Affirmative action in India via vertical and horizontal reservations

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AFFIRMATIVE ACTION IN INDIA VIA VERTICAL AND HORIZONTAL RESERVATIONS

TAYFUN SÖNMEZ AND M. BUMIN YENMEZ

ABSTRACT. Built into the country's constitution, one of the world's most comprehensive affirmative action programs exists in India. Government jobs and seats at publicly funded educational institutions are allocated through a Supreme Court-mandated procedure that integrates a meritocracy-based system with a reservation system that provides a level playing field for disadvantaged groups through two types of special provisions. The higher-level provisions, known as vertical reservations, are exclusively intended for backward classes that faced historical discrimination, and implemented on a "set aside" basis. The lower-level provisions, known as horizontal reservations, are intended for other disadvantaged groups (such as women, disabled, or the economically disadvantaged), and they are implemented on a "minimum guarantee" basis. We show that, the Supreme Court-mandated procedure suffers from at least four major deficiencies. First and foremost, it is not well-defined when candidates can qualify for multiple horizontal reservations, a phenomenon that has been increasingly more common in recent years. Moreover, while a candidate can never lose a position to a less meritorious candidate from her own group under this procedure, she can lose a position to a less meritorious candidate from a higher-privilege group. This loophole under the Supreme Court-mandated procedure causes widespread confusion in India, resulting in countless lawsuits, conflicting judgements on these lawsuits, and even defiance in some of its states. We propose an alternative procedure that resolves these two major deficiencies and two additional ones.

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1. Introduction

While the term "affirmative action" was first used in 1961 when President John F. Kennedy signed Executive Order 10925, the 1950 Constitution of India had already mandated affirmative action to the members of its so-called "backward classes." The intended groups were Scheduled Castes (SC), which is the official term for Dalits or "untouchables," whose members have suffered millenniums-long systematic injustice due to their lowest status under the caste system, and Scheduled Tribes (ST), which is the official term for the indigenous ethnic groups of India, whose members were both physically and socially isolated from the rest of the society. Built into the country's constitution, affirmative action has been implemented in India through a reservation system that earmarks a certain percentage of government jobs and university seats, initially only to the members of SC and ST, and eventually to the members of Other Backward Classes (OBC) as well. Indeed, Article 16(4) of the Constitution of India reads:

> Nothing in this article shall prevent the State from making any provision for the reservation of appointments or posts in favor of any backward class of citizens which, in the opinion of the State, is not adequately represented in the services under the State.

In addition to this article, under which the intended beneficiaries are exclusively the members of backward classes, certain provisions are allowed under Article 16(1) for other groups of disadvantaged individuals—such as disabled citizens—to promote equality of opportunity.

While embedded in its 1950 Constitution, the scale, scope, and mechanisms of affirmative action in India have always been highly contested. As a result, the judiciary has always taken an active role in its implementation and enforcement. The following statement from the Supreme Court judgement *Indra Sawhney and others v. Union of India (1992)*,[1] one of the most influential judgements in the history of India, summarizes the sentiment on this important topic:

> The questions arising herein are not only of great moment and consequence, they are also extremely delicate and sensitive. They represent complex problems of Indian Society, wrapped and presented to us as constitutional and legal questions...
>
> There are occasions when the obvious needs to be stated and, we think, this is one such occasion. We are dealing with complex social, constitutional and legal questions upon which there has been a sharp division of opinion in the Society, which could have been settled more

[1] The case is available at https://indiankanoon.org/doc/1363234/ (last accessed on 03/10/2019).

> satisfactorily through political processes. But that was not to be. The issues have been relegated to the judiciary...
>
> There are other reasons, of course - that cause governments to leave decisions to be made by Courts. They are of expedient political character. The community may be so divided on a particular issue that a government feels that the safe course for it to pursue is to leave the issue to be resolved by the Courts, thereby diminishing the risk it will alienate significant sections of the Community.

India is a federal union that consists of twenty-nine states with a unitary, three-tiered judiciary made up of lower trial courts, a high court for each state, and a Supreme Court above all courts. The Supreme Court is not only vested with original jurisdiction to issue writs in defence of the fundamental rights listed in the Constitution, but also with appellate jurisdiction from the high courts to review and change the outcomes of their decisions (Neuborne, 2003). As a result, the Supreme Court of India has always played a central role in matters of affirmative action.

In its historical judgement of *Indra Sawhney (1992)*, the Constitution bench of the Supreme Court formulated *vertical reservations* as a tool to implement the higher-level provisions enabled by Article 16(4), and *horizontal reservations* as a tool to implement the lower-level provisions enabled by Article 16(1). The scope and the mechanics of these two types of reservations were distinctly differentiated in this historical case as follows:

(1) *Vertical reservations*:
   (a) They are also referred to as *social* reservations.
   (b) They are the highest form of special provisions that are intended exclusively for members of backward classes SC, ST, and OBC.
   (c) Being the highest form of special provisions, these reserved positions are to be earmarked to the members of backward classes in the form of a "set aside," which means positions secured by members of these classes on the basis of their own merit are not counted against vertically reserved positions.
   (d) They cannot exceed 50% of the positions.[2]
   (e) They are considered to be "reservations proper."

(2) *Horizontal reservations*:[3]
   (a) They are also referred to as *special* reservations.

[2]Not all states follow the 50% upper bound for vertical reservations. Most notable example is Tamil Nadu with 69.5%. See *The Print* story "4 states have gone over SC-imposed 50 percent reservation cap. Will Rajasthan follow?" available at https://theprint.in/india/governance/will-rajasthan-exceed-sc-imposed-50-per-cent-reservation-cap/16965/ (last accessed on 3/27/2019).

[3]It is important to emphasize that, even though the special provisions covered by Article 16(1) are referred to as horizontal reservations, they are not considered reservations. In its judgement *Indra Sawhney (1992)* the judges of the Supreme Court clarifies this technical distinction as follows: "Article 16(4) being

(b) They are the lesser form of special provisions that are intended for other disadvantaged groups of citizens (disabled, women, etc.), and adjustments through them cannot interfere with the number of positions vertically reserved for the backward classes.

(c) They are provided as a "minimum guarantee," which means positions secured by horizontal reserve-eligible candidates on the basis of their own merit, and, thus, without using the benefit of horizontal reservations, nonetheless are counted against horizontally reserved positions.

(d) A member of a backward class (say SC) who benefits from a horizontal reservation has to be adjusted within the vertically reserved positions for her own class SC at the expense of another member of SC, even if she has a higher merit score than a general class candidate who benefits from a horizontal reservation within open positions.

(e) They are considered to be a weightage, a special provision.

In the absence of horizontal reservations, implementation of vertical reservations is a straightforward task. First, open positions are to be filled one at a time according to merit score (including those from SC, ST, and OBC), and next for each of the backward classes SC, ST, and OBC, the vertically reserved positions are to be filled one at a time with the remaining candidates of the given backward class, based on merit score.

In many applications, however, there are also horizontal reservations, and in this case how exactly to integrate these two types of provisions is less clear. While the principles that dictate the implementation of reservations were clearly laid out in *Indra Sawhney (1992)*, an explicit procedure to implement them was not provided. Perhaps due to a large number of lawsuits brought to high courts, as well as the Supreme Court, an explicit procedure was provided to this end in another landmark judgement of the Supreme Court, in *Anil Kumar Gupta v. State of U.P. (1995)*.[4] Famous for its usage of mechanism design, this judgement has been used as a main reference in virtually all subsequent legal disputes—thousands of them—on integrated implementation of vertical and horizontal reservations. Indeed, it is referred to as a "class by itself" by the judges of the Madras High Court in their case *K.R.Shanthi vs The Secretary To Government (2012)*:[5]

> `This judgment is a class by itself which clearly makes a demonstration as to how selection has to be made as against the open quota and the reserved quota for various reserved classes by applying the vertical reservation and`

part of the scheme of equality doctrine it is exhaustive of reservation, therefore, no reservation can be made under Article 16(1)." Hence the phrase reservation, when used alone, always refers to vertical reservation.

[4]The case is available at `https://indiankanoon.org/doc/1055016/` (last accessed on 03/10/2019).

[5]The case is available at `https://indiankanoon.org/doc/41866200/` (last accessed on 03/10/2019).

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special reservation for women, physically handicapped etc., by following
horizontal reservation.
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One of our main contributions in this paper is bringing to light that the reference procedure given in *Anil Kumar Gupta (1995)* and mandated throughout India suffers from a number of technical shortcomings when candidates potentially qualify for multiple traits of horizontal reservation. This is a very widespread scenario in India, and indeed it was also the case in the dispute brought to the Supreme Court in *Anil Kumar Gupta (1995)*, the very judgement in which the procedure itself was introduced. Details of the Supreme Court-mandated procedure, and examples of its technical vulnerabilities, are presented in Section 2. One of the most critical shortcomings of this procedure is that mathematically speaking it is not well-defined, in the sense that it may generate different outcomes depending on the processing sequence of horizontal reservation traits. This possible multiplicity also means that the procedure is potentially vulnerable to favoritism.

While the details of the second shortcoming are somewhat elaborate, the source of its complications is easy to explain. Remember that horizontal reservation is intended to provide minimum guarantees to target special groups, in an effort to minimally distort the meritorious outcome at each vertical category. Consider an application where there is one unit of horizontal reservation for women candidates, and one unit of horizontal reservation for disabled candidates. The Supreme Court's procedure may sometimes overadjust the meritorious outcome, failing to take into consideration that admission of a disabled woman would be counted against both types of horizontal reservations, deeming the dismissal of two meritorious candidates—one for each horizontal reservation—unnecessary.

In addition to these technical vulnerabilities, which are admittedly somewhat obscure, the Supreme Court-mandated procedure has two additional shortcomings that are highly visible, and responsible for countless lawsuits throughout India. While vertical and horizontal reservations are introduced to protect disadvantaged groups, the Supreme Court-mandated procedure allows for situations where a candidate from a disadvantaged group, despite being more meritorious, may still lose a position to a candidate from a more privileged group. We refer to this undesirable situation as a failure to eliminate *justified envy*. This failure is highly inconsistent with the principle of *inter se* merit built into the Constitution of India, whereby a candidate can never lose a position to a less meritorious candidate provided that they are from the same group. Therefore, under the Supreme Court-mandated procedure a candidate can never lose a position to a less meritorious candidate from the same group, but she can lose a position to a less meritorious candidate from a more privileged group. In addition to this highly implausible possibility, the Supreme Court-mandated procedure may also penalize candidates for reporting their vertical reserve-eligible backward class. In that sense, the procedure is not

incentive compatible. These two shortcomings not only result in countless lawsuits, but also provide a loophole in the procedure that can be used to discriminate against members of backward classes. In Section 6, we provide evidence that these shortcomings are responsible for widespread confusion in India, often resulting in legal action, and even defiance in some states through the illegal implementation of better-behaved versions of the mandated procedure.

Motivated by the shortcomings of the Supreme Court-mandated procedure, we propose an alternative choice rule. Fortunately, there exists a unique *merit-maximal* outcome that complies with the minimum guarantee requirements of horizontal reservations (see Theorem 1), which can be used to describe a well-defined choice rule that integrates horizontal reservations with vertical reservations, along the principles outlined in *Indra Sawhney (1992)*. While the procedure given in *Anil Kumar Gupta (1995)* produces a conceptually related outcome when each candidate qualifies for at most one trait of horizontal reservation, it may not be merit-maximal or even well-defined when candidates potentially qualify for multiple traits of horizontal reservations. Given that the Supreme Court-mandated procedure not only suffers from technical shortcomings, but also allows for situations contrary to the spirit of positive discrimination, it will benefit strongly from an amendment with two important features: First and foremost, a Supreme Court-mandated procedure has to be well-defined. This can be achieved either by adopting our proposed choice rule given in Section 5, or by explicitly requiring candidates to apply for at most one trait of horizontal reservation.[6] In our view, it is also important to adopt an incentive-compatible procedure that eliminates justified envy given the challenges due to the lack of these features. This can also be achieved by adopting our proposed choice rule.

