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Author: Dominique Brabant

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Dominique Brabant

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ESSAYS IN INTERNATIONAL ECONOMICS

By

Dominique Brabant

Dissertation Committee:

James E. Anderson (Co-Chair)

Rosen Valchev (Co-Chair)

Peter Ireland

Abstract

The effect of uncertainty on firms' behavior and on the macroeconomy is generally negative in the literature. Extensive research has also demonstrated that financial frictions limit the extent of firms' activities and growth prospects. In the first two chapters of this dissertation, I study both empirically and theoretically how a specific type of uncertainty, exchange rate uncertainty, interacts with financial frictions to affect the behavior of exporting firms. In line with the existing literature, I find in the first chapter that exports of manufacturing sectors in which firms are more financially constrained decrease by more in times of high uncertainty. Having more tangible capital, which can potentially be used as collateral, makes the effect of uncertainty less negative, especially im sectors where firms are large. Relying more on external financing, on the other hand, makes the effect more negative and affects sectors with small firms more.

Current theoretical models have little to say about the effect of uncertainty on heterogeneous firms. To address this issue, I introduce in the second chapter a model of financially-constrained heterogeneous exporting firms in which credit conditions depend on the degree of exchange rate uncertainty. Firms in different sectors face different types of financial constraints, and are therefore differently affected by uncertainty. I use the calibrated model to evaluate potential policies that could be implemented to alleviate the negative effect of exchange rate uncertainty on exports.

The uncovered interest parity puzzle is the empirical finding that countries with higher risk-free interest rates tend to see their currencies appreciate in the short run. Typical two-country macroeconomic models instead predict that high interest-rate currencies depreciate, with arbitrage opportunities eliminating profitable carry trade strategies. The international finance literature responded to this puzzle by providing several alternative theoretical models able to explain the puzzle. In the third chapter of this dissertation, I study how the predictions of two of these alternative models - the habit model of Verdelhan (2010) and the distorted belief model of Gourinchas and Tornell (2004) - are affected when re-cast in a standard dynamic stochastic general equilibrium framework. I investigate how the mechanisms rely on specific parameter values in order to find under which conditions, if any, they can explain the UIP puzzle. In addition, I obtain business cycle moments from model simulations and compare them to the moments obtained from a standard two-country DSGE model and from the data.

My results show that for the first model, the habit model, the UIP results disappear under realistic calibrations. For the second model, the distorted beliefs model, UIP properties remain under some calibrations. In addition, business cycle predictions remain close to empirical evidence.

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1.1 Introduction

The effect of uncertainty on firms' behavior and on the macroeconomy is generally negative in the literature. Extensive research has also demonstrated that financial frictions limit the extent of firms' activities and growth prospects. In this chapter, I study empirically how a specific type of uncertainty, exchange rate uncertainty, interacts with financial frictions to affect the behavior of exporting firms. In line with the existing literature, I find that exports of manufacturing sectors in which firms are more financially constrained decrease by more in times of high uncertainty.

The unpredictability of exchange rates dates from the collapse of the Bretton Woods agreement in 1973, when the currencies of advanced countries began to float against each other. Initial theories about the effect on international trade, including Ethier (1973) and Clark (1973), suggested that in the absence of effective financial hedging tools, an increase in exchange rate volatility and unpredictability would negatively affect risk-averse firms. Volatility was expected to result, other things equal, in a decrease in international trade. The question remains of interest today, as countries still consider joining or leaving currency unions, and central banks in several countries consider whether to target the exchange rate or the volatility of the exchange rate as part of their monetary policy.

The extensive empirical literature on the effect of exchange rate uncertainty on international trade has not yet reached definitive conclusions. Recent surveys conclude that the aggregate effects are generally not significant and that, when they are, they are small in magnitude.[1] The recent literature has found more significant results using disaggregated data: some industries are more negatively affected than others. This suggests more serious effects at the disaggregated level.

This chapter estimates the role that financial frictions play in the relationship between exchange rate uncertainty and trade in the context of financially developed countries. Canadian exports to the United States in 21 manufacturing industries are studied over the 1988Q1-2015Q4 period. Figure 1.1 plots the exchange rate and exchange rate uncertainty - measured as the average squared (log) change in the nominal exchange rate over the previous six months - during this period. Both series fluctuate widely. The exchange rate reaches lows under parity and highs above 1.5 CAD per USD. Such variations can seriously affect the profitability of firms operating in both countries.

Two measures of financial constraints are commonly used in the literature: asset tangibility and external finance dependence. On the one hand, firms with more tangible assets are typically seen as less constrained since they are able to offer more collateral to potential lenders. On the other hand, firms with higher external finance dependence are typically seen as more financially constrained since they are thought to be closer to their borrowing limit. Since Beck et al. (2008) find that smaller firms are significantly more financially constrained even when controlling for other observables, I also examine the effect of firm size as an indirect measure of financial constraints. In the main regressions, I interact the two direct measures of financial constraints with the sectoral share of exports that comes from small or medium firms.

[1] See, for example, McKenzie (1999) and Auboin & Ruta (2013).

To my knowledge, this is the first study to investigate the heterogeneous effects of financial constraints in this context.

I find that exchange rate uncertainty has on average a negative effect on Canadian exports to the U.S. and that this effect varies significantly across industries. As expected, sectors in which firms have more tangible assets see their exports decrease less in times of high uncertainty. This result is even stronger for sectors composed of relatively large firms. In sectors in which a higher share of exports comes from small or medium firms, external finance dependence acts as a financial constraint and makes the effect of uncertainty on exports more negative.

A feature of my analysis is the focus on Canada-U.S. trade and a time period in which 1) exchange rate volatility is high, 2) exchange rate volatility fluctuates significantly, and 3) hedging instruments are widely available, while simultaneously not targeting exclusively the recent financial and euro crises. The fact that Canadian exports to the United States are substantially affected by uncertainty about the CAD-USD exchange rate is even more striking when considering the importance of the U.S. market for Canadian exporters: during the time period studied, 80 percent of Canadian exports were destined for the United States. Almost all transactions happened in U.S. dollars. One would therefore expect Canadian firms to be fully aware of, and insured against, exchange rate risk. This suggests that the decision to hedge exchange rate risk is not as straightforward as often believed, and that the effect of this type of uncertainty on trade between different countries is potentially more negative.[2]

The remaining of the chapter is organized as follows: Section 1.2 discusses the existing literature; Section 1.3 describes the methodology; Section 1.4 presents the

[2]A recent survey performed by Export Development Canada shows that only 57 percent of the Canadian exporting firms who participated in the survey perform some type of hedging activity.

empirical evidence; and Section 1.5 performs robustness checks. Section 1.6 concludes.

1.2 Literature Review

The empirical literature on the effect of exchange rate uncertainty on international trade is extensive. Until recently, the consensus has been that the effect of exchange rate uncertainty on trade is typically not statistically significant and that, when it is, the effect is fairly small and can go in either direction. A survey by McKenzie (1999) compares the various methodologies used by different authors and concludes that the chosen measure for exchange rate uncertainty, the model specification and the estimation technique do not seem to impact results. A more recent survey by Auboin & Ruta (2013) concludes that researchers that have used disaggregated data have been more successful at finding statistically significant effects.

Klein (1990) looks at U.S. exports of nine categories of goods to seven countries and finds that the effect of exchange rate volatility is significant for two thirds of the categories. The direction of the effect also varies across categories. Byrne et al. (2008) find that exchange rate volatility has a negative effect on U.S. international trade, and that this effect is stronger for differentiated goods. More recently, Verheyen (2012) tests whether exports from euro-zone countries to the U.S. have been affected by the uncertainty that came with the financial crisis. He finds that sectors producing manufactured goods, machinery, and transport equipment were more negatively affected. None of these papers attempt to explain why different sectors are affected differently. One notable exception is Chor & Manova (2012), who find that during the recent financial crisis, U.S. imports from countries with tighter credit markets decreased more. They also find that U.S. imports decreased more in sectors that require more external financing, have limited access to trade credit or possess few collateralizable assets.

In recent years, researchers have been able to use firm-level data to confirm the importance of financial variables in determining the relationship between international trade and uncertainty. Bricongne et al. (2012) find that during the recent euro crisis, the French firms that reduced their exports the most were the firms operating in sectors with high external finance dependence. Demir (2013) finds that real exchange rate volatility negatively affects the growth of Turkish manufacturing firms, but that this effect is reduced if firms have access to equity financing. Héricourt & Poncet (2015) find that higher exchange rate volatility reduces the trade of Chinese firms, and that this effect is stronger for financially vulnerable firms but weaker for firms located in financially-developed regions of China. Other researchers have also used firm-level data to explore mechanisms other than financial frictions. For example, Héricourt & Nedoncelle (2018) find that French firms exporting to more destinations are able to better manage increases in exchange rate uncertainty by redistributing their exports across markets. This mechanism results in the firms being overall less negatively affected.

This chapter also relates to the growing literature on the interaction between financial frictions and international trade. Financial constraints have been found to reduce trade independently of exchange rate uncertainty in studies such as Manova (2008), Manova (2013), and Muûls (2015). Other studies, such as Amiti & Weinstein (2011) and Feenstra et al. (2014), investigate more closely the role of banks in allocating credit to financially-constrained exporters. Foley & Manova (2015) offer a review of how corporate finance has been found to affect firms' international production and trade decisions more generally.

Taking a wider view, this chapter contributes to the recent literature on the effect of economic uncertainty on the macroeconomy.[3] One aim of this chapter is to understand how specific economic agents, in this case exporting firms, react to a specific, measurable type of uncertainty that can be readily hedged with financial instruments.

1.3 Methodology

1.3.1 Identification

The existing literature has raised an array of issues that can potentially bias the estimated relationship between exchange rate uncertainty and international trade. A first concern is omitted variable bias. Omitted variables include all factors that affect trade between two countries, are not properly controlled for in the estimated regressions, and are correlated with the regressor of interest, in this case exchange rate risk. One frequently omitted variable is uncertainty about interest rates and inflation rates, which can both cause exchange rate uncertainty and affect international trade directly. Another important factor is the currency in which the international transaction takes place, which affects importers and exporters differently and which generally varies across transactions between the same country pair. Finally, another confounding factor in international trade more generally is the third-country effect, which describes instances in which trade between two countries may vary over time because of reasons pertaining to a third country.[4]

A second concern is endogeneity. Two countries trading a lot with one another could see their economies becoming more correlated over time, which could stabilize

[3]See, among others, Aghion et al. (2009), Bloom (2014), and Alessandria et al. (2015).

[4]For example, if Germany increases its production of good x, which it was previously importing from France, it may cause an increase in the export of good x from France to, say, Belgium. This increase in trade is not due to factors specific to France or to Belgium and hence is difficult to account for when explaining variations in bilateral trade flows.

their exchange rate. They could also decide to fix their exchange rate, by adopting a common currency or a pegged exchange rate. Endogeneity would cause the observed relationship between exchange rate uncertainty and trade to go in the opposite direction of that studied.

For all these reasons, I focus the empirical analysis in this article on a single country pair and study trade in a single direction: from Canada to the United States. Political risk is low in both countries. Their economies are highly correlated, which minimizes the possibility that differences in interest rate or inflation risk drive trade flows. Most importantly, almost all transactions involving goods being shipped from Canada to the U.S. are conducted in U.S. dollars and, in recent decades, 80 percent of Canadian exports went to the U.S.[5] These characteristics of the country pair greatly reduce concerns about omitted variable bias.

Choosing this country pair however raises another potential concern: the Canadian currency is highly correlated with the price of crude oil. If this fact is not properly accounted for, it could bias the estimated relationship since variations in exports due to oil price uncertainty could be mistakenly attributed to movements in exchange rate uncertainty. For this reason, oil price uncertainty is included as an additional control in the regressions.[6]

Choosing this country pair also addresses some of the endogeneity concerns, since both countries had freely floating currencies over the sample period. Another potential endogeneity issue however arises from the use of this specific country pair coupled

[5]Donnenfeld & Haug (2008) find that 93 percent of U.S. imports from Canada are made in U.S. dollars.

[6]Over the 1988Q1-2015Q4 period, the CAD-USD exchange rate and the price of oil (West Texas Intermediate, WTI) had a coefficient of correlation of -0.75. Over the same period, the volatilities of the two series were considerably less correlated, with a coefficient of 0.35.

with the strong relationship between the Canadian currency and the price of crude oil. If Canadian oil exports are important enough to drive the North American price of oil (WTI) and if the exchange rate responds to movements in the price of oil, then Canadian exports could cause movements in the exchange rate and exchange rate uncertainty, which would cause an endogeneity bias. It is however not a major issue in this instance since exports of crude oil are not included in any of the regressions.[7]

1.3.2 Model specification

Following the literature, I obtain the empirical results in a framework of monopolistic competition. The amount of good ω in sector s produced in country j and consumed in country i can be expressed as the demand function

$$q_{ijs}(\omega) = p_{ijs}(\omega)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}},$$

where $q_{ijs}(\omega)$ denotes the quantity demanded, $p_{ijs}(\omega)$ denotes the price of the good in country i's currency, θ_s is the share of income Y_i that country i spends on goods in sector s, P_{is} is the price in country i of a bundle of all goods in sector s, and ϵ is the elasticity of substitution between different goods in a given sector. The price that maximizes the expected profit of a risk-neutral firm is such that the firm's expected marginal revenue, expressed in its own currency, is equal to a constant markup over its expected marginal cost. The marginal cost includes production costs, trade costs and, potentially, financing costs. A risk-averse firm would instead have a markup that increases with the degree of uncertainty.

The first empirical specification tests for the average effect of exchange rate un-

[7]Only manufacturing exports are included in the regressions. Furthermore, all the results below hold if the Petroleum and Coal Products Manufacturing industry is excluded.

certainty on (log) Canadian exports to the United States over time and across sectors:

$$
\begin{aligned}
\log \text{Exports}_{st} = \alpha + \beta_1 \log \text{Uncertainty}_t + \sum_s \Big\{ & \beta_{2s} \log \text{Exch Rate}_t \times \text{Sector}_s \\
& + \beta_{3s} \log \text{PPI}_{US,st} \times \text{Sector}_s + \beta_{4s} \log \text{GDP}_{US,t} \times \text{Sector}_s \\
& + \beta_{5s} \log \text{Oil Price}_t \times \text{Sector}_s + \beta_{6s} \log \text{Oil Uncertainty}_t \times \text{Sector}_s \\
& + \beta_{7s} \text{Riskfree}_{CAN,t} \times \text{Sector}_s \Big\} + \text{Sector}_s + \text{Quarter}_t + \text{Year}_t + u_{st}.
\end{aligned}
\tag{1.1}
$$

The sectoral producer price index ($\text{PPI}_{US,st}$) controls for the price index P_{is} in the demand function; $\text{GDP}_{US,t}$ controls for U.S. income Y_i; and $\text{Riskfree}_{CAN,t}$, the risk-free interest rate in Canada, controls for financial costs common to all firms in all sectors. The oil price and oil price uncertainty variables are added because of the specificity of the relationship between the price of crude oil and the Canadian currency.

All controls are interacted with sector fixed effects, allowing exports from different sectors to react differently to changes in these variables, and, in the following specifications, allowing the heterogeneous effect of uncertainty to be properly identified. Sector, quarter, and year fixed effects additionally control for any unobserved sector-specific, quarter-specific, or year-specific factors, such as constant technological differences (e.g. production costs), seasonality, or other economic conditions. The errors are clustered at the sector level, but results are robust to clustering by year and to allowing for general heteroskedasticity. Additional specifications test for differentiated effects of uncertainty on exports of different sectors by interacting exchange rate uncertainty with either sector fixed effects or sector-specific financial characteristics (asset tangibility, external finance dependence, and share of exports coming from small or medium firms). All variables are expressed in natural logarithmic terms, except for the risk-free rate and the sectoral financial characteristics (when included), which are already expressed in percentage points.

This specification assumes that uncertainty perceived in the current period about next period's exchange rate affects exports in the current period. Exporters worry about current uncertainty when the goods they ship in the current period will be paid for in the next period.[8] The robustness section explores alternative timing assumptions.

In a second series of regressions I use as dependent variable the number of product varieties exported from Canada to the United States in a given manufacturing industry and in a given time period. I can thus test whether (and how) the extensive margin of trade (the number of product varieties exported in a given industry) is affected by exchange rate uncertainty, or whether all the adjustment happens at the intensive margin (the quantity of a given product variety being exported).[9] Choosing this dependent variable has the additional benefit of being less prone to measurement error. A given product is only recorded as exported or not (a binary outcome) rather than being recorded in value and then deflated using a price index.

1.3.3 Measuring exchange rate uncertainty

The measures of exchange rate uncertainty most used in the literature include past volatility (e.g. standard deviation) of the exchange rate and the volatility of the errors implied by a (G)ARCH model of exchange rate.[10] In this analysis, the preferred

[8] A survey performed by the Association for Financial Professionals distributed to Canadian and American senior-level corporate practitioners finds that open accounts and letters of credit are widely used by businesses. In both cases, the payment for the goods sold is received after the goods have been delivered.

[9] A similar methodology is used in Manova (2013).

[10] In his survey, McKenzie (1999) finds that the measure of exchange rate uncertainty used by researchers does not significantly affect their estimation results.

measure used is the square root of the average squared (log) change in the nominal exchange rate over the previous six months. For a given quarter t starting on the first day of month m, the measure of uncertainty is measured as

$$\sigma_t \equiv \left[\frac{1}{6} \times \sum_{i=1}^{6} \left(x_{m-i} - x_{m-1-i} \right)^2 \right]^{1/2}, \tag{1.2}$$

where x_m is the natural logarithm of the average nominal exchange rate during month m. The use of the average change in the previous two quarters ensures that the relevant exchange rate data is available to agents when they make their decisions and further alleviates endogeneity concerns. The real exchange rate, calculated using consumer price indexes, is highly correlated with the nominal exchange rate, with a coefficient of correlation of 0.95. The results of the analysis below can therefore be interpreted in terms of either the nominal or the real exchange rate. The nominal exchange rate is used to minimize measurement error.

In Section 1.5.2, I obtain regression results with four alternative measures of exchange rate uncertainty: past daily volatility, volatility predicted by a GARCH model, the degree of disagreement among forecasters and volatility implied by currency option prices. While past volatility and GARCH measures are widely used in the literature and are calculated only from historical exchange rate data, the two other measures have not been used in this literature and theoretically capture more precisely the concept of forward-looking uncertainty. The four measures are strongly but imperfectly correlated, suggesting that they capture slightly different phenomena. Nevertheless, the regression results obtained with the different measures of uncertainty are all broadly similar.

1.3.4 Data

I estimate the baseline regressions using a panel dataset containing quarterly data for 21 manufacturing industries. Over the sample period, manufactured goods represented around 65 percent of Canadian exports and exports represented 33 percent of Canadian GDP. Canadian sectoral exports to the United States are obtained from Statistics Canada's Canadian International Merchandise Trade Database.[11] Goods are classified according to the Harmonized System (HS) nomenclature at the 6-digit level. I use the concordance tables developed by Pierce & Schott (2012) to aggregate goods into 3-digit North American Industry Classification System (NAICS) manufacturing industries.[12] Exports are reported in Canadian dollars. I use the Canadian industrial product price indexes from Statistics Canada to deflate values. The resulting series covers the time period 1988Q1-2015Q4 and is expressed in terms of 2010 prices. Table 1.1 contains summary statistics by industry. By far the most important industry in terms of export value is transportation equipment (336). After adjusting for price, exports of most sectors have a standard deviation-to-mean ratio ranging from 0.35 to 0.45. The main exceptions are the relatively volatile clothing industry (315) and the stable paper manufacturing industry (322).

The number of product varieties exported from Canada to the United States in each manufacturing sector is measured as the number of HS-10 products with a value of exports greater than zero in a given NAICS-3 industry. Industries are assigned using the Pierce & Schott (2012) tables. The HS-10 trade data is obtained from the USA Trade Online database of the U.S. Census Bureau and is available annually from 1992.

[11] Data on Canadian exports to the U.S. are collected by the U.S. Census Bureau and then shared with Statistics Canada.

[12] I use the tables converting HS10 product codes into 2007 NAICS7 industries for U.S. imports. The product codes not included in these tables are converted manually.

I obtain sectoral price indexes for the United States from the U.S. Bureau of Labor Statistics' Producer Price Index database. I obtain the U.S. real GDP, the CAD-USD exchange rate, and oil price (WTI) data from the Federal Reserve Economic Data collected by the Federal Reserve Bank of St. Louis. Oil price uncertainty is constructed using the methodology described above for exchange rate uncertainty. I obtain the Canadian risk-free interest rate (the interest rate on a 3-month Treasury bill) from Statistics Canada. Figure 1.1 plots the exchange rate (left axis) and exchange rate uncertainty (right axis, dashed line) over the 1988Q1-2015Q4 period. High uncertainty does not seem to be caused by, or to cause, a strong or a weak Canadian currency. In fact, the correlation between the two series is only -0.18.

I use two direct measures of financial constraints: asset tangibility and external finance dependence. Asset tangibility is the ratio of a firm's property, plant and equipment to its total assets.[13] External finance dependence is the fraction of capital expenditures that cannot be financed by current operational cash flow ($\frac{\text{Capital expenditure - Cash flow}}{\text{Capital expenditure}}$). I construct these measures following the literature and using firm-level data from Compustat, which contains most publicly-traded U.S. firms.[14] For each firm that existed in the database at some point between 1988 and 2015, I compute the two measures by taking the median for the years for which data is available. I then take the median of each measure for all firms in a given sector.[15]

Table 1.2 contains the financial variables for different sectors. Industries dif-

[13]Intangible assets include, among others, research and development and goodwill.

[14]U.S. firms are used because the number of Canadian firms available in Compustat is too small in some industries to be representative. Furthermore, Rajan & Zingales (1998), who introduced the measure of external finance dependence, argue that firms' need for external finance is contingent on the industry to which they belong for technological reasons. The same argument can be made for asset tangibility. Using the measures for Canadian industries further assumes that the type of industry-specific technology used in U.S. firms in similar to the type of technology used in Canadian firms.

[15]The measures that I obtain are similar to those reported in Chor & Manova (2012), computed using U.S. firms between 1996 and 2005.

fer greatly. Miscellaneous products (339), Computer and electronic products (334), and Electrical equipment (335) are the industries that rely most heavily on external financing.[16] On the other hand, many industries do not need any external financing over the studied time period. The Paper (322) as well as Petroleum and coal products (324) industries have the most tangible assets whereas the Clothing (315), Leather products (316), and Computer and electronic products (334) industries have the least tangible assets. It is interesting to note that these two variables are negatively related, with a correlation coefficient of -0.38.

The last column of Table 1.2 contains the share of exports produced by SMEs (small and medium enterprises, defined as hiring 0-499 employees) for each industry. The annual data is obtained from Statistics Canada and is averaged over the 2010-2015 period.[17] Exporters in the Transportation equipment (336) industry are almost all large and those in the textile-related industries (313, 314, 315, 316) are almost all small.

1.4 Results

Tables 1.3 and 1.4 display the main results of the empirical analysis. Column 1 of Table 1.3 contains the estimated results for the first specification, equation (1.1). On average, an increase of one percent in exchange rate uncertainty leads to a decrease of 0.9 percent in exports. This average effect however obscures larger, heterogeneous effects across sectors. Columns 1 and 2 of Table 1.5 contain the results estimated using a second specification in which exchange rate uncertainty is interacted with sector fixed effects. The results show that nine out of the 21 sectors are significantly

[16]NAICS industry 325 (Chemical products) is excluded from all regressions since its measure of external finance dependence is extremely high, with a ratio of 11.

[17]The earliest available year is 2010. In the regressions, the share of exports produced by SMEs is assumed to be exogenous. In reality, it may reflect firms' heterogeneous reactions to exchange rate uncertainty over the previous years and may cause the estimates to be biased.

negatively affected by exchange rate risk. Among these sectors, a one-percent increase in exchange rate uncertainty leads to an average decrease of 6.53 percent in exports. Alternatively, an increase of one standard deviation in (log) uncertainty reduces exports by 3.2 percent. The remaining sectors are either not significantly affected or significantly positively affected. This confirms findings in the existing literature that aggregate effects can be small and go in either direction due to heterogeneous disaggregated effects.

One contribution of this chapter is to examine the reasons behind such heterogeneous results. The third specification tests for the role of two commonly used measures of financial frictions: external finance dependence and asset tangibility. Results are presented in the second column of Table 1.3. If being financially constrained prevents firms from efficiently handling exchange rate risk, as has been found in the literature, then external finance dependence should worsen (i.e., make more negative) the effect of uncertainty on exports whereas asset tangibility should make the effect more positive (or less negative). The regression results suggest that the effect of asset tangibility is indeed positive, but not statistically significant, while the effect of external finance dependence appears to be non-existent.

One possible explanation for these surprising results is that financial markets are more developed in Canada than in countries for which financial frictions have been found to matter, so that on average, these measures of financial frictions do not show a statistically significant effect. An alternative and potentially complementary explanation is that these measures of financial frictions do matter, but differently for different firms. In order to test this theory, the fourth specification tests for the possibility that firms could be differently affected by financial frictions because of their differences in size by further interacting uncertainty and measures of financial constraints with the share of sectoral exports coming from SMEs. The results are presented in the third column of Table 1.3.[18] The results show that external finance dependence has a significantly more negative effect on exports in sectors with smaller firms. In fact, in an industry composed only of large firms, a higher level of external finance dependence results in a significantly *less* negative effect of exchange rate uncertainty on exports (the coefficient on log $\text{Uncert}_t \times \text{Ext Fin Dep}_s$ is positive). In an industry composed only of SMEs, it makes the effect *more* negative (the sum of the coefficient on log $\text{Uncert}_t \times \text{Ext Fin Dep}_s$ and of $100\times$ the coefficient on log $\text{Uncert}_t \times \text{Ext Fin Dep}_s \times \text{Sh SME}_s$ is negative). These results suggest that external finance dependence is, as suggested in the literature, a measure of financial frictions, but not necessarily for larger firms. One possible explanation regarding why this effect has not been found in previous studies is that Canadian financial markets could be more developed (especially for larger firms) than those of countries previously studied. The results in column 3 also suggest that sectors with relatively larger firms benefit more from asset tangibility than those with relatively smaller firms. The theoretical model introduced in the second chapter of this dissertation proposes mechanisms explaining the different effects of financial constraints on firms of differing sizes.

The results obtained using the number of exported product varieties as dependent variable are presented in Table 1.4 and columns 3 and 4 of Table 1.5. They widely confirm the results obtained for total exports. On average, the effect of exchange rate uncertainty on the number of exported varieties is negative and highly significant. A one-percent increase in uncertainty results in a decrease of more than 3 percent in the number of exported varieties.[19] The sectoral results show that, for most industries,

[18] The coefficients on interacted variables appear small in magnitude since the three interaction variables (Ext Fin Dep_s, Asset Tang_s, and Share SME_s) are expressed in percentage terms.

[19] The coefficients contained in Table 1.3 cannot be directly compared with the coefficients of Table 1.4 because they come from datasets with different frequencies. The large average effect of exchange rate uncertainty does not necessarily reflect a larger adjustment at the extensive margin of trade and could reflect the accumulated effect of uncertainty over several quarters.

the effect of uncertainty on the extensive margin of exports is qualitatively the same as the effect on exports overall. This finding suggests that the effect on the extensive and intensive margins of exports typically go in the same direction. Interestingly, while asset tangibility plays a similar role in both sets of regression, external finance dependence does not seem to matter as much in the extensive margin of trade's response to exchange rate uncertainty.

Table 1.6 contains additional regression results obtained separately for sectors negatively (columns 1 and 3) and positively affected (columns 2 and 4) by exchange rate uncertainty identified from column 1 of Table 1.5 (disregarding statistical significance). I find that financial frictions play a much stronger role in sectors negatively affected by uncertainty and no role at all in sectors positively affected. Hence the heterogeneity in the effect of exchange rate uncertainty that can be identified from measures of financial frictions should be interpreted as applying only to negatively-affected sectors.

The estimation results also suggest that both the effect of exchange rate uncertainty on exports and the effect of financial frictions on firms' responses to uncertainty are economically significant. Figure 1.2 contains the predicted percentage change in exports for different hypothetical sectors when uncertainty increases by one standard deviation and shows that the effect can reach several percentage points. Figures 1.2a and 1.2b show the change in exports in sectors with the average degree of external finance dependence but with differing levels of asset tangibility, with either a low (20 percent) or a high (80 percent) share of exports coming from SMEs. The change in exports is more positive the higher the tangibility, especially for sectors dominated by large firms. Hence, asset tangibility appears to be crucial for large firms' ability to adapt to risk. Low tangibility results in a statistically and economically significant decrease in exports of several percentage points following the increase in uncertainty, regardless of the size distribution of exporters. Figures 1.2c and 1.2d show the change in exports in sectors with the average level of asset tangibility but with varying degrees of external finance dependence. The direction of the effect of external finance dependence depends on the composition of the sector: in a sector dominated by large firms, high external finance dependence leads to a more positive effect of uncertainty. The opposite is true of a sector composed of mostly small firms.

1.5 Robustness

1.5.1 Alternative timing assumptions

Existing theoretical models of the effect of exchange rate uncertainty on international trade can come to different conclusions when using different assumptions about the timing of the decisions made by the firm. More specifically, whether firms decide the quantity to export before or after deciding the quantity to produce can determine whether uncertainty has a positive or a negative effect on exports. On the one hand, if the production decision happens first, the firm may be able to observe the exchange rate (or the realization of the uncertain outcome) before deciding the quantity to be exported. In this case, higher uncertainty can lead to higher potential profits, and higher exports if the realization turns out favorable.[20] On the other hand, if the exporting decision must be made first, it must be made before the exchange rate is realized and more uncertainty can lead to lower exports.

The timing assumption made in the empirical analysis does not rely on whether it is the production or the exporting decision that happens first. It only relies on the payment for the exported goods being received *after* the goods have been produced.

[20]See, for example, Dellas & Zilberfarb (1993).

