising avenue for leveraging mathematical inquiry to increase equitable outcomes in mathematics spaces.

DEDICATION

For Max and Jay,
who make me better,
and love me just the way I am

ACKNOWLEDGEMENTS

While I have been looking forward to writing these acknowledgements across my doctoral journey, noting the acts of kindness, generosity, and grace that I have experienced, in the face of this historical moment, I appreciate them even more profoundly and am so thankful to be able to write them here. I am grateful to each person who shared a coffee, entertained my son, cheered me on, sparked a new idea, or challenged my thinking along the way. I must also acknowledge those who edited my writing, gave me more time when I needed it, suggested a new direction or performed one of thousands of other acts of kindness. Each of you shares in this accomplishment and has made this expedition a more joyful one.

The dissertation would not have been possible without my advisor and dissertation chair, Dr. Lillie Richardson Albert. She has been a consistent presence in my doctoral experience: kindly moving me forward, holding me accountable, and making me a better scholar every day. I have never met anyone who works as hard on behalf of her students. She is the teacher I aspire to be. To my committee members, Drs. Juliana Belding, David Scanlon, and Sarah Sword, your willingness to develop my dissertation concept from its infancy and respond to the unexpected changes that arose over the course of its growth has been invaluable. To the professors and doc students from whom I have learned during my time: each of you has taught me so much; I am better for having known you. To Dr. Jim Slotta who helped me complete the first two years of my program, your guidance and compassion was vital to my progress.

To the participants in this study whose willingness to share their ideas and work made it possible, I am deeply grateful and hope that this dissertation is a tribute to what you do. I am grateful to the scholars in the education field who have informed this dissertation. I also need to acknowledge the librarians of Boston College Libraries, particularly those in the Education Resource Center, whose response to learners’ needs is unparalleled. I thank you all.

In my academic endeavors, I owe a debt of gratitude to my family and friends that I can never hope to repay. To my grandparents, Geoffrey, John, Lillian, Mary Ann, and my great Aunt Gladys, I hope this would make each of you proud. To my parents, Charmaine and Spencer, my first and best teachers, who let me believe that I could do anything and made numerous sacrifices for the many forms in which my education came. I am lucky to know you are always in my corner. For that and so many other things: I love you and I thank you. To Dominic, my “baby” brother, thanks for the laughs and the truth telling. For my mother-in-law, Gale, whose hours of babysitting and restorative meals made my courses and this dissertation achievable; your interest in my classes and research has been sustaining. Thank you for loving us all through this and everything else. And to Allie, my old friend, it’s true, done is so, so good.

Finally, I must acknowledge my husband, best friend, and partner in this life, Jason. Thank you for holding things together, for making decisions when I couldn’t make one more, and loving me when I was being impossible. You deserve so much credit for this dissertation, and everything else you do. We did it. And to my wonderful child, Max, who is growing to be the best of us. Words cannot express how proud you make me or what an inspiration you have been to me along the way. I strive to be the mother you deserve. Do good, work hard, have fun, and learn a lot.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Statement of the Problem

In the past two decades, mathematics education research has turned toward a focus on providing equitable and just opportunities in mathematics for students (Gutiérrez, 2013a) in which teachers need to know how to address both issues of justice and social realities of their students (Martin, 2003) as well as supporting positive developments in their mathematical and racial identities (Martin, 2009a). Frameworks, like Culturally Relevant Pedagogy (CRP; Ladson-Billings, 1995) have been developed to support teachers and teacher educators to center the assets of historically marginalized student populations, in this case African American students (Ladson-Billings, 2014). These ideas have spurred the proposal of Culturally Sustaining Pedagogy, which suggests that not only should curriculum be tailored to meet the cultural needs of students, but also to support their own identity development within their cultural heritage (Paris, 2012). Further, white students need access to Culturally Disruptive Pedagogy (San Pedro, 2018), which helps them to unpack aspects of White Privilege in their classrooms. In mathematics education, in particular, certain kinds of "official knowledge" have previously been valued to the detriment of some students' experiential knowledge (Apple, 1992), and to the detriment of the discipline, which would benefit from many ways of knowing mathematics (Gutiérrez, 2002, 2008b).

In addition to these concerns, there is also an impetus from the mathematics education community to continue to increase opportunities for students to participate in mathematics as an act of doing, rather than the acquisition of a static body of knowledge (e.g., Schoenfeld, 2016). In this way, students are inducted into the mathematical practices that professional mathematicians do in undertaking their work. These mathematical practices have been described as: (1) asking deeply mathematical questions, (2) applying appropriate patterns and methods of reasoning with warranted conclusions, (3) furthering the goals of the existing discipline, (4) using language that is familiar to practitioners of the discipline, and (5) creating a body of knowledge through accepted findings (Kitcher, 1984). These practices are strongly connected to ideas that will serve students in their lives beyond schools, being useful to their future selves (Cuoco, Goldenberg, & Mark, 1996). Despite the importance of these practices, students do not often have the opportunity to pose or pursue their own mathematical questions, but rather are provided with a body of mathematical knowledge (Ernest, 2016). The promise of mathematical inquiry activities is creating opportunities for students to participate in problem posing and sense making experiences. In the past, these have been assumed to be for "exceptional" individuals and not appropriate at all levels of mathematics education or applicable to each group of students (Silver, 1994).

If students are to experience inquiry in the context of a more equitable classroom, the mathematics teacher education community faces a challenge. While not incompatible, teacher candidates must learn to provide both an equitable classroom experience that interrogates the origins of mathematical knowledge, and creates a way to do mathematics in community with each other and with other members of the discipline. Ball, Goffney, and Bass (2005) have noted that deploying some reform or inquiry practices, without attention to equity can reproduce the existing social experiences of oppression. So, teachers must be prepared to work within the existing framework of mathematics and support students in questioning practices, and even the discipline, that has excluded so many. Gutiérrez (2009b) has dubbed this the tension between “teaching students” and “teaching mathematics,” whereby teachers must teach into the space between the two, rather than choose between them (p. 14). Educators of these mathematics teachers face an even broader challenge: how to prepare teacher candidates (TCs) to hold both of these student needs in their minds in the many teaching contexts they will enter, how to provide ongoing support to in-service teachers, and how to focus on their own research goals. MTEs need to play each of these roles all while focusing on the tensions and supports presented by these interactions.

Thus, mathematics teacher educators (MTEs) need a complex set of skills, ideas, and agendas in order to undertake their work. Despite this, MTEs report needing more support to develop these conceptions and skills early in their careers (Yow, Eli, Beisiegel, Mccloskey, & Welder, 2016). Further, research focused on teachers transitioning to roles as teacher educators suggests that new teacher educators need professional learning opportunities to support the shift from classroom work with students learning mathematics content to classroom, research, and school-based work with teachers developing their skills to support students’ mathematics learning (Ping, Schellings, & Beijaard, 2018). Despite these initial findings, there is a dearth of research on what it means to be a mathematics teacher educator (L. Brown, Helliwell, & Coles, 2018). Thus an accounting of what the profession entails, what a vision for being a mathematics teacher educator looks like, and how one undertakes work in this complicated space of inquiry and equity will provide the field with some direction in considering the kind of support new MTEs require. A particularly useful approach to this work is to consider experienced MTEs professional vision for their work. Professional vision is an idealized vision of what work in the profession reflects (Hammerness, 2001). Therefore, professional vision provides a roadmap of how practice unfolds and is directed (Hammerness, 2001). Thus, understanding MTEs' visions for their role in supporting the ongoing development of K-12 teachers might provide guidance for novice MTE's own development in practice.

Purpose of the Study

The purpose of this project is to unpack how MTE's vision their role in supporting development of teachers across their many professional functions. In particular, I seek to describe MTE's professional vision for their work toward goals of equity and inquiry and in what contexts they enact these goals. In order to address this purpose, I have undertaken a case study that examines two major research questions:

1. How do MTEs choose, operationalize, and enact equity and inquiry goals in their professional visions for their work with K-12 teachers?
2. In what ways does the attention to inquiry and equity goals create a tension between or support for "teaching students" and "teaching mathematics" in their vision for work with K-12 teachers?

Context of the Study

This study is undertaken with four mathematics teacher educators (MTEs). The study is designed to understand how MTEs see their roles in working toward mathematical inquiry and equity goals in their own professional work. These MTEs have made a commitment to mathematical inquiry and equity in their practice and were selected for this reason. Participants included individuals who were appointed in both mathematics and education departments. In this analysis, MTE will be broadly construed to mean any individual who holds a primary role in providing teacher candidates or early career teachers with teaching in the discipline and/or pedagogy of mathematics.

Theory of Action

Figure 1.1 displays the theory of action for the impact of a Mathematics Teacher Educator on students. In the figure, the relationship between MTE and K-12 students is suggested. This model demonstrates how the MTEs' relationships with K-12 teachers and their students might impact connected aspects of classroom practice. Importantly, the arrows are double headed indicating the bi-directional relationships of MTEs, K-12 teachers, and their students. This indicates that MTEs play a role in supporting the development of K-12 teachers, first as their educators and later as partners in or out of schools through professional development and research. In the current study, MTEs will be the focus of analyses. The influence of each of these constituents, teachers and students, will impact how the MTEs envision their roles.

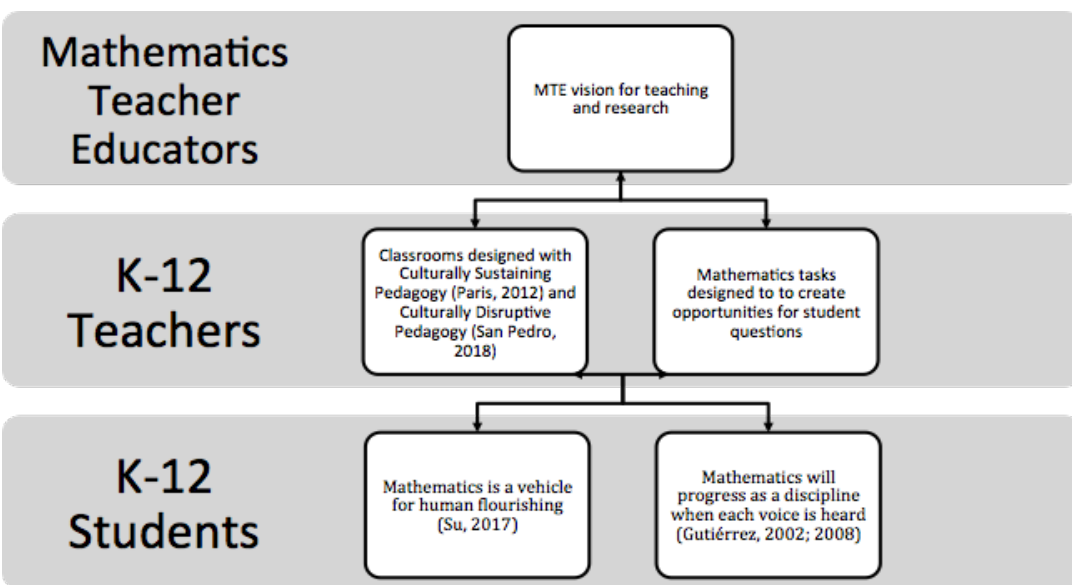

Figure 1.1 Theory of action for current study

The skills and expertise for K-12 teachers that are identified in this model are not expected to emerge in a single pre-service mathematics methods course, but in combination with learning in other portions of pre-service and in-service programs. MTEs vision for their role in teacher development will guide how early in their careers they envision that teachers might acquire some of these skills and which they believe to be most important. This Theory of Action provides a grounding context for the data analysis. In this study, I assume that MTEs are working with the ultimate goal of impacting K-12 students. However, part of what this study seeks to uncover are the multitude of ways that MTEs envision this work unfolding.

Importance to the Field

In what follows, I provide four separate case studies, using portraiture (Lawrence-Lightfoot, 2005), followed by a cross-case analysis. This work provides three interrelated strands in the worlds of inquiry, equity, and understanding the role of MTEs. First, each case study provides four separate stories of how MTEs envision their roles in the development of teachers with respect to inquiry and equity. While these stories are individual and institution-specific, concrete examples and will provide the field with a way to think about MTEs' role in the development of teachers and mathematics education. Second, the cross-case comparison provides some information about possible universal experiences and which might be case-specific in these MTE visions. Further, the cross-case comparison will provide insight into the extent to which inquiry and equity experiences are compatible. Finally, the study provides information about how MTEs envision the development of teachers and MTEs' role in supporting that development.

Commitments and Assumptions

For the study, I worked from the following perspectives and assumptions. Francis Su recently described his vision for the future of the discipline of mathematics. He stated:

> I want us as a mathematical community to move forward in a different way. It may require us to change our view of who should be doing mathematics and how we should teach it. But this way will be no less rigorous and no less demanding of our students. And yet it will draw more people into mathematics because they will see how mathematics connects to their deepest human desires. So if you asked me: Why do mathematics? I would say: Mathematics helps people flourish (Su, 2017, pp. 483–484).

I want to join Francis Su in making this commitment for this future of the mathematical discipline. I assume that each K-12 student has a right to a rigorous mathematics experience in which they are valued for who they are and welcomed to participate in the discipline of mathematics. I assume that mathematics can be one vehicle for "human flourishing." Further, I recognize that in the US, the system of schooling has excluded children from this experience based on their race, sex assigned at birth, gender identity, sexual identity, disability status, family constellation or linguistic diversity. Additionally, I believe that as a discipline mathematics has suffered from the loss of these voices. Given my understandings of the world, it is incumbent upon me to support changes to an unjust system, so that students, teachers, other mathematics educators, and I can all be more free.

Core Assumptions

There are many aspects of mathematics education in which equity must be explored, each inequitable practice and system deserves unique and complete consideration. For the purpose of this study, I have let the MTEs be my guide in which forms of equity they have chosen to identify. However, I will let the following assumptions guide the work of this study.

1. School mathematics has long been governed by institutional systems of tracking, rote teaching, and limited access to well-prepared teachers, that have restricted students' opportunities in mathematics because of their racial identities (Berry, Ellis, & Hughes, 2014). And because mathematics can act with unearned privilege in society and as a socialized proxy for intelligence (Gutiérrez, 2018), these institutional systems can perpetuate deficient views of students' abilities in mathematics.
2. Disciplinary mathematics is created and re-created by humans (Ernest, 1989). Despite this, students are often provided with a static vision of what mathematics is (e.g., Ernest, 2016). As a result, many students do not have the opportunity to experience the doing of mathematics in schools.
3. In mathematics classrooms, issues of inquiry and equity must be treated in concert if students are to benefit from them. Mathematics teaching practices that might be classified as "reform," implemented without attention to equity can reproduce existing social inequities in the classroom (Ball et al., 2005). As a result, while inquiry practices are important, they need to be interrogated in their implementation for any inequities they may perpetuate or create.

These assumptions influenced data generation efforts and data analysis activities across the project.

Key Terms

In the following section, I define the terms and the associated assumptions that guided my work and underlie the conclusions that I reach in what follows. These definitions are provided in alphabetical order to ease use during the course of reading the study. I want to underscore that this order does not indicate relative importance of terms.

The definitions themselves are provided to highlight how I think about these key terms. Participants were not provided with how I think about these ideas, but rather, were asked to provide their own understandings. Part of the work of this project is reconciling the various existing conceptions of these key terms in the mathematics education field and therefore it is important that I provide my own understandings before analysis.

Equity in Mathematics Education. For the purpose of this project, I will draw upon Aguirre's (2009) definition of equity. I draw upon her work because she emphasizes both the importance of embracing students in their full humanity and leveraging mathematics as an approach to empowering students. She wrote,

> To me equity means that all students in light of their humanity - personal experiences, backgrounds, histories, languages, physical and emotional well-being - must have the opportunity and support to learn rich mathematics that fosters meaning making, empowers decision making, and critiques, challenges, and transforms inequities/injustices. [...] equity demands that responsible and appropriate accommodations be made as needed to promote equitable access, attainment, and advancement for all students. [...] Equity and mathematics

> comprise a powerful dialectic that is continually being constructed. It is important to acknowledge that this work is always evolving because the work for equity and social justice is never a finished product (Aguirre, 2009, p. 296).

In addition to focusing on differing needs of students and the various uses of mathematics, Aguirre's definition also emphasizes the dynamic nature of approaching equity in mathematics education classrooms.

Inquiry Approaches to Curriculum and Pedagogy. Mathematical inquiry can encompass multiple approaches to teaching mathematics, including discovery learning (e.g., Tuovinen & Sweller, 1999), problem based learning (e.g., Savery, 2006), and constructivist approaches to sense-making (e.g., von Glasersfeld, 1983). In this definition Staples (2007) encompasses many of these ideas:

> Inquiry is a practice or stance, and indicates a particular way of engaging with and making sense of the world [...] Inquiry into mathematics involves delving into mathematical ideas and concepts and trying to understand the structure, power, and limitations of mathematics. Inquiry with mathematics involves using mathematics as a tool to make sense of problem situations and come to some reasonable resolution [...] Learning results from, and is evidenced by, student participation in both standard disciplinary practices (e.g., justifying, representing algebraically) and an array of other practices of mathematical communities (e.g., questioning, communicating, informal reasoning).

For me, one important aspect of the mathematics inquiry model is the inclusion of *problem posing* (Silver, 1994), which is often absent from curricular approaches. In problem posing, students construct new problems by asking questions about previous problem-solving activities or by identifying new mathematical questions from the environment applying creative processes (Silver, 1994). For me, inquiry approaches refers to the process of problem creation in response either to external mathematical situations or existing problem posing activities (Silver, 1994) in addition to other aspects of mathematical inquiry approaches.

Theoretical Position and Epistemological Framing

In undertaking the research for this project, I am taking a sociopolitical stance. Gutiérrez (2013a) defines sociopolitical researchers as "those who...seek not just to better understand mathematics education in all of its social forms but to transform mathematics education in ways that privilege more socially just practices" (p. 40). In this section, I define what I mean by "sociopolitical stance" and identify how the intersection of my many identities impacts this work. In Chapter 3, I demonstrate how this stance impacts my research methods.

The Necessity of a Sociopolitical Stance

A sociopolitical stance is an important part of the work that I am undertaking for two reasons. First, teaching is a political activity and undertaking it without recognizing the ways in which it is political can be damaging to students (Felton-Koestler & Koestler, 2017). Mathematics teachers in particular must understand how mathematics has been deemed societally neutral, but has worked to exclude students of color, linguistically-diverse students, and students with disabilities (Gutiérrez, 2013a). While I am not operating as a teacher educator or K-12 teacher in this case, I will be interacting with MTEs who will be grappling with these ideas and needs.

Second, just as teaching is political, research focused on equity and mathematics is also necessarily political (Pais & Valero, 2010). In 2012, four obstacles to tackling issues of race and ethnic equity in mathematics education were identified as: a) race and ethnicity conversations are not central to the work of mathematics education research as evidenced by leading journals publication records; b) race and/or ethnicity is treated as a static independent variable that influences students in predictable ways; c) race and/or ethnicity of the researcher is not explored as contributing to the way in which research might unfold; and d) even in literature where race and/or ethnicity are explored, authors tend to do so in a superficial way (Parks & Schmeichel, 2012). Thus, if I am to meaningfully address the issues that will emerge from the work as envisioned by participants, I need to attend carefully to issues of power and race as they have emerged from the study. While the Parks and Schmeichel (2012) study applies specifically to race and ethnicity, other kinds of diversity require a similar kind of attention.

Positionality

As I enter into this work, I take a cue from Albert (2005) recognizing my own positionality as a white middle-class cis-woman studying at a Predominantly White Institution (PWI). Further, given my commitment to a sociopolitical stance and my position as a novice to many of these topics, I am going to use the framework outlined by Wiley and Drake (2013), provided to novice teachers to understand their contexts relative to their identities, by examining their position and decisions at four levels: personal, interpersonal, institutional, and public. At the personal level, I need to interrogate their own privilege and power (Wiley & Drake, 2013). At the interpersonal level, I will seek to understand how I might disrupt oppression through interactions with others (Wiley &

Drake, 2013). At the institutional level, Wiley and Drake focus on how organizational structures can influence oppression, either by reinforcing or disrupting in. Finally, the cultural or public-sphere level is defined as the external structures, pressures, or norms that can influence liberation or oppression.

I describe two of these levels here, the personal and interpersonal, relative to my role as a researcher. I used the other two levels, institutional and public sphere, to guide decisions around analyses and the sharing of findings. On the personal level, Wiley and Drake (2013) suggest asking questions like, “How am I privileged?” (p. 67). On this level, I need to understand how my own upbringing in well-resourced, primarily white schools and universities has influenced my perspectives on schools and the schooling process. As a child and young person, my schooling experience was primarily positive, especially around experiences in mathematics. As an adult, I taught in several schools and have worked with some students who had similar experiences and those who had very different experiences from mine. In combination, my own experiences of schools as a child and an adult frame my perspectives as a researcher in mathematics education. In the course of my project, I have considered how my privilege has impacted the decisions that I have made.

At the interpersonal level, Wiley and Drake (2013) suggest future teachers ask, “How might I transform my critique of [someone I work with] into thoughtful, provocative, and action-driven questions?” (p. 67). In my case, I consider how my position as a graduate student who is studying the practice of MTE might be influenced by this question. That is, if, as a researcher, I have critiques or responses to choices made by MTEs, “how can I use them to support growth as a researcher and support the field’s development?”

Given my individual experiences and layers of privilege, I need to be particularly careful in the relationship as a researcher with respect to existing privilege as I explore issues of marginalization as suggested by Vakil, McKinney de Royston, Nasir, and Kirshner (2016). This is further complicated because no member of this project will necessarily be part of the communities that teachers will be serving. As a researcher, I have tried to be aware that this poses a challenge, particularly in terms of power.

Overview of the Chapters

This chapter offers an overview of the current study, including grounding assumptions, research objectives, and the importance of this study. The second chapter offers an overview of literature associated with the purpose of the current study and a framework for Knowledge for Mathematics Teaching. Chapter 3 explains the methods of this study including justification, design, and procedures. Chapters 4 and 5 outline the major findings from this study. Chapter Four provides a portrait of each participating MTE and Chapter Five explores a comparison of cases. Chapter 6 will provide a discussion of the major findings, conclusions and implications, the limitations of the current study, and future directions for this line of work.

CHAPTER 2

A REVIEW OF THE LITERATURE

In this chapter, I outline literature relevant to the proposed study. First, I will underscore some of the needs of K-12 mathematics classrooms and their teachers as a result of the current state of schooling, with attention to equity and inquiry. Next, I describe one way to conceive of the knowledge teachers require in order to address these needs. This conception of mathematics teaching knowledge will drive an understanding of choices that mathematics teacher educators make. Finally, I will outline the knowledge, practices, and learning opportunities that MTEs might draw upon in making their decisions for the work of teacher candidates. I include in this section information about professional vision for MTEs. The review of the literature will guide decisions for data generation and analysis outlined in Chapter 3.

The Needs of K-12 Mathematics Classrooms and their Teachers

In the United States, teachers serve increasingly diverse populations of students at all levels of schooling (Musu-Gillette et al., 2016). Further, these students are completing high school at higher rates and moving on to tertiary education at higher rates than ever before (Musu-Gillette et al., 2016). On the other hand, bachelor's degrees in STEM fields do not reflect the changing demographics of K-12 education (de Brey et al., 2019). In combination, this suggests that educators who prepare teachers for U.S. schools need to address the changing needs of future K-12 students as they move through their own education. In mathematics education, where outcomes are not the same for each population of students (Musu-Gillette et al., 2016) additional work is necessary.

The Current State of Mathematics Education

While mathematics has often considered a “neutral” or “cultureless” discipline, researchers have argued that its construction and resulting practice the United States has denied its multicultural, multi-ethnic roots and has constructed particular, white mathematical creators as more responsible for the discipline (Felton-Koestler, 2017). It is in this environment that in just the past decade, mathematics education research has turned in order to focus on providing equitable and just opportunities for students (Gutiérrez, 2013a) in which teachers need to know how to address both issues of justice and social realities of their students (Martin, 2003). The outcomes referenced in the opening paragraph reflect the systematic barriers that students have faced over time in the country. In fact, Gloria Ladson-Billings (2006) has argued that as a nation, the United States owes students of color an educational debt due to these generational barriers. While a full accounting of the barriers faced by students in their pursuit of their right to a mathematics education is outside the scope of this review, here recent developments in policy around mathematics education will be outlined to demonstrate the kinds of knowledge future teachers need.

Since 2001, No Child Left Behind (NCLB), the Mathematics for All movement, and Common Core State Standards (CCSS), have been the major drivers in mathematics education (Berry et al., 2014). According to Ellis (2008) NCLB, which purported to improve mathematics through “objective” science has instead continued to stratify students, promoting existing differential opportunities for students based on race. Additionally, the focus on “highly qualified” teachers has used the same problematic testing procedures to identify who is prepared to teach (Davis & Martin, 2018). Further, NCLB has supported the development of curricula that focus on “test preparation” to the detriment of other learning experiences (Musoleno & White, 2010).

The “Mathematics for All” movement purports to provide an egalitarian and outstanding experience for **all** students in mathematics (Martin, 2010); however, Martin argues that in this form, it preserves existing inequitable structures. Both neoliberal and neoconservative projects promote programs that work for all students at once rather than focusing on individual student experience within their own social position (Martin, 2010). From this perspective, the focus on international competitiveness and economic workforce means that students must conform to the dominant white culture, thus destroying cultural differences between students (Martin, 2010). In forcing students to assimilate to white culture, students of color are always depicted as deficient, thus propagating the existing system, as in the “achievement gap” literature which spans student development from pre-kindergarten through tertiary education (e.g., Harackiewicz, Canning, Tibbetts, Priniski, & Hyde, 2016; Lee & Reeves, 2012; Wang, 2008). Berry and colleagues (2014) indict the Common Core State Standards for the same reasons, that is, while the Common Core provides lip service to equitable experiences for students, the standards do not attend to the shared social experiences of particular student groups. While the impacts of the latest federal legislation, Every Student Succeeds Act (ESSA) are yet to be felt, there will be some lasting impacts in the years to come. Further, even as teachers need to better understand systematic challenges that their students face, they also need to be aware that each student’s demography is not their identity (Gutiérrez, 2013a).

Problem Posing in Schools

There is an impetus in the mathematics education community to treat the learning of the discipline as an act of "doing", rather than the acquisition of a static body of knowledge (e.g., Schoenfeld, 2016). From this perspective, students should be inducted into some of the central practices of "doing" mathematics. Kitcher (1984) describes these mathematical practices as: (1) asking deeply mathematical questions, (2) applying appropriate patterns and methods of reasoning with warranted conclusions, (3) furthering the goals of the existing discipline, (4) using language that is familiar to practitioners of the discipline, and (5) creating a body of knowledge through accepted findings. Each of these practices is both an active and collaborative experience. Further, it has been argued that through the mathematical practice of "doing," particularly of asking and pursuing mathematical questions, students become stronger problem solvers (Silver, 1994). Researchers argue that promoting these activities provide students with access to mathematical habits of mind, which are extensible ways of thinking that prepare them for a world beyond school (Cuoco et al., 1996; Cuoco, Goldenberg, & Mark, 2010).

However, traditionally mathematics classrooms in the United States have been characterized by a series of unconnected mathematical procedures which students must learn to reproduce, with little emphasis on sense-making and low-level challenge (Hiebert et al., 2005). These methods for teaching mathematics provide little space for students to gain authority over problem posing (Silver, 1994). Further, problem posing has been associated with exceptional mathematical talent and creativity, meaning that it has not been an activity that has been promoted for each student (Silver, 1994). Traditional approaches to mathematics teaching have proven resistant to change, perhaps because a teacher's experience as a student is more salient to them than the teacher's experience in preparation due to the cultural importance of the experience of being a student (Hiebert, 2013). Even in reform-oriented mathematics classrooms, much of what occurs focuses on sense-making and response to questions asked by the textbook or teacher, rather than students posing or pursuing their own mathematical questions (Ernest, 2016). These classroom experiences then do not leave room for students to participate in the first of Kitcher's (1984) practices in mathematics: posing mathematical questions.

In sum, the research on equity in mathematics classrooms, particularly meeting the needs of students who have diverse experiences of the culture in and around schools and shifting the problem posing opportunities in schools, requires teachers of mathematics to have a complex and nuanced set of tools and knowledge. This knowledge can be developed through preparation programs supported by mathematics teacher educators. It is the assumption of the project's theory of action (See Figure 1.1) that MTEs do impact development of this kind of knowledge. In the next section, I will outline how I conceive of Knowledge for Mathematics Teaching.

Knowledge for Mathematics Teaching

As it stands, Louie (2017) argues that mathematics teaching advances a particularly narrow view of what mathematics is and who can do it and is particularly influenced by the culture and larger cultural context. Thus, because of the developmental trajectory in mathematics education in the United States, teachers must be prepared for the political nature of their future work (Felton-Koestler & Koestler, 2017; Gutiérrez, 2013b). However, it is not always clear what researchers and educators mean by teaching as a political act, with impacts on equity or social justice (Bartell, 2012; Gates &

Jorgensen, 2009). Further definitions of what equity and social justice are, are not consistent across the mathematics education field (Lawler & Uy, 2017).

However, Dyches and Boyd (2017) have proposed an extension to Shulman's (1986) notion of the special kinds of knowledge needed for teaching including both subject-matter knowledge and pedagogical content knowledge (PCK), which organizes some of these previous ideas. Dyches and Boyd's (2017) framework proposes that social justice knowledge for teachers is comprised of general social justice knowledge which contains two subdomains: social justice pedagogical knowledge and social justice content knowledge. They named this framework Social Justice Pedagogical and Content Knowledge (SJPACK). In mathematics education, the "Mathematics Knowledge for Teaching" (MKT; Ball, Thames, & Phelps, 2008) framework, which was also derived from Shulman's has been widely applied; however, this framework does not attend explicitly to social justice..

In the remainder of this section, I will use SJPACK (Dyches & Boyd, 2017) and aspects of MKT (Ball et al., 2008) to organize a new framework that centers the knowledge teachers need to teach rigorous mathematics for social justice. In what follows, I will suggest how SJPACK can be amended to meet the disciplinary needs of mathematics and to align with assumptions outlined in Chapter 1. In addition, I will focus on some of the unsettled questions in mathematics teacher education. Because of the amalgam of several frameworks, rather than calling the newly proposed scheme "Social Justice Knowledge," I propose to call it "Knowledge for Mathematics Teaching." I do this to avoid the suggestion that "social justice teaching" should or can be treated as separate from "mathematics teaching."

Figure 2.1 describes the relationships between three major domains of Knowledge for Mathematics Teaching: Knowledge about mathematics and society, specialized pedagogical content knowledge for mathematics teaching, and mathematics subject matter for teaching. Briefly, knowledge about mathematics and society is the related understandings of the historical and current day positioning of mathematics in schooling. Specialized pedagogical content knowledge for mathematics teaching describes the knowledge teachers need to make decision and to implement equitable inquiry instruction in mathematics classrooms. The final domain, mathematics subject matter for teaching, encompasses mathematical disciplinary knowledge teachers need in order to do their jobs in current society. The domains are arranged in this order because I assume that knowledge about mathematics and society to be the broadest domain due to its interdisciplinary nature. I assume that specialized pedagogical content knowledge for mathematics teaching, while still interdisciplinary, is more specific to the discipline of mathematics. Finally, subject matter knowledge for teaching, while it includes a variety of kinds of knowledge, it is specific to the discipline of mathematics education.

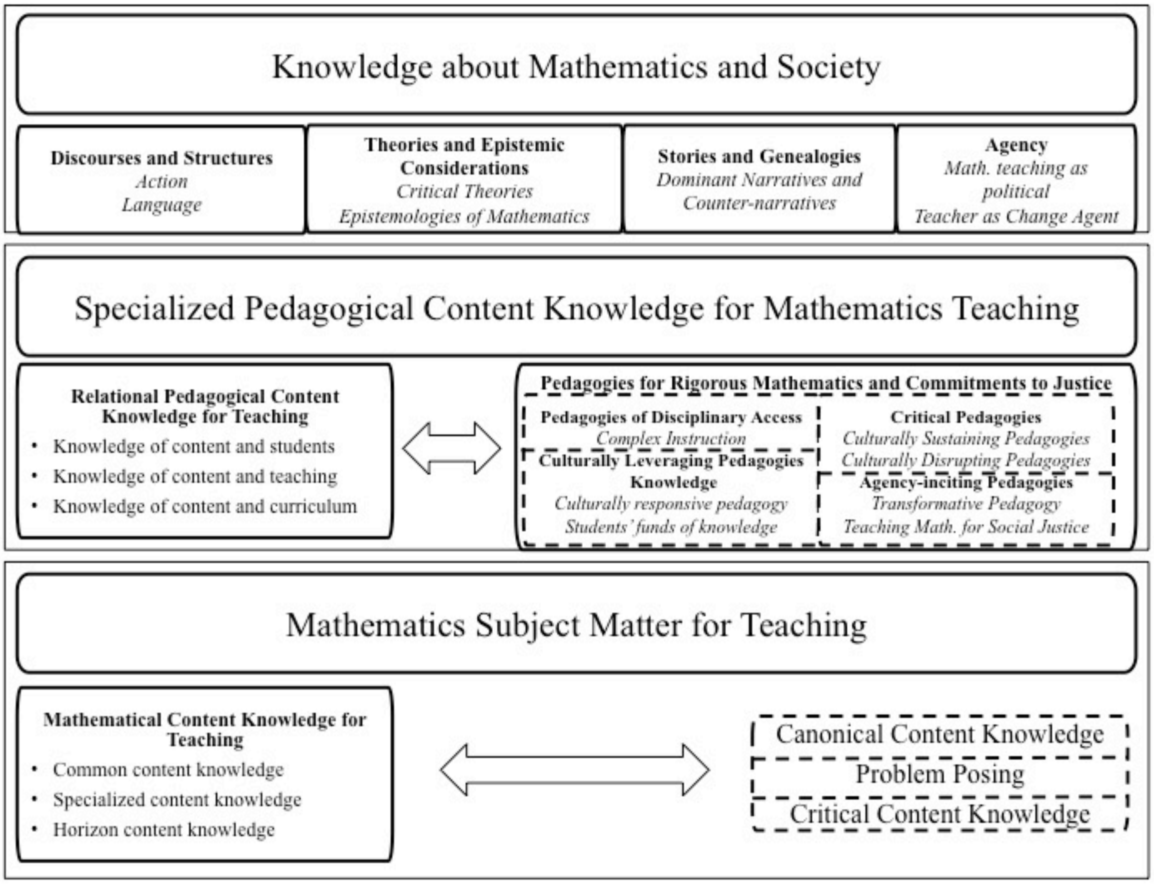

Figure 2.1. Knowledge for Mathematics Teaching. Adapted from "Mathematics Knowledge for Teaching" Framework (MKT; Ball, Thames, & Phelps, 2008) and Social Justice Pedagogical and Content Knowledge (SJPACK; Dyches & Boyd, 2017).

