Using Life Stories to Analyze Mathematics Teachers' Beliefs and Instructional Practices:

Author: Sunghwan Hwang

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Lynch School of Education

Department of
Teacher Education, Special Education, and Curriculum and Instruction

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USING LIFE STORIES TO ANALYZE MATHEMATICS TEACHERS' BELIEFS AND INSTRUCTIONAL PRACTICES

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USING LIFE STORIES TO ANALYZE MATHEMATICS TEACHERS' BELIEFS AND INSTRUCTIONAL PRACTICES

By Sunghwan Hwang

Lillie Richardson Albert, Ph.D., Chair

Abstract

Why do mathematics teachers' beliefs and instructional practices differ, and why are some teachers' beliefs aligned or misaligned with their instructional practices? This qualitative case study investigated how eight Korean elementary teachers' sociocultural life stories shaped their mathematical beliefs and practices. The specific aim was to explore through mathematics-related life stories the relationship between the elementary teachers' mathematical beliefs and instructional practices. The overarching research question was: "How does a theoretical model based on sociocultural theory (Albert, 2012; Vygotsky, 1978) explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?"

The findings of this study indicate that the teachers' attribution of their unsuccessful teaching experiences contributed to their perception about the value of continuing their own learning and development, which, sequentially, influenced the construction of their current beliefs about mathematics teaching and learning. Their pedagogical beliefs for teaching mathematics were likely to have an impact on their attitude toward implementing student-centered or teacher-centered instructional practices. Additionally, the teachers' knowledge and self-efficacy beliefs about teaching mathematics influenced this relationship, resulting in different levels of alignment and even misalignment. Thus, teachers used their past mathematics learning and teaching experiences to justify their current beliefs and practices and to explain their classroom

culture. These findings resonate with scholarship pertaining to mathematics teachers' knowledge, beliefs, and instructional practices and contribute further to their developing theory about teachers' life stories by illustrating how teachers' life stories play out in a complex mathematics classroom environment.

DEDICATION

This study is dedicated to

Hwayeong Jung
Seojun Hwang
My wife and son

Leeung Hwang
Chunhee Jang
Jinsuk Hwang
Jeehyun Hwang
Jaeho Hwang
Sungja Jun
My family

Dr. Lillie R. Albert
My mentor and friend

Friends and Teachers

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CHAPTER ONE

INTRODUCTION

A carpenter is known by his chips. (Jonathan Swift, 1667-1745)

Why do mathematics teachers' *beliefs* and *instructional practices* differ, and how do we ensure that we accurately understand a teachers' beliefs and intentions related to his/her instructional practices? Moreover, how can we be sure that we know a mathematics teacher's beliefs and practices? To answer these questions, I will first describe some limitations and challenges of previous research on mathematics teachers' beliefs, and then connect these implications to the relationship among teachers' beliefs, practices, and life stories. Because teachers' instructional practices and life stories are, in part, related to their beliefs (Barkatsas & Malone, 2005; Drake, 2006; Ernest, 1989), analyzing research on their beliefs will shed light on studies about their practices as well.

We, as outsiders, cannot have direct access to a teacher's beliefs (Philipp, 2007; Thompson, 1992). At the same time, the teachers themselves might not be able to accurately describe their own beliefs, especially because different types of beliefs can emerge simultaneously (e.g., beliefs about mathematics, teaching, and learning) (Cross, 2009; Pajares, 1992). In addition, beliefs are related to both cognitive and affective domains, such as knowledge, attitude, and emotion; belief structures are considered to be complex and their meaning is ambiguous (Skott, 2009; Speer, 2005). Thus, it is hard for teachers to identify their beliefs and to distinguish them from their emotions or knowledge (Hannula, 2012). Moreover, teachers' mathematical beliefs are likely to change at different times and in different sociocultural contexts (Hopkins & Spillane, 2015). At the same time, some beliefs resist change, regardless of outside factors (Pajares, 1992; Rousseau, 2004).

Adding to the complications, many studies have examined mathematics teachers' beliefs across three constructs: beliefs about the nature of mathematics, mathematics teaching, and students' mathematics learning (Cross, 2009; Ernest, 1989), usually without considering sociocultural factors. Researchers have simply bifurcated teachers' beliefs as student-centered or teacher-centered based on interview and survey data (Nathan & Koedinger, 2000; Raymond, 1997). However, it is now evident that these approaches are not sufficient enough to judge an individual's beliefs systems because they provide limited insight into teachers' inner perceptions (Handal, 2003; Raymond, 1997; Skott, 2009). How, then, are we able to analyze a mathematics teacher's beliefs, which might either differ from or be similar to others' beliefs that might be fixed or changing? In order to answer such a question, we should first understand how teachers' beliefs and instructional practices are constructed, and then identify the factors which influence the construction processes.

In terms of teachers' constructions of beliefs, the influence of social factors is a primary consideration (Albert, 2012; Kagan, 1992; Richardson, 1996). Mathematics teachers' beliefs are influenced by not only their personal perceptions about mathematics, but also school contexts, and students' backgrounds (Alba, 2001; Hopkins & Spillane, 2015). For example, Hopkins and Spillane (2015) reported that over time, a group of mathematics teachers working in the same district developed similar types of beliefs, which were aligned with district and school cultures. Also, teachers' former experiences as learners affect the construction of their beliefs (Cobb & Yackel, 1996; Drake, 2006; Lortie, 1975). During the thousands of hours spent in classrooms, they were observing and evaluating their teachers' instruction, which formed their beliefs about teaching practices and pedagogies. Additionally, teachers' own personal teaching experiences influenced their beliefs (Clarke & Hollingsworth, 2002; Tynjälä, 2008; Vermunt & Endedijk, 2011). Teachers use specific instructional methods aligned with their belies (e.g., drill or problem-solving activities), and the successes or challenges experienced during the instruction result in either reinforcement or changes in their previous beliefs.

These factors influencing the construction of beliefs provide insight into analyzing mathematics teachers' beliefs. First, given that their beliefs are influenced by contextual factors, we should analyze teachers' broad beliefs systems, including beliefs about students' abilities and motivation, schools' expectations, and mathematics teaching and learning. Second, teachers' life stories should be examined to see how they were able to acquire specific sets of beliefs, because one's development is mediated by social-cultural and historical contexts (Vygotsky, 1986, 1978). Thus, investigating teachers' experiences as learners, and as preservice and inservice teachers could provide valuable information for interpreting their contemporaneous beliefs and practices. Third, connecting teachers' life narratives to their mathematics learning experiences and current belief systems can help teachers vividly describe them in a way that helps researchers accurately understand their beliefs structures and instructional practices. Because discrepancies between beliefs and practices, in part, are caused by a lack of shared understanding between researchers and teachers about particular conceptualizations of beliefs (Speer, 2005), asking teachers to provide narrative about their beliefs, thoughts, and experiences, and then, analyzing such elements can help researchers better understand teachers' beliefs. In short, in order to know a mathematics teacher, we should know their mathematical and contextual beliefs, and their life stories regarding mathematics teaching and learning. These understandings may also help researchers verify and interpret the intentions of teachers' instructional practices. If we accept that teachers' mathematical beliefs can be analyzed in relation to their life stories, that their current contextual background can be understood in light of their narratives, and that their practices can be analyzed by connecting them to their broad belief structures, then we might understand the relationship between mathematics teachers' beliefs and practices in more precise ways. This understanding may also lead us to other understandings about mathematics teachers.

This study can be differentiated from previous research by its use of teachers' life stories as a tool for analyzing mathematics teachers' beliefs and practices. Although many studies analyzed the effects of teacher education programs (e.g., Philipp et al., 2007) and school context (e.g., Hopkins & Spillane, 2015) on the formation of teachers' beliefs and practice, only a few studies provided explanations about mathematics teachers' life experiences and their relationship to their current beliefs and practices. The limitation in the literature might be due to the influence of cognitive research related to mathematics teachers. For several decades, researchers have focused on teachers' stated mathematical beliefs without considering their social-cultural and -historical backgrounds (Atweh, Forgasz, & Nebres, 2001), a focus which pertains to studies about teachers' beliefs as well. Thus, it can be challenging to create a model to explain and predict the construction of mathematics teachers' beliefs and practices based on their life stories (Drake, 2006). More generally, however, using life stories to understand beliefs is not a new method. In particular, psychologists who study personality (e.g., Atkinson) have long been using life stories to gain insight into people's lives, identities, and behaviors, because these psychologically internalized narratives resonated with critical events in their lives, and the stories represent the outcomes of their' cognitive and affective development (McAdams, 1996, 2001).

Analyzing mathematics teachers' beliefs and instructional practices through their life stories has many theoretical benefits. Examining teachers' life stories facilitates researchers' understanding of the relationship between mathematics teachers' beliefs and practices, which could not be achieved by analyses based on previous limited perspectives. Such narratives by teachers can also help researchers understand and critique teachers' beliefs and instructional practices. The knowledge gain from this process can be used to analyze the development of teachers' unproductive teacher-centered beliefs and practices, which are not aligned with current student-centered reforms. In addition, identifying the kinds of instruction during their school experiences that are most memorable to teachers and the influences on their current teaching beliefs and practices can help researchers build a model describing how their beliefs and practices are formed and when they are developed or changed. The purpose of this study is to help provide such empirical and theoretical foundations.

Purpose of the Study

The purpose of this study is to investigate how eight Korean elementary teachers' sociocultural life stories shape their mathematical beliefs and practices. This study aims to explore through life stories the relationship between elementary teachers' mathematical beliefs and instructional practices. The goal is to apply a theoretical model to explain the relationship that exists among the elementary teachers' life stories, the development of their beliefs and their instructional practices. For the purposes of this study, the concept of life stories considers relevant events that the elementary teachers recall as they reflect on past and present experiences (e.g., at school) that constitute their beliefs about teaching and learning mathematics and instructional practices. Thus, a major assumption is that studying elementary teachers' beliefs through a contextual model, such as life stories, will contribute to our understanding of how teachers construct their beliefs about mathematics and practices. The current study will help us make sense of how elementary teachers make meaning of sociocultural life experiences that influence or shape their mathematics beliefs and practices.

Research Questions

This study draws on a sociocultural perspective to explore the relationship between mathematics teachers' life stories and their teaching beliefs and practices. The overarching research question is: "How does a theoretical model based on Vygotsky's sociocultural theory explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?" To meet the purposes outlined here, the following research questions will be investigated. Through the course of conducting this study, other questions that emerge will also be examined.

1) How do Korean elementary teachers' sociocultural life stories influence their pedagogical beliefs?
    a) What types of events do the participants describe as they recall certain life experiences?
    b) Are these events similar across the eight participants? If so, how are they similar and how are they different?
2) What is the relationship between Korean elementary teachers' pedagogical beliefs and their instructional practices?
    a) What are teachers' pedagogical beliefs?
    b) What are the mathematical classroom norms, tasks, and discourses identified among the participants?
    c) What beliefs are relevant to participants' instructional practices?

3) How does a theoretical model explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?

Importance of the Study

Many studies have investigated factors that influence mathematics teachers' instructional practices. These studies have generally focused on teacher knowledge (Ball, 1994), teacher beliefs (Philipp et al., 2007), student abilities (Rousseau, 2004), and school cultures (Hopkins & Spillane, 2015). However, these individual factors are related to only one aspect of teachers' current states, not teachers' overall cognitive, emotional, and sociocultural states. For example, although we can assume a relationship between mathematics teachers' knowledge and their instructional practices, we cannot explain all of their instructional practices, such as classroom discourse, management, and mathematical tasks, only in terms of their knowledge. In addition, studies that address teachers' instructional practices, in particular contexts, cannot explain or predict teachers' instructional practices in other environments. Rather, as Raymond (1997) argued, mathematics teachers' instructional practices can be fully understood only in terms of their past school experiences, beliefs, classroom situations, and lives outside of school. Similarly, Skott (2009) suggested that teachers' mathematics instruction is not only influenced by their school colleagues and students' mathematical attitudes, but also by their past learning experiences from college and the educational philosophy they have developed. Therefore, Drake (2006) argued that a teacher's life story of past and present experiences, particularly those connected to learning, "provides a more contextualized and integrated view of teachers' beliefs, knowledge, and prior experiences than can be achieved through a focus on any one of these components separately" (p. 580).

Understanding teachers' mathematical life stories in connection with their current beliefs and practices has various practical implications. First, telling their life stories can provide individual teachers' a more accurate understanding of their own instructional practices and teaching beliefs, which might otherwise be examined and analyzed. Second, school administrators and teacher education instructors might use observation of life experiences and narrating life stories as a way to improve inservice and preservice teachers' instructional practices. Last, as an outcome of these benefits, students might be provided high-quality mathematics instructions by their teachers, leading to increased mathematics achievement and participation, as well as the development of positive mathematical attitudes and beliefs.

Theoretical Framework

This study draws on sociocultural theory of learning and development (Cobb & Yackel, 1996; Vygotsky, 1986, 1978) for its theoretical framework to explore the relationship between mathematics teachers' life stories and their teaching beliefs and practices. Lev S. Vygotsky, a founder of sociocultural theory, argued that all human experiences are mediated by "the sociocultural, or social and historical, context" (Albert, 2012, p. 1). Wertsch (1985) identified three interconnected themes underlying Vygotsky's theoretical framework:

> (a) a reliance on a genetic or developmental method (*social interaction*), (b) the claim that higher mental processes in the individual have their origin in processes, (*mental processes*), and (c) the claim that mental processes can be understood only if we understand the tools and signs that mediate them (*mediation*) (p. 14-15).

In terms of mental processes, Vygotsky (1978) emphasized the importance of analyzing an individual's contextual background to understand his/her genetic development. For example, by investigating disruptions and interventions in the individuals' intellectual development, we are able to thoroughly understand the changes in their development. In alignment with sociocultural theory, Lave and Wenger (1991) put forth ideas relevant to situative learning theories, arguing that learning is strongly influenced by the specific social situation in it, which occurs through the interactions with others using tools and representations. Therefore, knowledge is not considered a static entity; instead, social engagement and context influence the interpretation and acquisition of knowledge. As an example, Lave and Wenger point out that based on the characteristics of learning environments, “speakers acquire regional accents…[and] students come to reproduce aspects of the performance style of a charismatic teacher” (p. 19).

Regarding sociocultural origins, such as learning how to use tools, produce speech, and interact with others, Vygotsky (1978) posited that external social interventions and structures influence the individual’s internal development. However, his account of internalization does not imply that individuals were passively molded by external interventions, but instead, the individual consciously internalizes them. For example, Wertsch (1985) maintains that speech, the most dominant means of social interaction, “combines within itself both the function of social interaction and the function of (individual’s) thinking” (p. 94). For Vygotsky (1978), internalization is the process by which people acquire pre-existing external realities from each other (inter-psychological) through consciousness interpretation (intra-psychological). Similarly, Lave and Wenger (1991) argue that learning is an interactive process, which means that students’ learning experiences may shift based on their personal characteristics.

Sociocultural theory implies that mathematics teachers’ current beliefs and practices are the product of their mathematics education and related life events, such as their interactions with and responses to their previous teachers and instructors, their students, and their colleagues, as well as how they interpret those experiences. A life story is, thus, an open arena representing one's teaching beliefs and philosophy (Ball, 1994; Drake, Spillane, & Hufferd-Ackles, 2001). When analyzing mathematics teachers' beliefs and instructional practices, it is necessary for researchers to examine their life stories as well. In sum, sociocultural theory provides a holistic perspective for analyzing the relationship between teachers' life stories and their beliefs and instructional practices.

Positionality

This research was connected to my personal experience in South Korea. When I worked as an elementary school teacher in South Korea for 10 years, I observed many teachers' mathematics instruction and realized that different teachers teach mathematics in extremely unique ways. While some teachers focused on student-discussion and manipulation, others were more concerned about accurately teaching procedures and having their students solve mathematical problems as fast as they could. These differences were interesting to me because at the time, South Korea had only one kind of mathematics textbook, curriculum, and teacher certification test at the elementary level. In addition, almost all elementary school teachers graduated from similar types of government-approved teacher preparation programs at government colleges. So, I asked the question, *why do Korean elementary school teachers have different mathematical beliefs and instructional practices although they share similar backgrounds*?

In order to answer this question, I am conducting this study and taking an insider's stance (Foote & Gau Bartell, 2011) because of the relationship I have had with the participants, some of whom I went to college with or worked in the same schools, and others whom I became acquainted with in professional development programs. My connections with these participants allowed them to describe their experiences, opinions, and practices without reticence and helped me collect and analyze rich data. Because I already knew some of their life stories and instructional practices, they openly shared material without withholding or manipulating information to satisfy researchers' purposes, or provide responses altered by the Hawthorne effect (Creswell & Creswell, 2017; Miles, Huberman, & Saldaña, 2014; Stake, 2010)

These long-standing relationships also enabled me to ask them for additional information. For example, when a teacher explained her nadir experiences in school because of her low mathematics achievement, I asked for her test scores and ranks in the classroom, as well as any related conflicts with her mother or mathematics teachers. The insider's stance is also helpful because it allowed me not only to collect truthful data and deeply analyze participants' responses, non-verbal cues, and their life stories but also to have them check my interpretations to improve the accuracy of data gathered.

However, taking an insider stance has several drawbacks. Because of my close relationship with the participants, my interpretations might be biased (Denzin & Lincoln, 2005). I might unintentionally take an advocacy position because of my affiliation with the participants (Yin, 2015). In order to avoid such risks and offset any related concerns that might be caused by my positionality, I practiced reflexivity, conceptualized by Yin (2015) as "describing as best as possible the interactive effects between researcher and participants, including the social roles as they evolve in the field but also covering advocacy positions" (p. 43). Based on the information disclosed, readers can critically read the findings and assess the integrity of the study.

Although I took the role of an insider, I still maintained the role of a third-party perspective. For example, I was an outsider in that I did not know any of the students in the classrooms I observed, and the teachers were able to select any lessons and teaching strategies based on their preferences. Also, I needed to synthesize various data sources to determine the relationships among teacher's life stories, beliefs, and instructional practices through triangulation. The most important factor that mitigated my insider's stance was that many teachers treated me as an outsider conducting research in their mathematics classroom regardless of our affiliations. In sum, while I did assume the role of an inside, I also took an outsider perspective in many ways as well.

Definition of Terms

This sections defines several key terms as they are used in this study, *life story* being the most salient term. I follow Atkinson's (1998) definition of *life story* as

> the story a person chooses to tell about the life he or she has lived, told as completely and honestly as possible, what the person remembers of it and what he or she wants others to know of it, usually as a result of a guided interview by another (p. 125).

Atkinson also differentiated life story from life history, in that the latter is an individual's recounting of certain events in the past, whereas the former broadly covers an individual' entire life. Life story also can be differentiated from autobiographical memory, which refers to someone's memories acquired through direct participation, and not indirect experiences (e.g., observation and instruction). Thus, historical events and figures are not regarded as elements of autobiographical memory (Robinson, 1976). Given both its breadth and depth, a life story extending from childhood to adulthood allows researchers to accurately understand an individual's characteristics and the development of his/her current identity (McAdams, 2005). However, a person's life story cannot completely explain or anticipate someone's actions. Rather, it provides related background information to help interpret the action (Bruner, 1990). This study uses both the singular (*life story)* and plural (*life stories)* forms, the plural because a teacher's life story is itself a collection of constituent stories, and the singular because these events coalesce into an overarching life story, the parts and the whole representing each other.

Beliefs is another important term in this study, a term that can be difficult to define. Some researchers have used beliefs and other affective related terms (i.e., attitude and emotions) interchangeably (Thompson, 1992). Others have differentiated beliefs from other affective dimensions (Philipp, 2007). Some researchers have identified beliefs as a sub domain of attitude (Hart, 1989) and others suggested emotion, attitude, and beliefs are individual elements of the affective domain (McLeod, 1992) as illustrated in Figure 1.1 (Hannula, 2012). Although there is a continuous controversy regarding the characteristics of beliefs, researchers have been differentiating beliefs from other affective elements (Hannula et al., 2016) and are regarding beliefs as having a more cognitive basis than the elements of attitude and emotions (Philipp, 2007).