However, if the decision to export at time t is made at time $t-1$, then exchange rate uncertainty perceived at time $t-1$ is relevant. Column 2 of Table 1.7 contains regression results obtained with the first lag of the uncertainty measure.

The payment for the exported goods may also be made at or around the time of shipping. In this case, exchange rate uncertainty matters only if the decision to export must be made ahead of time, possibly because production happens after the decision to export has been made. In this case, it is the measure of perceived uncertainty in the previous period $(t-1)$ that may affect exports. Hence, the first lag should also be used.

One last possibility comes from the way the measure of exchange rate uncertainty is built. The main analysis assumes that agents use past volatility of exchange rate in order to form their current perception of future exchange rate uncertainty. It could also be that agents use current volatility of the exchange rate instead to form their perception. In this case, the first lead of the measure of exchange rate uncertainty would matter for exports. Column 3 of Table 1.7 contains regression results obtained with the first lead of the uncertainty measure. The striking similarity between the results reached using different timing assumptions suggests that the order in which different steps happen has little impact on the way financial frictions shape the relationship between exchange rate uncertainty and exports.

1.5.2 Alternative measures of exchange rate uncertainty

In this section, I obtain regression results with four alternative measures of exchange rate uncertainty: past daily volatility, volatility predicted by a GARCH model, the degree of disagreement among forecasters and volatility implied by option prices.

The past daily volatility measure is calculated as the square root of the average daily squared (log) change in the nominal exchange rate over the quarter preceding quarter t. It measures the volatility of the exchange rate at a higher frequency than the measure used in the main analysis. It is therefore noisier, but also has the potential to capture events missed by the measure using monthly changes.

The GARCH measure is obtained from a GARCH(2,2) model. One advantage of this measure is that it takes into account that periods of high (or low) volatility tend to happen in clusters rather than randomly. It uses past volatility to predict future volatility. The GARCH model that I estimate is

$$x_t - x_{t-1} = \alpha_0 + \alpha_1(x_{t-1} - x_{t-2}) + \alpha_2[\log(OilPrice_t) - \log(OilPrice_{t-1})] + u_t \tag{1.3}$$

$$u_t = \sigma_t e_t,\ \ e_t \sim iidN(0,1) \tag{1.4}$$

$$\sigma_t^2 = \beta + \sum_{i=1}^{p} \beta_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \tag{1.5}$$

I include the price of oil in equation (1.3) since there is strong evidence that it affects the CAD-USD exchange rate. I estimate the variance equation (1.5) with $p = 2$ and $q = 2$ following the results of Engle's Lagrange-multiplier test. I find no evidence of serial correlation in the disturbances u_t. Fitted volatility $\hat{\sigma}_t^2$ proxies for exchange rate uncertainty.

I calculate forecasters' disagreement using the Blue Chip Financial Forecasts compiled by Aspen Publishers. This monthly survey contains forecasts made by financial institutions about what the CAD-USD exchange rate will be three months later. I calculate disagreement by computing for each month the difference between the highest and the lowest forecasts.[21] I then average this monthly measure within each quarter.

Finally, I obtain option-implied volatility of the CAD-USD exchange rate from Bloomberg. Option-implied volatility is calculated from option prices. Investors may purchase options to exchange Canadian dollars for U.S. dollars (or vice versa) at different exchange rates and at different dates in the future. The Black-Scholes formula predicts the theoretical price of a given option (for a risk-neutral investor) by taking into account the current exchange rate, the exchange rate secured in the option contract, the option's maturity and the expected volatility of the exchange rate until maturity. Since all variables, including option prices, are available in the data except expected volatility, the formula can be used to deduct the degree of volatility consistent with the observed prices. I use the series calculated from prices of options with a 3-month maturity to capture markets' uncertainty about the movements of the exchange rate in the following quarter.

The different measures of uncertainty are available for different time periods. While the volatility and GARCH measures are built using only historical exchange rate data (and oil price data in the case of the GARCH measure), the forecast disagreement measure is available from 1993Q1 and the implied volatility measure is available from 1999Q1. The five measures of exchange rate uncertainty used in this analysis, normalized to 100 in 1999Q1, are plotted in Figure 1.3 and Table 1.8 contains their bilateral coefficients of correlation. It is clear that while ex-post and forward-looking measures are positively correlated, they are not perfectly correlated and capture fairly different phenomena. Interestingly, the option-implied measure of uncertainty is highly correlated with the GARCH measure. This suggests that market participants base (or expect other market participants to base) their expectations of

[21] The small number of forecasters prevents the use of more sophisticated measures of disagreement.

future volatility on past volatility and incorporate little forward-looking information. The disagreement measure, on the other hand, suggests that forecasters do incorporate a fair amount of forward-looking (private) information in their forecasts of the level of the exchange rate so that this measure is the least correlated with the others.

Table 1.9 compares the results obtained from the most complete specification using the five measures of uncertainty for the 1999Q1-2015Q4 period.[22] Using this shorter sample has two main effects on the regression results. First, the smaller number of observations reduces the number of degrees of freedom, which generally tends to increase standard errors. Second, since the period dropped (1988-1998) saw low and less volatile levels of uncertainty, the effect can be better identified with the smaller sample. All measures of uncertainty confirm the qualitative results obtained with the larger sample. Hence, when accounting for the differential effects, the measure of uncertainty used seems to matter little.

1.5.3 Other robustness tests

One plausible alternative interpretation of the empirical results described in Section 1.4 is that Canadian firms could be uncertain about the future demand from U.S. consumers for their products. More financially-constrained firms could find it harder to manage this uncertainty. Hence it could be that the effect attributed to exchange rate uncertainty in fact captures the effect of demand uncertainty. One measure of macroeconomic uncertainty that has been frequently used in the macroeconomic literature is the VIX index. The VIX index is a measure of macroeconomic volatility computed and published by the Chicago Board Options Exchange that measures the volatility implied by prices for options to trade the S&P 500 stock index. A higher value of the VIX index indicates that financial markets are more uncertain about the

[22]This is the longest period for which the five measures are available.

future profitability of U.S. firms. This situation could both cause greater uncertainty about the CAD/USD exchange rate and affect Canadian exports to the U.S. directly, thereby resulting in omitted variable bias on the coefficients of interest in the regressions. The coefficient of correlation between the VIX index and the monthly volatility measure of exchange rate uncertainty however only comes out to 0.30. Furthermore, adding the VIX index as an additional control to the regressions, interacted with sector fixed effects, does not modify the qualitative effects found in the main empirical analysis and only slightly affects the magnitudes of the coefficients. The coefficients of interest are presented in column 2 of Table 1.10. Column 1 reproduces the baseline results presented in column 3 of Table 1.3.

Since the sample period contains the U.S. Great Recession and the financial crisis that preceded it, a second alternative interpretation of the main empirical results is that the relationship found between exchange rate uncertainty and Canadian sectoral exports held only for that period and that the effect estimated in the main analysis is in fact capturing the effect of the financial crisis on exports. As a second robustness check, I create a *crisis* dummy variable which takes the value of one between 2007Q1 and 2009Q4 and zero during all other periods. I interact this *crisis* variable with all regressors containing exchange rate uncertainty. As a third robustness check, I estimate the baseline regression for the pre-financial crisis period of 1988Q1-2006Q4. Finally, in order to ensure that other macroeconomic events are not driving the interactions found between exchange rate uncertainty and financial constraints, I estimate the regression with period-specific (Quarter $\times$ Year) fixed effects. The results of these three robustness tests are presented in column 3 of Table 1.10 and columns 2 and 3 of Table 1.11, and confirm that the heterogeneity in the effects of uncertainty is not caused by macroeconomic conditions.

A third alternative interpretation of the main empirical results is that Canadian firms could be affected by exchange rate uncertainty because of the inputs that they import from the U.S. If firms pay for their imported inputs after they have ordered them, higher exchange rate uncertainty causes higher uncertainty about costs and firms that are not as financially robust could find it harder to manage the risk. A reduction in imported inputs would then result in a reduction of production, which could result in a reduction of exports. Hence, the observed impact of exchange rate uncertainty on exports would at least partially be caused by the importing behavior of firms rather than their exporting behavior. As a robustness check, I use the ratio of exports to production by sector as the dependent variable. The identifying assumption is that all production (both for domestic sales and for foreign sales) relies on the same share of imported inputs. Hence, if exports are impacted by exchange rate uncertainty only because of a change in imported inputs instead of specific exporting decisions, exports and production would move exactly proportionally. Their ratio would then not be affected by exchange rate uncertainty. Column 2 of Table 1.12 contains the results of a regression for which the log of the ratio of exports to production is used as independent variable.[23] The coefficients of interest remain mostly similar, confirming that the heterogeneity in the response of exports to exchange rate uncertainty results from firms' exporting, not importing, decision.

A fourth alternative interpretation of the main empirical results is that the effect of firm size could be non-monotonic. Since firm size is measured as the share of exports coming from small and medium firms (1-499 employees), it is possible that medium-size firms drive the results associated to smaller firms and that small firms in fact behave more like large firms. In this robustness check, I replace the Share SME_s variable with the share of exports coming from small firms (Share Small_s), defined

[23]For data availability reasons, this regression does not include the Textile (313) and Textile Products (314) industries, and covers a slightly shorter period (1988Q1-2012Q3).

as firms with 1-100 employees. The results are presented in column 3 of Table 1.12 and show that small firms' differential effects of financial constraints are even larger, suggesting that the effect of firm size is monotonic.

Finally, in a last robustness check, I control for a potential third-country effect that could come from the distinct trade relationship between Canada, the United States, and Mexico. I add Mexican GDP, interacted with sector fixed effects, to the regression.[24] The results are presented in column 4 of Table 1.12 and remain unchanged.

1.6 Conclusion

The economic consequences of uncertainty are receiving growing attention from researchers. In this chapter, I study empirically how a specific type of uncertainty, exchange rate uncertainty, affects firms' exporting decisions. Building on progress made by the recent literature, I focus on the role of financial frictions. I use data on Canadian exports to the United States and find that exports in industries in which firms are more financially constrained are more negatively affected by exchange rate uncertainty. This result holds when looking at either the total margin or the extensive margin of exports. Sectors in which firms have more tangible assets see their exports decrease less in times of high uncertainty. This is even more true of sectors composed of mostly large firms. I also find that the effect of external finance dependence on the relationship between exchange rate uncertainty and trade is not homogeneous and depends on the size of the exporters. In sectors in which most exporters are small or medium firms, external finance dependence acts as a financial constraint and exports are more negatively affected by uncertainty. In sectors in which most exporters are large, a higher degree of external dependence has little effect.

[24] Historical sectoral production data are not freely available for Mexico.

The fact that exchange rate uncertainty and financial frictions interact to affect the behavior of Canadian exporting firms is especially unexpected considering the high level of financial development in the countries involved, including the wide availability of financial hedging instruments. This finding suggests that the effect of exchange rate uncertainty on trade between different country pairs is potentially significantly more negative.

This chapter confirms existing empirical results about the role of financial frictions in the relationship between exchange rate uncertainty and international trade, and uncovers new patterns. A natural next step is to design a theoretical model that suggests mechanisms to explain these findings. This is precisely what I do in the next chapter of this dissertation.

1.7 Figures

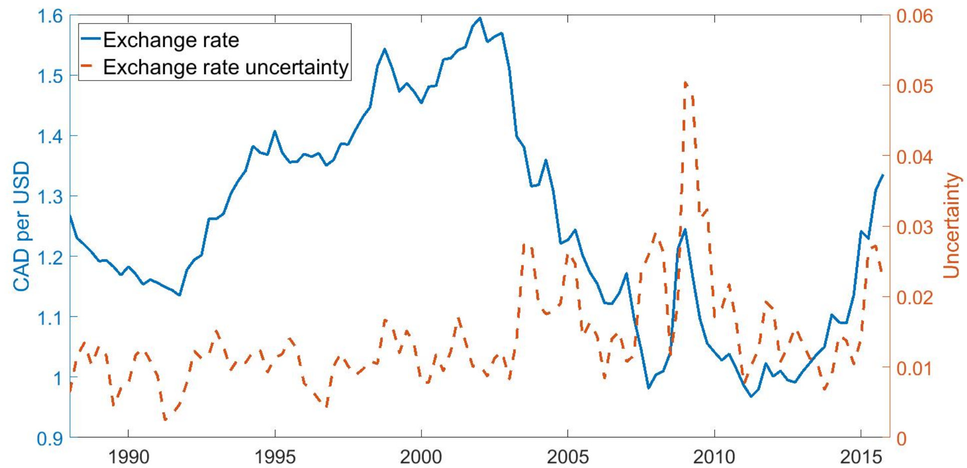
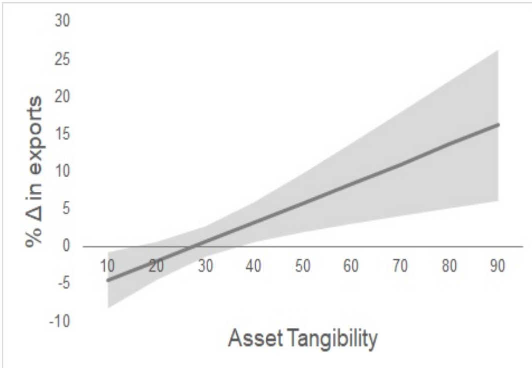
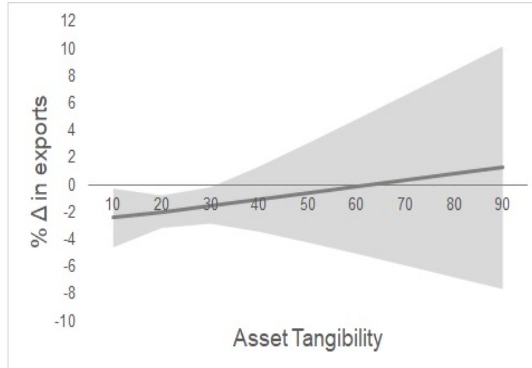


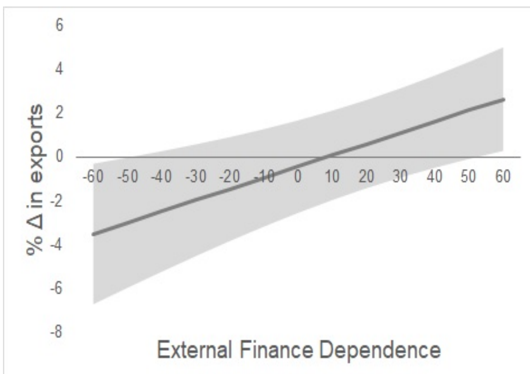
Figure 1.1: Exchange rate and exchange rate uncertainty, 1988Q1-2015Q4



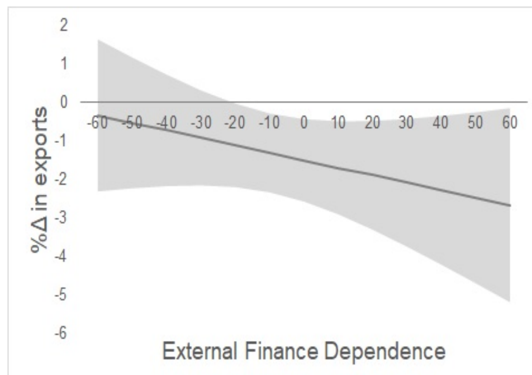
(a) Effect of asset tangibility,
share of SMEs = 20%



(b) Effect of asset tangibility,
share of SMEs = 80%



(c) Effect of external finance dependence,
share of SMEs = 20%



(d) Effect of external finance dependence,
share of SMEs = 80%

Figure 1.2: Percentage change in exports in response to a one-standard-deviation increase in exchange rate uncertainty

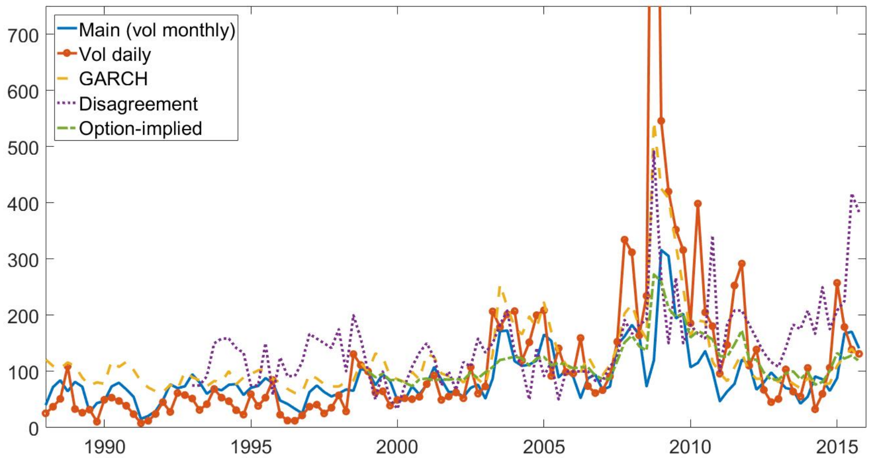


Figure 1.3: Different measures of exchange rate uncertainty

1.8 Tables

NAICS-3	Million CAD, 2010 prices Mean	St Dev	Min	Max
311 Food	2285	1010	596	4411
312 Beverage and tobacco	343	130	159	658
313 Textiles	235	114	54	449
314 Textile products	107	44	38	215
315 Clothing	323	212	69	760
316 Leather products	41	18	16	92
321 Wood products	2401	883	1071	4335
322 Paper	3631	639	2553	4774
323 Printing and related activities	320	134	120	605
324 Petroleum and coal products	2536	977	617	4223
325 Chemical	4879	1699	1515	7343
326 Plastics and rubber products	1712	639	557	2703
327 Non-metallic mineral products	567	216	251	1044
331 Primary metal	5403	1241	2249	7225
332 Fabricated metal products	1575	526	631	3030
333 Machinery	3237	917	1334	4528
334 Computer and electronic products	2394	857	1385	6195
335 Electrical equipment	1036	411	320	1778
336 Transportation equipment	13821	2979	7815	19376
337 Furniture and related products	1037	500	330	2000
339 Miscellaneous	467	209	132	928

This table contains summary statistics about Canadian sectoral exports to the U.S. over the 1988Q1-2015Q4 period. Exports values are extracted in HS-6 classification and translated into NAICS-3 classification. Exports are deflated using industrial product price indexes obtained from Statistics Canada.

Table 1.1: Summary statistics of Canadian manufacturing exports to the United States

NAICS-3	External Fin. Dependence (%)	Asset Tangibility (%)	Share of SMEs (%)
311 Food	-3.3	31.0	48.0
312 Beverage and tobacco	33.5	21.9	67.3
313 Textile	-9.0	33.5	98.7
314 Textile products	-16.7	29.9	95.5
315 Clothing	47.7	12.1	89.8
316 Leather products	-81.9	11.1	100
321 Wood products	-17.0	32.0	47.2
322 Paper	-8.3	44.6	38.3
323 Printing and related activities	-17.2	32.3	61.5
324 Petroleum and coal products	7.1	51.1	32.1
325 Chemical	11114.8	8.8	50.8
326 Plastics and rubber products	11.8	34.2	58.1
327 Non-metallic mineral products	3.9	43.0	71.5
331 Primary metal	0.3	35.6	53.2
332 Fabricated metal products	-24.6	26.0	82.4
333 Machinery	10.2	16.4	73.8
334 Computer and electronic products	73.8	11.3	65.9
335 Electrical equipment	61.2	18.0	60.2
336 Transportation equipment	-0.3	22.0	8.0
337 Furniture and related products	-108.4	25.0	79.3
339 Miscellaneous	164.5	12.0	39.4

This table contains financial indicators for 21 manufacturing industries. External finance dependence and asset tangibility are calculated from Compustat's database using U.S. public firms active at some point over the 1988Q1-2015Q4 period. External finance dependence is calculated as the share of capital expenditure that cannot be financed by current operational cashflow. Asset tangibility is the ratio of property, plant, and equipment to total assets. Both measures are first calculated at the firm level for every year of existence. I take the median over the years of existence, and then the median over all firms in a given industry to construct constant sectoral measures. Share of SMEs indicates the share of all Canadian exports in a given sector coming from small or medium firms (0-499 employees). The annual data is obtained from Statistics Canada. The average is taken over the 2010-2015 period.

Table 1.2: Financial characteristics of manufacturing industries

Dependent variable	log Exports (2010 prices)		
	1	2	3
log Uncert_t	-0.009*	-0.060	-0.184**
	(-1.83)	(-1.49)	(-2.59)
log $\text{Uncert}_t \times$ Ext Fin Dep_s		-0.00004	0.0015***
		(-0.14)	(3.51)
log $\text{Uncert}_t \times$ Asset Tang_s		0.0019	0.0067***
		(1.36)	(3.01)
log $\text{Uncert}_t \times$ Share SME_s			0.0016
			(1.50)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s			-0.00002***
			(-3.02)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s			-0.00007*
			(-1.95)
Sector FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Number obs	2240	2240	2240
R-squared(within)	0.918	0.918	0.919

This table presents estimates obtained using a panel dataset covering 21 manufacturing sectors and the 1988Q1-2015Q4 period. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices. The control variables include the log of exchange rate uncertainty as well as the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All controls except exchange rate uncertainty are interacted with sector fixed effects. In column 2, exchange rate uncertainty is interacted with sectoral measures of external finance dependence and asset tangibility (both in percentage). In column 3, exchange rate uncertainty is additionally interacted with sectoral measures of the share of exports coming from SMEs; and triply interacted with share of SMEs and external finance dependence, and share of SMEs and asset tangibility. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.3: Financial constraints, exchange rate uncertainty and the total margin of trade

Dependent variable	log Number of varieties		
	1	2	3
log Uncert_t	-0.036***	-0.100***	-0.119**
	(-3.06)	(-3.88)	(-2.39)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s$		0.0003	0.00006
		(1.51)	(0.15)
log $\text{Uncert}_t \times \text{Asset Tang}_s$		0.002**	0.005***
		(2.78)	(3.08)
log $\text{Uncert}_t \times \text{Share SME}_s$			0.0007
			(0.78)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s \times \text{Sh SME}_s$			0.000003
			(0.41)
log $\text{Uncert}_t \times \text{Asset Tang}_s \times \text{Sh SME}_s$			-0.00006*
			(-1.82)
Sector FE	Yes	Yes	Yes
Quarter FE	-	-	-
Year FE	No	No	No
Number obs	480	480	480
R-squared(within)	0.761	0.766	0.770

This table presents estimates obtained using a panel dataset covering 21 manufacturing sectors and the 1992-2015 period. The dependent variable is the log of the number of product varieties exported. The control variables include the log of exchange rate uncertainty as well as the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All controls except exchange rate uncertainty are interacted with sector fixed effects. In column 2, exchange rate uncertainty is interacted with sectoral measures of external finance dependence and asset tangibility (both in percentage). In column 3, exchange rate uncertainty is additionally interacted with sectoral measures of the share of exports coming from SMEs; and triply interacted with share of SMEs and external finance dependence, and share of SMEs and asset tangibility. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.4: Financial constraints, exchange rate uncertainty and the extensive margin of trade

Dependent variable		log Exports (2010 prices)		log Number of varieties	
NAICS	Manufacturing sector	Total effect	T-statistics	Total effect	T-statistics
		1	2	3	4
311	Food	0.029**	(2.33)	-0.032***	(-21.41)
312	Beverage and tobacco	-0.028*	(-2.08)	-0.012***	(-11.27)
313	Textile	-0.087***	(-5.72)	-0.129***	(-47.98)
314	Textile products	-0.041**	(-2.62)	-0.079***	(-30.77)
315	Clothing	-0.119***	(-8.35)	-0.093***	(-41.57)
316	Leather products	0.077***	(5.70)	-0.051***	(-29.97)
321	Wood products	-0.036**	(-2.49)	-0.032***	(-30.48)
322	Paper	0.034**	(2.53)	0.011***	(9.42)
323	Printing and related activities	0.094***	(6.88)	0.003**	(2.42)
324	Petroleum and coal products	0.062***	(4.77)	0.056***	(70.06)
326	Plastics and rubber products	-0.001	(-0.08)	0.003*	(1.74)
327	Non-metallic mineral products	0.010	(0.78)	-0.030***	(-22.26)
331	Primary metal	0.022	(1.56)	-0.031***	(-25.51)
332	Fabricated metal products	-0.032**	(-2.45)	-0.004**	(-2.74)
333	Machinery	0.012	(0.95)	-0.034***	(-21.27)
334	Computer and electronic products	-0.110***	(-7.05)	-0.049***	(-46.22)
335	Electrical equipment	-0.019	(-1.51)	-0.030***	(-17.12)
336	Transportation equipment	-0.071***	(-5.62)	-0.028***	(-17.76)
337	Furniture and related products	-0.064***	(-5.00)	-0.132***	(-81.29)
339	Miscellaneous	0.012	(0.90)	-0.046***	(-27.16)

This table presents estimates obtained using a panel dataset covering 20 manufacturing sectors and the 1988Q1-2015Q4 period for columns 1-2 and the 1992-2015 period for columns 3-4. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices in columns 1-2 and the log of the number of product varieties exported in columns 3-4. The control variables include the log of exchange rate uncertainty as well as the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All controls are interacted with sector fixed effects. Standard errors are clustered at the sector level. The reported coefficients are the total effect of uncertainty for each sector. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.5: Exchange rate uncertainty and sector-level trade

Dependent variable	log Exports (2010 prices)		log Number of varieties	
	Negatively affected	Positively affected	Negatively affected	Positively affected
	1	2	3	4
log Uncert_t	-0.360***	0.097	-0.298**	-0.118
	(-3.89)	(0.73)	(-2.58)	(-0.99)
log $\text{Uncert}_t \times$ Ext Fin Dep_s	0.0045**	0.00018	0.0034*	-0.00015
	(4.10)	(0.84)	(2.13)	(-0.70)
log $\text{Uncert}_t \times$ Asset Tang_s	0.0147***	-0.0012	0.0126*	0.0041
	(3.37)	(-0.40)	(2.42)	(1.59)
log $\text{Uncert}_t \times$ Share SME_s	0.0033**	-0.0016	0.0023	0.00094
	(2.74)	(-0.92)	(1.26)	(0.60)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s	-0.00006***	-0.000014***	-0.000037	0.000005**
	(-4.09)	(-4.53)	(-1.65)	(2.86)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s	- 0.00015**	0.000018	-0.00014	-0.000049
	(2.64)	(0.45)	(-1.64)	(-1.35)
Sector FE	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	-	-
Year FE	Yes	Yes	No	No
Number obs	1232	1008	264	216
R-squared(within)	0.933	0.910	0.812	0.698

This table presents estimates obtained using a panel dataset covering 20 manufacturing sectors and the 1988Q1-2015Q4 period for columns 1-2 and the 1992-2015 period for columns 3-4. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices in columns 1-2 and the log of the number of product varieties exported in columns 2-3. The control variables are the same as in Tables 1.3 and 1.4. Columns 1 and 3 include only sectors negatively affected by uncertainty identified from Table 1.5 (NAICS codes 312, 313, 314, 315, 321, 326, 332, 334, 335, 336, 337). Columns 2 and 4 include only sectors positively affected by uncertainty identified from Table 1.5 (NAICS codes 311, 316, 322, 323, 324, 327, 331, 333, 339). Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.6: Heterogeneous effect of uncertainty on negatively and positively affected industries

Dependent variable	log Exports (2010 prices)		
	Current	First lag	First lead
log Uncert_t	-0.184**	-0.130*	-0.134
	(-2.59)	(-1.82)	(-1.61)
log Uncert_t × Ext Fin Dep_s	0.0015***	0.0013***	0.0011**
	(3.51)	(3.31)	(2.39)
log Uncert_t × Asset Tang_s	0.0067***	0.0047*	0.0054**
	(3.01)	(1.88)	(2.42)
log Uncert_t × Share SME_s	0.0016	0.0015	0.0009
	(1.50)	(1.38)	(0.79)
log Uncert_t × Ext Fin Dep_s × Share SME_s	-0.00002***	-0.00002**	-0.00002**
	(-3.02)	(-2.78)	(-2.11)
log Uncert_t × Asset Tang_s × Share SME_s	-0.00007*	-0.00006	-0.00005
	(-1.95)	(-1.60)	(-1.39)
Sector FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Number obs	2240	2240	2240
R-squared(within)	0.908	0.919	0.919

This table contains estimates obtained using different timing assumptions. The panel dataset covers 20 manufacturing sectors and the 1988Q1-2015Q4 period unless otherwise indicated. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices. The non-reported control variables include the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All these controls are interacted with sector fixed effects. Sector, quarter and year fixed effects are always included. Column 1 contains the baseline regression results, presented in column 3 of Table 1.3. Column 2 includes instead the first lag of the measure of exchange rate uncertainty, and column 3 includes instead the first lead. Standard errors are clustered at the sector level. The reported coefficients are the total effect of uncertainty for each sector. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.7: Alternative Timing Assumptions

Measures	Vol monthly	Vol daily	GARCH	Disagreement	Option-implied
Vol monthly	1.00	0.78	0.79	0.37	0.71
Vol daily		1.00	0.85	0.33	0.73
GARCH			1.00	0.46	0.86
Disagreement				1.00	0.58
Option-implied					1.00

Table 1.8: Coefficients of correlation between different measures of exchange rate uncertainty

Dependent variable	log Exports (2010 prices)				
Measure of uncertainty	Volatility Monthly	Volatility Daily	GARCH	Dis-agreement	Option-implied
log Uncert_t	-0.148	-0.068	-0.156	-0.131*	-0.284
	(-1.63)	(-0.88)	(-1.09)	(-2.06)	(-1.07)
log $\text{Uncert}_t \times$ Ext Fin Dep_s	0.00097**	0.0007*	0.0011	0.0014**	0.0033**
	(2.22)	(1.93)	(1.47)	(2.59)	(2.10)
log $\text{Uncert}_t \times$ Asset Tang_s	0.0044	0.0022	0.0041	0.0034**	0.0085
	(1.50)	(1.11)	(1.09)	(2.29)	(1.23)
log $\text{Uncert}_t \times$ Share SME_s	0.0023*	0.00077	0.0017	0.0019*	0.0037
	(1.82)	(0.78)	(0.92)	(1.90)	(1.03)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s	-0.000015*	-0.0000083	-0.0000096	-0.000016*	-0.000038
	(-2.03)	(-1.56)	(-0.95)	(-1.75)	(-1.51)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s	-0.000073	-0.000030	-0.000055	-0.000054*	-0.00014
	(-1.50)	(-1.09)	(-0.97)	(-2.00)	(-1.33)
Sector FE	Yes	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Number obs	1360	1360	1360	1360	1360
R-squared(within)	0.912	0.912	0.912	0.913	0.913