Knowledge about Mathematics and Society

The top section of Figure 1 focuses on Knowledge about Mathematics and Society, because of its interdisciplinary nature. In order to understand how mathematics operates in schools, knowledge about history, sociology, and so on is required. Mathematics might be considered a triumph of the human experience, but it is also marked by the same social structures of power and oppression as society itself. Because mathematics is a reflection of the larger society, teachers need to understand both how issues of status, power, and oppression impact society, but particularly how these have emerged in and around mathematics and schools. This domain of knowledge requires that teachers come to understand how schools, the discipline of mathematics, and they themselves are shaped by culture and the historical development of mathematics as it is now practiced. This domain is subdivided into four subdomains: discourses and structures, theory and epistemic considerations, stories and genealogies, and agency.

Discourses and structures. The subdomain of discourses entails the various social artifacts (e.g., social roles, beliefs, values) reflect an individual's place(s) in the world (Dyches & Boyd, 2017). As such, an individual can be an insider or outsider in a social space and this impacts the view of equity (Dyches & Boyd, 2017). SJPACK divides discourses into two threads: actions and language. Both discourses are applicable to larger society and are part of what teachers need to know in order to be prepared to enter schools and systems of schooling. In what follows, I will provide a short, far from inclusive list, of how these discourses apply specifically to the mathematics classroom.

Action. These include institutional, systematic, and otherwise embedded phenomenon that act to maintain the status quo. Dyches and Boyd (2017) note that often in these cases, individuals can ignore the social realities of others and in this way contribute to systems of oppression. Take, for example, the case of ability tracking in United States mathematics classrooms, where it is part of the norm experienced by students (Boaler, 1999). Proponents argue that curricular tracking better allows teachers to meet the needs of their students (Ballón, 2008; Stiff, Johnson, & Akos, 2011). However, research suggests that the results are detrimental, particularly for students in the "lower" track (Boaler, 1999; Stiff et al., 2011). These results include: reduced numbers of opportunities to learn, less interesting mathematical coursework, lower expectations from teachers, and reinforcing the status of "better" and "worse" mathematics students (Boaler, 1999). Study results suggest that being in a “lower” track negatively impacts the desire to pursue goals in mathematics (Simzar, Domina, & Tran, 2015). This would be troubling enough if it were equally distributed across student populations; however, research shows that students of color who do not identify as Asian or Pacific Islander are disproportionality placed on lower tracks (Ballón, 2008; Oakes, Ormseth, Bell, & Camp, 1990). While some of the differences can be explained by lower achievement scores and differential school experiences (Ballón, 2008), research also shows that teachers predict lower scores for Black, Latinx, and Indigenous children, and thus suggest them for lower tracks at rates higher than their white or Asian peers (Stiff et al., 2011).

Another example of how action exists in mathematics classrooms is in the application of reform approaches to instruction. For example, “real world” contexts of problems may only make sense to particular students while others have to spend significant cognitive energy trying to unpack the meaning (Ball et al., 2005). Another common reform practice is when teachers are hesitant about explicitly telling students approaches, but rather having the students determine their own approach (Ball et al., 2005). However, this again may mean particular students gain access, while others do not; these differences often reflect existing social inequalities across racial or socioeconomic lines (Ball et al., 2005; Esmonde, Brodie, Dookie, & Takeuchi, 2009). Esmonde and colleagues (2009) argue, though, that rather than being a reason to create more homogenous groups, teachers should be prepared to pay particular attention to the ways in which social identities, including membership in a group that has been historically marginalized, but also friendships, and other student characteristics, impact how students participate.

Language can have the impact of constructing cultural norms that maintain power structures and accepted myths which obscure their oppressive nature (Dyches & Boyd, 2017). While this has a number of implications in mathematics education, two important examples in mathematics education include the prevalence of achievement gap language and the myth of meritocracy.

The language of “achievement gap” on the surface has been designed to improve the outcomes for students of color; for example, the National Center for Education Statistics (NCES) produces an annual report on the status and trends in education with regard to the education of racial and ethnic groups with the purported purpose of supporting greater achievement for students of diverse backgrounds (Musu-Gillette et al., 2017). However, the use of achievement gap language has several consequences for students. Gutiérrez (2008a) argues that simply by focusing on the achievement gap, equity becomes harder to achieve. The achievement gap can promote the following detrimental concepts: a static picture of student progress, stereotypes of students who have been systematically marginalized in schooling, a focus on identifying “technical” responses to the challenge, and a narrow idea of what equity and learning are (Gutiérrez, 2008a). The achievement gap can also provide the impression that there are “problems” with individual students, rather than entrenched social structures that perpetuate problems of power (Milner, 2013). Further, the use of achievement gap language perpetuates the normalization of Whiteness, where white students are the standard to which all other students are compared (Milner, 2013).

A second example of how language can be used to perpetuate power structures in mathematics classrooms is the construction of the “myth of meritocracy” (e.g., Cobb & Russell, 2015; Dyches & Boyd, 2017; Milner, 2013). The myth of meritocracy can be summed up as a belief that each individual is born with the same advantages and as a result, any success (or failure) that individuals achieve is due to personal characteristics such as hard work and intelligence (Cobb & Russell, 2015). The myth of meritocracy ignores any structural, institutional or other societal aspects that provide particular individuals with systematic privileges (Cobb & Russell, 2015). The myth of meritocracy is entwined with the tradition of tracking mathematics schooling; that is, it is assumed that individuals in higher tracks at schools have made it there through “hard work” or “intelligence” rather than opportunities that are systematically provided to some and denied to others (Cobb & Russell, 2015).

Theory and Epistemic Considerations. Teachers need knowledge of theories that work to uncover how systems of oppression are produced and reproduced in society (Dyches & Boyd, 2017). Here, I highlight a few critical theorists (e.g., Ladson-Billings & Tate IV, 1995; Martin, 2007); however, as this framework continues to be developed, LatCrit, critical disability theories, and queer theory should be included. Additional attention needs to be paid to how intersectionality of oppression, for example, that of race and gender (Crenshaw, 1991), can impact mathematics classrooms. Dyches and Boyd (2017) argue that teachers and teacher candidates with social justice knowledge for teaching need to have a particular view of the knowledge of their discipline as subjective, rather than neutral or objective. This is particularly important in the world of mathematics education, where mathematics is typically constructed as neutral or culture-free (Leonard, Brooks, Barnes-Johnson, & Berry, 2010).

Critical Theories in Mathematics Education. While a complete list of how critical theories have been applied to mathematics classrooms is not appropriate here, some of what understanding these theories can help mathematics teachers with will be delineated here. In their seminal piece, Ladson-Billings and Tate (1995) argued that race is ever present in the United States and has shaped the development of society and its schools. Tate (1997) suggests that there are five major elements of critical race theory (CRT) that support the goal of ending racial and other forms of oppression. These include: (1) recognizing that race is endemic in the United States, (2) borrowing from multiple theoretical traditions, (3) reinterpreting previous events (i.e., civil rights law, multicultural education) with a critical lens for additional impacts, (4) uncovering or spotlighting dominant claims of neutrality or objectivity within education or the discipline, and (5) challenging ahistorical or acontextual understandings of the experiences of students of color (Tate, 1997).

Crenshaw (1991) argues that for Black women whose identities put them at the intersection of race and gender oppression, the interaction of oppressive structures require attention. This can be a particularly salient issue for mathematics classrooms, where racist and sexist assumptions impact the experiences of girl students of color (e.g., Gholson, 2016; Gholson & Martin, 2014; Joseph, Hailu, & Boston, 2017). Ladson-Billings and Tate (1995) argue that through applications of CRT to schools and schooling, a radically new paradigm for (mathematics) education can be born. In classrooms, teachers can use these theories to understand how in the design of their classrooms they are contributing to mathematical and academic identities, but also are co-

constructing student racial identities (Martin, 2007). Without teachers who critically examine classroom practices, students can be harmed in the process of their mathematical education (Martin, 2007).

Epistemic Considerations. Mathematics is generally considered to be a neutral, culture-free discipline (Felton-Koestler & Koestler, 2017); however, mathematics is neither neutral nor culture free (Felton-Koestler & Koestler, 2017; Leonard et al., 2010; Nasir, Rosebery, Warren, & Lee, 2014). Volmink (1994) argues, for example, that the origins of geometry, which are often given “formal” roots in Greece and “informal” roots in Egypt and Mesopotamia, are much more complicated. In particular, geometry was not limited in use to social elites in any of those societies; there were formal procedures in Egypt and Mesopotamia, and the true roots of geometry run even deeper and are lost to history (Volmink, 1994). Further, in mathematics schooling, research has shown that particular forms of mathematical knowledge and approaches have been privileged and hold power over other forms (Carraher, Carraher, & Schliemann, 1985; Nasir & McKinney de Royston, 2013). For teacher knowledge, this stance and understanding of the epistemic nature of mathematics has several implications.

Stories and Genealogies. In the original SJPACK, this subdomain was named “history,” and while the authors of SJPACK were careful to note that history is not static, I want to emphasize the ways in which historical marginalization can be experienced as ongoing and so I have renamed the sub domain. There are dominant narratives that teachers need to be prepared to understand and disrupt including traditional pedagogies, student tracking, deficit perspectives of students, their families, and communities, narrow definitions of who does mathematics, and the positioning of mathematics as neutral

(Felton-Koestler & Koestler, 2017). In order to achieve these disruptions, teachers must learn/hold expansive, nuanced views of teaching, learning, disciplinary mathematics, and young people who have been systematically marginalized; recognize and develop novel interpretations of mathematics, social justice, student experiences, and teaching as a profession; adopt an advocacy position both in their profession and with marginalized young people; and finally, learn to creatively respond to external narrow conceptions of students and their families or communities, and the discipline of mathematics (Gutiérrez, 2016). Counternarratives uncover the ways in which teachers can support students who might be negatively impacted by dominant narratives. For example, Berry, Thunder, and McClain (2011) argue factors that supported students in their middle grades success could include: (1) fluency in mathematical computation by Grade 3; (2) external recognition for success; (3) personal connections to supporters; and (4) mathematics that provided a unique intellectual challenge. Teachers who have this kind of knowledge can act in the best interest of their students.

Agency. Dyches and Boyd (2017) argue that teachers must know how to act as change agents, where they have both an individual understanding of their profession as political and how they might use their political activity to effect change. While Dyches and Boyd provide particular dimensions of this subdomain, in mathematics education, the two seem intertwined. The first aspect, internal, requires that mathematics teachers come to see their job as political; further, recognizing that by ignoring the political nature of mathematics they participate in reproducing injustice (Felton-Koestler & Koestler, 2017). Felton-Koestler and Koestler (2017) argue that there are five major ways that teachers need to understand their political role which can be framed as a series of choices that teachers make. For example, teachers can decide to use teacher-centered dissemination approaches to providing mathematics to students or they can choose mathematical approaches that center student understandings and recognize that each of us is responsible for creating mathematics (Felton-Koestler & Koestler, 2017). The other choices teachers can make include: making rich mathematics available to students regardless of "ability," uncovering their own and others' deficit perspectives of students and students' communities, expanding notions of what mathematics does and who does it, and, finally, positioning mathematics as a mode for social justice rather than a neutral culture-free discipline (Felton-Koestler & Koestler, 2017). Gutiérrez (2013b) argues that teachers must understand these kind of choices well enough to manage their own classroom practices for work with students who need to be ready for an unjust and shifting landscape, and prevent deficit perspectives from sneaking into those classrooms. In fact, teachers have a moral imperative to act as change agents using their knowledge and understandings to drive this work according to Villegas and Lucas (2002).

Specialized Pedagogical Content Knowledge for Mathematics Teaching (SPCK-MT)

The subdomain of Specialized Pedagogical Content Knowledge for Mathematics (SPCK-MT) as drawn in Figure 1 tries to bring together different approaches to classifying pedagogies necessary for teachers to access as part of their work. It is in the center of the figure because it encompasses knowledge both of the discipline of mathematics and theories of learning, schooling, child development, context and so on. While it may require less interdisciplinary knowledge, there are aspects that cross multiple disciplines of teaching. As it stands, it provides one map of the variety of approaches to specialized pedagogical content knowledge for mathematics. In what follows, I will summarize what is outlined in the Figure and end with a conversation of the different aspects of SPCK-MT.

Relational Pedagogical Content Knowledge for Teaching. This set of pedagogical knowledge focuses on how teachers understand content in relation to students, teaching, and curriculum (Ball et al., 2008). Ball, Thames, and Phelps (2008) argue that teachers need to be able to predict what errors students will make, what they will find interesting or intriguing, and process and support students' ideas-in-the-making. Knowledge of content and teaching centers on issues of course or task design and content understandings (Ball et al., 2008). Finally, knowledge of curriculum and its relation to content can be considered the information teachers need to understand about their particular curricular resources and the required curriculum for their students (Ball et al., 2008).

Pedagogies for Rigorous Mathematics and Commitments to Justice. This set of pedagogies have primarily been explicated in the last three decades as issues of differential opportunities have come to the forefront of mathematics education. They are categorized here based on initial framework derived from Dyches and Boyd (2017) and Rubel (2017) into five major categories: pedagogies of disciplinary access, culturally leveraging pedagogies, critical pedagogies, agency inciting pedagogies, and democratic pedagogies. Rubel (2017) argues that what these pedagogies have in common is their commitment to providing students with a conceptual understanding of mathematics.

Pedagogies of Disciplinary Access. I have added this category to the initial framework to account for a group of pedagogical approaches that are designed to open the discipline of mathematics to students who have traditionally been marginalized.

Historically, mathematics has served a “gatekeeping” function in schooling that maintained and reinforced existing social stratification (Stinson, 2004). These pedagogical approaches seek to, in Stinson’s (2004) word, “empower” students by making “gatekeeping” inclusive. Multiple examples of these pedagogies exist, but for this purpose Complex Instruction can be highlighted. Complex Instruction suggests that classroom teachers provide rigorous, open-ended, group-worthy tasks for completion in small groups where students can benefit from each others’ expertise (Cohen, Lotan, Scarloss, & Arellano, 1999). Teachers pay particular attention to issues of status and deploy practices that support their students in addressing these issues (Cohen et al., 1999).

Culturally Leveraging Pedagogies. Dyches and Boyd (2017) called these culturally-accessing pedagogies, but the terminology has been changed to differentiate it from the pedagogies above in this framework. These pedagogies have a similar goal to disciplinary access pedagogical approaches, but these approaches explicitly focus on important dimensions of student culture (Averill et al., 2016) or can draw on student funds of knowledge (Aguirre et al., 2012) to support their learning of mathematics. One example of this is culturally responsive pedagogy, where teachers design classroom environments that provide students with educational experiences that address them from a position of their strengths and are relevant to their own cultural experiences (Gay, 2002). Further, it requires that teachers recognize that previous pedagogical approaches have required students who identify as people of color or lower SES, for example, to obscure their cultural experiences when performing the routines of schooling (Gay, 2002). In the case of these pedagogical approaches, students’ cultural or personal experience are welcomed in to the classroom, but the outcome of the work is not necessarily designed to transform the surrounding society or the discipline itself.

Critical Pedagogies. These pedagogical approaches help students to develop a critical literacy using mathematics and supports them in understanding and critiquing society and its associated structures (Dyches & Boyd, 2017). Frameworks, like culturally relevant pedagogy (CRP; Ladson-Billings, 1995) have been developed to support teachers and teacher educators to center the assets of historically marginalized student populations, in this case African American students (Ladson-Billings, 2014). These ideas have spurred the proposal of Culturally Sustaining Pedagogy, which suggests not only should curriculum be tailored to meet the cultural needs of students, but also to support their own identity development within their culture(s) and disciplines they are studying (Paris, 2012). Further, white students need access to Culturally Disruptive Pedagogy (San Pedro, 2018), which helps them to unpack the aspects of White Privilege in classrooms and schools and address what this means for society.

Agency-Inciting Pedagogies. These pedagogical approaches center on ways to disrupt existing structures of thinking about students and mathematics as well as creating opportunities for those students to go beyond participating to changing the discipline with their approaches (Gutiérrez, 2009b). In mathematics, in particular, certain kinds of “official knowledge” have previously been valued to the detriment of some students’ experiential knowledge (Apple, 1992), and to the detriment of the discipline, which would benefit from new ways of knowing (Gutiérrez, 2002, 2008b). These pedagogies emphasize the teachers’ use of their political knowledge and power to support students in action toward justice. These approaches include: transformative pedagogy (Aguirre, 2009), social justice pedagogies (Gutstein, 2012; McDonald & Zeichner, 2008) and Democratic Pedagogies (Dover, 2013; Ellis & Malloy, 2012). These approaches to mathematics teaching require an outcome that provides students with an opportunity to impact the world beyond the classroom.

The Relationship between Aspects of Specialized Pedagogical Content Knowledge for Mathematics Teaching. The relationship between the relational pedagogical content knowledge and pedagogical approaches for justice is still to be fully resolved. Figure 1 is designed to outline that these kinds of knowledge are in dialogue with each other. For example, a teacher would need to call on multiple forms of relational knowledge in order to make decisions about the pedagogical approaches for justice that would be appropriate at a particular moment and vice versa.

Further, it's not clear from the literature exactly which of the pedagogical approaches for justice are supportive of each other and which are in tension with each other. Rubel (2017) argues that some of these approaches – those categorized as disciplinary access and culturally leveraging pedagogies – are "dominant equity directing practices" (e.g., p. 90). While these approaches can support students to learn to "play the game" of school mathematics, they do not support students to "change the game" of school mathematics (Gutiérrez, 2009b) and may obscure what Rubel refers to as more critical practices. The tension in these approaches is echoed by Enyedy and Mukhopadhyay (2007) who argue that upon their implementation of a culturally relevant project, their pedagogical goals and mathematical goals were in tension.

Mathematics Subject Matter for Teaching

This is the final section in Figure 2 because it represents primarily disciplinary knowledge, as separate from the previous two interdisciplinary domains. The mathematics subject matter necessary for teaching does not encompass every aspect of mathematics, but it does require that teachers have access to the mathematics that they will teach as well as the procedures, skills, or habits that they want their students to be able to access. Further, they need to have a deep understanding of the various ways that connections across the discipline can be made.

Mathematical Content Knowledge for Teaching. Ball, Thames, and Phelps (2008) argue that there is a particular way in which teachers need to be able to "decompress" mathematical knowledge, that is different from what mathematicians need to do in the process of their work. They suggest that there are three subsets of this kind of mathematics content knowledge, including: common content knowledge, specialized content knowledge, and horizon content knowledge (Ball et al., 2008). Common content knowledge is mathematical knowledge that has applications outside of the realm of teaching (Ball et al., 2008). Specialized content knowledge is the mathematical knowledge that does not have typical applications outside of the teaching profession (Ball et al., 2008). Finally, horizon content knowledge is an understanding of the way mathematics interacts at the edge of the mathematics that is taught in schools, so that teachers can prepare their students in meaningful ways for what is to come (Ball & Bass, 2009).

Canonical Content Knowledge. In the original version of the SJPACK, this subdomain was named "traditional" content knowledge; however, given the ways in which mathematics has developed to exclude some traditions and embrace others, canonical seems to be a more appropriate term. This knowledge includes the set of "settled" mathematics that exists in the world, and in the context of schools is constrained by the standards, for example, the Common Core State Standards for Mathematics (NGA & CCSO, 2010), NCTM documents (e.g., NCTM, 2014) and other documents used by school districts (e.g., NRC, 2004). Aguirre (2009) argues that this kind of knowledge can be envisioned as the set that holds power and access for those who possess it, including access to economic advancement, educational opportunities, and other gatekeeping uses of mathematics. Further, it tends to center Western ways of understanding and does not focus on indigenous or ways of understanding mathematics from other traditions (Aguirre, 2009).

In addition to these sets of fixed knowledge, there are also sets of methods or habits of mind that are essential to the doing of mathematics (Cuoco et al., 1996). Sword and colleagues (2015) define Mathematical Habits of Mind as "the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways mathematicians employ" (p. 111). Given the argument made here that mathematics is a process of human doing, it is essential that teachers have some experience and exposure to these habits of doing mathematics. These habits provide teachers and students with ways to approach novel problems that serve them well in the process of coming to know mathematics.

Critical Content Knowledge. In addition to having access to the mathematical cannon, teachers must also be able to unearth the way in which knowledge has been developed over time, what has been included or excluded, and what controversies have existed in the development of the discipline (Dyches & Boyd, 2017). As a teacher applies this version of knowledge, they will provide students with the opportunity to problematize issues in the discipline which are otherwise accepted as truth (Dyches & Boyd, 2017). Aguirre (2009) suggests that there is a second layer to this knowledge; not only does it mean teachers need to understand how the discipline is acted on by society, teachers need to understand how the knowledge can be deployed to act on society.

Considering the Relationship between Aspects of the Knowledge for Mathematics Teaching Framework

Aguirre (2009) argues that one way to transform mathematics education, and perhaps the discipline itself, is by centering problem posing in school mathematics as compared to centering the typical methods providing problems for students to solve. However, while not incompatible, teachers and teacher candidates must then have the knowledge to deploy aspects of the Knowledge for Mathematics Teaching that provide for students to approach issues of justice, but also interrogates origins of mathematical knowledge. Further, teachers need to do this while creating a way to "do" and "create" mathematics in community with each other and other members of the discipline. Gutiérrez (2009a) proposes that rather than addressing tensions inherent in working at the intersection of these competing knowledges, individuals should embrace them. According to Gutiérrez, teaching mathematics from an equity stance requires that teachers both "teach mathematics" and "teach students" (Gutiérrez, 2009a). Here to "teach mathematics" means to focus on disciplinary inquiry, while to "teach students" means to center the work on the lived experiences of individuals in the classroom

(Gutiérrez, 2009a). In the embrace of the space between the two, teachers can become equitable teachers of mathematics for their students (Gutiérrez, 2009a).

Overall, Figure 2.1 illustrates one way of conceiving of the knowledge that teachers need in order to respond to the diverse needs of learners, as outlined in section 1, when implementing inquiry approaches to mathematics education. This frame offers me an organization and a theoretical tool to understand how MTEs professional vision impacts teachers. In particular, this will serve as an analytical tool for the proposed research study.

Preparing Teachers to Teach Rigorous Mathematics for Equity and Inquiry

There is limited research on how to prepare teachers to think about issues of equity and justice in the context of inquiry approaches to mathematics teaching. Those that have been reported include providing exploratory experiences in mathematical content (Crespo & Sinclair, 2008), identifying and targeting aesthetic experiences (Crespo & Sinclair, 2008), and critiquing existing problems (English, Cudmore, & Tilley, 1998). Further, in teacher candidate development of this skill, supporting them with collaborative problem solving experiences, providing an authentic audience, and offering experiences with new content have proven useful (Crespo, 2003). In what follows, I will outline the design principles that have been suggested for designing these preparation courses and associated initial findings from studies that do exist.

Experiences for K-12 Teachers that Promote Rigorous and Just Mathematics Teaching

In order to enact rigorous and just mathematics teaching, teacher candidates and in-service teachers need to have a set of nuanced and complicated understandings of the world (Leonard et al., 2010). In order to achieve these understandings, TCs need models of what it means to both be rigorous and to attend to social justice concerns from methods course (Leonard et al., 2010). In order to design these classrooms, Koestler (2012) proposed the following five assumptions: a) mathematics is an activity of "doing" and "sense making"; b) each person can participate in and contribute to mathematics (and all peoples can and have); c) students bring useful mathematical knowledge that should be leveraged and applied; d) mathematics teaching is a political act; e) this work requires reflection in order to understand one's place in the social, cultural, and political context. Further in the course of the class meetings, MTEs can revisit contexts that are familiar to the TCs, but provide new ways of seeing them through problems that provide multiple approaches or completing mathematics autobiographies for example (Ellis & Malloy, 2012).

Second, TCs and novice teachers need opportunities to apply and reflect on aspects of associated pedagogies from their practical experiences in classrooms and schools (Leonard et al., 2010). Teachers, broadly construed, can be supported to use a "what," "how," and "who" framework to examine teaching (Felton, Simic-Muller, & Menéndez, 2012). This framework is implemented by examining "what" a particular teaching episode or set of episodes says about the nature of mathematics, "how" mathematics concepts are connected to the real world through these teaching episodes, and "who" is represented in the mathematics contexts/experiences (Felton et al., 2012). This was used to examine their own teaching as MTEs, but it could also be used by K-12 teachers to examine similar issues (Felton et al., 2012).

Finally, teacher candidates and novice teachers need practice self-reflecting on their own identity relative to issues of power and marginalization (Leonard et al., 2010). They also need support to understand the cultural, social, political, and economic contexts of their students' learning (Leonard et al., 2010). Teacher candidates and novice teachers need to recognize the cultural richness of each culture and understand that these pedagogies do not have a one size fits all application to the classroom (Leonard et al., 2010). To hone this skill, teacher candidates could participate in two-way discourses by examining mathematics, learning, and just teaching about local schools and their communities (Ellis & Malloy, 2012).

Approaches to Supporting Teacher Candidates

Positive outcomes of a focus on the equitable nature of mathematics for teachers and students have been reported in the literature. By studying methods of teaching that promoted rigorous mathematics and just teaching, teachers learned to recognize the relevance of mathematics to their own understandings of society (Spielman, 2009) and how a focus on social justice could help them to take an inquiry stance(Esmonde & Caswell, 2010). Through their participation in such learning experiences, teacher candidates and in-service teachers both became better at drawing on students' mathematical funds of knowledge (Aguirre, Turner, et al., 2013; Esmonde & Caswell, 2010) and better at recognizing how to draw on this information to design instruction (Spielman, 2009). Further, teachers reported that participation in these professional activities helped them identify how school mathematics could be used to support and empower K-12 mathematics students (Gonzalez, 2012; Spielman, 2009). Finally, some teachers reported explicitly addressing issues of social justice in their mathematics classrooms (Esmonde & Caswell, 2010).

Challenges were also identified in the course of teachers and MTEs working together. Course and professional learning experiences can create the impression that issues of "social justice" are an add-on or serve as a hook for students rather than an essential part of mathematics learning (Bartell, 2012; Felton et al., 2012; Gonzalez, 2012). In fact, Felton, Simic-Muller, and Menéndez (2012) reported that teacher candidates felt uncomfortable with the political nature of teaching and worked toward depoliticizing their instructional choices. Further, both in-service teachers and teacher candidates reported that they felt they were unable to meet both social justice and mathematics goals simultaneously (Bartell, 2013; Felton et al., 2012) as a result, teachers continued to treat them as separate (Bartell, 2013). In a similar tension, it was reported that even after teacher candidates were able to identify that sociopolitical factors in their students lives impacted the classroom, they were unsure of how this might affect their instructional practices (I. A. Brown, Davis, & Kulm, 2011) or how they might be able to use the results of such practices to transform society (Felton et al., 2012). Finally, there is an inherent tension in implementing a university course that tries to teach democratic principles because there are authority structures at play in universities and classes and the degree to which democratic pedagogies can be implemented is difficult to determine (Ellis & Malloy, 2012).

This section of the literature highlights some of the approaches MTEs have previously chosen to meet their professional vision. The challenges or tensions identify what might be stumbling blocks for MTEs; of particular importance is the challenge of treating social justice and mathematics separately. Consequently, in the analysis of the data, I will consider how these tensions reflect the work of MTE Participants.

The Role of Mathematics Teacher Educators

While MTEs are members of a much larger system of educators who prepare teacher candidates for their future classrooms, they play a vital role in supporting teachers to develop the outlined Knowledge for Mathematics Teaching. And while there is a growing interest in the work of MTEs among researchers, the role of mathematics teacher educator is still being defined (L. Brown et al., 2018). In fact, mathematics educators within the same program may enforce different kinds of norms and classroom expectations (Güven & Dede, 2016).

Research on the experiences and professional learning for MTEs is still in relative infancy (Beswick & Goos, 2018). Here, I will outline what is known about how MTEs learn, the state of formal professional learning experiences for MTEs, and research on the state of informal professional learning experiences for MTEs. This section will emphasize what exists in the field and what still needs to be better understood. My study will contribute to this body of literature providing guidance about the kind of goals that MTEs have for their professional work.

MTEs' Knowledge and Professional Practices for Teaching

Shulman (Shulman, 1986) posited that in addition to subject matter knowledge (or mathematics content knowledge), teachers also need pedagogical content knowledge (PCK), a special kind of knowledge necessary to support classroom learning of a particular discipline. In this section, I argue that in addition to PCK and subject matter knowledge, teacher educators need additional knowledge related to addressing issues of systemic inequities that have been reproduced through institutions of school and schooling. Further, those who prepare teachers do not need an identical set of skills, but rather a separate and overlapping knowledge (Castro Superfine & Li, 2014; Chick & Beswick, 2017). In this section, I focus on the professional practices that MTEs are expected to achieve. While these are derived from the kinds of knowledge that practitioners need, the practitioner knowledge itself is not central to this study. Rather my overarching research question focuses on how they choose to enact these understandings. This section provides context from some MTEs experiences.

MTEs and Mathematical Knowledge for Teaching

Knowledge for Mathematics Teaching as outlined in the previous section is squarely in the realm of what mathematics teachers, not mathematics teacher educators need to have. Sztajn, Ball, and McMahon (2006) suggest that examining knowledge for mathematics teaching could serve as a starting point for supporting MTEs. However, the relationship between ideas of knowledge for mathematics teaching, broadly construed, and what teacher educators need to know need is not entirely clear.

Research suggests that while similar, the PCK necessary for teaching school mathematics and the PCK necessary for preparing teacher candidates to teach school mathematics are not entirely the same (Castro Superfine & Li, 2014; Chick & Beswick, 2017). For example, while teacher candidates will need to fully understand a particular curriculum to do their daily work, MTEs need a broader understanding of multiple curricula and their connection to education research (Chick & Beswick, 2017). Further, understanding expertise that teachers have as divergent from that of MTEs can spark a deeper understanding of teacher education processes (Mohammad, 2008). However, MTEs are not, nor should they be, all knowing in any of these realms (Jaworski, 2008).

One four-category framework, initially developed for use in K-12 mathematics classrooms categorizes specialized aspects of MKT for MTEs as different from necessary K-12 mathematics teaching (Muir, Fielding-Wells, & Chick, 2017). This framework is summarized in this paragraph and also in Table 2.1. The first category is “foundation” which is the mathematics content knowledge, knowledge of PCK for teachers, and beliefs about the nature of mathematics (Muir et al., 2017). The second, “transformation” is how MTEs model their knowledge and choose to underscore it for future teachers (Muir et al., 2017). In “Connection,” the third, the MTE provides coherence within and across lessons for teacher candidates (Muir et al., 2017). Finally, “contingency” describes how MTEs respond to unexpected events in the classroom (Muir et al., 2017). MTEs also need knowledge at the mathematical horizon of teacher educators, which includes an understanding of the kind of experiences that will provide teacher candidates with disequilibrium that will expand the teacher candidates horizon knowledge (Zazkis & Mamolo, 2018). These conceptions provide a framework from which to begin to understand what MTEs need to know in the realm of MKT, but this framework does not explicitly address the need for MTEs to have understandings of the historical and contextual factors of school and schooling.

Table 2.1

Categories of Mathematics Educator PCK from Muir, Fielding-Wells, and Chick, 2017

Category	Description
Foundation	Knowledge required for mathematics teaching at the K-12 level
Transformation	Pedagogical content knowledge modeled and emphasized
Connection	Knowledge needed to connect lessons across a course
Contingency	Knowledge to respond to unexpected outcomes in classroom spaces

MTEs' Professional Practices in Mathematics Classrooms

Explicit attention to the kind of professional practices MTEs need to undertake as part of their profession has not been widely studied; however a few key principles have been examined in the research literature and are reported here. Aspects of classroom practice for teacher candidates such as noticing (Aguirre, McDuffie, et al., 2013), opening curriculum space (Drake et al., 2015), and conducting interviews to understand student progress (Crespo & Nicol, 2003) have been identified, but a core curriculum that lays out a complete list of these practices for teacher candidates has yet to be defined (Ball & Forzani, 2011). Further, Tzur (2001) highlights that in the era of reform – at that time No Child Left Behind movements – MTEs need to be prepared to support changes to traditional mathematics teaching practices.

One key aspect of what MTEs are expected to do as part of their professional practice is to use theory to drive their classroom practices (García, Sánchez, & Escudero, 2007; Zaslavsky, 2008). MTEs are in a unique position to work to harmonize multiple theoretical positions and use them to support teacher candidates in defining solutions to pragmatic problems of classroom practice (F. L. Lin, Yang, Hsu, & Chen, 2018). In fact, MTEs' experiences as researchers and studiers of theory can provide guidance to their classroom task choices (Zaslavsky, 2008).

Additionally, a teacher educator needs to be able to provide a kind of meta-commentary about their own teaching in the process of their coursework. A MTE supports a teacher by making visible the teacher candidates own experience in such a way that it would be transferable to their future students (L. Brown et al., 2018). For example, MTEs should be able to apply the theories underlying professional noticing to teach teacher candidates to perform professional noticing (Amador, 2016). This process may include first getting to understand professional noticing by enacting it multiple times before working to understand how teachers learn to notice and translating this theory to practice (Amador, 2016). Methods for doing this work are less well defined.

MTEs' Professional Practices for Equitable Classrooms

The frameworks outlined in the previous sections are not explicit in addressing issues of historical and ongoing inequity in mathematics classrooms in particular. Educators suggest that there are some practices and kinds of knowledge that MTEs deploy to both create classrooms that address systemic inequity and prepare future teachers for those future spaces (Chao, Hale, & Cross, 2017; Drake et al., 2015; A. M. Marshall & Chao, 2017). These practices do not comprise "knowledge" per se, but rather practices that are derived from specialized knowledge that MTEs have acquired.