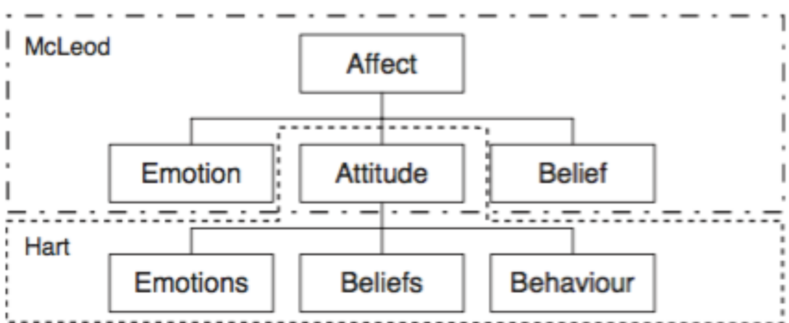


Figure 1.1 Different frameworks regarding beliefs (excerpted from Hannula, 2012, p. 140).

Researchers have also demarcated the characteristics of beliefs by distinguishing them from knowledge. Nespor (1987) argued that beliefs, unlike knowledge, reflect affective and subjective evaluations because they are related to personal experiences, culture, and even propaganda. For example, someone's knowledge of chess does not depend on his/her preference for chess. On the other hand, Kagan (1992) suggested that knowledge and beliefs cannot be separated and that "teachers' professional knowledge can be regarded more accurately as beliefs"

(p. 73) because teachers use specific knowledge from various alternatives, based on their judgment in relation to belief systems.

In order to conceptualize and understand mathematical beliefs, Furinghetti and Pehkonen (2002) selected nine characteristics of beliefs, based on the literature to be evaluated, using a five-point Likert scale by 18 different panels, most of which agreed with the statement, “beliefs… [are an] individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior,” and disagreed with the statement, “beliefs and conceptions are regarded as part of knowledge. Beliefs are the incontrovertible personal 'truths' held by everyone” (p. 47).

After a review of relevant findings for this study, Philipp’s (2007) definition, beliefs are “psychologically held understandings, premises, or propositions about the world that are thought to be true,” was adopted, indicating a narrow zone of convergence between the cognitive and affective domains, most of which remain separate from each other (see Figure 1.2).

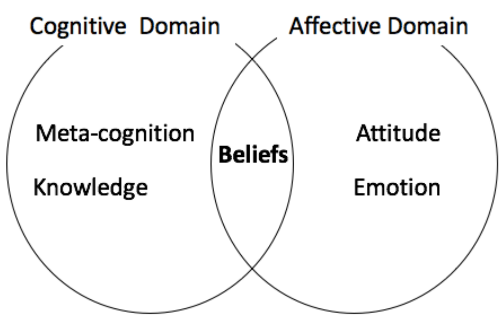


Figure 1.2 Characterization the characteristics of Beliefs

Overview of Chapters

This qualitative study of mathematics teachers’ life stories, beliefs, and practices is presented in the following chapters, including a review of literature, the methods and methodology, the research findings, and discussion and implications. Chapter two provides a review of the relevant theoretical and conceptual literature related to sociocultural theory in mathematics education, life stories and mathematics teachers, and mathematics teachers' beliefs and instructional practices. This chapter also presents the conceptual framework of this study. Chapter three outlines the methodology used in this study, which includes the research setting, participants, data sources, collection procedures, and methods of analysis. Chapters four and five present the findings of this study, which explain the relationships between teachers' sociocultural life stories and their mathematical beliefs and practices, and offer a theoretical model of the relationship. Chapter six summarizes and discusses the findings, implications, and limitations of the study.

CHAPTER TWO

REVIEW OF THE LITERATURE

This review of relevant literature is organized based on three constructs: sociocultural theory, life story, and mathematics teachers' beliefs and instructional practices. First, sociocultural theory is reviewed, in connection with the development of teachers' beliefs, instructional practices, and life stories. Second, life stories are discussed regarding mathematics teachers' beliefs and instructional practices and a research method. Next, the characteristics of traditional teacher-centered and non-traditional student-centered beliefs and practices are discussed, which also includes research on mathematics teachers' incongruences between beliefs and practices. The last section summarizes and synthesizes the literature to form the conceptual framework of this study.

Sociocultural Theory in Mathematics Education

History of Research on Sociocultural Theory

Before the establishment of mathematics education, mathematics and education were strictly separate. Mathematicians were concerned about the field of mathematics itself (pure mathematics), and psychologists studied peoples' intellectual abilities and children's' learning processes (Kilpatrick, 1992). These two disparate disciplines were eventually merged because of the need to extend the boundaries of both fields. Concern about students' low enrollment and achievement in mathematics courses motivated mathematicians to seek understanding of the ways students' mathematical thinking and reasoning development and how mathematics should be taught. Meanwhile, psychologists had been conducting studies of students' problem solving using mathematics activities because the discipline's hierarchical structures were useful for analyzing children's psychological and cognitive development, leading to such works as Thorndike's *The Psychology of Arithmetic* (Bidwell & Clason, 1970).

After the emergence of mathematics education, early mathematics educators focused on behavioristic approaches based on Thorndike's connectionism and Skinner's operant conditioning (Woodward, 2004). Behaviorists assumed that learning was a stimulus and response process and could be measured only through observable performances (Ertmer & Newby, 2013). In this approach, teachers' questions and students' answers were regarded as stimuli and responses respectively. At the time, the main goal mathematics educators had for their students was to build and reinforce strong connections between particular stimuli and responses through rote memorization and practice drills, which positioned students as passive learners, and their existing knowledge and mental processes were disregarded. Behaviorism was not able to explain complex cognitive thinking, such as problem solving and critical thinking (Bidwell & Clason, 1970; Ertmer & Newby, 2013).

In the 1970s, mathematics educators began to conduct research based on cognitivism, which, unlike behaviorism, stresses the importance of mental processes and structures, as well as problem solving ability (Ertmer & Newby, 2013; Woodward, 2004). They viewed learning as the development of knowledge through internal mental activity, and not as a visible response to a stimulus, and eventually became interested in the concepts of information processing, storage, and retrieval. Thus, the key research questions concern how learners came to know something (understanding). Although teachers attended to students' understanding and frequently used pictorial and concrete representations, these classroom cultures were not much different from behaviorist classrooms. Ertmer and Newby (2013) stated,

> The actual goal of instruction for both of these viewpoints (behaviorism and cognitivism) is often the same: to communicate or transfer knowledge to the students in the most efficient and effective manner possible. Two techniques used by both camps in achieving this effectiveness and efficiency of knowledge transfer are simplification and standardization. (p.52)

Although behaviorism and cognitivism objectified knowledge as an outside factor to be acquired by learners, researchers were finding that students' interpretations of knowledge, achievement, and classroom participation varied based on their experiences and sociocultural backgrounds (Atweh et al., 2001). Constructivism as an emerging theory viewed learning as a person's active interactions between the self and outside knowledge. Therefore, in the late 1980's, new types of mathematics studies based on constructivism, were being conducted. It is important to note that the categorization of constructivism varied based on researchers' study areas and perspectives because the theoretical position of constructivism originated from several earlier theories (Doolittle & Camp, 1999). For example, although Ertmer and Newby (2013) differentiated cognitivism and constructivism, Boghossian (2006) combined them and divided constructivism into four categories: cognitive, critical, radical, and social constructivism. Doolittle and Camp (1999) also argued that constructivism consisted of cognitive constructivism, social constructivism, and radical constructivism.

As a simplification, Powel and Kalina (2009) suggested two types of constructivism: cognitive or individual constructivism (Piaget), and social constructivism (Vygotsky). The former focuses on individuals' knowledge construction processes through representations (biological and psychological aspects), while the latter stresses the roles of interactions (social aspect) with others or with tools (e.g., language) for constructing knowledge. In sum, mathematics educators have been broadening the research field from an interest in what teacher and students do (behaviorism), to a pursuit of how student understand information (cognitivism) and then to a consideration of how sociocultural factors influence teaching and learning (constructivism). As sociocultural theory provides a framework for examining the complex relationships between individuals and their social/material environments, the following sections explain the tenets of the theory and the development of mathematics teachers' beliefs and practice processes based on sociocultural perspectives.

Sociocultural Theory and Beliefs and Practice Development

How does a teacher develop specific beliefs and practice? Which factors influence their development? How can we analyze these factors? The answers to these questions can be found in Vygotsky's (1978) concept of the Zone of Proximal Development (ZPD), a powerful theory in people's learning, which he described as:

> The distance between the [learner's] actual developmental level as determined by independent problem solving and the level of [his/her] potential development as determined through problem solving under adult [and others] guidance or in collaboration with more capable peers. (p. 86)

In his book, *Thought and Language*, Vygotsky (1986) explained ZPD by considering what he called scientific and spontaneous concepts. According to Vygotsky, these two concepts come from different kinds of educational settings. Scientific concepts are related to logical thinking and emerge from purposeful experiences in a structured setting, such as a classroom. On the other hand, spontaneous concepts originate from peoples' own reflections on their daily experiences. That is, individuals have their own unorganized empirical concepts, which can be systematically organized under the guidance of a person who has logical reasoning. The space connecting these two concepts is another manifestation of the ZPD.

According to the ZPD construct, learning can originate from others, cultural products (e.g., texts), and tools (e.g., language) within a symbolic space, and individuals wittingly and unwittingly interpret and interact within the symbolic spaces to acquire knowledge (Vygotsky, 1986, 1978). Therefore, teachers also benefit from collaboration with other teachers and from interactions with their students (Albert, 2012). For example, Warwick, Vrikki, Vermunt, Mercer, and van Halem, (2016) found that teachers' collaboration and observation of students' learning helped them understand their students' learning and their own teaching needs, and ultimately led to individual teachers' development.

The concept of ZPD helps educators and researchers understand how teachers' beliefs and instructional practice are developed. Based on the ZPD, for example, Warford (2011) theorized the ways in which pre-service teachers develop professionalism. According to Warford, pre-service teachers' characteristics are developed based on their prior school experiences as learners, observations during field experiences, interactions with students during student-teaching (practicum), and learning from teacher education programs. Similarly, Clarke and Hollingsworth (2002) proposed a model to explain teachers' professional development processes, which could explain how teachers' beliefs and perceptions are constructed. They argued that teacher growth results from multiple events points that are organized in continuing cycles.

As indicated in Figure 2.1, Clarke and Hollingsworth's (2002) framework consists of four domains: the external domain, personal domain, domain of practice, and domain of consequence. The last three domains are explicitly connected to teachers' individual worlds. The external domain is related to external sources of information and stimulation (e.g., learning experiences and social interactions), the personal domain includes teachers' knowledge, beliefs, and attitudes; the domain of practice is connected to personal experiences in the classroom and the domain of consequence refers to the salient outcomes of specific teachers' knowledge, beliefs, and practices. These domains are mediated by reflection and enactment. Compared to reflection, which hard to observe and analyze, enactment is visibly performed in the actual classroom environment. The underlying assumption of this model is that the change in one domain results in changes in the other domains, which means that one domain (e.g., external stimulus) affects the development or impediment of other domains.

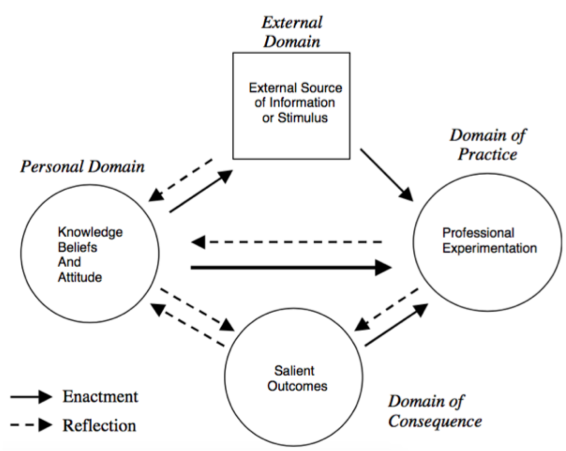


Figure 2.1 The interconnected model of professional growth (Excerpt from Clarke & Hollingsworth's (2002, p. 951)

Taking a similar approach, Vermunt and Endedijk (2011) proposed a teacher-learning model, where the key point is that personal and contextual factors influence teachers' beliefs and their learning. In addition, Tynjälä (2008) explained how people learn in the workplace, which can be summarized as (a) doing a work, (b) socializing and working with colleagues, (c) handling challenges, and (d) attending formal education. These frameworks for teachers' teaching and learning experiences and professional development could provide insight into how teachers' beliefs and practices are developed and explain teacher-centered and student-centered instructional practices in terms of their past experiences, including teacher education programs and professional development programs. Also, teachers continue to construct and internalize their beliefs and practices in connection with their students in specific school contexts. Therefore, we may assume that teachers' beliefs and practices are influenced not only by systematically organization programs (e.g., schools and teacher education programs) but also by unstructured daily experiences (e.g., conversations with other teachers and students).

Life Stories

McAdams (2001), a psychologist, claimed that people's beliefs and practices are co-constructed by themselves and their social contexts, so that we can understand people's sociocultural histories, their present beliefs, and their expectations for the future by connecting these contextual elements to their life stories. Some studies have found that people might react differently to similar situations and events, but, Atkinson (2007) claims, "their stories of what happened and what they did should be consistent within themselves (p. 134). From these perspectives, researchers have analyzed how teachers' experiences as learners and teachers (e.g., classroom or sociomathematical norms) influence their current beliefs and practices, as well as their identities (Drake, 2006; Drake et al., 2001; Foote & Gau Bartell, 2011; Kaasila, 2007b).

Among the many kinds of life-related stories, *critical events* are different from less life-altering experiences (Webster & Mertova, 2007). A *critical event* challenges people's worldview, and sometimes even changes them. Because a critical event accelerates one's learning and has long term effects, past *critical events* can be used to understand and predict people's current and future behaviors. In other words, past life stories can affect one's life path and lead to present values toward oneself and others (Bluck & Habermas, 2000). Because people are likely to maintain their worldviews, they regard some past life stories as an obligation or guidance for the future, and are motivated to move into a certain direction (McAdams, 2001). Similarly, mathematics teachers' mathematics-related life experiences influence their beliefs about teachers' and students' roles and teaching strategies (Ball, 1994). This argument is supported by Barlow and Cates' (2006) study showing that experiencing successful instruction gives rise to changes in teachers' beliefs. According to Barlow and Cate, teachers who have participated in the professional development program began to change their teacher-centered beliefs when they observed their students' success with reform-based teaching strategies.

Atkinson (1998) suggested chronological analysis with a thematic framework, which consists of examining the life story of one theme from childhood to adulthood. Because this method maintained a specific focus on one topic over a long period of time, it provided researchers with a detailed understanding of how this life story changed and developed (see Table 2.1). Similarly, McAdams (1995) proposed seven genres of life stories and questions by focusing on key scenes, such as critical events (see Table 2.2). McAdams also recommended that researchers select certain chapters of life stories, according to their research purposes.

Table 2.1

Types of Life Story and Sample Questions by Atkinson (1998)

Types of Life Story	Sample Questions
Family Origin	How would you describe your parents?
Cultural Setting	Was there a noticeable cultural flavor to the home you grew up in?
Social Factors	Were you encouraged to try new things, or did you feel held back?
Education	What are your best memories of school?
Love and Work	Did you achieve what you wanted to, or did your ambitions change?
Historical Events	What is different or unique about your community?

Table 2.2

Chapters of Life Story and Sub-Areas by McAdams (1995)

Chapter of Life	Areas or Issues
Key Scenes in the Life Story	High point Low point Turning point Positive childhood memory Negative childhood memory Vivid adult memory Wisdom event Religious, spiritual, or mystical experience
Future Script	The next chapter Life project.
Challenges	Life challenge Health Loss Failure and regret
Personal Ideology	Religious/ethical values Political/social values Change, development of religious and political views Single value Other

Mathematics Teachers and Life Stories

This section reviews studies that have analyzed mathematics teachers' beliefs and instructional practices in connection to their life stories. Generally, research on pre-service teachers has focused on the relationships between teachers' life stories and their beliefs about mathematics teaching and learning, while research on practicing teachers has been concerned with how their life stories affect their curriculum interpretations and implementations. I first review studies involving pre-service teachers, followed by those involving in-service teachers and mathematics educators.

Research on preservice teachers. Ellsworth and Buss (2000) examined 98 prospective elementary teachers' mathematics and science-related life stories from the period of their K-12 education. They asked participants to describe life experiences that had positively or negatively affected their perceptions of mathematics and sciences. After analyzing the teachers' responses, they proposed two major elements that influenced teachers' perceptions: their teachers' instructional practices and their family members. In terms of instructional practices, their perceptions were especially impacted by whether their teachers had used real-life situations, whether they emphasized skills and memorization, and in what ways they instructed mathematics content. However, participants perceived and interpreted similar events differently based on their familiarity with those activities and level of mathematics achievement. For example, students who were familiar with skill-based instructional practices and got good grades described such approaches favorably. On the other hand, students who had struggled with drill-based instruction perceived similar experiences as nadir moments. Ellsworth and Buss also found that pleasurable childhood experiences with family members positively influenced teachers' perceptions of mathematics and sciences. For example, family trips (e.g., visiting zoos), parents' math or science related vocations (e.g., as an engineer), and provision of support (e.g., helping with homework), had helped them understand and develop positive beliefs about mathematics and science. In contrast, their family's pressure regarding achievement had caused them to experience frustration and stress, resulting in negative perceptions of mathematics and sciences.

Hauk (2005) analyzed 67 preservice elementary teachers' written mathematical life stories and found that their mathematical experiences related to their ability, efficacy, and potential in mathematics influenced their perceptions of mathematics education and beliefs about mathematics teachers' roles. However, because their personal successes were connected to their experiences with external factors such as the authority of teacher and textbook and the nature of mathematics, these factors had more of a critical impact on students' mathematical experiences, opposed to personal factors. Taking a similar approach, Phelps (2010) interviewed 22 preservice elementary teachers about their past experiences to investigate what factors affected their self-efficacy beliefs and learning goals. They concluded that the participants' mathematics performance in school, vicarious experiences from friends and family members, verbal persuasion by teachers and parents, career goals, and experiences in mathematics classrooms (e.g., perceptions of classroom environments) affected the development of both their self-efficacy beliefs and their learning goals. Another interesting finding was that the effects of vicarious experiences and verbal expressions on teachers' self-efficacy beliefs were similar to their actual experiences.

While the three studies reviewed above focused on preservice teachers' beliefs and perceptions, the following studies addressed how preservice teachers' life experiences also affected their instructional practices during fieldwork. In order to re-examine Lortie's (1975) previous research finding that teachers' instructional practices were strongly influenced by their own teachers' practices, which were transmitted through an "apprenticeship of observation" (p. 61), Ebby (2000) examined three elementary preservice teachers' reports of their K-12 schools, coursework, and fieldwork experiences and found hidden resistance, which hindered preservice teachers' mathematical learning, especially their ability to practice student-centered teaching. Because of negative mathematics experiences, one preserve teacher had developed mathematics anxiety. She intentionally avoided learning mathematical concepts and pedagogies in mathematics and mathematics education courses. She stated,

> My whole life in school has been, math has been there and it's been like this *thing*, that ugh, I just didn't want to come up against... I'm 24 years old, I'm not about to let math make me feel inadequate, or just get me frustrated. I've *had* it, I'm not going to put myself in that position (p. 86)

However, her negative perceptions of learning mathematics were mitigated during fieldwork. She observed student performances, interacted with students through interviews, and taught classes, which helped her realize that mathematics instruction did not need to teach only certain correct procedures but that children had the ability to find various problem solving strategies themselves. Because her fieldwork experiences helped her dismantle negative perceptions of mathematics and construct positive experiences, she began to focus on her mathematics method courses to learn new mathematical ideas. In conclusion, Ebby (2000) argued that while past experiences might delimit pre-service teachers' learning in college coursework, new positive experiences could change their negative mathematical beliefs into positive approaches.

Similar to Ebby's (2000) study, Smith (2003) analyzed how the teacher-centered beliefs and practices of a female preservice teacher had developed, due to negative mathematical experiences, had changed during her fieldwork. She had struggled with learning mathematics. While she always wanted to know reasons behind mathematical concepts and procedures (e.g., why 2+2=4), nobody explained them to her but treated her "like an idiot" for asking. Such experiences led her to believe that teachers had the authority to provide knowledge and students had to reproduce it with minimal to no interactions. However, she started to form different mathematical beliefs after being involved in fieldwork. She learned to listen to her students' voices and observed their performances. As she understood students' mathematical thoughts and perceptions, she wanted to learn new mathematical teaching strategies, resulting in constructing student-centered mathematical beliefs and practices.