This table presents estimates obtained using a panel dataset covering 20 manufacturing industries and the 1999Q1-2015Q4 period. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices. The control variables include the log of exchange rate uncertainty as well as the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All controls except exchange rate uncertainty are interacted with sector fixed effects. Sector, quarter and year fixed effects are also included. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.9: Alternative measures of exchange rate uncertainty

Dependent variable	log Exports (2010 prices)		
	1	2	3
log Uncert_t	-0.184**	-0.228***	-0.138**
	(-2.59)	(-3.11)	(-2.32)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s$	0.0015***	0.0018***	0.0013***
	(3.51)	(4.11)	(3.46)
log $\text{Uncert}_t \times \text{Asset Tang}_s$	0.0067***	0.0082***	0.0061***
	(3.01)	(3.95)	(3.15)
log $\text{Uncert}_t \times \text{Share SME}_s$	0.0016	0.0021*	0.0010
	(1.50)	(1.93)	(1.16)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s \times \text{Sh SME}_s$	-0.000024***	-0.000027***	-0.000024***
	(-3.02)	(-3.50)	(-3.08)
log $\text{Uncert}_t \times \text{Asset Tang}_s \times \text{Sh SME}_s$	-0.000072*	-0.000094**	-0.000066**
	(-1.95)	(-2.66)	(-2.10)
VIX $\text{index}_t \times \text{Sector}_s$	No	Yes	No
log $\text{Uncert}_t \times \text{crisis}_t$			-0.057*
			(-2.01)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s \times \text{crisis}_t$			0.00022*
			(1.87)
log $\text{Uncert}_t \times \text{Asset Tang}_s \times \text{crisis}_t$			0.00046
			(0.91)
log $\text{Uncert}_t \times \text{Sh SME}_s \times \text{crisis}_t$			0.00046
			(1.71)
log $\text{Uncert}_t \times \text{Ext Fin Dep}_s \times \text{Sh SME}_s \times \text{crisis}_t$			-0.0000005
			(-0.29)
log $\text{Uncert}_t \times \text{Asset Tang}_s \times \text{Sh SME}_s \times \text{crisis}_t$			-0.000005
			(-0.53)
Sector FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Quarter × Year FE	No	No	No
Number obs	2240	2080	2240
R-squared(within)	0.908	0.924	0.920

This table contains estimates obtained for robustness checks. The panel dataset covers 20 manufacturing sectors and the 1988Q1-2015Q4 period unless otherwise indicated. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices. The non-reported control variables include the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All these controls are interacted with sector fixed effects. Sector, quarter and year fixed effects are always included. Column 1 contains the baseline regression results, presented in column 3 of Table 1.3. Column 2 includes as additional controls the VIX index interacted with sector fixed effects and covers the period 1990Q1-2015Q4. Column 3 interacts all controls including exchange rate uncertainty with an additional dummy variable, crisis_t which takes a value of 1 between 2007Q1 and 2009Q4 and zero otherwise. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.10: Robustness tests I: Macroeconomic conditions

Dependent variable	log Exports (2010 prices)		
	1	2	3
log Uncert_t	-0.184**	-0.133***	
	(-2.59)	(-3.37)	
log $\text{Uncert}_t \times$ Ext Fin Dep_s	0.0015***	0.00096 **	0.0015***
	(3.51)	(2.55)	(3.44)
log $\text{Uncert}_t \times$ Asset Tang_s	0.0067***	0.0062***	0.0067***
	(3.01)	(4.45)	(2.95)
log $\text{Uncert}_t \times$ Share SME_s	0.0016	0.00069	0.0016
	(1.50)	(0.283)	(1.46)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s	-0.000024***	-0.000017**	-0.000024***
	(-3.02)	(-2.43)	(-2.96)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s	-0.000072*	-0.000057**	-0.000072*
	(-1.95)	(-2.27)	(-1.91)
Sector FE	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Quarter $\times$ Year FE	No	No	Yes
Number obs	2240	1520	2240
R-squared(within)	0.908	0.943	0.921

This table contains estimates obtained for robustness checks. The panel dataset covers 20 manufacturing sectors and the 1988Q1-2015Q4 period unless otherwise indicated. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices. The non-reported control variables include the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All these controls are interacted with sector fixed effects. Sector, quarter and year fixed effects are always included. Column 1 contains the baseline regression results, presented in column 3 of Table 1.3. Column 2 covers only the pre-crisis period of 1988Q1-2006Q4. Column 3 has additional Quarter × Year fixed effects. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.11: Robustness tests II: Macroeconomic conditions

Dependent variable	log Exports (2010 prices)*			
	1	2	3	4
log Uncert_t	-0.184**	-0.175***	-0.143***	-0.180**
	(-2.59)	(-4.45)	(-2.99)	(-2.77)
log $\text{Uncert}_t \times$ Ext Fin Dep_s	0.0015***	0.00095***	0.0025***	0.0017***
	(3.51)	(3.25)	(3.68)	(3.70)
log $\text{Uncert}_t \times$ Asset Tang_s	0.0067***	0.0061***	0.0047***	0.0075***
	(3.01)	(4.17)	(3.52)	(3.81)
log $\text{Uncert}_t \times$ Share SME_s	0.0016	0.0020***		0.0017
	(1.50)	(3.13)		(1.72)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s	-0.000024***	-0.000015**		-0.000026***
	(-3.02)	(-2.30)		(-3.10)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s	-0.000072*	-0.000078***		-0.000089**
	(-1.95)	(-3.01)		(-2.63)
log $\text{Uncert}_t \times$ Share Small_s			0.0032	
			(1.70)	
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh Small_s			-0.00010***	
			(-3.46)	
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh Small_s			-0.00013*	
			(-1.96)	
log GDP $\text{Mexico}_t \times \text{Sector}_s$	No	No	No	Yes
Sector FE	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Number obs	2240	1782	2240	2240
R-squared(within)	0.908	0.886	0.908	0.925

* unless otherwise indicated

This table contains estimates obtained for robustness checks. The panel dataset covers 20 manufacturing sectors and the 1988Q1-2015Q4 period unless otherwise indicated. The dependent variable is the log of Canadian exports to the U.S. expressed in 2010 prices except for column 2. The non-reported control variables include the log of the exchange rate, sector-specific producer price index, U.S. GDP, oil price (WTI), oil price uncertainty and the Canadian risk-free rate. All these controls are interacted with sector fixed effects. Sector, quarter and year fixed effects are always included. Column 1 contains the baseline regression results, presented in column 3 of Table 1.3. Column 2 has the share of exports to total Canadian production as dependent variable, covers the period 1988Q1-2012Q3 and does not include the Textile (313) and Textile Products (314) industries. Column 3 uses the share of exports coming from small firms (1-99 employees) instead of the share of exports coming from SMEs (1-499 employees) as measure for firm size. Column 4 includes as additional control the (log) real GDP of Mexico interacted with sector fixed effects. Standard errors are clustered at the sector level. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 1.12: Robustness tests III: Other tests

2 The effect of exchange rate uncertainty on international trade: The role of financial frictions - A theoretical analysis

2.1 Introduction

The empirical literature on the effect of exchange rate uncertainty on international trade is extensive. The theoretical literature is however much more sparse. Early theoretical models on the relationship between exchange rate uncertainty and international trade predict a negative effect: in times of high exchange rate uncertainty and in the absence of hedging instruments, risk-averse entrepreneurs choose to export less and instead concentrate on the domestic market.[25] Ethier (1973) and Baron (1976) however predicted that in countries where financial markets were developed, the liberalization of exchange rates would not significantly affect trade since hedging instruments were available to firms or their shareholders.

Following puzzling empirical results, other researchers have designed models that can explain how increases in exchange rate risk could instead lead to increases in international trade. For example, Dellas & Zilberfarb (1993) show that if exports contracts are designed as options for the firm, an increase in exchange rate volatility increases the exporter's potential profit and can result in a higher level of exports, depending on the exporter's degree of risk aversion. More recent literature on the relationship between exchange rate uncertainty and trade lacks theoretical models.

In the first chapter of this dissertation, I provide additional empirical evidence

[25] See Ethier (1973) and Clark (1973).

about the role of financial frictions in the relationship between exchange rate uncertainty and exports. I find that manufacturing shipments from Canada to the United States are more negatively affected by this type of uncertainty in sectors where firms are more financially constrained. Having more tangible capital, which can potentially be used as collateral, makes the effect of uncertainty less negative, especially im sectors where firms are large. Relying more on external financing, on the other hand, makes the effect more negative and affects sectors with small firms more.

In this chapter, I introduce a theoretical model that rationalizes these empirical findings. The model extends that of Manova (2013). Risk-neutral firms must borrow from a lender to cover some of the costs of exporting. The lender charges an interest rate above the risk-free rate because the repayment by the firm is only partially guaranteed by collateral. In Manova (2013), uncertainty about the firm's repayment comes from limited contract enforcement. It is constant and independent of the firm's behavior. In contrast, in my model, uncertainty arises from exchange rate risk. The exporting firm agrees on a price with its foreign customer, but this price is set in the customer's currency. Hence, before the exchange rate is realized, the exporting firm cannot precisely predict its revenue in its own currency. This causes ex-ante uncertainty regarding the firm's ability to fully repay the lender after the exchange rate is realized. Importantly, the firm's pricing decision affects the likelihood that it cannot fully repay.

The model introduced in this chapter differs from the existing literature in that firms are negatively affected by exchange rate uncertainty even if they are risk-neutral. The level of uncertainty affects the interest rates firms can obtain, which in turn affect their decisions regarding production and exporting. If firms are assumed to be risk averse in addition to being financially constrained, the role of financial frictions in the relationship between exchange rate uncertainty and exports is dampened but remains qualitatively identical.

Exports from firms with varying degrees of external finance dependence and asset tangibility are differently affected by exchange rate uncertainty since they represent different potential levels of risk to their lender. Firms with more tangible assets are less risky because they have more collateral to offer. Firms depending more heavily on external finance can be more or less affected by exchange rate risk depending on their size. Such firms borrow a greater share of their costs, and hence are more likely to default. However, by borrowing larger amounts, they also offer higher potential returns to lenders. As in other heterogeneous-firm models, smaller firms are also less productive. In their case, greater external finance dependence leads their exports to decrease even more in times of high exchange rate uncertainty.

The firm-level predictions of the model are therefore consistent with the empirical results obtained in the first chapter of this dissertation. In order to verify that the model can also reproduce the sector-level empirical patterns, I calibrate the model to the Canadian economy, generate sectoral export series from the calibrated model, and run the main empirical regressions I run in the first chapter on the simulated data. The resulting coefficients are similar. As a further step, I test the model's mechanisms using data on syndicated lending to Canadian firms. I find that interest rates are indeed higher for firms with less tangible capital and in periods of high uncertainty. These results suggest that banks do take idiosyncratic and aggregate risks into account when deciding which interest rates to charge firms.

In the last stage, I use the calibrated model to answer further questions about the theoretical effect of exchange rate uncertainty on exports. First, I find that the amounts that different firms would be willing to pay in order to avoid exchange rate uncertainty are considerable in dollar terms, but represent a relatively small fraction of their profits. This suggests that the cost of hedging exchange rate exposure is perceived by firms to be rather high relative to the benefit. Second, I examine the effect of a hypothetical policy akin to providing additional collateral to financially-constrained exporters on the negative effect of exchange rate uncertainty on exports.[26] I find that this type of policy must be applied very widely to have a substantial effect.

The remaining of the chapter is organized as follows: Section 2.2 introduces the theoretical model; Section 2.3 describes the implications of the model; Section 2.4 details the calibration strategy and simulated regressions; and Section 2.5 presents a counterfactual analysis. Section 2.6 concludes.

2.2 Model

In this section, I outline a theoretical model that rationalizes the empirical results described in the first chapter of this dissertation. The model is inspired by Manova (2013), who designs a Melitz (2003)-type, heterogeneous-firm trade model in which different types of financial frictions reduce firms' ability to export.

As in Manova (2013), firms must borrow from a lender to cover some of the costs of exporting. The lender charges an interest rate above the risk-free rate because the repayment by the firm is only partially guaranteed by collateral. In Manova (2013), the uncertainty around the firm's repayment comes from limited contract enforcement. It is constant and independent of the firm's behavior. In my model, time-varying uncertainty arises instead from exchange rate risk. The exporting firm agrees on a price with its customer, but this price is set in the customer's currency. Hence, before the

[26]Note that this resembles an actual policy implemented by Export Development Canada.

exchange rate is realized, the exporting firm cannot predict perfectly its revenue in its own currency. This situation causes ex-ante uncertainty regarding the firm's ability to fully repay the lender after the exchange rate is realized. Additionally, firms are also assumed to face domestic risk, resulting in smaller firms being more likely to not repay their loan.

A key difference between this model and the model introduced in Manova (2013) is that in the former the riskiness of the firm, from the lender's perspective, depends on the firm's own behavior. By choosing how much to export, a firm chooses its exposure to exchange rate risk, which affects the risk premium charged by the lender. The firm internalizes this reaction by the lender and adjusts its optimal decision accordingly. In times of high uncertainty, a firm reduces its exports in order to pay lower interest rates. How much it does so however depends on its own financial constraints. This is the mechanism which creates heterogeneity in firms' reactions to varying levels of exchange rate uncertainty.

2.2.1 Consumers

Consumers in country i have Cobb-Douglas preferences over goods in S sectors, each identified by the letter s:

$$U_i = \prod_s C_{is}^{\theta_s}, \tag{2.1}$$

where θ_s is the fraction of country i's households' income spent on goods from sector s. Within each sector, consumers in country i have constant elasticity of substitution (CES) preferences over Ω_{is} varieties, each identified by ω:

$$C_{is} = \left[\int_{\omega\in\Omega_{is}} q_{is}(\omega)^{\alpha} d\omega\right]^{1/\alpha}, \tag{2.2}$$

where $\epsilon = \frac{1}{1-\alpha} > 1$ is the elasticity of substitution between different varieties in a given sector. The resulting demand for variety ω of sector s in country i is given by

$$q_{is}(\omega) = p_{is}(\omega)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}}, \tag{2.3}$$

where Y_i is total income in country i and the optimal consumer price index for sector s in country i, P_{is}, satisfies

$$P_{is} = \left[\int_{\omega \in \Omega_{is}} p_{is}(\omega)^{1-\epsilon} d\omega \right]^{\frac{1}{1-\epsilon}}. \tag{2.4}$$

Of all varieties ω in sector s, some are produced domestically (in country i) and some are produced abroad. Consumers do not have specific preferences regarding the country of origin of each variety.

2.2.2 Producers

Each period, potential firms in country j can pay a sector-specific fixed entry cost f_{ejs} in order to be able to draw a productivity level $1/a$ from cumulative distribution $G(a)$. Production happens in a constant-returns-to-scale fashion, so that each unit of good costs ac_{js} to produce for a firm operating in sector s, located in country j, and with productivity level $1/a$. Drawing a higher a implies a lower productivity level (a higher variable cost). Departing from traditional heterogeneous-firm trade models, I assume that a fraction of the variable cost represents expenditures on capital goods.[27] The cost parameter c_{js} is a technical parameter determining relative cost levels across countries and industries but independent from firm-specific technology a. There is no additional fixed cost to serve the domestic market, so that all producing firms are producing at least for their own market. The production for the domestic market is

[27] As emphasized below, these capital goods may be used as collateral.

not modeled in this chapter.

2.2.3 Exporters

In order to be able to export to country i, a firm in country j must first pay a fixed cost f_{ijs}, where $f_{ijs} > 0$ if $i \neq j$ and $f_{jjs} = 0$. In addition, an exporting firm must pay a variable trade cost: in order for one unit of good to reach its destination, the exporter must ship τ_{ij} units, where $\tau_{ij} \geq 1$ if $i \neq j$ and $\tau_{jj} = 1$. If it chooses to export, the firm in country j signs an exporting contract with an importing firm in country i. In this contract, quantity and price in the importing country's currency are fixed.

The exporting firm in country j with productivity level $1/a$ can decide of the terms (price $p_{ijs}(a)$ and quantity $q_{ijs}(a)$) of the contract, based on its profit-maximization problem (described below). The firm maximizes its expected profit subject to the demand function coming from country i, subject to its own financial constraint, and given its expectation of what the exchange rate will be at the time of payment by the customer.

The importing customer in country i pays country j's producing firm in country i's currency only after it receives the shipment. The producing firm can use revenues from domestic sales to internally finance the variable cost of producing the exported good as well as a portion $1 - d_s$ of the fixed cost of exporting. It must borrow the remaining portion d_s from a lender. The parameter d_s therefore captures the firm's degree of external finance dependence. After it has been paid by its customer, the exporting firm in country j uses the revenue to repay the lender. The timing of the exporting firm's operations can be summarized as follows. In period t, it finds a customer in country i, agrees on a price (in country i's currency) and quantity with this customer, and produces and ships the required quantity of goods. In period $t+1$, the exchange rate is realized, the customer receives the shipment and pays the exporting firm, and the exporting firm repays its lender.[28]

I assume that exporting firms are also subject to additional risk on the domestic market, and that this risk is decreasing in a firm's productivity level.[29] It is modeled as a probability $\lambda(a)$ that the firm is unable to repay anything to its lender, with $\lambda'(a) > 0$. This assumption captures the fact that small firms have been found to be more financially constrained even when controlling for observable variables. In effect, this assumption results in smaller firms facing higher interest rates.[30]

The exchange rate process The exchange rate X_t, defined in terms of the exporting country's currency per unit of the importing country's currency, is assumed to evolve according to

$$X_{t+1} = X_t \varepsilon_{t+1}, \tag{2.5}$$

where ε_{t+1} follows a triangular distribution with a median of 1, a lower bound of $1 - \iota_{t+1}$ and an upper bound of $1 + \iota_{t+1}$, and where $0 < \iota_{t+1} < 1$.[31] At time t, X_{t+1} is unknown but ι_{t+1}, which captures the degree of time-varying exchange rate uncertainty, is known. This implies that $E_t X_{t+1} = X_t$ so that higher uncertainty does not affect the agents' expectations of the future level of exchange rate.

[28] Whether the gap between periods t and $t+1$ is due to the time required for the goods to physically travel or to the delayed payment by the importing firm does not affect the model's predictions. The key requirement is that at the time of production, the exchange rate that will determine the exporting firm's revenue in its own currency has not been realized and remains unknown.

[29] As explained below, in this type of model, less productive firms also turn out to be smaller firms.

[30] See, for example, Beck et al. (2008) for evidence that small firms are more financially constrained. Section 2.3.4 provides evidence that small Canadian firms obtain higher interest rates on average.

[31] Choosing a different symmetric distribution such as a normal distribution would not alter the qualitative predictions of the model. The triangular distribution is chosen for its simplicity: it greatly simplifies the model's analytical outcomes and significantly shortens the time required to solve the model.

The borrowing contract The contract between the exporting firm and its lender is as follows. The firm borrows a portion d_s of the fixed cost of exporting, the amount of the loan is therefore $d_s f_{ijs}$. After it has been paid by its foreign partner in foreign currency, the firm converts its revenue into its own domestic currency. If the converted revenue is high enough, the firm repays its loan with gross interest rate $1 + R_{ijs}(a)$. If the firm cannot repay its loan fully in cash, the lender can seize a fraction $t_s \in (0, 1)$ of the firm's capital and immediately resell it. I assume that the firm's stock of capital is the sum of the fixed cost it has to pay in order to operate, f_{ejs}, and a fraction $1/b$ of the variable cost of producing the exported good, where $b > 1$. The lender can only seize capital up to the amount it is owed.

The lending market is perfectly competitive and lenders are risk neutral. The firm makes a take-it-or-leave-it offer to a lender for a loan of amount $d_s f_{ijs}$ with gross interest rate $1 + R_{ijs}(a)$. When the lender agrees to the loan, the current exchange rate is known by all, but the future exchange rate X_{t+1}, that will determine the firm's ability to repay its loan, is unknown. The lender can receive three different types of repayment. First, if the firm's profit turns out to be high enough and/or if the firm has enough tangible capital, the lender receives full repayment: $(1 + R_{ijs}(a))d_s f_{ijs}$. Second, if the firm's profit is not high enough and the firm's capital is not tangible enough, the lender receives partial repayment of the amount $\{X_{t+1}p_{ijs}(a)q_{ijs}(a) - ac_{js}\tau_{ij}q_{ijs}(a) - (1 - d_s)f_{ijs}\} + t_s f_{ejs} + t_s \frac{c_{js}\tau_{ij}}{b} q_{ijs}(a)$, where the term in curly brackets represents the firm's profit and the last two terms represent the firm's capital being seized. Finally, with probability $\lambda(a)$, the lender receives no repayment.

The conditional probability (from the perspective of period t) that the lender receives full repayment, denoted by λ^f, is given by the conditional probability that the sum of the firm's profit and tangible capital is higher or equal to the repayment amount agreed upon in the loan contract:

$$\begin{aligned}\lambda^f &\equiv \text{Conditional probability that the lender receives full repayment} \\ &= Prob_t\Big(X_{t+1}p_{ijs}(a)q_{ijs}(a) - ac_{js}\tau_{ij}q_{ijs}(a) - (1-d_s)f_{ijs} + t_s f_{ejs} + t_s\frac{c_{js}\tau_{ij}}{b}q_{ijs}(a) \\ &\qquad\qquad \geq (1+R_{ijs}(a))d_s f_{ijs}\Big) \\ &= 1 - F(A_1),\end{aligned}$$

where $F(\cdot)$ is the cumulative distribution function of the symmetric triangular distribution described above and where

$$A_1 \equiv [(1+R_{ijs}(a))d_s+(1-d_s)]\frac{f_{ijs}}{X_t p_{ijs}(a)q_{ijs}(a)}+\left(1-\frac{t_s}{ab}\right)\frac{ac_{js}\tau_{ij}}{X_t p_{ijs}(a)}-\frac{t_s f_{ejs}}{X_t p_{ijs}(a)q_{ijs}(a)}.$$ [32]

In the case where ε_{t+1} is low enough that the lender is not fully repaid (that is, in the case where $\varepsilon_{t+1} < A_1$), the lender receives all of the firm's profit and all of its tangible capital. In this case, the lender expects to receive

$$\begin{aligned}&X_t p_{ijs}(a)q_{ijs}(a)\int_{1-\iota}^{A_1} f(\varepsilon_{t+1})\varepsilon_{t+1}d\varepsilon_{t+1} \\ &- F(A_1)\left[ac_{js}\tau_{ij}q_{ijs}(a) + (1-d_s)f_{ijs} - t_s f_{ejs} - t_s\frac{c_{js}\tau_{ij}}{b}q_{ijs}(a)\right],\end{aligned}$$

where $f(\cdot)$ is the probability density function of the symmetric triangular distribution. The lender is risk neutral, and the gross return on its outside option (the risk-free

[32] The symmetric triangular distribution with median 1, lower bound $1-\iota$, and upper bound $1+\iota$ has the probability density function $f(x) = \frac{(x-(1-\iota))}{\iota^2}$ for $1-\iota \leq x < 1$, the cumulative distribution function $F(x) = \frac{(x-(1-\iota))^2}{2\iota^2}$ for $1-\iota < x \leq 1$, a mean of 1, and a variance of $\frac{\iota^2}{6}$.

rate) is defined as $(1+r)$. The lender's participation constraint is therefore given by

$$\begin{aligned}(1+r)d_s f_{ijs} \leq [1-\lambda(a)] \times \Bigg[&[1-F(A_1)](1+R_{ijs}(a))d_s f_{ijs} \\ &+ X_t p_{ijs}(a) q_{ijs}(a) \int_{1-\iota}^{A_1} f(\varepsilon_{t+1})\varepsilon_{t+1} d\varepsilon_{t+1} \\ &- F(A_1)\left[ac_{js}\tau_{ij}q_{ijs}(a) + (1-d_s)f_{ijs} - t_s f_{ejs} - t_s \frac{c_{js}\tau_{ij}}{b} q_{ijs}(a)\right]\Bigg].\end{aligned}$$

The repayment that the lender expects to receive from the firm must be greater or equal to the return it can obtain from its outside option.

The exporting firm's optimization problem The firm chooses the selling price $p_{ijs}(a)$ of the good it produces by maximizing its expected profit, $E_t\pi_{ijs}(a)$. The only unknown variable for the firm (as for the lender) is the realization of the exchange rate determined by ε_{t+1}. The firm's profit-maximization problem of exporting can be characterized as

$$\max_{p_{ijs}(a)} E_t\pi_{ijs}(a) = p_{ijs}(a)q_{ijs}(a)E_t(X_{t+1}) - ac_{js}\tau_{ij}q_{ijs}(a) - (1-d_s)f_{ijs} - (1+R_{ijs}(a))d_s f_{ijs} \tag{2.6}$$

subject to

$$q_{ijs}(a) = p_{ijs}(a)^{-\epsilon}\frac{\theta_s Y_i}{P_{is}^{1-\epsilon}} \tag{2.7}$$

$$\begin{aligned}(1+r)d_s f_{ijs} \leq [1-\lambda(a)] \times \Bigg[&[1-F(A_1)](1+R_{ijs}(a))d_s f_{ijs} \\ &+ X_t p_{ijs}(a) q_{ijs}(a) \int_{1-\iota}^{A_1} f(\varepsilon_{t+1})\varepsilon_{t+1} d\varepsilon_{t+1} \\ &- F(A_1)\left[ac_{js}\tau_{ij}q_{ijs}(a) + (1-d_s)f_{ijs} - t_s f_{ejs} - t_s \frac{c_{js}\tau_{ij}}{b} q_{ijs}(a)\right]\Bigg].\end{aligned} \tag{2.8}$$

The firm maximizes its expected profit subject to the demand function (constraint (2.7)) and the lender's participation constraint (constraint (2.8)). From the firm's perspective, the loan is always fully repaid because of the presence of collateral. The firm reckons that if its profit is insufficient to fully repay, at least a fraction of its capital will be seized to cover the difference. From the lender's perspective, however, capital is costly to recover and resell ($t_s < 1$): even if it seizes all of a firm's capital, the payoff to the lender may be less than the amount it was due.[33] The two constraints always bind. The first constraint states that the firm always chooses to sell the full quantity demanded for a given price and can be directly substituted into the problem. The second constraint determines the lowest possible interest rate the firm can obtain from a lender as a function of the price it chooses. Since the lending market is perfectly competitive, this constraint also always binds. The firm optimizes by choosing the price level that yields the highest expected profit. If the firm's resulting maximum expected profit from exporting, $E_t\pi_{ijs}(a)$, is positive, it selects into exporting. If it is negative, the firm chooses not to borrow and not to produce for the foreign market.

2.3 Implications of the model

If a firm's interest rate were independent of its chosen price, the firm would choose its expected-profit-maximizing price such that

$$X_t p_{ijs}(a) = \frac{\epsilon}{\epsilon - 1} a c_{js} \tau_{ij}.$$

Being in a monopolistically competitive market, the firm would choose a price (expressed in the firm's domestic currency) that is a constant markup $\frac{\epsilon}{\epsilon-1}$ over its marginal cost of production. However, in this model, the interest rate that the firm

[33]The firm's optimization problem reflects that capital is worth more for the firm than for the lender. Explicitly modeling two different valuations of the same capital stock yields the same mechanism but makes the model significantly less tractable.

faces is affected by its chosen price. The firm's optimal price condition is therefore given by

$$X_t p_{ijs}(a) = \frac{\epsilon}{\epsilon - 1} a c_{js} \tau_{ij} - \frac{1}{\epsilon - 1} d_s f_{ijs} p_{ijs}(a)^{1+\epsilon} \frac{P_{is}^{1-\epsilon}}{\theta_s Y_i} \frac{\partial(1 + R_{ijs}(a))}{\partial p_{ijs}(a)}, \tag{2.9}$$

where $\frac{\partial(1+R_{ijs}(a))}{\partial p_{ijs}(a)}$ reflects the effect of the firm's chosen price on the interest rate it can obtain, all else equal. This condition simplifies to

$$X_t p_{ijs}(a) = \frac{\epsilon}{\epsilon - 1} a c_{js} \tau_{ij} \left\{ \frac{\left(1 - \frac{t_s}{ab}\right) F(A_1)}{1 - \int_{1-\iota}^{A_1} f(\varepsilon_{t+1})[1 - \varepsilon_{t+1}] d\varepsilon_{t+1}} \right\}. \tag{2.10}$$

Hence, the firm's marginal cost is composed not only of a production cost, but also of a financial cost. The latter is captured by the term in curly brackets in equation (2.10).

For a given price chosen by a firm, a lender chooses an optimal interest rate. Since the lending market is perfectly competitive, the lender always chooses the lowest possible interest rate while satisfying its own participation constraint. All else equal, a firm's chosen price has several counteracting effects on the interest rate. First, for a given probability of the firm defaulting, a higher price charged by the firm results in a lower revenue for the firm, which results in a lower revenue to seize for the lender in case of default and puts upward pressure on the interest rate. Second, depending on the firm's cost structure and on the level of exchange risk, a higher price can result in a higher or lower probability of default for the firm. A higher probability of default results in the lender asking for a higher interest rate and vice versa.

In what follows, I assume that the model's parameters and variables are such that the price chosen by any given firm is higher than it would be in the absence of financial constraints (see equation (2.9)).[34] This assumption reproduces the empirical

[34]Only firms that are effectively financially constrained choose a higher price. In the model simulations below, some larger firms are in effect financially unconstrained, for example because

fact that financial constraints hurt international trade and implies that a higher price chosen by the firm results in a lower interest rate.[35]

The solution to the firm's optimization problem is given by the price-interest-rate pair that simultaneously solves equations (2.8) and (2.10).[36] As in other heterogeneous-firm models, more productive firms choose lower prices and export greater quantities, and all firms with a productivity level equal to or higher than a sector-country-pair-specific threshold ($1/a^*_{ijs}$) select into exporting.