If MTEs are to support the movement of pedagogy for access to a pedagogy for transformation, equity must be foregrounded in the practice of teacher preparation (Aguirre, 2009). To meet this goal, MTEs must be prepared to confront their own biases and those of their students, design environments where teacher candidates feel comfortable constructively challenging others, and continuously review and revise learning goals with respect to equity (Jett & Cross, 2016). Practically, this means that teacher candidates need support to understand the sociopolitical contexts that socialized them to hold deficit views and how they are symptomatic of larger institutional factors in oppression (Gorski, 2011).

Given the sociopolitical context of teaching, teacher educators need additional practices to support future teachers of color (Gist, 2014). First, successful TEs purposefully work as change agents in alliance with communities of color (Gist, 2014). Second, TEs challenge institutional barriers to success for teachers of color within teaching and teacher education (Gist, 2014). And finally, TEs deploy constructivist approaches to prepare future teachers of color with the many diversities of their own future student populations (Gist, 2014).

Particular practices have been proposed at the intersection of rigorous mathematics and equity. One example is a mathematical autobiography that can serve dual purposes for teacher candidates and MTEs (A. M. Marshall & Chao, 2017). First, the autobiography provides the MTE insight into teacher candidates' relationship to mathematics and second, the experiences contained can provide teacher candidates with rich ground to begin to interrogate issues of racism, sexism, ableism, or other institutional marginalization (A. M. Marshall & Chao, 2017). A second example that is often found in the teacher candidates mathematics education courses are cognitively guided interviews (CGI; Chao et al., 2017). CGI can either be used as a diagnostic tool to determine a student's location on a predetermined mathematics trajectory or they can be applied critically to support teacher candidates to understand a child's lived experience in mathematics learning (Chao et al., 2017). Thus, a MTE needs a set of practices and knowledge that supports the critical use of these kinds of tools (autobiography and CGI, for example) as well as its other applications.

Further, the idea of equitable mathematics practices must extend beyond classrooms and into the field at large. MTEs as a group of professionals remain overwhelmingly white and middle class, and as such need to interrogate what is valued in classrooms and for teacher candidates (Joseph, 2017). In this context and the context of schools and schooling, MTEs must address issues of power that are inherent in social structures in the U.S. (Hand & Goffney, 2013; Kalinec-Craig & Bonner, 2015). Hand and Goffney (2013) suggest that in trying to concretize "equity," MTEs must consider desirable outcomes, listen to and use ideas of other scholars, and determine what are the resulting affordances and constraints of the given definition. Even for MTEs who have been deeply involved educating for social justice, further professional learning is necessary (Ritchie, Cone, An, & Bullock, 2012).

In attempting to implement these practices, MTEs are constrained by attempting to shoehorn the task of interrogating Whiteness as it exists in mathematics and mathematics education into a single or two courses in a teacher candidates career (Gutiérrez, 2016). Further, these implementations can center white identities, which ultimately can require teacher candidates of color to do a lot of the teaching to white teacher candidates (Gutiérrez, 2016). To begin to meet these challenges, MTEs can take the mirror test, whereby MTEs ask themselves questions like, "Are there ways my methods instruction contributes to white supremacist capitalist patriarchy (hooks, 1994) or the white institutional space (Martin, 2013) of mathematics education? Even if I am neither white nor male?" (McCloskey, Lawler, & Chao, 2017, p. 331). MTEs can use similar tests to examine in community(ies) of MTEs and consider how the field is (or is not) developing (McCloskey et al., 2017).

The State of Professional Learning for MTEs

Clearly, undertaking this work necessitates a complex set of skills for mathematics teacher educators, but the mechanism for professional development has not been well studied. In this section, I consider how the field has understood the development of MTEs and the supports that exist to reinforce their development. In the current study, MTEs have provided key moments that are supportive of their own development. In considering those moments, this literature will be helpful.

Mechanisms for MTE Development

Tzur (2001) proposed a four-stage professional development trajectory that identified how individuals to move from mathematics learner to mathematics teacher to mathematics teacher educator to mathematics educator mentor learners. In the first stage, a learner focuses on aspects of mathematics like reasoning, communicating, connecting ideas, and computing (Tzur, 2001). Next, a learner learns to become a mathematics teacher by considering what doing mathematics means, how someone comes to this knowledge, and what experiences a teacher should provide to support this learning (Tzur, 2001). Third, the learner becomes a mathematics teacher educator by focusing on what teaching mathematics means, how someone comes to gain this knowledge, and what experiences a teacher educator should provide to support this learning (Tzur, 2001). Finally, the learner moves into a role as a mathematics teacher educator mentor by focusing on what teaching mathematics teachers means, how someone comes to gain this knowledge, and what experiences a teacher educator mentor should provide to support this learning (Tzur, 2001). Through reflection and experience, the learner can move from one experience to the next, while still deepening knowledge at the same level (Tzur, 2001). Researchers suggest that this is a lifelong developmental process that requires MTEs to switch between acting as learners and acting as coordinator of others' learning (Zaslavsky, Chapman, & Leikin, 2003).

Zaslavsky and Leikin (2004) have reported that a transformation from teacher to teacher educator can occur developmentally in the presence of knowledgeable community members and through performing the community practices. A number of possible foci have been proposed in this context. For example, these teams should examine the professional role of teacher educators, as it differs from the role of classroom teaching (Van Zoest, Moore, & Stockero, 2006). When teams are working together they can reflect through experiences of designing and understanding course goals for teacher candidates, determining and implementing tasks, and scaffolding and supporting teacher candidates to make mathematical claims (Masingila, Olanoff, & Kimani, 2018). Further, through the process of implementing these tasks, teacher educators can deepen their understanding of mathematics through implementing tasks with teacher candidates (Zazkis & Mamolo, 2018).

Van Zoest, Moore, and Stockero (2006) go as far as to suggest this form of mentoring and reflection should be a mandatory part of a doctoral education in preparation for becoming a MTE. Such a model could also be extended to take place across multiple communities of practice like those for disciplinary mathematicians and mathematics teacher educators (Bleiler, 2014). All of these mechanisms are summed up as development through reflection on professional activities of the MTE (Tzur, 2001).

In the field, however, MTEs report needing more support to develop these skills in both their educational training and their early careers (Yow et al., 2016). And, in transitioning from roles as teachers to teacher educators, research suggests that new teacher educators need professional learning opportunities to support this shift (Ping et al., 2018). In particular, teachers-turned-teacher-educators do not always feel that they are able to translate theory into practice (Maher, 2011), despite this being identified as a major strength of MTEs elsewhere (X. Lin et al., 1995).

Formal and Informal Professional Learning Opportunities

Despite this reported need, there is relatively little formal professional learning available to MTEs (Zaslavsky et al., 2003). The formal programs that do exist tend to focus on enhancing teacher leadership through mathematics education (Zaslavsky et al., 2003). Less formal, in-service programs including learning to teach through implementation and working with other researchers tend to be the norm (Zaslavsky et al., 2003). Other informal routes to development emphasize the interplay of the roles MTEs tend to play as both teacher educator and teacher researcher, whereby MTEs can use intensive reflection as a mode of growth (Zaslavsky, 2009).

Because of the typical professional learning arc, much has been written about professional learning for mathematics teacher educators centers work of MTEs as learners in their own contexts (Chapman, 2008). Approaches to this kind of learning includes: consistent reflection across interactions with pre-service or in-service teachers (García et al., 2007; Sakonidis & Potari, 2014), learning experiments (Chamberlin &

Candelaria, 2018), and performing action research in the work of teaching teacher candidates (Masingila et al., 2018; Zazkis & Mamolo, 2018). In doing this work, MTEs take on different aspects of teacher candidate instructional approaches including: studying children or students, studying teacher candidate's own experiences, studying cases of master teachers, examining mathematical tasks, centering student-centered interactions, and focusing on the intersection of practical experiences and the methods course (Chapman, 2008). Chapman (2008) characterized the outcomes for teacher candidates from these interventions in three ways: 1) as providing a change in teacher candidates, 2) emphasizing the value and importance of reflection, and 3) principles for teacher candidate instruction.

In reviewing the literature, Chapman (2008) found that in doing reflective research many MTEs report from an outside perspective and as a result it is not clear what the outcomes of this style of research is on the professional learning trajectory. A similar sentiment was reported in more modern research studies (Chamberlin & Candelaria, 2018), suggesting that the focus of MTE research report focuses on alternative outcomes.

Professional learning outcomes that have been reported suggest that, just as the model suggested early in this section, MTEs professional learning positions them as learners and allows them to improve their practice. For example, Sakonidis and Potari (2014) report that in the process of professional learning MTEs continue to navigate and re-negotiate their identities in relation to teachers and teaching. In another paper, Mohammad (2008) suggests that by positioning himself as a learner, he was better prepared to understand what philosophical assumptions were driving his decision; but further, he was able to address those that required attention.

The research base suggests that the professional learning activities for MTEs are currently limited, primarily to informal self-reflection. In addition, there is growing interest in uncovering and understanding the kinds of knowledge and practices that are required for MTEs in the course of performing their professional duties. This study will attempt to address a need for the field by uncovering which goals and objectives MTE Fellows choose, how they selected the goals, and how the goals were enacted in his classroom.

Professional Vision for MTEs

Two related understandings of the construct *vision* will be described in this section. In this study, I have attempted to unite these definitions as a tool to understand how MTEs choose or co-construct goals for the teachers, schools, and communities where they do their professional work. Just as MTEs need understandings of mathematics teaching that are intersecting and complementary, their professional vision must be considered this way as well. Since the field of research around MTEs is fairly small, this construct has not been studied extensively. However, the term vision encompasses the important idea of horizons beyond current practice (Munter, 2014), which is essential to the current study.

When Goodwin (1994) described the construct "professional vision," he suggested that it was a socially constructed, shared set of understandings within the professional context of MTEs. The act of calling on professional vision requires that an individual has access to a set of integrated knowledge about the professional topic at hand

and leverages it to understand and evaluate a situation (Sherin, 2002). Thus understanding this version of professional vision can support researchers to understand how MTEs make decisions in the course of their professional work.

Hammerness's (2001) construction of "teacher vision," describes how a teacher perceives of an ideal classroom, whether or not it reflects the teacher's practice. Teacher vision can provide both an impetus for moving forward and some direction for how to do so. On the other hand, vision can cause an individual disappointment when the individual fails to reach it.

Thus, a "professional vision for MTEs" is a socially co-constructed and reconstructed understanding of the field of mathematics education and the MTE's role within it (Goodwin, 1994). While it is socially constructed, the vision itself can be highly personal and individuals within mathematics education may or may not share it. This vision shapes both how MTEs see and understand the world and how they react to what they see and understand (Sherin, 2002). Further, the vision is not fixed, but dynamic and is often just beyond the horizon of current practice (Munter, 2014). Such a vision can be used both to drive the decisions a MTE might make and to narrow the scope of their current work.

Summary

In summary, there is a need for teachers of diverse populations of learners, especially for the diversity of racial and ethnic identities that are in today's schools to have an understanding of the socio-historical development of schools and mathematics education in particular (Martin, 2007). The ramifications for pedagogical approaches is not totally clear; however, there are some promising approaches that require teachers use both sophisticated classroom understandings and draw on deep in interconnected mathematics knowledge (Aguirre, 2009). In the current development of these pedagogies and knowledges, tension is experienced in meeting rigorous and just teaching goals (Gutiérrez, 2012). The role of MTEs in resolving these classroom tensions and supporting teachers is not entirely obviously, although some examples exist (Aguirre, McDuffie, et al., 2013; Chao et al., 2017; Drake et al., 2015; A. M. Marshall & Chao, 2017; McCloskey et al., 2017). In this study, by examining participants' professional vision for MTE, I will seek to understand the goals and objectives that a MTE Fellow chooses to address using the Knowledge for Mathematics Teaching framework to place these goals. And further, I will attempt to understand how these goals are supportive of or in tension with each other.

CHAPTER 3

RESEARCH AND DESIGN METHODS

This chapter will focus on the methods for the current study. Here, I will begin by briefly restating the purpose and research questions. I have made a conscious decision to take on a socio-political stance in this work, so I will open by describing the operationalization of this stance for research and the rationale for choosing a qualitative case study approach. Next, I will describe aspects of the research design including participants, setting, data descriptions and collection, instrumentation descriptions, and data analysis. I will end by outlining the trustworthiness of the study and the study limitations.

While taking a sociopolitical stance can result in many tensions, Gutiérrez (2009a) outlines one that is particularly important in the study: teaching mathematics from an equity stance requires that one attends to the relationship between what it means to "teach mathematics" and what it means to "not to teach mathematics" (p. 14). Here to "teach mathematics" means to focus on disciplinary mathematical inquiry within the boundaries of existing mathematics, while to "teach students" means to center the work on the lived experiences of individuals in the classroom, as well as their unique ways of knowing (Gutiérrez, 2009a). In the embrace of the space between the two, the tension itself, teachers can become equitable teachers of mathematics for their students (Gutiérrez, 2009a). The idea of existing at the in-between space is central to this project.

Research Questions

The purpose of this project is to unpack how MTE's envision their role in supporting development of teachers across their own professional lifespan with respect to the constructs of mathematical inquiry and equity. In particular, I seek to describe how MTE's envision their work toward goals of equity and inquiry and in what contexts they enact these goals. In addition, I will examine what frames, skills, or processes MTEs draw upon in order to undertake this work. In order to understand this:

1. How do MTEs choose, operationalize, and enact equity and inquiry goals in their professional visions for their work with K-12 teachers?
    a. How do MTEs envision the knowledge necessary for teaching mathematics as defined by the Knowledge of Mathematics Teaching Framework as evidenced by professional autobiography and through interview?
    b. How do MTEs implement vision their professional role in the development of knowledge necessary for teaching mathematics as defined by the Knowledge of Mathematics Teaching Framework as evidenced by professional artifact and through interview?
2. In what ways does the attention to inquiry and equity goals create a tension between or support for "teaching students" and "teaching mathematics" in their vision for work with K-12 teachers?
    a. How is the tension between and/or support provided by "teaching mathematics" and "teaching students" envisioned in their work as a teacher or professional developer as evidenced by professional autobiography, professional artifact, and through interview?
    b. How is the tension between and/or support provided by "teaching mathematics" and "teaching students" implemented in their work as a

researcher as evidenced by professional autobiography, professional artifact, and through interview?

Operationalizing the Socio-Political Stance for Research

As noted, I am taking a socio-political approach to this research study and in order to operationalize this approach, I will embrace four domains derived from sociocultural theory (Nasir & Hand, 2006) that will guide this socio-political research approach. These four domains are: concurrent analysis at multiple levels, centering cultural practices that including both classroom and social contexts, examining tools, ideas, and artifacts, and learning as a shift in social relations with relation to identity (Nasir & Hand, 2006). In what follows, I will define each of these dimensions in connection to socio-political theory and describe how they relate to the current project.

Concurrent Analysis at Multiple Levels

Nasir and Hand (2006) suggest that issues of power and marginalization, as well as culture issues in the classroom must be examined in the context of the immediate and broader society. Power circulates and is reshaped in its interaction with social exchanges at each level of society (Gutiérrez, 2013a). Thus in undertaking a sociopolitical endeavor, institutional structures at many levels must be studied and understood, in conjunction with the social structures that surround them. While the institutional and social structures in which MTEs exist vary, these levels of analysis are present in the current study. For example, professional biography and interview will provide information about how institutions have shaped MTE's goals. Further, the professional biography provides information about how a MTE has existed in the social world. Additionally, when considering the MTEs vision, particular attention was paid to how this vision interacted with the MTE's current institution and understandings of the social world.

Centering Cultural Practices as a Unit of Analysis

Socio-political positions focus on how community practices embody cultural practice (Nasir & Hand, 2006). By assessing the classroom norms, teacher expectations, and classroom organization, the researcher can unpack the ways in which issues of equity, status, and power in the larger society are reproduced or disrupted by the classroom (Nasir & Hand, 2006). Nasir and Hand suggest that by focusing on activity, it becomes apparent how student groups are positioned along with implicit issues of power and identity. In this project, I focus on how the MTEs work to make visible some of the underlying activities in schools that position particular students as deficient and others as proficient. In particular, examining a professional artifact makes the MTE's norms concrete.

Examining Tools, Ideas, and Artifacts

Researchers can see how communities are organized and who has access to participate and what the patterns of participation are by examining artifacts, tools used, and ideas that are promoted (Nasir & Hand, 2006). Culturally-held ideas are particularly important to unpack, as these can provide constraints or affordances to the learning processes (Nasir & Hand, 2006). In the context of mathematics education, the discipline of mathematics and ideas around what success is within and beyond the discipline require deconstruction and reconstruction in pursuit of greater justice for students (Gutiérrez, 2013a). In this project, mathematical inquiry and its location in the discipline as well as addressing identifying who gets to ask the questions will be centered.

Shifts in Social Relationships as a Mode of Learning

Individuals negotiate and manage their participation with relation to others, within and across different cultures present in a classroom, which creates a process of learning (Nasir & Hand, 2006). Thus, identity is constantly negotiated with oneself in relation to others' expectations and understandings in the center of multiple cultural practices (Gutiérrez, 2013a). In mathematics education, the question, "why are identities constructed and whom does the construction serve?" must be asked (Gutiérrez, 2013a). Central here will be how the MTEs consider the development of productive professional identities for K-12 teachers at all levels of development and what they believe their role in that development to be.

As outlined here, the hallmarks of socio-political theory in mathematics education are the dynamic nature of learning and teaching within the context of school, community, and world, the deconstruction of existing systems of schooling, and the inherent tensions that emerge from this reality (Gutiérrez, 2013a). Beyond this, a sociopolitical stance requires the researcher to work to transform these dynamics to provide a more just experience of mathematics education (Gutiérrez, 2013a). In the context of mathematics teacher education, these stances become both increasingly important because of the number of teachers who will learn with the MTEs, and increasingly challenging because of the distance between the MTE and K-12 students.

Research Design

Rationale for the Qualitative Case Design

In order to address the research questions posed at the start of this chapter, I will undertake a descriptive multiple-case study (Yin, 2009). Each participating MTE will be considered the unit of analysis. Because case study was borne of the constructivist tradition (Baxter & Jack, 2008); this approach is epistemologically appropriate given my research stance; this tradition allows for the construction of reality from the experience of the researcher and participants. It also lends itself to questioning the origins of (mathematical) knowledge. Further, a case study approach is suitable according to the criteria suggested by Yin (2012) because it addresses "how" questions, the conditions of study will not be changed by the researcher, and the contextual factors are important an important part of understanding the experience of study. A multiple case design allows for comparison for cases between different contexts (Baxter & Jack, 2008), including different institutional commitments and departmental responsibilities in the case of participating MTEs.

Context of the study

While the location of data generation will vary depending on the collection activity, the context for the study will be within the bounds of the MTEs' professional life. In order to honor the sociopolitical stance outlined above, the study will include examination of institutional and departmental experiences that have guided MTEs to their current professional visions.

Participants and Participant Selection

Four MTE participants were recruited by purposeful or judgment sampling technique; in particular, a key informant sample was chosen (M. N. Marshall, 1996). The key informant sample is chosen because it leverages MTEs' expertise (M. N. Marshall, 1996). In this study, mathematics teacher educators were identified who had made a significant commitment to implementing mathematical inquiry, in one of its many forms, with either in-service or novice teachers. In order to capture a broad band of mathematics teacher educators' experience, two individuals with appointments to mathematics departments and two individuals had been appointed to departments or schools of education were recruited. Capturing these experiences is important because they represent two different paths to becoming mathematics teacher educators and provide different perspectives on the nature of mathematical inquiry and equitable mathematics classroom spaces. Further, participants represented different genders, races, US-region, and institution types. Table 3.1 displays the demographic selection of the participants.

Table 3.1

Summary of Participant Selection

Participant pseudonym*	Institution details	Preferred Pronouns	Race/ Ethnicity	Department/College Appointment
Dr. Marie Adams	Large private research university in US Northeast	She/her/ hers	white	Mathematics Education
Dr. Dani Juan-sin-tierra	Large public research university	They/them /theirs	Latinx	Mathematics
Dr. Patrick Mahoney	Small private teaching college in US Northeast	He/him/ his	white	Education
Dr. Makoto Yoshida	Small private teaching college in US Midwest	He/him/ his	Asian	Mathematics and Statistics

Note: * Participant name is a pseudonym chosen or approved by participating MTE

Entry and Access

This data was collected from adult professional participants in a research study. These participants were not connected to a site. Procedures followed were governed by the Boston College Institutional Review Board (BC-IRB). Entry and access to data was provided by participants.

Data Sources

In what follows, I identify the data generation activities for this project. The data sources for the project include: interviews with MTEs, professional artifact, and professional biography. Table 3.2 identifies the alignment between data sources and the research questions.

Table 3.2

Research Questions and Associated Data Sources

RQ	Data Source
1. How do MTEs choose, operationalize, and enact equity and inquiry goals in their professional visions for their work with K-12 teachers?	
1a	Interview Professional Autobiography
1b	Interview Professional Artifact
2. In what ways does the attention to inquiry and equity goals create a tension between or support for "teaching students" and "teaching mathematics" in their vision for work with K-12 teachers?	
2a	Interview Professional Autobiography Professional Artifact
2b	Interview Professional Autobiography Professional Artifact

Interviews with MTEs

A semi-structured interview was conducted with the MTE following the collection of the first two sources of data. A semi-structured interview was selected because it allows participants to provide their own responses, but ensures that a particular space of data is covered by the researcher (Noor, 2008). Further, by employing a semi-structured interview approach, new information or ideas may emerge and be explored (Adams, 2010).

Interviews lasted between about 55 minutes and 90 minutes. Three were conducted via videoconference while one was conducted with the researcher and MTE physically co-located. All four were audio recorded. The interview was designed with three major goals in mind. The first goal was to understand how the participating MTE defined the constructs of inquiry and equity. As a result, each participant was provided with definitions of each of these constructs as a starting place. Second, the interview was designed to elicit how the participating MTE saw the two constructs as intertwined, overlapping, or otherwise intersecting, or in tension with each other. Finally, the interview was to unearth what the participating MTE identified as their goal(s) with regard to the constructs and their own role in moving these goals forward. All four MTEs were asked at least five identical questions to address these goals; the remainder of the interview was co-constructed by the researcher and participant. The items that were consistent across the participants and additional possible sample items can be found in Appendix A.

Professional Artifact

Yin (2009) suggests that case study should rely on a variety of different sources of information, including artifacts of the investigated case. Further, artifacts can provide an important source of data triangulation for the researcher (Noor, 2008). And while artifacts can include instruments, tools, or other physical objects (Tellis, 1997), I will focus on a professional artifact provided by the participant.

The professional artifact(s) collected for this project are objects, which took a form chosen by the participating mathematics teacher educator. Suggested forms included PowerPoint presentations from a recent conference, a task or lesson for a course for teacher candidates, or an artifact from a recent professional learning experience the MTE led or directed. All fours participants identified their professional artifacts from a pre-existing professional context. One participant provided a PowerPoint from a mathematics methods course. Two provided tasks that supported mathematics learners, both future teachers and other future careers. And finally, one participant provided a task that could be used at the elementary level, but was used to support professional learning of in-service teachers. Thus, these artifacts represent the implementation of the MTE's professional vision. Table 3.3 summarizes the artifacts provided by the participants in this study.

Table 3.3

Summary of Participant Artifacts

Participant	Artifact Description
Dr. Marie Adams	PowerPoint presentation for Elementary Mathematics Methods Course
Dr. Dani Juan-sin-tierra	Student task and lecture notes
Dr. Patrick Mahoney	Task developed for teacher learning and use in elementary classrooms
Dr. Makoto Yoshida	Task developed for content course

Professional Biography

Written documents are recognized data sources in many qualitative research projects (e.g., Labuschagne, 2003). Analysis of such documents can be used to minimize researcher bias and corroborate credibility; however the researcher must consider the purpose and context of the document when performing an analysis (Bowen, 2009). I have used this guidance as I requested that the participants provide a professional biography. The participants were asked to provide a written document that reflected how they envision their personal journeys to their current professional positions (see Appendix B for information on this). One biography was written with the express purpose of being used in this study and one biography was derived from existing or previously written stories. Finally, two biographies previously existed and were provided directly to the researcher without any alteration. The biographical data provides important contextual and institutional information about the institutional contexts in which an MTE developed their current professional vision. Table 3.3 summarizes the biographies provided by the participants in this study.

Table 3.4

Summary of Participant Biography

Participant	Biography Description
Dr. Marie Adams	Written professional biography for the study, full page; additional details in interview
Dr. Dani Juan-sin-tierra	Written professional biography for a different purpose, half page; personal biography details in interview
Dr. Patrick Mahoney	Written professional biography for a different purpose, two pages; additional details in interview
Dr. Makoto Yoshida	Written professional biography edited for the study, two pages; additional details in interview

Table 3.5 displays the relationship between the salient dimensions of sociopolitical theory as defined by Nasir and Hand (2006), the related study context or "location," research questions, and data sources. It is important to note that because the

dimensions, research questions, and data sources cannot easily be disentangled, research questions and data sources show up multiple times in the table.

Table 3.5

Aligning Aspects of Sociopolitical Theory to the Project

Dimension of socio-political theory	Location in study	RQs
Concurrent analysis at multiple levels	Institutional level and individual level	1 (b) 2 (a, b)
Centering cultural practices as a unit of analysis	Professional activities, norms, and expectations	1 (b) 2 (b)
Examining tools, ideas, and artifacts	Professional vision within the context of institution(s) and academia	1 (a) 2 (a, b)
Shifts in social relationships as a mode of learning	Relationships between MTE and mentors and/or other MTEs	1 (a) 2 (a, b)

Analysis Procedures

The analysis of these data was undertaken by borrowing approaches from thematic analysis (Braun & Clarke, 2006; Braun, Clarke, & Rance, 2014; Vaismoradi, Turunen, & Bondas, 2013) and content analysis (Elo et al., 2014; Elo & Kyngäs, 2008; Vaismoradi et al., 2013). My research stance lends itself to an inductive analysis strategy that promotes knowledge-building and situates the production of knowledge within society (Braun & Clarke, 2006). Content analysis particularly lends itself to deductive approaches using a codebook (Elo & Kyngäs, 2008). In combination, these qualitative approaches permitted analysis that both addressed the research questions and is consistent with the theoretical approach. Table 3.3 displays how the data analysis proceeded across each data source. The techniques will be discussed in more detail in what follows.

Table 3.6

Summary of Data Analysis Procedures

Analysis phase	Data source	Data analysis approach and outcome
Within case analysis	Interview with MTE	Inductive thematic analysis for production of codes; development of codebook; memo for each case
	Professional artifact(s) and professional biography	Deductive content analysis and codebook revision; memo for each participating MTE's case and location in the knowledge for mathematics framework
Cross-case analysis	Case memos	Examination for families of cases (Miles, Huberman, Saladaña, 2014)

Thematic Analysis of MTE Interview

The MTE interviews were examined using an inductive thematic analysis (Vaismoradi et al., 2013), with a focus on description, which was chosen because I want to understand the MTE's professional vision. The coding process took three rounds for each interview and used the steps described by Braun and Clarke (2006): (1) an in-depth examination of the interview, (2) an initial round of *in vivo* coding, (3) identifying "candidate" themes and subthemes from the codes, (4) revising and winnowing themes, and (5) naming and understanding the relationships between themes. The unit of analysis was a phrase up to a pair of sentences that formed a single thought. Following this coding process, I created a memo about each case. These memos described emergent themes. Additionally, they addressed how these themes might describe the tension between inquiry and equity in the MTE's professional practice. Based on these described analysis, I developed the initial memo that describes how tension or support provided by inquiry and equity are identified by the MTEs. This memo provided initial guidance for analyzing the other two data sources.

Deductive Qualitative Content Analysis

Following the completion of the thematic analysis of the MTE interviews and using the initial findings from the interview and resulting, I developed a set of codes to form a codebook for the deductive content analysis of remaining data sources. These data sources included the professional artifact and biography. These written data sources were analyzed at the sentence or phrase level, with a focus on the message conveyed by the written sentence.

Data was analyzed by case, rather than by data source, in order to gain a deeper picture of the full case from the analyses. The codebook for analysis was derived according to deductive content analysis procedures and include code name, definition, example, and coding rules (Mayring, 2000). The remaining data sources were examined using the process described in deductive content analysis, first applying the codes from the codebook and then revising the code book following each round of data analysis (Young Cho & Lee, 2014). A memo was first written and then revised following the completion of the coding of each data source. Following the completion of coding of all cases, an additional review of the *corpus* was performed with the revised codebook to ensure each data source was fully coded.

Within Case Analysis

Following the completion of the coding steps, a within case analysis was performed and a "thick description" in narrative form (Eisenhardt, 1989) in response to the research questions was completed. Further, for each case, I created a concept map that located: a) the identified goals from the MTE that center equity and inquiry in their professional vision; and b) demonstrated whether these goals were in tension or supportive of each other in this vision. In doing this work, I focused on the sociopolitical commitments of this project by: doing analysis at multiple levels, focusing on professional vision by examining the proffered tools, ideas, and artifacts, and focusing on how relationships between developing identities are described by the data (Nasir & Hand, 2006). Upon completion, these memos formed the foundation for the portraits in Chapter 4. Participants were provided with the opportunity to member check these portraits.

Cross-case Analysis

Following the completion of each within-case analysis, a cross-case analysis was conducted to determine emergent families of cases (Miles, Huberman, & Saldaña, 2014). Cross-case analysis can provide additional explanatory understanding (Miles et al., 2014). In particular, uncovered aspects of identity, power, and status as well as institutional commonalities and/or differences will be used to examine how the cases are related, as required by a sociopolitical research position. Cross-case analysis was performed by comparing how the themes emerged in the various data sets for each participant. Specifically, comparably coded segments were compared and the themes were further refined and examined for how the described the support and or tension between constructs of inquiry and equity emerged in the different cases.

Trustworthiness

I will apply Guba's (1981) four aspects of trustworthiness in the review of this project's data analysis and publication. The four aspects are: credibility, transferability, dependability and confirmability (Guba, 1981). This is an appropriate way to ensure the quality of the study as Guba argues and is aligned with a sociopolitical researcher position. Guba argues that the traditional measures of quality (i.e., reliability, internal validity, external validity) do not meet the necessary assumptions of a naturalistic inquiry that focuses on multiple realities, the relationship between researcher and researched, and multiple depictions of the "truth." This is in line with my stance as a sociopolitical researcher.

Credibility

This dimension of trustworthiness links the findings of a study to the reality it purports to report (Guba, 1981). Credibility will be achieved through multiple criteria: use of well-recognized research and analysis methods, familiarity with the culture of participating institutions, triangulation via different kinds of data, thick description of the phenomenon, and member checks (Shenton, 2004). The methods described here are well-established methods from the qualitative tradition. The context in which the MTEs developed their vision was well examined in the course of data analysis. I triangulated the data using multiple types and modes of data generation including both interview and written artifacts. A thick description was the result of the within-case analysis and MTE members checked the results of this analysis to determine if my perception was aligned with theirs. The analyses were adjusted appropriately.

Transferability

Transferability is the degree to which the findings are context-dependent or can be transferred to a different context (Guba, 1981). The responsibility of the researcher is to provide enough background information description of the phenomenon to ensure others can determine the aspects that are context-dependent and those that can be transferred

(Shenton, 2004). In this case, this information will be collected and provided in any reports associated with the project. In what is outlined, it can be identified in the within case memos.

Dependability

Dependability is the consistency of the research process over time; that is, the researcher must be conforming to standards of credibility over the course of the entire research project (Guba, 1981). Lincoln and Guba (1985, as cited in Shenton, 2004) point out that if credibility standards are met throughout the study, the dependability criteria are as well. In addition to the credibility steps I will take, I also maintained an audit trail in order to consistently ensure that I am coming to logical conclusions based on the data and meeting credibility standards. The audit trail includes a chronological log of data collection and analysis decisions as well as procedures completed (Creswell & Miller, 2000).

Confirmability

Confirmability is the degree to which the other researchers would come to the same conclusions based on the collected data (Korstjens & Moser, 2018). Shenton (2004) suggests several practical strategies for ensuring confirmability including providing researcher assumptions and beliefs, recognition of possible limitations of the study, in-depth of methodological approach, triangulation of data, and use of audit trail. I have previously outlined how these activities will be implemented, but I also include the following aspects in my reporting in my study: a list of my commitments and assumptions and a clear description of the methods applied (as outlined previously in this chapter).

Limitations of the Research Design

I will use Flyvbjerg's (2006) notion of misunderstandings and assumptions about case study to guide this section on the limitations of my own study. Flyvbjerg identifies five such misunderstandings; I will address how each one is situated in this study and what it means for the outcomes generated. I was careful to address these limitations throughout data collection and analysis to the extent possible.

First, Flyvbjerg (2006) asserts that case studies are not valued because they contribute less to what is considered important – (context-free theoretical knowledge –) and more to what is considered less important – (the content-dependent practical knowledge base). However, Flyvbjerg suggests that because human behavior cannot be predicted using only context-free theoretical knowledge, so context-dependent examples provide important contributions to the knowledge base. In the case of this study, attention was paid to the contextual factors for the MTEs professional vision including the institutional requirements, the population of future teachers, and their future jobs. These are conveyed in Chapters Four and Five of this text in order to contribute to this context-dependent knowledge base.

Secondly, case study has been criticized because it cannot be generalized beyond its own context (Flyvbjerg, 2006). Flyvbjerg (2006) challenges researchers to recognize that while case study does not generalize in the formal sense, individual cases can contribute to generalization or can provide counterexamples to accepted generalized theory. Case study adds to accepted scientific knowledge and contributes to generalization through the selection of cases that illuminate particular aspects of the phenomenon of study (Flyvbjerg, 2006). In the case of this study, cases will be selected to illuminate how MTEs envision their professional work and purpose.

Third, Flyvbjerg (2006) suggests that many researchers see case study as a conjecture-producing endeavor, rather than a hypothesis-testing design. While this is an important premise, this limitation does not apply to this study. That is, given that the study I am proposing is an exploratory project, I anticipate producing multiple hypotheses as a result of this work. Given what Flyvbjerg states, for the trajectory of this work it will be worth considering case study in the next stages.