Reservation policy remains to be one of the most debated topics in India. In part due to the highly contested nature of the reservation policy, and in part due to the highly technical nature of vertical and horizontal reservations, there have been thousands of court cases in India on this important topic. This includes numerous erroneous decisions, even at state high courts. Since the reference procedure itself is not well-defined, it may contribute to these legal challenges in applications where candidates can qualify for multiple horizontal reserves. Amending the Supreme Court's procedure, and further clarifying its mechanics, may possibly reduce the scale of these legal disputes, thereby relieving the enormous burden currently placed on the judicial system.

[6]While in some practical applications candidates are requested to apply for at most one trait of horizontal reservation, (see for example the Madhya Pradesh High Court case Ameer Khan vs State Of M.P. And Ors. (2002), available at `https://indiankanoon.org/doc/1298571/`, last accessed on 02/27/2019), there are also applications where some candidates are eligible for multiple types of horizontal reservation (see for example U.P. State Entrance Examination, Dr. A. P. J. Abdul Kalam Technical University, Uttar Pradesh Information Brochure, available at `https://upsee.nic.in/publicinfo/Handler/FileHandler.ashx?i=File&ii=215&iii=Y`, last accessed on 02/27/2019).

Focusing on the case where a candidate qualifies for at most one trait of horizontal reservation, the case in which the Supreme Court-mandated procedure is well-defined, we provide one additional result. This result, presented in Theorem 4, has to do with the impact of horizontal reservations on backward communities. When the principles that govern the reservation policy were formulated in *Indra Sawhney (1992)*, one important point was emphasized repeatedly: The primary beneficiaries of the reservation policy should be the members of the backward classes, and the introduction of additional reservations or any other special provisions, shall not dilute the benefits intended for these historically disadvantaged groups. This is one of the main reasons why the judges of the Supreme Court ruled: "No reservation can be made for any class other than backward class either under Article 16(1) or 16(4)." The philosophy underlying this ruling is given in the judgement as follows:

> These backward people and others in like positions of helplessness are the favoured children of the Constitution. It is for them that ameliorative and remedial measures are adopted to achieve the end of equality. To permit those who are not intended to be so specially protected to compete for reservation is to dilute the protection and defeat the very constitutional aim.

To this end, the judges of the highest court also sanctioned that the number of positions vertically reserved for any of the backward classes is not to be reduced due to horizontal reserves. The Supreme Court-mandated procedure complies with this principle. However, we show in Theorem 4 that, despite this important measure to protect their interests, the number of positions allocated to members of backward classes weakly decreases with the introduction or increase of any horizontal reservation. This is because while the number of positions they receive through vertical reservations remains the same, the number of open-category positions their meritorious members receive weakly decreases with the introduction or increase of horizontal reservations. Thus, there is a tension between the overall interests of each backward class and the interests of the disadvantaged group that is provided with special horizontal reservations. This tension disappears under our proposed choice rule.

In addition to our contributions to the field of market design, we also have a conceptual contribution. While allocation of positions at various institutions solely based on merit is not uncommon, in many applications the admitted group has to comply with minimum guarantee requirements. We introduce the concept of *merit maximality* to capture a minimal deviation from the meritorious outcome, while at the same time respecting these requirements, and show that it uniquely defines an adjusted outcome. As it turns out, when each candidate qualifies for at most one horizontal reserve, the procedure given in

Anil Kumar Gupta (1995) can be interpreted as a two-step implementation of this idea, one for open positions and one for vertically reserved positions for each of the three backward classes.

1.1. Related Literature. While there is a rich literature on affirmative action policies in India and elsewhere, to the best of our knowledge, our paper is the first one to formally analyze vertical and horizontal reservation policies when they are jointly implemented.

There are a number of recent papers on reservation policies, most in the context of school choice. Abdulkadiroğlu and Sönmez (2003) study affirmative action policies that limit the number of students of a given type at schools. Kojima (2012) shows that the policy of limiting the number of majority students can hurt minority students, the intended beneficiaries. To overcome the detrimental effect of affirmative action policies based on majority quotas, Hafalir et al. (2013) introduce affirmative action policies based on minority reserves. Echenique and Yenmez (2015) study when there can be reservations for every type of student and provide an axiomatic characterization of choice rules with horizontal reservations that provide a minimum guarantee for students. We show in Theorem 2 that when each candidate qualifies for at most one trait of horizontal reservation, the unique merit-maximal choice rule that we construct is the same as the choice rule of Echenique and Yenmez. Ehlers et al. (2014) study more general affirmative action policies that adjust the priorities of students depending on the number of admitted students with different types.

Two recent papers on reservation policies can be interpreted as special cases of our paper. In Dur et al. (2016), the authors study allocation of Chicago's elite public high school seats to eighth graders, and compare various reservation policies. In Chicago, students are partitioned into four socio-economic classes based on their home addresses, and 17.5% of the seats are reserved for each socio-economic class as a set aside (i.e., as a vertical reservation), while the remaining 30% is open to all students and priority is based on their composite scores. The authors show that under some distributional assumptions on composite scores, and fixing the numbers of reserve seats, Chicago's policy is the best possible policy for the members of the lowest socio-economic class. In Dur et al. (2018), the authors study allocation of Boston's public school seats to students, where 50% of the seats at each school are reserved for neighborhood students as a minimum guarantee (i.e., on a horizontal reservation basis). The authors show that implementation of walk-zone seats on a minimum guarantee basis was inconsistent with the stated allocation policy in Boston, and it had the unintended consequence of virtually eliminating its walk-zone priorities. Based on this result, and public testimonies of Pathak and Sönmez to the School Committee, the city has given up walk-zone priorities altogether starting with 2013-14 school year, in an effort to increase the system's transparency. Both of these models are

applications of the more general model in Kominers and Sönmez (2016), where the authors introduce a matching model with slot-specific priorities. Unless there is at most one trait of horizontal reservation, our model cannot be covered by Kominers and Sönmez (2016).

Three additional papers on reservation policies include Aygün and Turhan (2016, 2017), where the authors study admissions to engineering colleges in India, and Aygün and Bó (2016), where the authors study admissions to Brazilian public universities. While the application in Aygün and Turhan (2016, 2017) is closely related to ours, their analysis is independent because they assume away horizontal reservations altogether. The Brazilian affirmative action application studied by Aygün and Bó (2016) relates to ours in that it also includes multi-dimensional reserves, but unlike ours their application is a special case of Kominers and Sönmez (2016).[7]

More broadly, our paper contributes to market design, where economists are increasingly taking advantage of advances in technology to design new or improved allocation mechanisms in applications as diverse as entry-level labor markets (Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), spectrum auctions (Milgrom, 2000), kidney exchange (Roth et al., 2004, 2005), internet auctions (Edelman et al., 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), refugee matching (Jones and Teytelboym, 2017; Delacrétaz et al., 2016; Andersson, 2017), and interdistrict school choice (Hafalir et al., 2018).

2. Institutional Background on Vertical and Horizontal Reservations

In its landmark judgement in *Indra Sawhney (1992)*, the Constitution bench of the Supreme Court coined the terms *vertical reservation* and *horizontal reservation*, while emphasizing in the following statement how these two types of affirmative action tools are to interact with each other:

> A little clarification is in order at this juncture: all reservations are not of the same nature. There are two types of reservations, which may, for the sake of convenience, be referred to as 'vertical reservations' and 'horizontal reservations'. The reservation in favour of scheduled castes, scheduled tribes and other backward classes [under Article 16(4)] may be called vertical reservations whereas reservations in favour of physically handicapped [under clause (1) of Article 16] can be referred to as horizontal reservations. Horizontal reservations cut across the

[7]Other related papers on design of reservation policies include Westkamp (2013), Kamada and Kojima (2015), and Fragiadakis and Troyan (2017).

> vertical reservations -- what is called interlocking reservations. To be more precise, suppose 3% of the vacancies are reserved in favour of physically handicapped persons; this would be a reservation relatable to clause (1) of Article 16. The persons selected against his quota will be placed in the appropriate category; if he belongs to SC category he will be placed in that quota by making necessary adjustments; similarly, if he belongs to open competition (OC) category, he will be placed in that category by making necessary adjustments. Even after providing for these horizontal reservations, the percentage of the reservations in favour of backward class of citizens remains -- and should remain -- the same...
>
> It is, however, made clear that the rule of 50% shall be applicable only to reservations proper; they shall not be - indeed cannot be - applicable to exemptions, concessions or relaxations, if any provided to 'Backward Class of Citizens' under Article 16(4).

It is further emphasized in the judgement that vertical reservations in favor of backward classes SC, ST, and OBC (which the judges refer to as *reservations proper*) are earmarked for these classes, and they cannot be reduced due to positions allocated through horizontal reservations.

While horizontal reservations can be implemented either as *overall horizontal reservations* for the entire set of positions, or as *compartment-wise horizontal reservations* within each vertical category including the open category (OC), the Supreme Court recommended the latter in their judgement of *Anil Kumar Gupta (1995)*:

> We are of the opinion that in the interest of avoiding any complications and intractable problems, it would be better that in future the horizontal reservations are comparmentalised in the sense explained above. In other words, the notification inviting applications should itself state not only the percentage of horizontal reservation(s) but should also specify the number of seats reserved for them in each of the social reservation categories, viz., S.T., S.C., O.B.C. and O.C.

The compartment-wise implementation of horizontal reservations ensures that, unlike the aforementioned case, the distributional benefits of the special horizontal reservations extend to all segments of the society. Consistent with the Supreme Court's recommendation, many states in India have adopted compartment-wise implementation of horizontal reservations in their allocation of public positions. For example, in an effort to increase the participation of women in public employment, compartment-wise horizontal reservations for female candidates is mandated by government order in several states, including in Bihar with 35%, Andhra Pradesh with $33\frac{1}{3}$%, and Madhya Pradesh, Uttarakhand, Chhattisgarh, Rajasthan, and Sikkim with 30% each. As such, we will focus on

compartment-wise horizontal reservations in the rest of the paper. That is, the term horizontal reservation will indicate its compartment-wise implementation throughout the paper.

Another important distinction between the vertical and horizontal reservations is given in a third Supreme Court judgement *Rajesh Kumar Daria v. Rajasthan Public Service Commission and others (2007)* as follows:[8]

> But the aforesaid principle applicable to vertical (social) reservations will not apply to horizontal (special) reservations. Where a special reservation for women is provided within the social reservation for Scheduled Castes, the proper procedure is first to fill up the quota for scheduled castes in order of merit and then find out the number of candidates among them who belong to the special reservation group of 'Scheduled Castes- Women'. If the number of women in such list is equal to or more than the number of special reservation quota, then there is no need for further selection towards the special reservation quota. Only if there is any shortfall, the requisite number of scheduled caste women shall have to be taken by deleting the corresponding number of candidates from the bottom of the list relating to Scheduled Castes. To this extent, horizontal (special) reservation differs from vertical (social) reservation. Thus women selected on merit within the vertical reservation quota will be counted against the horizontal reservation for women.

These reference Supreme Court judgements imply that

(1) the social vertical reservations are mandated to be implemented as an *over-and-beyond* (or equivalently as a *set aside*) system where a position allocated to a candidate from a reserve-eligible social category (i.e., from SC, ST, or OBC) on his own merit is not counted against the positions reserved for his social category, whereas
(2) the special horizontal reservations within the vertical reservations are mandated to be implemented as a *minimum guarantee* system, where a position secured by a candidate within her vertical category by merit is counted against the special horizontal reserves within this vertical category.

It is important to emphasize that an open-category position secured by a meritorious reserve-eligible social category candidate—say from SC—through her merit score, without invoking either a vertical or horizontal reserve, is not counted against the horizontal reserves within SC. It would only be counted against the horizontal reserves within the vertical category SC if she receives one of the positions reserved for her vertical category

[8]The case is available at `https://indiankanoon.org/doc/698833/` (last accessed on 03/12/2019).