Proposition 1. *For any given sector s and country pair ij, (i) more productive firms charge lower prices and export greater quantities; and (ii) there exists a productivity threshold $1/a^*_{ijs}$ such that every firm with $a > a^*_{ijs}$ chooses not to export and every firm with $a \leq a^*_{ijs}$ chooses to export.*

Proof. See Appendix.

2.3.1 The effect of financial constraints

A firm's level of asset tangibility affects its lender's return in case of incomplete repayment and hence affects the interest rate it faces. A firm with highly tangible assets has access to a lower interest rate and hence chooses a lower price (and a greater quantity to export). A firm's level of external finance dependence affects its marginal cost mostly because it modifies the portion of cost on which interest must be paid, but also because it affects the interest rate the firm must pay. For a given chosen price, a higher d_s results in a higher probability of default, but also in a higher potential return for the lender since the loan amount is larger. Hence, a higher degree

they have enough collateral to guarantee their full loans. These firms are assumed to behave as unconstrained firms would.

[35] See, for example, Manova (2013).

[36] The firm's problem does not have a closed-form solution.

of external finance dependence can either increase or decrease the interest rate paid by the firm. In either case it results in a higher optimal price chosen by the firm, since interest is paid on a larger amount, and a smaller quantity of goods being exported.

Proposition 2. *Everything else equal, a firm with either (i) a higher degree of asset tangibility; or (ii) a lower degree of external finance dependence exports a larger quantity.*

Proof. See Appendix.

2.3.2 The effect of exchange rate uncertainty

The empirical analysis presented in the first chapter of this dissertation suggests that exchange rate uncertainty affects exporting behavior differently in the presence of different types of financial frictions. On the one hand, higher tangibility helps to mitigate the negative effect of uncertainty, especially in sectors composed of mostly large exporters. A higher degree of external finance dependence, on the other hand, worsens the effect of uncertainty in sectors with a relatively high share of SMEs. In this subsection, I describe the model's mechanisms that can explain these empirical features. It should be noted that the model can only explain a (weakly) negative effect of exchange rate uncertainty on exports.

An increase in exchange rate uncertainty affects the exporting firm only through its financial constraints and the interest rate it pays. A higher degree of uncertainty increases the probability that the firm defaults on its loan, which makes the risk-neutral lender demand a higher interest rate. As a result, the exporting firm increases its price in order to limit the increase in interest rate, and exports decrease.

Proposition 3. *Everything else equal, a higher degree of exchange rate uncertainty* *results in lower exports.*

Proof. See Appendix.

A firm's reaction to an increase in exchange rate uncertainty strongly depends on its financial constraints. As mentioned above, higher tangibility results in a lower interest rate since it reduces the likelihood that the lender receives less than it is due. Tangibility however matters differently for firms with different levels of production and, hence, for firms of different sizes. Figure 2.1a shows the percentage change in a firm's optimal price in response to an increase in exchange rate uncertainty for two different levels of asset tangibility and different levels of productivity.[37] An increase in uncertainty results in an increase in the optimal price, and hence in a decrease in the quantity exported for all types of firms. Highly productive (large) firms make high profits and are therefore very safe for lenders regardless of their degree of asset tangibility. They are relatively less negatively affected by exchange rate uncertainty. Asset tangibility benefits firms of all sizes. On the one hand, since smaller firms are riskier, their interest rates increase more with uncertainty. More tangible assets work to limit this increase. On the other hand, larger firms own more capital (since they have higher total variable costs), which makes a given degree of tangibility more valuable to them. Whether having higher asset tangibility benefits more smaller or larger firms is indeterminate in this model and depends on parameter values. The calibration strategy described below reflects the fact that asset tangibility offers greater help to larger firms in mitigating the negative effect of uncertainty more, as was found in the empirical analysis, even if it did not directly target this fact.

Figure 2.1b shows the percentage change in a firm's optimal price in response to an increase in exchange rate uncertainty for two different levels of external finance

[37] The figures in this section are obtained by simulating the model for a baseline calibration. This baseline calibration is for an average sector and an average time period as detailed in Section 2.4 below.

dependence. Again, an increase in uncertainty results in an increase in the optimal price, and hence in a decrease of quantity exported for firms will all levels of productivity and external dependence. However, for the less productive firms, higher external finance dependence results in a more negative effect of exchange rate uncertainty, that is, it results in a larger increase in the optimal price and hence in a larger decrease in the quantity exported. This effect comes from the fact that for smaller, less productive firms, the risk of default is higher. An increase in exchange rate risk therefore increases these firms' probability of default even more, which results in their lenders' requiring even higher interest rates. Since external finance dependence has several counteracting effects on financial frictions faced by firms, the net effect remains small overall.

Extensive margin The model predicts that an increase in exchange rate uncertainty does not only reduce the quantity of goods exported by each firm, but also reduces the number of firms selecting into exporting and hence the number of varieties being exported in a given industry. As described above, an increase in uncertainty leads to higher interest rates and higher optimal prices. For a firm, a higher interest rate results in higher costs, while a higher optimal price results in lower revenues (since the elasticity $\epsilon > 1$). Hence, the firm's profit is lower. Given the fixed costs, this outcome reduces the a^*_{ijs} threshold below which firms select into exporting. Since firms with more tangible capital represent less risk for lenders, their interest rates increase less and they are less affected by exchange rate uncertainty. The productivity threshold in sectors with more tangible capital is therefore less affected by uncertainty, as illustrated by Figure 2.2a.

As described above, firms with high external finance dependence reduce their exports by greater percentages in times of high uncertainty, especially smaller, less productive firms closer to the productivity threshold. The reason these firms subject their price to greater increases is that they wish to limit the increase in the interest rate they face, since it applies to a larger portion of their cost. As a result, an increase in exchange rate uncertainty decreases the productivity threshold equally regardless of the degree of external finance dependence, as illustrated by Figure 2.2b. Indeed, while a firm with a higher external finance dependence does increase its price by more in times of high uncertainty, thereby decreasing to a greater extent the quantity it exports, it also faces a smaller increase in interest rate. The combination of the larger increase in price (hence larger decrease in revenue) and lower increase in cost leads to an approximately equal decrease in profit and consequently to an approximately equal decrease in the productivity threshold a^*_{ijs}.[38]

Proposition 4. *Everything else equal, a higher degree of exchange rate uncertainty results in a lower threshold* a^*_{ijs} *below which firms select into exporting.*

Proof. See Appendix.

This model is therefore able to predict a behavior of exporting firms in response to exchange rate uncertainty and in the presence of financial constraints that is consistent with the sectoral evidence identified in the first chapter of this dissertation. In the absence of a closed-form solution, the model's predictions cannot be easily aggregated to the sector level. In Section 2.4, I calibrate the model and obtain sector-level predictions through simulations. I then demonstrate that the firm-level mechanisms of the model are sufficient to explain the sector-level empirical evidence of the first chapter.

[38] Although not visible on the graph, the decrease is slightly larger for a higher level of external finance dependence.

2.3.3 Alternative assumptions

In this subsection, I modify some of the assumptions made in the baseline model and study their effect on the model's predictions and ability to reproduce the empirical results described in the first chapter of this dissertation.

Financial (un)constraints If a firm is able to finance all of its production costs internally (that is, $d_s = 0$), its optimal pricing condition is given by

$$X_t p_{ijs}(a) = \frac{\epsilon}{\epsilon - 1} a c_{js} \tau_{ij}. \tag{2.11}$$

The optimal price is a fixed markup $\frac{\epsilon}{\epsilon-1}$ over marginal production cost, corrected for the expected exchange rate. Because of the presence of uncertainty, the firm's optimality condition is to set *expected* marginal revenue equal to a constant markup over the (perfectly known) marginal cost of production. The expected marginal revenue is given by $E_t(X_{t+1})p_{ijs}(a) = X_t p_{ijs}(a)$. If changes in exchange rate uncertainty are mean-preserving, exchange rate uncertainty has no effect on the firm's optimal price or on its expected profit or the (inverse) productivity threshold a^* below which firms choose to export.

Therefore, if firms are assumed to be risk neutral, the financial constraint assumption is necessary for the model to be able to predict that exchange rate risk has a negative effect on exports. This finding is analogous to earlier theories, such as in Ethier (1973), predicting that financially unconstrained firms are affected by risk only because of their risk-averse behavior. The case of risk-averse firms is analyzed below.

Risk aversion To gain intuition, it is useful to evaluate the effect of risk aversion on firms' behavior by first assuming that firms can internally finance all of their costs ($d_s = 0$). If a firm has mean-variance preferences, it solves the following maximization problem:

$$\max_{p_{ijs}(a)} V_t = E_t \pi_{ijs}(a) - \frac{\varrho}{2} var_t(\pi_{ijs}(a)) \tag{2.12}$$

where

$$\begin{aligned} E_t \pi_{ijs}(a) &= p_{ijs}(a) q_{ijs}(a) X_t - a c_{js} \tau_{ij} q_{ijs}(a) - c_{js} f_{ij} \\ var_t(\pi_{ijs}(a)) &= \left[X_t p_{ijs}(a) q_{ijs}(a)\right]^2 var_t(\varepsilon_{t+1}), \end{aligned}$$

and subject to

$$q_{ijs}(a) = p_{ijs}(a)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}},$$

where $var_t(\cdot)$ denotes the variance operator conditional on information known at time t and ϱ denotes the firm's degree of risk aversion. In this case, the firm's optimal-pricing condition is given by

$$p_{ijs}(a) = \frac{\epsilon}{\epsilon - 1} \frac{a c_{js} \tau_{ij}}{X_t} \left[1 - \varrho p_{ijs}(a)^{1-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}} X_t \frac{\iota^2}{6}\right]^{-1} \tag{2.13}$$

and the productivity threshold below which firms choose to export is given by [39]

$$a^* = \frac{\epsilon - 1}{\epsilon - 2} \frac{X_t p_{ijs}(a)}{c_{js} \tau_{ij}} - 2 \frac{\epsilon - 1}{\epsilon - 2} \frac{f_{ij}}{\tau_{ij} p_{ijs}(a)^{-\epsilon}} \frac{P_{is}^{1-\epsilon}}{\theta_s Y_i}. \tag{2.14}$$

These conditions imply that the optimal price is increasing both in the degree of risk aversion and in the level of exchange rate uncertainty. Intuitively, higher risk aversion decreases the firm's expected risk-corrected profit V_t but only to the extent that its

[39] In this case, I assume that firms choose to export as long as their utility $E_t \pi_{ijs}(a) - \frac{\varrho}{2} var_t(\pi_{ijs}(a))$ is greater or equal to zero. If $\varrho = 0$, the optimal-price condition simplifies to equation (2.11).

profit is uncertain. Since the exchange rate risk only applies to its revenue, the firm can only reduce the variance of its profits by decreasing the variance of its revenue. It does so by decreasing its revenue altogether, via an increase in price. The intuition is similar for the effect of exchange rate uncertainty. The firm reacts to an increase in risk by decreasing the exposure of its revenue to this risk: it increases its price, which decreases the quantity it exports.

Whether an increase in risk aversion or in exchange rate risk leads to a decrease in the productivity threshold a^* depends on whether the firm's overall expected risk-corrected profit decreases. By itself, an increase in risk aversion decreases V_t. However, it also prompts the firm to increase its price, which decreases its expected profit but also decreases the conditional variance of its profit. The overall effect on the firm's V_t (and on the productivity threshold a^*) therefore depends on the relative strength of these three effects. An increase in exchange rate risk generates the same three effects.

If a firm is risk averse *and* financially constrained ($d_s > 0$), the lender is not directly affected by the firm's degree of risk aversion. Hence, for a given price chosen by the firm, the lender demands the same interest rate as in the case where $\varrho = 0$. A positive degree of risk aversion however does affect the firm's pricing decision.[40] More risk-averse firms charge higher prices, which decreases their probability of default, which in turn decreases the interest rate they can obtain. In fact, making firms risk averse does not affect qualitatively how financial frictions affect the reaction of firms' exporting behavior in response to varying levels of uncertainty. However, because firms charge higher prices when they are risk averse, they reduce their own riskiness

[40] In this case, the firm's optimal-pricing condition is given by
$X_t p_{ijs}(a) = \frac{\epsilon}{\epsilon-1} a c_{js} \tau_{ij} \frac{1-(t_s/a)F(A_1)}{1-F(A_1)+\int_{1-\iota}^{A_1} f(\varepsilon_{t+1})\varepsilon_{t+1} d\varepsilon_{t+1} - [1-F(A_1)]\varrho \frac{\iota^2}{6} X_t p_{ijs}(a)^{1-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}}}$.

and are in effect less financially constrained. The role of financial frictions in the relationship between exchange rate uncertainty and exports is therefore dampened.

2.3.4 Empirical evidence supporting the model

The model introduced in this chapter relies on financial frictions to rationalize the results obtained in the first chapter of this dissertation: when exchange rate uncertainty increases, exporting firms are more likely to default on their loans and as a result their lenders require higher interest rates. The model also predicts that firms with higher asset tangibility are able to obtain lower interest rates, but makes no such clear predictions for the degree of external finance dependence. Finally, the model predicts that lenders demand higher interest rates in times of higher uncertainty. In this section, I provide evidence supporting the model's financial mechanisms by using syndicated loan data for Canadian firms.

In order to validate the mechanisms of the model, I test how the interest rates that Canadian firms obtain from banks depend on the different measures of financial constraints and risk. I use the WRDS-Thomson-Reuters' LPC DealScan dataset covering syndicated lending in the world. The database contains credit deals ("facilities") obtained by firms from a syndicate of lenders, including information on the borrower, the lender(s) and the deal itself (interest rate, maturity, etc.).[41] For all regressions described and presented in this section, the dependent variable is the facility spread, which is the premium (including fees) over the risk-free rate (typically the LIBOR or prime rates).[42]

[41]I keep only the facilities in which the borrower (and the borrower's "ultimate parent") is based in Canada and does not operate in the finance, insurance, real estate, or public administration industries (SIC codes in the 6000 and 9000). I also keep only the facilities that are either term loans or credit lines. Finally, I keep only the facilities whose type is described as a "Term Loan", "364-Day Facility", "Demand Loan", "Limited Line", or "Revolver".

[42]It should be noted that the results presented in this section should be compared to the empirical results about exports only with great care. Syndicated loans or credit lines are available for high

In a first regression, I test for the effect of firm and facility characteristics on the interest rate. In terms of firm characteristics, I include tangibility of assets, external finance dependence and profitability (defined as gross profits over sales). In the absence of detailed information about firms in the DealScan database, I construct these variables at the NAICS-3 industry level from U.S. firms using the Compustat database. Additional controls include the amount of the facility, the maturity of the facility, as well as dummies for term loans (as opposed to credit lines), public firms, multiple lead arrangers, and currency of the facility. I also include province fixed effects. The results are presented in column 1 of Table 2.1. I find that asset tangibility and facility amount lead to a lower interest rate whereas facility maturity and profitability lead to a higher interest rate.[43] External finance dependence has a positive but much less significant effect on average.[44]

In a second regression (column 2 of Table 2.1), I test for the effect of economic uncertainty on the interest rate charged by banks. Since time fixed effects cannot be included, I add U.S. GDP growth and Canadian GDP growth as additional controls to capture macroeconomic events. I also add industry fixed effects. I find that both exchange rate uncertainty and macroeconomic uncertainty (measured by the VIX index) have statistically significant positive effects on interest rates. This suggests that banks do take macroeconomic risk into account when making decisions about interest rates allocated to specific firms for specific loans or credit lines. This further suggests

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amounts, which are more likely to be needed by large firms. In my dataset, only two facilities are under 150,000 USD and the average facility amount is above 300 million USD. These results may therefore not reflect the average relationships between firms and their creditors.

[43]It is surprising to find that profitability results in a higher interest rate. One potential explanation is that lenders may evaluate a firm's profitability relative to profitability of other firms in the same industry. In that case, the sectoral-level measure of profitability would capture other unobserved sectoral characteristics.

[44]It should be noted that the coefficients on the external finance dependence variable in these regressions have to be interpreted differently from the model's implications since here the debt amount is known and explicitly controlled for.

that firms do not completely hedge their exposure to these risks and that their level of risk aversion is not high enough to completely eliminate the risk for their lenders.

Even though no disaggregated data covering non-syndicated credit are publicly available, Statistics Canada publishes aggregated data from surveys sent to Canadian SMEs. The responses to the Survey on Financing of Small and Medium Enterprises for the years 2000, 2004, 2007, 2011, and 2014 show that among SMEs it is harder for smaller firms to obtain credit (a smaller share of their requested credit is likely to be accepted) and that they obtain less favorable interest rates. The latter tends to be true both for the total credit obtained and within each debt category.[45] A subset of the surveys' aggregated responses are presented in the Appendix.

2.4 Calibration and simulated regressions

In this section, I describe the strategy used to calibrate the model. Most parameters are obtained directly from data or from the existing literature. The remainder are calibrated by matching sector-level moments to their data counterparts. In the absence of a closed-form solution, the model must be simulated to obtain sector-level predictions. I then simulate the calibrated model using the true processes of variables over the 1988Q1-2015Q4 period and aggregate the resulting exports at the sector level. Lastly, I run the regressions of the empirical analysis (presented in the first chapter of this dissertation) on the simulated data and demonstrate that the model is able to replicate the results.

[45] The available data does not allow to uncover the reasons behind these results, i.e. whether smaller firms tend to be less profitable, have less tangible assets, etc.

2.4.1 Calibration

The model contains 14 parameters to be calibrated when the processes of the variables Y_i, P_{is}, X_t, $1 + r$ and ι are taken as given. The first four parameters are obtained from data. The construction of external finance dependence d_s and asset tangibility t_s is described in the empirical analysis in the first chapter of this dissertation. External finance dependence is rescaled to take only values between 0.2 and 0.8.[46] Additionally, the parameter b, which captures the share of variable cost composed of physical capital, is set to 4.[47] Parameter θ_s, which measures the fraction of U.S. income spent on goods and services from sector s, is obtained by calculating the ratio $\frac{\text{U.S. domestic consumption of goods and services from sector } s}{\text{U.S. GDP}}$ where the numerator is equal to the sum of U.S. production and net imports (imports minus exports) in sector s. All variables are in nominal terms. The measure is calculated quarterly over the 2002Q1-2015Q4 period for each sector. Since the ratio does not vary much over time, the parameter θ_s is assumed to be constant and equal to the average between 2002Q1 and 2015Q4.

The elasticity of substitution between varieties, ϵ, is set to 3.8 following Bernard et al. (2003). To obtain this value, the authors match the size advantage of U.S. exporting firms: a high elasticity results in higher sensitivity to price on that part of consumers, benefiting larger, more productive firms able to charge lower prices. The iceberg trade cost τ_{ij} is set to 1.2, following the existing literature. The parameter $\lambda(a)$ measures the probability that a firm with productivity parameter $1/a$ does not repay its loan and that the lender is unable to recover any collateral. It is assumed

[46] In the model, d_s captures a share of the export cost and must hence be between 0 and 1. A slightly smaller range is chosen since extreme values of d_s result in extreme model predictions.

[47] Measures of asset tangibility and physical capital intensity of the production process reported in Manova (2013) and estimated from U.S. firm-level data by Braun (2003) show that the latter is highly correlated with the former, but on average four times smaller. I therefore use the fraction $\frac{t_s}{4}$ to capture the share of variable costs that can be attributed to physical capital, i.e. that is fit for being posted as collateral.

to be increasing in parameter a, or decreasing with the firm's level of productivity: smaller, less productive firms are more likely to go bankrupt. $\lambda(a)$ is assumed to take the functional form $\lambda(a) = 0.1\left(\frac{a}{a^*_{ijs}}\right)^4$. In each sector, the least productive exporting firm (for which $a = a^*_{ijs}$) has a 10-percent probability of bankruptcy.[48] Parameter f_{ejs} measures the fixed cost of producing (or existing) for a firm in sector s. It reflects part of the physical capital that the firm can use as collateral to obtain outside financing. While the firm's existence decision is not modeled in this chapter, several papers attempt to measure the relative sizes of f_{ijs} and f_{ejs}. I follow Impullitti et al. (2013) and assume that $f_{ijs} = 1.43 f_{ejs}$. Assuming a constant ratio between f_{ijs} and f_{ejs} also ensures that the effects of t_s (multiplying f_{ejs}) and d_s (multiplying f_{ijs}) can be distinguished from the effects of the fixed cost parameters.

In order to make predictions at the sector level, firm-level predictions must be aggregated. For example, a random potential exporter in sector s is expected to export $\int_{a_L}^{a_H} q_{ijs}(a)dG(a)$, where $G(a)$ is the cumulative distribution function from which the firm's productivity parameter a is drawn, and a_L and a_H are the lower and upper bounds of the support of this distribution, respectively. Total exports in sector s are given by $EX_{ijs}(a) \equiv \int_{a_L}^{a_H} q_{ijs}(a)dG(a) \cdot n_{exporters}$, where $n_{exporters}$ is the total number of potential exporters.[49] Hence, parameters a_L, a_H and $n_{exporters}$, as well as any parameter governing the shape of distribution $G(a)$, must be calibrated to obtain sectoral predictions and appropriately evaluate the model's ability to explain the empirical evidence detailed in the first chapter. a_L and a_H are normalized to 1 and 10, respectively. The number of potential exporters $n_{exporters}$ is set to 10,000. In comparison, data from Statistics Canada indicates that the number of existing firms (not necessarily exporting) in any given manufacturing sector between 2011

[48] In comparison, data from Statistics Canada suggest that in a given quarter, about 2.5 percent of Canadian manufacturing firms declare bankruptcy.

[49] The term "potential exporters" is used since it is generally not profitable for all firms to export.

and 2015 was between 198 and 8,089.[50] Following the existing literature, I assume that the parameter $1/a$ follows a bounded Pareto distribution defined over $[a_L, a_H]$ with sector-specific shape parameter k_s, so that $G(a) = \frac{a^{k_s} - a_L^{k_s}}{a_H^{k_s} - a_L^{k_s}}$.

Since the model does not have a closed-form solution, EX_{ijs} is obtained by simulating the model for a large number of productivity level a and discretely adding the resulting firm-level exports $q_{ijs}(a)$ weighted by their probability density function $g(a) = G'(a)$. The $[a_L, a_H]$ interval is divided into 1,000 sections. The model is simulated for the values of a at the center of each of these sections and EX_{ijs} is computed as $\sum_{a_L}^{a_H} q_{ijs}(a)g(a)$.[51]

The three remaining sector-specific parameters - k_s, c_{js}, and f_{ijs} - are calibrated to simultaneously match three sector-specific moments obtained from data.[52] The first moment is average exports over the sample period. The second moment is the share of exports coming from SMEs. This moment is strongly influenced by the parameter of the Pareto distribution: a higher k_s results in a higher weight given to smaller firms (firms with a larger technology parameter a). The third moment is the number of exporting firms. In the model, this moment is given by $G(a^*_{ijs}) \cdot n_{exporters}$ and strongly depends on the fixed cost of exporting f_{ijs}. In the data, this moment is obtained from Statistics Canada. The number of exporters is available annually from 2010 and averaged over the 2010-2015 period.

For each potential calibration, the model is simulated for the average period in the sample. The U.S. GDP (Y_i), U.S. sector-level producer price index (P_{is}), CAD-

[50]The earliest year available is 2011.

[51]For values of $a > a^*_{ijs}$, $q_{ijs}(a)$ is set to zero since these firms do not select into exporting. If a^*_{ijs} turns out greater than a_H, it is re-set to a_H.

[52]The absence of closed-form solutions causes all simulated moments to depend on *all* variables and parameters.

USD exchange rate (X_t), and Canadian risk-free interest rate $(1+r)$ variables are set to their average value. The remaining variable, exchange rate uncertainty, captured by the parameter ι of the triangular distribution, is set to 0.4. This value is chosen to ensure that the predicted interest rates faced by firms remain reasonable.[53] Because the model aims to explain the negative effect of uncertainty on firms' decision to export, I consider only the 11 sectors which have been found in the first chapter to react significantly negatively to uncertainty. The resulting sector-level parameters are contained in Table 2.2.

2.4.2 Simulated Regressions

In order to verify that the model can reproduce the sector-level patterns uncovered in the first chapter of this dissertation, I simulate data from the calibrated model using the true processes of the four variables. The measure of exchange rate uncertainty is rescaled to take only values between 0.2 and 0.8 and to have a mean of 0.4.[54] I then run the main regressions on the simulated export series (total margin) and on the simulated number of exporters series (extensive margin).[55] Tables 2.3 and 2.4 contain the results for both actual and simulated data, and for both the total and extensive margins regressions. The resulting coefficients are quantitatively and qualitatively similar. Unsurprisingly, the mechanism is better identified with the

[53]The symmetric triangular distribution around 1 used to capture the exchange rate uncertainty process can only take values of ι greater than zero and smaller than one. As described below, for simulations, the measure of exchange rate uncertainty used in the empirical analysis is re-scaled to take values between 0.2 and 0.8 only since the model behaves unrealistically when ι takes on extreme values. Assuming, additionally, a mean value of 0.4 balances a trade-off between smaller firms' interest rates being unrealistically high and the mean value of ι being unrealistically close to the lower bound of 0.2.

[54]The log of the resulting series is highly correlated with the log of the original series, with a coefficient of correlation of 0.975. To be able to compare the coefficients' magnitudes, the original series is used in the regressions on simulated data.

[55]The variables Oil Price$_t$ and Oil Price Uncertainty$_t$ are excluded from the regressions on simulated data since they are not part of the theoretical model. For the extensive margin regressions, I aggregate the quarterly simulated number of exporters series to an annual series and keep only the years for which the true data is available.

simulated data. This confirms that even though the model predicts only a very small effect of external finance dependence on both export margins, it is sufficient to reproduce empirical results.[56]

The model introduced in this chapter is therefore able to explain the patterns of international trade in response to exchange rate uncertainty identified earlier. Higher exchange rate uncertainty results in lower exports, but this decrease is not homogeneous across sectors. Different types of financial frictions result in different sectors being differently affected by exchange rate risk. Sectors in which firms have more tangible capital are unambiguously less negatively affected by uncertainty both at the intensive and extensive margins. Larger firms, producing larger volumes and owning more capital, benefit even more for having highly tangible assets. Sectors in which firms rely more on external finance are more negatively affected by uncertainty. Since less productive firms are riskier, they are even more negatively affected. Hence, external finance dependence results in a larger decrease in exports in sectors with relatively more small firms. This result comes from the intensive margin of trade (exports of a given variety of good) rather than the extensive margin of trade (number of varieties of goods exported). External finance dependence does not have a significant effect on how much the number of varieties of goods exported is affected by exchange rate uncertainty. These predictions of the model fit with the empirical features documented in the empirical analysis presented in the first chapter of this dissertation.

2.5 Counterfactual analysis

In this section, I use the calibrated model to answer additional questions about the effect of exchange rate uncertainty on the behavior of exporting firms. I first

[56] One caveat of the model however is that it overestimates the total effect of exchange rate uncertainty on exports, even when considering only the sectors negatively impacted.

estimate the amount that different types of firms would be willing to pay in order to avoid uncertainty or, equivalently, the reduction in their profit that is attributable to this type of risk. Second, I examine how policymakers can mitigate the costs of exchange rate uncertainty. In the model, risk affects exports because of the presence of financial frictions. Alleviating these frictions therefore results in a smaller effect of risk. As an experiment, I increase the tangibility of firms' capital by 50 percent. In effect, this reproduces a policy (implemented, for example, by Export Development Canada) of providing additional collateral to financially-constrained exporters. The goal of this type of policy is to reduce exporters' borrowing costs or increase their borrowing limit, and allow them to export more.

2.5.1 The willingness to pay to avoid uncertainty

In theory, exporting firms that are negatively affected by exchange rate uncertainty should be willing to pay in order to reduce this uncertainty, for example by hedging their exposure to the exchange rate. In this counterfactual exercise, I use the calibrated model to quantitatively assess how much firms would be willing to pay. I estimate the profit loss due to the presence of uncertainty for different hypothetical firms for each of the 11 sectors found to be negatively affected by exchange rate uncertainty. For each hypothetical firm, I calculate the difference in profit between two situations: 1- all variables, including exchange rate uncertainty, are set to their average value over the sample period, and 2- all variables are set to their average value except exchange rate uncertainty, which is set to zero. Columns 1, 2, and 3 of Table 2.5 contain the estimated difference in profit for firms at the 25th, 50th and 75th percentiles of the sector-specific $[a_L,\ a^*_{ijs}]$ spectrum, respectively. The results show that the difference in profit is slightly lower for firms of medium size, that it is the highest for larger firms, and that it is generally between 200,000 and 400,000 CAD per firm per quarter. These results follow directly from the model's implications. On the one hand, large firms increase their price by less than other firms in the presence of uncertainty, but these prices apply to large quantities. This results in the largest decreases in profit in absolute terms and hence to the largest willingness to pay to avoid uncertainty. On the other hand, smaller firms sell small quantities but decrease their price by more, resulting also in large profit losses. Consequently, firms of medium size have the smallest profit losses and the smallest willingness to pay.

Columns 4, 5, and 6 of Table 2.5 express these different willingness to pay in terms of the share of the firms' profit that they represent. The calibrated model predicts that relatively large firms would be willing to pay less than one percent of their profits to completely avoid exchange rate uncertainty. Medium firms would be willing to pay around two percent, and smaller firms, around eight percent. The fact that profit differences are relatively small compared to firms' simulated profits offers some potential answers regarding why many firms choose to ignore exchange rate uncertainty risk. One such potential answer is that the profit differences are too small for managers to take notice, even if they are economically significant. A second potential answer is that the (perceived) cost of hedging exchange rate exposure is higher than the firms' willingness to pay. In this case, estimates in columns 1-3 of Table 2.5 provide lower bounds on firms' (perceived) cost of hedging.