Fourth, case study has been indicted for containing verification bias, by which the researcher identifies what they expect to find (Flyvbjerg, 2006). Flyvberg (2006) suggests that while this indictment is generally made of all qualitative research studies and can be made of quantitative studies as well, case study offers the opportunity to study a phenomenon in depth. Intense study can provide the researcher with the opportunity to recognize how the data may be different from their own understandings of the phenomenon (Flyvbjerg, 2006). To prepare to undertake this kind of scrutiny of the data, I have laid out my assumptions in Chapter One. These will be carefully considered as I undertake the data coding. I used these assumptions to re-examine the code book following the initial round of deductive data coding for ways in which my own verification bias

Finally, case study has been criticized for being hard to understand the outcomes and understand the takeaways (Flyvbjerg, 2006). Flyvberg (2006) suggests that this is more of a function of how messy telling stories about reality is than a critique of the method itself. Further, it means that summarizing a case can be difficult and does not explain the breadth of the research undertaken (Flyvbjerg, 2006). In the case of this study, I provide a thick narrative description of each of the cases, while also providing some key information for the reader in the cross-case comparison.

Summary

In this study, I undertook a multiple case study (Yin, 2009) of four mathematics teacher educators' visions for their professional vision as it relates to the constructs of inquiry and equity. Data generation included: a professional artifact, a professional autobiography, and an interview. These case studies have produced a set of portraits. Comparisons across them provided both practical and theoretical information to understand the professional role of Mathematics Teacher Educators. Throughout the process of analysis, my own biases and assumptions were revisited to avoid verification bias.

CHAPTER 4

PORTRAITS OF MATHEMATICS TEACHER EDUCATORS

Introduction

This chapter will both provide a portrait of each participating mathematics teacher educator and also address this study's first research question. The research question is: *How do MTEs choose, operationalize, and enact equity and inquiry goals in their professional visions for their work with K-12 teachers?* Data sources including participant biography, interview, and tasks informed these portraits.

I have chosen to describe the participant cases as portraiture because portraiture lends itself to my sociopolitical research stance. Portraiture embraces both the rigors of scientific inquiry and the reality of the researcher as an instrument in that inquiry (Lawrence-Lightfoot, 2005). Lawrence-Lightfoot and Davis (1997) argue, "Portraitists seek to record and interpret the perspectives and experiences of the people they are studying, documenting their voices and their visions – their authority, knowledge, and wisdom" (p. xv). In this way, these portraits seek to provide a snapshot of the wisdom of the participating mathematics teacher educators, while honoring their voices.

Each portrait is derived from the methods discussed in Chapter 3 and were member-checked by participants. These portraits were not designed to highlight uniting themes, which will be discussed in Chapter 5, but rather to provide a descriptive summary of each teacher educator's experience. The portraits are provided in order of participating MTEs' last names[1]: Dr. Marie Adams, Dr. Dani Juan-sin-tierra, Dr. Patrick Mahoney, and Dr. Makoto Yoshida. The order of these participants does not imply anything about the study or the researcher's ideas and purposefully interleaves

[1] All participants' names and universities are pseudonyms either chosen or approved by the participant

individuals with appointments in mathematics departments and education schools or departments. Table 4.1 summarizes the participant characteristics and their primary teaching focus for reference.

Table 4.1

Summary of Participant Characteristics

Participant Name	Pronoun preference	Race/ Ethnicity	Department Appointment	Primary teaching focus
Dr. Marie Adams	She/her/ hers	white	Mathematics education	Mathematics content for teachers, mathematics methods for teachers
Dr. Dani Juan-sin-tierra	They/them /theirs	Latinx	Mathematics	General mathematics, mathematics content courses for teachers
Dr. Patrick Mahoney	He/him/his	white	Education	Mathematics and science methods for teachers, school leadership
Dr. Makoto Yoshida	He/him/his	Asian	Mathematics	General mathematics, mathematics methods courses for teachers

Note: Participant name is a pseudonym chosen or approved by participating MTE

Dr. Marie Adams, Saltfleet University

Dr. Marie Adams holds a position in the department of mathematics education, focusing in elementary mathematics at Saltfleet University. Saltfleet University is a large, private research university located in Saltfleet City. Saltfleet City is a large urban center that contains several universities and is located in the Northeastern U.S. Dr. Adams identifies her race and gender as a white woman. Dr. Adams uses the pronouns she, her, and hers.

Portrait of Dr. Adams

Dr. Adams relayed that her love of mathematics in elementary school, combined with how much she enjoyed spending time with children as a teenager, meant that a career in elementary school teaching seemed like a good fit. In her elementary years as a student, Dr. Adams described a traditional understanding of mathematics. She described a typical I-do, we-do, you-do sequence with worksheets that showed bare number problems followed by two word problems.

> The teacher talks for some bit of time and shows you something and then you work on it mostly by yourself. Maybe you get to talk to the person next to you about it a little bit and then you just do like so many problems. And then there's the two problems at the bottom of the page on the right side that are word problems, which I loved. I'm like, "Let me get there!"

In this passage, she describes several computational problems followed by story problems that were mathematics context. Dr. Adams enjoyed this instructional routine provided in her elementary mathematics learning experience. This routine provided her child-self with the idea that mathematics learning was individual and rigidly hierarchical. Master skills with number only problems are taught first and then students are taught to apply these foundational skills to real work.

As an undergraduate student during her elementary teacher preparation program, Dr. Adams was not provided with an alternate view of how mathematics could be taught. Nonetheless, she still continued to enjoy doing mathematics; she claims, "We did a lot of problems, which I thought was really fun. And I found the one other person that really liked math in there […] and so I would sit near [my peer] and we would do the problems." While Dr. Adams did not formulate a new vision of mathematics teaching from this experience, she described her experiences doing mathematics with another person.

In addition to her coursework, Dr. Adams completed a pre-practicum in sixth and seventh grade mathematics and science. This interest led her to consider changing her major from elementary education to middle grades mathematics. Dr. Adams's advisor suggested that she could do so, but it would take a fifth year as an undergraduate. Further, her advisor suggested that it was possible to do a Master's degree to earn licensure in middle school mathematics.

Dr. Adams decided to earn a degree in elementary education as a pathway toward a Master's program in middle grades mathematics educations. However, where Dr. Adams lived such a program did not exist. And so, Dr. Adams eventually earned a position teaching several sections of fifth grade mathematics. She recalled that the curriculum she used "had some interesting stuff" and that she "saw some higher quality tasks." During the course of her teaching fifth grade mathematics, Dr. Adams described her teaching as aligned with her experiences of learning mathematics. "I was just teaching the way I experienced[mathematics] as a kid and I didn't know that I was all that curious about the different strategies that kids would use." Thus, her own classroom closely resembled the classroom where she learned mathematics as a child.

Dr. Adams continued to seek out opportunities to understand how students learned mathematics. When reflecting on her experiences as a teacher, she "was curious about why some kids seemed to be understanding mathematical ideas and being able to answer questions […] and why some kids weren't." Dr. Adams was noticing that her approaches seemed to work for some students, but not for all of her students. Dr. Adams eventually moved into a full-time mathematics coaching position, where she noted that she did not have the opportunity to explore these questions. She prioritized learning how to successfully do her new job in a different environment with new colleagues in a challenging context.

Ultimately, Dr. Adams decided to pursue doctoral level study, having already completed a Master's degree. Dr. Adams identified two major reasons she made this choice: first, there were limited opportunities to learn more about students' mathematical thinking outside of doctoral programs and second, Dr. Adams wanted to learn more about how to teach mathematics. Regarding her decision to go to graduate school, she said, "I never thought I'd be a professor or get a PhD, but I couldn't get what I needed unless I went to get my Ph.D."

It was in graduate school that Dr. Adams began to understand how mathematics could be taught and learned in a different way from her previous experiences. In her graduate school experience, Dr. Adams spent each semester observing students in an elementary mathematics methods courses. During her very first experience in one of these classes she said, "that's when I started like really understanding the kids had interesting mathematical ideas and you could teach math in a really different way, in a way that responds to those ideas. In that the task was so critical. And that the teacher had such a critical role, I started learning about discourse and talk moves." In this way, Dr. Adams began to learn about the importance of the task, how to attend to students' individual mathematical ideas, and how to facilitate and sustain discussions.

Dr. Adams described one particularly important experience as a doctoral student. As part of a course studying mathematics teaching and learning, Dr. Adams and one of her peers performed a small design experiment in a second grade classroom. A candy factory context supported students' understanding of place-value; in this context, students learned about regrouping where there were single candies, rolls of 10, boxes of 100, and crates of 1000. Dr. Adams noted that this experience was "fascinating" for a number of reasons. First, it was one of her first opportunities observing students making sense of quantities. And second, Dr. Adams had the chance to see how design experiments looked in action. Finally, Dr. Adams noted that she also paid attention to how the teacher interacted with students. Dr. Adams described it like this: the teacher "was helping us call on people and she was helping us kind of ask, 'Did anybody else think of it that way?'" Dr. Adams suggested that the teacher's role in the design experiment indicated that the teacher supported student development of mathematical ideas.

Dr. Adams recounted that she continued to attend the class following the end of the candy factory design of experiment experience. She noticed that at the conclusion of the experiment, the teacher returned to her traditional instructional methods.

> It was a geometry unit, and she went right back to teaching in a very direct instruction way. I was like, "Oh wow, that's not what I was expecting." All of ways we were talking to kids, and all of the ways we were inviting them into conversations and all the ways we were asking them questions. And it was like, "Nope, can't do that in geometry" or "I don't know how to do that in geometry."

While Dr. Adams noted that this was an early experience for her in inquiry mathematics spaces, she emphasized how a teacher might revert to earlier experiences. This incident underscored how change in teaching practice is "fragile."

When she completed her doctoral degree, Dr. Adams took a post-doctoral position in an urban center in the Northwestern region of the United States. Dr. Adams spent much of her time deeply embedded in a local elementary school, where she supported teachers' professional learning in mathematics. Dr. Adams noted that this experience was enriching for both her and the teachers, but also that she "saw the incredibly complex nature of trying to teach math and learn math. This reinforced Dr. Adams's graduate student experiences, where changing instructional practices is difficult and the changes are not always permanent; however, Dr. Adams emphasized that change is possible.

Currently, Dr. Adams works as an assistant professor in a mathematics education department and is the Co-PI and PI on two grants, both focused on elementary grades. Dr. Adams also teaches mathematics content and methods courses for future elementary students. Dr. Adams considers her current research grant work and doctoral student advising work as frames for how she currently thinks about equity in mathematics spaces. Dr. Adams works closely with a doctoral student who supports Dr. Adams's grant and co-teaches her methods courses. Dr. Adams's doctoral student is experienced with mathematical inquiry as well as a focus on social justice and Dr. Adams credits their interactions with her own growth as a professional.

Dr. Adams and her research group are "thinking about how elementary teachers run discussions across both math and science and ELA that both lead to deep disciplinary learning, but also attend to the issues of power within the discussion." The current work of the project is to begin to unpack how teachers are using their spaces to promote disciplinary ideas. One major theme from this project has been the idea of *settled expectations*, derived from the ideas of Harris (1993, 1995). Harris (1995) defines *settled expectations* as, "assumptions, privileges, and benefits that accompany the status of being white […] that whites have come to expect and rely on across […] many contexts" (p. 277). Further, *settled expectations* can have the pretense of being neutral, but have real and negative consequences for individuals who are people of color. In the project's case, a classroom-focused view of *settled expectations*, as assumptions and orientations toward teaching and learning, which can, for example, impact how teachers interpret students' behavior and what counts as a response in a classroom discussion. Dr. Adams describes how *settled expectations* might be visible in their project. She explains:

> Some settled ideas within mathematics would be to be good at math, you are fast at doing it. You don't make mistakes. […] It might be a settled expectation that white boys have the right and the privilege to have their ideas heard or that they can say, "No, you are wrong."

In this excerpt, Dr. Adams emphasizes how teachers' assumptions might impact the choices they make about who gets space and time, during a class discussion. In particular, she stresses the racialized and gendered ways that power is distributed throughout the classroom and how teachers might promote this distribution of power.

Dr. Adams has been using what she has learned to change how she works with teacher candidates in her elementary mathematics methods courses. In particular, she has increased the number of readings and videos that relate to teaching mathematics for social justice, with a lens on the role of power dimension in classrooms. She also considered how she might use questions to approach the issues of justice and power. A few questions that she raised with teacher candidates included, "How do we know when race is coming into play, if the student is being disrespectful or inappropriate? How do you know when it is appropriate to react? What cues guide your response?" With these questions and other similar questions, Dr. Adams' hopes to support her students' development of reflective practices around equity and justice in which they move away from privileging traditional *settled expectations* of mathematics.

Dr. Adams' Vision of Equity and Inquiry in her Professional Work

Dr. Adam's vision of equity and inquiry in her professional work was characterized by attention to how disciplinary mathematics is closely linked to understandings of recognizing students' mathematical ideas and positioning them as competent in developing mathematical reasoning. Particularly, in her artifact, Dr. Adams asked her novice teachers to examine an instance of mathematical inquiry in a classroom. Novice teachers were first introduced to the mathematical problem and then asked to watch a video episode of the same problem. Novice teachers are then challenged to understand "How do kids position themselves? How do kids position each other? How do teachers position kids? How do systems in society position kids?" These questions are indicative of a professional vision that embraces both activities that promote mathematical inquiry and how to create equitable spaces within the context of mathematics education.

In her interview, Dr. Adams described where she is on her journey toward equity in mathematics spaces. She relayed that her own research and experience has been focused on ambitious teaching practices; however, she also stated that ambitious teaching practices are not enough to ensure equity. In order to implement ambitious teaching practices with equity, teachers need to be oriented toward how power and privilege operate in mathematics classrooms. In her biography, Dr. Adams underscored this point,

> In working to enact ambitious teaching practices (e.g., orienting students to each other's thinking), I have come to see them as tools that can sustain power and

> white forms of knowing, or can disrupt power as is circulates through classroom life. In my work preparing novice teachers in elementary math methods, I have worked to center power in our readings, discussions, and reflections of teaching. We have drawn on writings of scholars of color in our field to better understand their perspectives about the multiple mathematical knowledge bases; mathematics identity and racial identity; what is dehumanizing and (re)humanizing in mathematics classrooms; and consider the spaces in which teachers make decisions about what to say or not say.

Dr. Adams discussed how she wants to integrate her understanding of ambitious and rigorous teaching practices with her new understandings of equity in mathematics spaces. Specifically, Dr. Adams hopes to support teacher candidates to take steps toward a practical understanding of how these ideas can be implemented with equitable outcomes for students.

Dr. Adams' focus in her mathematics methods courses is primarily in the area of Knowledge about Mathematics and Society. In particular, Dr. Adams reported on leveraging the existing literature and video of mathematics teacher educators to support teachers in developing their understandings of the narratives in mathematics education. Further, this work supported her students' development of their understandings of their future profession as a political endeavor. Dr. Adams' coursework is further focused on practices that are married to these ideas.

More broadly in her work, Dr. Adams hopes to work with practitioners to think about how to implement both the ambitious teaching practices and her new understandings of equity to the benefit of children. Further, Dr. Adams described leveraging her understandings of how other disciplines unfold for children to consider how to implement problem posing in mathematics contexts at the elementary level.

Dr. Dani Juan-sin-tierra, Somewhere University

Dr. Dani Juan-sin-tierra is a professor in the mathematics department at Somewhere University, where they focus on mathematics and statistics education. Somewhere University is a large, public research university located in a small city. Dr. Juan-sin-tierra identifies as a Latinx person who uses pronouns they/them/theirs in this study.

Portrait of Dr. Juan-sin-tierra

Dr. Juan-sin-tierra grew up with their family and two parents, who “were teacher educators. My mom [taught] at the elementary level, and my dad at the secondary, but none of them in mathematics, though.” While Dr. Juan-sin-tierra stated that their parents were important influences in their life, it was their fourth-grade teacher who “told me ‘you're good at mathematics’ and I believed her from then on.” From these conversations, it became apparent that this teacher influenced Dr. Juan-sin-tierra deeply and as a result of their experiences with their parents and their teachers, Dr. Juan-sin-tierra decided to become a secondary teacher.

Dr. Juan-sin-tierra described their experience in mathematics, articulating, “there was no inquiry whatsoever. I mean, nothing was discussed and nothing was connected. Nothing was, we didn't stop to think about it. We did not create anything; we did not use what we knew to get to the next one.” Hence, their experience of learning mathematics in school was perceived as a set of disconnected facts, whereby students learned discrete aspects of the mathematical cannon without making connections across ideas. Dr. Juan-sin-tierra elaborated that this experience was one during which "I loved, loved the procedure, the algorithm, the memorization, the circling the right answer, the whole thing, so, it was joy to me." School mathematics for Dr. Juan-sin-tierra, with the experiences of learning a set of facts to be used appropriately, was a place where they felt success and joy.

Dr. Juan-sin-tierra described the sorting experience in schools at that time. In particular how schools sorted individuals into groups. They stated that there was a "very, very compartmentalized ways of putting people, especially in the intellectual. So you're good at this, so you will become X and forget about everything else." So not only was their internal experience of mathematics joyful and fulfilling, but because of the design of the schools they attended, they was encouraged to pursue mathematics by the school structures. Dr. Juan-sin-tierra explained the sorting philosophy in the following way, suggesting that these schools structures communicated, "Why are you going to do this other thing, if you're not good at and they know it, or you're setting [yourself] up for failure." It was therefore assumed that the structures of schooling should help students to play to their strengths and not focus on their weaknesses.

Dr. Juan-sin-tierra earned a scholarship to study in the United States for two years when they were 19-years-old. They had hoped they could use this experience to earn their degree in secondary mathematics teaching. However, when they arrived in the United States, they did not "know how to speak English. Not one word. Well, 'how are you?' 'Very well, thank you.'" Dr. Juan-sin-tierra did not allow this to deter them, recounting that, "I was never seen as someone that could not succeed. I was placed already and I had achieved a level of mathematics." Their status as an emergent bilingual learner, which could have resulted in deficit views from teachers and professors, did not negatively impact the assumption that they would be a success. However, in two years of scholarship they could not achieve the level of English proficiency to complete a teacher preparation program.

Dr. Juan-sin-tierra graduated with a bachelor's degree in mathematics and minor in computer science, "and I said, I want to be a teacher." Dr. Juan-sin-tierra was advised by their mentors as an undergraduate in the Mathematics Department to apply for a Master's degree in Statistics and a teaching assistant position and went on to earn a Master's degree in Statistics at the same university. However, they still wanted to achieve their goal of becoming a secondary mathematics teacher. Then, "my wonderful mentor […] said, 'Why don't you get a PhD in math education?' I said, 'What is that?'" After their mentor's explanation, Dr. Juan-sin-tierra decided to pursue this avenue.

Dr. Juan-sin-tierra and their mentor identified a large, public, research university. Once Dr. Juan-sin-tierra had identified the program, they thought, "Oh, maybe I have a chance. And despite my GRE scores being so low, […] [Ph.D. granting university] gave me a chance." Accordingly, Dr. Juan-sin-tierra decided to pursue their Ph.D. in mathematics with a focus on mathematics education.

It was at their Ph.D.-granting University where Dr. Juan-sin-tierra describes their first experience with mathematical inquiry. It was in a course designed to explore school mathematics, particularly geometry. Dr. Juan-sin-tierra classified their response to this first inquiry experience as, "Oh my God! Lord! Oh, it was just like, how could I not have known and learned all these things? So my passionate love [of mathematics] went off the roof and also my decision to become a mathematics educator was reaffirmed." The inquiry experience in this case provided a new and exciting way to understand mathematics – Dr. Juan-sin-tierra was able to add to what they already knew and loved about mathematics.

Dr. Juan-sin-tierra earned their Ph.D. in mathematics, “And so finally, you know, after I got out of [Ph.D.-granting university] and I was like, okay, can I teach now?” And they do teach mathematics and mathematics content courses for teachers at Somewhere University, but of their desire to be a secondary teacher, they stated, “It has not yet being fulfilled inside of me. I cannot say I am a teacher […] it just brings back those memories.” Dr. Juan-sin-tierra emphasized that this desire is still with them and they have considered how they might still achieve this dream. On the other hand, they did note that their experience of not yet achieving this goal has shaped some of what they do and how they think about the world.

Across their career in Dr. Juan-sin-tierra’s work as a teacher of mathematics courses, they came to realize that it “was just a subset of those learners for which the non-inquiry based approach is fulfilling.” They identified that some learners did not feel the joy they felt as a child when a new algorithm came along, but rather felt something quite different. They branded the purpose of inquiry as two-fold. First, inquiry was a way for some learners to take advantage of mathematical opportunities. They stated,

> And so I started seeing inquiry as a way for others to enjoy [mathematics], and also what I do.” And that is not to say that I left my other piece forgotten where I have now the pleasure to meld them both and, and I reach way more students that way.

Thus, they see inquiry as a mode of mathematical engagement and understanding to learners who have otherwise been excluded from that experience. In particular, it provided others with a chance to have the same experience of enjoyment in mathematics that they had experienced all their life.

Second, mathematical "inquiry is about, wondering. It's being curious about why certain things work the way they do," they stated. Thus, mathematical inquiry also serves the purpose of making explicit the connections between underlying ideas in mathematics. Dr. Juan-sin-tierra classified this aspect of mathematical inquiry as a way to provide, "more access to not just mathematics, but also, to mathematical practices." Therefore, mathematical inquiry is a process for individuals to participate in the doing of mathematics, not just the receiving of it.

Dr. Juan-sin-tierra went on to explain one common classroom practice that is aligned with their approach to mathematical inquiry. They described this approach, which begins with a concrete example of a situation and derives a general formula, explaining,

> We do a problem and the problem is easily accessible by what [the students] know from before. It just requires a little bit of guidance in how to think about it or just making it clear what we're trying to do. And the students themselves, […] participate and sometimes they don't. So I guide them and then from that exemplar-example, if that's such a thing, carefully, like they can easily derive the general formula [themselves].

Through this approach, students are able to ground abstract ideas into concrete examples regarding what these ideas mean. Dr. Juan-sin-tierra noted that they could take the more traditional approach, suggesting,

> Here's the theorem; here's the justification of why. Let's do a problem; there's nothing wrong with that. It does teach them the content and achieve this goal. But it doesn't get to this awesomeness of the discussion and how it works.

Again, Dr. Juan-sin-tierra emphasizes how this approach serves multiple mathematical and equity goals. First, it provides students with an understanding of how mathematical ideas fit together and shows an inductive approach to understanding the mathematics. Second, this approach to mathematical inquiry, in concert with others, achieved their goal of reaching those students who had previously felt that mathematics was not for them. Dr. Dr. Juan-sin-tierra explains,

> Their perception of learning mathematics changes, especially for those that have had, for whatever reason, an awful experience and continue to have it and they have yet to have one where they enjoy being here.

Thus, they are able to bring the kind of intrigue and interest that they experienced when learning mathematics to their own teaching and students.

In their teaching, Dr. Juan-sin-tierra differentiates how they talk about mathematical inquiry with their teacher candidates from those taking mathematics courses for alternative purposes. In particular, they stated,

> For my preservice teachers, it's almost like a double standard along with developing [mathematical inquiry] practice. I want to model good [teaching] practice…I do certain things enough times during the semester that I want them to see a pattern in the way I approach the inquiry process.

That is, they hope their teacher candidates can start to learn how to interweave the practice of mathematical inquiry into their teaching. Dr. Juan-sin-tierra suggested that understanding inquiry for mathematics teaching is a slightly different construct from understanding inquiry for doing mathematics. They stated that, "inquiry for teaching mathematics is slightly different, because the students engage in the inquiry process. [It] also goes to […] the teaching and learning of mathematics." This approach to inquiry is one they consider in their work, which goes beyond the classroom.

In addition to their teaching duties, Dr. Juan-sin-tierra is a PI on several grants that focus on emergent bilingual learners in mathematics classes in the U.S. and mathematics and statistics education in Central and South America. They serve on committees throughout their department and university. In this work, they note that their attention to mathematical inquiry and practices have served to broker conversations and broaden understandings. Dr. Juan-sin-tierra notes that in conversations with others, they uses their "mathematical mind […] to differentiate between […] when something is being generalized or, or is it just a hypothesis. Are you claiming this […] or were you wondering [if] we need more evidence?" In this way, their mathematical inquiry stance supports their and their colleagues to clarify what mathematical assumptions are being made.

Further, Dr. Juan-sin-tierra argues that taking an inquiry-stance to the world, in general and their students in particular, can result in a more equitable experience for students. Mathematical inquiry and an inquiry stance require that one repeatedly asks "why" both to unearth assumptions and to deepen understandings. They stated,

> We don't ask ourselves, […] 'why is this student not speaking?' 'Why is this student not writing what is supposed to be writing?' Or 'why is this student too is not participating?' Or 'why just sitting there? No, we don't ask why. We just say,

> ‘Oh they are irresponsible.’ ‘Oh, they don't know that language.’ ‘Oh, they didn't study last night.’ Or worse. ‘They can't read what I wrote’ or ‘they didn't hear me.’ Oh my God! Haven't you yourself being in that position?

Dr. Juan-sin-tierra was discussing how teachers and professors in learning spaces can make assumptions about students based on only what is visually observable, such as, how students present themselves in class, what students look like, and what is written on exams. Dr. Juan-sin-tierra was urging the mathematics and mathematics education communities to take an inquiry stance to each student instead, both to unearth assumptions and to understand better the learners in their classrooms.

Dr. Juan-sin-tierra’s Vision of Equity and Inquiry in their Professional Work

Dr. Juan-sin-tierra’s deep love of mathematics and education has led them to a place where they understand that mathematical inquiry strengthens their students understanding and sense of belonging in mathematics classrooms. Furthermore, their inquiry-stance toward the world at large helps them to understand each of their students as individuals who exist within a complex set of social constructs. Especially, when they take this stance in their work, it encourages their colleagues to broker greater understanding.

Dr. Juan-sin-tierra’s professional vision can be categorized by their dedication to developing their students’ understanding of how mathematics is developed. These developments help novice teachers to understand how humans developed mathematics over time using both deductive and inductive understandings of disciplinary ideas. Further, Dr. Juan-sin-tierra builds on these constructions to help their novice teachers develop mathematical canonical knowledge. In Dr. Juan-sin-tierra’s artifact, novice teachers were provided with numerical examples of a mathematical phenomenon upon which to perform calculations. In their lecture notes, Dr. Juan-sin-tierra supported the development of general forms from the numerical examples.

Thus, Dr. Juan-sin-tierra described goals that most closely align to those of canonical content knowledge as well as epistemic considerations in mathematics. In particular, Dr. Juan-sin-tierra identified goals that were related to coming to understand how to make mathematical sense through inquiry, and coming to understand mathematics as a human endeavor. In their enacted work, Dr. Juan-sin-tierra emphasized that the goals for their students centered the doing of mathematics. While they were not explicit about it, from what they shared, it became apparent that they valued students' sense-making experiences and indicated mathematics was a human-driven activity. Dr. Juan-sin-tierra described two classroom practices that moved their goals forward. First, they described a practice of "reverse-engineering" where they used concrete examples to deduce and understand a general mathematical rule. Second, Dr. Juan-sin-tierra described how they provided students with tasks that can be addressed using multiple methods and with different entry points. Dr. Juan-sin-tierra pointed out that these tasks might not appear to be particularly salient, but learner responses uncovered interesting aspects of the mathematics. Further, when Dr. Juan-sin-tierra described their broader work, they used similar terms, specifically an inquiry stance that helps one to understand the world as well as issues of equity.

Dr. Patrick Mahoney, Farnham College

Dr. Patrick Mahoney directs programs in mathematics education and leadership at Farnham College, where he teaches courses to teacher candidates, in-service teachers, and graduate students. Farnham is an undergraduate historically women's and post-graduate co-ed institution with an excellent reputation located in a rural area of a Northeastern U.S. state. Dr. Mahoney identifies his race and gender as a white man.

Portrait of Dr. Mahoney

Dr. Mahoney began his teaching career at an elementary school, which he described as, "98% white, and middle, upper middle class." Dr. Mahoney noted that his second grade classroom was "often known as the special education room. I would have a class of about 20 students. I think in one year [there] was 12 students on IEPs out of 20 students, which is pretty significant, when more than half your class is on IEPs." He explained the reason for this significant portion of students with IEPs, stating, "it was always a work in progress to figure out what's the way […] for Christopher, what's the way for Steph, what's the way for Janelle? I think that alone just had the effect on more and more special education teachers, and classroom teachers knew that by sending kids who had diverse needs into my classroom, that their needs were more likely to be met." Dr. Mahoney emphasized this to portray his thoughts about diversity during his time as a teacher.

In his practice as an elementary school teacher, he noted that, "I was actually literacy- and science-focused. Mostly because I didn't have great experiences with math and so my first couple years teaching, I never really liked math." This is significant because Dr. Mahoney's initial orientation toward mathematics at the start of his career is very different from how he describes it now. In his early years of teaching, he described his reasons for this mathematical orientation, "I was taught in a really procedural way.

And so that's how I taught." Dr. Mahoney reflected an experience of mathematics that depicted it as a set of facts and procedures to learn.

Dr. Mahoney can point to the moment early in his teaching career when he saw a shift in his orientation toward mathematics. He recalled that, fairly early on, he applied for a job that would take him out of the classroom to work as a literacy coordinator who would run a grant, where his primary responsibilities would be to train teachers in guided reading. Dr. Mahoney stated that he was not given the job because he was "too young to be taken seriously." He recounted that following this disappointment, his colleague from across the hall

> Came over to console me a little bit and said, "Well since you're not doing this, they need another field test teacher for [NSF-funded, elementary, progressive curriculum]?" Would you be interested in doing that? And I had no interest at all 'cause it's math and you have to commit to teaching the [curriculum]. And I didn't really understand it.

Dr. Mahoney underscored that due to his orientation toward mathematics and a lack of preparation to teach the curriculum, he was not interested in pursuing this line of work. His colleague continued to urge him to try this opportunity. Eventually, she informed him that, "they offered a stipend if you did it. And I, I was young teacher, pretty poor. So I was like, 'Yeah, I'll do it then.' So that was the day that that changed everything because part of the agreement of being a field as teachers, you had to do professional learning." Dr. Mahoney emphasized that missing out on the literacy job and reluctantly deciding to participate in field-testing changed the course of his professional life.

In the professional learning program, Dr. Mahoney had the opportunity to work with many leaders in the mathematics education field, stating, “You cannot be unchanged working with those brilliant women.” These educators provided Dr. Mahoney with another way to understand mathematics. He started, “to tell other people why couldn't I have learned math this way. If I had learned math this way, I would've had a whole different outlook on it.” The professional learning experience associated with the curriculum shifted Dr. Mahoney’s understanding of what mathematics was and what it could be. Further it changed how he approached teaching mathematics. Dr. Mahoney stated that, “Once you see that, you can't un-see it. So then I couldn't go back and teach in any other way than through inquiry and problem based learning and exploring and trying students' ideas.” This was in stark contrast to his previous practices that focused on the procedural aspects of mathematics.

Following fourteen years in the elementary classroom, Dr. Mahoney moved on to Farnham College where he orchestrates several strands work in mathematics education and leadership. Currently, Dr. Mahoney directs the graduate mathematics education programs and travels to provide both on- and off-site professional learning experiences for teachers and school districts in mathematics pedagogy.

Dr. Mahoney described a key event early in his time at Farnham College that influenced how he thinks about equity in mathematics spaces. Early in his career, Dr. Mahoney began attending a major national conference for mathematics teachers. He described that experience like this, articulating,

> The first couple of times I didn't even know it was, you know, it's just bananas there. I became friends with [presenters], and, and so I went to all of their sessions

> and loved their work and I got really excited about it. Because nobody out in [rural part of home state] knew who any of these people were. […] If you're in a classroom you can't go to these conferences. So, I decided to design a course on bringing all of these people in. So, I designed this course with all white men and women and I got called out on it and that was that. I just did not have that lens.

In trying to bring his positive experience of the conference to Farnham, Dr. Mahoney had accentuated white voices that were often highlighted in mathematics education spaces. When someone pointed this out to him, it provided him with space to reflect on why he had attended these sessions. He questioned himself, asking, “Why did I go to all of their sessions? Because they're white and I identify with them? Did I go, because it's sort of like your sphere of influence? […] Is it because of that or was it because I have a bias, but I'm not aware of it?” Reflecting on these questions served as a catalyst that led to shifts in Dr. Mahoney’s thinking and action, including changing the syllabus for the course to increasing his “sphere of influence” to include racially and ethnically diverse voices in mathematics education. His action illustrates how important racial and ethnic diversity is to his teaching of mathematics.

In reflecting on this experience, Dr. Mahoney emphasized that in the case of his professional learning of mathematical inquiry, suggesting “ once you see it, you can't unsee it. I have that lens, when I'm invited to […] present at a conference […], I now ask, who the other speakers are and are they, are they being mindful to have a diverse pool of speakers?” Dr. Mahoney stressed that while this was a relatively new way of looking at the world for him, in which he is continuing to learn and grow, it is something that he looks for regularly.

Currently, Dr. Mahoney teaches two courses to graduate students that focus on leadership in mathematics education, particularly, how to “become an advocate in the math world, both at the local level, national level” and andragogy with a focus on adult learning. These courses point to an important part of Dr. Mahoney’s work in mathematics education, which includes being an advocate for teacher learning.

His experience in putting together the initial course based on his visit to that national conference is showcased in his mathematics leadership course. Particularly, he provides students with an experience that parallels what his own was.

> I show all the different speakers and people get really excited about it. Then, once I have all, I pop up the different and they said, we- the first time we did effective practices and this is about designing it and I pop-up, Dan Meyer and Robert Kaplinsky and people are like, “Oh! Look at this star studded cast” and then when they have all the faces up there. I say to the students, "Take a moment…what do you notice? What do you wonder?"

Dr. Mahoney noted that in his own case, the graduate students often have an emotional response to this experience. It helps them to unearth blind spots they would not otherwise be able to uncover.

In his course and programmatic work, Dr. Mahoney also focuses on unearthing the stories of participating teachers and helping them to think about their own experiences in mathematics. He believes that unearthing traumatic experiences in mathematics provides a lens for understanding current students’ experiences. He wants teachers to think about how to, “ensure that future generations experience less trauma, less judgment and less exclusion in mathematics.”