SC. This distinction is at the heart of the judgement in *Dwarika Prasad Patel and Others vs. State of Chhattisgarh (2017)* by the High Court of Chhattisgarh.[9]

2.1. Implementation of Vertical and Horizontal Reservation: SCI-VHR Choice Rule. The judges of the Supreme Court did not merely specify the principles that govern the implementation of the social vertical and special horizontal reserves; they also provided a procedure to implement these reserves in their judgement of *Anil Kumar Gupta (1995)*. The procedure provided by the Supreme Court in this case, using the court's own wording, is as follows:

> The proper and correct course is to first fill up the O.C. quota (50%) on the basis of merit: then fill up each of the social reservation quotas, i.e., S.C., S.T. and B.C; the third step would be to find out how many candidates belonging to special reservations have been selected on the above basis. If the quota fixed for horizontal reservations is already satisfied - in case it is an over-all horizontal reservation - no further question arises. But if it is not so satisfied, the requisite number of special reservation candidates shall have to be taken and adjusted/accommodated against their respective social reservation categories by deleting the corresponding number of candidates therefrom. (If, however, it is a case of compartmentalised horizontal reservation, then the process of verification and adjustment/accommodation as stated above should be applied separately to each of the vertical reservations...

The adjustment phase of the procedure for special horizontal reserves is further elaborated in the Supreme Court judgement *Rajesh Kumar Daria (2007)* as follows:

> If 19 posts are reserved for SCs (of which the quota for women is four), 19 SC candidates shall have to be first listed in accordance with merit, from out of the successful eligible candidates. If such list of 19 candidates contains four SC women candidates, then there is no need to disturb the list by including any further SC women candidate. On the other hand, if the list of 19 SC candidates contains only two woman candidates, then the next two SC woman candidates in accordance with merit, will have to be included in the list and corresponding number of candidates from the bottom of such list shall have to be deleted, so as to ensure that the final 19 selected SC candidates contain four women SC candidates. [But if the list of 19 SC candidates contains more than four women candidates, selected on own merit, all of them will continue in the list and there is no question of deleting the excess women candidate on the ground that 'SC-women' have been selected in excess of the prescribed internal quota of four.]

[9]The case is available at https://indiankanoon.org/doc/63082828/ (last accessed on 03/12/2019).

We refer to this choice rule as the *Supreme Court of India Vertical & Horizontal Reservations choice rule*, or *SCI-VHR choice rule* in short.

2.2. Shortcomings of the SCI-VHR Choice Rule. One of our objectives in this paper is to show that while these Supreme Court judgements are the main references for integrating special horizontal reservations with social vertical reservations, the SCI-VHR choice rule sanctioned by the Supreme Court throughout India has three important technical shortcomings the way it is defined in *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)*. Of these three shortcomings, the first one is likely an oversight, and straightforward to fix. The latter two shortcomings, on the other hand, are more substantial and require further analysis. Our first example illustrates the first—easy to fix—shortcoming.

Example 1. Consider a set of candidates with two general-category men m_1^G, m_2^G, two backward class men m_3^R, m_4^R, one general-category woman w_1^G, and one backward class woman w_2^R.

There are four positions to be allocated. Two of these positions are set aside for candidates with backward classes. The remaining two positions are to be allocated as open positions, for which all candidates are qualified. In addition, one of the open-category positions and one of the positions for backward classes are each horizontally reserved for women within their vertical categories.

Candidates have the following ranking according to their merit scores:

$$m_1^G > m_3^R > m_2^G > w_1^G > w_2^R > m_4^R.$$

Let us execute the SCI-VHR choice rule following the steps given in *Anil Kumar Gupta (1995)*:

Step 1 (Fill up the open-category positions): Candidates m_1^G and m_3^R are on hold for the open positions.

Step 2 (Fill up the vertical social reservation positions): The only remaining vertical reserve-eligible candidates are m_4^R and w_2^R. Both are on hold for the positions reserved for backward classes.

Step 3 (Adjustment for the horizontal woman reservation): Neither of the two candidates on hold for open positions from Step 1 is a woman. Therefore, an adjustment is necessary. The female candidate w_1^G receives the second open position, displacing the candidate with lower score on hold from Step 1, namely m_3^R. For the backward class positions, the woman reserve is already satisfied, and thus no further adjustment is necessary.

Thus the outcome of SCI-VHR choice rule is finalized as $\{m_1^G, w_1^G, m_4^R, w_2^R\}$. The anomaly here is that the higher merit score male backwards class candidate m_3^R fails to receive a

position, while the lower merit score male backwards class candidate m_4^R receives one despite belonging the same vertical reserve category and having the same set of horizontal traits. □

SCI-VHR choice rule, as it is described in *Anil Kumar Gupta (1995)*, fails to adjust the (relatively higher merit score) social reserve-eligible candidates, who may be displaced in Step 3, from their tentative open position assignment of Step 1, due to a special horizontal reserve adjustment made in Step 3. The lack of this adjustment results in the failure of the observance of *inter se* merit among candidates of identical social category and special traits, contradicting the Supreme Court judgement in *Anurag Patel v. Uttar Pradesh Public Service Commission (2004)*:[10]

> The authorities should have compared the candidates who are to be appointed on general merit as also candidates who are to be appointed as against the reserved vacancies and while making appointments the inter se merit of the reserved candidates should have been considered and they must have been given the option treating each service separately. As this exercise was not followed, less meritorious candidates got appointment to higher positions whereas more meritorious candidates had to be satisfied with lower positions.

This shortcoming, however, is easily fixed by

(1) first tentatively assigning the open positions based on merit scores,
(2) next implementing the adjustments for special horizontal reservations finalizing the allocation of open positions,
(3) then tentatively assigning the vertically reserved positions to remaining social reserve-eligible candidates based on their merit scores, and
(4) finally implementing the adjustments for special horizontal reservations within vertically reserved categories.

In this way, the allocation of open positions is finalized before the allocation process for social reserve categories (i.e. for SC, ST, and OBC) starts. Since this anomaly seems to be an oversight, the term SCI-VHR choice rule will be used to indicate the corrected version in the rest of the paper.

The next two shortcomings are more substantial, and they have to do with the accommodation of horizontal reservation constraints at the adjustment stage. We will next show that the SCI-VHR choice rule can result in the dismissal of too many meritorious candidates, in an effort to accommodate the minimum guarantee requirements of horizontal reservations, and moreover its outcome may not even be uniquely determined. That is,

[10]The case is available at `https://indiankanoon.org/doc/1962361/` (last accessed on 03/10/2019).

strictly speaking the SCI-VHR choice rule is not always well-defined. Importantly, these anomalies are peculiar to applications where a candidate can qualify for more than one trait of special horizontal reservation, a very common occurrence throughout India. In fact, there are five horizontal reservation traits in the lawsuit which was decided in *Anil Kumar Gupta (1995)*, the very judgement which resulted in the introduction of the SCI-VHR choice rule. The horizontal reservation traits in this judgement are:

(1) dependents of freedom fighters (5%),
(2) children of deceased/disabled soldiers (2%),
(3) physically handicapped candidates (2%),
(4) candidates belonging to hill areas (3%), and
(5) candidates belonging to Uttaranchal areas (3%).

Clearly a candidate can qualify for two or more of these traits, such as a handicapped candidate who belongs to an Uttaranchal area. If such a candidate is allocated a position, he would count towards not only the horizontal reservation for handicapped candidates, but also the horizontal reservation for candidates who belong to Uttaranchal areas. This simple observation is at the heart of the two more substantial failures of the SCI-VHR choice rule. One vertical category is sufficient to illustrate these failures, and hence we will illustrate them through two examples with open positions only.

Example 2. Consider a set of candidates with four men m_1, m_2, m_3, m_4 and two women w_1, w_2. Candidates w_2 and m_4 are disabled. There are three open positions. There is one horizontal reservation for the female candidates and one horizontal reservation for the disabled candidates. Starting with the candidate with the highest merit score, the candidates are ranked according to their merit scores as follows:

$$m_1 > m_2 > m_3 > w_1 > m_4 > w_2.$$

The SCI-VHR choice rule works as follows: Initially the three highest merit score candidates m_1, m_2, m_3, are selected for the three open positions. Since all of these candidates are men and none of them is disabled, neither the minimum guarantee for female candidates nor the minimum guarantee for disabled candidates is satisfied. While the adjustment process to accommodate the horizontal reservations is clearly indicated in *Rajesh Kumar Daria (2007)* for a single trait of horizontal reservations, this reference court case fails to specify how to proceed with the adjustment process when there are multiple traits of horizontal reservations. While it is not always the case, in this example the processing order of the horizontal reservation traits is immaterial. Thus, suppose that the adjustment process starts with the horizontal reservation constraint for female candidates. In that case, the highest merit score female candidate w_1 has to be chosen at the expense of the male candidate m_3 by the mechanics of the adjustment process given in *Rajesh Kumar Daria (2007)*

described above. Next, the disabled horizontal reservation constraint is accommodated by including the highest score disabled candidate m_4 in the choice set, at the expense of a second displaced candidate m_2. At this point, both horizontal reservation constraints are satisfied, and the outcome of the SCI-VHR choice rule is finalized as

$$\{m_1, w_1, m_4\}.$$

Observe that the same outcome is obtained if the disabled horizontal reservation is accommodated first and the female horizontal reservation is accommodated next.

Therefore, through the adjustment phase, two higher merit-score candidates m_2 and m_3 are removed from the original merit-based choice set. We argue that the removal of the candidate m_2 is unjustified since both horizontal reservation constraints could have been accommodated with only one adjustment, namely by including the disabled female candidate w_2 at the expense of the candidate m_3. When the SCI-VHR choice rule was originally introduced, the judges of the Supreme Court in *Anil Kumar Gupta (1995)* indicated that, for the purpose of accommodating the horizontal reservations "the requisite number of special reservation candidates shall have to be taken and adjusted/accommodated against their respective social reservation categories by deleting the corresponding number of candidates therefrom." Since both of the special horizontal reservations can be satisfied with the inclusion of the disabled female candidate w_2, we argue that the requisite number is only one. The outcome that has to be selected with only one adjustment is

$$\{m_1, m_2, w_2\}.$$

But this outcome cannot be achieved by accommodating the horizontal reservation types one at a time. Instead, a forward-looking approach is needed for the adjustment phase. □

In their judgement *Indra Sawhney (1992)*, the judges of the Constitution bench of the Supreme Court urged that horizontal reservations be used in a restrictive way: "Preferential treatment in shape of weightage etc. can be given to those who are covered in Article 16(1) but that too has to be very restrictive." In contrast, the SCI-VHR choice rule may result in an over-adjustment due to its myopic processing of horizontal reservation traits.

Our third example illustrates in our view the most critical anomaly, caused by the adjustment stage of the SCI-VHR choice rule.

Example 3. Consider an application with four horizontal reservation traits referred to as t_1, t_2, t_3, t_4, and suppose that, based on the allocation of the open positions on the basis of merit in Step 1, the minimum guarantee fails to hold for each of these four categories by one candidate. Suppose that the highest merit score candidates among those not chosen

in Step 1 are candidates a, b, c, d, and starting with the candidate with highest score they are merit-ranked as $a > b > c > d$. Furthermore, each candidate qualifies for two of the horizontal reservation traits indicated under the agent as follows.

a	b	c	d
t_1	t_1	t_2	t_3
t_2	t_3	t_4	t_4

For example, candidate a qualifies for the horizontal reservation in traits t_1 and t_2. In order to show that the SCI-VHR choice rule is not well-defined, we will carry out the adjustment process with two sequences of traits: $t_1 - t_2 - t_3 - t_4$ and $t_4 - t_3 - t_2 - t_1$.

Case 1 ($t_1 - t_2 - t_3 - t_4$) : Of the four candidates, candidates a and b are the only ones who qualify for horizontal reservation in trait t_1. Having a higher merit score than candidate b, candidate a will be the first beneficiary of the adjustment process. Observe that candidate a qualifies not only for the horizontal reservation in trait t_1, but also for the horizontal reservation in trait t_2. Therefore, with his inclusion the minimum guarantee is satisfied both in trait t_1 and also in trait t_2. Hence, there is no further need for an additional adjustment for trait t_2. Next, consider the adjustment for trait t_3. Of the remaining three candidates, candidates b and d are the only ones who qualify for horizontal reservation in trait t_3. Having a higher merit score than candidate d, candidate b will be the second beneficiary of the adjustment process. By this point the minimum guarantee is satisfied in traits t_1, t_2, and t_3. Finally, consider the adjustment for trait t_4. Of the remaining two candidates c and d, each one qualifies for horizontal reservation in trait t_4. Having a higher merit score than candidate d, candidate c will be the third beneficiary of the adjustment process. Since all minimum guarantees are satisfied by this point, no further adjustment is needed and the beneficiaries of the adjustment process are candidates a, b, and c.