Kaasila (2000) also examined mathematics teachers' life stories in order to understand their beliefs about the nature of mathematics, mathematics teaching, and students' roles. Contrary to Ebby's (2000) and Smith's (2003) studies, however, Kaasila found that some preservice teachers' negative experiences positively impacted their mathematical beliefs. Specifically, teachers, who have personally struggled with mathematics themselves could easily understand the challenges experienced by marginalized students in learning mathematics. Thus, they try to design mathematics lesson to fit such students' needs and abilities, such as providing tasks related to pupils' previous experiences and current daily lives

In another study, Kaasila (2007a) found that sometimes positive experiences in school did not benefit preservice teachers' development of student-centered pedagogy. When pre-service teachers had received high scores and positive feedback from their teachers who implemented teacher-centered mathematics instruction, the preservice teachers were likely to believe that teacher-centered practices were helpful for learning mathematics. When method course instructors asked preservice teachers to analyze teacher-centered instructional practices after watching related video clips, a preservice teacher having had positive experiences with teacher-centered instruction, naturally accepted this instructional style without any criticism. Because her positive school experiences provided the lens through which she analyzed mathematics classrooms, she could not recognize the drawbacks of teacher-centered practices, such as lack of student participation and development of mathematical reasoning processes. After method courses and fieldwork, however, she had developed student-centered beliefs and teaching practices. In particular, learning from supervising teachers in fieldwork and interacting with students helped her understand the values of using manipulatives, reflective thinking, and problem solving activities.

Along the line with Kaasila's (2000, 2007a) studies, Lutovac & Kaasila (2014) argued that although some preservice teachers had negative experiences with mathematics learning, the impact of such events was quite different regarding how they interpreted events and developed their identity as mathematics learners and teachers. Some preservice teachers, dubbed *decisive cases*, invested considerable time and effort to become effective teachers, despite being afraid of learning and teaching mathematics. However, other teachers, *irresolute cases*, manifested their fears of mathematics because of their perceived lack of mathematical abilities and did not have clear goals for teaching mathematics.

Research on preservice and inservice teachers. Upon the release of the NCTM's 1991 document emphasizing student-centered instructional practices, Smith (1996) described how preservice and beginning teachers responded to this document. Because teacher-centered practices were predominantly being implemented in mathematics classrooms, which resonated with the teachers' school experiences, many were ignoring or resisting the guidelines suggested by the NCTM. In contrast with reform-based student-centered pedagogy, some preservice teachers had learned mathematics from teachers' direct instruction and drills. While these preservice teachers admitted the value of mathematical reasoning, discourse, and understanding, they were not likely to implement such approaches because of their familiarity with teacher-centered practices. Beginning mathematics teachers often felt that implementing student-centered practices might undermine their authority and decrease their sense of teaching efficacy. Because teachers had limited knowledge of and experience with student-centered practices, they wanted to avoid implementing them for fear of losing control over students and mathematics instruction.

Lutovac and Kaasila (2018) analyzed the practices of a male elementary teacher, for two decades, from his pre-service training through his inservice teaching. As a preservice teacher beginning his teacher education program, he believed that teacher-centered practices were helpful for mathematics learning, and competition regarding mathematics achievement was useful to enhance motivation for learning, which originated from his interactions with family members, friends, and teachers. For example, competing with his brother calculate dart-game scores as quickly as possible made him create strong bonds with mathematics. Based on such experiences, he believed that teachers should explicitly teach the most effective mathematics procedures to their students. Through learning in teacher preparation courses, from other teachers, and while interacting with mixed-grade students in one classroom, however, he started to develop student-centered beliefs and criticized competition-based instruction. In particular, he learned that individual students had different mathematical abilities and ideas, as well as the benefit of using real-life examples while teaching. Consequently, over two decades, his teacher-centered beliefs and practices gradually shifted toward student-centered perspectives.

Research on inservice teachers. Drake et al. (2001) analyzed how elementary school teachers constructed their identities and developed instructional practices by connecting with their negative life stories. Based on observation and interview data, they categorized teachers' responses to negative experiences into three groups: *failures, turning points, and roller coasters.* Teachers in the *failures* group wanted to use traditional teacher-centered practices, although their school used student-centered textbooks. Having had negative mathematical experiences (e.g., low grades), these teachers developed negative mathematical identities and lacked confidence in teaching mathematics. They were reluctant to use materials that increased student participation and felt uncomfortable when they did use them. On the other hand, although teachers in the *turning point* group had had negative mathematical experiences, their more recent and positive experiences enabled them to accept some student-centered practices. Trying hard to develop their new identities as mathematics teachers, the teachers of this group implemented student discussion and manipulations during class. However, their interests were limited to certain teaching strategies, not mathematical content. Last, *roller coaster* teachers were concerned about both teaching strategies and mathematical content knowledge. They assumed that increasing their mathematical knowledge might lead to the development of student mathematical understanding. While teachers in this group have also had negative mathematical experiences, they resolved these nadir experiences through positive attitudes. Some positive experiences have helped them mitigate the negatives in their mathematical experiences.

Using a similar approach, Drake (2006) investigated 20 elementary school teachers' mathematics experiences and found that teachers who had similar life experiences were likely to have similar beliefs and instructional practices with regard to a new, reform-based curriculum. Interview questions elicited stories about challenges, critical events, and peak and nadir experiences, in relation to teaching and learning mathematics and their influences on present beliefs and practices. Some experiences helped teachers accept reform-based curriculum and practices, while others were barriers. Drake and Sherin (2006) also found that the teachers' life stories that influenced their mathematical beliefs and practices were related to not only their early school experiences and their students, but also their interactions with family members. By raising her own children, for example, a female teacher realized that children needed a lot of time to learn mathematical concepts through trial and error. This family-related experience helped her to develop student-centered beliefs and practices. Consequently, she spent considerable time teaching a single mathematics concept and provided multiple resources to encourage students' investigations. Similar to researchers discussed above, Foote and Gau Bartell (2011) interviewed 26 emerging scholars studying equity issues in mathematics education and concluded that their participants shared similar experiences regarding marginalization and inequity. Some had experienced being marginalized in a group, and some had witnessed such discrimination against others. In short, specific events in teachers' lives influenced the construction of their beliefs, instructional practices, and identities. Table 2.3 summarized the research on mathematics teachers' life stories.

Table 2.3

Summary of Research on Mathematics Teachers' Life Stories

Study Participants	Findings
Ellsworth & Buss (2000) 98 [a]PET	- Their teachers' instructional practices and their family members influenced preservice teachers' perceptions - Participants interpreted similar events differently based on their familiarity with those activities and level of mathematics achievement
Hauk (2005) 67 PET	- Their mathematical experiences related to their ability, efficacy, and potential in mathematics influenced their mathematical beliefs - External factors (i.e., the authority of teacher) had more of an impact on students' mathematical experiences rather than personal factors
Phelps (2010) 22 PET	- Teachers' mathematics performance in school, vicarious experiences, verbal persuasion, career goals, and experiences in classrooms affected the development of both their self-efficacy beliefs and their learning goals
Ebby (2000) 3 PET	- Because of negative mathematics experiences, a perservice teacher avoided learning mathematical concepts and pedagogies in mathematics courses. - New positive experiences could change their negative mathematical beliefs into positive beliefs
Smith (2003) 1 PET	- Negative experiences led a teacher to believe that teachers have the authority to provide knowledge and students have to reproduce it - Teacher started to form different mathematical beliefs through fieldwork
Kaasila (2000) PETs	- Some preservice teachers' negative experiences positively impacted their mathematical beliefs - Teachers who had struggled with mathematics may understand the challenges experienced by marginalized students in learning mathematics
Kaasila (2007a) 1 PET	- When preservice teachers had received high scores and positive feedback from their teachers who implemented teacher-centered mathematics instruction, the preservice teachers were likely to believe that teacher-centered practices were helpful for learning mathematics

Lutovac & Kaasila (2014) 6 PET	- Although some preservice teachers had negative experiences with mathematics learning, the impact of such events was quite different regarding how they interpreted events and developed their identity
Smith (1996) PETs & [b]IETs	- While preservice teachers admitted the value of mathematical reasoning, and understanding, they were not likely to implement such approaches because of their familiarity with teacher-centered practices - Teachers felt that implementing student-centered practices might undermine their authority and decrease their sense of teaching efficacy
Lutovac & Kaasila (2018) 1 teacher from PET to IET	- A preservice teacher beginning his teacher education program believed that teacher-centered practices were helpful for mathematics learning, which originated from his interactions with family members, friends, and teachers - Through learning in teacher preparation courses, from other teachers, and while interacting with mixed-grade students in one classroom, however, he started to develop student-centered beliefs
Drake et al. (2001) 10 IET	- While teachers had had similar negative mathematical experiences, individual teachers responded to such experiences differently - Some teachers developed negative mathematical identities, while others resolved their nadir experiences through positive attitudes
Drake (2006) 20 IET	- Teachers who had similar life experiences were likely to have similar beliefs and instructional practices with regard to reform-based curriculum.
Drake & Sherin (2006) 20 IET	- Teachers' life stories that influenced their mathematical beliefs and practices were related to not only their early school experiences and their students, but also to their interactions with family members
Foote & Gau Bartell (2011) 26 mathematics researchers	- Participants shared similar experiences regarding marginalization and inequity. Some had experienced being marginalized in a group, and some had witnessed such discrimination against others - Specific events in researchers' lives influenced the construction of their mathematical beliefs and identities

Note. [a] Preservice elementary school teachers, [b] Inservice elementary school teachers

In this section, I have looked at the ways in which preservice and inservice teachers perceived and interpreted their life stories, and how their interpretations influenced their mathematical beliefs and instructional practices. The research reveals that not only teachers' past school experiences, but also their interactions with their family members, friends, and students affected the construction of their mathematical identities. In addition, these were not limited to direct personal experiences; they included vicarious experiences and verbal persuasion. Given that teachers were inclined to interpret past experiences based on their abilities and efficacy in and attitudes toward mathematics, when analyzing teachers' life stories, it is important to examine how participating teachers interpret both internal and external contextual factors relating to their life stories. Because most studies have found that teachers' experiences continuously influence the development of their mathematical identities, it is also important to examine how their life stories have continued and are continuing to influence the development of their mathematical beliefs and practices.

Mathematics Teachers' Beliefs and Instructional Practices

History of Research on Mathematics Teachers' Beliefs

Research on beliefs and attitudes emerged in the field of social psychology in the early 1900s. Most of the research at this time was focused only on the influence of beliefs on people's behaviors, and theoretical cohesiveness among such studies was lacking (Thompson, 1992). Around 1930, psychologists Thurson and Allport provided theoretical foundations, including definitions and research methods, for research on beliefs and attitudes (Jones & Carter, 2007). Because of the prevalence of the associationism and behaviorism, which focused on peoples' explicitly observable behaviors within the process-product paradigm, peoples' affective domains were neglected at the time (Thompson, 1992). However, interest in the behaviorist perspective faded as studies in cognitive science emerged, since behaviorism could not explain people's thought and decision-making processes, At the National Conference on Studies in Teaching in 1984, one of the 10 panel group released a report including the following statement.

> To the extent that observed or intended teacher behavior is "thoughtless," it makes no use of the human teacher's most unique attributes. In so doing, it becomes mechanical and might well be done by a machine. If, however, teaching is done and, in all likelihood, will continue to be done by human teachers, the question of the relationships between thought and action becomes crucial (Clark & Peterson, 1984, p. 5)

Aligned with the growing interest in peoples' cognitive processes, some mathematics researchers started to analyze teachers' beliefs again, which might shed light onto teachers' thoughts and behaviors in their mathematics classrooms (Nespor, 1987).

In the first published handbook of research on mathematics teaching and learning, Thompson (1992) synthesized previous studies and suggested characteristics of beliefs by describing inquiry-based definitions and constructs. Thompson strongly emphasized the importance of research on the relationship between teachers' mathematical beliefs and their instructional practices, and also called for additional studies because the findings of extant studies on this relationship, though numerous, were not consistent. In the second handbook on mathematics teaching and learning, Philipp (2007) suggested four critical areas of beliefs that researchers should address: definition of beliefs, measurement of beliefs, relationships between beliefs and practices, and changes in beliefs. Moreover, Philipp recommended using sociocultural and participatory theories, which might help explain the complex relationships between teachers' beliefs and practices.

In 2016, the 13th International Congress on Mathematical Education (ICME-13) published a volume of proceedings to highlight current topics in mathematics education. In one chapter, Hannula et al. (2016) proposed two research topics, which might represent current research on teachers' beliefs: (1) relationships between beliefs and practices and (2) changes in teachers' beliefs. Given the importance of contextual elements in teachers' beliefs and practices, Hannula et al. also argued that " [further] research should focus on teachers' contexts and the actual and virtual communities of practice teachers live in, not only on beliefs" (p. 12).

Constructs of Mathematics Teachers' Beliefs

Although there is no consensus on the definition of beliefs, researchers generally agree with the constructs of mathematics teachers' beliefs (Cross, 2009; Philipp, 2007; Thompson, 1992). Building upon Ernest's (1989) study, many researchers have described mathematics beliefs as having three components: beliefs about the nature of mathematics, about instructional practices, and about students' learning. According to Ernest, the first component is teachers' beliefs about the philosophy of mathematics, the second and third components are concerned with teachers' roles and students' learning experiences and outcomes, respectively. Ernest also argued that beliefs about the nature of mathematics can be categorized as the instrumentalist view, the Platonist view, and the problem-solving view (see Table 2.4). These three views of the nature of mathematics form a hierarchical structure. At the lowest level, the instrumentalist view considers mathematics as a body of facts and rules. At the next level, the Platonist view regards mathematics as a logically connected structure. At the highest level, the problem solving view assumes that mathematics is related to social and cultural contexts, and is continually expanding as a living human phenomenon.

Table 2.4

Components of Mathematics Teachers' Beliefs Suggested by Ernest (1989)

Beliefs about the nature of mathematics	- Instrumentalist view - Platonist view - Problem-solving view.
Beliefs about teaching mathematics	- Instructor - Explainer - Facilitator
Beliefs about students' mathematics learning	- Compliance with teachers' instruction and mastery skills - Acceptance of suggested knowledge models - Actively construction of their own models for understanding - Autonomous exploration to pursue their own interest

In terms of teachers' beliefs about teaching mathematics, Ernest (1989) suggested three different roles: instructor, explainer, and facilitator. The instructor focuses on students' mastery of skills and procedures, and the explainer is concerned with students' conceptual understanding as evidenced in their explanations. The facilitator assumes that teachers should increase students' confidence and autonomy in problem solving activities. Concurrent with these three roles, Ernest proposed four patterns of teachers' beliefs about students' mathematic learning by focusing on levels of student authority and action: compliance with teachers' instruction and mastery skills, acceptance of suggested knowledge models, active construction of their own models for understanding, and autonomous explorations to pursue their own interest.

Teacher-centered and Student-centered Mathematical Beliefs

While Ernest's (1989) framework provides a valuable and rich description about mathematics teachers' beliefs, researchers need a more simplified categorization to systematically analyze them. Therefore, many current researchers have divided mathematics teachers' beliefs into traditional and non-traditional beliefs as these are aligned with NCTM's (1989) documents (Philipp, 2007; Raymond, 1997). In addition, because some elements of mathematics teachers' beliefs about mathematics teaching and student learning overlap, some researchers have proposed the terms *teachers' pedagogical beliefs* and *beliefs about teaching and learning mathematics* (e.g., Lloyd, 2002).

Instead of using the general notion of non-traditional beliefs, other researchers have used more specific terms that are in line with their research perspectives, such as student-centered (Remillard & Bryans, 2004), reform-based (Nathan & Koedinger, 2000), and productive beliefs (NCTM, 2014). The implications of those different terms, however, are similar, and are well described in the NCTM's recent document (2014). Teachers with unproductive beliefs (teacher-centered beliefs) focus on instructing correct procedures and ask students to remember standard algorithms. They provide simple problems to avoid discouraging their students. These teachers also believe that students should memorize the procedures they present and should be allowed to solve highly demanding tasks only after mastering basic skills. However, teachers with productive beliefs (student-centered beliefs) focus on developing students' mathematical understanding and expect students to devise various problem-solving strategies themselves. They are more likely to provide challenging and context-related problems to enhance student's mathematical reasoning and problem-solving abilities. These teachers also promote students' active involvement in mathematical activities and justification of their ideas through mathematical discourse.

Because teachers' belief structures consist of different domains (Ernest, 1989), a teacher might have contradictory beliefs (Cross, 2009; Green, 1971; Pajares, 1992). Therefore, it is possible for a teacher with traditional beliefs about the nature of mathematics to have student-centered beliefs about mathematics teaching (Conner, Edenfield, Gleason, & Ersoz, 2011). Conner et al. (2011) claimed that beliefs about the nature of mathematics are relatively stable while beliefs about mathematics teaching are likely to be challenged, based on both educational experiences and external factors. Other studies (Philipp et al., 2007) have also found that teachers' beliefs about the nature of mathematics is less likely to change regardless of their educational experiences. Given that one of main goals of this study is to investigate how teachers' life stories are related to their mathematics beliefs, examining teachers' beliefs about the nature of mathematics might not be appropriate because of their instability.

In addition, it is hard to dichotomize mathematics teachers' beliefs as either teacher- or student-centered. Because teachers' beliefs are gradually developed and change through interactions with others, there is a transitional area between the two poles, which means the existence of several points on the spectrum of teacher- and student-centered beliefs (Ambrose, 2004). Therefore, this study will analyze only mathematics teachers' beliefs about mathematics teaching and learning (pedagogical beliefs) using the following scale: strongly teacher-centered, moderately teacher-centered, moderately student-centered, and strongly student-centered.

Teacher-centered and Student-centered Instructional Practices

Like teachers' beliefs, teachers' instructional practices have been categorized as teacher-centered or student-centered. Since 1991, when the NCTM initiated the movement to convert US mathematics teachers' instructional practices from a traditional to a student-centered orientation, student-centered practices have been aligned with the recommendations of the NCTM's (1989, 2000, 2014) documents, and referred to as either non-traditional or reform-based. Smith (1996) referred to traditional instructional practices as "telling." When teachers dominated the discourse in the mathematics classroom, teachers were telling in order to transmit the knowledge of the textbook and dictating step-by-step procedures so that student could rapidly solve mathematics problems. If students misunderstood or misapplied a procedure, teachers would provide additional problems to have them master it through repetitions. At the same time, students were asked to listen carefully to the teacher's telling and strictly follow the directions s/he gave them. Students' active participation and questions were usually discouraged, unless the teacher allowed them. Both teachers and students regarded the mathematics textbook as a bible which embodied mathematical truth. Therefore, the main goal of mathematics classrooms was for students to be able to solve textbook problems efficiently (Franke, Kazemi, & Battey, 2007).

Several scholars have proposed a set of dimensions of instructional practices to determine whether mathematics teachers' instructional practices were student-centered, which focused on mathematical understanding. Carpenter and Lehrer (1999) suggested tasks or activities presented to students, tools to be used in the classroom, and mathematical norms regulating student activity as ways to evaluate characteristics of teachers' instructional practices. Similarly, Franke et al. (2007) proposed that analyzing classroom discourse, classroom norms, and relationships between teacher and students and students and mathematics could help researchers understand teachers' instructional practices. In addition to tasks, tools, and mathematics classroom norms, Carpenter et al., (1997) analyzed the roles of mathematics teachers in relation to equity and associability to investigate students' participation in their classrooms.

In order to develop a rubric to systematically evaluate high-quality mathematics instruction, Munter (2014) conducted interviews with more than 900 teachers, coach teachers, principals, and district administrators. The rubric consisted of four dimensions: roles of teachers, classroom discourse, mathematical tasks, and student engagement in classroom activity. With the exception of the last dimension, which consisted of two levels, all dimensions were categorized from level 4 (high-quality) to level 1 (low-quality). In particular, Munter stated that teachers at level 1 privileged "traditional forms of classroom activity [in which] students should be in their seats, listening, taking notes" (p. 609). It is important to note that the high-quality and low-quality instructional practices (HLIP), suggested by Munter, were aligned with the student-centered and teacher-centered instructional practices suggested by NCTM (1989, 2000, 2014), as summarized in Table 2.5.