2.5.2 A policy experiment

In a last exercise, I estimate by how much firms' profit losses and export reductions due to exchange rate uncertainty can be alleviated by relaxing their financial constraints. For each sector, I increase the tangibility parameter t_s by 50 percent to simulate a hypothetical economic policy of providing collateral to exporting firms. Results are displayed in Table 2.6. I find that profit losses can be reduced on average by 19 percent, with the greatest gains for larger firms. I also find that increased tangibility leads on average to a decline of 13 percent in the decrease in exports due to uncertainty. Again, the largest firms make the highest relative gains.[57] These numbers suggest that providing additional collateral to firms is a sensible policy, but that it must be applied very widely to have a large effect on exports. Furthermore, this type of policy disproportionately benefits larger firms.

2.6 Conclusion

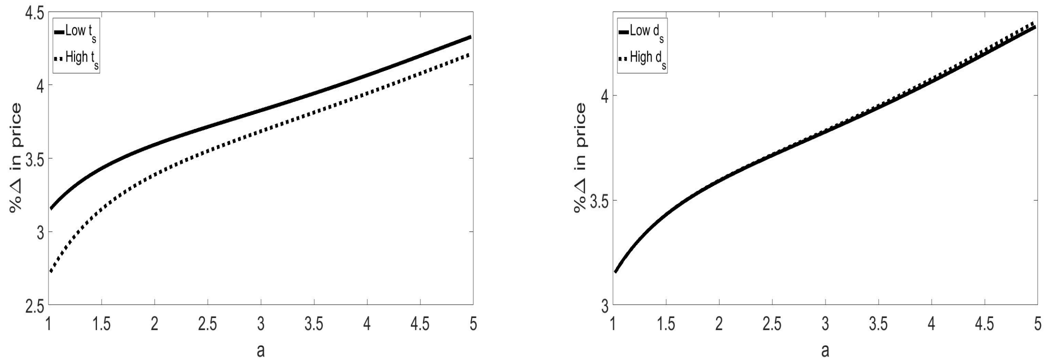
In this chapter, I study theoretically how a specific type of uncertainty, exchange rate uncertainty, affects firms' exporting decisions. Building on progress made by the recent literature, I focus on the role of financial frictions. I introduce a theoretical model that provides mechanisms rationalizing the empirical findings described in the first chapter of this dissertation. In the model, risk-neutral firms must borrow from a lender to cover some of the costs of exporting. The lender charges an interest rate above the risk-free rate because the repayment by the firm is only partially guaranteed by collateral. The uncertainty about the firm's repayment arises from exchange rate risk. The price agreed upon by the exporting firm and its foreign customer is set in the customer's currency. Hence, before the exchange rate is realized, the exporting firm cannot precisely predict its revenue in its own currency, which causes ex-ante uncertainty regarding its ability to fully repay its loan. Exports from firms with different degrees of external finance dependence and asset tangibility are differently affected by exchange rate uncertainty since they represent different potential levels of risk for their lender.

The calibrated model reveals in a counterfactual analysis that the amount that firms would be willing to pay in order to avoid exchange rate uncertainty is a relatively

[57] In the model, large firms benefit more from asset tangibility by construction because of their larger capital stock.

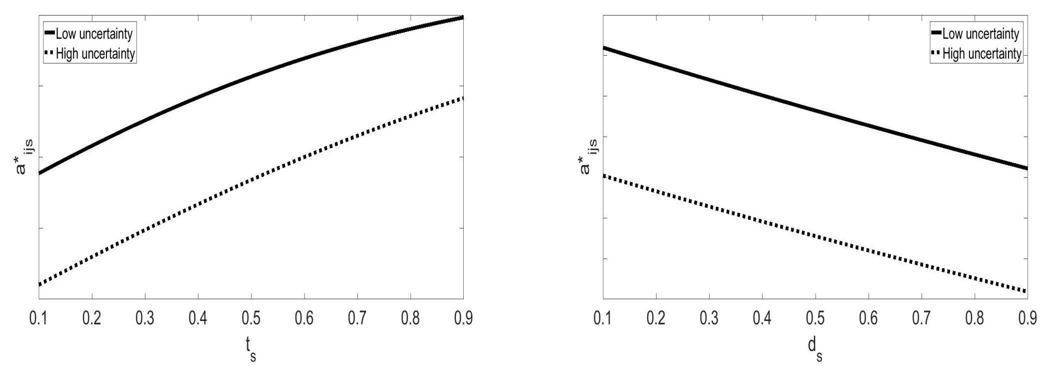
small fraction of their profits and that a policy of providing additional collateral to exporters would have limited effects on smaller firms. The model therefore sheds light on the effect of potential ways for policymakers to alleviate the negative effect of uncertainty on trade.

2.7 Figures



(a) Asset tangibility (b) External finance dependence

Figure 2.1: Change in price in response to an increase in exchange rate uncertainty



(a) Asset tangibility (b) External finance dependence

Figure 2.2: Change in productivity threshold in response to an increase in exchange rate uncertainty

2.8 Tables

Dependent variable	(log) spread over risk-free rate (basis points)	
	1	2
Asset Tang	-0.0034***	
	(-4.82)	
Ext Fin Dep	0.00010*	
	(1.67)	
Profitability	0.716***	
	(5.85)	
log Exch Rate Uncertainty		0.282***
		(7.10)
log VIX		0.170***
		(3.22)
log Loan amount	-0.242***	-0.141***
	(-23.62)	(-12.30)
log Loan maturity	0.125***	0.115***
	(5.60)	(4.54)
Quarter × Year FE	Yes	No
Industry FE	No	Yes
Obs	2076	1879
R-squared	0.549	0.422

This table presents estimates obtained using data on syndicated lending to Canadian firms for the 1988Q1-2015Q4 period. The dependent variable is the (log) spread over the risk-free rate in basis points. The control variables include the (log) loan amount, the (log) loan maturity, as well as dummy variables for term loans, public firms and the presence of multiple lenders. Province fixed effects are also added. In the first column, sectoral measures of asset tangibility, external finance dependence and profitability are added, as well as quarter-year fixed effect. In the second regression, time fixed effects are dropped and industry fixed effects are added, and additional controls include the log of exchange rate uncertainty and of the VIX index, as well as U.S. GDP growth and Canada GDP growth. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 2.1: Determinants of interest rates for Canadian firms

Param.	312	313	314	315	321	326	332	334	335	336	337
τ_{ij}	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
ϵ	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
b	4	4	4	4	4	4	4	4	4	4	4
θ_s (%)	0.24	0.06	0.07	0.16	0.19	0.37	0.55	0.78	0.24	1.39	0.17
d_s	0.51	0.42	0.40	0.54	0.40	0.46	0.38	0.60	0.57	0.44	0.20
t_s	0.22	0.33	0.30	0.12	0.32	0.34	0.26	0.11	0.18	0.22	0.25
c_{js}	237	120	173	204	203	259	259	275	245	381	180
f_{ijs}	0.0030	0.0033	0.0013	0.0014	0.0027	0.0023	0.0020	0.0048	0.0026	0.0078	0.0026
f_{ejs}	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$	$\frac{f_{ijs}}{1.43}$
k_s	6.0	11.3	9.7	8.0	4.8	5.3	6.9	5.8	5.5	2.8	6.8

Table 2.2: Calibrated parameters by sector

Dependent variable	log Exports (2010 prices)	
	Actual data	Simulated data
	1	2
log Uncert$_t$	-0.360***	-0.637***
	(-3.89)	(-14.87)
log Uncert$_t$ × Ext Fin Dep$_s$	0.0045***	0.0063***
	(4.10)	(7.97)
log Uncert$_t$ × Asset Tang$_s$	0.0147***	0.0255***
	(3.37)	(13.89)
log Uncert$_t$ × Share SME$_s$	0.0033**	0.0055***
	(2.74)	(9.83)
log Uncert$_t$ × Ext Fin Dep$_s$ × Sh SME$_s$	-0.00006***	-0.000086***
	(-4.09)	(-8.25)
log Uncert$_t$ × Asset Tang$_s$ × Sh SME$_s$	-0.00015**	-0.00037***
	(2.64)	(-15.90)
Sector FE	Yes	Yes
Quarter FE	Yes	Yes
Year FE	Yes	Yes
Number obs	1232	1232
R-squared(within)	0.933	0.999

Column 1 reproduces the results from column 1 in Table 1.6. Column 2 contains the results of the identical regression run on simulated data obtained from the calibrated model. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 2.3: Regressions on simulated data - Total margin of trade

Dependent variable	log Number of varieties	
	Actual data	Simulated data
	1	2
log Uncert_t	-0.298**	-0.332***
	(-2.58)	(-4.35)
log $\text{Uncert}_t \times$ Ext Fin Dep_s	0.0034*	0.0024**
	(2.13)	(2.70)
log $\text{Uncert}_t \times$ Asset Tang_s	0.0126**	0.0097**
	(2.42)	(2.95)
log $\text{Uncert}_t \times$ Share SME_s	0.0023	0.00057
	(1.26)	(0.52)
log $\text{Uncert}_t \times$ Ext Fin $\text{Dep}_s \times$ Sh SME_s	-0.000037	-0.000033**
	(-1.65)	(-2.71)
log $\text{Uncert}_t \times$ Asset $\text{Tang}_s \times$ Sh SME_s	-0.00014	-0.00015**
	(-1.64)	(-2.98)
Sector FE	Yes	Yes
Quarter FE	-	-
Year FE	No	No
Number obs	264	264
R-squared(within)	0.812	0.991

Column 1 contains the results of a regression identical to that of column 3 in Table 1.6. Column 2 contains the results of the identical regression run on simulated data obtained from the calibrated model. T-statistics are in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

Table 2.4: Regressions on simulated data - Extensive margin of trade

	Million CAD			Share of profits (%)		
	Large	Medium	Small	Large	Medium	Small
Sector	1	2	3	4	5	6
312 Beverage and tobacco	0.41	0.27	0.32	0.86	2.11	8.37
313 Textile	0.47	0.27	0.30	0.69	1.73	6.97
314 Textile products	0.19	0.11	0.12	0.73	1.81	7.21
315 Clothing	0.32	0.16	0.17	1.00	2.17	8.36
321 Wood products	0.34	0.22	0.25	0.69	1.80	7.26
326 Plastics and rubber products	0.32	0.19	0.21	0.68	1.71	6.80
332 Fabricated metal products	0.39	0.19	0.20	0.81	1.85	7.24
334 Computer and electronic products	1.10	0.56	0.60	1.02	2.20	8.46
335 Electrical equipment	0.46	0.26	0.29	0.92	2.11	8.15
336 Transportation equipment	0.91	0.68	0.82	0.88	2.25	8.95
337 Furniture and related products	0.43	0.24	0.26	0.81	1.95	7.77

This table contains the profit loss arising from the presence of exchange rate uncertainty per firm per quarter. In columns 1-3 I calculate the difference between a firm's predicted profit in case of average uncertainty and its predicted profit in case of no uncertainty. Columns 4-6 express these losses as percentages of firms' profits. A large firm is defined as a firm whose productivity parameter a is on the 25th percentile of the sector-specific $[a_L, a^*_{ijs}]$ spectrum. Medium and small firms are on the 50th and 75th percentiles, respectively.

Table 2.5: Profit loss from uncertainty

	Reduction in profit loss (%)			Reduction in exports decrease (%)		
	Large	Medium	Small	Large	Medium	Small
Sector	1	2	3	4	5	6
312 Beverage and tobacco	23.3	17.6	18.0	17.40	12.36	11.08
313 Textile	29.7	22.7	24.7	22.41	15.87	15.26
314 Textile products	27.2	20.7	22.6	20.41	14.45	13.82
315 Clothing	9.9	7.6	8.5	7.25	5.20	4.97
321 Wood products	31.4	23.7	25.1	23.76	16.71	15.62
326 Plastics and rubber products	30.8	23.4	25.5	23.27	16.42	15.76
332 Fabricated metal products	19.8	15.6	18.1	14.73	10.64	10.75
334 Computer and electronic products	9.2	7.1	7.9	6.78	4.86	4.60
335 Electrical equipment	16.8	12.8	13.6	12.47	8.87	8.20
336 Transportation equipment	25.4	19.2	19.2	19.04	13.61	11.96
337 Furniture and related products	22.0	17.0	18.8	16.39	11.73	11.33

This table contains the reduction in profit loss (columns 1-3) and export reduction (columns 4-6) arising from the presence of exchange rate uncertainty per firm per quarter when asset tangibility is increased by 50 percent. A large firm is defined as a firm whose productivity parameter a is on the 25th percentile of the sector-specific $[a_L, a^*_{ijs}]$ spectrum. Medium and small firms are on the 50th and 75th percentiles, respectively.

Table 2.6: Reduction in losses from increased tangibility

2.9 Appendix A: Proofs

Note that these proofs assume that:

1. $A_1 < 1$ since if, in expectation, the firm is expecting to default, it will not select into exporting.

2. $A_1 > 1 - \iota$, that is, the realization of the exchange rate which would make a firm to default is on the range of possible realizations of the exchange rate.

3. $X_t p_{ijs}(a) > \frac{\epsilon}{\epsilon-1} a c_{js} \tau_{ij}$, that is, the presence of financial frictions and exchange rate uncertainty decreases exports compared to the frictionless case.

2.9.1 Proposition 1

Proposition 1 (i) The quantities part follows from demand: $q_{ijs}(a) = p_{ijs}(a)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}}$. Need to show that $\frac{\partial p_{ijs}(a)}{\partial a} > 0$, taking into account the effect on the interest rate. First note that the firm's FOC can be expressed as

$$(1-\epsilon) + \frac{\epsilon a c_{js} \tau_{ij}}{X_t p_{ijs}(a)} = \frac{[(1-\iota) - A_1]^2}{6\iota^2 - 3[(1-\iota) - A_1]^2} \left[(\epsilon - 1)[(1-\iota) + 2A_1] - 3\left(1 - \frac{t_s}{ab}\right) \frac{\epsilon a c_{js} \tau_{ij}}{X_t p_{ijs}(a)} \right].$$

Take the total differential with respect to $p_{ijs}(a)$ and a, taking into account that A_1 depends on both variables both directly and via the interest rate $(1 + R_{ijs}(a))$, to obtain

$$\frac{dp_{ijs}(a)}{da} = \frac{\frac{\epsilon c_{js} \tau_{ij}}{X_t p_{ijs}(a)} \frac{[6\iota^2 - 3(t_s/ab)[A_1 - (1-\iota)]^2]}{6\iota^2 - 3[A_1 - (1-\iota)]^2} - \{\cdot\} \frac{\partial A_1}{\partial a}}{\frac{\epsilon a c_{js} \tau_{ij}}{X_t p_{ijs}(a)^2} \frac{[6\iota^2 - 3(t_s/ab)[A_1 - (1-\iota)]^2]}{6\iota^2 - 3[A_1 - (1-\iota)]^2} + \{\cdot\} \frac{\partial A_1}{\partial p_{ijs}(a)}}$$

where

$$\{\cdot\} =$$
$$\left\{\frac{36\iota^2[A_1-(1-\iota)]}{[6\iota^2-3[A_1-(1-\iota)]^2]^2}\left\{(\epsilon-1)A_1-(\epsilon-1)\frac{[A_1-(1-\iota)]^3}{6\iota^2}-\left(1-\frac{t_s}{ab}\right)\frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)}\right\}\right\},$$

$$\frac{\partial A_1}{\partial a}=\left(1-\frac{t_s}{ab}\right)\frac{c_{js}\tau_{ij}}{X_t p_{ijs}(a)}\left[1+X_t p_{ijs}(a)^{1-\epsilon}\frac{\theta_s Y_i}{P_{is}^{1-\epsilon}}\frac{3[A_1-(1-\iota)]^2}{6\iota^2-3[A_1-(1-\iota)]^2}\right]$$
$$+\frac{\lambda'(a)(1+r)}{[1-\lambda(a)]^2}\frac{d_s f_{ijs}6\iota^2}{6\iota^2-3[A_1-(1-\iota)]^2},$$

and

$$\frac{\partial A_1}{\partial p_{ijs}(a)}=$$
$$\left\{\frac{(\epsilon-1)A_1}{p_{ijs}(a)}-\left(1-\frac{t_s}{ab}\right)\frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)^2}+\frac{(\epsilon-1)}{p_{ijs}(a)}\frac{[A_1-(1-\iota)]^3}{6\iota^2}\right\}\frac{6\iota^2}{6\iota^2-3[A_1-(1-\iota)]^2}.$$

This term can be shown to be negative as long as (i) we are not in a situation where risk is very low and probability of default is very high (unlikely to be delivered by the model) and (ii) financial constraints are strong enough so that the optimal price is sufficiently above the pure monopolistic competition markup over marginal cost of production. This implies that $\{\cdot\}$ is also negative. $\frac{\partial A_1}{\partial a}$ is positive. As a result, $\frac{dp_{ijs}(a)}{da}>0$.■

Proposition 1 (ii) A firm chooses to export only if $E_t\pi_{ijs}(a)>0$. Hence, it suffices to show that $E_t\pi_{ijs}(a)$ is strictly decreasing in a (since the fixed cost $(1-d_s)f_{ijs}$ does not depend on a).

$$\frac{\partial E_t \pi_{ijs}(a)}{\partial a} = \frac{\partial E_t \pi_{ijs}(a)}{\partial p_{ijs}(a)} \frac{\partial p_{ijs}(a)}{\partial a}$$

$$- c_{js}\tau_{ij} p_{ijs}(a)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}} - d_s f_{ijs} \left[\frac{\partial(1+R_{ijs}(a))}{\partial a} \right]^{\text{holding } p_{ijs}(a) \text{ fixed}}$$

where $\frac{\partial E_t \pi_{ijs}(a)}{\partial p_{ijs}(a)} = 0$ by firm's FOC and

$$\left[\frac{\partial(1+R_{ijs}(a))}{\partial a} \right]^{\text{holding } p_{ijs}(a) \text{ fixed}} =$$

$$\frac{\left(1 - \frac{t_s}{ab}\right) c_{js}\tau_{ij} p_{ijs}(a)^{-\epsilon} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}} \frac{3[A_1-(1-\iota)]^2}{6\iota^2} + \frac{\lambda'(a)(1+r)}{[1-\lambda(a)]^2} d_s f_{ijs}}{\frac{6\iota^2 - 3[A_1-(1-\iota)]^2}{6\iota^2} d_s f_{ijs}} > 0. \blacksquare$$

2.9.2 Proposition 2

Proposition 2 (i) To show that $\frac{\partial p_{ijs}(a)}{\partial t_s} < 0$, take total differential of the firm's FOC with respect to $p_{ijs}(a)$ and t_s to obtain

$$\frac{dp_{ijs}(a)}{dt_s} = \frac{-\frac{3[A_1-(1-\iota)]^2}{6\iota^2 - 3[A_1-(1-\iota)]^2} \frac{\epsilon c_{js}\tau_{ij}}{X_t p_{ijs}(a)} - \{\cdot\} \frac{\partial A_1}{\partial t_s}}{\frac{6\iota^2 - 3(t_s/ab)[A_1-(1-\iota)]^2}{6\iota^2 - 3[A_1-(1-\iota)]^2} \frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)^2} + \{\cdot\} \frac{\partial A_1}{\partial p_{ijs}(a)}}$$

where $\{\cdot\}$ is as defined above and where, taking into account the effect of t_s on $(1+R_{ijs}(a))$, but holding $p_{ijs}(a)$ fixed, one can obtain

$$\frac{\partial A_1}{\partial t_s} = \frac{-6\iota^2}{6\iota^2 - 3[A_1-(1-\iota)]^2} \left[\frac{c_{js}\tau_{ij}}{X_t p_{ijs}(a)} + \frac{f_{ejs}}{X_t p_{ijs}(a)^{1-\epsilon}} \frac{P_{is}^{1-\epsilon}}{\theta_s Y_i} \right] < 0. \blacksquare$$

Proposition 2 (ii) To show that $\frac{\partial p_{ijs}(a)}{\partial d_s} > 0$, take total differential of the firm's FOC with respect to $p_{ijs}(a)$ and d_s to obtain

$$\frac{dp_{ijs}(a)}{dd_s} = \frac{-\{\cdot\} \frac{\partial A_1}{\partial d_s}}{\frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)^2} \frac{6\iota^2 - 3(t_s/ab)[A_1-(1-\iota)]^2}{6\iota^2 - 3[A_1-(1-\iota)]^2} + \{\cdot\} \frac{\partial A_1}{\partial p_{ijs}(a)}}$$

where $\{\cdot\}$ is as defined above and where, taking into account the effect of d_s on $(1+R_{ijs}(a))$, but holding $p_{ijs}(a)$ fixed, one can obtain

$$\frac{\partial A_1}{\partial d_s} = \frac{f_{ijs}}{X_t p_{ijs}(a)^{1-\epsilon}} \frac{P_{is}^{1-\epsilon}}{\theta_s Y_i} \frac{6\iota^2}{6\iota^2 - 3[A_1 - (1-\iota)]^2} \left\{ \frac{(1+r)}{1-\lambda(a)} - 1 \right\} > 0. \blacksquare$$

2.9.3 Proposition 3

To show that $\frac{\partial p_{ijs}(a)}{\partial \iota} > 0$, take total differential of the firm's FOC with respect to $p_{ijs}(a)$ and ι to obtain

$$\frac{dp_{ijs}(a)}{d\iota} = \frac{-\{\cdot\}\frac{\partial A_1}{\partial \iota} + \frac{(\epsilon-1)[A_1-(1-\iota)]^2}{6\iota^2-3[A_1-(1-\iota)]^2} - \left[(1-\epsilon) + \frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)}\right]\left[\frac{12\iota[\iota-[A_1-(1-\iota)]]}{[A_1-(1-\iota)][6\iota^2-3[A_1-(1-\iota)]^2]}\right]}{\frac{\epsilon a c_{js}\tau_{ij}}{X_t p_{ijs}(a)^2}\left[\frac{6\iota^2-3(t_s/ab)[A_1-(1-\iota)]^2}{6\iota^2-3[A_1-(1-\iota)]^2}\right] + \{\cdot\}\frac{\partial A_1}{\partial p_{ijs}(a)}}$$

where $\{\cdot\}$ is as defined above and where, taking into account the effect of ι on $(1+R_{ijs}(a))$ but holding $p_{ijs}(a)$ fixed, one can obtain

$$\frac{\partial A_1}{\partial \iota} = \frac{[A_1-(1-\iota)]^2}{\iota} \frac{3\iota - 2[A_1-(1-\iota)]}{6\iota^2 - 3[A_1-(1-\iota)]^2} > 0$$

since $\iota > A_1 - (1-\iota)$.$\blacksquare$

2.9.4 Proposition 4

It was already showed that $E_t\pi_{ijs}(a)$ is strictly decreasing in a. Hence, to show that a^*_{ijs} is decreasing in ι, I only need to show that $E_t\pi_{ijs}(a)$ is strictly decreasing in ι for any given a.

$$\frac{\partial E_t\pi_{ijs}(a)}{\partial \iota} = \frac{\partial E_t\pi_{ijs}(a)}{\partial p_{ijs}(a)} \frac{\partial p_{ijs}(a)}{\partial \iota} - d_s f_{ijs} \left[\frac{\partial(1+R_{ijs}(a))}{\partial \iota}\right]^{\text{holding } p_{ijs}(a) \text{ fixed}} < 0$$

since $\frac{\partial E_t \pi_{ijs}(a)}{\partial p_{ijs}(a)} = 0$ by the firm's FOC and

$$\left[\frac{\partial(1+R_{ijs}(a))}{\partial \iota}\right]_{\text{holding } p_{ijs}(a) \text{ fixed}} = \frac{X_t p_{ijs}(a)^{1-\epsilon}}{d_s f_{ijs}} \frac{\theta_s Y_i}{P_{is}^{1-\epsilon}} \frac{[A_1-(1-\iota)]^2}{\iota} \frac{3\iota+2[(1-\iota)-A_1]}{6\iota^2-3[A_1-(1-\iota)]^2} > 0. \blacksquare$$

2.10 Appendix B: Financial constraints of small and medium firms in Canada

		Number of employees				
2004		0	1-4	5-19	20-99	100-499
Demand or short-term loan	Interest rate	5.8	6.2	6.2	6.6	7.1
	Length of term (m)	9	10	7	10	9
Term loan	Interest rate	6	6.3	6.3	6.2	5.9
	Length of term (m)	53	71	56	65	83
New line of credit	Interest rate	6	6.2	6.3	6.4	5
New credit card	Interest rate	17.9	17.1	17.3	18.8	-
2007						
Term loan or mortgage	Interest rate	7.5	7.3	7.3	6.8	6.7
	Length of term (m)	69.4	61.4	72	66.2	66.7
Operating line	Interest rate	7.8	7.5	7.3	6.9	6.8
2011						
All credit	Interest rate	-	7.2	6.7	5.8	4.9
	Length of term (m)	-	64	79	70	57
2014						
Term loan	Interest rate	-	5.3	5.5	4.7	4.3
	Length of term (m)	-	63	59	57	50
Line of credit	Interest rate	-	5.6	5.1	4.7	4.8
Credit card	Interest rate	-	17.2	17	18.2	18.6

This table contains a subset of the answers to Statistics Canada's Survey on Financing of Small and Medium Enterprises for the years 2004, 2007, 2011, and 2014. The table reports the average interest rate and length of the term (in months, where relevant) of different types of credit obtained by Canadian SMEs, classified by size.

Table 2.7: Average interest rate and maturity for different firm sizes

	Number of employees				
	0	1-4	5-19	20-99	100-499
2000	-	68	76	85	-
2004	-	81	87	83	79
2007	85	75	90	89	98
2011	-	90	86	97	99
2014	-	83	81	88	98

This table contains a subset of the answers to Statistics Canada's Survey on Financing of Small and Medium Enterprises for the years 2000, 2004, 2007, 2011, and 2014. The table reports, by firm size category, the ratio of total (over all firms and financing instruments) credit authorized to credit requested.

Table 2.8: Ratio (%) of credit authorized to credit requested

3 Uncovered Interest Rate Parity Models in a Dynamic Stochastic General Equilibrium Framework

3.1 Introduction

One of the main puzzles in international finance is the uncovered interest rate parity (UIP) puzzle, also referred to as the foreign premium puzzle. The UIP puzzle is the empirical finding that countries with a higher risk-free interest rate tend have currencies that appreciate in the short run. Typical two-country macroeconomic models instead predict that high interest-rate currencies should depreciate, thus eliminating profitable carry trade strategies. If a high interest rate currency is expected to appreciate against a low interest rate currency, then it is on average profitable for an investor to borrow in the low interest rate currency and invest in a high interest rate currency. In a typical representative-agent model with time-separable preferences and rational expectations, these profitable opportunities are arbitraged away, and the high interest rate currency depreciates. Hence, the extensive empirical literature demonstrating that high interest rate currencies tend to appreciate on average has given rise to the UIP puzzle.

The international finance literature has responded with several alternatives to typical two-country models in order to address the UIP puzzle. These alternatives have been constructed to specifically address the UIP puzzle and are generally set in endowment economies.[58] That is, real variables such as production, consumption, or interest rates follow exogenous processes. These alternative models provide useful intuition about where UIP violations could come from. However, it remains unclear whether they can successfully match UIP evidence when re-cast in a general equilib-

[58]One exception is Valchev (forthcoming).

rium framework with endogenous production and consumption.

In this chapter, I test the robustness of the UIP predictions of two of the main models that provide explanations to the UIP puzzle. I re-cast these models (separately) in a standard dynamic stochastic general equilibrium (DSGE) framework and examine how the UIP predictions are impacted. I investigate how the mechanisms rely on specific parameter values in order to find under which conditions, if any, they can explain the UIP puzzle. In addition, I obtain business cycle moments from model simulations and compare them to the moments obtained from a standard two-country DSGE model and from the data.

The two models that I investigate in this chapter are the habit model of Verdelhan (2010) and the distorted beliefs model of Gourinchas & Tornell (2004). I find that for the first model, the UIP results disappear under realistic calibrations. For the second model, UIP properties remain under some calibrations. In addition, business cycle predictions remain close to empirical evidence.

There is an extensive literature on the empirical violation of the theoretical UIP relationship. Early work in the field began when advanced economies abandoned the gold standard in the 1970s and exchange rates became free to float. Early studies focused on testing the efficiency of the new currency markets by estimating whether on average, the forward exchange rate was equal to the realization of the future exchange rate. Market efficiency was broadly rejected. The rejection of the currency market efficiency hypothesis lead economists to consider the existence of risk premia as an explanatory factor. When investing in foreign risk-free bonds, an investor does not face a risk in terms of nominal returns in foreign currency, but instead a risk in terms of nominal returns in his or her own currency, given that the future exchange rate is unknown. Hence, it is reasonable to expect a risk-averse investor to demand a risk premium, that is, a return greater than the nominal risk-free rate, when investing in foreign bonds. Fama (1984) estimates simultaneously how variations in the risk premium and in expectations of future spot rates can explain forward rates. He finds that variations in the risk premium are by far the most important explanation. However, using data on actual market expectations on the exchange rate, Froot & Frankel (1989) are able to test whether UIP deviations are due to risk premia or to errors in expectations. They find that expectation errors are able to explain all of the deviations, thus questioning the mere existence of a risk premium. Most existing theoretical models explaining the UIP puzzle follow one of these two views. One of the models that I investigate in this chapter adopts the risk premium explanation whereas the other relies on biased expectations of agents.

Testing for UIP can be done using the following regression:

$$s_{t+m} - s_t = \beta_{m0} + \beta_{m1}(r_{t,m} - r^*_{t,m}) + \varepsilon_{t+m} \tag{3.1}$$

where * denotes the Foreign country, s_t denotes the natural logarithm of the spot exchange rate such that an increase in s_t represents a depreciation of the Home currency, and $r_{t,m}$ denotes the natural logarithm of the m-period risk-free nominal interest rate. The null hypothesis is $\beta_{m0} = 0$, $\beta_{m1} = 1$, meaning that if the Home interest rate is higher than the Foreign interest rate, the Home currency depreciates on average. The literature typically finds β_{m1} to be significantly smaller than one. Most papers in fact find a negative β_{m1}. This implies that on average, a country's currency *appreciates* when its interest rate is relatively high. A risk premium explanation to the UIP puzzle requires that an additional term be missing from this equation, capturing a time-varying risk premium. A constant risk premium would be captured by β_{m0} and would not bias β_{m1} even if omitted. The explanation to the UIP puzzle that relies on expectation errors, such as that suggested by Froot & Frankel (1989), requires that the error term ε_{t+m} be non-random, that is, that it varies systematically with the behavior of the exchange rate.