Case 2 ($t_4 - t_3 - t_2 - t_1$) : Of the four candidates, candidates c and d are the only ones who qualify for horizontal reservation in trait t_4. Having a higher merit score than candidate d, candidate c will be the first beneficiary of the adjustment process. Observe that candidate c qualifies not only for horizontal reservation in trait t_4, but also for horizontal reservation in trait t_2. Next, consider the adjustment for trait t_3. Of the remaining three candidates, candidates b and d are the only ones who qualify for horizontal reservation in trait t_3. Having a higher merit score than candidate d, candidate b will be the second beneficiary of the adjustment process. Since candidate b qualifies not only for horizontal reservation in trait t_1 but also for horizontal reservation in trait t_3, all four minimum guarantees are satisfied by this point. Hence, no further adjustment is needed, and the beneficiaries of the adjustment process are candidates b and c.

Since two different group of candidates benefit from the adjustment process, the outcome of the SCI-VHR choice rule depends on a detail that is not included in the description of the rule. □

It is important to observe that the last anomaly cannot be fixed even if the selection of the candidates is carried out in a forward-looking manner, to minimize the number of meritorious candidates who are displaced due to adjustments. Under this modification, the mechanics of Case 1 will change, but the mechanics of Case 2 will remain the same since it already achieves the minimum number of adjustments. Under this "forward-looking" version of Case 1, candidate d will be chosen as the second beneficiary after candidate a since his admission counts towards both of the minimum guarantees in traits t_3 and t_4. But in this case the beneficiaries of the adjustment process under Case 1 will be candidates a, d, which is still different than the beneficiaries of the adjustment process under Case 2 who are candidates b, c.

Based on the anomalies presented in Examples 2 and 3, the adjustment process for horizontal reserves shall not be carried out one at a time. But if the adjustments cannot be carried out one horizontal reservation trait at a time, how shall it be carried out? Is there even a well-defined method to carry out the adjustment process while respecting its original philosophy in the Supreme Court judgement *Indra Sawhney (1992)*? The answer to this important question is yes, but it requires a formal analysis. To this end, we next introduce a model. This will allow us not only to fix the deficient adjustment phase of the SCI-VHR choice rule, but also to present two additional, and this time highly-visible shortcomings of the SCI-VHR choice rule which have been exposing its implementation to numerous lawsuits throughout India. Our proposed choice rule provides a remedy for all these shortcomings as well.

3. Model and Preliminary Results

Consider a finite set of candidates $\mathcal{A}$ who apply for q positions. There are three social categories of backward class candidates referred to as "Scheduled Castes" (SC), "Scheduled Tribes" (ST), and "Other Backward Classes" (OBC). For each of these social categories, a number of positions is earmarked exclusively for its members.[11] The set of these reserve-eligible categories is denoted by $\mathcal{R} = \{\text{SC}, \text{ST}, \text{OBC}\}$. A candidate who does not belong to one of the three reserve-eligible categories in $\mathcal{R}$, belongs to the

[11] In some applications in India, vertical reservation-eligible categories are further divided into smaller groups. We focus on the generic case for notational simplicity only, and the entire analysis extends easily regardless of the number of reservation-eligible categories.

"General" category (G). Unlike the reserve-eligible categories, there are no positions earmarked for the members of the general category. The set of all categories is denoted by $\mathcal{C} = \{\text{SC}, \text{ST}, \text{OBC}, \text{G}\}$.

We use the function ρ to indicate category membership. For a candidate $c \in \mathcal{A}$ and a reserve-eligible category $X \in \mathcal{R}$, $\rho(c) = \{X\}$ indicates that candidate c is a member of the reserve-eligible category X. For a candidate $c \in \mathcal{A}$, $\rho(c) = \emptyset$ indicates that candidate c is a member of the general category.

In addition to being a member of a category, each candidate also has a (possibly empty) set of traits. Each trait represents a disadvantage in the society, and the government may provide the candidates who have this trait with easier access to positions to level the playing field. For example, being a woman and having a disability are traits that are commonly used by state governments. The set of traits is finite and denoted by $\mathcal{T}$. The set of traits of candidate c is denoted by $\tau(c) \subseteq \mathcal{T}$.

Finally, each candidate has a distinct merit score, where the score of candidate c is denoted as $\sigma(c) \in \mathbb{R}_+$.

An **allocation problem** is given by a tuple $\langle \mathcal{A}, \mathcal{C}, \mathcal{T}, \rho, \tau, \sigma \rangle$.

3.1. Vertical Reservations Only. Affirmative action for the backward categories ST, SC, and OBC is implemented by setting aside a number of positions for each of these categories. These are called *vertical or social reservations*. Let r^{SC}, r^{ST}, and r^{OBC} denote the number of positions set aside for SC, ST, and OBC candidates, respectively. The rest of the positions are open for all candidates. Let r^O denote the number of open-category positions, so $r^O = q - (r^{SC} + r^{ST} + r^{OBC})$. These positions are open to candidates from all categories and allocated to them based on their merit scores. When a candidate from a reserve-eligible category receives an open-category position on his own merit, that is not counted against the vertical reservations for his social category. This is the sense in which vertically reserved positions are "set aside" for members of reserve-eligible social categories, regardless of who receives open-category positions.

When only vertical reservations exist, a **choice rule** is a function C such that for any set of candidates $A \subseteq \mathcal{A}$ and parameters q and $(r^X)_{X \in \mathcal{R}}$,

$$C(A|q, (r^X)_{X \in \mathcal{R}}) \subseteq A \text{ with } \left| C(A|q, (r^X)_{X \in \mathcal{R}}) \right| \leq q.$$

In words, for a given number of positions, q, a profile of the number of positions for each reserve-eligible social category, $(r^{SC}, r^{ST}, r^{OBC})$, and a set of candidates A who are applying for the positions, the choice rule C produces a subset of candidates who are allocated these positions. When there is no ambiguity about the parameters, we denote the set of chosen candidates by $C(A)$.

In the absence of horizontal or special reservations, which we study in the next section, the aforementioned principles on reservation policies define a unique subset of candidates that must be chosen from any set of applicants. By law, an allocation must respect *inter se* merit: if a category-X candidate with a lower score is given a position, then a category-X candidate with a higher score must also be given a position. Furthermore, when a candidate with a reserve-eligible category claims an open-category position by merit, she does not count for the vertical reservations for her category. Therefore, candidates with the highest merit scores must be allocated the open-category positions first, to determine which candidates are eligible for these positions. Then, positions reserved for socially backward categories can be allocated to the remaining candidates, again based on their merit scores. More formally, this choice rule can be described as follows.

Choice Rule C^{ver}

Step 1: Choose the candidates with the r^O highest merit scores for the open-category positions.

Step 2: For each of SC, ST, and OBC category, choose the remaining candidates from that category with the highest merit scores up to the number of positions reserved for that category, i.e., r^{SC}, r^{ST}, and r^{OBC}, respectively.

Since there is no overlap between SC, ST, and OBC candidates, the order in which the three reserve-eligible categories are processed in Step 2 does not matter.

3.2. Horizontal Reservations Only. With the interpretation of Article 16(1) in Supreme Court judgement *Indra Sawhney (1992)*, disadvantaged candidates with certain traits were provided with some lower-level special provisions referred to as *horizontal* or *special reservations*. These reservations provide a minimum guarantee for the number of candidates with these traits who are allocated positions. Let $r_t > 0$ be the number of reserved positions for trait t. Assume that the sum of positions reserved for candidates with different traits is less than q, so $\sum_{t\in\mathcal{T}} r_t \leq q$.

While horizontal reservations are provided at each vertical category, in this subsection we consider the more basic case when there are no vertical reservations. In this case, when only horizontal reservations exist, a **choice rule** is a function C such that for any set of candidates $A \subseteq \mathcal{A}$ and parameters q and $(r_t)_{t\in\mathcal{T}}$,

$$C(A|q,(r_t)_{t\in\mathcal{T}}) \subseteq A \text{ with } |C(A|q,(r_t)_{t\in\mathcal{T}})| \leq q.$$

As in the previous subsection, when there is no ambiguity about the parameters, we denote the set of chosen candidates by $C(A)$.

Let $A \subseteq \mathcal{A}$ be a set of candidates who apply for a position. Say that $A' \subseteq A$ **satisfies trait-t reservations for A**, if, whenever the number of trait-t candidates in A' is strictly less than r_t, then there exists no trait-t candidate in $A \setminus A'$. In other words, when A' satisfies

trait-t reservations for A, then either the number of trait-t candidates in A' is at least r_t or all trait-t candidates in A are selected in A'. Say that $A' \subseteq A$ **satisfies the horizontal reservations for A** if, for every trait t, A' satisfies trait-t reservations for A.

Let A be the set of candidates who are applying for the positions. Consider the following choice rule.

Choice Rule C^{hor}

Step 1: Consider all subsets of A that satisfy the horizontal reservations for A. Choose the candidate with the highest merit score who is in any of these subsets. Let A_1 denote the set including only this candidate.

Step k ($\mathbf{k} \in [2, q]$): Consider all subsets of A that include A_{k-1} and satisfy the horizontal reservations for A. If the only such subset is A_{k-1}, then stop and return this set. Otherwise, from $A \setminus A_{k-1}$, choose the candidate with the highest merit score who is in any of these subsets. Let A_k denote the set of candidates chosen so far.

When the number of applicants is less than q, then this procedure chooses all the applicants. However, if there are more than q applicants, then it stops at Step q, and returns A_q which has q candidates.

Consider two different sets of candidates A and A'. Say that A **dominates** A' if, there exists a candidate in $A \setminus A'$ with a merit score that is strictly greater than the merit scores of all candidates in $A' \setminus A$. Domination is a binary relation that we use to compare different sets based on merit scores of candidates. It is easy to see that domination is a *strict partial order*.[12]

Given a set of applicants, there are typically multiple subsets of these applicants that satisfy the horizontal reservations. We use the domination relation to determine which subset of applicants are "more deserving" of the positions.

Definition 1. *A choice rule C is **merit maximal** if, for every set of candidates A,*

(1) $C(A)$ satisfies the horizontal reservations for A, and

(2) $C(A)$ dominates A' for any other set $A' \subseteq A$ that satisfies the horizontal reservations for A.

We are ready to present our first result:

Theorem 1. *C^{hor} is the unique merit-maximal choice rule.*

Note that this result holds without making any assumptions about the traits of candidates. However, when each candidate has at most one trait, there is a simpler choice rule that gives the unique merit-maximal outcome. We refer to this alternative choice rule as

[12] A binary relation is a strict partial order if it is *irreflexive*, *transitive*, and *asymmetric*.

C^{mg}, since it first accommodates the "minimum guarantees" for each trait, and then fills all the remaining positions.

Theorem 2. *Suppose that each candidate has at most one trait. Then C^{hor} is equivalent to the following choice rule.*

Choice Rule C^{mg}

Step 1: *For each trait $t \in \mathcal{T}$, if the number of trait-t candidates is less than r_t, then choose all of them. Otherwise, if this number is at least r_t, then choose trait-t candidates with the r_t highest merit scores.*

Step 2: *For the unfilled positions, choose the remaining candidates with the highest merit scores.*

Note that C^{mg} is not well-defined when candidates have multiple traits, because the order in which traits are processed in Step 1 is not specified. However, the order becomes immaterial when each candidate has at most one trait, and in this case, C^{mg} is the same as the unique merit-maximal choice rule C^{hor}.[13] In Section 4.1, we present a lawsuit from Chhattisgarh, where the result in Theorem 2 would have been helpful if known at the time by the judges of the court.