Because teachers' instructional practices include various aspects, however, it was hard to classify and describe a given teacher's classroom practices as either teacher- or student-centered. For example, after analyzing 25 teachers who reported that they were implementing student-centered (reform-based) practices, Spillane and Zeuli (1999) found that their instructional practices were quite varied. Although participating teachers acknowledged the importance of talking and reasoning, conceptual understanding, and multiple representations, some teachers implemented traditional discourse patterns and tasks in their actual instruction. They used rule-based games and activities in order to increase student discourse, and failed to attend to the accuracy of student' mathematical reasoning and justifications. In terms of classroom discourse, the teachers focused on the frequency rather than the quality of student discourse, and used Yes or No questions. Spillane and Zeuli thus found that teachers' instructional practices may not be divisible into teacher-centered or student-centered categories. Instead, teachers could implement a mix of teacher-centered and student-centered practices simultaneously, a concept Spillane and Zeuli (1999) dubbed "Conceptually Oriented Tasks and Procedure-Bounded Discourse" (p. 11).

From a similar perspective, Stein, Grover, and Henningsen (1996) examined 144 mathematical tasks used in reform-based classrooms. Although participating teachers claimed that they were implementing reform-based (student-centered) instructional practices, characteristics of individual task features (e.g., number of problem-solving strategies and representations) and cognitive demands varied. Even during instruction, some teachers, intentionally or unintentionally, changed the cognitive demand of tasks, resulting in the need for a more nuanced determination of the nature of a task as teacher- or student-centered.

While the characteristics of teacher- and student-centered practices were generally clear as concepts, categorizing a teachers' instructional practices as one or the other was challenging, especially because a mathematics teacher's instructional practices could be analyzed across several dimensions (e.g., tasks, discourses, and tools). In addition, within student-centered instructional practices, there were some variations in terms of intensity, such as strongly or moderately student-centered (e.g., Munter, 2014). Therefore, a teacher's instructional practice should not be dichotomized as teacher- or student-centered. Rather, it should be analyzed across several stages on Likert scales, such as strongly teacher-centered, moderately teacher-centered, moderately student-centered, and strongly student-centered.

Table 2.5

High-quality and Low-quality Instructional Practices (HLIP) Rubric Suggested by Munter (2014)

Roles of teachers	H: Teachers act as facilitators. They support student discussion and investigation by posing questions. They ask students to explain their reasoning and work together to solve perplexing problems. Teacher share authority with students.
	L: Teacher act as deliverers of knowledge. They are usually concerned with the accuracy and clarity of their explanations and neglect students' discourse. They directly instruct mathematical procedures and students' participation are very limited, and authority resides only with the teacher or textbook.
Classroom discourse	H: Student-to-student conversations are promoted. Teachers respect student-initiated discourse and emphasize mathematical arguments, justifications, and multiple problem-solving strategies. Students evaluate their peers' ideas and ask them to provide additional explanations.
	L: The primary discourse patterns are IRE (teacher initiate-student response-teacher evaluation) and IRF (teacher initiate-student response-teacher feedback). Student-to-student conversations are very limited.
Math tasks	H: Teachers provide challenging tasks that can be solved in multiple ways. Students are expected to discuss and compare ideas with others to find better solutions. The tasks are related to students' lives, and are intended to increase students' insights into mathematical concepts.
	L: Teachers provide tasks that allow students to practice mathematical procedures acquired from their teachers and textbooks.
Student engagement	H: Students are engaged in mathematical investigations and move within the classroom to discuss with their peers. They make graphs or tables and use manipulatives to express their ideas.
	L: Students are expected to be in their seats and listen carefully to what the teacher is saying. They are allowed only to do the activities suggested by the teacher.

The Relationship Between Beliefs and Instructional Practices

In the above section, I briefly described the relationship between mathematics teachers' beliefs and practices. In this section, I introduced the findings of related empirical researches. Some studies found alignment between mathematics teachers' beliefs and their instructional practices. Remillard and Bryans (2004), for example, argued that elementary teachers' beliefs affect the way in which they use curriculum materials, as well as their mathematics teaching. Similarly, Cross (2009) found that high school teachers' beliefs influenced their classroom practices, including their interactions with students and assessment of their understanding. However, other studies have found a misalignment between teachers' beliefs and practices (e.g., Ambrose, 2004; Raymond, 1997), although this misalignment has not been as well documented as the alignment. Thus, this section looks more closely at inconsistencies between beliefs and practices.

One of the most cited studies about the misalignment between mathematics teachers' beliefs and instructional practices was Raymond's (1997) study of six novice elementary school teachers for ten months using interviews, observations, and lesson plan as data. Raymond found that because the participants had not anticipated the constraints of actual classroom circumstances, due to their lack of teaching experience, their initial beliefs were idealistic and superficial. Although the participating teachers had student-centered beliefs, they implemented traditional teacher-centered practices when they faced challenges, such as a lack of resources for teaching or students' poor performance. As a recommendation for further studies, Raymond argued that teachers' beliefs should be analyzed in relation to actual classroom environments, local social teaching norms, and teachers' past experiences as mathematics learners and in teacher education programs.

Barkatsas and Malone (2005) analyzed one elementary mathematics teacher' beliefs and practices. Generally, she had non-traditional, constructivist beliefs, but her actual practices were close to traditional teaching methods. The researchers attributed the discrepancy to the pressure of traditional social norms (e.g., expectations of administrators and parents) and classroom contexts. These findings concur with Turner, Warzon, and Christensen's (2011) year-long study of three middle school teachers, in which they found that teachers' instructional practices were influenced by not only their mathematical beliefs, but also their perceptions of students' abilities and their own teaching efficacy. Alba (2001) concluded, from a literature review, that teachers' incongruences between beliefs and practices were due to (a) the students' preparation for standardized tests, (b) lack of resources and time, (c) pressure of school administrators to use a specific pedagogy, (d) classroom management issues, and (e) insufficient student effort. Similarly, Handal (2003) proposed that unpredicted classroom environments, external pressures from administrators (school cultures), lack of time, and pressure for exam preparation were impediments to mathematics teachers' implementation of instructional practices aligned with their beliefs.

From a similar perspective, Skott (2009) argued that the inconsistencies found between teachers' beliefs and practices were caused by two factors. First, many researchers disregarded the importance of social perspectives on teachers' practices. Because mathematics teachers' beliefs included not only mathematical beliefs, but also beliefs about working communities' cultures and contexts, it was inevitable that they would find inconsistencies when analyzing solely the relationship between mathematical beliefs and practices. If researchers also considered teachers' beliefs about their community, these discrepancies could be explained. Second, the inconsistencies existed solely from the researchers' perspectives, as the teachers perceived themselves as making reasonable decisions by considering their contexts and beliefs. They stated, "Inconsistency… is an observer's perspective that does justice neither to the complexity of teaching, nor to teachers' attempts to relate sensibly to this complexity" (p. 44). Given the complexity of mathematics teaching in a particular community, Skott suggested analyzing teachers' various related communities, such as the actual classroom community, the expected classroom norms implicitly developed within a school culture, and teachers' previous experiences in other schools and their teacher education programs.

In addition to the external (social cultural factors) or internal (teachers' life histories) factors, the incongruities might reflect limitations of research methods (Philipp, 2007; Thompson, 1992) or teachers' insufficient knowledge about specific beliefs and practices (Richardson, 1996). Richardson, for example, argued that teachers could not implement student-centered practice accurately because a "teacher does not know how to develop or enact a practice that meshes with a new belief" (p. 114). In terms of research methodology, Speer (2005) critiqued methodological issues in analyses of the relationships between beliefs and practices. Because researchers usually collected beliefs and practices data separately, and then analyze the relationship between them, the beliefs teachers held at specific times and places might not be the same as those stated outside these contexts, and therefore, they appear misaligned with observed practices. In addition, Speer was concerned about lack of common understanding between researchers and teachers,

> T(t)he perceived discrepancy between professed and attributed beliefs may actually be an artifact of the methods used to collect and analyze relevant data and the particular conceptualizations of beliefs implicit in the research designs. In particular, the apparent

> dichotomy can be the result of a lack of shared understanding between teachers and researchers of the meaning of terms used to describe beliefs and practices (p. 362)

In order to overcome these constraints, Speer proposed using *video clip-based interview* methods, which uses teachers' reflections on videos of their own practices as data to analyze their beliefs. In this design, before watching the classroom video, teachers were asked to describe their mathematical beliefs contextualized within their actual classrooms. Then, teachers were asked to explain their specific practices in the classroom episode while watching the video with the researchers, mainly addressing teaching methods and the goals of the instruction, as well as the teachers' expectations of the students' participation. Because teachers narrated their beliefs in connection to specific practices, the method helped researchers understand teachers' contextualized beliefs and achieved shared understanding with participating teachers.

Conceptual Framework

In the first section, I discussed how teachers' beliefs and instructional practices are constructed based on sociocultural theory. In the next section, I describe the significance and types of life stories. Last, I provide the structure for analyzing mathematics teachers' beliefs and practices and the relationships between them, as well as ways in which misalignments might be resolved. Taken together, individual elements and their relationships are presented in Figure 2.2. This framework is used to analyze individual teachers' cases.

I approached this study with three assumptions based on the review of relevant research. The assumptions were that 1) teachers' past mathematics learning experiences positively and negatively influence their pedagogical beliefs; 2) teachers' current sociocultural context affects their pedagogical beliefs; and 3) teachers' past mathematics learning experiences, current social cultural context, and pedagogical beliefs influence their current instructional practices. I further assumed that the incongruences between teachers' pedagogical beliefs and instructional practices are caused by limitations of contextual factors.

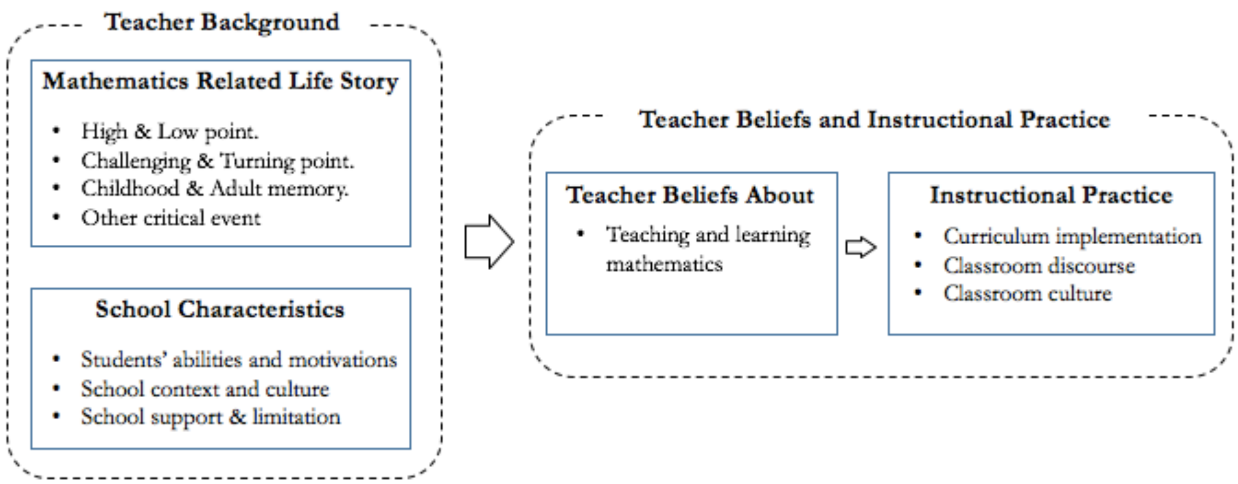


Figure 2.2 Conceptual framework

Chapter 3

METHODOLOGY

Introduction

This chapter introduces research methods, including participants, access and entry, data sources, data analysis, and the limitations of this study. In particular, various sections provide rationale for using qualitative research methods to answer following research questions.

a) How do Korean elementary teachers' sociocultural life stories influence their pedagogical beliefs?

b) What is the relationship between Korean elementary teachers' pedagogical beliefs and their instructional practices?

c) How does a theoretical model explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?

Design of the Study

To holistically investigate participants' experiences, beliefs, and interactions with curriculum materials and students, I have adopted a qualitative research methodology. Unlike quantitative research, which deals largely with broad trends and probabilities, qualitative research focuses on people's specific experiences and perceptions. Qualitative researchers do not design experiments and manipulate data; instead, they seek to understand how people interact with other people, elements of the environment, and/or information in a natural setting (Miles et al., 2014). As Sandelowski (2000) states, the primary purpose of qualitative research methods is to understand "how people think and act …in a natural and unobtrusive manner" (p. 7). These purposes are aligned with the goal of this study, which is to understand the relations among mathematics teachers' beliefs, instructional practices, and life stories. Thus, the findings and conclusions of this research are derived from data acquired from the teachers' narratives in their own words and their own social contexts, and not framed in predetermined hypotheses or models. Moreover, each participant's data are regarded as valuable because each teacher's classroom environment and experiences are unique.

In particular, using qualitative research methods has several benefits for this study. First, as an exploratory research method, a qualitative approach is useful when theory is absent or undeveloped and when related research is sparse (Creswell & Creswell, 2017). Given the lack of studies and concepts about mathematics teachers' life stories (Drake et al., 2001), a qualitative approach meets this criterion. By using inductive inquiry, for example, this study might discover various phenomena related to teachers' life stories concerning their instructional practices in terms of classroom discourse patterns, types of tasks, group organization, and classroom management. That is, qualitative research methods allow for the exploration of the relationships between participants' experiences and their perspectives about mathematics teaching, while the researcher attends to complex and dynamic life events and classroom environments of these participants.

The second reason for using a qualitative approach is related to the types of data sources. For this study, personal data are collected, such as the accounts of teachers' their experiences, which includes as students their failures in learning mathematics, and as teachers their conflicts with parents, students, and principals. Given that these experiences are hard to share, especially with standardized survey instruments to be examined by unknown persons, quantitative methods would not be suitable (Denzin & Lincoln, 2005). However, as Stake (2010) suggests, qualitative research allows the researcher to interact with participants who are sympathetic, resulting in looking at "things closely, becoming sensitive to, even vicariously experiencing, the [participants'] feelings, thoughts, and happenings" (p. 46). Consider the participants' feeling, the researcher can encourage them to comfortably share their personal experiences and allow the researcher to observe them in their natural classroom environment.

Lastly, qualitative research can increase the accuracy of data interpretation (i.e., validity) by involving participants' active participation and member checking throughout the process. As suggested by Creswell and Creswell (2017), the accuracy of research findings can be assured during the data collection and data analysis processes. With established cooperative relationships, participants may not only provide accurate and extensive information during data collection but also correct and evaluate the interpretation of data, which leads to the improved findings.

Case study research. Among several qualitative research approaches, a case study methodology is used in this study. A case study addresses a specific phenomenon in a real context (Yin, 2015). Thus, each case is delimited by space and time boundaries (Gerring, 2006). As Creswell (1998) explained, "a case study is an exploration of a *bounded system* or a case (or multiple cases) over *time* through detailed, in-depth data collection involving multiple sources of information rich in *context*" (p. 61, italics added). If researchers investigate teachers working in two different schools, for example, the two cases are delimited mainly by space. On the other hand, if a study analyzes a teacher's life stories, connecting various events through his/her life, the case is defined mainly by time. Also, the former type of study can involve cross-case analysis because two or more cases are compared. Given that individual cases are strongly influenced by their temporal and spatial boundaries, the selected cases cannot be claimed to represent a population. Therefore, the findings of a case study might not be applicable to other phenomena, or to the same phenomena, if the research is conducted with different participants at different places and times.

The advantages of using case study methods are similar to those of other qualitative research methods. As explained by Gerring (2006), it allows researchers to holistically analyze cases with rich data collected in real-life contexts without manipulation, and secure construct and internal validity through triangulation. Compared to other qualitative methods, however, a case study has additional advantages (Creswell, 1998): (a) the time to collect data and become accustomed to cases is relatively shorter than with ethnographic methods; (b) the method does not need to present a theory as in grounded theory research; (c) it is usually easier to find participants than in phenomenological studies, which analyze individuals who represent a particular phenomenon; and (d) the method allows researchers to use various data, unlike narrative research methods. From a broad perspective, Yin (2003) argued that a case study is a “comprehensive research strategy” (p. 14), encompassing all types of qualitative research methods, including data collection and analysis methods. In particular, Yin proposed six sources of evidence for case study: documentation, archival records, interviews, direct observation, participant observation, and physical artifacts, which may be used in all other qualitative research methods.

Case studies can be categorized as exploratory, descriptive, or explanatory, according to the purpose of the inquiry (Yin, 2003). The exploratory study is mainly concerned with *what* and *how* questions, such as what instructional strategies do high-quality teachers use in classroom? This type of study seeks to provide a justifiable rationale and can be used as a preliminary study, or an aid in developing hypotheses for other studies. In addition, the exploratory study might exert some control over events being explored. The descriptive case study focuses on *what*, *who* and *where* questions to describe naturally occurring phenomena. For example, a study may seek to describe the differences between high-quality and low-quality mathematics instruction. The explanatory case study poses *how* and *why* questions. This method goes beyond description to explain underlying dynamics of a phenomenon. Having distinguished these three types of case studies, Yin cautions that the boundaries are not firm, and a study could include elements of more than one type.

Based on the case study literature, this study can be referred to as a cross-case descriptive and explanatory study because it involves *why* and *how* research questions, and it compares multiple cases using various types of evidence (i.e., observations, interviews, and field notes). In addition, the purposes of this study are both to describe the beliefs, instructional practices, and life stories of individual teachers and explain how such elements are interrelated.

Narrative research. While this study is defined as a descriptive explanatory case study, the narrative research method is applied, given that the narrations of teachers' life stories are the sole data source. A life story includes not only the narrator's life and identity, but also others who are part of his/her stories and the surrounding sociopolitical circumstances, because peoples' beliefs and behaviors are formed within certain spatial and temporal boundaries, and are built upon coherently connected past experiences (Sarbin, 1986). Because people often think and speak about their lives in the form of a story (Atkinson, 1998), the way a life story is narrated and connects past and present events helps construct and define the individual's identity. Therefore, individuals' narrations of past events allow researchers to analyze their beliefs, knowledge, and practices (Clandinin & Connelly, 2000; Drake, 2006; McAdams, 2005).

The idea of participants' narratives in research has been implemented in many fields as narrative unity, narratology, and narrative analysis (Clandinin & Connelly, 2000). In education, narrative inquiry was first introduced by Connelly and Clandinin (1990), who, in *Stories of Experience and Narrative Inquiry*, claimed that a narrative is the combination of phenomenon and method in that the method of collecting information about the phenomenon of interest was the phenomenon (story) itself. Thus, narrative inquiry could be described as both *inquiry into narrative* (people's narrative life experiences) and *narrative inquiry* (use of narrative methods) without a clear distinction between them. Hence, Connelly and Clandinin (1990) explained that narrative studies are based on the premise that "people by nature lead storied lives and tell stories of those lives, whereas narrative researchers describe such lives, collect and tell stories of them, and write narratives of experience" (p. 2).

Unlike other fields, in which narrative inquiry may focus only on an individual's history, development, and emotions, narrative inquiry in education is concerned with both individuals and their social contexts. Because education is implemented in social contexts through active interactions among participants, people cannot narrate their school experiences without tapping into their socio-cultural environments (Webster & Mertova, 2007). In particular, Clandinin (2006) and Connelly and Clandinin (2006) proposed three essential dimensions for understanding a life story: *temporality, sociality, and place*. Temporality refers to understanding of people, place, and event over time. Sociality includes understanding the narrator's personal and emotional conditions, the relationship between researchers and participants, and contributing factors. Place refers to understanding the concrete and physical environment relating to events.

Additionally, narrative inquiry is especially useful for analyzing critical events in people's lives, such as nadir experiences. Because people are not likely to share negatively emotional experiences, they might not have experience in describing them. Thus, such stories might be illogically connected and narrated without any intended meanings. Through logically organized interview questions and appropriate narrative inquiry skills, however, narrators can be guided to connect critical events and retrospect on how they felt and what they learned from the experiences and how they influenced their identity (Connelly & Clandinin, 2006; Webster & Mertova, 2007).

To achieve thematic coherence in narrative analysis, Bluck and Habermas (2000) suggested analyzing critical events. Because such events are more accurately stored and retrieved more frequently than other ordinary events, they provide an opportunity to evaluate an individual's trajectories. Thus, this study sought to understand how teachers' critical life stories (i.e., high, low, and turning-point events) influence their current beliefs and instructional practices. In particular, the events revealed in the narratives were analyzed to determine how participants were challenged by and overcame events by considering temporality, sociality, and place aspects.