Dr. Mahoney credits his experiences in the past several years with changing his view of equity in mathematics education. He described the experience of working at Farnham as “broadening my world, my scope.” In his own experience, his inquiry stance in mathematics has affected how he thinks about equity in the mathematics classroom and in society in the U.S. In particular, he emphasized that taking an inquiry stance, not just to mathematics, may also reveal societal assumptions and presumptions. He declared,

> Much of our society has been constructed by powerful white men and, and the only way these systems get broken down and become more inclusive is when we, as individuals in society, ask questions and push back on assumptions and presumptions and, and biases and stereotypes and explicit and implicit racism and sexism. And so, I feel like inquiry is a great way to approach mathematics. And I think it's also a great way for people to learn to question and to learn, not accept authority. I don't know that the result of this problem is because you did the thing I told you to do. I want you to know, because you took the time to make sense of it and to question it and to push back and to analyze. And I want you to take those same behaviors and I want you to look at our world with that same lens.

Dr. Mahoney emphasized that an important part of inquiry was to ask questions repeatedly until the mathematics or social objects makes sense, but also to notice when social norms do not make sense. This is closely aligned with the “notice and wonder” routine that is common in many inquiry-based classrooms.

Dr. Mahoney’s Vision of Equity and Inquiry in his Professional Work

In Dr. Mahoney’s story, he emphasized two major events that shaped how he thinks today. The first provided a new way for him to think about mathematics and the second offered him a new way to think about equity. It seems that Dr. Mahoney believes that mathematical inquiry can be a way in for students who have been traditionally excluded from mathematical spaces. In particular, it can provide opportunities or experiences to build on students' knowledge, which might help them make sense of the mathematical world. If teachers and students take an inquiry stance based on the mathematical model, it can support them to examine and question their worlds.

Dr. Mahoney described two enactments of his goals. The first was related to the task he provided. The task itself was designed with novice teachers and teachers in mind, despite the fact it could be used in classrooms. The task had little language associated with it to ensure learners with different home languages could participate. And further, the task was action-focused to provide learners with opportunities to identified and determined how to narrow their own mathematical questions. Dr. Mahoney's second enacted goal was described in detail in earlier.

Dr. Mahoney's vision centered on supporting teachers' development of mathematics teaching as a political act, as well as the development of relational content knowledge for teaching. In order to support the first strand of work, Dr. Mahoney described course activities, how he has been thinking about his professional development work, and future speaking engagements. Further, Dr. Mahoney described how he supported students to develop an understanding of multiple levels of inquiry.

Dr. Makoto Yoshida, Anselm's College

Dr. Makoto Yoshida currently holds an appointment in the mathematics, statistics, and computer science departments at a small, religious, highly ranked, liberal arts college in the Midwestern part of the United States, Anselm's College. Dr. Yoshida identifies his race and gender as an Asian man.

Portrait of Dr. Yoshida

Dr. Yoshida described his own high school experience as similar to the high school where he taught after completing his graduate degree, Pleasanton High. "I grew up in what one might call a middle class kind of school very much, like [Pleasanton] […] So you can imagine how white and Asian it was. Very white and very Asian. And so, I grew up in that environment."

He described his experience of learning mathematics in high school as,

> I had good teachers, but they are very traditional, for lack of a better phrase. They would kind of teach you these things, in a very direct way. And I understood them and I tried to make sense of them, and you know, it's not- 'Okay, this all makes sense and I can solve most of the problems that were posed to me and then move onto the next topic.'

In combination, these experiences provided Dr. Yoshida with a fairly traditional – and perhaps typical – view of mathematics in U.S. schools. Dr. Yoshida was not exposed to inquiry as an approach to thinking about mathematics. In retrospect, Dr. Yoshida identified that his high school training in mathematics had resulted in a "very compartmentalized way of looking at mathematics." In his high school experience, he was taught mathematics as a collection of concepts, without having to understand how they fit together. Dr. Yoshida characterized his learning as, "not like I was just memorizing stuff, but I really didn't know how to create my own mathematical understanding when I was in high school." His high school experiences did not provide him with an image of what it means to do mathematics, particularly how to create connections between the various concepts he learned.

Following high school, Dr. Yoshida went on to attend a selective, well-regarded technical university in the northeastern part of the United States, where he majored in mathematics. Speaking of his undergraduate mathematics experience, he noted, "When I was asked to prove something for homework [as an undergraduate] I kind of skimmed through the textbook for something that was kind of closely related to…what chapter is this problem from? So I can look at it." Similar to high school experience, Dr. Yoshida was taught mathematics in a way that did not promote skills for mathematical sense making. Dr. Yoshida suggested that professors reinforced this idea, and acted as "Some great thinkers, you know, espousing their wisdom to these wunderkinds if you will. Unfortunately, I wasn't one of them." In this case, the mathematical authority resided with the professor, but in both cases, mathematics was a body of knowledge to be transmitted to individuals who were "wunderkind" enough to understand it.

When reflecting on his experience as an undergraduate, Dr. Yoshida described that his experience in mathematics classes resulted in a feeling of having "hit [his] limit."

> My peers in college were people […] who are kind of famous in the math world nowadays […] And so, you know, you come from high school and you think you're pretty good and you go to college and you realize, well, okay, I'm kind of scraping to keep up with most of these folks. And so I think at the end of college, […] I definitely enjoyed my college learning experience and I wouldn't change anything about it, but I felt like I had kind of hit my limit.

Dr. Yoshida left his undergraduate with two messages that continue to be essential to his work as a mathematics teacher and mathematics teacher educator. First, he had "hit [his] limit" mathematically. And second, while he saw mathematics was a set of logically connected ideas, he did not understand the procedures or reasoning that would support his own development of these connections.

Following his undergraduate degree, Dr. Yoshida worked in industry for two years before seeking his Master's Degree in teaching from another well-known university in the same part of the United States, Saltfleet University[2]. While there, Dr. Yoshida participated in a professional learning experience that focuses on inquiry-type mathematical practices for future and current teachers. In particular, participants in the program spent time *immersed* in mathematics, where they approached new problems and made connections. He credits the professional learning program in which he participated at Saltfleet as being where he "learned how to *do* mathematics---to explore, to conjecture, to wonder." Dr. Yoshida's view of mathematics dramatically shifted as a result of his involvement with this professional learning program. Further, Dr. Yoshida noted that how he thought about what was necessary for learning mathematics also shifted. Prior to his experience at Saltfleet University, he believed that "helping students develop understanding through their own work was not at the forefront of the way [he] thought about teaching before."

Following the completion of his degree, Dr. Yoshida continued to work with the program in the summer; first as the equivalent of a teaching assistant and later as the director of the program.

---

[2] Dr. Marie Adams is now a professor at Saltfleet University. Dr. Adams was not at Saltfleet at the same time as Dr. Yoshida; however, their respective association with the university and its philosophies may have some implications for findings in Chapter 5.

Once he completed his Master's degree, Dr. Yoshida went on to teach in a suburb of Saltfleet at Pleasanton High School. Pleasanton is a medium-sized, wealthy city that participated in a school choice program with students from Saltfleet City attending Pleasanton. Dr. Yoshida said,

> [T]his is not at all to diminish what [Pleasanton High does], but they do like to talk about their diversity, which it's true because they, they have the […] kids bussed in. Um, but you know, come on. My calculus classes are all, they're all white and Asian kids. […] Right? Kids [who participated in the school district sharing program] never took calculus. […] where the diversity showed up were some of my lower level classes that I taught.

Dr. Yoshida pointed to this experience as one where he noticed who was given the message that they belonged in mathematics and who was not. After three years, Dr. Yoshida decided to pursue his graduate education in mathematics at Saltfleet University. Dr. Yoshida wrote in his biography, "Leaving [Pleasanton High] was hard to do, but the urge to do more math was too strong."

Dr. Yoshida highlighted that his experiences of mathematics in his graduate program were different from his previous experiences. He classified his mathematical experience in graduate school courses through this example,

> my advisor […], probably the best math teacher I've ever had, all he did was just lecture for like an hour. […] We never had discussion, you know, like during class, right? Some student might ask a question but he gave these beautiful lectures, just absolutely beautiful lectures, you know, just showing you all these

> connections and just the perfect examples. But he just stood there and talked for an hour.

Dr. Yoshida shared this example to highlight how different a suitable teaching approach might be in graduate school, once students have identified that they belong in mathematics and have the skills to make sense of the material. In particular, Dr. Yoshida described this learning experience,

> The way he taught was just right for his audience. He knew that we could go home and make sense of the material on our own. He also gave us these beautifully crafted sets of problems for homework, and I learned so much through working on them. […] He […] welcomed one-on-one discussions outside of class time, and those informal conversations were very different from his lectures.

Dr. Yoshida provided this clarification to underscore that the teaching supported the development of his mathematical understanding, but also that there were supports outside of the classroom at the graduate school level.

Dr. Yoshida graduated from Saltfleet and completed the equivalent of a post-doc in a mathematics education program, where he taught mathematics content courses for teacher candidates and supported an NSF-grant in mathematics education. Dr. Yoshida arrived at Anselm's following his post-doctoral work. He describes Anselm's as "very homogeneous, mostly all white, not even many Asians."

Dr. Yoshida describes his current body of work as three interconnected strands. First, he teaches courses in mathematics and mathematics methods at his current institution. Second, he works closely with his regional affiliate of NCTM. He has served on the board of that organization and is currently working with the committee that focuses on supporting early career teachers and teacher candidates. And finally, Dr. Yoshida is involved with ongoing research in mathematics education, primarily in teacher education.

Central to what Dr. Yoshida does in his mathematics content and mathematics methods classes is derived from the understanding of mathematics he gained from experiences at Saltfleet's professional learning program. At Anselm's, content teachers take mathematics courses along side all other mathematics majors. Dr. Yoshida classified his pedagogical outlook by suggesting schools "give students the kind of time and space to work with concrete examples and have those experiences that lead…to sense-making and even posing their own follow up questions or something like that." From conversations with Dr. Yoshida about his classroom artifact, it was apparent that inquiry practices were prominent in the classroom with the expectation that students would undertake sense making as a result of the experience. In fact, Dr. Yoshida stated,

> When I teach a proof class, like abstract algebra, which I often teach, I tell my students 'well, you shouldn't prove something that you don't already believe is true.' And the reason why you might believe something is true is because you've had experiences with it and you've worked with concrete examples to see some patterns and make some generalizations, and then you can go and justify those.

Dr. Yoshida is emphasizing here that mathematical inquiry provides an opportunity for students to make sense of their own experience in their own ways and then fit their new understandings into the discipline of mathematics. Further, Dr. Yoshida highlights that for teacher candidates, inquiry is an important part of learning.

At the university level, Dr. Yoshida contrasted his experience at Anselm's to that at his undergraduate institution. He noted that his department takes a big tent approach to mathematics: "anyone who is interested and invested in doing math should be able to do math […] they shouldn't be made to feel like only the elite of the elite can do math." Dr. Yoshida recounted that in some mathematics departments, "there's an assumption that by the time you get there, you already have [an inquiry-oriented] way of thinking about math," underscoring the idea that mathematics can be a space where students without certain ways of thinking are not welcome. Additionally, Dr. Yoshida noted that for the public, there are cultural notions of who can do mathematics, which impact the students at all levels. He stated, "And if you mentioned mathematics at all, they love to talk about how they were not good at it, right? […] it's kind of assumed that math has this- like you're- You have to be a genius." Dr. Yoshida is highlighting this cultural tension, whereby there is a public assumption that math is hard and therefore, only particular "geniuses" can be successful in mathematics. Dr. Yoshida's department is attempting to counteract this idea by creating opportunities where students can act as sense-makers in mathematics.

Dr. Yoshida's Vision of Equity and Inquiry in his Professional Work

As a culmination of his experiences and professional learning in mathematics and mathematics education, Dr. Yoshida has come to a place where he understands that mathematical inquiry is an essential part of supporting learners to understand mathematics as a practice-based discipline. And further, participation in inquiry allows individuals to break through perceptions of society that perpetrate the belief that mathematics is not a place for everyone. The discipline of mathematics would ultimately be opened to new ideas from traditionally excluded individuals.

Dr. Yoshida's professional vision is characterized by his dedication to enacting learning for his students through mathematical inquiry and using these developments to create space for learners outside of societal restrictions. Dr. Yoshida's artifact highlights this vision by providing mathematics learners with the opportunity to develop their own understandings of mathematical ideas by recognizing underlying structure. Further, Dr. Yoshida's task supports learners to focus on his goals. For example, in one question Dr. Yoshida asked how many solutions there were to a particular item and why. In the margin he provided the learners with the solution, but not the reason why. Thus Dr. Yoshida is directing the learners' attention to that "why" rather than the "what" item.

In his enactment of these goals, he outlined a "workshop" model of mathematics instruction. In this instructional model, the learners begin by working on mathematics problems that address the class day's mathematical objective. Dr. Yoshida visits with each group of students as they work to unpack and understand the mathematics. At the end of the session, Dr. Yoshida, in conjunction with learners, provides a summary that often generalizes the mathematics from the course day.

Dr. Yoshida's goals and work align with epistemic considerations and canonical content knowledge. In particular, Dr. Yoshida's portrait displays a desire for his students to understand mathematics through sense making. In his discussions, he emphasized that he hoped teachers would be able to teach in a way that would "pepper" in these modes of understanding the discipline.

Summary

This chapter presented a portrait of each participating mathematics teacher educator. Important aspects of these portraits showed how individuals came to their current understandings of inquiry and equity. While Chapter 5 will provide a detailed comparison of themes that emerged from the participants' stories, and make note of how notable relationships transpired. In examining the participants' stories, it became apparent that their experiences as young people tremendously influenced how they came to understand teaching. Their own experiences as K-12 students were used as models for teaching mathematical content in their own contexts. While the participants came to inquiry at different points in their professional trajectory, their K-12 experiences proved to be foundational in their initial conceptualization about teaching. For some of the participants, their K-12 experiences provided them with a love of mathematics that they carried into their adult lives. In all four cases, the participants identified episodes or experiences that brought them to their current conceptualization of inquiry. In each case, these experiences were described in their own professional learning contexts, rather than in more informal venues.

The participants' experiences in inquiry seem to have impacted their beliefs and ideas around issues of equity and justice in their practice as mathematics teacher educators. Inquiry seems to play a role in the way in which they understand equity and social justice in the contexts of K-12 classrooms and their own practices as mathematics teacher educators. The participants suggested that inquiry provides means of doing mathematics. In addition, they suggested that aspects of inquiry that attend to problem posing can also be a forum for learners to better understand the world as they create their own world views. In combination, these impacted the types of equity pedagogy put forward in content or methods courses that the participants offered to the novice teachers with whom they worked. In each portrait, I briefly classified how each of the four participants identified their learning goals for teacher candidates and teachers. In Figure 4.1 below, I have located each of these goals in the Knowledge for Mathematics Teaching Framework. This provides a comparison of the focus of each of the portraits. The themes will be explored more thoroughly in Chapter 5.

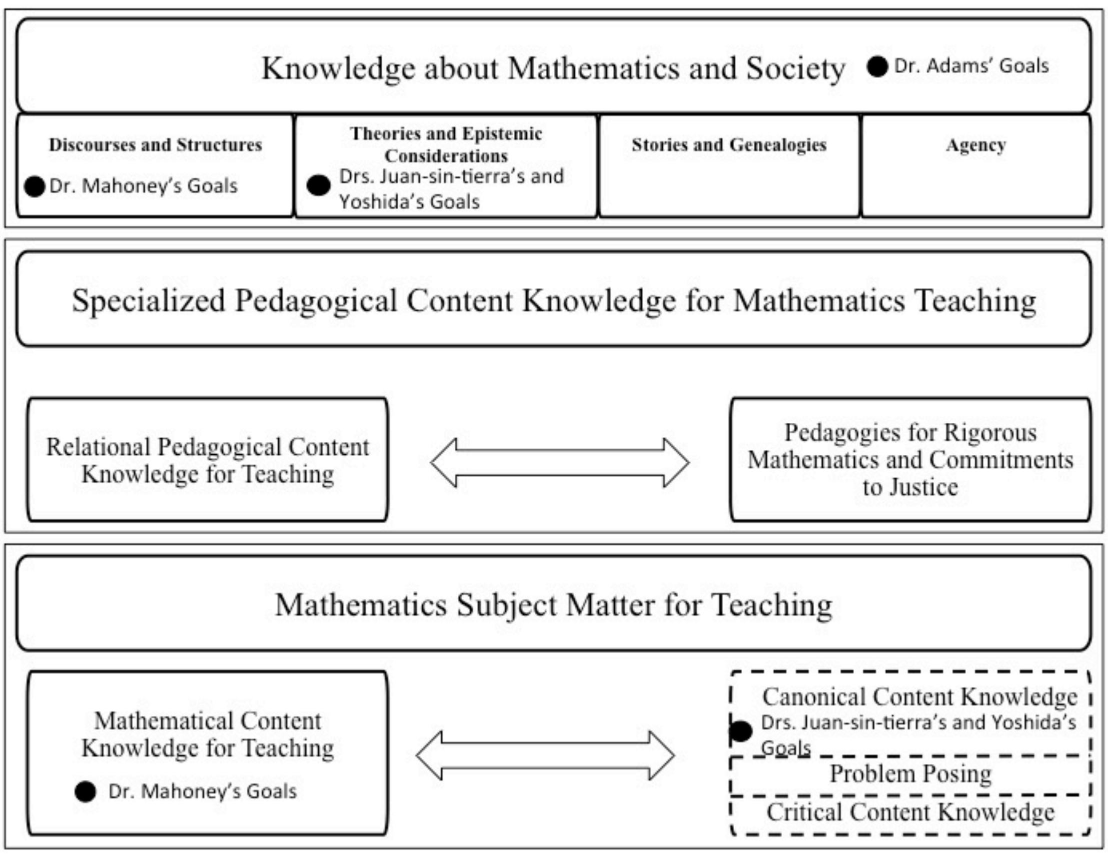

Figure 4.1. Operationalized Goals for MTEs

In the figure above, the points are placed to represent the area that each of the goals identified by MTEs described. Dr. Adams' goals were primarily in the area of knowledge about mathematics and society, broadly. That is, in her description of her artifact, she targeted multiple areas. The remaining MTEs targeted specific aspects of the larger category. Drs. Juan-sin-tierra and Yoshida focused on epistemic considerations in mathematics. In particular, the idea that mathematic emerges from human ideas. Dr. Mahoney's goals focused on supporting teachers and teacher candidates to understand these careers as political. Drs. Juan-sin-tierra, Mahoney, and Yoshida all expressed goals that fell into the broad category of Mathematics Subject Matter of Teaching. In particular, both Drs. Juan-sin-tierra and Yoshida described goals that fell into the area of Canonical Content Knowledge. While inconclusive, it is notable that both Drs. Juan-sin-tierra and Yoshida focus on mathematics content courses, as opposed to teaching primarily mathematics methods courses. It would make sense that the operationalized goals then would fall in these categories. Dr. Mahoney's goals also fell in this area, but his focus was more on the mathematical content knowledge for teaching. In particular, he focused on supporting teachers and teacher candidates to unpack their pre-existing understandings of mathematics.

All of the participants described their current experiences as mathematics teacher educators as trajectories. In particular, participants described that they still have a lot to learn both in thinking about inquiry and understanding equity in mathematics and beyond. However, taking an inquiry stance on the world can provide the participants with a mode to advance their ideas.

CHAPTER 5

COMPARING THE EXPERIENCES OF PARTICIPANTS

Introduction

This chapter will provide a comparison of the themes that emerged from the participants' stories. Following a description of the themes, this chapter addresses the study's second research question: *In what ways does the attention to inquiry and equity goals create a tension between, or support for, "teaching students" and "teaching mathematics" in their vision for work with K-12 teachers?* Data sources including participant biography, interview, and tasks informed these portraits. In contrast to Chapter 4, which focused on individual stories that were considerably more descriptive, this chapter approaches the research question from the examination of generalized patterns rather than individual experiences; thus, the findings are primarily interpretive.

The themes presented in this chapter take into account the tension and support between *teaching mathematics* and *teaching students, consisting of* three distinct categories: (1) learning and understanding of mathematics, (2) mathematics teaching, and (3) inquiry as a stance in one's professional life. These contexts highlight how issues of inquiry and equity emerged and how they were intertwined. The image displayed in Figure XX shows the three major categories and their accompanying themes. The various categories and their subthemes are related, but they are not causal. In particular, the categories displayed here are first most general as related to life and then most specific to learning mathematics. In what follows, I describe these themes in detail as they emerge from the various data sources. They are described, in detail, beginning with the theme most closely related to the MTE's professional life in the form of learning and understanding mathematics and ultimately, how they relate to how MTEs see their inquiry and equity stances as related to their larger professional lives.

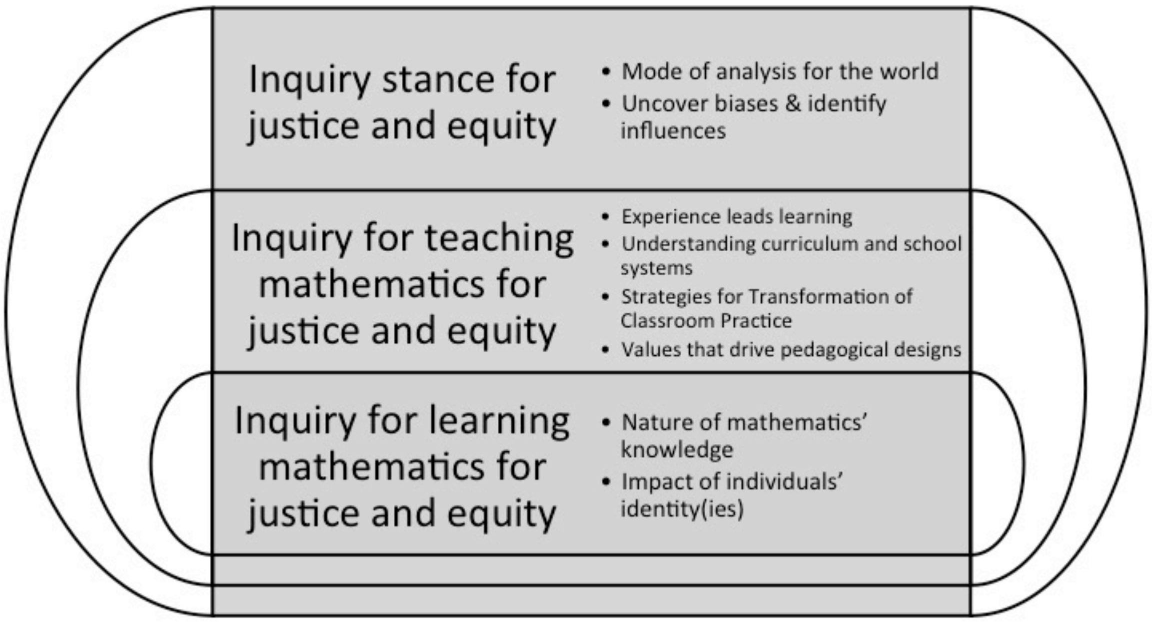

Figure 5.1. Categories and themes from across cases

In each of these categories, major themes provide ways in which particular methods of knowing and doing mathematics can improve fair outcomes in classrooms and where these practices might be in tension with equity goals.

Inquiry for Learning and Understanding Mathematics for Equity and Justice

I loved, loved the procedure, the algorithm, the memorization, the circling the right answer, that the whole thing, it was joy to me.

– Dr. Juan-sin-tierra

All four participants described an understanding of school mathematics from their own schooling that might be labeled as traditional, as outlined by the portraits in Chapter 4. In this image, mathematics was identified as a fixed set of information transmitted from teacher to students. Both Drs. Adams and Juan-sin-tierra described how much they enjoyed this experience of mathematics in their childhoods, as demonstrated by the opening quote to this section. However, in all four cases, this was not how the MTEs described what they wanted their students to understand about the practices of mathematics. In considering the learning experiences of the MTEs and their students, two major themes emerged: the nature of mathematics knowledge and the impact of a learner's many identities on their experience of mathematics.

Nature of Mathematics' Knowledge

Many studies have examined the epistemic underpinnings of what is commonly accepted as disciplinary mathematics (e.g., Cellucci, 2013; Ernest, 2016; Pais, 2011). And while a full review of these is outside the scope of this study, questions about the nature of mathematics emerged from this data as fundamentally related to issues of inquiry and justice in the learning and understanding of mathematics.

MTEs' mathematics school experiences were well aligned with both what Ernest (1989) calls the instrumentalist or Platonist view of mathematics. The Platonist view supposes that mathematics is a fixed product that continues to be discovered, not created (Ernest, 1989). The instrumentalist view, on the other hand, suggests that mathematics is a set of useful, but unrelated, skills and procedures (Ernest, 1989). Dr. Yoshida described his high school experience as "compartmentalized," and Dr. Juan-sin-tierra stated, "nothing was connected." In both of these cases, mathematics appeared to be a set of discrete procedures or skills, rather than a set of connected ideas or experiences.

The MTEs described the mathematical discipline quite differently from their experiences in schooling. While MTEs used different ways of describing mathematical phenomena, there was a clear expectation regarding underlying patterns and mathematical structures and the importance of understanding that mathematical ideas can and should be connected. Dr. Mahoney described the difference between his own school experience and this conception of mathematics as the difference between learning how to read a story and learning how to read individual words.

These understanding of mathematics represent two previously described views of mathematics: Platonist and problem solving (Ernest, 1989). This third view, problem-solving, envisions mathematics as an ever-expanding, creative discipline, up for constant revision (Ernest, 1989). These two have very different implications for learners and participants in mathematics, particularly related to equity and justice. From a Platonist standpoint, mathematics is a static body of knowledge that does not change as a result of new voices. However, from the problem-solving perspective, all mathematics is up for revision and as a result, new voices might fundamentally change the discipline. The participants were not explicit in their stances on whether mathematics was discovered or created; however, there were some suggestions that the field has some fundamental questions related to what mathematics is.

For several of the participating MTEs, it became apparent that leveraging existing mathematical ideas to understand the world might be a mode that supports mathematical inquiry as issues move toward a more just state. Thus, existing mathematics and associated mathematical structures could be used to change injustices in the world. In the following quote, Dr. Adams points to this way of using mathematics, but also acknowledges a tension at the elementary level.

> Elementary math better prepares you to be able to do that. You know you still have to come to learn the number system. You still need to have lots of

> opportunities to practice, lots of opportunities to combine quantities in all sorts of different ways. To learn about why that works in addition, but that doesn't work in subtraction. Or why it works in multiplication and addition but not in subtraction and division. You know, it's like you still have to learn all of that to be able to ask these questions. But can you ask these questions along the way?

In this passage, the tension of understanding the existing mathematics in order to ask and answer questions that will have an impact on the larger world is emphasized. And while Dr. Adams addresses its importance at the elementary level, a similar argument might be made at the secondary and even undergraduate level. This is a tension that learners of mathematics experience throughout their formal and informal schooling.

This tension to other questions about the very nature of the knowledge is shared with learners in formal settings. *Ethnomathematics,* as described by D'Ambrosio (2001) in the context of young children's learning, is understanding the cultural influences (ethno) on the experiences of mathematical pattern seeking and sense-making (mathematics). Coming to see the cultural influences on disciplinary mathematics was a major theme for Dr. Adams, who questioned the influence of white, European values on the knowledge that is considered the body of mathematics.

While none of the participants seemed to have fully resolved these questions of the nature of mathematics, they focused heavily on mathematics as a set of practices for understanding rather than a singular set of knowledge for memorization. In particular, the participants highlighted the underlying approaches to understanding mathematics as key for moving mathematical understanding forward. Dr. Mahoney outlined how traditional learning experiences of mathematics do not necessarily emphasize these practices.

> Lots of kids and adults […] think about math with procedures and algorithms that they remember- that they memorize and they can get answers. But, if they can't create a representation to show why that works or they can't build a model to show why that works, then there's a piece […] missing.

Participating MTEs all seemed to espouse this belief that knowledge of algorithms or mathematical skill without understanding the practices that developed these algorithms provide an incomplete picture of the discipline.

Given that the participating MTEs believed that practices in mathematics were crucial to learning and understanding mathematics, there were two prevalent approaches to practices of mathematics that inquiry introduced which were identified in this study: mathematizing the world and problem posing. Inquiry supports both of these practices and each can be seen as furthering the goals of equity. They will initially be discussed separately and then in combination.

In the ethnomathematics literature, *mathematizing* is the process by which individuals, in light of their own cultural experiences, develop tools and practices that support understandings of the phenomena through these tools and practices (Rosa & Orey, 2010). In this case, ethnomathematics is an appropriate lens because the participating MTEs called into question the knowledge that has traditionally been transmitted at schools as mathematics. Additionally, mathematizing is broadly construed to include both models of general practices that result from the act of making mathematical sense. Inquiry can "empower […] kids to start realizing that you can mathematize the world and ask questions that are mathematical and then use mathematics to answer those questions." That is, inquiry as a practice provides a starting place for learners to begin to understand what questions can and cannot be answered using disciplinary mathematics. Absent from this data was a consideration of how different cultural funds of mathematics might mathematize ideas in different ways. While approaches from different traditions would not necessarily be in tension with each other, mathematics in the western cannon has been given more status in many mathematical spaces.

Even now, the unsettled nature of mathematics creates several opportunities for learners of mathematics to move forward with equity goals in the context of inquiry by using mathematics as a lever to change the world and to change the nature of the discipline of mathematics to include more forms of mathematical thought. However, mathematical inquiry can have tensions with the increasing diversity of voices if mathematics is seen as a fixed set of knowledge that is waiting to be discovered by using previously accepted techniques. Also, even if mathematics is seen as created by humans, the decisions about whose voices are heard are subject to preexisting power and privilege structures in society.

Impact of Individuals' Identity(ies)

While learners' identities are constructed by many attributes including gender, race, ethnicity, disability status, and family SES-status, they are also constructed by experiences in schools and particular subjects. All of these different characteristics and experiences interact with each other in constructing the learner's identity. In this section, I will outline how the participating MTEs thought about identities related to the discipline of mathematics. In particular, MTEs described how narratives about mathematics from popular culture permeate the learning experiences of the discipline, how these narratives shape the stories of learners in mathematics, and how systems that support mathematics learners can or cannot be put into place.

Myths about mathematics and who can do it are pervasive in mathematics education and have real consequences for teachers and learners (e.g., Barlow & Reddish, 2006; Clements & Sarama, 2018; Phelps-Gregory, Frank, & Spitzer, 2020; Sheffield, 2017). In the current study, MTEs suggested that there is a requirement that “you have to be a genius” in order to be good at mathematics. Further, Dr. Yoshida stated, “And if you mention mathematics at all, they love to talk about how they were no good at it, right?” This points to a common outlook in popular culture that mathematics can only be mastered by the few individuals who possess innate mathematical skills and talent (Kogelman & Warren, 1978 as qtd in Barlow & Reddish, 2006; Clements & Sarama, 2018) and that many individuals are not capable of doing mathematics.

Mathematics myths not only reproduce the idea that there are individuals who are innately capable of mathematics, but they can reproduce dangerous racial and gender stereotypes about who can and cannot do mathematics. Sheffield (2017) highlighted the cultural account that white and Asian men are better at mathematics than other populations of students and how hazardous these assumptions can be for learners. The MTEs participating in the current study also recounted stories of learners who “thought they weren't mathematical because of their gender, because of their race, because of their language.” While these narratives are not the only reason that learners feel shut out of mathematics, they do point to a distinctive form of structural inequities that is pervasive in the discipline.

In addition, mathematics classrooms can reproduce forms of oppression that are prevalent in the rest of society (Ball et al., 2005). Dr. Adams described how in learning mathematics, there are "expectations [that] are socially constructed by people and humans. They're settled because in our society, [in the] United States, white, middle class ways of talking, speaking, acting are privileged." These expectations can be especially impactful regarding how learners experience mathematics in the presence of myths about who is capable of learning mathematics, especially, those that promote the idea that white and Asian individuals are the most capable of learning mathematics.

Participating MTEs suggested that an additional repercussion of these myths is that in mathematics departments, it is assumed that students can do mathematics when they arrive at the university. In describing this phenomenon in his undergraduate experience, one MTE stated, "the expectation [in my undergraduate program] was that you already kind of knew how to do math when you got there." This quotation stresses the prevalent belief that students should arrive in mathematics learning environments fully formed. Again, this echoes the assumption that individuals have an innate mathematical ability. In this case, Dr. Yoshida argued that learning mathematics through inquiry approaches counteracts these pervasive myths.

However, if inquiry is to successfully address some of these myths, then the approaches themselves need to be examined for what messages about identities they convey. Dr. Adams pointed to an example of how pervasive some of these ideas are in STEM disciplines. She noted,

> Megan Bang wrote a piece about Native Americans and how they don't categorize things as living and unliving, but that's a settled piece of knowledge or

> information because [it was produced from a] Cartesian, kind of European ways of thinking about classifying things.

In this way, scientists have made a decision based on white, European experiences to classify objects as living or non-living. These kinds of ideas are also embedded in practices of mathematical inquiry; thus, inquiry approaches also need to be questioned. In using inquiry as a learner of mathematics, it is important that “it's not about supporting Brown and Black children to have access to white mathematics or it's not about supporting those children to be successful [in which they] enact privileged ways [that are] typically determined by white, middle class folks.” Mathematical inquiry approaches cannot simply be about accessing existing ways of doing mathematics, rather they must bear revisiting and revising existing mathematical ideas.

Thus there are pervasive social myths about disciplinary mathematics that impact the development of learner identities in mathematics. Mathematical inquiry can support dispelling some of these myths because they support learners in acquiring practices of mathematics, rather than assuming learners can or cannot perform in mathematical spaces. However, if inquiry practices are to promote equity and work against these dominant narratives, then the set of inquiry practices need to be interrogated and revised to meet the needs of an increasingly diverse set of voices, rather than those, which have been deemed successful by dominant narratives.

Inquiry for Teaching Mathematics for Justice and Equity

We can talk about the connection between inquiry and equity, but I don't know if you can really feel its value until you go to an actual school…

– Dr. Yoshida

Dr. Juan-sin-tierra proposed a new kind of inquiry that teacher candidates needed to understand. They called it inquiry for mathematics teaching. This construct is a meta-understanding of the value of inquiry as a mode for increasing access to mathematics, developing individual identities in mathematics, and designing classrooms with both of these in mind. This section deals primarily with what it means to teach for inquiry and equity, rather than what it means to learn and understand mathematics. In what follows, I will first discuss an underlying assumption of inquiry approaches to teaching: experience leads learning. In the second section, I will discuss what MTEs reported teachers need to know in order to navigate school systems. Finally, I will discuss how MTEs supported their teacher candidates to reach these goals.