More recent empirical evidence also rejects the UIP hypothesis. Burnside et al. (2006) test the relationship for nine currencies against the British pound, for the period between 1976 and 2005.[59] They find that for *all* currency pairs, the relationship is negative and that it is significantly negative in most cases. Other recent empirical literature on the UIP puzzle tests whether carry-trade portfolios are profitable. Lustig & Verdelhan (2007), for example, show that carry-trade strategies, when performed using groups of assets, are highly profitable and of low risk.

The two main theoretical models explaining the UIP puzzle with time-varying risk premia are Verdelhan (2010) and Colacito & Croce (2013). Verdelhan (2010) includes external habit formation introduced by Campbell & Cochrane (1999) to an otherwise frictionless two-country endowment economy. Because of habits, the agents' degree of risk aversion varies over time, so that the risk premium that they demand to invest abroad also varies over time. Hence, a high interest rate currency may be expected to appreciate, but highly risk-averse agents from the low interest rate country may still not be willing to invest there. Colacito & Croce (2013) modify the one-country model of Bansal & Yaron (2004) that includes recursive preferences and long-run risk by also setting it in a frictionless two-country endowment economy. Recursive preferences cause the agents' degree of risk aversion to vary over time, similar

[59] The authors test the relationship $\left(\frac{S_{t+1}}{S_t} - 1\right) = \alpha + \beta\left(\frac{F_t}{S_t} - 1\right) + \varepsilon_t$ where F_t is the one-period forward exchange rate. Forward rate are sometimes used instead of risk-free interest rates because they are available for more maturities and more currencies, and because they are more comparable across currency pairs. The authors and numerous other researchers find that the covered interest parity relationship $\frac{F_t}{S_t} = \frac{1+R_t}{1+R_t^*}$ generally holds in their sample.

to the external habit model. However, the implications of these two models for the behavior of interest rates are quite different. The external habit model of Verdelhan (2010) relies on the risk-free interest rate being low for the economy that is in relatively "bad times" compared to the other country. The recursive preferences model of Colacito & Croce (2013) relies on the risk-free interest rate in the country that is expected to grow more, that is the country that is in relatively "good times," to be lower.

The other side of the literature instead questions the rationality of expectations or behavior of agents. Gourinchas & Tornell (2004) model agents' beliefs about the conduct of monetary policy as slightly distorted: agents believe that there is a constant probability that monetary authorities are subject to a one-time shock, and are unable to distinguish such shocks from actual persistent monetary shocks. This leads them to react slowly to monetary shocks, as they learn their true nature, and can cause a high-interest rate currency to continue to appreciate for several periods. Bacchetta & van Wincoop (2010) instead argue that agents make infrequent portfolio decisions. The result is similar to the distorted beliefs mechanism: even if agents perfectly observe shocks, they can only react to it slowly. Other models of biased expectations include Ilut (2012), who assumes that agents are ambiguity-averse and have access to limited information, and Lansing & Ma (2017), who use a model of bounded rationality of agents to explain the UIP puzzle.

In the first part of this chapter, I find that the UIP results disappear when the habit model is re-cast is a standard DSGE framework. Verdelhan's (2010) results rely on the households' precautionary saving motive being stronger than their intertemporal smoothing motive in the determination of interest rates. When this is the case, interest rates are procyclical, so that interest rates and risk premia are negatively correlated. This makes the UIP coefficient smaller than zero. Verdelhan obtains this result by assuming that the consumption process is a random walk. When the consumption process is instead mean-reverting (for example, because technology or the monetary shock process are mean-reverting) or when consumption growth is time-varying but predictable for other reasons (such as the presence of sticky prices), the intertemporal smoothing motive dominates. As a result, interest rates are counter-cyclical and the UIP coefficient is close to one.

In the second part of this chapter, I find that re-casting the distorted beliefs model in a DSGE framework does not alter its theoretical UIP properties. However, the assumptions about the shock processes are crucial to obtain a negative UIP coefficient. Monetary shocks must be more persistent than is typically assumed, and beliefs must be highly distorted. These conditions are necessary to obtain a coefficient significantly below one. This model is able to explain the UIP puzzle and preserve other moments in a DSGE framework.

The remaining sections are organized as follows. In Section 3.2 I describe the baseline two-country DSGE framework and its UIP properties. Sections 3.3 and 3.4 contain the description of and results for the habit and distorted beliefs models, respectively. Section 3.5 concludes.

3.2 Baseline Model

Each of the UIP mechanisms investigated in this chapter is introduced in an otherwise standard two-country DSGE framework inspired from Clarida et al. (2002) (CGG). In this section, I describe this framework and demonstrate why it cannot account for UIP empirical evidence.

The two countries, Home and Foreign, are symmetric except for the shocks they face. In each country, a representative household gets positive utility from consumption and negative utility from working. Each country produces two types of goods. Intermediate differentiated goods are produced using domestic labor and technology. Final goods are produced using country-specific intermediate goods. Households consume final goods produced domestically and abroad, but may prefer the goods produced in their own country. Nominal prices of intermediate goods are sticky and monetary policy is based on a Taylor-type rule. Markets are complete both nationally and internationally, which implies that all agents in all countries are affected equally by stochastic shocks.

3.2.1 Households

There is a measure one of households in each country. The representative household in the Home country maximizes the present discounted value of its lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{3.2}$$

where β is the household discount factor. The intra-period utility function is given by

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \tag{3.3}$$

where C_t is the consumption level, N_t is the number of hours worked, σ is both the degree of risk aversion and the inverse of the intertemporal elasticity of substitution, and ϕ is the inverse of the Frisch elasticity of labor supply. The assumption of complete financial markets insures that all households within a country are identical and allows the analysis of the representative household.

The household's consumption basket is composed of two final goods: one produced domestically and one produced abroad. The household has constant elasticity of substitution (CES) preferences over these two final goods, which implies that the Home household's consumption basket satisfies

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{3.4}$$

where $0 \leq \gamma \leq 1$ is a preference parameter such that there is home bias if $\gamma > 0.5$ and θ is the elasticity of substitution between the Home and the Foreign final goods. Minimizing the cost of a basket containing one unit of Home consumption C_t yields the following consumer price index:

$$P_t = \left[\gamma (P_{H,t})^{1-\theta} + (1-\gamma)(P_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.5}$$

where $P_{H,t}$ is the price paid for the final good produced in Home and $P_{F,t}$ is the price paid (in Home currency) for the final good produced in Foreign.

The representative household's utility-maximization problem is given by

$$\max_{\{C_t, N_t, D_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}\right] \tag{3.6}$$

subject to the sequence of budget constraints

$$P_t C_t + E_t\{Q_{t,t+1} D_{t+1}\} = W_t N_t + D_t - T_t + \Gamma_t \qquad \forall t = 0, 1, 2, ... \tag{3.7}$$

where $Q_{t,t+1}$ is the household's stochastic discount factor, D_{t+1} is the payoff at $t+1$ of the portfolio purchased by the household at time t, W_t is the nominal wage, T_t is lump sum taxes raised by the government and Γ_t is lump sum profits coming from domestic firms producing intermediate goods.[60]

The first-order conditions for consumption and labor imply:

$$C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t \tag{3.8}$$

$$C_{F,t} = (1-\gamma) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t \tag{3.9}$$

$$\frac{N_t^{\phi}}{W_t} = \frac{C_t^{-\sigma}}{P_t}. \tag{3.10}$$

The household's nominal stochastic discount factor $Q_{t,t+1}$ satisfies

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{3.11}$$

and the nominal risk-free interest rate satisfies

$$\frac{1}{1+R_t} = E_t\{Q_{t,t+1}\}. \tag{3.12}$$

The Euler equation is therefore given by:

$$\frac{1}{1+R_t} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]. \tag{3.13}$$

The Foreign representative household faces a symmetric utility-maximization problem, that is to maximize

$$\max_{\{C_t^*, N_t^*, D_{t+1}^*\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^*)^{1-\sigma}}{1-\sigma} - \frac{(N_t^*)^{1+\phi}}{1+\phi} \right] \tag{3.14}$$

[60]Firms producing the final good are perfectly competitive and earn no profit.

subject to the sequence of budget constraints

$$P_t^* C_t^* + E_t\{Q_{t,t+1}^* D_{t+1}^*\} = W_t^* N_t^* + D_t^* - T_t^* + \Gamma_t^* \qquad \forall t = 0, 1, 2, ... \tag{3.15}$$

where

$$C_t^* = \left[(1-\gamma)^{\frac{1}{\theta}}(C_{H,t}^*)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}(C_{F,t}^*)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3.16}$$

$$P_t^* = \left[(1-\gamma)(P_{H,t}^*)^{1-\theta} + \gamma(P_{F,t}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.17}$$

The first-order conditions imply

$$C_{H,t}^* = (1-\gamma)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta} C_t^* \tag{3.18}$$

$$C_{F,t}^* = \gamma\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} C_t^* \tag{3.19}$$

$$\frac{(N_t^*)^{\phi}}{W_t^*} = \frac{(C_t^*)^{-\sigma}}{P_t^*} \tag{3.20}$$

and

$$\frac{1}{1+R_t^*} = E_t\{Q_{t,t+1}^*\} \tag{3.21}$$

where

$$Q_{t,t+1}^* = \beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*}. \tag{3.22}$$

The terms of trade, TOT_t, are defined as the ratio of the price of the Foreign good to the price of the Home good, both expressed in Home's currency:

$$TOT_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{3.23}$$

The consumer price indices can then be expressed as

$$\left(\frac{P_t}{P_{H,t}}\right)^{1-\theta} = \gamma + (1-\gamma)TOT_t^{1-\theta} \tag{3.24}$$

$$\left(\frac{P_t^*}{P_{F,t}^*}\right)^{1-\theta} = \gamma + (1-\gamma)TOT_t^{\theta-1} \tag{3.25}$$

and the consumers' demand functions can be expressed as

$$C_{H,t} = \gamma\left(\gamma + (1-\gamma)TOT_t^{1-\theta}\right)^{\frac{\theta}{1-\theta}} C_t \tag{3.26}$$

$$C_{F,t} = (1-\gamma)\left((1-\gamma) + \gamma TOT_t^{\theta-1}\right)^{\frac{\theta}{1-\theta}} C_t \tag{3.27}$$

$$C_{H,t}^* = (1-\gamma)\left((1-\gamma) + \gamma TOT_t^{1-\theta}\right)^{\frac{\theta}{1-\theta}} C_t^* \tag{3.28}$$

$$C_{F,t}^* = \gamma\left(\gamma + (1-\gamma)TOT_t^{\theta-1}\right)^{\frac{\theta}{1-\theta}} C_t^*. \tag{3.29}$$

Prices are assumed to be set in the producer's currency, and the law of one price implies

$$P_{H,t} = S_t P_{H,t}^* \tag{3.30}$$

$$P_{F,t} = S_t P_{F,t}^* \tag{3.31}$$

where S_t is the nominal exchange rate defined as the amount of Home currency that can be exchanged for one unit of Foreign currency. An increase in S_t corresponds to a depreciation of the Home currency. The terms of trade can then be expressed as

$$TOT_t = S_t \frac{P_{F,t}^*}{P_{H,t}}. \tag{3.32}$$

Risk sharing Under complete financial markets, the intertemporal marginal rates of substitution of households, or stochastic discount factors $Q_{t,t+1}$ and $Q_{t,t+1}^*$, are equalized across countries.[61] These variables are expressed in different units: Home's stochastic discount factor is expressed in Home currency this period per unit of Home currency unit next period, whereas Foreign's stochastic discount factor is expressed in Foreign currency this period per unit of Foreign currency unit next period. Equalizing the intertemporal marginal rates of substitution therefore requires a nominal exchange rate adjustment:

$$Q_{t,t+1} = \frac{S_t}{S_{t+1}} Q^*_{t,t+1}. \tag{3.33}$$

Thus risk sharing determines the exchange rate process. Note that this equation holds exactly, not only in expectation. Because of the presence of complete international financial markets, perfect risk sharing is achieved in every period.

The behavior of the real exchange rate RER_t is determined similarly, equalizing the real stochastic discounts factors $RQ_{t,t+1}$ and $RQ^*_{t,t+1}$ across countries:

$$\frac{RER_{t+1}}{RER_t} = \frac{RQ^*_{t,t+1}}{RQ_{t,t+1}}$$

where $RQ_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$ and $RQ^*_{t,t+1} = \beta \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}$. This implies the usual definition of the real exchange rate

$$RER_t = \frac{S_t P^*_t}{P_t}. \tag{3.34}$$

3.2.2 Firms

In each country, production happens in two stages. Intermediate good producers produce differentiated intermediate goods in a monopolistically competitive market.

[61] Because of home bias in preferences, consumption baskets differ across countries and complete risk sharing does not imply equal consumption growth across countries.

They use the available domestic technology and labor, and face price-setting rigidities à la Calvo (1983). Final good producers produce final goods by combining all domestic intermediate goods. They operate in a perfectly competitive market and face no price rigidity.

Final good producers There is a measure one of final good producers in each country. Each of them combines the intermediate goods in a CES fashion:

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \tag{3.35}$$

where Y_t is the amount of final good produced, $Y_t(f)$ is the amount of differentiated intermediate good f used as input, and ξ is the elasticity of substitution between intermediate goods produced domestically. There is a measure one of intermediate goods being produced in each country.

Cost-minimization by final good producers yields the producer price index

$$P_{H,t} = \left(\int_0^1 P_{H,t}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}} \tag{3.36}$$

where $P_{H,t}(f)$ is the price of a specific intermediate good f, and the demand function

$$Y_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} Y_t. \tag{3.37}$$

The corresponding equations for the Foreign country are

$$Y_t^* = \left(\int_0^1 Y_t(f^*)^{\frac{\xi-1}{\xi}} df^* \right)^{\frac{\xi}{\xi-1}} \tag{3.38}$$

$$P_{F,t}^* = \left(\int_0^1 P_{F,t}^*(f^*)^{1-\xi} df^* \right)^{\frac{1}{1-\xi}} \tag{3.39}$$

$$Y_t(f^*) = \left(\frac{P_{F,t}^*(f^*)}{P_{F,t}^*} \right)^{-\xi} Y_t^*. \tag{3.40}$$

Intermediate good producers Each intermediate good producer f combines labor $N_t(f)$ and the available technology A_t according to

$$Y_t(f) = A_t N_t(f) \tag{3.41}$$

and faces a subsidy rate of τ. The cost-minimization problem for the choice of labor yields

$$MC_t = (1-\tau)\frac{W_t}{A_t} \tag{3.42}$$

where MC_t is the nominal marginal cost of the intermediate firm.[62]

Intermediate firms are operating in a monopolistically competitive market. Following Calvo (1983), they can only reset their price with a probability of $1-\alpha$ in each period. When receiving a signal that they can change their price, intermediate firms choose the optimal price $P_{H,t}^O$ by maximizing their profit:

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j}(f) \left[P_{H,t}^O - MC_{t+j} \right] \tag{3.43}$$

[62]As noted by CGG, constant returns to scale in technology insures that the marginal cost is the same for all intermediate firms, regardless of their level of production. This assumption insures tractability in the presence of price rigidities.

subject to the demand function (3.37) coming from final good producers. The resulting optimal price satisfies the condition

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j}(f) \left[P_{H,t}^O - \left(1 + \frac{1}{\xi - 1}\right) MC_{t+j} \right] = 0, \tag{3.44}$$

that is, on weighted average over all periods during which $P_{H,t}^O$ is expected to remain the firm's price, this price must cover the marginal cost plus a constant markup of $\frac{1}{\xi-1}$.[63] Firms that cannot reset their price keep their price from the previous period. By the law of large numbers, the price index $P_{H,t}$ satisfies

$$P_{H,t} = \left[\alpha (P_{H,t-1})^{1-\xi} + (1-\alpha)(P_{H,t}^O)^{1-\xi}\right]^{\frac{1}{1-\xi}}. \tag{3.45}$$

Combining equations (3.44) and (3.45) yields an expression in terms of $t-1$, t, and $t+1$ only:

$$P_{H,t-1} \left(\frac{1}{1-\alpha} (1 + \pi_{H,t})^{1-\xi} - \frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\xi}} = \frac{NN_t}{DD_t} \tag{3.46}$$

where

$$NN_t = \left(1 + \frac{1}{\xi - 1}\right) P_{H,t}^{\xi} Y_t MC_t + \alpha E_t Q_{t,t+1} NN_{t+1} \tag{3.47}$$

$$DD_t = P_{H,t}^{\xi} Y_t + \alpha E_t Q_{t,t+1} DD_{t+1} \tag{3.48}$$

$$(1 + \pi_{H,t}) = \frac{P_{H,t}}{P_{H,t-1}}. \tag{3.49}$$

[63] Note that if prices were fully flexible ($\alpha = 0$), the optimal price would be $P_{H,t}^O = (1 + \frac{1}{\xi-1}) MC_t$.

The corresponding equations for the Foreign country are

$$Y_t(f^*) = A_t^* N_t(f^*) \tag{3.50}$$

$$MC_t^* = (1-\tau)\frac{W_t^*}{A_t^*} \tag{3.51}$$

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j}^* Y_{t+j}(f^*) \left[P_{F,t}^{O*} - \left(1 + \frac{1}{\xi - 1}\right) MC_{t+j}^* \right] = 0 \tag{3.52}$$

$$P_{F,t}^* = \left[\alpha (P_{F,t-1}^*)^{1-\xi} + (1-\alpha)(P_{F,t}^{O*})^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{3.53}$$

$$P_{F,t-1}^* \left(\frac{1}{1-\alpha}(1+\pi_{F,t}^*)^{1-\xi} - \frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\xi}} = \frac{NN_t^*}{DD_t^*} \tag{3.54}$$

$$NN_t^* = \left(1 + \frac{1}{\xi - 1}\right)(P_{F,t}^*)^{\xi} Y_t^* MC_t^* + \alpha E_t Q_{t,t+1}^* NN_{t+1}^* \tag{3.55}$$

$$DD_t^* = (P_{F,t}^*)^{\xi} Y_t^* + \alpha E_t Q_{t,t+1}^* DD_{t+1}^* \tag{3.56}$$

$$(1+\pi_{F,t}^*) = \frac{P_{F,t}^*}{P_{F,t-1}^*}. \tag{3.57}$$

3.2.3 Monetary policy

In each country, a central bank sets the nominal interest rate $1 + R_t$ in response to current consumer-price inflation and past nominal interest rate. Monetary policy is subject to a (log) first-order autoregressive innovation MS_t:

$$(1+R_t) = (1+R_{t-1})^{\phi_i} \left(\frac{1}{\beta}\right)^{1-\phi_i} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\phi_i)\phi_\pi} MS_t \tag{3.58}$$

$$(1+R_t^*) = (1+R_{t-1}^*)^{\phi_i} \left(\frac{1}{\beta}\right)^{1-\phi_i} \left(\frac{P_t^*}{P_{t-1}^*}\right)^{(1-\phi_i)\phi_\pi} MS_t^*. \tag{3.59}$$

3.2.4 Market clearing

Market clearing in the goods markets implies

$$Y_t = C_{H,t} + C^*_{H,t} \tag{3.60}$$

$$Y^*_t = C_{F,t} + C^*_{F,t}. \tag{3.61}$$

Market clearing in the labor markets implies

$$\int_0^1 N_t(f)df = N_t \tag{3.62}$$

$$\int_0^1 N_t(f^*)df^* = N^*_t. \tag{3.63}$$

3.2.5 Equilibrium

Combining the labor market clearing condition (3.62) to the demand for intermediate inputs (3.37) and the production function (3.41) yields

$$A_t N_t = Y_t V_t, \tag{3.64}$$

where

$$V_t = \int_0^1 \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\xi} df \tag{3.65}$$

is a measure of the price dispersion.

Similarly for the Foreign country,

$$A_t^* N_t^* = Y_t^* V_t^* \tag{3.66}$$

$$V_t^* = \int_0^1 \left(\frac{P_{F,t}^*(f^*)}{P_{F,t}^*} \right)^{-\xi} df^*. \tag{3.67}$$

Using equations (3.45) and (3.53), it can be shown that

$$V_t = \alpha(1 + \pi_{H,t})^{\xi} + (1 - \alpha) \left[\frac{1 - \alpha(1 + \pi_{H,t})^{\xi-1}}{1 - \alpha} \right]^{\frac{\xi}{\xi-1}} \tag{3.68}$$

$$V_t^* = \alpha(1 + \pi_{F,t}^*)^{\xi} + (1 - \alpha) \left[\frac{1 - \alpha(1 + \pi_{F,t}^*)^{\xi-1}}{1 - \alpha} \right]^{\frac{\xi}{\xi-1}}. \tag{3.69}$$

An equilibrium for this model is then defined as an allocation for the variables N_t, N_t^*, C_t, C_t^*, W_t, W_t^*, P_t, P_t^*, R_t, R_t^*, $C_{H,t}$, $C_{H,t}^*$, $C_{F,t}$, $C_{F,t}^*$, $P_{H,t}$, $P_{F,t}^*$, Y_t, Y_t^*, V_t, V_t^*, $\pi_{H,t}$, $\pi_{F,t}^*$, MC_t, MC_t^*, NN_t, NN_t^*, DD_t, DD_t^*, $Q_{t,t+1}$, $Q_{t,t+1}^*$, TOT_t, and S_t that satisfies the equations (3.10), (3.11), (3.12), (3.20), (3.21), (3.22), (3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.32), (3.33), (3.42), (3.46), (3.47), (3.48), (3.49), (3.51), (3.54), (3.55), (3.56), (3.57), (3.58), (3.59), (3.60), (3.61), (3.64), (3.66), (3.68), and (3.69), given the exogenous processes for A_t, A_t^*, MS_t, and MS_t^* given by:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t} \tag{3.70}$$

$$\log A_t^* = \rho_A \log A_{t-1}^* + \varepsilon_{A,t}^* \tag{3.71}$$

$$\log MS_t = \rho_R \log MS_{t-1} + \varepsilon_{R,t} \tag{3.72}$$

$$\log MS_t^* = \rho_R \log MS_{t-1}^* + \varepsilon_{R,t}^* \tag{3.73}$$

$$\begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{A,t}^* \end{bmatrix} \sim N(0, \Sigma_A \Sigma_A') \tag{3.74}$$

$$\begin{bmatrix} \varepsilon_{R,t} \\ \varepsilon_{R,t}^* \end{bmatrix} \sim N(0, \Sigma_R \Sigma_R'). \tag{3.75}$$

3.2.6 UIP implications

Taking logs and expectation at time t of the risk sharing equation determining the behavior of the real exchange rate, I obtain[64]

$$E_t(\Delta rer_{t+1}) = E_t rq_{t,t+1}^* - E_t rq_{t,t+1}$$

Using the Euler equation, I can express the log of the real interest rate (denoted rr_t) as $rr_t = -log E_t RQ_{t,t+1}$. If the stochastic discount factor follows a log-normal distribution, then

$$rr_t = -E_t rq_{t,t+1} - \frac{1}{2} var_t(rq_{t,t+1})$$

[64]A lower case letter indicates the natural log of a variable: $x_t = \log X_t$.

where $var_t(x_{t+1})$ indicated the expected variance of x_{t+1} from the perspective of period t.[65] Then, the expected change in the exchange rate can be expressed as

$$E_t(\Delta rer_{t+1}) = rr_t - rr_t^* + \frac{1}{2}\left[var_t(rq_{t,t+1}) - var_t(rq_{t,t+1}^*)\right]. \tag{3.76}$$

In this model, a UIP coefficient different from one ($\beta^{UIP} \neq 1$) can only be obtained if the variance of the stochastic discount factors is allowed to be time-varying. Hence, the model must be solved using at least a third-order approximation.

From the definition of the stochastic discount factor, $var_t(rq_{t,t+1}) = \sigma^2 var_t(\Delta c_{t+1})$, so that

$$E_t(\Delta rer_{t+1}) = r_t - r_t^* + \frac{\sigma^2}{2}\left[var_t(\Delta c_{t+1}) - var_t(\Delta c_{t+1}^*)\right] \tag{3.77}$$

To obtain an estimated $\beta^{UIP} \neq 1$, it must be that

$$cov\left(r_t - r_t^*, \frac{\sigma^2}{2}\left[var_t(\Delta c_{t+1}) - var_t(\Delta c_{t+1}^*)\right]\right) \neq 0.$$

A necessary condition to obtain a negative UIP coefficient is that this covariance must be strictly negative.[66] In this baseline model, however, there is no mechanism that causes this relationship to be negative. How C_{t+1} is expected at time t to react to shocks coming at time $t+1$ is independent from the state of the economy at time t. Hence, $var_t(\Delta c_{t+1}) - var_t(\Delta c_{t+1}^*)$ does not systematically covary with $rr_t - rr_t^*$, and $\beta^{UIP} = 1$ even when the model is solved using a third-order approximation.

[65] The log-normality assumption is not imposed on the model when solving or simulating it, or when estimating UIP coefficients. It is only made in the text in order to describe the approximate effect of the first and second moments of the stochastic discounts factors on the exchange rate.

[66] If the covariance is strictly negative, then the estimated β^{UIP} is negatively biased due to the omitted variable.

3.2.7 Calibration

Table 3.1 contains the benchmark calibration, which is fairly standard. This is a quarterly model. I assume that α, the degree of price stickiness, is equal to 0.75, which implies that prices hold on average for four quarters. The household's discount factor, β, is set to 0.99, which implies an annualized rate of around 4 percent on the one-period nominal bond in steady state. The elasticity of substitution between the intermediate goods produced within the same country, ξ, is set to 7.66, which implies a markup over marginal cost in steady state of about 15 percent. The subsidy parameter τ is calibrated so that firms charge the perfect competition price ($(1-\tau)(1+\mu) = 1$). The elasticity of substitution in consumption between the Home and the Foreign bond is a parameter about which there is disagreement in the literature. It is common to set it to a value between 1 and 2 when using a two-country macroeconomic DSGE model. I choose a value of 1.5. The degree of home bias in consumption γ is set to 0.85, implying an import-to-consumption ratio of 15 percent in steady state.[67] The inverse of the Frisch labor supply elasticity, ϕ, is set to 1.5. The households' degree of risk aversion, and the inverse of their intertemporal elasticity of substitution, is set to 2. The monetary authority is assumed to smooth interest rate, $\phi_i = 0.8$, and to target inflation with a coefficient of $\phi_\pi = 2$, following the results in Smets & Wouters (2007). The persistence of the log of technology, ρ_A, is set to 0.95 and the coefficient for the monetary shock, ρ_R, is set to 0.12, both also following the results in Smets & Wouters (2007). The standard deviations of the technology and monetary shocks are 0.0075 and 0.0040, respectively. The size of the monetary shock is chosen to match the on-impact effect on the nominal interest rate found in Eichenbaum & Evans (1995). The relative magnitude of the technology shock (σ_A/σ_R) is chosen to match results found in Smets & Wouters (2007).

[67] According to data from the US International Trade Commission and from the St. Louis Fed FRED database, the annual average of the ratio of imports for consumption to total household consumption between 1973 and 2015 is 0.15.

3.2.8 Baseline model impulse response functions

Figure 3.1 contains the impulse responses of the main variables of the model to a positive technology shock in the Home country.[68] An increase in technology in Home increases the productivity of the workers in Home and, therefore, increases their real wage. The higher real wage has two effects on hours worked: a substitution effect, according to which leisure is now more expensive, prompting workers to work more, and an income effect according to which a higher real wage means that workers can consume the same by working less, prompting them to work less. According to my calibration (intertemporal elasticity of substitution of 0.5), the income effect dominates, and workers in Home reduce their hours worked. However, hours decrease less than technology improves, so that Home output increases. Because of the lower marginal cost, the price of the Home good decreases. A lower relative price makes consumers in both Home and Foreign to increase their consumption of the Home good.

Risk sharing ensures that changes in the marginal utility of consumption are equalized across countries. Therefore, since the technology shock made the Home households better off, the Foreign households need to be compensated. The Foreign currency appreciates against the Home currency, so that it is now cheaper for Foreign consumers to buy the Home good. This decreases the Foreign consumption price level, increasing the Foreign households' real wage. Since the income effect of the change in real wage dominates, hours and production decrease.

Because the relative price of the Home good is lower, both countries want to shift their consumption towards the Home good. Hence, Home consumers increase

[68]These impulse responses have been computed using a third-order approximation of the model. The impulse responses obtained using the first-order approximation of the model are not significantly different. Variables in all impulse responses are expressed in natural logarithmic differences from their steady state.

their consumption of the Home good and decrease their consumption of the Foreign good. However, because of Home bias, Foreign households actually increase their consumption of both goods (they have a higher purchasing power because real wage increases more than hours worked decrease), but that of the Home good increases more. Overall, both countries' consumption prices decrease, so that both monetary authorities choose to decrease the nominal interest rate.

Figure 3.2 contains the impulse responses of the main variables of the model to a contractionary monetary shock in the Home country when prices are sticky. In a flexible-price world, a monetary shock affects only nominal variables: a positive shock on the nominal interest rate induces agents to save more, and thus it decreases their demand for goods. Home producers are able to decrease their price enough so that real demand is not affected: production continues at the same level. Nominal wages follow the nominal prices, and the real wage is not affected. Foreign producers do not face a decrease in demand for Foreign consumers, so they do not decrease their price. However, risk sharing requires the nominal exchange rate to decrease (Home currency appreciates). Since Foreign goods are now relatively cheaper for Home consumers, but Home consumers want to consume less because of the upward pressure on the interest rate, the demand remains the same and Foreign production remains the same.