Access and Entry

Because I was an elementary school teacher for more than 10 years in South Korea, I can easily find individual participants for this study in Korea. In order to receive permission from school principals, however, I sent an email or visit the school to explain my research purpose and request his/her support. In terms of recruiting participants and collecting data, teachers were willing to participate in this study and provided comprehensive information because many of these teachers had an interest in improving teaching quality and examining their beliefs. Also, my elementary school teaching experience helped me accurately understand what they are saying in interviews, and what I am seeing during observations.

This study is a part of a large mixed-method research study on South Korean elementary teachers' mathematical beliefs about teaching and learning mathematics regarding the meaning attributed to their beliefs and the nature of mathematics, which was conducted by Prof. Albert. Prof. Albert provided the right to access the data.

Setting and Participants

Eight elementary teachers at four different schools in Korea participated in the study. Three (Schools A, B, and C) were located in the same city, and one (School D) in another city. I convenience selected teachers based on their teaching experiences, grade levels, degrees, and gender. The teachers in the study were Kim and Lee from School A, Yang and Choi from School B, Woo and Ko from School C, and Sim and Jung from School D (all names are pseudonyms). All School information is obtained from the following two websites operated by the Ministry of Education of South Korea: https://www.schoolinfo.go.kr and https://kess.kedi.re.kr. The information of all participants and schools are shown in Table 3.1.

School A

While School A was located in big city, geographically, it was closer to a neighboring rural city. Most of the students' parents were blue-collar workers, and their socio-economic status was far below the national average. According to the 2011 national elementary students' mathematics and language assessment of the 600 elementary schools in the city, school was ranked at around 500. Because of School A's poor student achievement record, parents were likely to move to areas with better schools. While a classroom in the city averaged 23-24 students, in School A the average was around 20 students.

Mr. Kim held a certificate in elementary education and master's degree in elementary mathematics education and had been teaching for 25 years. At the time of the study, Mr. Kim was pursuing a Ph.D. in elementary mathematics education. At the school principal's request, his main duties were currently administrative, and he was teaching mathematics and science as a subject teacher, which meant that he did not have his own classroom and students. Moreover, he was a former instructor for pre- and in-service teachers and a member of the Elementary Mathematics Teacher Organizations. Kim had also written several national elementary mathematics textbooks and led workshops for mathematics teachers.

Table 3.1

Summary of Schools' and Teachers' Backgrounds

School characteristics		Teacher characteristics		
Name School district	[a] School ranks [b] Class size	Name	[c] Final degree	Years of teaching
A Suburb Low SES	Around 500 of 600 schools 20 Students	Kim Male	Pursuing a Ph.D. in elementary mathematics education	25
		Lee Female	B.A. in Korean language at the middle school level	11
B Rural Low SES	Around 350 of 600 schools 22 Students	Yang Male	M.A in counseling psychology	15
		Choi Female	B.A. in elementary education	2
C Urban High SES	Around 100 of 600 schools 25 Students	Woo Female	M.A in civic education	5
		Ko Female	B.A. in elementary education	5
D Rural Low SES	Around 20 of 250 schools 28 Students	Sim Male	M.A in elementary mathematics education	10
		Jung Female	Pursuing a M.A. in elementary mathematics education	7

Note. [a:] The data is based on the 2011 national elementary students' mathematics and language assessment. [b] Average numbers of students per classroom. [c] Except for teacher Lee, all teachers graduated from an elementary teacher preparation program at a government-approved college

Ms. Lee, who had been teaching for 11 years, did not graduate from an elementary teacher preparation program at a government-approved college. Instead, she had studied the Korean language and received certification in language teaching at the middle school level. However, due to a teacher shortage, she was recruited as an elementary school teacher by the

Ministry of Education. Except for a few professional development experiences, therefore, she was not trained in elementary mathematics pedagogy and did not have confidence in teaching mathematics. She preferred to teach early grade level students.

School B

The school was located far from the center of the city. The parents of students at this school were almost evenly divided between blue- and white-collar occupations. Although the school was located in one of the poorest towns in the city, student achievement was relatively high within its area, ranking around 350 on the 2011 national mathematics and language test. Averagely, the number of students in a class was 22, half of whom attended cram schools. In both schools A and B.

Mr. Yang had been teaching for 15 years. He held a bachelor of elementary education and a master's degree in counseling psychology. Yang had never attended mathematics-related professional development programs because he felt that they were not effective. However, he had confidence in teaching mathematics and liked to share his teaching strategies with colleagues.

Ms. Choi had been teaching mathematics for only one year. While she had been deployed to School B two years ago, she only taught English and Music in the first year. Because Choi liked to study mathematics education, she took additional mathematics-related courses in her undergraduate elementary teacher preparation program. Also, throughout the school year, Choi attended several professional development programs related to mathematics education and was planning to apply to a master's program in mathematics education. Despite her endeavors, however, Choi reported that she is afraid of teaching mathematics because sometimes her students, whom she perceived disliked studying mathematics, disrupted her lessons by dominating classroom discourse with irrelevant topics.

School C

The school was located in one of the top three most affluent towns in the city. Most parents were white-collar employees or professionals and actively participated in school events. Because parents were concerned about their children's achievement, almost all students were receiving private tutoring or extra instruction. More than 85% of the students lived in the same big apartment complex, so they knew each other's family information, such as jobs, cars, and family members. However, their mathematics achievement was relatively low, ranking only around 100 among 600 elementary schools in the 2011 assessment. The average number of students per classroom is 25.

Ms. Woo had been teaching for five years. She had completed a bachelor's degree in elementary education and a master's degree in civic education. She generally taught the high grades and School C is her first school. It is important to note that all elementary school teachers in South Korea are rotated among the schools in the same city every five years.

Ms. Ko had been teaching for five years in School C, her first school. After taking additional mathematics education courses in her preservice teacher education program, she had lost her enthusiasm for mathematics education and no longer wanted to study it. During the previous four years, she had taught only six graders. For this year, however, she was a third grade teacher for the first time. Both Ms. Woo and Ms. Ko had attended only one mathematics-related PD.

School D

School D had been founded several years ago in a new town in the state, which was developed by the government to support IT research institutions and companies. Therefore, most of the residents of the town were young and affluent adults. Because the school acquired a reputation for high student achievement, however, some families moved into the town, regardless of their jobs, for the sake of their children's education. The ranking of school D is around 20, out of 250 elementary schools, in the 2011 assessment. Averagely, the number of students per classroom was over 28. The teachers had voluntarily organized a mathematics education research group to improve their instructional practices. They met every Wednesday after class and shared their ideas for teaching mathematics, new information from their reading, and new technological devices.

Mr. Sim, who had been teaching for 10 years, had a bachelor's degree in elementary education and a master's degree in mathematics education. Sim generally taught the higher grades and led the mathematics education research group in his school. Outside of school, he actively interacted with mathematics education scholars and teachers. For example, he led another mathematics education study group consisting of teachers from several towns, which held a mathematics education festival for elementary students. In this group, he worked as both an instructor and an organizer. Out of the school subjects, he enjoyed teaching mathematics the most and had the most confidence in his teaching.

Ms. Jung had been teaching for seven years and, like teacher Sim, actively participated in several mathematics education related organizations. When she was in college, she took additional English courses because of personal interests. After becoming an elementary school teacher, however, she realized the importance of mathematics education and pursued further study, so she had only two semesters left to complete a master's degree in mathematics education. Jung liked to discuss her mathematics teaching with other teachers and receive feedback from them.

Data Sources and Collection

Data sources include pre- and post- observation interviews, classroom observations, and field notes. Once participants had decided to participate in this study, I determine a schedule for data collection based on their dates of availability. While I had a prepared interview protocol, I changed some of them based on participants' response. I also did not restrict the focus of my observations or specify the format of instructional practices in order to avoid creating additional work for participating teachers.

Interview Data

The qualitative interview is usually either un- or semi-structured and open-ended so that researchers may acquire an in-depth understanding of participants. Taylor, Bogdan, and DeVault (2016) state that verbal accounts can be used for "understanding informants' perspectives on their lives, experiences, or situations as expressed in their own words" (p. 102). The participants of this study were asked open-ended interview questions to encourage them to make meaning of their everyday mathematics teaching and learning in relation to their mathematical beliefs. They were also asked to explain their life stories in chronological order. Each interview took around one and a half hours so participants and researchers could interact casually. Also, as a researcher, I gained information and impressions that went beyond the interview questions. Each interview was audio-recorded and transcribed with prospective findings documented as memos.

The purpose of the pre-observation interview was to examine the teachers' beliefs about mathematics and mathematics teaching and learning. Interview questions, useful when determining whether teachers have traditional (teacher-centered) or non-traditional (student-centered) beliefs, were selected and modified from those developed by Bahr, Monroe, and Shaha (2013), Gaffney (2014), NCTM documents (2000, 2014), and Tatto et al. (2008). I also asked about their classroom practices and definitions of good instructional practices, students, and teachers in mathematics classroom. The interview questions also elicited their beliefs about and perceptions of their school environments and students' mathematical abilities and motivations, because teachers' beliefs might be influenced by the characteristics of their school and students (Alba, 2001; Hopkins & Spillane, 2015). The full interview protocol is given in Appendix A.

The purpose of the post-observation interview was to investigate the connections among their beliefs, instructional practices, and life stories. I first asked participants to describe their instructional goals, and then whether or not they had achieved their goals. If my classroom observations were not aligned with their stated intentions, I encouraged teachers to explain what factors hindered their accomplishment of their original goals. To remind teachers of particular moments about which I was asking and helped them understand my questions, I showed them relevant video clips of their classrooms, following Speer's (2005) *video clip-based interview* methods. I also asked participants to explain what factors they wanted to change in order to achieve their intended goals. After completion of questions related to classroom observations, I posed questions to elicit their life stories. Following a chronological order, I asked about their experiences before becoming mathematics teachers, such as those in their school and college mathematics courses, and progressed to their becoming mathematics teachers and subsequent experiences with teaching mathematics, professional development activities, and interactions with colleagues.

The life story related questions were developed based on Atkinson (1998), McAdams (1995), and Sun (2017). According to McAdams (1995), the life story interview consisted of six parts: key scenes in the life story, future script, challenges, personal ideology, life theme, and reflection. From the pilot study, however, I found that teachers' responses to questions were quite overlapping. For example, the answers concerning challenging experiences were similiar to their nadir experiences elicited by the question asking for *key scenes in the life story,* and their responses concerning future scripts were similar to the turning point of *key scenes in the life story*. Therefore, I decided to focus questions about *the key scenes in the life story*, that is, those which stood out in the participants' experiences. McAdams (1995) defined a key scene as a moment that "stands out for a particular reason - perhaps because it was especially good or bad, particularly vivid, important, or memorable" (p. 2), such as a particularly positive or negative experience in a mathematics classroom.

Other key scenes might be related to outside of school factors (professional development or personal experiences). Given these circumstances, although I asked the same questions to all participants, I did not ask them to restrict their answers to the mathematics classroom in school. However, the life story questions largely consisted of three dimensions: experiences in mathematics classrooms as school students, as preservice teachers, and as inservice teachers. Participants also were encouraged to explain their stories, including temporality, sociality, and place information as suggested by Connelly and Clandinin (2006). Sample interview questions are as follows, and the full life story interview protocol is given in Appendix B.

1. When you were in school, what was your most memorable experience with a mathematics teacher and mathematics classroom? Is it a good or bad memory? What happened and why were you involved? What did you learn and feel? How is the event connected to your current mathematical beliefs and practice?

Observation and Field Notes Data

Observations allowed me to directly collect information on the participants' actions and the research site, which was used to triangulate the interview data and check for possible

Table 3.2

Mathematics Teachers' Instructional Practice (MTIP) Rubric Developed by Star and Strickland (2008, p. 113)

Category	Description
Classroom Environment	Includes physical setting such as desk arrangements, materials and equipment available and utilized, demographics of students and teacher, class size, grade level, and course title
Classroom Management	Includes the ways the teacher deals with disruptive events, pace changes, procedures for calling on students or handling homework, and the teacher's physical presence (e.g., patterns of moving around the classroom, strategies for maintaining visibility, tone and volume of voice)
Task	Refers more generally to activities students do in the class period (e.g., warm-ups, worksheets, taking notes, presentations, passing out papers) or future activities such as homework or upcoming quizzes
Mathematical Content	Includes representation of the mathematics (graphs, equations, tables, models), examples used, and problems posed
Classroom discourse	Refers to student-to-student as well as teacher-to-student talk and includes questions posed, answers or suggestions offered, and word choice

discrepancies between what participants said and what they actually did. Thus, strengthening the validity of the data (Creswell, 2007). I observed each teacher's mathematics classroom two times and video- or audio recorded and transcribed all lessons observed for later analysis. When observing classrooms, I focused on the relationship between the interview and the teacher's practices using check lists. For example, I examined teachers who claimed to use student-centered pedagogy, focusing on whether they actually taught students following their beliefs or how their life stories related to their instructions. In this process, I took an outsider stance as described in the Positionality section. Through these observations, I sought similarities and differences across teachers because, as Miles et al. (2014) stated, “observing one class of events invites comparison with another; and understanding one key relationship in the setting reveals facets to be studied in others” (p. 30).

During an observation, I scripted important events in my notes. These field notes help record factors that could not be captured by either video or audio. For example, at a micro-level, I noted students’ behavior and small talk, which might hinder or enhance mathematical learning, and at a macro-level, classroom culture, which includes information about classroom norms and environments. In addition, I used field notes to record important events and environmental factors relating to mathematics instruction. In particular, I used a descriptive rubric developed by Star and Strickland's (2008), which consisted of five constructs: classroom environment, classroom management, tasks, mathematical contents, and classroom discourses (see Table 3.2). The present study referred to the rubric as Mathematics Teachers’ Instructional Practice (MTIP).

Data Analysis

I organized data analysis to address the research questions. To address research question 1 (*How do Korean elementary teachers’ sociocultural life stories influence their pedagogical beliefs?)*, I first analyzed each teacher’s life story guided by a content analysis framework developed from previous research (Atkinson, 2007; Sun, 2017), consisting of four dimensions: period, level, subject, and event. Period refers to when a certain event happened: pre-college, college or preservice education, and post-college or inservice teaching. Level refers to the extent to which the event was experienced as positive or negative and how. Subject refers to whether a certain event was related to mathematics itself, mathematical pedagogy (e.g., the teacher’s learning style) or other social-cultural factors (e.g., family members). Event refers to whether the event was primarily a teacher-centered, student-centered, or neutral experience and how (see

Table 3.3). This analysis of each participant's life story focusing on their beliefs and practices shed light onto research question 1. For example, a participant's story about an elementary school teacher who emphasized drill and practices discouraging his/her interest in mathematics was analyzed as a childhood->negative-> mathematical pedagogy -> drill and practices-> discouraging his/her interest in mathematics.

Table 3.3

Content Analysis Framework for Analyzing Life Story

Period	Level	Subject	Event
Childhood	Positive	Mathematics	Teacher-centered
Preservice teacher	Negative	Mathematical pedagogy	Student-centered
Inservice teacher		Sociocultural factors	Neutral

Based on my analysis corresponding to research question 1, each participant's interview data were analyzed further to determine whether he/she held student- or teacher-centered beliefs. As this was a case study, participants' responses to individual interview questions were not rated. Rather, I categorized each teacher's mathematical beliefs by synthesizing his/her interview data. In addition to the teachers' mathematics beliefs, teachers' beliefs about students' abilities and motivation, schools' and parents' expectations are analyzed to improve the accuracy of classification. If a teacher responded positively to interview questions and emphasized students' participation, abilities, creativity, and collaboration, he/she was classified as having student-centered beliefs. If a teacher generally responded negatively to the same questions and strongly emphasized teachers' dominant roles without considering students' contributions, the teacher was classified as having teacher-centered beliefs.

During the data analysis process, however, I realized the necessity of more precise criteria to better represent the balance between teacher-centered and student-centered beliefs.

Therefore, I used a four-level scale to categorize participants' mathematical beliefs: strongly teacher-centered, moderately teacher-centered, moderately student-centered, and strongly student-centered. Although I used a four-point scale to differentiate participants' mathematical beliefs, my categorization was not determined quantitatively by scores or specific ratings but rather qualitatively by my interpretation of the teacher's mathematical beliefs based on his/her words. Thus, my analysis did not yield statistical information but placed the participant teachers on a continuum from strongly teacher-centered to strongly student-centered beliefs.

The observations and field notes data were analyzed together by using the MTIP rubric by Star and Strickland (2008) and *HLIP* rubric by Munter (2014). Like the interview data analysis, individual teachers' instructional practices were classified into one of the four categories. Again, it is important to note that the teachers' instructional practices across the two rubrics were evaluated holistically, and not quantitatively. Figure 3.1 shows the relationship between interview data and observation and field notes data, or between teachers' statements of their beliefs and their observed practices. In this figure, the horizontal line represents the continuum of teachers' mathematical beliefs from student-centered on the right and teacher-centered on the left. The vertical line represents the continuum of their observed instructional practices, from student-centered at the top and teacher-centered at the bottom, resulting in four quadrants represent different belief orientations. This analytic framework was used to answer research question 2, *What is the relationship between Korean elementary teachers' pedagogical beliefs and their instructional practices?* The pedagogical beliefs of teachers in quadrants 1and 3 were aligned with their mathematical instructions, and those of teachers in quadrant 2 and 4 had misalignment with their practices. The framework also allowed me to compare individual teachers (cross-case analyses), which was useful for identifying patterns or differences among the data.

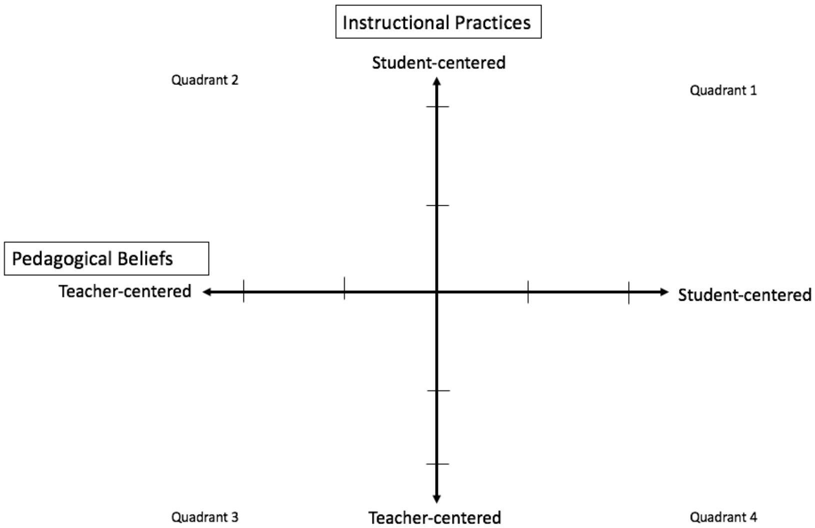


Figure 3.1 Analysis of the relationship between teachers' pedagogical beliefs and instructional practices

To address research question 3 (*How does a theoretical model explain the relationship among teachers' life stories, the development of their beliefs, and their instructional practices?*), the findings of research questions 1 and 2 were analyzed together. This study generally focused on how well a particular life event aligned with teachers' current pedagogical beliefs and practices and why. I also explored participants' life stories to explain misalignments between their beliefs and practices (i.e., teachers in quadrant 2 and 4). That is, I tried to identify certain events on their life stories that could be used to explain the discrepancies between their stated pedagogical beliefs and instructional practices.

Limitation

This study has several limitations related to its research design. To analyze teachers' beliefs and practices in conjunction with their life stories, the study has examined eight Korean elementary school teachers working in four different schools through observations, field notes, and interviews. Although this study might shed light regarding this relationship, there are some limitations. The first limitation of the study is connected with the data collection process. In order to collect sufficient data, at least three observations are recommended (Wilhelm & Kim, 2015), but teachers in this study were observed only twice, which might have resulted in insufficient information to accurately interpret the teachers' practices. In addition, teachers' life stories were collected only through teachers' narration. Because I did not use other data sources to augment the teachers' life stories (e.g., their official transcription or interviews with their previous teachers), and some narrated stories had happened several decades earlier, I cannot guarantee the accuracy of the narrated life stories. Given that I had close relationships with participants and asked them to narrative only events that they could vividly remember, I only assume that they narrated their most memorable life stories without manipulation.