Experience Leads Learning

Each participating MTE expressed an assumption that an experience of mathematics should lead the inquiry experience. And further that experience in teaching inquiry mathematics should also be led by experience in mathematics inquiry. Dr. Mahoney extended his book metaphor to make this point, stating

> If we're going to have a book talk in a classroom and we never actually read the book, it's not going to be a really enriching book talk. You might ask really surface level questions and as kids respond, we're not even able to really ask good follow up questions because we're not familiar with the material.

Here he points to the idea that in order to teach reading, first the teacher reads the book, and then they plan for supporting a student experience of reading the book. In the case of mathematics, Dr. Mahoney argued, teachers and teacher candidates are rarely given the chance to "read the book of math." That is, while they may have learned mathematics, their experiences of inquiry are still limited. In recent research, teacher-centered pedagogies continue to be central to students learning experiences in mathematics (Kurniati & Surya, 2017). This suggests that even though more inquiry-focused mathematics approaches are being provided to teacher candidates, much of teacher candidates learning experiences were of a more traditional version.

This points to the second major reason for these inquiry experiences. In working with teacher candidates, an inquiry experience provided teacher candidates with a new "image" of what learning mathematics might be. Because inquiry is assumed to be a practice, rather than a set of skills, it is only by doing it that teacher candidates can understand it, and it might be quite different from their own schooling experiences. This image is different from what many teachers have had opportunities to complete in previous mathematics courses.

> Often teachers have rarely had opportunities to engage with mathematics in a way that wasn't trying to show that they know the procedure for a particular problem. It's not really about engaging with quantities or making sense of what's happening to those quantities or all the different ways that you might engage with those quantities, really understanding different properties of operations.

In this way, MTEs provide an experience that is different from more traditional mathematics settings, where teacher candidates may be exposed to a new way of thinking about mathematics.

Just as all of the participating MTEs reported their teaching was heavily influenced by their schooling experiences, an experience of inquiry can mean that teachers and teacher candidates "perception of learning mathematics changes." Literature reinforces this, suggesting that teachers' prior assumptions about mathematics teaching influence their classroom practices (Maass, 2011). Dr. Mahoney described how a similar experience that he had impacted him – once he understood the power of an inquiry experience for learners it was an essential part of his teaching practice.

> [O]nce you see that you can't unsee it. So then I couldn't go back and teach in any other way than through inquiry and problem-based learning and exploring and trying students' ideas. And, and so that's what brought me to where I am today.

This was echoed across the data sources from the participating MTEs; in order to understand the power of mathematical inquiry, an experience in the mathematics tasks was a necessary, although not sufficient condition, as is supported by literature (Swars, Smith, Smith, Carothers, & Myers, 2018).

However, an experience in inquiry not only fulfilled disciplinary goals, but could also serve as a new way to consider student reasoning. If teacher candidates have a chance to be sense-makers, they can come to believe their own students can also be sense-makers in mathematics. One MTE stated that if teachers and MTEs "can develop these math ideas ourselves, we can then also assume that kids can do those things, with the right opportunities." Thus experience in inquiry can provide teachers with another important lesson about mathematics and sense making, that mathematics skills can be taught through a series of activities, rather than as a set of skills. Additionally,

Finally, MTEs expressed that experience served as a device by which they model how inquiry can be taught in the contexts of mathematics or mathematics methods courses. Dr. Juan-sin-tierra conveyed this notion when they noted, "inquiry for teaching mathematics is slightly different because besides having the students engage in the inquiry process […] but also the teaching and learning of mathematics. […] It's sort of like a composition function. It's almost like an inquiry of the inquiry." Thus mathematics inquiry in the context of teacher education provides an additional opportunity for teacher candidates to see a model of how inquiry can provide a learning opportunity. MTEs noted that in their methods courses they were explicit about drawing attention to these practices. Swars and colleagues (2018) suggest that this can be impactful in the context of mathematics teacher education.

Briefly, participating MTEs expressed their assumption that experience leads learning from an inquiry stance. Experience can serve multiple purposes in the context of teacher education and teacher learning. First, disciplinary mathematics inquiry supports teachers in understanding mathematics as a cohesive set of practices. Second, it can provide an alternate image of what a mathematics learning experience might be and how a student might participate in it. Finally, it can serve as a model of how a classroom that employs these strategies can look.

Understanding School Systems and Mathematics Curriculum

Participating MTEs explained that from an inquiry standpoint, teacher candidates need a set of knowledge related not just to the processes of inquiry, but also one formed in relation to state standards and curricular expectations, as well as how systems of schooling exist and impact the classroom.

MTEs are well aware that their teacher candidates and the teachers with whom they work are under immense pressure to cover the state-targeted standards and benchmarks. And in fact, MTEs are concerned that these pressures can impact how novice teachers choose to implement inquiry-focused lessons in their first years of practice. Dr. Yoshida noted that teacher candidates as early as a classroom placement are, "so stressed with [because] they have to cover so much stuff for the- for what purpose? I don't know. Cause it's in the standards…" Thus the teacher candidates respond to external pressure by returning to more traditional approaches to mathematics teaching. Similar research has shown that in-service teachers choose to move away from more reform-oriented mathematics under pressure to follow a curriculum in a strict manner (Jong, 2016).

In order to combat this pressure, MTEs felt strongly that teacher candidates should have an understanding of how to evaluate provided curriculum when they enter their own classrooms. In particular MTEs in this case discussed how curricula could be adjusted to be used for mathematical inquiry. For example, Dr. Mahoney asserted that he facilitates "looking at the [curricular] materials […] to think about how you could take a textbook lesson and strip away things to actually increase the level of inquiry." In this way, teacher candidates can learn to work within the existing framework to provide inquiry opportunities.

While the participating MTEs suggested that mathematical inquiry in the form of problem posing, created opportunities for equitable opportunities for K-12 students to participate in mathematics classrooms, they noted these were hard to envision. Dr. Adams noted that, "when I look in elementary math classrooms, I don't see them choosing topics and don't see them posing problems. I see that as being very fixed by the curriculum and driven by the curriculum." Again, the curriculum frames the choices that teachers make, rather than being driven to support students to ask questions. Further, the experience of mathematics, even in the case of reform-focused curricula tend to focus on "children […] being encouraged to create different approaches to the problem and discovering mathematical ideas," but not posing their own problems.

Additionally, Dr. Adams highlighted that there are models for promoting student problem solving at the secondary level, but there appears to be a dearth of examples for the elementary level. She wondered what an elementary student would do if they were asked to pose questions in the context of a task on peas and carrots. "[W]here would a six year old go from there? What do they want to know next? Do they want to add a different vegetable in there? Do they think that that's like such a boring task that like they don't care about peas and carrots?" In this way, Dr. Adams was posing an inquiry on the mathematical inquiry as is Dr. Juan-sin-tierra had suggested. However, there still was not a clear way forward for supporting teachers and candidate teachers to perform these problem-posing activities in their classrooms. Further, how to account for issues of classroom equity in the doing of problem posing were not clear. However, other MTEs suggested that while there were challenges to problem posing in classrooms, modes for supporting teachers to include this practice in their own classrooms do exist. They will be discussed in detail in the upcoming pedagogical section.

In sum, participating MTEs proposed that in order to take an inquiry stance to mathematics teaching, students needed to understand the curricular and systematic demands on their work as mathematics teachers. Further, teacher candidates required support to meet both of these demands and approaches to mathematical inquiry. In particular, problem posing was put forward as an approach that could support both inquiry and equity goals; however, there is a dearth of approaches to doing this work.

Additionally, the act of problem posing can be in tension with the needs to meet particular standards in a given year.

Strategies for Transformation of Classroom Practice

Multiple sources of data suggested MTEs suggested that considerations for mathematical inquiry represented a shift in many current teaching practices. For example, Dr. Adams described it this way: “I think my age group has been socialized to think that math teaching only looks a very particular way, right? And so we're trying to disrupt an image of that, to say it could look this other way.” That is, approaching mathematics teaching through reasoning feels like a significant departure from what has previously been identified as mathematics teaching.

Further, MTEs suggested that while issues of equity have been prevalent in the mathematics education field and for some in their own work for many years, there seems to be something important about this particular moment in history. However, both teachers and MTEs identified that there is an ongoing change was important in equitable mathematics spaces. Dr. Mahoney identified one of the challenges with supporting change for individuals in trying to understand equitable mathematics spaces like this:

> I think sometimes race and equity and access and these conversations can be uncomfortable, so people avoid them. And when we do, we don't grow. And so I'm trying to be in these spaces a lot more and I'm trying to make other people aware of the importance of going into these spaces as well to have these kinds of conversations because we are only going to get better as a result of it.

In the current study, the majority of participating MTEs identified that they and the teachers with whom they work needed to continue to grow and understand how issues of mathematical inquiry and equitable mathematics spaces interact. And further, the reality that some of this might be uncomfortable might present an additional barrier.

This work is challenging though, in part because it does not always have a straightforward answer. Dr. Adams outlined this by stating that social inequities are apparent in classrooms: “but how is it showing up and how are ways to disrupt that? How do I think about that differently? […] I'm developing more understandings of what it looks like in classrooms, to center power.” So just as MTEs are continuing to develop their own ideas of how to ensure equity in mathematics spaces, there are still questions about how to address these issues. And in particular, Dr. Adams went on to say that “It's not easy and it's not, often not like a clear answer.” So MTEs face a challenge, to support the teachers they work with to understand their own spaces.

If then, as the data from this study and others suggest (e.g., Hiebert, 2013), some ongoing change to typical classroom practices needs to be implemented, how do MTEs go about supporting this kind of change? As stressed in the previous section, all MTEs suggested that experience leads learning. Thus providing an experience that embraces inquiry and equity might be a path forward; however, according to participating MTEs, an experience alone cannot induce change. Dr. Mahoney suggested that based on his reading of *Switch* (Heath & Heath, 2010), “if you want people to change, you can't just make them think something. You have to make them feel something.” That is, an experience that evokes a feeling might be an impetus to change. However, implementing and sustaining change requires more than an impetus.

While none of the MTEs had a failsafe approach to supporting this change in their approach to classroom practice, there were some factors that seemed to be vital in the

MTEs experience. The first was confidence in their ability to facilitate classrooms that focused both on inquiry and promoted equitable classroom spaces. The second was inquiring about and adjusting orientations towards teaching. Dr. Adams recounted that many of the mathematics teachings practices that have been associated with inquiry-learning and student sense-making are practices that should continue to be taught; however, they might be implemented in different ways in light of understandings about structural inequities that exist in US classrooms: "But like the orientations that the teachers have towards mathematics, towards learners, towards teaching towards lots of other things. When you have a different orientation to teaching, you're going to use those core practices and those talk moves in a different way." Thus orientation toward inquiry and equity can work together to promote equitable opportunities to question mathematics, but also be in tension if aspects of equity are not attended to when implementing these teaching practices.

Finally, while both confidence and orientations toward teaching, among others, can spur change, from this study and others, it is apparent that change is "fragile." That is, the changes that are implemented are not necessarily sustained. It was not apparent to the MTEs why these changes were not necessarily sustainable and could include a shift in content taught, an issue in a teacher's personal life, or a shift in the school culture. Dr. Adams suggested that a teacher might feel as if "I've got to know all 'the things' to be able to do this. And if you take one of 'the things' away, I might not know how to do this." As a result, changes to practice are not sustained in the classroom. Research suggests that taking an inquiry-stance on ones own teaching creates a self-sustaining change (Farmer, Gerretson, & Lassak, 2003). In the current study, taking an inquiry stance toward ones professional life is identified as a third category and is considered in the final major section of this chapter.

In this section, the evidence from this study provides several important assumptions about changing teacher practices as related to inquiry and equity. First, MTEs expressed the idea that some change is needed in many mathematics classrooms with respect to how learning is experienced. Second, if those changes are to be undertaken, they need to be led by an experience of mathematics that produces a new feeling about how mathematics can be learned and what impacts learners. Additionally, MTEs need to know that they are impacted by confidence in the material and orientations toward teaching and learning. In particular, understanding pedagogies of teaching for mathematical inquiry in the absence of recognition of issues of equity does not create a change that promotes equitable outcomes for students. Finally, any changes that do occur are fragile and can be upset in the absence of additional protective factors.

Values that Drive Pedagogical Designs

The MTEs that participated in this study reflected three major pedagogical values in implementing inquiry for equitable mathematics spaces. These goals were creating space for teacher candidates within mathematics spaces and beyond, creating opportunities for teacher candidates to act as sense-makers and reflect on those experiences, and for teacher candidates to understand how privilege and power interact in the K-12 classrooms that they might eventually enter. Their classroom and task designs resulted from these foundational values.

Each MTE described a slightly different way that they show that they value creating space for their teacher candidates mathematical thinking and ideas. Further, they want their teacher candidates to bring their full human experience to the classroom. In particular, MTEs described how they are addressing the needs of their teacher candidate–students, by learning about their stories, interrogating any assumptions they have about students, and sending messages that mathematics is a space where many individuals can be successful, not just an elite few. In the literature, supporting teachers and teacher candidates to leverage existing curriculum spaces to include their students' multiple mathematics knowledge bases has been identified as an equity-practice (Drake et al., 2015). In general, MTEs were referring to leveraging their classrooms to make space for teacher candidates – MTEs students – and this value is an equity-promoting one, which might be implicitly transmitted to students.

The second value that MTEs seemed to share was one of mathematics as a sense-making activity. Each of the participants described slightly different understandings of what sense making might mean in the context of their own courses. They all shared the idea that teachers and teacher candidates needed opportunities to make sense of mathematics on their own, as in experience leads learning. And that they needed to be supported in their sense making. For example, Dr. Yoshida wanted to provide sense making opportunities that were in the teacher and teacher candidates' zones of proximal development (Vygotsky, 1978), as compared to activities that might prove frustrating because they were not yet ready for them. Because future mathematics teachers, especially at the elementary level, are often re-learning the mathematics in a new way, Castro Superfine and colleagues (2020) have argued that sense-making can support this process.

Two of the participating MTEs described how they are explicitly centering power and privilege in their classroom activities. Dr. Adams described how her work with students might be considered "learning to kayak on still waters," echoing the ideas of Grossman and colleagues (2009). In particular, Dr. Adams described how teacher candidates in her mathematics methods were exposed and participated in several activities over the course of the semester that supported them to recognize issues of race, power, and privilege in the classroom. Toward the end of the semester, teacher candidates began to consider questions like, "How do we know when a student, when race is coming into play, where are there times when it's not being disrespectful or inappropriate? How do you know when appropriate to react or not? What cues guide your response?" Dr. Adams noted that this was an overhaul of her previous course designs. As the course went on, Dr. Adams emphasized her teacher candidates were working in different ways to make sense of their own elementary students thinking.

> It's not that I haven't had individuals do that in classes before, but they wanted kids' names up next strategies, they wanted like kids offering ideas that was really big and important to them. And the other thing is they let mistakes kind of live for a while. Not like, "Oh I gotta fix that." They're like they kind of let it sit there and then they kind of came back to it. So this idea of like getting the right answer or like this very privileged, one way of doing it [was not present].

In this way, Dr. Adams students were using what they had learned in terms of sense making, but also understandings of the racial and historical positioning of children in mathematics spaces to display values for multiple ways of thinking together mathematically.

Dr. Mahoney described a different course design, by which he leveraged his own experience to support his teacher and teacher-candidate students to understand how privilege and power emerge in mathematics education. In particular, he tells his own story of designing a course based on conference attendance and then has his students reflect critically on that.

> I show all the different speakers and people get really excited about it. And then once I have all I pop up like the different, like, and they said, we- the first time we did effective practices and this is about designing it and I pop-up, Dan Meyer and Robert Kaplinsky and all the- and people are like, oh! look, look at this star studded cast and then when they have all the faces up there. I say to the students, "take a moment and say, what do you notice? What do you wonder?"

Thus, Dr. Mahoney provides the teachers and teacher candidates he works with, with an experience similar to his own. While this will have a different impact on different individuals based on their own many identities, it provides an image of how Dr. Mahoney values identifying and naming power and privilege in mathematics education spaces.

The MTEs participating in the current study shared a set of common values that they hoped to instill in the teachers and teacher candidates with whom they worked. These values would support both the goals of mathematical inquiry and equity in mathematics spaces. The values were named here as making space in mathematics spaces, supporting sense making in mathematics spaces, and naming how power and privilege have worked in these spaces. As described, these values do not suggest a tension between mathematical inquiry and equity in mathematics spaces, but again they do suggest that without consideration of "what counts" mathematically, there can be a fundamental mismatch between these goals.

Taking Inquiry as Stance for Equity and Justice

Much of our society has been constructed by powerful white men and, and the only way these systems get broken down […] is when we as individuals in the society ask questions and push back on assumptions and presumptions and, and biases and stereotypes and explicit and implicit racism and sexism.

– Dr. Mahoney

Three of the four participating MTEs were explicit in describing that their approach to the world using their understandings of mathematical inquiry practices led to how they came to understand the world and how issues of equity and inequity are experienced. All four were implicit in this description; each took on a stance of using questioning to understand and from this position to make decision. Further this stance supported MTEs to undercover their own biases. And moreover, these stances supported them to understand their own influences in the profession and beyond.

Mode of Analysis for the World

The participating MTEs outlined how they used their inquiry practices not just in their understanding of mathematics, but also in how they approach coming to understand the world. Dr. Juan-sin-tierra described this phenomenon: "I'm constantly, constantly asking what do we mean by phrases, labels, constructs, definitions, um, that we assume are of common knowledge and just like mathematical inquiry." Here they are stating that in their work as a mathematics educator both in their classroom and as part of their extended duties, they use the same practices they use in undertaking mathematical inquiry for themselves to understand and analyze the world. Cochran-Smith and Lytle (2009) describe *inquiry as stance*, a phrase I borrow for this section of the thematic results. I chose to apply this term in particular because the approach is classified "a worldview and a habit of mind" (Cochran-Smith & Lytle, 2009, p. viii). While the majority of work has been in the context of practicing K-12 teachers' classrooms, this phrase seems relevant here, in the broader context of an MTE's professional life. In this case, taking *inquiry as stance* can serve many of the same purposes as it served for in-service K-12 teachers. Specifically, *inquiry as stance* supports those who practice it to problematize current social and school systems, interrogate the origins of knowledge and assumptions, and can lead to making changes in those problematic issues that are uncovered (Cochran-Smith & Lytle, 1999). One significant difference between the Cochran-Smith and Lytle conception, is that in this case MTEs did not necessarily describe taking on this inquiry position in the context of a community; however, they were not

In addition to taking this stance, Dr. Mahoney suggested that as a matter of pedagogy, he hoped that teachers and teacher candidates might be able to take on this stance as well. He described how an inquiry stance might support the understanding of the outcomes of inequitable systems in US society.

> And I want you to take those same [mathematics inquiry] behaviors and I want you to look at our world with that same lens. You question why does the democratic primary candidates [consist of so many] white [candidates]. And why is it that we have two Rhodes scholars running for the democratic nomination, but

> only the white one in the media gets named as a Rhodes scholar and the African American one doesn't.

While this example is specific to the current time and place, Dr. Mahoney's point stands. In taking an inquiry stance to any public or civic events, as well as more local or school-based events, individuals can begin to uncover how systems around them function. And beyond this individuals can begin to formulate responses to these identified inequities.

Briefly, the participating MTEs suggested that by taking *inquiry as stance* (e.g., Cochran-Smith & Lytle, 1999, 2009), they can operate in a way that supports their learning about systems that exist in greater society. This provides a basis for the practices that can be used to come to understand how inequities are perpetrated at the international, national, local, and school- or university-level. This stance could further be developed with teachers and teacher-candidates for the benefits of supporting K-12 students.

Uncover Biases and Identify Influences

MTEs suggested that taking inquiry as stance provided them with two major outcomes. First, in taking *inquiry as stance*, MTEs used inquiry to uncover their own and others' assumptions and biases. Second, taking this standpoint supported MTEs to identify and diversify the influences in their professional and personal spheres.

MTEs described how they uncovered biases and assumptions in professional conversations. Principally, MTEs described that they could use inquiry to both uncover their own and others' biases and assumptions in conversation. In the following quotation, Dr. Juan-sin-tierra described how they used inquiry to "challenge" herself as well as their colleagues.

> I'm always trying to challenge myself and challenge others, but more myself and my understanding of the structures and the things that we work on. There's way too many assumptions because we're experts and all kind of understand what we mean.

In this quotation, they are referring to social and institutional structures, rather than mathematics structures. In this passage, Dr. Juan-sin-tierra also identifies that assumptions need to be uncovered despite being in the role of MTE who are experts in their respective fields.

As a result of unpacking one's biases, MTEs also identified who their personal and professional influences are. In Chapter 4, Dr. Mahoney described how he had this experience through his design of his course around his work at the national conference. He stated, "I've had my echo chamber and I've now broadened [who I interact with on social media]. And so there's different chats #Cleartheair is one that I follow and it's conversations and perspectives that I had not had before." Thus, uncovering assumptions had the impact of MTEs identifying who their personal and professional influences were.

In summary, MTEs reported two outcomes from taking inquiry as stance as a mode of analysis. MTEs identified uncovering their own assumptions and biases, as well as those of individuals with whom they interact. In addition, MTEs described how uncovering these biases led to understanding more about who influences their professional and personal experiences. It is notable that participating MTEs described these two outcomes, but this was not an explicit question that was addressed during data collection. Thus, there are probably other outcomes from taking this stance. Just as in the previous section, inquiry provided support for equity goals. However, MTEs did not provide insight about the tensions that exist.

Summary and Looking Forward

Three categories presented in this chapter take into account the tension and support between *teaching mathematics* and *teaching students,* consisting of three distinct categories: (1) learning and understanding of mathematics, (2) mathematics teaching, and (3) inquiry as a stance in one's professional life. In the first category, MTEs described how the nature of mathematical knowledge is important for mathematical inquiry and whether it can be considered supportive or in tension with equitable spaces in mathematics. In addition, MTEs identified that some of the pervasive social myths about what it means to do mathematics can impact learners' identity-development.

In the category of mathematics teaching, MTEs reported that they wanted teachers and teacher candidates to understand how experience can lead learning of mathematics and inquiry-focused instruction. Second, MTEs suggested that for teachers and teacher candidates to take an inquiry stance to mathematics, an understanding of the curricular and systematic demands of the work of teaching is necessary. Finally, in the implementation of a mathematical inquiry-focused teaching, the MTEs articulated three interrelated values: making space in mathematics spaces, supporting sense-making in mathematics spaces, and naming how power and privilege have and continue to work in these spaces.

Finally, in the category of taking an inquiry-stance to one's professional life, MTEs described two connected ideas. First, MTEs described how the inquiry practices can be a mode for understanding the world that can be used to promote equity. And second, MTEs identified how uncovering these biases and assumptions can be used to identify who MTEs include in their spheres of influence.

In all four cases, the participants highlighted that they are still learning, both about issues of inquiry and issues of equity. For example, Dr. Mahoney said, "the learning of it never ends. I'm still learning about inquiry. I'm still learning about equity. And, I will 'til I'm no longer here." His feelings were echoed in different ways by each of the participants; however, each identified ongoing growth as a key aspect of how they looked at their own work.

CHAPTER 6

DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

Introduction

The previous two chapters detailed the major findings from this study, including descriptive instances from the data *corpus*. Chapter 4 provided short portraits of the four participating MTEs and addressed the first research question of this study. Chapter 5 provided categories and compared the themes that emerged from analysis of data generated. This chapter will focus on a discussion of the major findings and their implications for mathematics teacher education, mathematics teacher professional development providers, and mathematics departments that prepare future teachers. First, I will provide a summary of the study and a review of the major findings. I will then present a discussion previously described, and end with the limitations of the study and recommendations for future research.

Summary of the Study

Leaders in the mathematics education field have suggested that in mathematics education, issues of equity are imperative for progress in the mathematics discipline and for K-12 students (e.g., Gutiérrez, 2013a, 2013b; Martin, 2009b; Nasir, 2016), and also approaches to mathematics teaching should focus on the supporting students in practices of *doing* mathematics (e.g., Cuoco et al., 1996; Cuoco, Goldenberg, & Mark, 2011; Ernest, 2016; Schoenfeld, 2016a). In the context of mathematics teacher education, this requires a complex set of skills, goals, and agendas in the practice of their work to support teacher candidates and teachers' professional growth. However, little research exists on how MTEs undertake their multifaceted work (L. Brown et al., 2018).

In order to understand the sometimes competing, sometimes supporting schemes of equity and inquiry, I conducted the current study. Additionally, this study was undertaken in the context of MTEs in order to partially rectify the dearth of research in the space of mathematics teacher educators. Thus, the purpose of the study was to describe and understand mathematics teacher educators' professional visions for mathematical inquiry in equitable mathematics spaces. And further, to describe how and in what contexts they enact the identified learning goals.

In order to address this purpose, I completed a case study that examined two major research questions:

1. How do MTEs choose, operationalize, and enact equity and inquiry goals in their professional visions for their work with K-12 teachers?
2. In what ways does the attention to inquiry and equity goals create a tension between or support for "teaching students" and "teaching mathematics" in their vision for work with K-12 teachers?

Before doing this work, I identified the following three core assumptions that were fundamental to how I made sense of the work of data generation and analysis activities. The first is that mathematics is a human and creative pursuit (Ernest, 1989); however, students are often presented complete results in a static body of knowledge (e.g., Ernest, 2016). Thus, students do not always have the opportunity to develop images of what it means to do mathematics. Second, mathematics, in particular, has been directed by institutional systems, such as tracking and rote teaching (Berry, Ellis, & Hughes, 2014), which has resulted in restrictions at the highest level of disciplinary mathematics. Finally, in mathematics classrooms, issues of inquiry and equity must be treated in tandem if students are to benefit from them.

Four mathematics teacher educators participated in the case study research. Some important characteristics of the participants are outlined in Table XX, including gender, racial identity, departmental appointment and so on. Each participant contributed a professional artifact, a short professional biography, and an approximately hour-long interview. Interviews were analyzed using a series of iterative inductive methods to develop codes. Developed codes were applied and iteratively revised with the remaining data sources following code development. The next section presents a discussion of the major findings of this study.

Discussion of Major Findings

In this section, the findings from the two major research questions are reviewed and connected to existing literature, and I then provide some additional conclusions. Major findings were reported for the first research question in Chapter 4 in the form of portraiture for each participating MTE. Then, the experiences of MTEs were compared to elicit themes that were described in Chapter 5. The findings both echoed some of the assumptions outlined early in the study and the associated research about issues of mathematical inquiry and equity in mathematics spaces. Further, some of the findings were surprising to the author and build upon the research related to mathematics teacher educators and how they think about the two constructs of study. A thorough discussion of these findings follows.

Results from the Portraits of Participating MTEs

The following section addresses the multiple parts of the first research question. First, I will address what drove the MTEs' choices in identifying goals in the context of mathematical inquiry and equity in mathematics spaces. Second, I will provide a summary of the major findings using the Knowledge for Mathematics Teaching Framework and their operationalization of the teaching and learning goals. Finally, I will provide a summary of how MTEs decided to enact these goals.

Each of the MTEs expressed values that promoted both mathematical inquiry and equity in mathematics spaces. In all four cases, the participants described that mathematical inquiry was something different from their own school mathematics knowledge. Further, each of the participants described a different kind of experience that developed their value of mathematical inquiry. Just as all four participants identified values in mathematical inquiry, they also highlighted that mathematics spaces should be more equitable for learners. However, in this case, the participants did not pinpoint a particular activity in their own life stories that supported this value.

These findings are supported by existing research about the work of MTEs. In a study that focused on mathematics content courses for teachers, MTEs choose goals that aligned with learner-centered, sense-making approaches to understanding mathematics (Li & Castro Superfine, 2018). Further learning goals that address issues of (in)equity in mathematics spaces are becoming more and more prominent in the mathematics teacher education (Felton-Koestler, 2020). Jackson and colleagues (2020) describe values as, "judgements [sic], based on research and experience, about what is important when teaching courses for prospective teachers" (p. 553). Further, they suggest that values can be used to drive and identify goals (Jackson et al., 2020). In the case of this study, these values seem to be essential to engaging in mathematical inquiry in equitable mathematics spaces. However, how the MTEs choose to operationalize and enact these goals was notably different from each other.

While the MTEs identified how mathematical inquiry and equity in mathematics spaces might be in tension with each other, they did not describe how the various goals for teacher candidates and teachers might fit together across their course work. Smith and Bretscher (2018) argue that the operationalized goals, or pedagogic messages, that novice teachers receive in their mathematics courses provide a “pivot” between deepening mathematical knowledge and reflecting on instruction. If these operationalized goals are provided consistently, they could support the development of the targeted mathematical knowledge for teaching.

The enactment of these goals was described differently for the participating MTEs. Because of the descriptive nature of this study, it is difficult to say if participating MTEs chose to enact their operationalized goals differently due to personal and institutional factors or the teaching context described. I will summarize and offer a commentary for each of the major enactment approaches that participants described.

Dr. Adams described a focus on the category of Knowledge about Mathematics and Society in her conversation. In particular, she described a series of activities that supported novice teachers in her elementary mathematics methods courses to unpack cultural aspects of mathematics, the experiences of students in mathematics classrooms, and their roles as change agents. Felton-Koestler (2020) described an aligned strand of work, but noted that he is beginning a process, which may not result in immediate impacts on teacher candidates' work. However, Dr. Adams did see a change that emerged in her novice teachers' responses to students from previous iterations of the course, suggesting that the enactment may have some immediate impact on classroom practice; however, a revision of curriculum, as Felton-Koestler describes, is beyond the scope of Dr. Adams observations.

In their work, Dr. Juan-sin-tierra emphasized that the goals for their students centered the doing of mathematics. Dr. Juan-sin-tierra described two classroom practices that pushed their goals forward. First, they described a practice of "reverse-engineering," where they used concrete examples to deduce and understand a general mathematical rule. Second, Dr. Juan-sin-tierra described how they provided students with tasks that can be addressed using multiple methods and with different entry points. Dr. Juan-sin-tierra pointed out that these tasks might not appear to be particularly salient, but learner responses uncovered the interesting aspects of the mathematics.

Dr. Mahoney described two separate enactments of his goals. The first was related to the task he provided. The task itself was designed with novice teachers and teachers in mind, despite the fact it could be used in classrooms. The task attended to similar design principles as those described by Dr. Juan-sin-tierra; it had multiple points of entry and many methods for reasoning through the solutions. In addition, the task itself required minimal language use so that learners with different home languages could participate. Additionally, the task was action-focused to provide learners with opportunities to identify their own mathematical questions. This approach to task design echoes some of the major aspects of problem posing and creativity described by Silver (1997). Dr. Mahoney's second enacted goal was described in detail in his portrait; but in summary, it provides learners with the opportunity to uncover biases in who is promoted as an authority in mathematics education. Just as Felton-Koestler (2020) defined his work in methods courses as a first step, Dr. Mahoney's second enacted goal can help develop a sociocultural consciousness for teachers from identity groups who have not been marginalized, for example, cis-gender, neurotypical, non-disabled, white, and so on (Villegas, Ciotoli, & Lucas, 2017)., and so on) to develop a sociocultural consciousness (Villegas et al., 2017). As a result, this activity might be a first step toward a deeper understanding of the role of mathematics in the world.

Dr. Yoshida's goals were similar to those of Dr. Juan-sin-tierra. He enacted a workshop model in his classroom. Research suggests that teacher educators do not always provide *congruent* teaching experiences for future teachers, where teacher educators model classroom practice, explain their classroom practices, and link those classroom practices to relevant theory (Villegas et al., 2017). Dr. Yoshida demonstrates a model of inquiry-focused teaching in his content courses and is explicit in describing the value of mathematical inquiry.

In combination, these findings reinforce that values are important drivers for MTEs in identifying and selecting goals for their professional lives. Further, in operationalizing these goals, as no single MTE was able to target the entire framework outlined in Chapter 2, further research about how different aspects of teacher education can operate in unison to achieve the goals for future teachers. Finally, the MTEs' enactments of their goals provide a range of approaches that MTEs are taking. These may serve as examples for the field.

Results from the Comparison of Themes

In Chapter 5, I presented three major categories in which the tension and support between *teaching mathematics* and *teaching students* was experienced by participating MTEs. In this section, I revisit three categories: learning and understanding of mathematics, mathematics teaching, and inquiry as a stance in one's professional life. Then, I connect them to existing mathematics education and teacher education literature.

In the category of learning and understanding mathematics, the nature of mathematical knowledge was recognized as essential for mathematical inquiry. Furthermore, the description of the nature of mathematics can determine if it is in support of or in tension with equitable spaces in mathematics. MTEs also identified how mathematical knowledge is experienced promotes pervasive social myths about who can do mathematics, which impact learners' identity development. While this aspect was addressed more extensively in Chapter Four4, the various possible understandings of the nature of mathematics can lead to cultural myths that enshrine marginalizing beliefs. For example, Sheffield (2017) described the cultural account that that white and Asian men are better at mathematics than other racial or ethnic student groups and how hazardous these assumptions are for many learners, particularly those who are excluded by this account. If it is believed that only certain groups of people are capable of doing mathematics, and that this groups of people have particular racial and gender identities, than others can be excluded. On the other hand, if mathematics is seen as a discipline where many individuals can creatively contribute, than the reasons for exclusion needs to be challenged.

Second, for the category of teaching mathematics, MTEs endorsed the idea that experience leads learning, in mathematics, but also in other learning spaces. Further, MTEs suggested that in order for teachers and teacher candidates to take an inquiry stance to mathematics teaching, they needed to understand the system and curricular demands of their work. Research suggests teacher education does not always connect theoretical learning from preparation programs to the practices of teaching (Grossman et al., 2009). Further, mathematics teacher education has suggested that mathematics teachers need political knowledge (Felton-Koestler & Koestler, 2017; Gutiérrez, 2013b). Thus, research suggests that an experience of inquiry or considerations of equity in mathematics spaces is not enough to drive a practice-based understanding of inquiry teaching or how to approach the political teaching role. However, the MTEs in this study did not suggest that experience was enough; rather, they proposed it as a first step in understanding teaching.