4. SCI-VHR Choice Rule under Vertical and Horizontal Reservations

We are now ready to analyze the model in its full generality, with both vertical and horizontal reservations. For any social category $X \in \mathcal{R}$, we refer to the positions vertically reserved for its members as category-X positions. Similarly, positions open for candidates from all categories are referred to as open-category positions. For any trait $t \in \mathcal{T}$ and reserve-eligible social category $X \in \mathcal{R}$, let r_t^X denote the number of category-X positions horizontally reserved for trait-t candidates. Only category-X candidates are eligible to use these positions. In addition, let r_t^O denote the number of open-category positions horizontally reserved for trait-t candidates. Candidates from reserve-eligible social categories can receive these positions by their merit only, without invoking the benefits of any reservation, but not by invoking the benefits of trait-t horizontal reservations. Only the general-category candidates can invoke these benefits. These horizontal reservations are provided on a minimum guarantee basis, and

[13] This choice rule was first introduced in Echenique and Yenmez (2015) and is conceptually related to the slot-specific choice rule defined by Kominers and Sönmez (2016). The relation can be seen as follows. For each trait t, let $q_t \equiv \min\{r_t, \{c \in A | t \in \tau(c)\}\}$ be the number of slots reserved for trait-t candidates and $q^O \equiv q - \sum_t q_t$ be the number of open slots. For trait-t slots, only trait-t candidates are ranked, while for open slots all candidates are ranked according to their merit scores. First, slots reserved for traits are filled, after which open slots are filled. In Kominers and Sönmez (2016), the types of slots are fixed, whereas in this construction the types of slots depend on the trait distribution of candidates. In other words, C^{mg} can be thought of as an application of the slot-specific choice rules once the types of slots can be determined endogenously, depending on the trait distribution of candidates.

(1) whenever a candidate from a reserve-eligible category X with trait-t receives a category-X position, this position is counted against the category-X trait-t horizontal reservations,
(2) whenever a candidate from the general category with trait-t receives an open-category position, this position is counted against the open-category trait-t horizontal reservations, and
(3) whenever a candidate from a reserve-eligible category X with trait-t receives an open-category position by his merit, this position is counted against the open-category trait-t horizontal reservations (and not against category-X trait-t horizontal reservations).

For each category, assume that the sum of horizontal reservations for this category is less than the number of positions set aside for this category, i.e., for every category $X \in \mathcal{C}$, $\sum_{t\in\mathcal{T}} r_t^X \leq r^X$.

A **choice rule**, when both horizontal and vertical reservations exist, is a function C such that for any set of candidates $A \subseteq \mathcal{A}$ and parameters q, $(r^X)_{X\in\mathcal{R}}$, and $(r_t^X)_{t\in\mathcal{T},X\in\mathcal{R}}$,

$$C(A|q,(r^X)_{X\in\mathcal{R}},(r_t^X)_{t\in\mathcal{T},X\in\mathcal{R}}) \subseteq A \text{ with } \left|C(A|q,(r^X)_{X\in\mathcal{R}},(r_t^X)_{t\in\mathcal{T},X\in\mathcal{R}})\right| \leq q.$$

As in the previous subsections, we denote the set of chosen candidates simply by $C(A)$ when there is no ambiguity about the parameters.

We next provide a formal definition of the SCI-VHR choice rule. For a set of candidates who are allocated category-X positions, say that trait-t is **saturated for X** if the number of trait-t candidates assigned to category-X positions is strictly more than r_t^X. Say that a candidate c who is assigned a category-X position is **exposed** if either she does not have a trait or her trait $\tau(c)$ is saturated for X.

SCI-VHR Choice Rule $C^{\mathbf{SCI}}$

Step 0: Construct the set of open-category horizontal reservation eligible candidates A_1 as the union of the set of the candidates with the r^O highest merit scores and the set of general-category candidates.

Step 1(i): Tentatively choose the candidates with the r^O highest merit scores for the open-category positions.

Step 1(ii): If all open-category horizontal reservations are satisfied for A_1, then proceed to Step 2(i). Otherwise, consider a trait t such that open-category trait-t reservations are not satisfied for A_1. Replace

- the exposed candidate with an open-category position who has the lowest merit score among such candidates with
- the unchosen general-category trait-t candidate who has the highest merit score among the unchosen general-category trait-t candidates in A_1.

Update the set of chosen candidates. Repeat Step 1(ii) until all open-category horizontal reservations are satisfied for A_1.

Step 2(i): For each reserve-eligible social category $X \in \mathcal{R}$, consider the set of category-X candidates who are not chosen yet. Denote this set by A_2^X. Tentatively choose the candidates in A_2^X with the r^X highest merit scores for the category-X positions.

Step 2(ii): For each trait t, whose category-X reservations are not satisfied for A_2^X, replace

- the exposed candidate with a category-X position who has the lowest merit score among such candidates with
- the unchosen category-X trait-t candidate who has the highest merit score among the unchosen trait-t candidates in A_2^X.

Update the set of chosen candidates for category X. Repeat Step 2(ii) until all category-X horizontal reservations are satisfied for A_2^X.

This process ends in finite time, because, there can only be a finite number of iterations at Steps 1(ii) and 2(ii), and a distinct candidate is chosen at each iteration.

Recall that in Example 3 of Section 2.2, we have shown that the Supreme Court-mandated choice rule SCI-VHR may fail to be well-defined if candidates are allowed to have multiple traits. Hence, in the rest of this section, we assume that each candidate has at most one trait.

For each category, C^{SCI} starts by tentatively choosing candidates with the highest merit scores eligible for positions in this category. Then it makes the necessary adjustments for horizontal reservations that are not satisfied. In the next result, we provide an equivalent choice rule that starts by filling the positions that are reserved horizontally. Therefore, this choice rule does not need any adjustment steps.

Theorem 3. *Suppose that each candidate has at most one trait. Then the Supreme Court choice rule C^{SCI} is equivalent to the following.*

Choice Rule $\mathbf{C_{1h}^{SCI}}$

Step 0: *Construct the set of open-category horizontal reservation-eligible candidates A_1 as the union of the set of the candidates with the r^O highest merit scores and the set of general-category candidates.*

Step 1: *Choose $C^{hor}(A_1|r^O, (r_t^O)_{t\in\mathcal{T}})$ for the open-category positions.*

Step 2: *For each reserve-eligible social category $X \in \mathcal{R}$, apply $C^{hor}(\cdot|r^X, (r_t^X)_{t\in\mathcal{T}})$ to the category-X candidates who are not chosen in Step* 1.

The order in which reserve-eligible social categories are processed in Step 2 of C_{1h}^{SCI} does not matter, because each candidate has at most one social category.

We next relate our analysis in this section to a court case from Chhattisgarh High Court.[14]

4.1. Case Study: Ashish Sharma & Ors. vs. State Of Chhattisgarh & Ors. on August 18th, 2003. In this Chhattisgarh High Court case, the petitioners challenge the implementation of horizontal reservations for women at a Chhattisgarh Medical School. There are 42 open seats, of which 13 are horizontally reserved for women, one is horizontally reserved for soldiers, and one is horizontally reserved for freedom fighters. In order to allocate the 42 open seats, the respondents followed a procedure that is mechanically different from the procedure for SCI-VHR choice rule C^{SCI}: They first allocated 13 seats to the highest merit score women, next allocated 27 seats to the remaining highest score candidates bringing the total to 40, and since horizontal reserves for soldiers and freedom fighters were not satisfied by this point, they assigned one seat each to the remaining candidates with the highest merit who has one of these two traits.[15] In addition to the 13 seats allocated to women in the first step, an additional 12 seats were also allocated to women among the 27 seats allocated in the second step, for a total of 25 seats. Observe that by Theorems 2 and 3, the procedure followed by the respondents gives the same outcome as the choice rule C^{SCI}. However, failing to observe this equivalence, the male petitioners challenged the procedure used by the respondents. This equivalence was not explained clearly by counsel, which in turn resulted the judges of the high court siding with the petitioners, requiring them to repeat the allocation process using the SCI-VHR choice rule.

This case illustrates that, despite its prominent role in India, even the most basic features of horizontal reservations are not well understood.

4.2. A Tension Between Beneficiaries of Vertical Reservations and Horizontal Reservations. We conclude this section by providing a comparative statics result for C^{SCI}, which illustrates a tension between the beneficiaries of vertical reservations and horizontal reservations.

Theorem 4. *Suppose that each candidate has at most one trait. Then, for every reserve-eligible social category $X \in \mathcal{R}$, the introduction or increase of any horizontal reservations weakly decreases the number of positions allocated by C^{SCI} to category-X candidates.*

This result provides a tradeoff between horizontal and vertical reservations, which are used to promote different types of socially backward groups in the society. Introduction

[14]The case is available at `https://indiankanoon.org/doc/820122/` (last accessed on 03/05/2019).

[15]The exact treatment of the one unit of horizontal reserve for soldiers and one unit of horizontal reserve for freedom fighters is not described in the case, and this last step is our interpretation from the description in the case.

of horizontal reservations for a group, say women, obviously improves the welfare of female candidates. However, this comes at the cost of socially backward reserve-eligible categories because they cannot benefit from the horizontal reservations in the open category, and, furthermore, they may lose some of the open-category positions that they claim by merit because of these horizontal reservations.

5. Additional Shortcomings of the Supreme Court-Mandated SCI-VHR Choice Rule: Elimination of Justified Envy and Incentive Compatibility

We have shown that an easy fix for the technical shortcomings of the choice rule C^{SCI} presented in Section 2.2 is made by simply restricting the number of horizontal traits for each candidate to a maximum of one. However, even in that case, the SCI-VHR Choice Rule suffers from two additional vulnerabilities, and unlike the technical shortcomings presented before, these vulnerabilities are highly visible and have resulted in numerous lawsuits.

The source of these vulnerabilities can be easily understood by paying attention to the Step 0 of C^{SCI}_{1h} (or Step 0 of C^{SCI}), the step that determines the eligibility for open-category horizontal reservations. Candidates with reserve-eligible social categories are ineligible for open-category horizontal reservations, and they can receive open-category positions by merit only, unless of course, they do not declare their reserve-eligible category and apply as a candidate from the general category. While this option may give an inferior outcome in most instances, as we demonstrate in the next example that is not always the case.

Example 4. Consider a set of candidates with two general-category men m^G_1 and m^G_2, one general-category woman w^G_1, one SC man m^{SC}_3, and one SC woman w^{SC}_2. Suppose that there are two open-category positions and one SC position available. Only one open-category position is reserved for women. Suppose the candidates have the following ranking according to their merit scores:

$$m^G_1 > m^G_2 > m^{SC}_3 > w^{SC}_2 > w^G_1.$$

When all candidates apply, C^{SCI} works as follows. At Step 1(i), m^G_1 and m^G_2 are tentatively chosen for the open-category positions. The horizontal reservation for women is not satisfied because no woman is allocated a general-category position and there is a rejected general-category woman. Therefore, an adjustment is made at Step 1(ii) and m^G_2 is replaced with w^G_1. At Step 2(i), m^{SC}_3 is tentatively chosen for the SC position. Since there are no women reservations for SC, no adjustment is made. The set of chosen candidates is $\{m^G_1, w^G_1, m^{SC}_3\}$.

There are two fundamental issues here. The first one is that even though w^{SC}_2 has a higher merit score than w^G_1, and w^{SC}_2 has a reserve-eligible category while w^G_1 does not,

w_2^{SC} is rejected while w_1^G is chosen. Woman w_2^{SC} has envy towards w_1^G and her envy is justified because w_2^{SC} has the same horizontal trait as w_1^G, she has a reserve-eligible category while w_1^G does not, and her merit score is higher than that of w_1^G.

The second issue is that if w_2^{SC} does not declare her category SC, then she will be considered a general-category female candidate and she will be allocated an open-category position at Step 1(ii) because her merit score is higher than that of w_1^G. Therefore, w_2^{SC} has incentives to not declare her caste status and participate as a general-category candidate. □

We next formalize these two conceptual issues with the SCW-VHR choice rule. To this end, first consider the following basic fairness property:

Definition 2. *A choice rule C **respects inter se merit** if, for any set of candidates A and candidates $c, c' \in A$ with $\rho(c) = \rho(c')$, $\tau(c) = \tau(c')$, and $\sigma(c) < \sigma(c')$,*

$$c \in C(A) \text{ implies } c' \in C(A).$$

A choice rule respects *inter se* merit, if a candidate with a higher merit score never loses a position to a lower merit score candidate with an identical category and set of traits. It is easy to see that the choice rule C^{SCI} respects *inter se* merit, a concept that is mandated by several Supreme Court judgements, and deeply interwoven into modern Indian legal thought.