The next limitation is related to qualitative data analysis. I am the only person who interpreted the data and drew conclusions. In this process, my personal bias or educational background might have influenced my interpretations (Creswell & Creswell, 2017). In particular, this study examined eight teachers (three males and five females) who had different educational backgrounds and teaching experiences and who, moreover, were not selected randomly but through their personal relationships with me. In order to reduce such bias, therefore, I analyzed several data sources taking the precaution of explicating my positionality (Darlington, Scott, & Scott, 2002) as outlined in the positionality section. Despite this effort, the findings of this study might include my presumptions about the participants and the Korean elementary school system. Therefore, readers should cautiously interpret the descriptions of teachers, as well as their beliefs and instructional practices. In short, while the number of the participants is not uncommon in qualitative case studies, it does narrow the level of generalizations across the population of elementary Korean teachers. However, the findings and conclusions of this study about the eight participants provides researchers an opportunity to study and interpret the findings and use them to design measures that are inclusive of a larger number of participants.

Generalizability

Creswell and Creswell (2017) pointed out that the meaning of generalizability in qualitative research is different from that of quantitative research in that the findings of a study could not be extended directly to other settings represented by the sample. Schostak (2005) also claimed that the gift of qualitative research is its particularity in describing uniqueness and otherness of subjects, because the purpose of qualitative research is not to reduce individual subjects to part of a homogenous base but to treat them as an individually meaningful, stating that "there is no place where the otherness of the other can be simply reduced to *being just the same as* me at some fundamental level" (p. 12, italics added). From a different perspective, Wengraf (2001) contends that moving from the particularities of case studies to generalizability is not a logical jump. Because certain general concepts are used to interpret data and draw inferences, it is still meaningful to derive implications and conclusions which might be transferred to other settings. Yin (2003), who equates external validity with generalizability, differentiates the analytical generalization of case studies from the statistical generalization of quantitative studies, arguing that the purpose of the former is "to generalize a particular set of results to some broader theory" (p. 37). For example, a case study about neighborhood change might be directly replicated in studies analyzing other neighborhood changes. Then, the accumulation of similar studies would support the generalization of a population transition theory. For this level of generalization, however, Yin (2003) recommends implementing multiple-case rather than single case studies because if multiple cases representing various circumstances arrive at similar conclusions, their findings of the study can combine with those of other multiple-case studies to build a foundation for developing a theory. In addition, the richer the documentation called for in the research design, the better for application to other settings. A well-articulated theoretical framework, for example, enables other researchers to implement the framework in studies of other settings, which can lead to the development of relevant theory.

As a multiple-case study of eight elementary school teachers from four schools, this study has limited generalizability and the findings of this study might not be applicable to other topics and settings. Rich description of the theoretical framework, research methods, and findings, however, might contribute to the development of broader theory describing the relationships between teachers' beliefs, practices, and life stories. In addition, analyzing teachers in four different school settings that are located in different SES sectors of two cities might be useful to demonstrate the replicability of research findings.

CHAPTER FOUR

TEACHERS' LIFE STORIES AND BELIEFS

The purpose of this chapter is to explore teachers' mathematics-related life stories. Each teachers' life experiences are unique regarding period, place, subject, and level (Atkinson, 1998; McAdams, 1996, 2001). In this description, each teachers' life events are presented chronologically from K-12 through college to their current elementary mathematics teaching, in consideration of their social-cultural backgrounds in order to compose a coherent portrait of their trajectories. This chapter is divided into two parts. First, to address the research question 1-a: *What types of events do the participants describe as they recall certain life experiences?* a portrait of each teacher's positive and negative mathematics-related life stories are presented. This is because, portraiture, as a blend of art and science, is useful for articulating the complex events of individuals' lives by revealing their voices and perspectives (Lawrence-Lightfoot, 2005). Next, for the research question 1-b: *Are these events similar across the eight participants? If so, how are they similar and how are they different?* several emerging themes, are developed by synthesizing data across individuals, will be addressed.

Portraits of Participants' Mathematics-Related Life Stories and Beliefs

Mr. Kim

K-12 school period. When Mr. Kim was young, he had very low linguistic and mathematics abilities. Because he lived in a rural area, where there were not any cram schools or preschools, and his parents did not complete elementary school, he received no preparation for school before beginning the first grade. He experienced a turning point in his mathematics learning with his fourth grade teacher, whom he described as an educator who taught mathematics like a dictator. The teacher asked students to memorize mathematics formulas and practice similar problems repeatedly. When students answered incorrectly, the teacher administered corporal punishment, frightening him so much he started to study mathematics on his own.

> Like I said, my fourth grade teacher demanded immediate feedback on a mathematics problem, and if we couldn't give that, we were punished with getting our hands hit. I was scared that I might get hit again. So, for the first time, I studied on my own during the breaks.

Due to the lack of prerequisite mathematics knowledge, he had great difficulty understanding the mathematics concepts in the textbook. However, after several hours of struggling with a division problem, he found his own way to solve it. Unlike the usual method of using multiplication for solving a division problem (e.g., 84 divided by 12 is equal to 7 because 7 times 12 is equal to 84 [84÷12=7 and 7×12=84]), he used a subtraction method, which had not been introduced by his teacher (e.g., 84 divided by 12 is equal to 7 because seven sets of 12 must be subtracted from 84 to reach 0 [84-12-12-12-12-12-12-12=0]). When his mathematics discovery resulted in personal accomplishment, he started to develop confidence in learning mathematics. Thus, his mathematics learning was not wholly driven by his teacher, but by his own endeavors to investigate problems, and he perceived mathematics learning as a process of examining problems and connecting them with his previous knowledge to make sure that he could understand them. These experiences made him believe that solving complex problems through personal investigations would improve problem solving abilities. Despite this insight and his own high mathematics performance, in high school, Mr. Kim's beliefs about teaching and learning mathematics were relatively traditional. He believed that listening to others' ideas was a waste of time and considered discussion and collaboration useless.

His next turning point came in the 12th grade, when his mathematics teacher asked him to justify his own methods and compare them with his peers, which led to his awareness that others' ideas could be more reasonable than his own and his peers might have a more extensive repertoire of problem solving strategies than he had. In addition, the teacher cognitively challenged the students by giving them two or three problems to find as many potential strategies for solving for these problems. Through these processes, Mr. Kim became skeptical of his closed-minded mathematical beliefs, and started to listen to the ideas of his peers. By comparing various problem solving strategies, he realized that his peers thought of methods that he had never even considered, some of which were much better than his own ideas. As a result, he changed his traditional mathematical beliefs into more productive thinking that was in line with student-centered ideas.

Teacher preparation period. Mr. Kim's positive and negative mathematics-related experiences during his teacher preparation program were related to learning pedagogical knowledge. As a sophomore, he completed several teaching modules relating to lesson goals, such as a concept learning module, a principal investigation module, and a problem solving module. The instructor was a mathematics coach teacher in an elementary school, so he not only taught pedagogical knowledge, but also taught how to use the knowledge in the actual classroom that aligned with curriculum goals. This course helped Mr. Kim to realize how to organize a mathematics classroom and introduce mathematics concepts in order to fulfill curricular goals and increase student participation. For example, his fourth grade mathematics teacher had just drawn a triangle on the blackboard and then asked students to memorize the figure. In contrast, the instructor explained several ways in which students could develop mathematical reasoning skills, such as making triangles with various materials and contrasting them with non-triangles that were similar to triangles. But while Mr. Kim enjoyed learning pedagogical knowledge, he had negative experiences in a course on psychology in mathematics education. The professor of this course introduced psychological theory without explaining how the concepts could be applied in elementary schools teaching. Because Mr. Kim wanted to acquire knowledge related to actual classroom environments, he lost interest in the course, which reinforced his understanding of the importance of connecting mathematics theories to actual practices.

Elementary mathematics teaching period. From the early stage of his instructional practice, Mr. Kim focused on introducing more complex problems to increase students' reasoning and problem solving abilities. In the mathematics classroom, he provided students with three or four *big* problems and encouraged them to solve them as his high school mathematics teacher had done. He primarily relied on a mathematics textbook that included advanced mathematics problems, rather than the school curriculum and drew on the knowledge he had gained in pure mathematics courses during his teacher preparation program. He believed that the school mathematics textbook was too easy for his students, and using it would not improve their problem solving abilities.

> I engaged in teaching a mathematics class that valued problem solving. I collected all of the difficult problems from the mathematics subject test and had the students solve one problem every morning. If they weren't able to solve the problem, I forced my students to stay after class with me to help them with what they were struggling with. I thought that I was doing well in teaching mathematics because my students had good mathematics grades.

His instructional practices were again changed by a student's simple questions. One day, when he had taught the formula of the area of a rhombus, one student asked him about the word *rhombus*, “Why do people call a rhombus a rhombus? Where does the term rhombus originate from? These basic questions about mathematics terms challenged his previous instructional practices, which focused primarily on investigating advanced problems. He recognized that he had asked students to solve problems without properly understanding the meanings and origins of certain mathematics vocabulary and concepts. The student’s question precipitated another turning point in his instructional practices, and he started to listen to students’ questions and perspectives.

> When a problem about solving the area of the rhombus was approached, a student in my class asked me why the name of the rhombus was a rhombus. This question has ultimately changed my life about how to teach mathematics because even though I taught the concept of a rhombus for so many years, I never asked why the name of a rhombus was called a rhombus. This means that I never gained the curiosity about what I was teaching while I was teaching this to others. I realized that there were too many moments where I honestly didn’t know what I was teaching to the students. So, I acted like I knew what I was teaching. After that, I searched for the definitions of all mathematics vocabulary and I searched with my own students. From then on, I transformed my view mathematics as a perspective of discovery.

Also, in order to study mathematics education more deeply, he enrolled in master’s and doctoral programs in mathematics education, during which he developed a specific pedagogical pattern that involved connecting mathematics concepts to students’ daily lives and trying to understand students’ perceptions of mathematical ideas in order to keep students interested. Mr. Kim strongly believed that the mathematics classroom should focus on investigating mathematics concepts, not just manipulating tools without understanding. In accordance with this perspective, he took responsibility for students' failures in mathematics, instead of blaming the outcomes on students' lack of mathematical abilities, and he sought ways to develop his teaching skills.

Table 4.1

Mr. Kim's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
4th G Negative MP [a]	A teacher implemented teacher-centered instructional practices with corporal punishment	Started to study mathematics to avoid punishment and found that he could understand mathematics through personal investigation
12th G Positive MP	A teacher implemented student-centered instructional practices with discussion and reasoning	Realized that discussion in mathematics classroom could be useful
College Positive MP	An instructor introduced practical knowledge aligned with curriculum goals and teaching strategies	Understood the importance of curriculum and module (strategies)
College Negative MP	A professor instructed psychological theory without explaining how the concepts could be used in elementary schools	Understood the importance of connecting mathematics theory with practice
EST [b] Negative MP	He could not answer a student's questions about the meaning and origin of a mathematical concept	Enrolled in Master's and Ph.D. program for further study, and started to design mathematics lessons from the students' perspective

Note. [a] MP: Mathematics pedagogy-related events, [b] EST: Elementary school teacher

It was about the year 2008 when I started to feel more like a mathematics expert and I was overconfident with myself to the point where even when there was a class

> observation, I didn't prepare a lot. About 4-5 teachers came to observe my class. The students were too nervous to the point that even when I centered the class on discovery, it just didn't work. I was disappointed that I couldn't provide them with the opportunity of discovery. I realized that teaching mathematics well does not come from just me purely teaching well, but from good cooperation between student and teacher based on the understanding of students' intellectual development and learning. I was too full of myself to not think about such a thing.

Ms. Lee

As described in the methods section, Ms. Lee did not graduate from an elementary teacher program at a government-approved college, but she sought preparation in another subject area, so there was no discussion of the preparation phase of her elementary mathematics teaching career. However, she was the only female teacher who had children so, as in Drake's (2006) study, her mathematics-related life stories were discussed by relating it to experiences she had while raising her children.

K-12 school period. Ms. Lee could not recall a specific teacher or instructional practice that might have positively or negatively influenced her mathematics learning and teaching. She simply recalled disliking studying mathematics. She felt that she did not gain anything useful through studying mathematics and merely considered it a boring and futile subject. In addition, most of her mathematics teachers could not properly engage her in mathematics learning. They used to ask her to memorize formulas in order to solve problems. Unlike her attitude toward mathematics, she loved studying Korean language, which interested her because it provided her with new experiences and knowledge. Also, in the Korean language classroom, she could actively interact with teachers and peers through discussion and express her ideas with writing.

Vastly preferring to study Korean language more than mathematics, she spent more time studying Korean and, consequently, got high Korean language test scores and low mathematics test scores. Based on her positive experiences studying Korean, she decided to be a middle school Korean language teacher. Ms. Lee stated,

> I didn't enjoy math and it was one of the subjects I hated. So, there really weren't any math teachers I remembered, in particular. I had no interest in mathematics and got really bored doing math classes. All I consistently did was solve problems and I don't think I had math classes that were meaningful to me. When learning in classes about other subjects, I learned something meaningful at the end of class. For instance, during a Korean literature class, there were emotional moments, and I experienced and learned new things.

Elementary mathematics teacher period. Because she completed a teacher preparation program for secondary Korean language teachers, Ms. Lee had never learned how to teach mathematics to young children. Her mathematics teaching strategies in the elementary classroom, therefore, were similar to the methods she had hated in her own school years. She believed that her role was to deliver knowledge accurately and was mainly concerned with the clarity of her explanations. Thus, she cared more about mathematics procedures than students' understanding.

> At the beginning, I learned math in a certain way, where it was pure transfer of knowledge and formulas. I initially taught which techniques and formulas to my students. When teaching the upper grades, I mostly taught my students how to get an answer as quickly as possible, which was how I learned it during my school years.

These instructional practices, however, had changed after she began working with Mr. Kim at the same school. Because Ms. Lee was not satisfied with her mathematics instructional practices during her past ten years teaching, she asked Mr. Kim to instruct her on how to teach mathematics. Mr. Kim recommended that she design her classroom to increase students' participation and discussion and to facilitate students' mathematical investigations. Working with Mr. Kim initiated a turning point for Ms. Lee. Among several instructional practices suggested by Mr. Kim, she adopted story-telling methods that resonated with her interest in language, and as an avid reader, she knew various stories. Although combining mathematics stories with teaching mathematics was not easy, she tried her best to incorporate stories involving mathematics learning in order to increase students' investigation and discussion. As her students became active participants in learning and began to develop positive perceptions of mathematics, Ms. Lee felt confident enough to design her own mathematics lesson plans using story-telling. Interestingly, these experiments with using the new methods developed her interest in teaching and learning mathematics. She said that she felt like a student who had never learned the concepts previously because now, it was enjoyable investigating the concepts from different perspectives. She stated,

> Ironically, when I learned mathematics, it was a really boring subject. But as I teach mathematics, I really enjoy exploring mathematical concepts together with my students. I don't know if this is easy for me, but exploring mathematical concepts and solving problems together with my students really make me happy. I think I have experienced joy in mathematics as a teacher, which I never experienced as a student. I think that is why I like mathematics.

Table 4.2

Ms. Lee's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
K-12 Negative M [a] & MP [b]	She could not recall a specific teacher and instructional practice. She disliked studying mathematics and the teacher-centered practices her teachers implemented.	Pursued studying the Korean language, in which students could participate through discussion and writing, so the lessons were meaningful.
EST [c] Negative MP	She usually implemented teacher-centered practices as her previous teachers did, although she felt dissatisfied with them.	Asked for help from another teacher to suggest methods to increase students' participation and discussion.
EST Negative MP	She raised a son who had low mathematics achievement and was unmotivated to learn.	Her new understanding of unmotivated low-achievers led to using manipulatives and story-telling to help them.

Note. [a] M: Mathematics-related events, [b] MP: Mathematics pedagogy-related events, [c] EST: Elementary school teacher

Ms. Lee's second turning point happened while raising her son. Before she had children, she could not understand low-achievers. Given that mathematics achievement was very important for students' future school and career success in Korea, Ms. Lee believed that students had to study mathematics whether or not they found it boring. In addition, she could not understand students who took a long time solving mathematics problems because she used to explain problem solving procedures directly so they should have been easy for students to apply. As a mother raising a son who had low mathematics abilities and was unmotivated to study mathematics, however, Ms. Lee realized that not all students could easily learn mathematics, and they needed different amounts of time to learn new mathematics concepts. Her interactions with her son helped her to appreciate the value of using manipulations and story-telling, which might help less motivated students understand new mathematics concepts and develop positive attitudes toward the subject, stating,

> There is a huge difference in how I treated my students before I married and how I treat my students now. My son is really mischievous. Taking care of such children really makes me grow… How I view my students has drastically changed. I try to understand struggling students who can't listen, who can't focus, and are chaotic by thinking about my own son's face… I realized that there are children who are late on these tasks (mathematics).

Mr. Yang

While Mr. Yang had taken several mathematics education courses during college, he could not recall any courses that had influenced his mathematics instructional practices. When I referred to specific names of courses to facilitate his recall, he just said that all his courses were useless and boring. Thus, his mathematics-related life stories during his teacher preparation program were not further pursued and were not discussed below.

K-12 School period. As a young student, Mr. Yang enjoyed studying mathematics. He described studying mathematics as similar to finding a gem in a mine because although studying the subject itself was challenging, the outcome was valuable and would last forever. He especially believed that reasoning, justifying, and probing were the core of mathematics. These positive perceptions toward mathematics investigation were triggered by his personal struggles with complex mathematics problems. At his first encounter, he said, all problems seemed to be obscure, but he could understand them by applying his cognitive abilities to investigations.

Despite his positive attitude toward mathematics itself, however, he hated learning mathematics in the school classroom, where teachers were the only people who could speak, and students were expected to sit in their chairs and take notes following their teachers' instruction. To reduce the time for solving problems, blackboards were filled with simple mathematics problems for students' rote practice. These negative experiences led to his disengagement from mathematics-related courses during college and professional development (PD). Indeed, during more than 15 years of teaching, he had not taken any mathematics-related PD. He believed that mathematics was a relatively personal subject requiring personal investigation and reasoning, and outside instruction and procedures for learning mathematics were meaningless.

Elementary mathematics teacher period. Because Mr. Yang was committed to developing students' reasoning, he used to give them two or three big mathematics problems to investigate by themselves and find their own problem solving strategies. He preferred not to explain mathematics concepts directly; instead, he was more likely to introduce problems which might improve students' interests in mathematics. He believed that mathematics teachers should focus on mathematics itself, and no other resources. At the time, however, he became frustrated by his students' low reasoning abilities and struggles in mathematics instruction because, except for a few, most of them did not participate in the reasoning processes he had hoped to invoke.

One day, he taught only mathematics throughout the day (around six hours), at the expense of other subjects, to help his students understand mathematical concepts through investigation and to realize the value of reasoning. Despite his endeavor and passion, however, some students could not understand why they had to do reasoning. Rather, they wanted Mr. Yang to directly explain mathematics concepts and procedures. Mr. Yang reasoned,

> Guiding difficult students is the most difficult thing to do. When teaching six graders, every day I had the difficult students remain after class for an extra 30 minutes, and sometimes, they stayed till 8 pm. If they didn't do well, I was disappointed. I tried everything I could but there were students who had difficulties understanding basic math principles. I concluded that there are students who just don't get it and I can't teach every student. For those difficult students, I have them study mathematics with memorization.

Through negative experiences with regard to implementing an investigative curriculum in his classroom, he felt that he had to change his instructional practices in consideration of low achievers' academic struggles and started to see that inductive reasoning alone was not always the best way to learn mathematics. Therefore, he used to implement two approaches for the same topic, one for the student-centered investigation and the other for teacher-centered instruction and drill. For example, he said,

> I realized that investigating problems did not always allow students to gain anything from it. If I provide them with a complex mathematics problem, what can they really gain from it? They can't even solve the problems on their own. Sometimes, I spend about 15 minutes explaining to students and then have them solve similar problems.

Using an investigative curriculum in the classroom, he provided students with sufficient time to ensure their mathematics learning with complex reasoning. However, in the practice-based classroom, he just provided simple practice problems and asked students to solve them as best as they could. He believed that those instructional practices would be useful for low achievers. Overall, he remained less interested in the mathematics tools, stories, and manipulatives, believing instead that students could enjoy learning mathematics through their own mathematics investigations without external aids. Regarding a question about roles of teacher and students, he said:

> There is a lot but for struggling students, the goal is to pressure them to study more. For students who are trained to only mechanically solve problems based memorization, they need to rethink or reconsider the problem they solved. And for those who don't think in that way, you should have them solve it in a mechanical manner in order to understand the pattern…The role of students is to complete things within a given time. I think a good student is someone who completes the problem within the given time.