In the implementation of a mathematical inquiry-focused teaching, the MTEs articulated three interrelated values: making space where mathematics is being learned, supporting sense-making in mathematics spaces, and naming how power and privilege have been maintained– and continue – to work in mathematics spaces. As examined in the previous section, values can be used to drive the development of goals for the classroom (Jackson et al., 2020). These particular goals are related to creating equitable learning spaces in mathematics. Researchers have proposed the idea of curriculum spaces where teachers can create opportunities for their students to draw on their multiple mathematics knowledge bases (Felton-Koestler & Koestler, 2017; Gutiérrez, 2013b). While this research focuses on how elementary teachers can open spaces for their students, the MTEs expressed creating a space for learners to draw on their own funds of knowledge as a pedagogical principle for inquiry for teaching. Sense-making, as a value for students in mathematics courses, is promoted by the Standards for Mathematics Practices (NGA & CCSO, 2010) and NCTM's (2014), *Principles to Action.* However, research suggests that a focus on sense-making must be supported by attending to differences between teachers' and students' race, culture, and language (Warren & Rosebery, 2011). Particularly, a teacher must be prepared to open space and respond to sense making in ways that attend to how power and privilege play out in mathematics classrooms (e.g., Drake et al., 2015; Land et al., 2019). While this research focuses on how elementary teachers can open space for the children they teach, the MTEs expressed creating space for learners to draw on their own funds of knowledge as a pedagogical principle for inquiry for teaching. Sense making as a value for students in mathematics courses is promoted by the Standards for Mathematics Practices (NGA & CCSO, 2010) and NCTM's (2014) *Principles to Action.* However, research suggests that a focused sense-making must be supported by attending to differences between teachers' and students' races, cultures, and languages This leads to the final value that was described in this study: attending to the manifestation of power and privilege exist in mathematics classrooms and school spaces. Parker, Bartell, and Novak (2017) proposed two major aspects of culturally responsive ways of knowing: (a) cultural awareness and (b) cultural responsiveness. Cultural awareness is classified as those perspectives where individuals recognize the role of culture, power, and privilege in schools and the discipline (Parker et al., 2017). Culturally responsiveness can be defined as dispositions grounded in cultural awareness that lead to teachers to using that part of their work is to come to understand their students’ backgrounds, and to use these understandings to support students’ disciplinary and cultural competence learning (Parker et al., 2017). The MTEs in the current study primarily referred to supporting the development of cultural awareness with respect to power and privilege in classrooms. However, participating MTEs might anticipate that this is a first step, along a longer professional development trajectory.

In the final thematic category, participants took an inquiry-stance to their professional lives. First, MTEs described how the inquiry practices can be a mode for understanding the world and that can be used to promote equity. Second, MTEs identified that using this inquiry stance can uncover biases and assumptions, and; further, it can be used to identify who are the major influences are in MTEs’ professional lives. Cochran-Smith (2003) has suggested that a formalized process of taking *inquiry as stance* can support the professional learning of teacher educators, as described here. In particular, taking inquiry as stance as a teacher educator requires that the teacher educator goes through processes of learning and unlearning (Cochran-Smith, 2003). While the learning could be characterized in many ways, Cochran-Smith suggests, in one sense, a teacher educator is learning how to be a change agent and to support the development of change agents. On the other hand, teacher educators could leverage their inquiry stance to unlearn or probe their own assumptions about race, culture, disability, and so on. This process of unlearning was described as the process of uncovering biases and identifying influences.

These categories and associated themes suggest that the participating MTEs envision their professional work as cohesive across their understandings of mathematics, teaching mathematics, and approaches to the professional life. Further, they suggest that mathematical inquiry comes into conflict with equity goals when equity considerations are not taken into account. Explicit attention to equity is required if social issues of marginalization are going to be addressed, in any mode of teaching, but particularly in those that promote idea exchange.

Conclusions and Implications

This study suggests implications for multiple individuals in the mathematics education field, particularly anyone who supports teacher education in any capacity. In this section, I will provide some conclusions and implications for those who provide pre-service teacher education and in-service professional learning.

For mathematics teacher educators who teach mathematics content courses for teacher candidates, this study suggests that attending to inquiry can provide students either with a different understanding of the nature of mathematics or a different way to make sense of mathematics, rather than relying on an external authority to do so. To enact an inquiry agenda, MTEs should consider how to provide an experience of mathematics before providing the connections or doing the sense making for the learners. However, teaching from an inquiry-stance, in the absence of attending to issues of equity that emerge in mathematics learning spaces, will not serve learners, especially learners who might be institutionally marginalized due to their racial, gender, (dis)ability, or bilingual identities. Further, these learning experiences can support teacher candidates or teachers to dismantle assumptions about the nature of mathematics that can lead to assumptions about who can and cannot do mathematics.

This study suggests that a major driver of how individuals teach mathematics can be their own experiences in mathematics courses, particularly those as learners. Research supports this phenomenon (Lortie, 1975). The implications for mathematics teacher educators who teach mathematics methods courses as well as individuals who provide professional learning for in-service teachers is that many mathematics teacher candidates and teachers need an image that portrays what it means to teach mathematics for inquiry. Thus, MTEs need to provide them with experiences that facilitate their development of alternative images. Further, in providing an alternative image, teachers and teacher candidates need support to identify concrete responses to the inequities that might emerge from teaching using inquiry approaches.

Notably in this study, participants focused on how equity needed to be addressed in the context of mathematical inquiry. Each of the participants made clear that they had paid explicit attention to mathematical inquiry in their classes for at least a decade. And while all of the participating MTEs expressed that each of them had a long-standing dedication to equity in mathematics spaces, three of them described how they had come to their current understandings more recently. Further, all four described that they are making how they understand equity more explicit for their learners. Thus, just as teachers must be provided with long-term supports in attending to both equity and inquiry, in the training of MTEs opportunities to attend to both are essential. And these opportunities must support change with long-term supports.

Finally, this study identifies that skills in mathematical inquiry might be leveraged to support teachers and teacher candidates to probe existing institutional and social structures for inequities. Therefore, for any individual who supports the professional learning of teacher candidates or teachers, considerations for how mathematical inquiry can support the interrogation of social structures should be shared. Specifically, teacher candidates and teachers need assistance to consider how they might use their knowledge about mathematical inquiry to understand more about the world. In that way, teachers and teacher candidates can develop their own *inquiry as stance* (e.g., Cochran-Smith & Lytle, 1999, 2009) to the world.

Limitations

The primary limitations posed by the research design were addressed in Chapter 3 of this study. Here I will attempt to address the limitations that became obvious as the study was undertaken and concluded. These include: the limitations of the researcher as storyteller, the decision to favor breadth over depth, and the challenge of comparing across participants.

As is required by qualitative research, I, as the researcher, act as an instrument. In particular, I acted as the co-constructor of the interview and data generation, the data analyst, and the writer. In this role, I tried to tell the stories of the participating MTEs from their points of view. I recognize that this is an imperfect science, but in an attempt to mitigate any emergent bias, I laid out my assumptions before recruitment began. I used the participants' own words to highlight as many of the major points as possible, and I provided participants with the opportunity to member check their stories. However, the reader should be aware that these stories still represent my best, yet flawed, attempt at telling the stories of others. This still remains an important endeavor, nonetheless, as it addresses the void of research from an outside perspective of a MTE's practice (Beswick & Goos, 2018).

In the design of this study, I determined that using a multiple case study approach was appropriate because it provided a breadth of mathematics teacher educator experiences. However, there was a tradeoff; a single case study would have provided a deeper dive into a mathematics teacher educator's work. This decision is justified by the fact that there is not an abundance of literature in the world of mathematics teacher educators' professional roles. In particular, I do not know of an additional set of case studies that examine their professional vision in this way.

Finally, in this case study, the participants represented a range of views and identities as MTEs. Two were from large research universities, while two were faculty at smaller liberal arts focused colleges. Two identify as women, while two identify as men. Two identify as People of Color, while the other two identify as white people. The differences between the cases could continue to be delineated. As a result, while some themes were shared across cases, in the cases where differences came to light, it was hard to discern if there were either institutional or structural differences that might have influenced the modifications. Again, this decision was justified based on the need for the field to have access to descriptive and broad information about MTEs professional views. And in addition, while individual cases are not generalizable, cases can contribute to a general theory (Flyvbjerg, 2006). While this small number of cases cannot create a new theory, they be used to add to existing theory. In particular, the data from this study, suggests MTEs are considering how mathematical inquiry might or might not be compatible with the necessity for equitable mathematics spaces.

Recommendations for Future Research

The recommendations for future directions for this line of work take three forms. The first set of recommendations emerges from a desire to be able to more purposely compare the stories of MTEs. The second set of recommendations are derived from the findings in this study, particularly as relate to classroom practices in mathematics teacher education. The final set of recommendations responds to the limitations of this study and would promote a more robust picture of the landscape for MTEs.

In order to continue to grow this line of work, research that limits some of the institutional and structural difference between participants would provide additional insight into the obligations and external pressure that emerges from universities and colleges. One example of this might be selecting several participants with roles at a single large university with a research focus. A second direction that would be helpful to the field would be to study the differences between MTEs' goals for those who work with undergraduate teacher candidates, graduate teacher candidates, and in service teachers. This would provide the field with additional information about how MTEs envision the trajectory for professional learning across these various professional learning stages.

The major findings from this study suggest several next steps for this work; here I propose two: long-term data generation and examination in teacher preparation classrooms and explicit study of inquiry as stance in mathematics teacher educators. First, in this study, part of the data generation activities included collection of a single professional artifact. And while these artifacts were illuminating, a long-term examination of classroom practice and artifacts would provide a richer picture of how inquiry and equity goals are being enacted in teacher educator. Such an element and could inform the field about how multiple goals for mathematics teacher learning are being achieved, and specially, if these goals are complementary or repetitive. Through this examination, a better sense of the expected trajectory with respect to these goals would be clearer. Second, while studies about *inquiry as stance* (e.g., Cochran-Smith &

Lytle, 1999, 2009) have existed in the teacher education literature for two decades, this study suggests that there is fertile ground for similar work with mathematics teacher educators as participants. Specifically, researchers should examine how considerations of mathematics as inquiry may or may not lead to inquiry processes in other facets of a MTEs' professional life. This line of work might provide some information about alignments between mathematical inquiry stances and social justice inquiry stances.

Although the previous two suggestions also address some of the limitations of the current study, a final line of inquiry that might be useful would be to employ additional qualitative methodology in the recruitment and study of new participants. Because case study is intensive and requires a good deal of researcher and participant resources, document analysis of existing syllabi for mathematics content or methods course in the teacher would provide an alternative way of gauging MTEs' goals in areas of inquiry and equity. Other methods, such as focus groups, surveys of teacher candidates' experiences of the mathematical inquiry and equity goals laid out by their MTEs, might also prove to be useful. In combination, these methods would add to the research base in ways that the case study alone would not.

Closing Thoughts

Implicit is this study is the assumption that I share with at least some of the participants that mathematics can be a discipline through which individuals can achieve intellectual joy. Beyond the economic and social power that is wielded in the knowledge of mathematics, it can provide an intellectual challenge can be self-actualizing for some individuals. This echoes some of Francis Su (2017)'s ideas of mathematics for "human flourishing" expressed early in this study. And if this process of "flourishing" is to occur in a just world, it is required of mathematics that not only do more to provide access to the discipline, but that the discipline of mathematics and those with authority in the field grow from these new ideas and viewpoints. In this study, each of the participants described a vision for how equitable practices in mathematics classrooms might look as relate to mathematical inquiry. In particular, how some mathematics can provide K-12 and teacher learners with insight into the aspect of how mathematics was built. This fundamentally human aspect of mathematics marries the constructs of mathematical inquiry and ideas of humanizing the discipline, which are tied to equitable practices.

The visions described in this study represent only four participants, but they identified themselves as teachers, instructional designers, outreach coordinators, classroom supervisors, educational researchers, learners, and co-constructors of knowledge, among other roles. In this study, I have only scratched the surface of what constitutes the broad and varied kinds of work that mathematics teacher educators undertake in their professional roles. Moreover, I did not even begin to address other professionals whose work might intersect with that of MTEs. More precisely, in addition to participants in this study who identify their professional role as, at least in part, as that of a mathematics teacher educator, individuals who hold roles primarily in mathematics research, in-service professional development, and school-based coaching could also be identified as mathematics teacher educators. Further, mathematics classroom teachers who act as mentors and guides to teacher candidates in their pre-practicum and practicum experiences also contribute as mathematics teacher educators for those they mentor. Thus, not only does this career choice encompass a number of roles for the professionals who identify as MTEs, but it also includes intersections with many other professions. As a relative newcomer, I feel lucky to be entering a profession that encompasses such a wide variety of perspectives. As I look toward a career wherein which I can support my drive for continued learning and commitment to a more just world, I look forward to opportunities to learn alongside each of these perspectives.

REFERENCES

Adams, E. (2010). The joys and challenges of semi-structured interviewing. *Community Practioner*, *83*(7), 1–9.

Aguirre, J. M. (2009). Privileging mathematics and equity in teacher education: Framework, counter-resistance strategies, and reflections from a Latina mathematics educator. In B. Greer, S. Mukhopadhyay, A. B. Powell, & S. Nelson-Barber (Eds.), *Culturally Responsive Mathematics Education* (pp. 295–319). New York, New York, USA: Routledge, Taylor & Francis Group.

Aguirre, J. M., McDuffie, A. R., Land, T., Bolson, C., Drake, C., Bartell, T. G., … Turner, E. E. (2013). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education*, *17*(3), 245–270. https://doi.org/10.1007/s10857-013-9257-0

Aguirre, J. M., Turner, E. E., Bartell, T. G., Drake, C., Foote, M. Q., & McDuffie, A. R. (2012). Analyzing effective mathematics lessons for English learners: A multiple mathematical lens approach. In S. Celedón-Pattichis & N. G. Ramirez (Eds.), *Beyond good teaching: Advancing mathematics education for ELLs* (pp. 207–221). Reston, VA: National Council of Teachers of Mathematics.

Aguirre, J. M., Turner, E. E., Bartell, T. G., Kalinec-Craig, C. A., Foote, M. Q., McDuffie, A. R., & Drake, C. (2013). Making Connections in Practice: How Prospective Elementary Teachers Connect to Children's Mathematical Thinking and Community Funds of Knowledge in Mathematics Instruction. *Journal of Teacher Education*, *64*(2), 178–192. https://doi.org/10.1177/0022487112466900

Albert, L. R. (2005). An exploration of a consciousness of difference through positionality: The role of the miner's canary. *Qualitative Research Journal*, *5*(1), 51–63.

Amador, J. (2016). Professional Noticing Practices of Novice Mathematics Teacher Educators. *International Journal of Science and Mathematics Education*, *14*(1), 217–241. https://doi.org/10.1007/s10763-014-9570-9

Apple, M. (1992). Do the Standards Go Far Enough ? Power, Policy, and Practice in Mathematics. *Journal for Research in Mathematics Education*, *23*(5), 412–431.

Averill, R., Anderson, D., Easton, H., Maro, P. Te, Smith, D., Hynds, A., … Hynds, A. (2016). Culturally Responsive Teaching of Mathematics: Three Models from Linked Studies, *40*(2), 157–186.

Ball, D. L., & Bass, H. (2009). With an Eye on the Mathematical Horizon: Knowing Mathematics for Teaching to Learners' Mathematical Futures. In *Keynote address at the 43rd Jahrestagung für Didaktik der Mathematik*. Oldenburg, Germany. Retrieved from http://www.mathematik.tu-dortmund.de/ieem/cms/media/BzMU/BzMU2009/Beitraege/Hauptvortraege/BALL_Deborah_BASS_Hyman_2009_Horizon.pdf

Ball, D. L., & Forzani, F. M. (2011). Building a common core for learning to teach and connecting professional learning practice. *American Educator*, *Summer*, 17–39.

Ball, D. L., Goffney, I. M., & Bass, H. (2005). The Role of Mathematics Instruction in Building a Socially Just and Diverse Democracy. *The Mathematics Educator*, *15*(1), 2–6.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Ballón, E. G. (2008). Racial Differences in High School Math Track Assignment. *Journal of Latinos and Education*, *7*(4), 272–287. https://doi.org/10.1080/15348430802143428

Barlow, A. T., & Reddish, J. M. (2006). Mathematical myths: Teacher candidates' beliefs and the implications for teacher educators. *Teacher Educator*, *41*(3), 145–157. https://doi.org/10.1080/08878730609555380

Bartell, T. G. (2012). Is This Teaching Mathematics for Social Justice? Teachers' Conception of Mathematics Classroom for Social Justice. In A. A. Wager & D. W. Stinson (Eds.), *Teaching Mathematics for Social Justice Conversations with Educators* (p. 213). Reston, VA: National Council of Teachers of Mathematics.

Bartell, T. G. (2013). Learning to Teach Mathematics for Social Justice: Negotiating Social Justice and Mathematical Goals. *Journal for Research in Mathematics Education*, *44*(1), 129–163. https://doi.org/10.5951/jresematheduc.44.1.0129

Baxter, P., & Jack, S. (2008). The Qualitative Report Qualitative Case Study Methodology: Study Design and Implementation for Novice Researchers. *The Qualitative Report*, *13*(2), 544–559. https://doi.org/citeulike-article-id:6670384

Berry III, R. Q., Ellis, M. W., & Hughes, S. (2014). Examining a history of failed reforms and recent stories of success: Mathematics education and Black learners of mathematics in the United States. *Race Ethnicity and Education*, *17*(4), 540–568.

Berry III, R. Q., Thunder, K., & McClain, O. L. (2011). Counter Narratives: Examining the Mathematics and Racial Identities of Black Boys who are Successful with

School Mathematics. *Journal of African American Males in Education*, *2*(1), 10–23.

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education*, *21*(5), 417–427. https://doi.org/10.1007/s10857-018-9416-4

Bleiler, S. K. (2014). Increasing awareness of practice through interaction across communities: The lived experiences of a mathematician and mathematics teacher educator. *Journal of Mathematics Teacher Education*, *18*(3), 231–252. https://doi.org/10.1007/s10857-014-9275-6

Boaler, J. (1999). Stuck in the slow lane: How American grouping systems perpetuate low achievement. In *What's Math Got to Do with It?: How Parents and Teachers Can Help Children Learn to Love their Least Favorite Subject* (pp. 106–122). New York, New York, USA: Penguin Books.

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal*, *9*(2), 27–40. https://doi.org/10.3316/QRJ0902027

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., Clarke, V., & Rance, N. (2014). How to use thematic analysis with interview data. In A. Vossler & N. Moller (Eds.), *The Counselling and Psychotherapy Research Handbook* (pp. 183–197). London, U.K.: Sage. https://doi.org/10.4135/9781473909847.n13

Brown, I. A., Davis, T. J., & Kulm, G. (2011). Pre-service Teachers' Knowledge for Teaching Algebra for Equity in the Middle Grades: A Preliminary Report. *The Journal of Negro Education*, *80*(3), 266–283.

Brown, L., Helliwell, T., & Coles, A. (2018). Working as mathematics teacher educators at the meta-level (to the focus of the teachers on developing their teaching). *AIEM*, *13*, 105–122.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology*, *3*(1), 21–29. https://doi.org/10.1111/j.2044-835X.1985.tb00951.x

Castro Superfine, A., & Li, W. (2014). Exploring the Mathematical Knowledge Needed for Teaching Teachers. *Journal of Teacher Education*, *65*(4), 303–314. https://doi.org/10.1177/0022487114534265

Castro Superfine, A., Prasad, P. V., Welder, R. M., Olanoff, D. E., & Eubanks-Turner, C. (2020). Exploring mathematical knowledge for teaching teachers: Supporting prospective elementary teachers' relearning of mathematics. *The Mathematics Enthusiast*, *17*(2 & 3), 367–402.

Cellucci, C. (2013). Top-Down and Bottom-Up Philosophy of Mathematics. *Foundations of Science*, *18*, 93–106.

Chamberlin, M. T., & Candelaria, M. S. (2018). Learning from Teaching Teachers: A Lesson Experiment in Area and Volume with Prospective Teachers. *Mathematics Teacher Education and Development*, *20*(1), 86–111.

Chao, T., Hale, J., & Cross, S. B. (2017). Experiences using clinical interviews in mathematics methods courses to empower propsective teachers: A conversation among three critical mathematics educators. In S. E. Kastberg, A. M. Tyminski, A. E. Lischka, & W. B. Sanchez (Eds.), *Building Support for Scholarly Practices in Mathematics Methods* (pp. 117–131). Charlotte, NC, U.S.A.: Information Age

Publishing, Inc.

Chapman, O. (2008). Mathematics teacher educators' learning from research on their instructional practice: A cognitive perspective. In K. Beswick & O. Chapman (Eds.), *International Handbook of Mathematics Teacher Education* (2nd ed., pp. 115–134). London, U.K.: Brill Sense.

Chick, H. L., & Beswick, K. (2017). Teaching teachers to teach Boris: A framework for mathematics teacher educator pedagogical content knowledge. *Journal of Mathematics Teacher Education*, *21*(5), 1–25. https://doi.org/10.1007/s10857-016-9362-y

Clements, D. H., & Sarama, J. (2018). Myths of early math. *Education Sciences*, *8*(2), 1–8. https://doi.org/10.3390/educsci8020071

Cobb, F., & Russell, N. M. (2015). Meritocracy or complexity: problematizing racial disparities in mathematics assessment within the context of curricular structures, practices, and discourse. *Journal of Education Policy*, *30*(5), 631–649. https://doi.org/10.1080/02680939.2014.983551

Cochran-Smith, M. (2003). Learning and unlearning: The education of teacher educators. *Teaching and Teacher Education*, *19*(1), 5–28. https://doi.org/10.1016/S0742-051X(02)00091-4

Cochran-Smith, M., & Lytle, S. (1999). Relationships of Knowledge and Practice: Teacher Learning in Communities. *Review of Research in Education*, *24*(24), 249–305.

Cochran-Smith, M., & Lytle, S. L. (2009). *Inquiry as stance: Practitioner Research for the Next Generation*. New York City, New York, USA: Teachers' College Press.

Cohen, E. G., Lotan, R. A., Scarloss, B. A., & Arellano, A. R. (1999). Complex instruction: Equity in cooperative learning classrooms. *Theory Into Practice*, *38*(2), 80–86. https://doi.org/10.1080/00405849909543836

Crenshaw, K. (1991). Mapping the Margins: Intersectionality, Identity Politics, and Violence against women of color. *Stanford Law Review*, *43*, 1241–1299.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, *52*(3), 243–270. https://doi.org/10.1023/A:1024364304664

Crespo, S., & Nicol, C. C. (2003). Learning to Investigate Students' Mathematical Thinking: The Role of Student Interviews. In *Proceeding of the International Group for the Psychology of Mathematics Education, 27* (Vol. 2, pp. 261–268).

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, *11*(5), 395–415. https://doi.org/10.1007/s10857-008-9081-0

Creswell, J. W., & Miller, D. L. (2000). Determining Validity in Qualitative Inquiry. *Theory Into Practice*, *39*(3), 124–130. https://doi.org/10.1207/s15430421tip3903_2

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of minds: An organizing principle for mathematics curriculum. *The Journal of Mathematical Behavior*, *15*, 375–402. https://doi.org/10.1016/S0732-3123(96)90023-1

Cuoco, A., Goldenberg, E. P., & Mark, J. (2010). Organizing a Curriculum around Mathematical Habits of Mind. *The Mathematics Teacher*, *103*(9), 682.

Cuoco, A., Goldenberg, P., & Mark, J. (2011). Curricular coherence through a focus on mathematical thinking 1. *Mathematics Teacher*, 1–11.

D'Ambrosio, U. (2001). What is ethnomathematics, and how can it help children in schools? *Teaching Children Mathematics*, *7*(6), 308–310. Retrieved from http://www.jstor.org/stable/41199887

Davis, J., & Martin, D. B. (2018). Racism, assessment, and instructional practices: Implications for mathematics teachers of African American students. *Journal of Urban Mathematics Education*, *11*(1&2), 45–68.

de Brey, C., Musu, L., McFarland, J., Wilkerson-Flicker, S., Diliberti, M., Zhang, A., … Wang, X. (2019). *Status and Trends in the Education of Racial and Ethnic Groups (NCES 2019-038)*. *U.S. Department of Education*. Washington, D.C. https://doi.org/10.1037/e571522010-001

Dover, A. G. (2013). Teaching for Social Justice: From Conceptual Frameworks to Classroom Practices. *Multicultural Perspectives*, *15*(1), 3–11. https://doi.org/10.1080/15210960.2013.754285

Drake, C., Land, T. J., Bartell, T. G., Aguirre, J. M., Foote, M. Q., McDuffie, A. R., & Turner, E. E. (2015). Three strategies for opening curriculum spaces. *Teaching Children Mathematics*, *21*(6), 346–353. https://doi.org/10.5951/teacchilmath.21.6.0346

Dyches, J., & Boyd, A. (2017). Foregrounding Equity in Teacher Education: Toward a Model of Social Justice Pedagogical and Content Knowledge. *Journal of Teacher Education*, *68*(5), 476–490. https://doi.org/10.1177/0022487117705097

Eisenhardt, K. M. (1989). Building Theories from Case Research. *The Academy of Management Review*, *14*(4), 532–550. https://doi.org/10.5465/AMR.1989.4308385

Ellis, M. W. (2008). Leaving No Child Behind Yet Allowing None Too Far Ahead:

Ensuring (In)Equity in Mathematics Education Through the Science of Measurement and Instruction. *Teachers College Record*, *110*(6), 1330–1356.

Ellis, M. W., & Malloy, C. E. (2012). Preparing Teachers for Democratic Mathematics Education. In *Action in Teacher Education* (Vol. 11, pp. 5–10). https://doi.org/10.1080/01626620.1989.10462708

Elo, S., Kääriäinen, M., Kanste, O., Pölkki, T., Utriainen, K., & Kyngäs, H. (2014). Qualitative Content Analysis. *SAGE Open*, *4*(1), 1–10. https://doi.org/10.1177/2158244014522633

Elo, S., & Kyngäs, H. (2008). The qualitative content analysis process. *Journal of Advanced Nursing*, *62*(1), 107–115. https://doi.org/10.1111/j.1365-2648.2007.04569.x

English, L. D., Cudmore, D., & Tilley, D. (1998). Problem posing and critiquing: How it can happen in your classroom. *Mathematics Teaching in the Middle School*, *4*(2), 124–129.

Enyedy, N., & Mukhopadhyay, S. (2007). They don't show nothing I didn't know: Emergent tensions between culturally relevant pedagogy and mathematics pedagogy. *The Journal of the Learning Sciences*, *16*(2), 139–174. https://doi.org/10.1080/10508400701193671

Ernest, P. (1989). The knowledge, beliefs, and attitudes of a mathematics teacher: A model. *Journal of Education for Teaching*, *15*(1), 13–33.

Ernest, P. (2016). The problem of certainty in mathematics. *Educational Studies in Mathematics*, *92*(3), 379–393. https://doi.org/10.1007/s10649-015-9651-x

Esmonde, I., Brodie, K., Dookie, L., & Takeuchi, M. A. (2009). Social Identities and

Opportunities to Learn: Student Perspectives on Group Work in an Urban Mathematics Classroom. *Journal of Ur*, *2*(2), 18–45.

Esmonde, I., & Caswell, B. (2010). Teaching mathematics for social justice in multicultural, multilingual elementary classrooms. *Canadian Journal of Science, Mathematics and Technology Education*, *10*(3), 244–254. https://doi.org/10.1080/14926156.2010.504485

Farmer, J., Gerretson, H., & Lassak, M. (2003). What Teachers Take from Professional Development: Cases and Implications. *Journal of Mathematics Teacher Education*, *6*(4), 331–360. https://doi.org/10.1023/A:1026318709074

Felton-Koestler, M. D. (2017). Mathematics education as sociopolitical: prospective teachers' views of the What, Who, and How. *Journal of Mathematics Teacher Education*, *20*(1), 49–74. https://doi.org/10.1007/s10857-015-9315-x

Felton-Koestler, M. D. (2020). Teaching sociopolitical issues in mathematics teacher preparation: What do mathematics teacher educators need to know? *The Mathematics Enthusiast*, *17*(2&3), 435–468. Retrieved from https://scholarworks.umt.edu/tme

Felton-Koestler, M. D., & Koestler, C. (2017). Should Mathematics Teacher Education Be Politically Neutral? *Mathematics Teacher Educator*, *6*(1), 67–72. https://doi.org/10.5951/mathteaceduc.6.1.0067

Felton, M. D., Simic-Muller, K., & Menéndez, J. M. (2012). "Math isn't just numbers or algorithms": Mathematics for social justice in preservice K-8 content courses. In L. J. Jacobsen, J. Mistele, & B. Sriraman (Eds.), *Mathematics teacher education in the public interest: Equity and social justice* (pp. 231–252). Charlotte, NC, U.S.A.:

Information Age Publishing, Inc.

Flyvbjerg, B. (2006). Five misunderstandings About Case-Study Research. *Qualitative Inquiry*, *12*(2), 219–245.

García, M., Sánchez, V., & Escudero, I. (2007). Learning through reflection in mathematics teacher education. *Educational Studies in Mathematics*, *64*(1), 1–17. https://doi.org/10.1007/s10649-006-9021-9

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education*, *12*(3), 161–170. https://doi.org/10.1007/s10857-009-9105-4

Gay, G. (2002). Preparing for Culturally Responsive Teaching. *Journal of Teacher Education*, *53*(2), 106–116.

Gholson, M. L. (2016). Clean corners and Algebra: A critical examination of the constructed invisibility of Black girls and women in mathematics. *The Journal of Negro Education*, *85*(3), 290. https://doi.org/10.7709/jnegroeducation.85.3.0290

Gholson, M. L., & Martin, D. B. (2014). Smart girls, black girls, mean girls, and bullies: At the intersection of identities and the mediating role of young girls' social network in mathematical communities of practice. *Journal of Education*, *19*(1), 19–34.

Gist, C. D. (2014). The Culturally Responsive Teacher Educator. *Teacher Educator*, *49*(4), 265–283. https://doi.org/10.1080/08878730.2014.934129

Gonzalez, L. (2012). Valuing the process: Working with teachers around mathematics for social justice. In Anita A Wager & D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 127–143). Reston, VA, U.S.A.: National Council of Teachers of Mathematics.

Goodwin, C. (1994). Professional Vision. *American Anthropologist*, *96*(3), 606–633.

Gorski, P. C. (2011). Unlearning Deficit Ideology and the Scornful Gaze: Thoughts on Authenticating the Class Discourse in Education. In R. Ahlquist, P. C. Gorski, & T. Montaño (Eds.), *Assault on Kids: How Hyper-Accountability, Corporatization, Deficit Ideologies, and Ruby Payne Are Destroying Our Schools* (pp. 152–173). New York, New York, USA: Peter Lang Publishing Inc.

Grossman, P. L., Compton, C., Igra, D., Rondfeltdt, M., Shahan, E., & Williamson, P. W. (2009). Teaching Practice: A Cross-Professional Perspective. *Teachers College Record*, *111*(9), 2055–2100.

Guba, E. (1981). Criteria for Assessing the Trustworthiness of Naturalistic Inquiries. *Educational Communication and Technology*, *29*(2), 75–91.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning*, *4*(2&3), 145–187. https://doi.org/10.1207/S15327833MTL04023_4

Gutiérrez, R. (2008a). A "Gap-Gazing" Fetish in Mathematics Education? Problematizing Research on the Achievement Gap. *Journal for Research in Mathematics Education*, *39*(3), 357–364.

Gutiérrez, R. (2008b). Realizing the potential of Chicanos and Native Americans: Engaging identity and power issues in teaching students mathematics and science. *SACNAS News*, *11*(1), 10–11.

Gutiérrez, R. (2009a). Embracing the inherent tensions in teaching mathematics from an equity stance. *Democracy & Education*, *18*(3), 9–16.

Gutiérrez, R. (2009b). Framing Equity: Helping Students "Play the Game" and "Change the Game. *Teaching for Excellence and Equity in Mathematics*, *1*(1), 4–8.

Gutiérrez, R. (2012). Embracing Nepantla: Rethinking "Knowledge" and its Use in Mathematics Teaching. *REDIMAT – Journal of Research in Mathematics Education*, *1*(1), 29–56. https://doi.org//10.4471/redimat.2012.02

Gutiérrez, R. (2013a). The Sociopolitical Turn in Mathematics Education. *Journal for Research in Mathematics Education*, *44*(1), 37–68. https://doi.org/10.5951/jresematheduc.44.1.0037

Gutiérrez, R. (2013b). Why (Urban) Mathematics Teachers Need Political Knowledge. *Journal of Urban Mathematics Education*, *6*(2), 7–19.

Gutiérrez, R. (2016). Nesting in Nepantla: The importance of maintaining tensions in our work. In N. M. Joseph, C. Haynes, & F. Cobb (Eds.), *Interrogating Whiteness and Relinquishing Power: White Faculty's Commitment to Racial Consciousness in STEM Classrooms*. New York, New York, USA: Peter Lang Publishing Inc.

Gutiérrez, R. (2018). Political Conocimiento for Teaching Mathematics: Why teachers need it and how to develop it. In *Building Support for Scholarly Practices in Mathematics Methods* (pp. 11–37). Charlotte, NC, U.S.A.: Information Age Publishing, Inc.

Gutstein, E. (2012). Connecting community, critical, and classical knowledge in teaching mathematics for social justice. *Alternative Forms of Knowing (in) Mathematics: Celebrations of Diversity of Mathematical Practices*, 299–311. https://doi.org/10.1007/978-94-6091-921-3

Güven, N. D., & Dede, Y. (2016). Examining Social and Sociomathematical Norms in Different Classroom Microcultures: Mathematics Teacher Education Perspective.

Educational Sciences: Theory & Practice, *17*(1), 13–16. https://doi.org/10.12738/estp.2017.1.0383

Hammerness, K. (2001). Teachers' visions: The role of personal ideals in school reform. *Journal of Educational Change*, *2*, 143–163. https://doi.org/10.1023/A:1017961615264

Hand, V., & Goffney, I. M. (2013). "All for One and One for All": Negotiating Solidarity Around Power and Oppression in Mathematics Education. *Journal of Urban Mathematics Education*, *6*(1), 28–34.