Given the importance of *inter se* merit in India, one would expect that the following stronger (but even more plausible) principle would also be respected under a Supreme Court-mandated procedure that implements the provisions for positive discrimination.

Definition 3. *There is an instance of **justified envy** for a choice rule C if there exist a set of candidates A and two candidates $c, c' \in A$ with $\rho(c) \subseteq \rho(c')$, $\tau(c) \subseteq \tau(c')$, and $\sigma(c) < \sigma(c')$ such that $c \in C(A)$ and $c' \notin C(A)$. A choice rule C **eliminates justified envy** if it never allows for an instance of justified envy.*

In words, there is an instance of justified envy for a choice rule whenever there exist two candidates c and c' such that

(1) either c and c' have the same category or c is a general-category candidate,
(2) c' has any trait that c has,
(3) c' has a higher merit score than c, and
(4) c' is rejected from a set of candidates while c is chosen.

Observe that candidate c' is either from a more disadvantaged category than candidate c, or belongs to a more disadvantaged group of citizens possessing additional horizontal traits; and yet she loses a position to candidate c despite having a higher merit score.

Clearly this is a highly implausible situation. As such, eliminating justified envy is even more important than respecting *inter se* merit, at least in the context of positive discrimination.

If a choice rule eliminates justified envy, then it also respects *inter se* merit. But even though C^{SCI} respects *inter se* merit, Example 4 shows that it does not eliminate justified envy because w_2^{SC} is rejected while w_1^G is chosen when all five candidates apply. In Section 6, we will present several challenges this shortcoming creates.

The second issue is that it is against the philosophy of reservation policies that declaring your reserve-eligible category or traits can hurt you in the allocation process. Before introducing this concept, we define the following auxiliary notion that we need.

A candidate **withholds some of her reserve-eligible privileges** if she does not declare either her backward category membership (in case she belongs to one), some of her traits, or both. For example, a SC candidate with a disability can withhold some of her reserve-eligible privileges by not declaring her SC membership or her disability.

Definition 4. *A choice rule C is **incentive compatible** when, for every set of candidates A and candidate $c \in A$, if c is chosen from A by withholding some of her reserve-eligible privileges, then $c \in C(A)$.*[16]

Incentive compatibility states the following: No agent should be able to get a position by withholding some of her reserve-eligible privileges that she cannot get by declaring her backward class membership and all of her traits. Example 4 shows that C^{SCI} is not incentive compatible because if w_2^{SC} is treated as a general-category female candidate, then she will be chosen when all five candidates apply whereas she is not chosen when she is treated as a SC woman.

In closing this section, we provide a natural modification of the Supreme Court's choice rule that preserves all of the principles laid down by the Constitution and the Supreme Court, while addressing the two fundamental issues that we have identified.

Choice Rule $\mathbf{C_{2s}^{hor}}$
Step 1: Apply $C^{hor}(\cdot|r^O, (r_t^O)_{t\in\mathcal{T}})$ to the set of all candidates to allocate the open-category positions.
Step 2: For each reserve-eligible social category $X \in \mathcal{R}$, apply $C^{hor}(\cdot|r^X, (r_t^X)_{t\in\mathcal{T}})$ to the category-X candidates who are not chosen in Step 1.

Since the source of the complications were hidden in Step 0 of C_{1h}^{SCI}, a remedy can be obtained by simply deeming every candidate eligible for open-category horizontal reservations, essentially removing Step 0.

[16] Incentive compatibility of a choice rule was first introduced in Aygün and Bó (2016) in the context of affirmative action in Brazilian colleges.

Theorem 5. *C_{2s}^{hor} eliminates justified envy and is incentive compatible.*

Before we discuss the benefits of adopting C_{2s}^{hor} to allocate positions in Section 7, we highlight the challenges of implementing the Supreme Court-mandated choice rule.

6. Challenges on Implementation of the Supreme Court-Mandated Choice Rule

As we have shown in Section 5, the SCI-VHR choice rule allows for justified envy. Moreover, it fails to be incentive compatible due to backward class candidates losing their access to horizontally reserved positions in the open category by declaring their backward class status. Since at least half of the positions are open, for any given trait, there are typically considerably more horizontally reserved positions at the open category than at any backward class category. As such, the choice between declaring one's backward class status or special trait may not be an easy one, and burdens these candidates—who are often from the most vulnerable groups of the society—with making an informed choice.

While the shortcomings of the SCI-VHR choice rule we presented in Section 2.2 are technical in nature, and may not be easy to challenge, its failure to eliminate justified envy is fairly straightforward to observe. All it takes is a declined backward class candidate to realize that her merit score is higher than an accepted general-category candidate, even though she has all the horizontal traits the admitted candidate does. In other words, observing instances of justified envy is fairly straightforward.

Focusing on complications caused either by the presence of justified envy or the lack of incentive compatibility, we next present a number of challenges on implementation of the SCI-VHR choice rule.

6.1. High Court Cases Related to Justified Envy. The failure of SCI-VHR choice rule to eliminate justified envy has resulted in countless court cases throughout India, and since the presence of justified envy in the system is highly implausible, these legal challenges often result in controversial rulings. In addition, there are also cases where authorities who implement a better-behaved version of the choice rule, one that does not suffer from this shortcoming, are nonetheless challenged in court, on the basis that their adopted choice rules differ from the one mandated by the Supreme Court. These court cases are not restricted to lower courts, and include several cases argued in state high courts. Even at the level of state high courts, the judgements on this issue are highly inconsistent, mostly because of the confusion caused by the possibility of justified envy under the SCI-VHR choice rule. In cases where a state insists on a version that eliminates justified envy—which coincides with the version we present in Section 5—they are often forced by the courts to revert to the Supreme Court-mandated version. Here is a small sample of high profile cases, each from a different state:

(1) *Mamta Bisht vs State of Uttaranchal And Others, 26 October, 2005*, Uttarakhand High Court.[17][18] In this case, there are 42 civil judge positions to be allocated in Uttaranchal. The petitioner, Mamta Bisht, is eligible for horizontally reserved positions for Uttaranchal women, but her merit score is not high enough to secure a position either through the open category, or from Uttaranchal-women category. She files a petition based on the following instance of justified envy in the announced outcome: The merit score of the lowest score candidate who secured a position in the open category is lower than the score of Neetu Joshi, who is the highest merit score candidate who benefitted from horizontally reserved positions for Uttaranchal women.[19] The petitioner argues that, Neetu Joshi has to receive the last open-category position due to the fact that her merit score is higher than that of the lowest score candidate admitted for one of these positions, and that candidate, having the highest merit score among remaining Uttaranchal-women candidates, has to receive the horizontally reserved position Neetu Joshi no longer needs to occupy. The high court allows her petition, and in its decision grants her a position based on the following justification:

> In view of above, Neetu Joshi, (SI. No. 9, Roll No. 12320) has wrongly been counted by respondent No. 3 / Commission against five seats reserved for Uttaranchal Women General Category as she has competed on her own merit as general candidates and as 5th candidate the petitioner should have been counted for Uttaranchal Women General Category seats.

This erroneous high court judgement was later overruled by the Supreme Court in their civil appellate case *Public Service ... vs Mamta Bisht And Ors on 3 June, 2010*,[20] but not before setting a precedent for several subsequent lawsuits.

(2) *Rajeshwari vs State (Panchayati Raj Dep) Ors, 15 March, 2013*, Rajasthan High Court.[21] This case was brought to the Rajasthan High Court by a large number of petitioners against the state government, on the basis that reserve category women are allowed to benefit from open-category horizontally reserved positions for women. The high court rules that the state is at fault, and it must abandon its choice rule,

[17]The case is available at https://www.casemine.com/judgement/in/56b494fb607dba348f01036a (last accessed on 03/07/2019).

[18]The name of this state was changed from Uttaranchal to Uttarakhand in 2007. We use Uttaranchal in our dicussion because this is the name used in the court case.

[19]This situation was possible due to an additional position horizontally reserved for ex-military personnel.

[20]The case is available at https://indiankanoon.org/doc/518824/ (last accessed on 03/07/2019).

[21]The case is available at https://indiankanoon.org/doc/128221069/ (last accessed on 03/07/2019).

adopting the one mandated by the Supreme Court. The following quote is from a story published in The Times of India covering this court case:[22]

> In a judgment that would affect all recruitments in the state government, the Rajasthan high court has ruled that posts reserved for women in the open/general category cannot be filled with women from reserved categories even if the latter are placed higher on the merit list...
> Women candidates who contested for different positions in at least three government departments, including the panchayati raj, education and medical, last year had challenged the government move to allow ''migration'' of reserved category women to fill the open category seats. The positions applied for included that of teachers Grade-II and III, school lecturers, headmasters and pharmacists.

Ironically, while the high court's decision is correct, it also means that the better-behaved version of the choice rule has to be abandoned throughout the state.

(3) *Ashish Kumar Pandey And 24 Others vs State Of U.P. And 29 Others on 16 March, 2016*, Allahabad High Court.[23] In a case that mimics the aforementioned Rajasthan High Court case, this lawsuit was brought to Allahabad High Court by 25 petitioners, disputing the mechanism employed by the State of Uttar Pradesh—the most populous state in India with more than 200 million residents—to apply the provisions of horizontal reservations in their allocation of more than 4000 civil police and platoon commander positions. Of these positions, 27%, 21%, 2% are each vertically reserved for backward classes OBC, SC, and ST, respectively, and 20%, 5%, and 2% are each horizontally reserved for women, ex-servicemen, and dependents of freedom fighters, respectively. While only 19 women are selected for open-category positions based on their merit scores, the total number of female candidates is less than even the number of open-category horizontally reserved positions for women, and as such all remaining women are selected. However, instead of assigning them positions from their respective backward class categories (as it is mandated by the Supreme Court), all of them are assigned positions from the open category. Similarly, backward class candidates are deemed eligible to use horizontal reservations for dependents of freedom fighters and ex-serviceman as well. The counsel for the petitioners argues that not only did the State of U.P. make an error in their implementation of horizontal reservations, but also that the error was intentional. The following quote is from the court case:

[22]The Times of India story is available at `https://timesofindia.indiatimes.com/city/jaipur/Womens-seats-on-open-merit-cant-be-filled-from-SC/ST-quota-High-court/articleshow/19101277.cms` (last accessed on 03/07/2019).

[23]The case is available at `https://indiankanoon.org/doc/74817661/` (last accessed on 03/07/2019).

> Per contra, learned counsel appearing for the petitioners would submit that fallacy was committed by the Board deliberately, and with malafide intention to deprive the meritorious candidates their rightful placement in the open category. The candidates seeking horizontal reservations belonging to OBC and SC category were wrongly adjusted in the open category, whereas, they ought to have been adjusted in their quota provided in respective social category. The action of the Board is not only motivated, but purports to take forward the unwritten agenda of the State Government to accommodate as many number of OBC/SC candidates in the open category.

The judge of the case sides with the petitioners, and rules that the State of Uttar Pradesh must correct their erroneous application of the provisions of horizontal reservations. The judge further emphasizes that the State has played foul, stating:

> There is merit in the submission of the learned counsel for the petitioners that the conduct of the members of the Board appears not only mischievous but motivated to achieve a calculated agenda by deliberately keeping meritorious candidates out of the select list. The Board and the officials involved in the recruitment process were fully aware of the principle of horizontal reservations enshrined in Act, 1993 and Government Orders which were being followed by them in previous selections of SICP and PC (PAC), but in the present selection they chose to adopt a principle against their own Government Orders and the statutory provisions which were binding upon them...
> I am constrained to hold that both the State and the Board have played fraud on the principles enshrined in the Constitution with regard to public appointment.

What is especially surprising is, despite the heavy tone of this judgement, the State goes on to appeal in another Allahabad High Court case *State Of U.P. And 2 Ors. vs Ashish Kumar Pandey And 58 Ors, 29 July, 2016*,[24] in an effort to continue using its preferred method for implementing horizontal reservations. Perhaps unsurprisingly, this appeal was denied by the High Court.