Table 4.3

Mr. Yang's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
K-12 Positive M[a]	He liked to study mathematics, believing that its outcome was valuable and would last forever	Spent time for solving problems on his through probing, reasoning, and justifying.
K-12 Negative MP [b]	He usually learned mathematics in teacher-centered mathematics classrooms	Believed that external aids were meaningless and mathematics was a personal subject
College Negative M & MP	He could not recall a specific course that was impactful for him, but he remembered that all courses were useless and boring	
EST [c] Negative MP	His students could not engage in reasoning processes	Realized that reasoning was not the best method for some students and direct instruction could be useful

Note. [a] M: Mathematics-related events, [b] MP: Mathematics pedagogy-related events, [c] EST: Elementary school teacher

Ms. Choi

K-12 School period. Ms. Choi liked to study mathematics and enjoyed the feeling of accomplishment gained from investigating complex problems. At first sight, mathematics problems were challenging but she could usually solve them, even if it meant several hours of struggling. Thus, she spent personal time on studying mathematics. Because of her impression of previous mathematics teachers, Ms. Choi remembered classrooms in which she was too afraid to ask questions. One of her elementary mathematics teachers did not let anyone pose questions and required students to follow his procedures. When Ms. Choi asked for an additional explanation, the teacher criticized her, saying, “What were you doing when I explained it? You should focus on my words without doing anything when I speaking” Fearing that she would be criticized again, she never again asked any questions in his classroom. Unlike this teacher, her fifth grade teachers used several mathematical tools for supporting students’ learning and increasing their participation. At this time, she found that she could learn mathematics from not only reading a mathematics textbook but also by using tools, such as,

> When I was in fifth grade, my teacher used various tools to teach math. The most meaningful thing he did while teaching math was having us use various tools. I think I really enjoyed learning math because the teacher taught it in a way where math can be learned through using our hands and not just training our brain and eyes on math knowledge.

With regard to positive mathematics learning experiences, she also remembered a high school teacher who wanted his students to solve problems in various ways and to discuss their different methods to figure out which were most effective. The teacher also used several methods to explain a mathematics problem. Because he introduced several ways to access the problem, all students, including low achievers could enjoy engaging in mathematics activities. However, Ms. Choi generally did not enjoy studying mathematics in her secondary school period. To get a high score on the test, she had to solve several problems without any sense of feeling accomplished.

Teacher preparation program period. Ms. Choi's negative and positive mathematics-related experiences were connected to mathematics pedagogy courses and pure mathematics courses, respectively. As a negative experience, Ms. Choi recalled a professor who forced his beliefs on his students without considering their ideas. While the professor frequently implemented discussion and encouraged student participation, he did not accept any ideas that were different from his own. The professor only gave high points to students who shared his opinions. Ms. Choi observed that presenting a new idea was meaningless in his classroom, so she just kept quiet during his lectures, which sparked her desire for more evenly distributed authority among classroom members.

> My professor emphasized the importance of debating, but mostly the professor provided good scores to students who had ideas that were similar to his/hers and kept saying ideas that opposed his were wrong. That class could have been very insightful if we could have debated about new ideas, but because I could only talk about an answer already given, I really hated it.

Her most favorite mathematics education course, the *History of Mathematics*. In this course, the professor provided interesting and engaging background information about concepts, such as the history of fractions as they were used by Egyptians during the construction of the pyramids. As she developed a greater understanding of mathematics concepts, her interest in learning mathematics grew. Her second positive experience happened while she was student teaching. Before teaching young children, she did not realize the difficulties involved in teaching mathematics because all concepts in the elementary mathematics curricula were very easy for her. When she taught first graders to count to ten, however, she found that it could be challenging even to teach the simplest concepts to young children, especially with their level of cognitive development. Ms. Choi overcame her difficulties with the assistance of a co-teacher in the classroom. The teacher suggested that Ms. Choi implement physical activities and used manipulatives to support students' mathematics reasoning. These experiences helped her learn how to design a classroom to engage students in mathematics learning.

> It was a class during my first year, and the topic of the class was *Knowing 10*. If we think about ten, we might think that counting to ten is an easy task. But in the minds of students, I recognized that it wasn't easy for them. The teacher prepared us by using various tools, such as based-ten blocks. As a student, I realized how difficult it was to teach first graders how to count, which helped me in my preparation for teaching math.

Elementary mathematics teaching period. Based on her past mathematics learning experiences, Ms. Choi wanted to design a mathematics classroom that encouraged students' participation. She tried to use various materials and provide mathematical context by connecting content and problems to students' daily lives. However, when she elicited students' feedback on her mathematics classroom, some students responded that they did not enjoy it because most of their classes were primarily comprised of teacher talk and the manipulation of tools, which did not give them sufficient time to share their ideas. In addition, although Ms. Choi did not realize it, she used relatively teacher-centered methods to explain various mathematics tools (e.g., Pantomino), with which students were unfamiliar. In order to resolve these issues, Ms. Choi asked her mentor teacher to help her, and she also frequently observed his classroom. Conversations with her mentor teacher regarding mathematics instructional practices paired with her students' negative responses created a crossroads, which pushed her to change her perceptions of teaching and learning mathematics.

Table 4.4

Ms. Choi's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
3rd G Negative M [a]	Her teacher did not allow students to pose questions and asked students to mimic his methods.	Never posed questions in the mathematics classroom
5th G Positive MP [b]	Her teacher used several mathematical tools to support students' learning and participation	Realized that she could learn mathematics from not only reading the textbook, but also manipulating tools.
High S [c] Positive MP	A teacher wanted students to solve problems in various ways and justify their methods	Learned how to engage low achievers in a mathematics classroom.
High S Negative M	She had to solve many meaningless problems to prepare for tests	Lost interest in studying mathematics
College Negative MP	A professor only accepted students' ideas if they aligned with his own.	Kept quiet during lectures and wished for distributed authority among classroom members.
College Positive M	An instructor taught about the historical backgrounds of mathematics concepts	Understood the importance of providing mathematics contexts for increasing students' understanding
EST [d] Negative MP	Her students did not enjoy her mathematics classroom because of relatively teacher-centered practices	Allowed students enough time to investigate problems and participate in PD for further learning.

Note. [a] M: Mathematics-related events, [b] MP: Mathematics pedagogy-related events, [c] S: School, [d] EST: Elementary school teacher

> Previously, I talked with a student who told me that my classes were not fun. After having a conversation with that student, I realized that it would have been great to think about being in my student's shoes. Now, I try to teach class where the students enjoy and understand the mathematics. I realized that there are some topics that students might think are tedious, while I think they are fun. So, I have a lot of conversations with my students in order to know what thoughts they have about mathematics and what topics they are interested in.

She realized that allowing students enough time to investigate mathematics problems was more important than introducing mathematical tools. Because elementary students were too young to understand mathematics concepts without personal reflection, simply providing manipulations did not guarantee their understanding. Although she still sometimes introduced mathematics tools, she started to focus on mathematics discussion and reasoning by presenting a few complex mathematics problems which could increase students' cognitive engagement. She also allowed students to propose and initiate mathematical tasks as a way to increase their participation. Additionally, she participated in several PD activities to increase her knowledge-base of mathematics teaching and learning.

Ms. Ko

K-12 school period. Ms. Ko claimed that her most positive and influential mathematics experiences occurred in the eighth grade, when her mathematics teacher encouraged reflection, rather than simple memorization. Up to that point, she had negative perceptions of mathematics learning in school because of teacher-centered instructional practices, in which she was asked to memorize mathematics procedures quietly at her seat and practice similar problems to reduce problem solving time. However, the eighth grade mathematics teacher used student-centered methods that made a great impression on Ms. Ko and led to changes in her perspective on mathematics learning. The teacher encouraged students to discuss problems and solve them with various ways. The teacher also introduced a reform-based mathematics curriculum developed by U.S. mathematics educators and mathematical tools to support students' mathematical investigations. Thus, Ms. Ko experienced mathematical investigation, discussion, and reasoning with her peers on a daily basis. She explained,

> I liked my teacher during my second year of middle school. The teacher personally helped me a lot and mathematics became enjoyable. Specifically, using an entertainment way, the teacher explained the things I didn't understand well and I also participated in a lot of activities relevant to math…I liked activities done by the [reform-based] book. And I also really liked that we discussed and presented how to solve a math problem.

These learning experiences deeply influenced her perception of the nature of mathematics, which shaped her as a mathematics investigator and resulted in increased mathematics achievement. Previously, her mathematics achievement was low (55 out 100 on her sixth grade mathematics test) and her perception of mathematics was fairly negative, consisting only of various operations, such as the multiplication of decimal and division problems, which she disliked doing. However, with student centered curriculum, tools, and classroom environments, she engaged in meaningful mathematics learning with her peers and eventually developed a positive attitude toward mathematics. Discovering that mathematics was not a set of standard procedures, but that problems could be solved in different ways, she realized the joy of mathematics investigation, and the importance of mathematics reasoning processes. Her experiences as a mathematical investigator, along with observations of her teacher as a co-constructor of mathematics knowledge, increased her mathematics achievement and made her believe that she could be an effective mathematics teacher similar to her teacher. As a result, she double majored in mathematics education and elementary education in her teacher education program.

Teacher preparation program period. Her most memorable mathematics-related experiences occurred during her teacher preparation program. Because of her double major, she had to take additional mathematics education courses at a college. Despite her expectations about learning mathematics pedagogy, however, some courses, such as calculus, focused only on pure mathematics, for which Ms. Ko had not been prepared. Also, she was shocked that the college calculus course was comprised of traditional classroom practices, consisting of the instructor's and textbook's explanation of mathematics problems, followed by practicing similar problems. As a result, she gave up further study of mathematics education.

> I did well in [high school] mathematics because I liked mathematics. However, college mathematics was totally different. I had to learn advanced calculus as soon as I entered college. I didn't learn mathematics at a deep level during high school because I was a liberal arts major. So, I didn't understand the content and that was difficult.

Although most of her experiences in the mathematics education program were negative, however, she described one positive experience. As a course requirement, one instructor asked the students to construct a mathematics curriculum handbook which briefly summarized the entire elementary mathematics curriculum. Through this project, she learned how mathematics concepts were related across grade levels and how to introduce the concepts to her prospective students.

Elementary mathematics teacher period. Ms. Ko was the only teacher who said that there had been little change in her instructional practices since her novice period. Although she experienced some positive and negative experiences as a mathematics teacher, her mathematics teaching philosophy and instructional practices were established before she became an elementary school teacher, and she did not experience critical events or influential individuals, which could have changed her mathematics beliefs and instructional practices after graduating from college.

From her first year of teaching, she had believed that learning mathematics was not about acquiring procedural knowledge, but about how to develop conceptual understanding of concepts, how the concepts were related to prior knowledge, and how the mathematics investigation process could be used to acquire new mathematics concepts. Thus, she encouraged students to explore mathematics concepts and provided various tools to support their investigations. However, many students who had already learned mathematics concepts procedurally in cram schools were not interested in mathematics investigations, and just wanted to practice by solving as many mathematics problems as possible. Despite these students' complaints, she strengthened her resolve to implement student-centered mathematics activities and continued to ask students to pursue mathematics practices she believed to be valuable, such as explaining and justifying their mathematics reasoning with classmates. These activities inspired some students' mathematics curiosity and encouraged them to try various methods for solving a mathematics problem. Therefore, she developed confidence in designing and implementing student-centered activities and did not feel the need to change her instructional practices. I asked her whether she liked to teach mathematics and she said,

> The reason why I like math class is being able to observe students who think about how to solve the problems when I simply assign problems. There are students who, after much

> thought, actually provide ways of solving the problem which I never would have thought of. These are the things that I really enjoy.

Another one of her negative experiences in implementing student-centered, investigation-based methods was related to the gaps among individual students' achievement levels. Although most of her students had attended private institutions to learn mathematics concepts in advance, their readiness to learn further mathematics was quite different. When she introduced new mathematics tools to investigate new mathematics concepts, for example, some students felt bored because they already knew what they were supposed to learn from the activities. Other students, however, did not understand the purpose of the activities because they could not connect them to mathematics concepts. To accommodate students' various achievement levels, she provided open-ended questions and tasks which allowed them to select problem solving strategies based on their abilities.

> I like an instructional practice that allows students to actively solve and participate in the problem, which includes having students construct the problems or having them explain the problems. They need something that allows them to actively participate in solving a problem. So, I like students to participate in various activities, which allows them to understand basics concepts.

Additionally, a question about the traits of a good student she said,

> A student who is curious. Even if it isn't utilizing another process to solve the problem, I think students who are asking why they need to solve a problem in a certain way is a good student. Oh, I forgot! A student who is confident about getting an answer wrong. A lot of the students are embarrassed about that.

Table 4.5

Ms. Ko's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
Elementary S [a] Negative M [b] & MP [c]	Her mathematics achievement was low and teachers implemented teacher-centered practices	Had negative perceptions toward mathematics learning and studying mathematics personally
8^{rd} G Positive MP	A teacher encouraged students to discuss and solve problems in various ways and used student-centered textbooks.	Developed positive perception of learning mathematics with investigation and discussion
College Negative MP	Some courses only focused on pure mathematics without providing pedagogical knowledge	Gave up studying mathematics education any further
College Positive M	An instructor asked her to make a mathematics curriculum handbook	Learned how mathematics concepts were related across grade levels
EST [d] Negative MP	Students had already learned mathematics concepts from cram schools, so they were not likely to participate in class	Strengthened her desire to implement student-centered mathematics activities more actively
EST Positive MP	Some students enjoyed investigating mathematics problems and found unexpected reasoning	Realized that she could help students develop mathematical reasoning abilities.
EST Negative MP	Students had achievement gaps and low achievers could not engage in mathematics investigation	Provided diverse levels of problems to increase all students' participation

Note. [a] S: School, [b] M: Mathematics-related events, [c] MP: Mathematics pedagogy-related events, [d] EST: Elementary school teacher

In terms of her PD experiences, she had attended only one mathematics-related PD, indicating she was not very concerned about learning new knowledge to develop her mathematics teaching skills.

Ms. Woo

K-12 school period. Ms. Woo disliked studying mathematics when she was young because of her low mathematics achievement and mothers' admonishments regarding her low performance. For example, her mother was angry when she scored only 48 percent on the sixth grade mathematics achievement test. Thus, she hated mathematics and decided to give up studying the subject any further. However, her brother was able to convince her to continue her study of mathematics. One day he said to her, "I don't care about your low mathematics achievement, but if you get a low score on the test, your friends might jeer at you. Why don't' you study mathematics because it might help you develop self-confidence?" With her brother's encouragement, she started to take extra mathematics classes outside of school and to study mathematics on her own. Despite her improved mathematics achievement, however, she still had a negative perception of mathematics; she viewed mathematics as a subject consisting of unrelated facts and procedures. She summarizes her experience, stating,

> During elementary school, there was a mathematics competition test and on the test, I scored 48 percent. Forty-eight percent out of 100 percent! After I received my test score, I gave it to my mom. After she looked at my score, she was really angry. So, mathematics became my enemy. At the time, I wished my mother had understood my feelings about my lack of interest in studying mathematics as a young kid. However, since my mother couldn't understand my emotions, I became upset and I built a barrier between mathematics and me.

Her eighth grade mathematics teacher was the most influential person in her mathematics-related life stories. This teacher's eagerness to engage students in mathematics learning by using storytelling methods made mathematics interesting to Ms. Woo, and she began to develop a positive perspective on learning mathematics. The teacher's interesting and entertaining stories were effective in helping students to learn new mathematics concepts. For example, the teacher shared his expenditures on dating his girlfriend to provide mathematics problems related to algebra equations. The teacher's teaching strategies helped Ms. Woo encouraged her to pursue more advanced mathematics learning. Ms. Woo commented,

> The teacher really made the class entertaining. He joked a lot and he joked about things that were relevant to the math class. He talked about girlfriends he had broken up with and connect that to mathematics. For example, when discussing about money, he talked about money used for dating.

Teacher preparation program period. Her positive mathematics learning experiences with her eighth-grade mathematics teacher influenced her to become an elementary school teacher and to study mathematics education as a double major. Taking additional mathematics courses in her teacher preparation program, however, made her hate mathematics. Despite her desire to learn mathematics teaching strategies, many courses focused on learning pure mathematics. Because instructors introduced advanced mathematics concepts directly without providing mathematical contexts which might provoke students' curiosity, she could not understand the concepts, so she eventually dropped some mathematics education courses in favor of non-mathematics courses. She offered this reason:

> I liked research about how to best teach mathematics, but when I went to college, I learn about Algebra and Topology…Basically, I didn't study and got a C. When learning

> Topology, I was taught to draw a line and then draw a circle and was told this was Topology. I didn't realize what Topology was. So, I memorized what I drew and then ended up getting everything wrong.

The only mathematics-related courses to which she positively responded was a course introducing mathematical tools and materials (e.g., board games) that could be used in her future classroom to increase students' participation and investigation. After graduating from the teacher preparation program, Ms. Woo was still willing to study new mathematics teaching methods, but she was concerned that she would only be exposed to pure mathematics in graduate school. Therefore, she studied civics education for her master's degree and participated in social groups playing board games which could be used in the mathematics classroom.

Elementary mathematics teaching period. Ms. Woo continued to introduce mathematics-related board games and storytelling in her classroom, and found that her students were excited to engage in such activities. Some students even said that their mathematics classroom was similar to recess time because they could manipulate various games and listen to interesting stories. Also, students discovered how mathematics concepts were applied to board games and how to change the rules of games based on new mathematics concepts. Given such positive feedback from her students, Ms. Woo intended to continue to introduce new games and stories. Her positive attitude toward using games and stories with mathematics investigation, however, was challenged by her new second grade students, who could not properly understand place value systems, although she used base-ten blocks and provided mathematical contexts. When she asked about the sum of 1000 and 3000, for example, her students responded that the answer was 1300. This negative experience prompted a turning point with regard to her instructional practices. She found that sometimes she had to use direct, teacher-centered instructional strategies rather than implementing student-centered discovery practices. Therefore, instead of only using games, she began to ask students to memorize basic addition strategies with drills and repetitive practice. She realized that memorization could be used as teaching strategies and implementing board games was meaningless in some cases.

Table 4.6

Ms. Woo's Mathematics-Related Life Experiences.

Period Level Subject	Event and experiences	Outcomes
Elementary S [a] Negative M [b]	Her mathematics achievement was low and evoked criticism from her mother.	Attended cram schools at the suggestion of her older brother.
8^{rd} G Positive MP [c]	Teacher focused on providing mathematics context with story-telling methods.	Developed interests in learning mathematics and pursuing more advanced mathematics problems.
College Negative M	She could not understand the concepts presented in pure mathematics courses.	Dropped some pure mathematics courses
College Positive MP	An instructor introduced mathematical tools and materials that could be used in class	Realized the importance of using mathematical tools.
EST [d] Positive MP	Students positively responded to learning mathematics with storytelling methods and board games.	Intended to continue to introduce new games and stories
EST Negative MP	Her students could not understand the addition through the use of manipulatives only.	Asked students to memorize addition strategies with drill and practices

Note. [a] S: School, [b] M: Mathematics-related events, [c] MP: Mathematics pedagogy-related events, [d] EST: Elementary school teacher

> During my time as a second year homeroom teacher, I was teaching how to add four-digit numbers, my students didn't understand the basic concepts of it, so I had to go over that for 3-4 hours…I think the students didn't know what a place system was at the time. For example, when adding 1000 and 3000 the students answered 1300.…I ask them to memorize how to add four-digit numbers. I also provided them a similar math problem to solve before going home. I asked my students one-by-one 'How much is something + something?' before they said their goodbyes.