Harackiewicz, J. M., Canning, E. A., Tibbetts, Y., Priniski, S. J., & Hyde, J. S. (2016). Closing achievement gaps with a utility-value intervention: Disentangling race and social class. *Journal of Personality and Social Psychology*, *111*(5), 745–765. https://doi.org/10.1037/pspp0000075

Harris, C. (1993). Whiteness as Property. *Harvard Law Review*, *106*(8), 1707–1791.

Harris, C. (1995). Whiteness as Property. In K. Crenshaw, N. Gotanda, G. Peller, & K. Thomas (Eds.), *Critical Race Theory* (pp. 276–291). New York City, New York, USA: New Press.

Heath, C., & Heath, D. (2010). *Switch: How to change things when change is hard*. New York, New York, USA: Random House.

Hiebert, J. (2013). The Constantly Underestimated Challenge of Improving Mathematics Instruction. In K. R. Leatham (Ed.), *Vital Directions for Mathematics Education Research* (pp. 45–54). New York, New York, USA: Springer Science+Business Media.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., …

Gallimore, R. (2005). Mathematics Teaching in the United States Today (and Tomorrow): Results From the TIMSS 1999 Video Study. *Educational Evaluation and Policy Analysis*, *27*(2), 111–132. https://doi.org/10.3102/01623737027002111

Jackson, B., Hauk, S., Tsay, J.-J., & Ramirez, A. (2020). Professional development for mathematics teacher education faculty: Need and design. *The Mathematics Enthusiast*, *17*(2&3), 537–582.

Jaworski, B. (2008). Development of the mathematics teacher educator and its relation to teaching development. In K. Beswick & O. Chapman (Eds.), *International Handbook of Mathematics Teacher Education* (2nd ed., Vol. 4, pp. 335–361). London, U.K.

Jett, C. C., & Cross, S. B. (2016). Teaching about Diversity in Black and White: Reflections and Recommendations from Two Teacher Educators. *The New Educator*, *12*(2), 131–146. https://doi.org/10.1080/1547688X.2015.1058448

Jong, C. (2016). Linking reform-oriented experiences to teacher identity: The case of an elementary mathematics teacher. *Journal of Educational Research*, *109*(3), 296–310. https://doi.org/10.1080/00220671.2014.947398

Joseph, N. M. (2017). The Promises of Deconstructing and Disrupting for Critical Transformations, 155–158.

Joseph, N. M., Hailu, M., & Boston, D. (2017). Black Women's and Girls' Persistence in the P–20 Mathematics Pipeline: Two Decades of Children, Youth, and Adult Education Research. *Review of Research in Education*, *41*(March), 203–227. https://doi.org/10.3102.0091732X16689045

Kalinec-Craig, C. A., & Bonner, E. (2015). Seeing the world with a new set of eyes:

(Re)examining our identities as white mathematics education researchers of equity and social justice. In *Interrogating Whiteness and Relinquishing Power: White Faculty's Commitment to Racial Consciousness in STEM Classrooms* (pp. 92–111). New York, New York, USA: Peter Lang Publishing Inc.

Kitcher, P. (1984). *The nature of mathematical knowledge*. Oxford, UK: Oxford University Press.

Koestler, C. (2012). Beyond apples, puppy dogs, and ice cream: Preparing teachers to teach mathematics for equity and social justice. In *Teaching mathematics for social justice: Conversations with Educators* (pp. 81–97). Reston, VA: National Council of Teachers of Mathematics.

Korstjens, I., & Moser, A. (2018). Series: Practical guidance to qualitative research. Part 4: Trustworthiness and publishing. *European Journal of General Practice*, *24*(1), 120–124. https://doi.org/10.1080/13814788.2017.1375092

Kurniati, I., & Surya, E. (2017). Student's Perception of their Teacher Teaching Styles. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, *33*(2), 91–98.

Labuschagne, A. (2003). The Qualitative Report Qualitative Research -Airy Fairy or Fundamental? *The Qualitative Report*, *8*(1), 100–103. Retrieved from http://nsuworks.nova.edu/tqr%5Cnhttp://nsuworks.nova.edu/tqr/vol8/iss1/7

Ladson-Billings, G. (1995). Toward a Theory of Culturally Relevant Pedagogy. *American Educational Research Journal*, *32*(3), 465–491. https://doi.org/10.3102/00028312032003465

Ladson-Billings, G. (2006). From the achievement gap to the education debt:

Understanding achievement in US Schools. *Educational Researcher*, *35*(7), 3–12.

Ladson-Billings, G. (2014). Culturally Relevant Pedagogy 2.0: a.k.a. the Remix. *Harvard Educational Review*, *84*(1), 74–84. https://doi.org/10.17763/haer.84.1.p2rj131485484751

Ladson-Billings, G., & Tate IV, W. F. (1995). Toward a Critical Race Theory of Education. *Teachers College Record*, *97*(1), 47–68. https://doi.org/10.1080/10282580701850413

Land, T. J., Bartell, T. G., Drake, C., Foote, M. Q., Roth McDuffie, A., Turner, E. E., & Aguirre, J. M. (2019). Curriculum spaces for connecting to children's multiple mathematical knowledge bases. *Journal of Curriculum Studies*, *51*(4), 471–493. https://doi.org/10.1080/00220272.2018.1428365

Lawler, B. R., & Uy, F. (2017). Equity & Social Justice Working Group. In W. M. Smith, B. R. Lawler, & L. Augustyn (Eds.), *Proceedings of the sixth annual Mathematics Teacher Education Partnership conference* (pp. 69–76). Washington, D.C.: Association of Public and Land-Grant Universities.

Lawrence-Lightfoot, S. (2005). Reflections on portraiture: A dialogue between art and science. *Qualitative Inquiry*, *11*(1), 3–15. https://doi.org/10.1177/1077800404270955

Lawrence-Lightfoot, S., & Davis, J. H. (1997). *The art and science of portraiture*. San Francisco, CA: Jossey-Bass.

Lee, J., & Reeves, T. (2012). Revisiting the Impact of NCLB High-Stakes School Accountability, Capacity, and Resources: State NAEP 1990-2009 Reading and Math Achievement Gaps and Trends. *Educational Evaluation and Policy Analysis*, *34*(2), 209–231. https://doi.org/10.3102/0162373711431604

Leonard, J., Brooks, W., Barnes-Johnson, J., & Berry III, R. Q. (2010). The nuances and complexities of teaching mathematics for cultural relevance and social justice. *Journal of Teacher Education*, *61*(3), 261–270. https://doi.org/10.1177/0022487109359927

Li, W., & Castro Superfine, A. (2018). Mathematics teacher educators' perspectives on their design of content courses for elementary preservice teachers. *Journal of Mathematics Teacher Education*, *21*(2), 179–201. https://doi.org/10.1007/s10857-016-9356-9

Lin, F. L., Yang, K. L., Hsu, H.-Y., & Chen, J. C. (2018). Mathematics teacher educator-researchers' perspectives on the use of theory in facilitating teacher growth. *Educational Studies in Mathematics*, *98*(2), 197–214. https://doi.org/10.1007/s10649-018-9804-9

Lin, X., Bransford, J. D., Hmelo, C. E., Kantor, R. J., Hickey, D. T., Secules, T., … The Cognition and Technology Group. (1995). Instructional Design and Development of Learning Communities: An Invitation to a Dialogue. *Educational Technology*, *35*(5), 53–63.

Lortie, D. C. (1975). *Schoolteacher; a sociological study*. Chicago: University of Chicago Press.

Louie, N. L. (2017). The Culture of Exclusion in Mathematics Education and Its Persistence in Equity-Oriented Teaching. *Journal for Research in Mathematics Education*, *48*(5), 488. https://doi.org/10.5951/jresematheduc.48.5.0488

Maass, K. (2011). How can teachers' beliefs affect their professional development? *ZDM*

- *International Journal on Mathematics Education*, *43*(4), 573–586. https://doi.org/10.1007/s11858-011-0319-4

Maher, N. (2011). From Classroom to Campus: The Perceptions of Mathematics and Primary Teachers on Their Transition From Teacher to Teacher Educator. In J. Clark, B. Kissane, T. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices* (pp. 491–499). Adelaide, SA, Australia: AAMT-MERGA.

Marshall, A. M., & Chao, T. (2017). Using mathematics autobiography stories to support emerging elementary mathematics teachers' sociopolitical conciousness and identity. In S. E. Kastberg, A. M. Tyminski, A. E. Lischka, & W. B. Sanchez (Eds.), *Building Support for Scholarly Practices in Mathematics Methods* (pp. 279–293). Charlotte, NC, U.S.A.: Information Age Publishing, Inc.

Marshall, M. N. (1996). Sampling for qualitative research Sample size. *Family Practice*, *13*(6), 522–525.

Martin, D. B. (2003). Hidden assumptions and unaddressed questions in mathematics for all rhetoric. *The Mathematics Educator*, *13*(2), 7–21. https://doi.org/10.1167/iovs.10-7062

Martin, D. B. (2007). Beyond Missionaries or Cannibals: Who Should Teach Mathematics to African American Children? *The High School Journal*, *91*(1), 6–28.

Martin, D. B. (2009a). Does Race Matter? *Teaching Children Mathematics*, *16*(3), 134–139.

Martin, D. B. (2009b). Researching race in mathematics education. *Teachers College Record*, *111*(2), 295–338.

Martin, D. B. (2010). Not-so-strange bedfellows: Racial projects and the mathematics education enterprise. In *Proceedings of the sixth international mathematics education and society conference* (Vol. 1, pp. 57–79). Berlin: Freie Universitat Berlin.

Masingila, J. O., Olanoff, D., & Kimani, P. M. (2018). Mathematical knowledge for teaching teachers: Knowledge used and developed by mathematics teacher educators in learning to teach via problem solving. *Journal of Mathematics Teacher Education*, *21*(5), 1–22. https://doi.org/10.1007/s10857-017-9389-8

Mayring, P. (2000). Qualitative content analysis. *Forum: Qualitative Social Research*, *1*(2), Article 20.

McCloskey, A., Lawler, B. R., & Chao, T. (2017). The "Mirror Test": A tool for reflection on our sociopolitical identities as mathematics teacher educators. In S. E. Kastberg, A. M. Tyminski, A. E. Lischka, & W. B. Sanchez (Eds.), *Building Support for Scholarly Practices in Mathematics Methods* (pp. 325–340). Charlotte, NC, U.S.A.: Information Age Publishing, Inc.

McDonald, M. A., & Zeichner, K. M. (2008). Social Justice Teacher Education. In W. Ayers, T. M. Quinn, & D. Stovall (Eds.), *Handbook of Social Justice in Education* (pp. 595–610). New York, New York, USA: Routledge.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). Fundamentals of Qualitative Data Analysis. In *Qualitative Data Analysis: A Methods Sourcebook* (3rd ed., pp. 399–404). Washington, D.C.: SAGE Publications Ltd.

Milner IV., H. R. (2013). Rethinking Achievement Gap Talk in Urban Education. *Urban Education*, *48*(1), 3–8. https://doi.org/10.1177/0042085912470417

Mohammad, R. F. (2008). *Becoming a Mathematics Teacher: Processes and Issues*. (K. Beswick & O. Chapman, Eds.), *The Handbook of Mathematics Teacher Education* (2nd ed.). London, U.K.: Brill Sense.

Muir, T., Fielding-Wells, J., & Chick, H. L. (2017). Developing an understanding of what constitutes Mathematics Educator PCK: A case study of a collaboration between two teacher educators. *Australian Journal of Teacher Education*, *42*(12), 60–79.

Munter, C. (2014). Developing Visions of High-Quality Mathematics Instruction. *Journal for Research in Mathematics Education*, *45*(5), 584–635. https://doi.org/10.5951/jresematheduc.45.5.0584

Musoleno, R. R., & White, G. P. (2010). Influences of High-Stakes Testing on Middle School Mission and Practice. *RMLE Online*, *34*(3), 1–10. https://doi.org/10.1080/19404476.2010.11462076

Musu-Gillette, L., de Brey, C., McFarland, J., Hussar, W., Sonnenber, W., & Wilkinson-Flicker, S. (2017). *Status and Trends in the Education of Racial and Ethnic Groups 2017 (NCES 2017-051)*. *U.S. Department of Education*. Washington, D.C. https://doi.org/10.1037/e571522010-001

Musu-Gillette, L., Robinson, J., McFarland, J., KewalRamani, A., Zhang, A., & Wilkinson-Flicker, S. (2016). Status and trends in the education of racial and ethnic groups 2016 (NCES 2016-007). *U.S. Department of Education, National Center for Education Statistics.*, 1–188. https://doi.org/10.1037/e571522010-001

Nasir, N. S. (2016). Commentary: Why should mathematics educators care about race and culture? *Journal of Urban Mathematics Education*, *9*(1), 7–18.

Nasir, N. S., & Hand, V. M. (2006). Exploring Sociocultural Perspectives on Race, Culture, and Learning. *Review of Educational Research*, *76*(4), 449–475. https://doi.org/10.3102/00346543076004449

Nasir, N. S., & McKinney de Royston, M. (2013). Power, Identity, and Mathematical Practices Outside and Inside School. *Journal for Research in Mathematics Education*, *44*(1), 264–287.

Nasir, N. S., Rosebery, A. S., Warren, B., & Lee, C. D. (2014). Learning as a cultural process: Achieving equity through diversity. In K. Sawyer (Ed.), *The Cambridge Handbook of The Learning Sciences* (2nd ed., pp. 489–504). New York, New York, USA: Cambridge University Press. https://doi.org/10.1192/bjp.bp.106.029678

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to Actions*. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices [NGA], & Council of Chief State School Officers [CCSO]. (2010). Common Core State Standards for Mathematics. Common Core State Standards Initiative. Retrieved from http://www.corestandards.org/the-standards/mathematics

National Research Council [NRC]. (2004). *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations*.

Noor, K. B. M. (2008). Case study: A strategic research methodology. *American Journal of Applied Sciences*, *5*(11), 1602–1604. https://doi.org/10.3844/ajassp.2008.1602.1604

Oakes, J., Ormseth, T., Bell, R., & Camp, P. (1990). *Multiplying inequities: The effects of race, social class, and tracking on opportunities to learn mathematics and science*. Santa Monica, CA.

Pais, A. (2011). Criticisms and contradictions of ethnomathematics. *Educational Studies in Mathematics*, *76*, 209–230. https://doi.org/10.1007/s10649-010-9289-7

Pais, A., & Valero, P. (2010). Beyond Disavowing the Politics of Equity and Quality in Mathematics Education. In B. Atwah, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping Equity and Quality in Mathematics Education* (pp. 35–48). Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-90-481-9803-0_3

Paris, D. (2012). Culturally Sustaining Pedagogy: A Needed Change in Stance, Terminology, and Practice. *Educational Researcher*, *41*(3), 93–97. https://doi.org/10.3102/0013189X12441244

Parker, F., Bartell, T. G., & Novak, J. D. (2017). Developing culturally responsive mathematics teachers: secondary teachers' evolving conceptions of knowing students. *Journal of Mathematics Teacher Education*, *20*(4), 385–407. https://doi.org/10.1007/s10857-015-9328-5

Parks, A. N., & Schmeichel, M. (2012). Obstacles to Addressing Race and Ethnicity in the Mathematics Education Literature. *Journal for Research in Mathematics Education*, *43*(3), 238. https://doi.org/10.5951/jresematheduc.43.3.0238

Phelps-Gregory, C. M., Frank, M., & Spitzer, S. M. (2020). Prospective Elementary Teachers' Beliefs About Mathematical Myths: A Historical and Qualitative Examination. *Teacher Educator*, *55*(1), 6–27. https://doi.org/10.1080/08878730.2019.1618423

Ping, C., Schellings, G., & Beijaard, D. (2018). Teacher educators' professional learning: A literature review. *Teaching and Teacher Education*, *75*, 93–104. https://doi.org/10.1016/j.tate.2018.06.003

Ritchie, S., Cone, N., An, S., & Bullock, P. (2012). Teacher education for social change: Transforming a content methods course block. *Current Issues in Comparative Education*, *15*(2), 63–83. https://doi.org/10.1109/SISPAD.2013.6650609

Rosa, M., & Orey, D. C. (2010). Ethnomodeling as a Pedagogical Tool for the Ethnomathematics Program. *Revista Latinoamericana de Etnomatemática*, *3*(2), 14–23.

Rubel, L. H. (2017). Equity-directed instructional practices: Beyond the dominant perspective. *Journal of Urban Mathematics Education*, *10*(2), 66–105.

Sakonidis, C., & Potari, D. (2014). Mathematics teacher educators'/researchers' collaboration with teachers as a context for professional learning. *ZDM - International Journal on Mathematics Education*, *46*(2), 293–304. https://doi.org/10.1007/s11858-014-0569-z

San Pedro, T. (2018). Abby as Ally: An Argument for Culturally Disruptive Pedagogy. *American Educational Research Journal*, *55*(6), 000283121877348. https://doi.org/10.3102/0002831218773488

Savery, J. R. (2006). Overview of Problem-based Learning: Definitions and Distinctions. *Interdisciplinary Journal of Problem-Based Learning*, *1*(1), 9–20. https://doi.org/10.7771/1541-5015.1002

Schoenfeld, A. H. (2016a). 100 Years of Curriculum History, Theory, and Research. *Educational Researcher*, *45*(2), 105–111. https://doi.org/10.3102/0013189X16639025

Schoenfeld, A. H. (2016b). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint). *Journal of Education*, *196*(2), 1–38. https://doi.org/10.1177/002205741619600202

Sheffield, L. J. (2017). Dangerous myths about "gifted" mathematics students. *ZDM - Mathematics Education*, *49*(1), 13–23. https://doi.org/10.1007/s11858-016-0814-8

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2004), 63–75. https://doi.org/10.1111/j.1744-618X.2000.tb00391.x

Sherin, M. G. (2002). When Teaching Becomes Learning When Teaching Becomes Learning. *Cognitivion and Instruction*, *20*(2), 119–150. https://doi.org/10.1207/S1532690XCI2002

Shulman, L. S. (1986). Those who understand: Knowledge and growth in teaching. *Educational Researcher*, *15*(2), 4–14. https://doi.org/http://www.jstor.org/stable/1175860

Silver, E. A. (1994). On Mathematical Problem Posing. *For the Learning of Mathematics*, *14*(1), 19–28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *International Journal on Mathematics Education*, *29*(3), 75–80. https://doi.org/10.1007/s11858-997-0003-x

Simzar, R. M., Domina, T., & Tran, C. (2015). Eighth Grade Algebra Course Placement and Student Motivation for Mathematics. *HHS Public Access*, *2*(1), 37–54. https://doi.org/10.1016/bs.mcb.2015.01.016.Observing

Smith, C., & Bretscher, N. (2018). Designing teacher education for pre-university mathematics: Articulating and operationalizing pedagogic messages. *Teaching Mathematics and Its Applications*, *37*(2), 84–97.

https://doi.org/10.1093/teamat/hry008

Spielman, L. J. (2009). Mathematics education in the public interest: Preservice teachers' engagement with and reframing of mathematics. In *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 408–415).

Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction*, *25*(2–3), 161–217. https://doi.org/10.1080/07370000701301125

Stiff, L. V., Johnson, J. L., & Akos, P. (2011). Examining what we know for sure: Tracking in middle grades mathematics. In W. F. Tate IV, K. D. King, & C. K. Rousseau Anderson (Eds.), *Disrupting tradition: Research and practice pathways in mathematics education* (pp. 63–77). Reston, VA: National Council of Teachers of Mathematics.

Stinson, D. W. (2004). Mathematics as "Gate-Keeper" (?): Three Theoretical Perspectives that Aim Toward Empowering All Children With a Key to the Gate. *The Mathematics Educator*, *14*(1), 8–18.

Su, F. E. (2017). Mathematics for Human Flourishing. *The American Mathematical Monthly*, *124*(6), 483–493. https://doi.org/10.4169/amer.math.monthly.124.6.483

Swars, S. L., Smith, S. Z., Smith, M. E., Carothers, J., & Myers, K. (2018). The preparation experiences of elementary mathematics specialists: examining influences on beliefs, content knowledge, and teaching practices. *Journal of Mathematics Teacher Education*, *21*(2), 123–145. https://doi.org/10.1007/s10857-016-9354-y

Sword, S., Matsuura, R., Gates, M., Kang, J., Cuoco, A., & Stevens, G. (2015). Secondary teachers' mathematical habits of mind: A paper and pencil assessment. In C. Suurtamm (Ed.), *Annual perspectives in mathematics education: Assessment to enhance tearning and teaching* (pp. 109–118). Reston, VA: NCTM.

Sztajn, P., Ball, D. L., & McMahon, T. A. (2006). Designing learning opportunities for mathematics teacher developers. In K. Lynch-Davis & R. L. Rider (Eds.), *The Work of Mathematics Teacher Educators* (pp. 149–162). San Diego, CA, USA.

Tate IV, W. F. (1997). Critical race theory and education: History, theory, and implications. *Review of Research in Education*, *22*, 195–247.

Tellis, W. (1997). Introduction to Case Study. *The Qualitative Report*, *3*(2), 1–14. Retrieved from https://nsuworks.nova.edu/tqr/%0Ahttp://nsuworks.nova.edu/tqr%5Cnhttp://nsuworks.nova.edu/tqr

Tuovinen, J. E., & Sweller, J. (1999). A comparison of Cognitive Load Associated With Discovery Learning and Worked Examples. *Journal of Educational Psychology*. https://doi.org/10.1037/0022-0663.91.2.334

Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, *4*, 259–283. https://doi.org/10.1023/A:1013314009952

Vaismoradi, M., Turunen, H., & Bondas, T. (2013). Content analysis and thematic analysis: Implications for conducting a qualitative descriptive study. *Nursing and Health Sciences*, *15*(3), 398–405. https://doi.org/10.1111/nhs.12048

Vakil, S., McKinney de Royston, M., Nasir, N. S., & Kirshner, B. (2016). Rethinking

Race and Power in Design-Based Research: Reflections from the Field. *Cognition and Instruction*, *34*(3), 194–209. https://doi.org/10.1080/07370008.2016.1169817

Van Zoest, L. R., Moore, D. L., & Stockero, S. L. (2006). Transition to teacher educator: A collaborative effort. In K. Lynch-Davis & R. L. Rider (Eds.), *The Work of Mathematics Teacher Educators* (pp. 133–146). San Diego, CA, USA: Association of Mathematics Teacher Educators.

Villegas, A. M., Ciotoli, F., & Lucas, T. (2017). A Framework for Preparing Teachers for Classrooms That Are Inclusive of All Students. In L. Florian & N. Pantič (Eds.), *Teacher Education for the Changing Demographics of Schooling* (pp. 133–148). Cham, Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-54389-5_10

Villegas, A. M., & Lucas, T. (2002). Preparing culturally responsive teachers: Rethinking the curriculum. *Journal of Teacher Education*, *53*(1), 20–32. https://doi.org/10.1177/0022487102053001003

Volmink, J. (1994). Mathematics by all. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 51–67). Dordrecht, The Netherlands: Kluwer Academic Publishers.

von Glasersfeld, E. (1983). Learning as Constructive Activity. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the 5th Annual Meeting of the North American Group of Psychology in Mathematics Education* (Vol. 1, pp. 41–101). Montreal: PME-NA.

Vygotsky, L. S. (1978). *Mind in Society*. Cambridge, MA, USA: Harvard University Press. https://doi.org/10.1007/978-3-540-92784-6

Wang, A. H. (2008). A pre-kindergarten achievement gap? Scope and implications. *US-China Education Review*, *5*(9), 23–31.

Warren, B., & Rosebery, A. S. (2011). Navigating interculturality: African American male students and the science classroom. *Journal of African American Males in Education*, *2*(1), 98–115.

Wiley, C., & Drake, C. (2013). Advocating for Equitable Mathematics Education: Supporting Novice Teachers in Navigating the Sociopolitical Context of Schools. *Journal of Urban Mathematics Education*, *6*(1), 58–70.

Yerushalmy, M., Chazan, D., & Gordon, M. (1990). Mathematical problem posing: Implications for facilitating student inquiry in classrooms. *Instructional Science*, *19*(3), 219–245. https://doi.org/10.1007/BF00120197

Yin, R. K. (2009). *Case study research: Design and methods*. Thousand Oaks, CA: Sage Publications.

Yin, R. K. (2012). A (very) brief refresher on the case study method. In *Applications of Case Study Research* (pp. 3–20). Sage Publications. https://doi.org/10.1088/1751-8113/44/8/085201

Young Cho, J., & Lee, E. (2014). Reducing confusion about grounded theory and qualitative content analysis: Similarities and differences. *The Qualitative Report*, *19*(32), 1–20.

Yow, J. A., Eli, J. A., Beisiegel, M., Mccloskey, A., & Welder, R. M. (2016). Challenging Transitions and Crossing Borders: Preparing Novice Mathematics Teacher Educators to Support Novice K-12 Mathematics Teachers. *Mathematics Teacher Education and Development*, *18*(1), 52–69.

Zaslavsky, O. (2008). Meeting the Challenges of Mathematics Teacher Education Through Design and Use of Tasks That Facilitate Teacher Learning. In B. Jaworski & T. Wood (Eds.), *The Mathematics Teacher Educator as a Developing Professional* (pp. 93–114). Rotterdam, The Netherlands: Sense Publishers.

Zaslavsky, O. (2009). Mathematics Educators' Knowledge and Development. In R. Even & D. L. Ball (Eds.), *The Professional Education and Development of Teachers* (pp. 105–111). Springer Science + Business Media. https://doi.org/10.1007/978-0-387-09601-8_12

Zaslavsky, O., Chapman, O., & Leikin, R. (2003). Professional Development of Mathematics Educators: Trends and Tasks. In A. J. Bishop, M. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education* (pp. 877–917). Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-010-0273-8_28

Zaslavsky, O., & Leikin, R. (2004). Professional Development of Mathematics Teacher Educators: Growth Through Practice. *Journal of Mathematics Teacher Education*, *7*(1), 5–32. https://doi.org/10.1023/B:JMTE.0000009971.13834.e1

Zazkis, R., & Mamolo, A. (2018). From disturbance to task design, or a story of a rectangular lake. *Journal of Mathematics Teacher Education*, *21*(5), 501–516. https://doi.org/10.1007/s10857-016-9361-z

APPENDIX A

SAMPLE INTERVIEW PROTOCOL

Thank you for taking the time to talk to me today. As a reminder, I'm a doctoral candidate at Boston College with an interest in understanding the professional trajectories of mathematics teacher educators. I'm particularly interested in understanding how mathematics teacher educators understand the dual constructs of "mathematical inquiry" and "equity in mathematics classrooms" in their vision for their professional work. This interview is designed to elicit information about this topic. We will begin by discussing how you think about mathematical inquiry, then how you think about equity in mathematics classrooms, and then how you think about them together. At each stage, I will ask you how these constructs relate to your professional vision. I hope you feel comfortable saying what you really think and how you really feel. I will de-identify data and store it in a secure location, as noted in the consent form. I will also audio -record our conversation. If that's all OK with you, we can get started. I will open with a question about the audio recording, just to get your verbal consent on the recording.

1. Is it OK to audio record this interview today?

On my screen [or in an email], I am sharing two definitions of mathematical inquiry. I will give you a moment to read them.

In using "inquiry", we want to bring to the reader's mind the process of learning employed by creative people at the forefront of their fields - people interested in a particular area and continuously motivated to learn more about it, who set themselves problems; design methods to explore them; and then try to create solutions [...] More specifically, inquiry teaching in mathematics might mean that students should learn mathematics by choosing a topic, posing problems, creating approaches to the problems, and recreating historical discoveries. (Yerushalmy, Chazan, & Gordon, 1990)

Inquiry is a practice or stance, and indicates a particular way of engaging with and making sense of the world [...] Inquiry into mathematics involves delving into mathematical ideas and concepts and trying to understand the structure, power, and limitations of mathematics. Inquiry with mathematics involves using mathematics as a tool to make sense of problem situations and come to some reasonable resolution [...] Learning results from, and is evidenced by, student participation in both standard disciplinary practices (e.g., justifying, representing algebraically) and an array of other practices of mathematical communities (e.g., questioning, communicating, informal reasoning). (Staples, 2007)

2. How do these definitions align or not align with how you think about mathematical inquiry in mathematical classroom spaces? Can you describe how you understand the term mathematical inquiry either with support from these definitions or as separate from these definitions?*
3. If you had no barriers to success, how would your definition of mathematical inquiry drive your professional role as a mathematics teacher educator?
    a. *Probe:* What do you think your role is in the support and development of pre-service teachers with regards to mathematical inquiry?
    b. *Probe:* I'm primarily interested in your role in teacher education; although, I recognize teacher education must be done in concert with schools and communities. How does you understanding of your professional role fit in the larger educational landscape with regards to mathematical inquiry?
    c. *Probe:* What barriers to success do you see in reaching your professional vision with regards to mathematical inquiry?
    d. *I believe you brought a copy of a biography with you today,* can you point to places where this understanding of inquiry has driven your past choices?
        i. How has your professional vision with regards to mathematical inquiry changed as you have continued to develop as a mathematics teacher educator? (Since graduate school, for example).
4. Can you describe how your definition of mathematical inquiry fits into your current professional life?*
    a. *I believe you brought a professional artifact with you today,* can you describe how your professional artifacts might be an example of your conception of mathematical inquiry?
    b. *Probe*: Do you see these related to the education of pre-service teachers and continued professional learning of in-service teachers? If so, how?
5. Do you have a sense of the development trajectory of teachers' understandings of mathematical inquiry as you have defined it? If so, can you describe such a trajectory?
    a. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should have access to before they enter the classroom with regards to mathematical inquiry?
    b. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should gain by the middle of their career with regards to mathematical inquiry?

* All participants were asked these questions

c. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should gain by the time they begin to move into leadership roles, as mentor teachers, for example with regards to mathematical inquiry?

On my screen [or in an email], I am sharing a definition of mathematical equity. I will give you a moment to read it.

[…]equity means that all students in light of their humanity - personal experiences, backgrounds, histories, languages, physical and emotional well-being - must have the opportunity and support to learn rich mathematics that fosters meaning making, empowers decision making, and critiques, challenges, and transforms inequities/injustices. [...] equity demands that responsible and appropriate accommodations be made as needed to promote equitable access, attainment, and advancement for all students. [...] Equity and mathematics comprise a powerful dialectic that is continually being constructed. It is important to acknowledge that this work is always evolving because the work for equity and social justice is never a finished product (Aguirre, 2009, p. 296).

6. How does this definition align or not align with how you think about equity in mathematical classroom spaces? Can you describe how you understand the term equity in mathematics classrooms either with support from these definitions or as separate from these definitions? *
7. If you had no barriers to success, how would your definition of equity in mathematics classrooms drive your professional role as a mathematics teacher educator?
   a. *Probe:* What do you think your role is in the support and development of pre-service teachers with regards to equity in mathematics classrooms?
   b. *Probe:* I'm primarily interested in your role in teacher education; although, I recognize teacher education must be done in concert with schools and communities. How does you understanding of your professional role fit in the larger educational landscape with regards to equity in mathematics classrooms?
   c. *Probe:* What barriers to success do you see in reaching your professional vision with regards to equity in mathematics classrooms?
   d. *I believe you brought a copy of a biography with you today,* can you point to places where this understanding of inquiry has driven your past choices?
      i. How has your professional vision with regards to equity in mathematics classrooms changed as you have continued to develop

as a mathematics teacher educator? (Since graduate school, for example).

8. Can you describe how your definition of equity in mathematics classrooms fits into your current professional life? *
   a. *I believe you brought a professional artifact with you today,* can you describe how your professional artifacts might be an example of your conception of equity in mathematics classrooms?
   b. *Probe*: Do you see these as related to the education of pre-service teachers and continued professional learning of in-service teachers? If so, how?
9. Do you have a sense of the development trajectory of teachers' understandings of equity in mathematics classrooms as you have defined it? If so, can you describe such a trajectory?
   a. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should have access to before they enter the classroom with regards to equity in mathematics classrooms?
   b. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should gain by the middle of their career with regards to equity in mathematics classrooms?
   c. *Probe*: What do you think are the essential kinds of knowledge, practices, skills, or conceptions that teachers should gain by the time they begin to move into leadership roles, as mentor teachers, for example with regards to equity in mathematics classrooms?

10. What is the relationship between mathematical inquiry and equity in mathematics classrooms as you have described them here? Please be as specific as possible.*
    a. *Probe:* Is this relationship present in your classroom artifact? If so, can you describe how you see it in this context?
    b. *Probe:* Has this relationship been present in your professional vision? How would you describe it in terms of the scope of your current work?
    c. *Probe:* How do you envision that this relationship might exist in your future professional work? What barriers to success do you envision? What would support success for your future endeavors?

11. What else would you like me to know either about your professional vision, how you think about mathematical inquiry or equity in mathematics classrooms, or any other contextual issues?

Thank you very much for taking the time to talk to me today. I will be preparing this case over the next few months and will share it with you for your review if you would like.

APPENDIX B

SAMPLE PARTICIPANT RECRUITMENT EMAIL

The following email served as a recruitment tool and data generation protocol for the artifacts and biography.

Dear [Participant Name],

I hope this email finds you well. I am contacting you because I am currently seeking participants for my dissertation research study and [my contact's name] suggested that you might be willing to participate. In the study, I am seeking to better understand the breadth of work that mathematics teacher educators undertake in their professional work. In particular, I hope to understand better how mathematics teacher educators understand equity in mathematics classrooms and mathematical inquiry and what role those constructs play in your professional vision for your work.

I am asking participants for three major sources of information as part of this study. First, a professional artifact that is an enactment of your professional vision with regards to inquiry and equity. For example, the slides from a recent presentation that you have given at a conference or lecture, or a task that you designed for use with pre- or in- service teachers in a class or at a workshop. Second, a brief biography that describes your professional journey. This may be something that you have already written that you might annotate or if you feel like it, you can write something new. And finally, about an hour-long audio recorded interview about your professional vision as a mathematics teacher educator as it relates to inquiry and equity.

If you are interested in participating in the study, would you like to set up a brief phone call to discuss details and any questions you have about participation? Unfortunately, I am unable to offer any compensation at this time; however, I hope that you might consider participation as I believe this work will contribute to the field. There is currently a shortage of research on the work of mathematics teacher educators and I'm hoping together we can contribute to meeting that need.

Sincerely,

Miriam Gates