This particular case clearly illustrates that there is a strong resistance in at least some of the states to implementing the provisions of horizontal reservations in their Supreme Court-mandated form. While this resistance most likely reflects the

[24]The case is available at `https://indiankanoon.org/doc/71146861/` (last accessed on 03/07/2019).

political nature of this debate, the arguments of the counsel for the State to maintain their preferred mechanism to implement the provisions of horizontal reservations are mostly based on the presence of justified envy under the Supreme Court-mandated version. The following quote from the appeal illustrates that this was the main argument used in their defense:

> The arguments that have been advanced on behalf of State and private appellant with all vehemence that women candidates irrespective of their social class i.e. SC/ST/OBC are entitled to make place for themselves in an open category on their inter-se merit clearly gives an impression to us that State of U.P and its agents/servants and even the private appellants are totally unaware of the distinction that has been time and again reiterated in between vertical reservation and horizontal reservation and the way and manner in which the provision has to be pressed and brought into play.

(4) *Asha Ramnath Gholap vs President, District Selection Committee & Ors. on March 3rd, 2016*, Bombay High Court.[25] In this case, there are 23 pharmacist positions to be allocated; 13 of these positions are vertically reserved for backward classes and the remaining ten are open for all candidates. In the open category, eight of the ten positions are horizontally reserved for various groups, including three for women. The petitioner, Asha Ramnath Gholap, is a scheduled caste woman, and while there is one vertically reserved position for SC candidates, there is no horizontally reserved position for SC women. Under the SCI-VHR choice rule, she is not eligible for any of the horizontally reserved women positions at the open category. Nevertheless, she brings her case to the Bombay High Court based on an instance of justified envy, described in the court records as follows:

> It is the contention of the petitioner that Respondent Nos. 4 & 5 have received less marks than the petitioner and as such, both were not liable to be selected. The petitioner has, therefore, approached this court by invoking the writ jurisdiction of this court under Article 226 of the Constitution of India, seeking quashment of the select list to the extent it contains the names of Respondent Nos.4 and 5 against the seats reserved for the candidates belonging to open female category.

There is no merit to this argument, because the choice rule mandated by the Supreme Court allows for justified envy. However, the judges sided with the petitioner on the basis that a candidate cannot be denied a position from the open

[25]The case is available at https://indiankanoon.org/doc/178693513/ (last accessed on 03/08/2019).

category based on her backward class membership, essentially ruling out the possibility of justified envy under a Supreme Court-mandated choice rule, which is designed to allow for positive discrimination for the vulnerable groups in the society.[26] Their justification is given in the court records as follows:

> We find the argument advanced as above to be fallacious. Once it is held that general category or open category takes in its sweep all candidates belonging to all categories irrespective of their caste, class or community or tribe, it is irrelevant whether the reservation provided is vertical or horizontal. There cannot be two interpretations of the words 'open category' ...

(5) *Uday Sisode vs Home Department (Police) on 24 October, 2017*, Madhya Pradesh High Court.[27] In another case parallel to those at Bombay and Uttarakhand High Courts, the judges of Madhya Pradesh High Court issued a questionable decision by siding with a petitioner who filed this lawsuit based on another instance of justified envy.

6.2. Wrongful Implementation of the Supreme Court-Mandated Choice Rule. It is bad enough that the Supreme Court-mandated choice rule is not incentive compatible, forcing some candidates to choose between declaring their social reservation-eligible backward class status and their special reservation-eligible horizontal traits. To make matters worse, in some cases candidates are denied access to open-category horizontally reserved positions even when they do not submit their backward class status, giving up their eligibility for vertically reserved positions for their reserve-eligible class. Therefore, even when the candidate applies for a position as a general-category candidate, the central planner processes the application as if the backward class status was claimed, denying the candidate's eligibility for open-category horizontally reserved positions for her trait. The central planners are able to do this, because last names in India are, to a large extent, indicative of a caste membership. This type of misconduct seems to be fairly widespread, and it is the main cause of the lawsuit in each of the following cases:

[26] In a very similar Bombay High Court case *Rajani Shaileshkumar Khobragade ... vs The State Of Maharashtra And ... on 31 March, 2017* where the petitioner filed a lawsuit based on another instance of justified envy, the judges of the same high court dismissed the petition. This case is available at https://indiankanoon.org/doc/7250640/, last accessed on 03/09/2019. Indeed, there seem to be several conflicting decisions at the Bombay High Court on this very issue, including a series of cases reported in a July 18, 2018 dated *The Times of India* story "MPSC won't issue job letters till HC hears plea on quota issue" available at https://timesofindia.indiatimes.com/city/aurangabad/mpsc-wont-issue-job-letters-till-hc-hears-plea-on-quota-issue/articleshow/65029505.cms (last accessed on 03/09/2019).

[27] The case is available at https://indiankanoon.org/doc/196750337/ (last accessed on 03/08/2019).

(1) *Vinod Kadubal Rathod And Another vs Maharashtra State Electricity ... on 17 February, 2017*, Bombay High Court.[28]
(2) Original Applications 1007, 1052, 1056, 1057 & 1070/2017 dated 29.11.2017, Maharashtra Administrative Tribunal, Mumbai Bench.[29]
(3) Original Application 529 of 2017 dated 28.09.2017, Maharashtra Administrative Tribunal, Mumbai Bench.[30]
(4) Original Applications 944, 945 & 220/2017 dated 20.07.2018, Maharashtra Administrative Tribunal, Mumbai Bench at Aurangabad.[31]

Moreover, not all decisions in these lawsuits are made in accordance with the Supreme Court-mandated procedure. For example, in the last lawsuit given above, two petitioners each applied for a position without declaring their backward class membership, with an intention to benefit from open-category horizontal reservations. Following their application, these petitioners were requested to provide their school leaving certificates, which provided information on their backward class status. Upon receiving this information, the petitioners were declined eligibility for the provisions of open-category horizontal reservations, even though they never claimed the benefits of backward class vertical reservations. Hence, they filed the fourth lawsuit given above. Remarkably, their petition was declined on the basis of their backward class membership. Here we have a case where the authorities not only go to great lengths to obtain the backward class membership of the candidates, and wrongfully decline their eligibility for special horizontal reservations, but they also manage to get their lawsuits dismissed. The mishandling of this case is consistent with the concerns indicated in the February 2006 issue of *The Inter-Regional Inequality Facility* policy brief:[32]

> Another issue relates to the access of SCs and STs to the institutions of justice in seeking protection against discrimination. Studies indicate that SCs and STs are generally faced with insurmountable obstacles in their efforts to seek justice in the event of discrimination. The official statistics and primary survey data bring out this character of justice institutions. The data on Civil Rights cases, for example, shows that only 1.6% of the total cases registered in 1991 were convicted, and that this had fallen to 0.9% in 2000.

[28]The case is available at https://indiankanoon.org/doc/162611497/ (last accessed on 03/09/2019).

[29]The case is available at https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A%201007.17%20and%20ors%20DB,%2029.11.17,%20Chairman.PDF (last accessed on 03/09/2019).

[30]The case is available at https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A%20529.17%20Appointment%20challenged,%20DB.0917.PDF (last accessed on 03/09/2019).

[31]The case is available at https://mat.maharashtra.gov.in/Site/Upload/Pdf/944%20945%20&%20220%20of%202017.pdf (last accessed at 03/09/2019).

[32]The policy brief is available at https://www.odi.org/sites/odi.org.uk/files/odi-assets/publications-opinion-files/4080.pdf (last accessed 03/09/2019).

6.3. Loss of Access to Horizontal Reservations without any Access to Vertical Reservations. The main justification offered in various Supreme Court cases for denying backward class members the provisions of horizontal reservations for open-category positions is avoiding a situation where an excessive number of positions are reserved for members of these classes. In several cases, however, members of these classes are denied access to horizontally reserved positions even when their reserve-eligible vertical category is not earmarked for those positions. This is the case in the following two court cases:

(1) *Tejaswini Raghunath Galande v. The Chairman, Maharashtra Public Service Commission and Ors. on 23 January 2019*, Writ Petition Nos. 5397 of 2016 & 5396 of 2016, High Court of Judicature at Bombay.[33]
(2) Original Application No. 662/2016 dated 05.12.2017, Maharashtra Administrative Tribunal, Mumbai.[34]

In both of the above cases, while both petitioners declared their backward class status, there was no position vertically reserved for their class. Yet they both lost access to horizontally reserved positions in the open category for their traits. In the first case, the petitioners' lawsuit to benefit from horizontal reservations was initially declined by a lower court, resulting in the appeal at the High Court. The lower court's decision was overruled in the High Court, and her request was granted. The second petitioner's similar request was declined by the Maharashtra Administrative Tribunal. What is more worrisome in the second case is that initially three positions were announced to be vertically reserved for the petitioner's backward class, but after her application these positions were withdrawn. Therefore, the candidate declared her backward class status, giving up her eligibility for several horizontally reserved women positions at the open category, presumably to gain access to vertically reserved positions for her backwards class, only to learn that she had given up her eligibility for nothing.

7. Recommended Choice Rule

Our recommended choice rule is C^{hor}_{2s}. While this choice rule is very similar in spirit to C^{SCI}, it escapes all the aforementioned shortcomings of the Supreme Court-mandated choice rule. It is well-defined and incentive compatible, it eliminates justified envy, and its outcome is merit maximal within each vertical category. Essentially there are two differences between these two choice rules:

[33]The case is available at `https://www.casemine.com/judgement/in/5c713d919eff4312dfbb5900` (last accessed on 03/09/2019).

[34]The case is available at `https://mat.maharashtra.gov.in/Site/Upload/Pdf/O.A.662%20of%202016.pdf` (last accessed on 03/09/2019).

(1) In contrast to all candidates who are eligible for open-category horizontal reservations under C^{hor}_{2s}, only the following candidates are eligible for open category horizontal reservations under C^{SCI}:
   (i) general-category candidates, and
   (ii) meritorious backward class candidates who would receive an open-category position in the absence of any horizontal reservations.

(2) For any given vertical category, while the adjustments for horizontal reservations are carried out in a merit-maximal way through a forward-looking procedure under C^{hor}_{2s}, they are carried out myopically one candidate at a time for some processing sequence of horizontal reservation traits under C^{SCI}.

The Supreme Court-mandated choice rule C^{SCI} is neither well-defined nor merit maximal because of the second difference. However, this difference becomes immaterial when each candidate qualifies for at most one horizontal trait (Theorem 3). Hence, one way to make sure choice rule C^{SCI} is well-defined and merit maximal is by limiting the maximum number of horizontal trait declarations by one for each candidate.

On the other hand, the choice rule C^{SCI} fails incentive compatibility and elimination of justified envy, due to the first difference. Under this choice rule, backward class candidates who also qualify for a special horizontal reservation are forced to choose between vertical reservation (along with horizontal reservation within this vertical category) or open-category horizontal reservation. The choice rule C^{SCI} fails incentive compatibility precisely because the latter option may be the only way to secure a position. Furthermore, it results in justified envy, whenever a candidate chooses the first option even though the latter option is the only way to secure a position.

We believe an ideal amendment involves both adjustments, but if this is considered excessive for any reason, either one can be carried out independently. Alternatively, the second difference can be eliminated altogether by limiting the maximum number of horizontal trait declarations by one for each candidate.

We also have a final, less ambitious policy recommendation, for a scenario in which the current choice rule is maintained. As we have seen in Section 6.3, there are cases where backward class candidates lose eligibility for open-category horizontal reservations, even when there is no vertical reservation for their class. In those cases, we believe these candidates should automatically be considered members of the general category; otherwise, the sole role of backward class membership becomes discrimination against these candidates under the choice rule C^{SCI}. There is no need for such a "precaution" under any choice rule that is incentive compatible.

8. Conclusion

With the passing of the *124th Constitution Amendment Bill* in January 2019, granting 10% reservation for economically weaker sections of the general category in both houses of the parliament, the reservation system was once again in the headlines in India. According to a story in *The Times of India*:[35]

> It also said the reservation would be "in addition to the existing reservations and subject to a maximum of 10 per cent of the total seats in each category".