In a world where prices are sticky, however, Home intermediate producers are not able to decrease their price enough so that the price of the Home final good is "artifically" relatively high. Home consumers are unable to save, so that if they decrease consumption of the Home good, they have to increase their consumption of the Foreign good. This is achieved in equilibrium by an appreciation of the Home currency: the Foreign good is now relatively cheaper to Home consumers even though Foreign producers actually increase their price. Since the price of the Home good cannot be decreased enough while the Foreign currency depreciates, Foreign consumers' demand for the Home good decreases. As a result, Home production decreases and Foreign production increases: both types of consumers switch towards the Foreign good and away from the Home good. Since the consumer price index decreases in Home and increases in Foreign, monetary authorities in Home decrease the Home nominal interest rate and Foreign authorities increase their rate. Overall, the Home monetary authority's action is weaker than the initial shock, so that the nominal interest increases on impact.

UIP predictions A positive technology shock in Home causes negative inflation in both countries, but more so in the country directly hit by the shock. Therefore, the Home monetary authority decreases the nominal interest rate by more. In the same time, the Home currency depreciates in both nominal and real terms before re-appreciating towards its steady-state value. A contractionary monetary shock in Home causes the interest rates to increase more in Home than in Foreign. On impact, the Home currency appreciates in both nominal and real terms, and then depreciates back towards its steady-state value. These impulse responses make clear that when the Home interest rate is higher than the Foreign interest rate, the Home currency is expected to depreciate, and vice versa. There is no mechanism in this model that could explain why it is high interest rate currencies that tend to appreciate on average.

3.2.9 Baseline model simulation results

Since one of the models analyzed in this chapter needs to be approximated to a third order, simulation results for the baseline model are obtained from both a first-order and a third-order approximation. Columns 2 and 3 of Table 3.2 contain selected simulation moments for the first- and third-order approximations, respectively. The order of approximation has only a small impact on the moments obtained. The simulated moments are compared to data moments obtained for the United States and, where relevant, an aggregate of the non-US G7 countries: Japan, United Kingdom, Germany, Italy, France, and Canada.

Both for data and simulations, the nominal UIP coefficient is the coefficient β^{UIPnom} resulting from the following regression:

$$\log(S_{t+1}) - \log(S_t) = \alpha^{UIPnom} + \beta^{UIPnom}[\log(1 + R_t) - \log(1 + R_t^*)] + e_t$$

The real UIP coefficient is obtained similarly. Note that the expectation operator does not appear on the left-hand side of the UIP regression equation. The expectation error is therefore included in the error term e_t. Since this is a model of rational expectations, this expectation error is zero on average by construction.

The simulation results show that the baseline model reproduces some of the data moments rather well: consumption and interest rates are less volatile than GDP, and GDP, consumption, and interest rates are very autocorrelated. The volatility of the change in exchange rates is also well predicted by the model. The baseline model fails to predict the relatively high volatility of hours worked. The model predicts negative correlation of output across countries, while this correlation is high and positive in the data. In the model, shocks are uncorrelated across countries so that risk sharing implies the negative correlation of outputs. A positive correlation could be obtained by (realistically) making technology shocks positively correlated across countries. Risk sharing in the model also implies that consumption is more positively correlated across countries than GDP, whereas the data show the opposite. This is known as the international consumption correlation puzzle.[69] The model does the most poorly in the exchange rate moments. The autocorrelation of changes in exchange rated is positive in the data, but negative in the model: impulse responses above show that on impact of a shock, exchange rates react in one direction, and then immediately revert back towards the opposite direction. The data suggest that this is not the correct predicted behavior. Finally, UIP coefficients are very close to one in the model, but zero or negative in the data.

3.3 Model with External Habits

In this section I first summarize the model with external habits introduced in Verdelhan (2010) and describe how it solves the UIP puzzle. I then insert habit preferences into the baseline model introduced in the previous section and examine the implications for the UIP puzzle. I discuss the (extreme) conditions required for this version of the model to solve the UIP puzzle.

3.3.1 The habit model of Verdelhan (2010)

In a model inspired by Campbell & Cochrane (1999), Verdelhan (2010) suggests a solution to the UIP puzzle that relies on the introduction of external habits in households' preferences. The utility that households get from consumption depends on how the current consumption level relates to an external habit level. The habit level is called external because it depends on (country-specific) *aggregate* consumption.[70] For a given level of consumption, the household gets more utility if this consumption level is much greater than the current habit level than if it is only slightly greater. As a result, the household is more risk averse when its consumption level is close to

[69] See Backus et al. (1992).

[70] This implies that when maximizing utility, agents ignore the impact of their own consumption on the level of habit.

the habit level (but still above it) than when it is farther away from it.[71]

The presence of external habits in the utility function implies that risk aversion varies over time, which in turn implies that the risk premium demanded by households to invest in risky assets also varies over time.[72] If the risk premium demanded by Home households to invest in Foreign bonds is very high at a time where the risk-free rate on the Home bond is lower than the rate on the Foreign bond, then the Home households may decide not to take advantage of this apparent profit opportunity. Hence, it is possible to obtain a negative UIP coefficient: it can be that the Foreign interest rate is relatively high and the Foreign currency is simultaneously expected to appreciate. This requires risk-free rates to be procyclical: when Home is in relatively "bad" times and risk aversion is higher in Home, interest rates must be lower in Home.

Verdelhan (2010) shows that in his endowment-economy model, this condition requires precautionary saving motives to be stronger than intertemporal smoothing motives. In order to smooth consumption, households have to save in good times, putting downward pressure on the interest rate. However, if households are more risk averse in bad times, they also have more incentives to save in bad times, also putting downward pressure on the interest rate. The cyclicality of interest rates, and therefore the ability of the model to explain the UIP puzzle, depends on the relative strength of these two opposing forces.

3.3.2 The DSGE model with external habits

This version of the model combines the DSGE model described in Section 3.2 with the preference specification introduced by Campbell & Cochrane (1999) and

[71] By construction, the consumption level cannot fall below the habit level.

[72] "Risk-free" Foreign assets are risky from the perspective of Home households since the exchange rate at the time of the asset's maturity is uncertain at the time of purchase.

used by Verdelhan (2010) in the context of the UIP puzzle. The only difference from the baseline model is the presence of external habits in the households' preferences.

Households The representative household in the Home country maximizes the present discounted value of its lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t, H_t, N_t)] \tag{3.78}$$

where the intra-period utility function is now given by

$$U(C_t, H_t, N_t) = \frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}, \tag{3.79}$$

where H_t is the external habit level. The external habit level depends indirectly on domestic consumption. Let the surplus consumption ratio, X_t, be defined as

$$X_t \equiv \frac{C_t - H_t}{C_t}. \tag{3.80}$$

Following Verdelhan (2010) and Campbell & Cochrane (1999), the log of the surplus consumption ratio, x_t, is assumed to follow a first-order autoregressive process:

$$x_{t+1} = (1-\nu)\overline{x} + \nu x_t + \lambda(x_t)(\Delta c_{t+1} - g), \tag{3.81}$$

where g is the average consumption growth rate. Verdelhan (2010) assumes consumption growth to be exogenous, with mean g. In this chapter, I assume a setting in which consumption is endogenous to the model and where there are no permanent technological shocks so that $g = g^* = 0$. ν is a parameter between zero and one, $\overline{x}$ is the natural log of the steady state surplus consumption ratio $\overline{X}$, and $\lambda(x_t)$ is a function of x_t that determines how sensitive the surplus consumption ratio is to consumption growth.

Following Campbell & Cochrane (1999) and Verdelhan (2010), the sensitivity function $\lambda(x_t)$ is defined as

$$\lambda(x_t) = \begin{cases} \frac{1}{\overline{X}}\sqrt{1-2(x_t-\overline{x})}-1 & \text{when } x_t \leq x_{max}; \\ 0 & \text{elsewhere} \end{cases} \tag{3.82}$$

where x_{max} is an upper bound on x_t. Hence, the function $\lambda(x_t)$ is decreasing in x_t. In good times, x_t is much greater than $\bar{x}$ and consumption growth affects the surplus consumption ratio relatively weakly. In bad times, $\lambda(x_t)$ is high and the surplus consumption ratio is strongly affected by consumption growth.

The consumption-labor intratemporal optimality condition is now given by

$$N_t^{\phi} = (C_t - H_t)^{-\sigma}\frac{W_t}{P_t} \tag{3.83}$$

whereas the household's nominal and real stochastic discount factors are now given by

$$Q_{t,t+1} = \beta\left(\frac{C_{t+1}-H_{t+1}}{C_t-H_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}} = \beta\left(\frac{X_{t+1}}{X_t}\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \tag{3.84}$$

$$RQ_{t,t+1} = \beta\left(\frac{C_{t+1}-H_{t+1}}{C_t-H_t}\right)^{-\sigma} = \beta\left(\frac{X_{t+1}}{X_t}\frac{C_{t+1}}{C_t}\right)^{-\sigma} \tag{3.85}$$

The Foreign representative household faces a symmetric utility-maximization problem. where the surplus consumption ratio evolves according to

$$x^*_{t+1} = (1-\nu)\overline{x}^* + \nu x^*_t + \lambda(x^*_t)(\Delta c^*_{t+1} - g^*) \tag{3.86}$$

$$\lambda(x^*_t) = \begin{cases} \frac{1}{\overline{X}^*}\sqrt{1-2(x^*_t - \overline{x}^*)} - 1 & \text{when } x^*_t \leq x^*_{max}; \\ 0 & \text{elsewhere} \end{cases} \tag{3.87}$$

The first-order conditions imply

$$(N^*_t)^{\phi} = (C^*_t - H^*_t)^{-\sigma}\frac{W^*_t}{P^*_t} \tag{3.88}$$

$$Q^*_{t,t+1} = \beta\left(\frac{C^*_{t+1} - H^*_{t+1}}{C^*_t - H^*_t}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}} = \beta\left(\frac{X^*_{t+1}}{X^*_t}\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}} \tag{3.89}$$

$$RQ^*_{t,t+1} = \beta\left(\frac{C^*_{t+1} - H^*_{t+1}}{C^*_t - H^*_t}\right)^{-\sigma} = \beta\left(\frac{X^*_{t+1}}{X^*_t}\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}. \tag{3.90}$$

Risk sharing In this version of the model, risk sharing still implies that stochastic discount factors are equalized across countries. The only difference is the definition of these factors, which now includes terms for the surplus consumption ratios.

Firms, monetary policy, and market clearing The rest of the model is identical to the baseline model. The stochastic discount factor used by intermediate firms to discount expected future profits comes from the household's optimization problem, and therefore involves the surplus consumption ratio.

Equilibrium An equilibrium for this model is defined as an allocation for the variables N_t, N^*_t, C_t, C^*_t, X_t, X^*_t, $\lambda(x_t)$, $\lambda(x^*_t)$, W_t, W^*_t, P_t, P^*_t, R_t, R^*_t, $C_{H,t}$, $C^*_{H,t}$, $C_{F,t}$, $C^*_{F,t}$, $P_{H,t}$, $P^*_{F,t}$, Y_t, Y^*_t, V_t, V^*_t, $\pi_{H,t}$, $\pi^*_{F,t}$, MC_t, MC^*_t, NN_t, NN^*_t, DD_t, DD^*_t, $Q_{t,t+1}$, $Q^*_{t,t+1}$, TOT_t, and S_t, that satisfies the equations (3.81), (3.82), (3.83), (3.84), (3.86), (3.87), (3.88), (3.89), and the equations identical to the baseline model (3.12), (3.21), (3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.32), (3.33), (3.42), (3.46), (3.47), (3.48), (3.49), (3.51), (3.54), (3.55), (3.56), (3.57), (3.60), (3.61), (3.58), (3.59), (3.64), (3.66), (3.68), and (3.69), given the exogenous processes for A_t, A_t^*, MS_t, and MS_t^* given by (3.70), (3.71), (3.72), (3.73), (3.74), and (3.75).

3.3.3 UIP implications

As in the baseline model, the expected change in the real exchange rate satisfies[73]

$$E_t(\Delta rer_{t+1}) = rr_t - rr_t^* + \frac{1}{2}\left[var_t(rq_{t,t+1}) - var_t(rq_{t,t+1}^*)\right].$$

In this model the stochastic discount factor includes the change in the surplus consumption ratio in addition to the change in consumption. Using the law of motion for the log of the surplus consumption ratio x_t (equation (3.81)) I obtain

$$var_t(rq_{t,t+1}) = \sigma^2[1 + \lambda(x_t)]^2 var_t(\Delta c_{t+1}),$$

so that the expected change in the exchange rate can be expressed as

$$E_t(\Delta rer_{t+1}) = rr_t - rr_t^* + \frac{\sigma^2}{2}\left([1 + \lambda(x_t)]^2 var_t(\Delta c_{t+1}) - [1 + \lambda(x_t^*)]^2 var_t(\Delta c_{t+1}^*)\right). \tag{3.91}$$

Hence in this model, even if the conditional variance of the change in consumption is approximately constant, the omitted variable bias present in the typical UIP regressions is non-zero. The effect of the conditional variance terms on the exchange rate depends on the current state of the economy, that is, whether each country has a high or low surplus consumption ratio. As a result, the sign of the correlation between relative interest rates $(rr_t - rr_t^*)$ and relative states $(x_t - x_t^*)$ governs the sign of the bias.

[73]The UIP implications of this model are analyzed in real terms for simplicity. The nominal UIP implications are analogous.

In order to understand how a UIP coefficient smaller than one, or even negative, can be obtained in this model, it is useful to study the behavior of interest rates. Using again the log-normality of stochastic discount factors assumption to expose the intuition more clearly, the real interest rate satisfies

$$rr_t = -\log(\beta) - \sigma(1-\nu)(x_t - \bar{x}) + \sigma[1+\lambda(x_t)]E_t(\Delta c_{t+1}) - \frac{\sigma^2}{2}[1+\lambda(x_t)]^2 var_t(\Delta c_{t+1}). \tag{3.92}$$

The term $\sigma(1-\nu)(x_t - \bar{x})$ captures the households' intertemporal smoothing motive. Because the surplus consumption ratio is mean-reverting, a low ratio means that the state of the economy will improve and agents have incentives to borrow now, putting upward pressure on the interest rate. The term $\sigma[1+\lambda(x_t)]E_t(\Delta c_{t+1})$ also captures the households' intertemporal smoothing motive. For a given value of x_t, an expected increase in consumption makes the households want to borrow, which again increases the interest rate. Note that in a model where real variables are mean-reverting, an expected increase in consumption is associated with low current levels of consumption and surplus consumption ratio. This magnifies the effects of the expected growth in consumption on the interest rate since $\lambda(x_t)$ is decreasing in x_t.

The last term, $-\frac{\sigma^2}{2}[1+\lambda(x_t)]^2 var_t(\Delta c_{t+1})$, instead captures the households' precautionary saving motives. Assume that $var_t(\Delta c_{t+1})$ is constant for simplicity. When the economy is in a bad state (low x_t), $\lambda(x_t)$ is high, households are more sensitive to risk, and hence a given conditional variance leads to a lower interest rate. This is because the presence of habits makes households dislike negative shocks more when they are in a bad state. Hence, even though they are aware of the mean-reverting process driving the economy on average, they are concerned about the fact that bad shocks happen also in bad times. As a result, they are tempted to save and alleviate the effects of potential future bad shocks even when the economy is in a bad state, putting downward pressure on the interest rate.[74]

The two intertemporal smoothing terms contribute to make interest rates countercyclical ($cov(rr_t, x_t) < 0$), whereas the precautionary saving term contributes to make them procyclical. Verdelhan shows that in the context of his endowment economy model, procyclical interest rates are sufficient to obtain a negative UIP coefficient. By assuming a random-walk process for consumption, Verdelhan is in effect muting most of the intertemporal smoothing motive, since $E_t(\Delta c_{t+1}) = 0$.[75] Given a constant $var_t(\Delta c_{t+1}) = \sigma_c^2$, the simple parameter condition $-\sigma(1-\nu) + \frac{\sigma^2 \sigma_c^2}{\overline{X}^2} > 0$ is sufficient to obtain procyclical interest rates and a negative UIP coefficient. This condition can be achieved by assuming a parameter ν very close to one (if the surplus consumption ratio is very persistent, it is less mean reverting, dampening further intertemporal smoothing motives) and high risk aversion or variance of consumption growth.

In a model with mean-reversion in the real variables, obtaining procyclical interest rates is a lot more challenging. There is no simple parameter condition that ensures the procyclicality. To uncover the relative strength of the intertemporal smoothing and precautionary saving motives, the model must be simulated.[76]

Asset pricing in a production economy with external habits Chen (2017) investigates whether empirical asset-pricing moments such as the mean and variance

[74]In a model with external habits and endogenous production, Chen (2017) obtains a volatility of consumption that is higher in bad times. Because of precautionary saving behavior, consumption is more affected by shocks when the economy is in a bad state. In the model discussed here, households cannot save across periods, but they can smooth the effect of shocks by trading with the other country. The time-varying behavior of consumption growth must be examined from model simulations. Preliminary results suggest no such significant systematic behavior.

[75]Verdelhan in fact assumes a process of random walk with a drift for consumption, so that $E_t(\Delta c_{t+1})$ is equal to a constant, which then enters the constant term in the equation for the interest rate.

[76]Making the sensitivity function $\lambda(x_t)$ even more strongly decreasing in x_t does not help making interest rates procyclical. While it strengthens the precautionary saving motive of household, it also strengthens their intertemporal smoothing motive.

of the equity premium can be predicted by a general-equilbrium model featuring the type of external habits introduced by Campbell & Cochrane (1999). Chen's model includes production with capital and quadratic capital adjustment costs. His results show that the model can generate strong precautionary saving motives even when $\lambda(x_t)$ is assumed to be a constant. This comes from the conditional variance of consumption growth $var_t(\Delta c_{t+1})$ being countercyclical. In bad times, the precautionary saving motive is stronger, reducing the intertemporal smoothing of consumption. This makes consumption more volatile in bad times and therefore increases the conditional variance of consumption growth. This mechanism relies on households being able to save across periods, which is achieved from the presence of endogenous physical capital in the economy. As predicted by Campbell and Cochrane, the empirically relevant asset pricing properties of their model remain when endogenous production is added, as long as agents are able to save across time periods.

Solving the model The model must be approximated to at least a third order in order to capture time-varying risk premia and potential UIP properties. The model is solved and simulated using Dynare and the pruning algorithm provided by Andreasen et al. (2017).

3.3.4 Habit model simulation results and discussion

As a first step, I reproduce the results in Verdelhan (2010) by calibrating the model to obtain a process for (log of) consumption that is completely exogenous and (almost) a random walk. This involves making the following changes: 1- prevent trade in goods across countries by making households extremely home-biased ($\gamma = 0.99999$); 2- fix labor supply to its steady state value ($N_t = \bar{N}$, $N_t^* = \bar{N}^*$); 3- make prices completely flexible ($\alpha = 0$); and 4- make the process for (log of) technology (almost) a random walk ($\rho_A = 0.99999$). The first change insures that each country consumes exactly what it produces. The second change insures that the output of each intermediate good producer varies only with demand (from final good producers) and technology. The third change insures that all intermediate good producers are identical. This is required to get *aggregate* production to follow the technology process (otherwise, labor can be reallocated across intermediate good producers even when aggregate labor remains fixed). It also eliminates the real effects of monetary shocks. The fourth change insures that technology, output, and consumption follow a random walk process. The remaining parameters are set as in the baseline model described in Section 3.2. The two additional parameters, the persistence of the surplus consumption ratio process, ν, and the steady-state value of the surplus consumption ratio, $\overline{X}$, are calibrated to 0.99 and 0.07, respectively, following Verdelhan (2010).

Key simulated moments are presented on the first line of Table 3.3.[77] The surplus consumption ratio is positively correlated with the real interest rate and the UIP coefficient is negative.[78] This confirms the results obtained by Verdelhan (2010): as long as $\sigma(\nu-1)+\frac{\sigma^2\sigma_A^2}{\overline{X}^2}>0$, the precautionary saving motive dominates the intertemporal smoothing motive, and the model predicts a negative UIP coefficient.

In a second step, I relax one at the time the four changes described above in order to see which assumptions are necessary to obtain a UIP coefficient that is smaller than one. Simulated moments are presented on lines 2-5 of Table 3.3. The first result to note is that the random-walk process of technology (and hence of consumption) and fully flexible prices are necessary to obtain a negative UIP coefficient (see lines

[77]Because agents' responses to shocks depend on the current state of the economy, impulse responses are not a useful tool to analyze the predictions of this model.

[78]In Verdelhan's model, the consumption level can be eliminated completely from the expression for the interest rate, so that the cyclicality of interest rates is determined based on the surplus consumption ratio. In the general-equilibrium version of the model, the consumption level and the surplus consumption ratio both appear in the expression. The cyclicality of interest rates is therefore not as clearly defined and both the correlation of interest rates with the surplus consumption ratio and with the consumption level matter for the UIP coefficient.

2 and 3 of Table 3.3). When the consumption process is a random walk, the expected change in consumption is not affected by a given shock realization. Hence, the intertemporal smoothing behavior is weak. The precautionary saving motive is however affected. A good shock makes the household less risk averse by increasing the surplus consumption ratio. As described above, this decreases the incentives to save and increases the interest rate. When the process is mean-reverting, the expected change in consumption is time-varying, and the intertemporal smoothing motive is stronger. Interest rates become countercyclical, and the UIP coefficient is positive.

The same logic applies to the price stickiness assumption. When prices are fully flexible, the economy adjusts to shocks instantaneously.[79] When prices are sticky, real variables such as consumption adjust slowly.[80] Hence, any shock to the technology process affects not only current consumption but also the expected change in consumption, thereby strengthening intertemporal smoothing and making interest rate countercyclical.

When countries are allowed to trade (the home-bias parameter γ is reduced to 0.85; line 4 of Table 3.3), the resulting UIP coefficient is again negative. Opening trade has two main effects: households can reduce their exposure to their own countries' technological shocks by consuming the other country's goods, but this also makes them more exposed to technological shocks in the other country. Overall, however, the consumption process remains a random walk, so that intertemporal smoothing is still weak.[81]

[79] For this simulation, monetary shocks have been turned off ($\sigma_R^2 = 0$) to show the effect of price stickiness only.

[80] Under rational expectations, households are aware that prices take time to adjust.

[81] The magnitudes of the regression coefficients obtained with different calibrations should not be compared since the changes in assumptions cause important changes in the variance of the variables.

When households are able to vary their supply of labor, the level of production (and consumption) is less directly connected to the level of technology. However, the economy still adjusts to shocks instantaneously. Hence, as long as technology follows a random walk process, so does consumption. What differs from Verdelhan's model is the magnitude of the response of consumption to a technology shock. If, as in the benchmark calibration, the Frisch elasticity of labor supply is such that the wealth effect dominates the substitution effect, households work less in response to a good shock. Hence the response of production is dampened, lowering the volatility of consumption and weakening precautionary saving. Recall the condition required to obtain a negative UIP coefficient: $-\sigma(1-\nu)+\frac{\sigma^2\sigma_c^2}{\overline{X}^2}>0$. When labor is fixed, $\sigma_c^2=\sigma_A^2$. When labor is flexible and the wealth effect dominates, $\sigma_c^2<\sigma_A^2$. Hence the volatility of technology shocks must be increased to be able to obtain a negative UIP coefficient. For example, if home bias is (almost) complete, prices are flexible, technology is (almost) a random walk and the rest of the parameters are calibrated as in the benchmark case (including $\phi=1.5$), obtaining $\sigma(\nu-1)+\frac{\sigma^2\sigma_c^2}{\overline{X}^2}>0$ requires $\sigma_A>0.0592$ since simulations produce a ratio $\frac{\sigma_c}{\sigma_A}\approx 0.0836$.

Unsurprisingly, when the model is calibrated similarly to the baseline model, with auto-regressive technology, sticky prices, international trade, and flexible labor, it is unable to produce a negative UIP coefficient (line 6 of Table 3.3). Table 3.4 contains more simulated moments for the model with external habits under Verdelhan's (column 3) and the benchmark (column 4) calibration. Results show that Verdelhan's calibration does improve the UIP prediction, but does worse than the baseline model in matching other macroeconomic moments, especially the auto-correlation of real variables (due to the presence of the random walk) and the volatility of the real exchange rate. The habit model calibrated under the benchmark calibration, on the other hand, does not improve on the baseline model's UIP predictions but does improve other moments, especially those related to the volatility and auto-correlation of interest rates. It however does worse on the correlation across countries of GDP and consumption, worsening the international consumption correlation puzzle.

3.4 Model with Distorted Beliefs

In this section I first summarize the model with distorted beliefs introduced by Gourinchas & Tornell (2004) and describe how it solves the UIP puzzle. I then insert the distorted beliefs into the baseline model introduced in Section 3.2. I discuss the conditions required for this version of the model to solve the UIP puzzle.

3.4.1 The distorted beliefs model of Gourinchas & Tornell (2004)

Gourinchas & Tornell (2004) suggest a solution to the UIP puzzle, and to other exchange rate puzzles, using biased expectations. In their model, agents misinterpret shocks to the interest rate differentials between the two countries: they believe that the shocks are more transitory (i.e., less persistent) than they actually are. They do not learn about their bias, so that it persists over time. Note that the agents always perfectly observe the interest rate differential. What is biased is what they *expect* the differential to be in the future. Hence, this model yields expectation errors that are not zero on average.

Since the agents underestimate the persistence of interest rate shocks, the response of the exchange rate to these shocks is smaller than under rational expectations. As future periods unfold and agents realize their mistake, the exchange rate's behavior reflects this learning and moves towards the path it would have taken if agents had had correct expectations.

In the authors' setting, the interest rate differential between the two countries follows a first-order autoregressive process and is directly subject to shocks. The agents believe that for any observed shock, there is a non-zero probability that the shock is transitory. However, in reality, all shocks are persistent and the probability of a shock being transitory is zero. Expectations are formed by agents using the Kalman filter. Their expectations of the future interest rate differential is a weighted average of their previous expectation of the differential, and the actual current differential. If agents expect the interest rate differential to go back towards steady state faster than it actually does, the exchange rate reacts less on impact of a given shock.

Following a (true) positive shock to the interest rate differential $r_t - r_t^*$, there are two different effects that drive the exchange rate. The usual effect is that following a relative increase in the Home interest rate that is expected to be persistent, the no-arbitrage condition requires the Home currency to be expected to depreciate: since markets are complete, there should not be any expected profit from investing in Home rather than in Foreign. Therefore, on impact of the shock, the exchange rate decreases (the Home currency appreciates), and then increases back towards steady state as the (true, persistent) shock goes away. This is the "rational" behavior of the exchange rate and it predicts a UIP coefficient equal to exactly one.

The second effect is the "irrational" behavior of the exchange rate: since agents do not believe that the high interest rate differential will last, they do not believe that the profit opportunity will last either. Therefore, the subjective no-arbitrage condition requires the exchange rate to decrease less than in the rational case. In the next period, the agents observe that the interest rate differential is much greater than they expected. Therefore, the exchange rate does not increase back towards steady state as fast as agents expected. Under some parameterizations, the exchange rate can continue to decrease, so that we can have a country with a high interest rate, whose currency is appreciating. Gourinchas and Tornell show that this can lead to a negative UIP coefficient. Under any parameterization, as long as agents give a non-zero probability to the shock being transitory, the UIP coefficient is smaller than one due to the initial response of the exchange rate being too small.

3.4.2 The DSGE model with distorted beliefs

This version of the model combines the DSGE baseline model described in Section 3.2 with the distorted beliefs and learning behavior described in Gourinchas & Tornell (2004). The model of Gourinchas & Tornell (2004) is in reduced form, and the shock about which agents are progressively learning hits the interest-rate differential directly. I adapt this feature by making beliefs about the shocks to monetary policy rules distorted, so that in my model, there are two elements about which agents are uncertain, one in each country.

The consumption and production sides of the model are identical to the baseline model. However, since expectations are biased due to the distorted beliefs, the forward-looking behavior of consumers and firms are impacted. The symbol E_t^s denotes the subjective, or biased, expectations of agents whereas E_t continues to denote rational, unbiased, expectations.

Monetary policy The monetary policy rules are still given by

$$(1+R_t) = (1+R_{t-1})^{\phi_i} \left(\frac{1}{\beta}\right)^{1-\phi_i} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\phi_i)\phi_\pi} MS_t \tag{3.93}$$

$$(1+R_t^*) = (1+R_{t-1}^*)^{\phi_i} \left(\frac{1}{\beta}\right)^{1-\phi_i} \left(\frac{P_t^*}{P_{t-1}^*}\right)^{(1-\phi_i)\phi_\pi} MS_t^* \tag{3.94}$$

where

$$\log MS_t = \rho_R \log MS_{t-1} + \varepsilon_{R,t} \tag{3.95}$$

$$\log MS_t^* = \rho_R \log MS_{t-1}^* + \varepsilon_{R,t}^* \tag{3.96}$$

and where $\varepsilon_{R,t}$, $\varepsilon_{R,t}^* \sim \text{iid}N(0, \sigma_R^2)$.