This quote suggests that the 10% reservation to the economically weak in general category will likely be horizontal.[36] Prior to the *124th Constitution Amendment Bill*, a 3% horizontal reservation for the disabled was already mandated by the Supreme Court in their judgement of *Union Of India & Anr vs National Federation Of The Blind & ... on 8 October, 2013*.[37] Another disadvantaged group whose members received constitutional rights for horizontal reservation in the last few years is the group of transgender persons, whose constitutional rights as equal citizens were recognized by the Supreme Court in 2014 (Kothari 2018).[38] Even prior to these high profile reforms, the use of horizontal reservations has already been extensive in some states. For example, in the state of Sikkim, the reservation system was restructured in June 2018, including the following six horizontal reservation traits:[39]

(1) women (30%),
(2) sports persons and artisans of excellence (5%),
(3) below poverty line families (5%),
(4) ex-servicemen (3%),
(5) physically challenged (3%), and
(6) paramilitary forces and Assam Rifles (2%).

[35]This story is available at https://timesofindia.indiatimes.com/india/10-reservation-for-economically-weak-in-general-category-comes-into-force/articleshow/67528010.cms (last accessed on 03/14/2019).

[36]According to a story in *The Indian Express*, however, "it is not clear in the Bill if the proposed reservation will apply vertically or horizontally." This story is available at https://indianexpress.com/article/opinion/columns/124th-constitutional-amendment-bill-reservation-5534333/ (last accessed on 03/14/2019).

[37]The case is available at https://indiankanoon.org/doc/178530295/ (last accessed on 03/14/2019).

[38]CLPR policy brief is available at https://clpr.org.in/wp-content/uploads/2018/12/Policy-Brief-2018-Implementing-Reservations-for-Transgender-and-Intersex-Persons.pdf (last accessed on 03/14/2019).

[39]See June 23, 2018 dated Sikkim Express story "Govt job reservation structure revised," which is available at http://www.sikkimexpress.com/NewsDetails?ContentID=11017 (last accessed on 03/14/2019).

With the increased use of horizontal reservations with several overlapping categories, the exposure of the reservation system to arbitrariness and legal challenges further increases, and it becomes even more essential to close the loopholes in the Supreme Court-mandated procedure to implement the reservation system. Not only can this goal be achieved by adopting of our proposed choice rule C^{hor}_{2s}, but also two of its highly contested vulnerabilities—the presence of justified envy and lack of incentive compatibility—can also be corrected.

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Appendix A. Proofs

In this appendix, we prove our results.

Proof of Theorem 1. First, we show that C^{hor} is well-defined. Consider a set of candidates A. In the construction of $C^{hor}(A)$, at every step, we consider the subsets of A that satisfy the horizontal reservations for A. In the first part of the proof, we construct a subset of A that satisfies the horizontal reservations for A, to show that there exists at least one such subset.

Consider all candidates in A who have at least one trait, say $\tilde{A}_1$. If $\tilde{A}_1$ is empty, then choose one candidate in A. Otherwise, choose one candidate from $\tilde{A}_1$ and decrease the number of reserved positions for the traits of this candidate by one. Consider the set of remaining candidates in $\tilde{A}_1$ who have at least one trait with a positive reserved position, say $\tilde{A}_2$. If $\tilde{A}_2$ is empty, then choose one of the remaining candidates from A. Otherwise, if $\tilde{A}_2$ is not empty, then choose a candidate from $\tilde{A}_2$. Continue this procedure so that the number of chosen candidates is $\min\{q, |A|\}$. We claim that the chosen subset, say A', satisfies the horizontal reservations for A. Suppose, for contradiction, that it does not. Then there exists a trait t such that the number of candidates with trait t in A' is less than r_t and that there is at least one candidate in $A \setminus A'$ with trait t. In this case, $|A'| = q$ because $A \setminus A'$ is nonempty. Since the number of remaining reserved positions for trait t is positive, and a candidate with this trait is rejected at the last step, a candidate with a trait that has a positive reserved position is accepted at every step. But this is a contradiction to the assumption that $\sum_{t \in \mathcal{T}} r_t \leq q$. Therefore, there exists at least one subset of A that satisfies the horizontal reservations for A.

Let $A' = \{a'_1, \ldots, a'_n\}$ be a subset of A that satisfies the horizontal reservations for A and $C^{hor}(A) = \{a_1, \ldots, a_m\}$. Suppose that $A' \neq C^{hor}(A)$. Re-order candidates in each set so that candidates with a lower index have higher merit scores than candidates with a higher index. We claim that $C^{hor}(A)$ dominates A'. Let k be the minimum index such that $a_k \neq a'_k$. By construction of $C^{hor}(A)$, a_k has a higher merit score than all candidates in $A' \setminus C^{hor}(A)$ because at Step k candidate a_k is chosen by C^{hor}. Therefore, $C^{hor}(A)$ dominates A'. □

Proof of Theorem 2. Before we start the proof, we introduce some notation. For any set of candidates $A \subseteq \mathcal{A}$ and trait $t \in \mathcal{T}$, let $A_t \equiv \{c \in A | t \in \tau(c)\}$. In words, A_t is the set of all candidates in A who have trait t. We use the following lemma in the proof.

Lemma 1. *$\bar{A} \subseteq A$ satisfies the horizontal reservations for A if, and only if, $\left|\bar{A}_t\right| \geq \min\{r_t, |A_t|\}$ for every trait $t \in \mathcal{T}$.*

Proof. First we show sufficiency. Let $\bar{A}$ be such that $\left|\bar{A}_t\right| \geq \min\{r_t, |A_t|\}$ for every trait t. Fix a trait t. If $\left|\bar{A}_t\right| < r_t$, then $\left|\bar{A}_t\right| \geq |A_t|$. Since $\bar{A} \subseteq A$, this implies $\bar{A}_t = A_t$. Therefore, there exists no candidate in $A \setminus \bar{A}$ who has trait t. Therefore, $\bar{A}$ satisfies the horizontal reservations for A.

For necessity, let $\bar{A} \subseteq A$ satisfy the horizontal reservations for A. Then, for every trait t, either $\left|\bar{A}_t\right| \geq r_t$ or $\bar{A}_t = A_t$. This implies $\left|\bar{A}_t\right| \geq \min\{r_t, |A_t|\}$ for every trait $t \in \mathcal{T}$. ■

Consider a set of candidates A. We show that $C^{hor}(A) = C^{mg}(A)$. First, for every trait t, the number of trait-t candidates in $C^{mg}(A)$ is at least $\min\{r_t, |A_t|\}$ because in the first step $\min\{r_t, |A_t|\}$ trait-t candidates are chosen. Therefore, by Lemma 1, $C^{mg}(A)$ satisfies the horizontal reservations for A. Let A' be the set of candidates chosen out of A by C^{mg} in Step 1. $C^{hor}(A)$ must include all the candidates in A' because by Lemma 1 the number of trait-t candidates in $C^{hor}(A)$ is at least $\min\{r_t, |A_t|\}$. Furthermore, by the construction of C^{hor}, whenever a trait-t candidate is chosen, it always selects the trait-t candidate with the highest merit score from the available set, so $C^{hor}(A) \supseteq A'$.

Now, if $C^{mg}(A) \setminus A' \neq C^{hor}(A) \setminus A'$, then $C^{mg}(A) \setminus A'$ would dominate $C^{hor}(A) \setminus A'$ by the construction of C^{mg} because it selects candidates with the highest merit score in Step 2. Therefore, $C^{mg}(A)$ would dominate $C^{hor}(A)$ because adding or subtracting a set of candidates preserves the domination relationship. But this cannot hold because C^{hor} is merit maximal and so $C^{hor}(A)$ dominates any subset of A different from $C^{hor}(A)$ that satisfies the horizontal reservations for A. Therefore, $C^{mg}(A) \setminus A' = C^{hor}(A) \setminus A'$, and thus $C^{mg}(A) = C^{hor}(A)$. □

Proof of Theorem 3. Denote the union of the set of candidates with the highest r^O merit scores and the set of all general-category candidates by A_1. In Step 1 of C^{SCI}_{1h}, $C^{hor}(A_1|r^O, (r^O_t)_{t\in\mathcal{T}})$ is chosen for the open-category positions. We first show that the set of candidates chosen for the open-category positions by C^{SCI} is the same set.

When the open-category positions are allocated according to C^{SCI} in Steps $1(i)$ and $1(ii)$, only candidates in A_1 are considered. Furthermore, the chosen set, call it A', satisfies the open-category horizontal reservations $(r^O_t)_{t\in\mathcal{T}}$ for A_1 because for each trait t either the number of chosen trait-t candidates is at least r^O_t or all trait-t candidates in A_1 are chosen. Likewise, $C^{hor}(A_1|r^O, (r^O_t)_{t\in\mathcal{T}})$ also satisfies the open-category horizontal reservations $(r^O_t)_{t\in\mathcal{T}}$ for A_1 by Theorem 1.

By Lemma 1, for each trait t, the number of trait-t candidates in A' and $C^{hor}(A_1|r^O,(r_t^O)_{t\in\mathcal{T}})$ are at least $\min\{r_t^O,|\{c\in A_1|\tau(c)=t\}|\}$. In both choice rules, a trait-t candidate with a low merit score is never chosen before a trait-t candidate with a higher merit score, so for every trait t, trait-t candidates with the highest $\min\{r_t^O,|\{c\in A_1|\tau(c)=t\}|\}$ merit scores in A' and $C^{hor}(A_1|(r_t^O)_{t\in\mathcal{T}})$ are the same. Furthermore, for the rest of the candidates chosen in Step 1, both rules choose candidates with the highest merit scores remaining in A_1, so they must choose the same set of candidates.

Next, we show that the set of candidates chosen for each reserve-eligible category $X\in\mathcal{R}$ is the same in C_{1h}^{SCI} and C^{SCI}. First note that the set of category-X candidates considered for positions at the second step are the same in both choice rules. The rest of the proof is analogous to the discussion above, as the same set of candidates are considered and the same procedures are applied at this step. □

Proof of Theorem 4. Step 1(i) of C^{SCI} does not depend on the horizontal reservations, so the number of tentatively-accepted candidates with the r^O highest merit scores remains the same regardless of the changes in the horizontal reservations. In Step 1(ii), the introduction or increase of any horizontal reservations weakly increases the number of adjustments that are made. For each such adjustment, the number of chosen category-X candidates weakly decreases because the candidate that is replaced can be a category-X candidate whereas the candidate replacing is a general-category candidate. Therefore, the number of category-X candidates allocated to open-category positions weakly decreases with the introduction or increase of any horizontal reservations.

In Steps 2(i) and 2(ii), no category-X candidate is rejected unless all category-X positions are allocated, regardless of the horizontal reservations. Therefore, with lower horizontal reservations, if r^X category-X candidates are chosen in Steps 2(i) and 2(ii), then the conclusion follows because r^X category-X candidates will also be chosen with higher horizontal reservations in Steps 2(i) and 2(ii). However, if the number of category-X candidates allocated to category-X positions is smaller than r^X, with lower horizontal reservations, then all category-X candidates must have been chosen with lower horizontal reservations, which implies the desired conclusion that the introduction, or increase, of horizontal reservations weakly decreases the number of positions allocated to category-X candidates.

□

Proof of Theorem 5. To show elimination of justified envy, consider a set of candidates A and two candidates $c,c'\in A$ with $\rho(c)\subseteq\rho(c')$, $\tau(c)\subseteq\tau(c')$, and $\sigma(c)<\sigma(c')$. At any step when c is considered by C_{2s}^{hor}, c' is also considered. Furthermore, by the construction of C^{hor}, which is used at every step of C_{2s}^{hor}, a candidate with a lower merit score and set

of traits τ is never chosen before another candidate with a higher merit score and set of traits τ' where $\tau' \supseteq \tau$. Therefore, C_{2s}^{hor} eliminates justified envy.

To show incentive compatibility, consider a set of candidates A and a candidate $c \in A$ such that $c \notin C_{2s}^{hor}(A)$. Fix every other candidate's category and set of traits. First note that C^{hor} does not use the categories of candidates, so modifying the category of c from a reserve-eligible category to general can only hurt c, as he will only be considered at the first step. Furthermore, declaring a set of traits $\tau \subseteq \tau(c)$ instead of $\tau(c)$ can only make this candidate worse off, because if he is considered with set of traits τ to satisfy some constraints, then he will also be considered with set of traits $\tau(c)$ to satisfy the same constraints. Therefore, C_{2s}^{hor} is incentive compatible. □