However, agents have beliefs about the monetary policy shocks that differ from the reality. While all monetary policy shocks MS_t and MS_t^* are persistent, agents believe that some shocks are transitory. Their beliefs about $\log MS_t$ and $\log MS_t^*$ are represented by

$$\log MS_t = z_t + \upsilon_t, \qquad z_t = \rho_R z_{t-1} + \varepsilon_{R,t}, \qquad \upsilon_t = \varepsilon_{\upsilon,t} \tag{3.97}$$

$$\log MS_t^* = z_t^* + \upsilon_t^*, \qquad z_t^* = \rho_R z_{t-1}^* + \varepsilon_{R,t}^*, \qquad \upsilon_t^* = \varepsilon_{\upsilon,t}^* \tag{3.98}$$

where $\varepsilon_{\upsilon,t}, \varepsilon_{\upsilon,t}^* \sim iidN(0, \sigma_\upsilon^{s2})$. Therefore, according to the agents' beliefs, a given change in MS_t could be caused either by a transitory innovation $\varepsilon_{\upsilon,t}$ or by a persistent innovation $\varepsilon_{R,t}$. The monetary shocks MS_t and MS_t^* are perfectly observed by all agents, but z_t, z_t^*, υ_t, υ_t^*, $\varepsilon_{R,t}$, and $\varepsilon_{R,t}^*$ are unobservable. In reality, the true $\sigma_\upsilon^2 = 0$.[82]

Learning Agents learn about the true nature of the shocks using the Kalman filter. Each period, they extract information from the realization of the monetary shocks MS_t and MS_t^*. They form their expectations about the future realizations of MS_{t+1}

[82] Moran & Nono (2018) also insert incomplete information about monetary policy into a two-country DSGE model and examine the implications for the UIP puzzle. Their monetary policy rule however involves a time-varying inflation target. Agents cannot immediately distinguish between transitory policy shocks and persistent shocks to the inflation target.

and MS^*_{t+1} by giving a weight to the current realizations and a weight to what they previously expected the current realizations to be. Agents' beliefs therefore evolve according to

$$E_t^s \log MS_{t+1} = (1-\kappa)\rho_R E_{t-1}^s \log MS_t + \kappa \rho_R \log MS_t \tag{3.99}$$

$$E_t^s \log MS^*_{t+1} = (1-\kappa)\rho_R E_{t-1}^s \log MS^*_t + \kappa \rho_R \log MS^*_t \tag{3.100}$$

where κ is the learning parameter given by

$$\kappa = \frac{1+\Delta-\eta^s(1-\rho_R^2)}{1+\Delta+\eta^s(1-\rho_R^2)}$$

where $\Delta = [\eta^s(1-\rho_R^2)+1]^2 + 4\eta^s\rho_R^2$ and where $\eta^s = \frac{\sigma_v^{s2}}{\sigma_R^2}$. The learning parameter κ is derived from the Kalman filter.[83] It is easy to see that κ is decreasing in η^s and increasing in ρ_R: when beliefs are more distorted, agents put less weight on their current observation, and when the true shock is more persistent, agents put more weight on their current observation since the current observation contains more information about the future observation.

Under rational expectations, agents would instead expect

$$E_t \log MS_{t+1} = \rho_R \log MS_t$$

$$E_t \log MS^*_{t+1} = \rho_R \log MS^*_t.$$

The distorted beliefs imply that if agents give positive probability to the realization of transitory monetary shocks, then $E_t^s \log MS_{t+1} < E_t \log MS_{t+1}$ since $\kappa < 1$. Agents expect the disturbance to be less persistent than it actually is.

[83]Details about the Kalman filter are included below.

Equilibrium An equilibrium for this version of the model is defined as an allocation for the variables N_t, N_t^*, C_t, C_t^*, W_t, W_t^*, P_t, P_t^*, R_t, R_t^*, $C_{H,t}$, $C_{H,t}^*$, $C_{F,t}$, $C_{F,t}^*$, $P_{H,t}$, $P_{F,t}^*$, Y_t, Y_t^*, V_t, V_t^*, $\pi_{H,t}$, $\pi_{F,t}^*$, MC_t, MC_t^*, NN_t, NN_t^*, DD_t, DD_t^*, $Q_{t,t+1}$, $Q_{t,t+1}^*$, TOT_t, and S_t that satisfies the equations (3.10), (3.11), (3.12), (3.20), (3.21), (3.22),(3.24), (3.25), (3.26), (3.27), (3.28), (3.29), (3.32), (3.33), (3.42), (3.46), (3.47), (3.48), (3.49), (3.51), (3.54), (3.55), (3.56), (3.57), (3.58), (3.59), (3.60), (3.61), (3.64), (3.66), (3.68), and (3.69), given the exogenous processes for A_t, A_t^*, MS_t, and MS_t^* given by (3.70), (3.71), (3.72), (3.73), (3.74), and (3.75) and given that agents believe that the monetary shocks processes are given by (3.97) and (3.98).

UIP implications In Gourinchas & Tornell (2004), the interest rate differential between the two countries is exogenous, so that the UIP predictions of their model can be analyzed directly. The authors are able to derive the asymptotic value of the UIP coefficient. In this general-equilibrium version of the model, such a simple condition cannot be derived and the UIP coefficient has to be estimated using model simulations. However, it is still possible to show how and why the UIP coefficient is expected to be different from one.

Recall that in the baseline model, a positive (contractionary) monetary shock in Home makes the Home interest rate increase more than the Foreign rate. In the same time, since this is a bad shock for Home, the Home currency appreciates (the exchange rate decreases), in accordance with the risk sharing condition.

When beliefs are distorted, agents expect the monetary shock to be less persistent than it actually is. Hence, the interest rate differential is smaller and the Home currency appreciates less. In the next period, as agents observe that the Home monetary shock MS_t is higher than expected, they start realizing their error and, if beliefs are distorted enough, the Home currency keeps appreciating. It is therefore possible to observe a relatively high Home real interest rate coexisting with a Home currency that is appreciating.

Solving the model The model with distorted beliefs is log-linearized since the mechanism introduced to solve the UIP puzzle, biased expectations, affects the variables at the first order.

In order to predict accurately the agents' behavior, their beliefs must be used to solve the model. That is, the model that is solved is the model as it exists in the agents' mind, where $\log MS_t$ and $\log MS_t^*$ are replaced with $z_t + v_t$ and $z_t^* + v_t^*$, respectively. This version is log-linearized and solved using Dynare. Let Y_t be a vector containing all control variables, X_t be a vector containing all state variables and ε_t be a vector containing all real and imagined shocks, so that $\varepsilon_t = [\varepsilon_{A,t}, \varepsilon_{A,t}^*, \varepsilon_{R,t}, \varepsilon_{R,t}^*, \varepsilon_{v,t}, \varepsilon_{v,t}^*]'$. Dynare solves the model and provides the matrices A, B, C, and D in the system:

$$\begin{aligned} Y_t &= AX_{t-1} + B\varepsilon_t \\ X_t &= CX_{t-1} + D\varepsilon_t \end{aligned} \tag{3.101}$$

Note that X_t contains elements that are not observed, z_t and z_t^*, and that ε_t also contains elements that are not observed. Because the model is approximated to the first order, certainty equivalence applies: the agents do not take into account that their beliefs may not be true, even if they know that their beliefs are biased. When forming their beliefs about shocks, agents are aware of their bias, as can be seen from the definition of the learning parameter κ. However, in response to these shocks, they behave according to their expectations only, and are not affected by the degree of bias.

The model is simulated using the agents' beliefs rather than the actual value of the variables. The values of the unobserved variables and shocks are replaced by the values the agents believe they have. The model is simulated as follows:

$$Y_t = AX_{t-1|t} + B\varepsilon_{t|t}$$
$$X_t = CX_{t-1|t} + D\varepsilon_{t|t}. \tag{3.102}$$

By certainty equivalence, the matrices A, B, C, and D are identical in the systems (3.101) and (3.102). The "believed" values of the Home variables and shocks are obtained from a simple version of the Kalman filter and smoother containing only three variables: MS_t, z_t, and υ_t. Since MS_t is perfectly observed and completely absorbs the behavior of the two believed shocks z_t and υ_t, agents learn about z_t and υ_t by observing only MS_t. They do not get any additional information about these two shocks than the information contained in MS_t. This allows me to apply the Kalman filter to these three variables only and obtain the series $\{z_{t|t}\}_{t=0}^T$ and $\{\upsilon_{t|t}\}_{t=0}^T = \{\varepsilon_{\upsilon,t|t}\}_{t=0}^T$. The agents learn about Foreign monetary shocks in an identical way, using a Kalman filter containing only MS_t^*, z_t^* and υ_t^*. Agents do not have more information about their own country's monetary shocks than about the other country's.

The values of $z_{t-1|t}$ and $\varepsilon_{R,t|t}$ are then obtained using the Kalman smoother. Since $z_t = \rho_R z_{t-1} + \varepsilon_{R,t}$, consistency of beliefs implies

$$z_{t|t} = \rho_R z_{t-1|t} + \varepsilon_{R,t|t}.$$

The Kalman smoother yields the series $\{z_{t-1|t}\}_{t=0}^T$. The series $\{\varepsilon_{R,t|t}\}_{t=0}^T$ is obtained from the residual.

The Kalman filter equations are (following the notation in Hamilton (1994)):

$$
\begin{aligned}
P_{t|t-1} &= FP_{t-1|t-1}F' + Q \\
P_{t|t} &= P_{t|t-1} - P_{t|t-1}H\Omega_{t|t-1}^{-1}H'P_{t|t-1} \\
x_{t|t-1} &= Fx_{t-1|t-1} \\
x_{t|t} &= x_{t|t-1} + P_{t|t-1}H\Omega_{t|t-1}^{-1}(X_t - X_{t|t-1}) \\
X_{t|t-1} &= H'x_{t|t-1} \\
\Omega_{t|t-1} &= H'P_{t|t-1}H
\end{aligned}
$$

where $X_t = MS_t$, $x_t = \begin{bmatrix} z_t \\ v_t \end{bmatrix}$, $H' = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $F = \begin{bmatrix} \rho_R & 0 \\ 0 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} \sigma_R^2 & 0 \\ 0 & \sigma_v^{s2} \end{bmatrix}$, $P_{1|0}$ is such that $P_{1|0} = FP_{1|0}F' + Q$, and $x_{1|0} = 0$.

The Kalman smoother equation is

$$x_{t-1|t} = x_{t-1|t-1} + P_{t-1|t-1}F'P_{t|t-1}^{-1}(x_{t|t} - x_{t|t-1}).$$

Following Gourinchas & Tornell (2004), I assume that the learning process has been going for long enough that the values of $P_{t|t}$ and $P_{t|t-1}$ have converged. This yields the learning parameter κ introduced earlier.

3.4.3 Distorted beliefs model simulation results and discussion

The benchmark calibration of this model is the exact same as the calibration described in Section 3.2.7. The only additional parameter is σ_v^{s2}, the standard deviation of the (completely) transitory shock to which agents wrongly believe monetary policy could be subject. To obtain the results presented and analyzed below, I use several values for σ_v^{2s} and discuss how this parameter impacts the model's UIP predictions.[84] The benchmark value is to set $\sigma_v^{2s} = \sigma_R$ so that both types of monetary shocks are drawn from the same distribution.

Figure 3.3 contains the impulse responses of selected variables to a contractionary monetary shock in Home.[85,86] It shows that under the benchmark calibration, real variables respond to the monetary shock almost exactly as in the baseline model. The response of nominal variables differs slightly, but patterns are identical. Column 3 of Table 3.5 contains the simulation results for this benchmark calibration. The UIP coefficients are below but close to one. This is not very surprising. What distinguishes the two types of monetary shocks is that one is persistent and the other one is completely transitory. However, under the benchmark calibration, the persistent shock is not very persistent: the autocorrelation, ρ_R, is only 0.12. Since both shocks resemble each other, the fact that agents take time to learn which type of shock is actually happening does not change their behavior a lot: they react similarly regardless of the type of the shock. Therefore, this version of the model resembles the baseline model and UIP coefficients are close to one.

Under these circumstances, modifying only the size of the transitory shock, σ_v^s, does not impact significantly the UIP predictions of the model. Increasing the persistence of the (true) persistent shock, however, has major impacts on the results. When $\rho_R = 0.7$ and $\sigma_v^s = \sigma_R$, the real UIP coefficient is down to 0.50, which is significantly below one. Increasing the size of the transitory shock to $\sigma_v^s = 7\sigma_R$ reduces it further

[84] Gourinchas & Tornell (2004) do not suggest a reasonable value for this parameter.

[85] Impulse responses for the technology shocks are not presented for this version of the model since there is perfect information about this type of shocks and agents react in the exact same way as they do in the baseline model.

[86] Results in this section, especially for nominal variables, are preliminary.

to 0.24. When the persistent shock is made more persistent, the agents' response to it is more different from their response to the transitory shock. When the transitory shock is larger than the persistent shock, agents learn more slowly about the type of shock that occurred. It is harder for them to learn whether a persistent shock or repeated transitory shocks have occurred. Hence, the larger σ_v^s, the more "plausible" it is for them that another transitory shock has happened and the more slowly they learn the true nature of the shock.[87]

The simulation results and impulse responses obtained with $\rho_R = 0.7$ and $\sigma_v^s = 7\sigma_R$ are presented Column 5 of Table 3.5 and in Figure 3.4, respectively, and show the learning process of agents. On impact of the shock, the nominal interest rate in the Home country increases more than under perfect information.[88] Since households perceive the shock to be less contractionary (because less persistent) than it actually is, consumption in Home decreases less and prices also decrease less. Hence, the monetary authority has less incentive to decrease the policy rate. In the same time, since the shock is first perceived to be less contractionary than it actually is, risk sharing implies that the Home currency does not appreciate as much as in the baseline model (RER_t decreases less). In the following period, agents learn that the shock is more persistent than expected, so that the Home currency *continues to appreciate*. Hence, on impact, the Home interest rate is higher than the Foreign interest rate, yet the Home currency will still appreciate in the next period. This is sufficient to make the real UIP coefficient significantly below one, at 0.24.

The nominal UIP coefficient, at -1.65, is very negative under this calibration. Because agents underestimate the persistence of the monetary shocks, prices do not

[87]This is true to some extent. For very large values of $\sigma_{v,t}^s$, agents are quickly able to learn the true nature of a shock looking only at its size.

[88]The nominal interest rate actually decreases under perfect information, that is, the action of the monetary authority fully overturns the initial shock.

respond as much and the Home nominal interest rate's response is positive, even though it is negative under perfect information. As a result, the interest rate differential is positive and remains so for several periods, while the Home nominal exchange rate appreciates on impact and keeps appreciating to a new steady state.

Despite modifying significantly the behavior of variables in response to monetary shocks, the inclusion of distorted beliefs generates moments that are very similar to those generated by the baseline model. It even replicates better the high autocorrelation of interest rates.

3.5 Conclusion

Several mechanisms have been suggested by the existing literature to explain the UIP puzzle. To simplify the analysis, they are typically set in endowment-economy settings. In this chapter, I investigate how these mechanisms perform when introduced into an otherwise standard two-country DSGE model.

The first mechanism relies on a time-varying risk premium and has been suggested by Verdelhan (2010). The presence of external habits in households' preferences makes them more risk averse, and hence more prone to saving, in bad times. Verdelhan shows that when consumption is assumed to follow a random-walk process, this is sufficient to obtain procyclical interest rates and a negative UIP coefficient. I show that when consumption does not follow a random-walk process, that is, when expected consumption growth is time-varying, households' consumption smoothing is stronger. The effect of time-varying risk aversion on interest rates is dampened and the UIP coefficient is positive and close to one.

The second mechanism relies on biased expectations and has been suggested by

Gourinchas & Tornell (2004). Because agents systematically underestimate the persistence of monetary shocks, they under-react to such shocks. As a result, it is possible to obtain a high-interest rate currency that appreciates for several periods as agents learn the true nature of the shock. I show that in a DSGE framework, the mechanism is intact, but dampened. As a result, (true) monetary shocks must be assumed to be very persistent. Furthermore, agents' expectations must be assumed to be strongly biased to have sizable effects.

Hence in this chapter I demonstrate that models that can explain the UIP puzzle in endowment-economy frameworks do not perform as well when re-cast in a standard DSGE framework. The forces that bring the UIP coefficient towards unity in DSGE models are difficult to counteract. My results however show that the two types of models, those relying on time-varying risk premia and those relying on biased expectations, are differently affected by the change in framework. For the latter type, the mechanism is weakened, but not completely overturned. Further research is needed to explore how these two types of models would perform in the presence of endogenous physical capital.

3.6 Figures

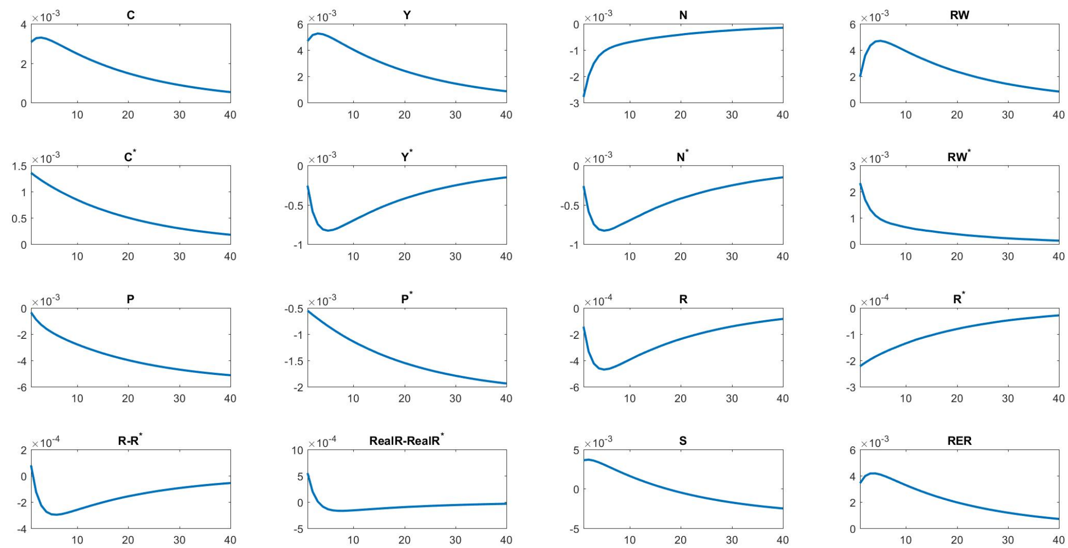


Figure 3.1: Baseline model, technology shock in Home

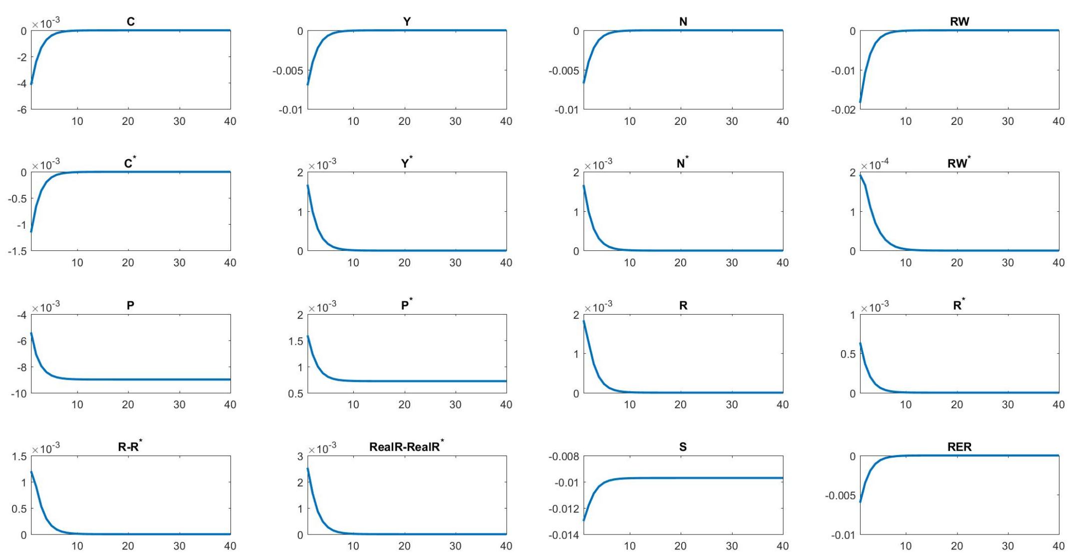


Figure 3.2: Baseline model, monetary shock in Home

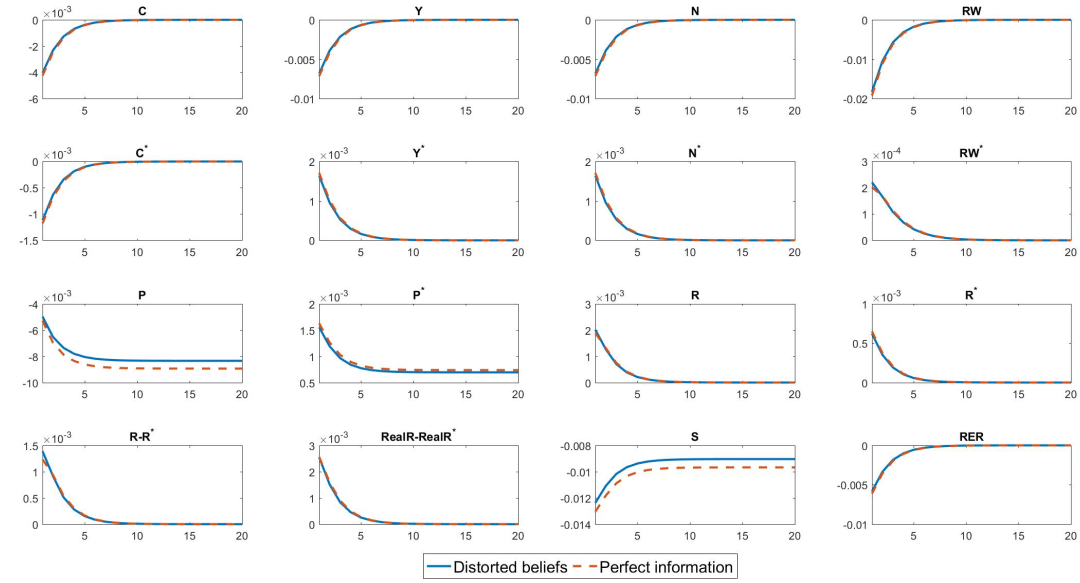


Figure 3.3: Contractionary monetary shock in Home, distorted beliefs model, benchmark calibration

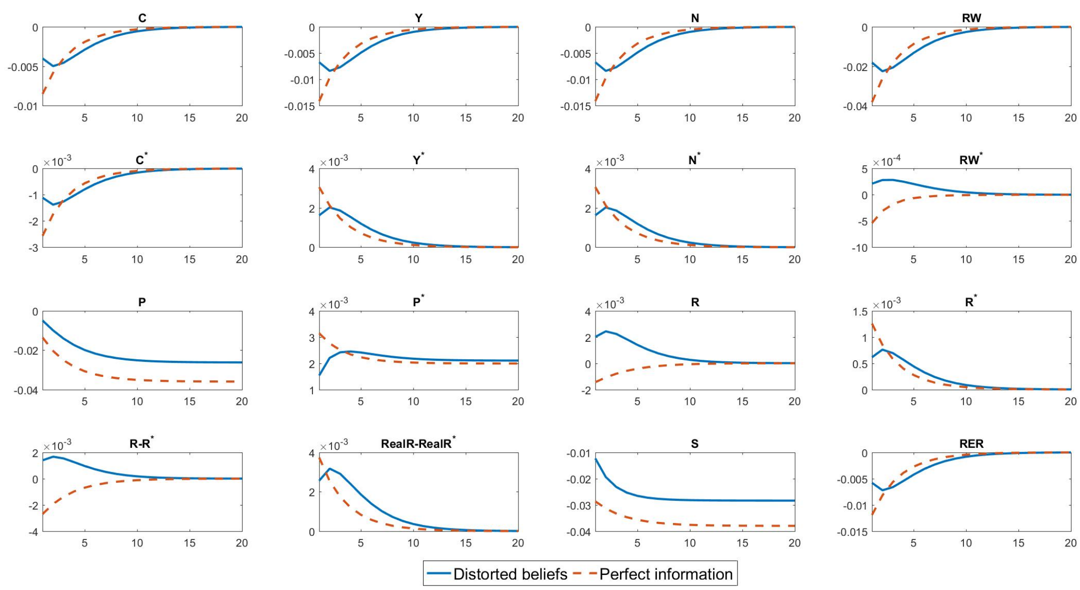


Figure 3.4: Contractionary monetary shock in Home, distorted beliefs model, alternative calibration

3.7 Tables

Param.	Description	Value	Explanation
α	Degree of price stickiness	0.75	Prices hold on average for 4 quarters
β	Households' discount factor	0.99	Annualized risk-free interest rate of 4 %
ξ	Elasticity of substitution in production between the intermediate goods produced within a country	7.66	Mark-up of 15%
θ	Elasticity of substitution in consumption between the Home and Foreign goods	1.5	Standard
γ	Degree of home bias	0.85	Import-to-consumption ratio of 0.15
ϕ	Inverse of Frisch labor supply elasticity	1.5	Standard
σ	Degree of risk aversion	2	Standard
ϕ_i	Degree of interest-rate smoothing	0.8	Smets & Wouters (2007)
ϕ_π	Degree of inflation targeting	2	Smets & Wouters (2007)
ρ_A	Persistence of technology shock	0.95	Smets & Wouters (2007)
ρ_R	Persistence of monetary shock	0.12	Smets & Wouters (2007)
σ_A	Standard deviation of technology shock	0.0075	σ_A/σ_R from Smets & Wouters (2007)
σ_R	Standard deviation of monetary shock	0.0040	Eichenbaum & Evans (1995)

Table 3.1: Benchmark calibration

Moment	Data	Baseline 1st-order	Baseline 3rd-order
$\beta^{UIP,nom}$	-0.23	1.00	0.99
$\beta^{UIP,real}$	-0.01	1.00	1.00
$\sigma(C_t)/\sigma(Y_t)$	0.87	0.63	0.64
$\sigma(N_t)/\sigma(Y_t)$	1.24	0.69	0.50
$\sigma(R_t)/\sigma(Y_t)$	0.79	0.20	0.15
$\sigma(RealR_t)/\sigma(Y_t)$	0.79	0.34	0.22
$\rho(Y_t, Y_{t-1})$	0.77	0.80	0.90
$\rho(C_t, C_{t-1})$	0.89	0.81	0.91
$\rho(R_t, R_{t-1})$	0.84	0.68	0.76
$\rho(RealR_t, RealR_{t-1})$	0.75	0.60	0.61
$\rho(Y_t, Y_t^*)$	0.55	-0.33	-0.33
$\rho(C_t, C_t^*)$	0.36	0.58	0.61
$\sigma(\Delta S_t)$	0.04	0.04	0.02
$\sigma(\Delta RER_t)$	0.04	0.02	0.01
$\rho(\Delta S_t, \Delta S_{t-1})$	0.09	-0.08	-0.07
$\rho(\Delta RER_t, \Delta RER_{t-1})$	0.06	-0.18	-0.12

Table 3.2: Simulated moments, baseline model

	Calibration				Simulated moments		
	γ	Aggr. labor	α	ρ_A	$\rho(rr_t, c_t)$	$\rho(rr_t, x_t)$	β^{UIP}
1	0.99999	fixed	0	0.99999	0.22	0.30	-0.16
2	0.99999	fixed	0	0.95	-0.44	-0.61	1.24
3	0.99999	fixed	0.75	0.99999	-0.25	-0.74	1.00
4	0.85	fixed	0	0.99999	0.10	-0.05	-1.45
5	0.99999	flexible	0	0.99999	-0.12	-0.98	1.49
6	0.85	flexible	0.75	0.95	-0.40	-0.54	1.00

Table 3.3: Selected simulated moments under different calibrations, habit model

Moment	Data	Baseline 3rd-order	Habit Verdelhan	Habit Benchmark
$\beta^{UIP,nom}$	-0.23	0.99	-	0.99
$\beta^{UIP,real}$	-0.01	1.00	-0.17	1.00
$\sigma(C_t)/\sigma(Y_t)$	0.87	0.64	1.00	0.14
$\sigma(N_t)/\sigma(Y_t)$	1.24	0.50	-	1.09
$\sigma(R_t)/\sigma(Y_t)$	0.79	0.15	-	0.27
$\sigma(RealR_t)/\sigma(Y_t)$	0.79	0.34	0.10	0.33
$\rho(Y_t, Y_{t-1})$	0.77	0.90	1.00	0.91
$\rho(C_t, C_{t-1})$	0.89	0.91	1.00	0.97
$\rho(R_t, R_{t-1})$	0.84	0.76	-	0.83
$\rho(RealR_t, RealR_{t-1})$	0.75	0.60	0.99	0.67
$\rho(Y_t, Y_t^*)$	0.55	-0.33	0.07	-0.97
$\rho(C_t, C_t^*)$	0.36	0.61	0.07	0.74
$\sigma(\Delta S_t)$	0.04	0.02	-	0.02
$\sigma(\Delta RER_t)$	0.04	0.01	0.32	0.01
$\rho(\Delta S_t, \Delta S_{t-1})$	0.09	-0.07	-	-0.07
$\rho(\Delta RER_t, \Delta RER_{t-1})$	0.06	-0.12	-0.01	-0.10
$\rho(RealR_t, X_t)$	-	-	0.31	-0.53

Table 3.4: Simulated moments, habit model

Moment	Data	Baseline 1st-order	Beliefs Benchmark calibration	Beliefs $\rho_R = 0.7$ $\sigma_v^s = 7\sigma_R$
$\beta^{UIP,nom}$	-0.23	1.00	0.84	-1.65
$\beta^{UIP,real}$	-0.01	1.00	0.92	0.24
$\sigma(C_t)/\sigma(Y_t)$	0.87	0.63	0.65	0.63
$\sigma(N_t)/\sigma(Y_t)$	1.24	0.69	0.37	0.68
$\sigma(R_t)/\sigma(Y_t)$	0.79	0.20	0.12	0.21
$\sigma(RealR_t)/\sigma(Y_t)$	0.79	0.34	0.14	0.35
$\rho(Y_t, Y_{t-1})$	0.77	0.80	0.95	0.94
$\rho(C_t, C_{t-1})$	0.89	0.81	0.95	0.94
$\rho(R_t, R_{t-1})$	0.84	0.68	0.83	0.90
$\rho(RealR_t, RealR_{t-1})$	0.75	0.60	0.64	0.90
$\rho(Y_t, Y_t^*)$	0.55	-0.33	-0.32	-0.38
$\rho(C_t, C_t^*)$	0.36	0.58	0.62	0.58
$\sigma(\Delta S_t)$	0.04	0.04	0.01	0.02
$\sigma(\Delta RER_t)$	0.04	0.02	0.01	0.01
$\rho(\Delta S_t, \Delta S_{t-1})$	0.09	-0.08	-0.01	0.56
$\rho(\Delta RER_t, \Delta RER_{t-1})$	0.06	-0.18	0.01	0.30

Table 3.5: Simulated moments, distorted beliefs model

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