Mr. Sim

K-12 School period. In elementary school, Mr. Sim's mathematics-related experiences had been negative. He felt that his mathematics teachers were incompetent and apathetic. Most teachers could not explain mathematics concepts properly and asked students to memorize mathematics procedures without providing mathematical context. In addition, some of them replaced teaching mathematics with non-mathematics activities during the mathematics class hours. For example, Mr. Sim's sixth grade teacher, who was interested in gardening, took students to playgrounds and asked them to collect flowers and leaves during the mathematics lesson time. When he did teach mathematics, the teacher only taught procedures that he required the students to memorize and practice. While Mr. Sim could solve some problems during class, he forgot most of the procedures afterward because he learned concepts through memorization. He stated,

> During sixth grade, my teacher was retiring soon and all he did everyday was have the students go outside and engage in hobbies he enjoyed. He grew a tree and the students provided food for the tree. The students sometimes did classwork. In the mathematics classroom, we mostly memorized what was in the textbook. If someone had appropriately

> taught me how to measure the volume of a cylinder, I could have then figured out how to measure the volume of a sphere in order to solve problems involving the volume of a sphere. But the teacher just gave me a bunch of numbers and told me to figure it out. I was able to do it during those times, but not afterwards.

To make up for his low mathematics achievement and feelings of learning alone, he took extra lessons at a cram school. However, the cram school teachers' instructional practices were not much different from those of his elementary school teachers. Nevertheless, the cram school teachers were more kind to the students. They always welcomed students' questions and seemed concerned about students' mathematics learning. Being able to get assistance whenever he wanted, he invested a lot of time into studying mathematics. Therefore, he was able to correct mathematics misunderstandings, which led to him enjoying learning mathematics. Mr. Sim found that he could learn mathematics on his own with minimal outside support and developed his own strategies for studying mathematics. As a result, he got high scores on mathematics tests, which led to increased self-efficacy for mathematics. During his middle and high school periods, he rarely listened to teachers' instruction or asked them for help. He viewed teachers' instructional practices just as mere replications of textbooks and these practices did not support his study of mathematics. He believed his personal investigations were more effective for understanding mathematics concepts and procedures.

Teacher preparation program period. Mr. Sim recalled only one useful mathematics course during his teacher preparation program, suggesting that the other courses were useless for him. In a mathematics pedagogy course, a professor introduced various mathematics tools and activities which could increase students' participation. Through this course, Mr. Sim learned several mathematics teaching strategies and mathematics classroom management skills to increase students' mathematical understanding. Another reason why he liked the class was related to the personality of the professor. The professor liked to listen to students' concerns and help them. Because Mr. Sim got personal advice with regard to his future career, he undertook a Master's program in mathematics education while working with this professor.

Elementary mathematics teaching period. During his first several years of teaching, Mr. Sim's instructional practices were linear, which meant that he followed the textbook's procedures without modification. For example, he first introduced lesson goals and then presented activities followed by providing sample problems. As he learned new mathematics education theory and practices in graduate school, he started to reflect on his instructional practices patterns, such as questioning why the teacher should always introduce lesson goals first. What if students discovered the lesson goals through investigation? Were there any issues with changing lesson goals to follow students' investigations? In order to answer to these questions, he conducted an experiment by changing his instructional practices.

> My instructional practices were very simple, at the beginning. I provided the goals of the lesson first, and then I introduced the activities to be completed by the students. I think four years ago I began to recognize the need to change the structure of my classes. I started to think if it is right to provide class goals before engaging students in activities, and if it were appropriate to have students already know what activities they would be doing. I have recognized the possibility that students would enjoy discovering on their own what the goal of the class is. As I began to think in this way, my class structure became less rigid and varied much more.

While Mr. Sim experienced several challenges, he found that his students may be more engaged in investigating mathematics when textbook activities are modified to align with the levels of students' mathematical understanding. These successful positive experiences allowed him to develop various instructional practices according to certain types of lesson goals. When he introduced new mathematics concepts, for example, he liked to spontaneously devise additional unplanned problems by considering students' abilities and motivations.

Table 4.7

Mr. Sim's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
Elementary S [a] Negative M [b] & MP [c]	His mathematics achievement was low and teachers implemented teacher-centered practices	Studied mathematics on his own with the support of his cram school teachers and found that he could learn mathematics by himself
College Positive MP	A mathematics professor introduced various mathematics tools and activities which could increase students' participation	Because of the professor, he wanted to study mathematics more
EST [d] Neutral MP	He questioned his instructional practice pattern, which was to proceed linearly from introducing lesson goals to solving problems	Developing various instructional practices to accommodate types of mathematics lessons and students' abilities
EST Negative MP	He found that some students could not understand mathematics concepts through investigation	Realized that relatively directive instruction could be more effective for students

Note. [a] S: School, [b] M: Mathematics-related events, [c] MP: Mathematics pedagogy-related events, [d] EST: Elementary school teacher

In addition, he realized that, in some cases, relatively directive instruction might be more effective for students than learning through personal investigations. Because he found that some students could not understand mathematics concepts through investigation, he viewed that sometimes teacher-centered instructional practices might be useful strategies. For instance, he said,

> For many years, as I guided students about the basic understanding of ratios and proportions, I realized that based on their own discovery of these concepts, students had difficulty understanding them. Based on the student's level of intellectual development, it is difficult for them to understand the content. So, what I say to my students is that even if they don't understand it now, they don't have to try too hard to fully understand it. I try to lead the class by engaging students in funny stories, which also allows them to use memorization.

Ms. Jung

K-12 School period. Throughout her K-12 school years, Ms. Jung liked to study mathematics because mathematics problems always had a correct answer. Despite her confidence in solving mathematics problems, however, Ms. Jung generally had negative mathematics learning experiences. Her teachers taught mathematics without considering students' different levels of abilities and interests. They used to initiate discourse and then limited student discussion by having them practice similar problems. Because teacher-centered instructional practices were not helpful for her, she was a low achiever in high school who struggled with advanced mathematics concepts, so she sought help from a private tutor, rather than her school teachers. Indeed, she referred to the private tutor as the most influential person in her mathematics-related experiences. The tutor explained mathematics concepts, and then encouraged Ms. Jung to explain the concepts to him in her own words, in verbal and written explanations and provided feedback. Through these learning experiences, Ms. Jung came to the realization that learning mathematics involved expressing mathematical understanding in various ways.

> I didn't like the classes of my school teachers. The teachers only explained and forced students to repetitively solve problems, which was not my style. But my tutor taught differently. He explained it to me first, and then I explained it to him again. Because I usually couldn't understand it the first time, I tried to explain it again on my own. Then my tutor checked to see if my explanation was correct. I really liked to explain what I learned on my own and confirm with someone else because I instantly received feedback.

Teacher preparation program period. Although she had taken several mathematics courses in her teacher education program, she could not recall any that significantly influenced her mathematics beliefs and instructional practices. She just said that all of her courses were uninteresting to her. Ms. Jung only mentioned one teacher during her field experiences, who used a web-based program for teaching mathematics instead of a textbook. In the classroom, the teacher accurately followed the linearly designed webpages and rarely implemented mathematics activities. This teacher also explained mathematics concepts quickly by clicking buttons on the program, and asked students to practice similar problems. Ms. Jung criticized the teacher's practices because she believed that students should construct mathematical understanding through investigations, while cooperating with peers.

Elementary mathematics teaching period. The first school that Ms. Jung taught at had students from low SES who were not interested in learning and lacked understanding of basic mathematics concepts. Because most of her students had low mathematics abilities, she did not have any ideas about how to teach them. She said that when she explained the basic concepts in the textbook, only three out of twenty students were engaged in the lessons, and the other students could not understand due to lack of prerequisite knowledge. Thus, she considered her mathematics teaching at the first school a failure, and she wanted to improve her teaching at the second school. She enrolled in a graduate program in mathematics education and participated in mathematics-related PD. However, her mathematics instructional practices still did not meet her expectations. While students in the second school had high mathematics achievement, most of them had learned mathematics through drill and practice in private institutions before she taught the concepts in her own classroom. Expecting her mathematics instruction to be similar to that of their private instructors, her students disrespected her methods and just wanted to solve mathematics problems.

Moreover, despite learning from private institutions and getting high test scores, most of her students could not properly explain mathematics concepts. Because private instructors focused only on procedural knowledge, students had not acquired conceptual knowledge. For example, when she asked students to explain the concept of one third and one fourth in the fraction lessons, many students could not explain them, but they could solve related problems. She commented,

> I think most students learned mathematics in cram schools through paper [and drill]. They know mathematical principles, but when asked to prove them with tools, they can't do it. For example, when teaching fractions to fifth grade students, I told them to get origami papers and cut them into ½ and then all the way up to 1/9. I observed that the students couldn't present 1/3 with the papers. I thought learning mathematics using only paper is problematic. So, in order to teach mathematics, I began to focus on using other objects.

Table 4.8

Ms. Jung's Mathematics-Related Life Experiences

Period Level Subject	Event and experiences	Outcomes
Elementary S [a] Negative MP	Teachers disregarded students' interests and asked students to practice similar problems.	Studied mathematics alone and disregarded school teachers' mathematics classroom.
High S Positive MP	Her tutor encouraged her to explain mathematics concepts orally and in writing	Believed that learning mathematics involved expressing mathematics concepts in various ways.
College Neutral M [b] & MP [c]	She could not recall any courses that were influential, and responded that all courses were uninteresting for her.	
College Negative MP	A teacher during the teacher practicum explained mathematics concepts by only clicking buttons via websites and asked students to practice similar problems.	Criticized the teacher's practices because of her view that mathematics knowledge should be constructed by investigations in cooperation with peers.
EST [d] Negative MP	Most students had low mathematics abilities, so she had no idea how to teach them mathematics.	Enrolled in a graduate program in mathematics education and participated in mathematics-related PD
EST Negative MP	Students had learned mathematics from cram schools with drills and practices and disrespected her methods.	Introduced various mathematics tools and then asked students to use them to justify their reasoning.

Note. [a] S: School, [b] M: Mathematics-related events, [c] MP: Mathematics pedagogy-related events, [d] EST: Elementary school teacher

In order to make students understand their actual problem solving abilities, she introduced various mathematics tools and then asked students to use them to justify their reasoning. Also, she encouraged students to find mathematics-related stories and objects in their lives. By cooperating with parents, Ms. Jung expected students to apply mathematics concepts they learned to real life issues, such as understanding bills with decimal numbers. Moreover, she organized professional teacher groups with her colleagues to study different ways of teaching mathematics. Interestingly, during the interview, she criticized her co-teachers, due to the lack of their interest in mathematics education. Because they were not interested in mathematics, she had to prepare all of the lessons and materials without external supports.

> Preparing for class is the most difficult task. I have to think and prepare a lot. It is a little easier to prepare lesson with the teachers of the same grade, but most of the teachers who teach the same grade are not interested in mathematics, which makes it difficult. So, I get help from the parents. It might be nice to get ideas from two or three teachers.

Themes in Participants' Life Stories

Common Themes in Participant's Life Stories

Why are teachers willing to teach mathematics in certain ways? How do they improve their instructional practices? Why do they change their instructional practices? To address these questions, eight teachers' mathematics-related life stories, including people, events, achievements, and experiences, as narrated by the participants, were analyzed for common themes, which are discussed below. It is important to note that these themes were not explicitly mentioned by the teachers, but emerged from the analysis of interview data, indicating critical life events, outcomes, and reflections on them. From the ten initial themes, four major themes were common across participants and related to the research questions: *Self-mastery*, *Responsibility and caring for students, Love for teaching mathematics,* and *Perseverance.* These themes would explain why they wanted to improve and change their instructional practices, despite adversity. Second, I elaborated on different themes, which implicitly and explicitly influence teachers' turning points: *attribution of their unsuccessful teaching experiences* and *perceptions of continued learning*. These themes would justify why responses/outcomes of adversity were different, and why individual teachers advocate different types of mathematical beliefs.

Self-mastery. Self-mastery was related to the teachers' desire to perfect themselves as professionals. Having the motivation to achieve self-mastery was likely to strengthen their abilities and will to learn new skills in order to become wiser and more powerful (McAdams, 1993). These teachers' propensity for self-mastery in mathematics learning extended into their careers as elementary school teachers. They wanted to know how to teach mathematics to be good mathematics teachers. As a consequence of their self-mastery processes, all but one described how their instructional changes, resulting from their quest for self-development, aligned with students' increased participation, mathematical understanding, and achievements. While other professionals, such as school principals or mentor teachers, did not explicitly ask them to change their instructional practices, these teachers felt the necessity to change their instructional practices for self-improvement. That is, they continuously self-evaluated and revised their instructional practices, taking into account their students' cognitive and emotional development with regard to mathematics.

When I asked Mr. Yang to talk about his instructional practices, he divided the development of them into three stages: students' mathematical reasoning, students' problem solving activities, and direct instructional methods. Mr. Yang's desire to improve was motivated by his inner desire to pursue self-mastery as a teacher. Similarly, Ms. Lee personally felt the necessity to improve her instructional practices, and so requested assistance from her colleagues in order to develop her teaching skills. When asked to describe how she acquired teaching strategies and ideas, Ms. Jung referred to the mathematics teacher organization in which she voluntarily participated in for professional growth.

Responsibility and care for students. The theme of responsibility and care for students was a strong motivator for participants to develop their own teaching practices and beliefs (McAdams, 1993). They were concerned about students' mathematics achievements, learning experiences, and enjoyment of mathematics, as well as their overall happiness in school. Although individual teachers' mathematics teaching goals differed, their sense of responsibility and concern for their students led them to organize mathematics lessons where students could receive support in learning mathematics. At the same time, caring for their students was a major factor in the teachers' decisions to change their previous teaching practices and adopt new teaching strategies. They assumed their share of responsibility for students' mathematics learning with their endeavor to make learning mathematics exciting and meaningful. Teachers believed that they could impact their students' perceptions of mathematics learning and help them improve their mathematics abilities. When the teachers' students had low mathematics achievement and lost interest in learning mathematics, they felt guilty about their students.

In the face of obstacles, they sought solutions in order to help not only students' mathematics learning but also their daily school lives. Much of Mr. Kim's story was related to his role as a caretaker for his students. He described competent mathematics teachers as teachers who take care of their students' emotional and social needs, as well as their cognitive development. He believed that mathematics teachers should be concerned about their students' daily lives and make sure that they were not stressed because of other factors that may hinder their mathematics learning.

Love for teaching mathematics. Love for teaching mathematics, which referred to teachers' desire to teach mathematics in the face of challenges, was common across all of the teachers, who strongly emphasized this aspect of their role as elementary school teachers. This conviction did not mean that they disliked teaching other subjects or that they believed mathematics was the most important subject. Regardless of their majors in their teacher preparation or graduate programs, however, they all loved teaching mathematics, which they believed was a valuable service in which they could make an important difference in their students' lives. Also, teachers assumed that they could easily modify activities in their mathematics textbooks to make them feel more realistic for their students.

Mr. Yang, who majored in educational counseling, stated that compared to other subjects, which were generally concerned with book knowledge, he could teach mathematics more meaningfully to increase students' investigative abilities. Similarly, Ms. Lee, who had disliked studying mathematics throughout the K-12 period, enjoyed teaching mathematics. In describing her instructional practices, Ms. Lee emphasized her love for teaching mathematics, which would enhance her understanding of mathematics and mathematics education. She learned many things from teaching mathematics and her interactions with students that she had not known before teaching them. Because teaching provided her with rich learning experiences, she loved teaching mathematics, although she had hated it when she was a student.

Perseverance. The last theme portrayed in participants' life stories was perseverance. Teachers believed that facing challenges provided opportunities for further development (McAdams, 1993). Aligned with this thinking, these teachers were resolved to face the challenges in their mathematics classrooms. Most of the teachers could appreciate negative experiences as a way to change their instructional practices and become more effective mathematics teachers. That is, teachers generally perceived their learning goals as attainable, although there were some limitations. They also believed in their ability to triumph over adversities and benefit from challenging events. To a degree, their tendency to persevere was related to their beliefs about their abilities. The teachers assumed that they had adequate mathematics and pedagogical knowledge to guide their students' learning. At the same time, these teachers challenged their students while encouraging them to persevere and develop a positive attitude.

When asked to discuss her difficulties with regard to teaching students, Ms. Jung stated that although she was not a good mathematics teacher at her first school, she wanted to improve as a teacher in her second school. To achieve this goal, she attended PD sessions and enrolled in a master's program. In discussing her nadir or novice experience as a mathematics teacher, Ms. Woo said that her students could not understand basic addition problems. However, she devised activities to support students' understanding. Similarly, Ms. Ko initially felt frustrated with some students who were disrespectful towards her, because her instructional practices differed from those of their cram school teachers. However, these negative attitudes reinforced her desire to achieve her teaching philosophy and encouraged her to design additional activities that might increase students' participation and desire to explore the content.

Different themes in participant's life stories

The purpose of the preceding section was to explain common themes among the participants' accounts of how their mathematics beliefs and instructional practices were constructed and modified. Besides these common themes in the teachers' life events, however, there were variations with regard to how they responded to and resolved similar events. Most of the mathematics teachers' instructional practices and beliefs were substantially changed because of critical mathematics-related life events that were unique to their situations and experiences. These experiences aligned with their individual *attributions of their unsuccessful teaching experiences* and *perceptions of continued learning,* so the consequences of these events were different. This analysis was guided by the teachers' professional growth model suggested by Clarke and Hollingsworth (2002), which highlights the interrelationships among teachers' instructional practices, beliefs, learning experiences, and salient outcomes. With regard to the goals of this research, this section focuses primarily on the life events and outcomes that occurred while they were elementary school teachers. The findings revealed that while there were variations in outcomes and perspectives among teachers in the same group, they could be categorized as *Proceeding* or *Retreating* types concerning how the teachers negotiated and resolved challenges (see Figure 4.1).

Figure 4.1 Teachers' responses to mathematically challenging experiences

Proceeding type. Teachers categorized as proceeding types viewed students' struggles with mathematics as opportunities to devise and implement student-centered practices. Mr. Kim, Ms. Lee, Ms. Choi, and Ms. Ko were teachers in this group. They were inclined to believe that students' faulty understanding of mathematics and low mathematics outcomes could be attributed to teacher-centered instructional practices, and if properly guided in the use of manipulations, collaboration, and investigation, students could actively engage in meaningful mathematics learning (Rousseau, 2004; Smith, Smith, & Williams, 2005; Turner et al., 2011). When students were unmotivated in class, these teachers assumed that their instructional practices were not sufficiently student-centered and failed to respect their students' ideas and participation. In accordance with this perspective, they took responsibility for students' failures in mathematics, rather than blame the outcomes on students' lack of mathematical abilities. As a result, they sought ways to develop their teaching skills and instructional practices to better meet the needs of all students.

Mr. Kim attributed his students' negative attitudes and lack of motivation to his single-minded approach that did not accommodate their different ways of learning. When he described his nadir experiences during the open classroom, he regretted his belief that his way of teaching was not best for his students, rather than blaming students for low participation. He also stated if he showed concern about students' daily lives and happiness at the same time, his students could focus on studying mathematics. Similar to Mr. Kim, Ms. Choi indicated that students' lack of interest in mathematics resulted from her inexperience as a teacher. When her students complained about her instructional style, she listened and was willing to change her practices by observing another teachers' class. Additionally, these teachers implemented changes in their instructional practices not only in response to their students' outcomes, but also by reflecting on their own mathematical learning and teaching experiences.

Most of the teachers mentioned the value of learning from others, as well as from books. They were aware that to face challenges and resolve problems, they must continue to improve their mathematical knowledge and instructional practices, despite the additional workload that continuous learning required. Therefore, they took advantage of additional mathematical learning opportunities. Ms. Lee and Ms. Choi, for example, expressed the benefits of learning from PD and other teachers. They recalled how such experiences facilitated their mathematical learning and helped them adopt more student-centered classroom practices. In particular, Ms. Lee admitted that because she had not graduated from an elementary teacher education program, she would sometimes think that she was not qualified to be an elementary mathematics teacher. Her learning, however, helped her to understand how to elicit students' thinking and help them develop mathematical ideas, which resulted in improved mathematics teaching efficacy. Mr. Kim also showed commitment to acquiring new mathematical knowledge. Although he was a national mathematics curricula developer and mentor teacher, he continued to invest time and effort into improving his practices by enrolling in a Ph.D. program in mathematics education. In sum, these teachers had a growth mindset (Boaler, 2013) that motivated them to continue to seek out opportunities to learn new skills and information and develop their content and pedagogical knowledge.

Ms. Ko was an unusual case in this group. Her instructional practices had changed little since her first year of teaching. Although she rarely attended mathematics-related PD, she always implemented student-centered practices, which she believed were indispensable for achieving her educational goals. When she faced challenges, her response was to implement student-centered practices more actively, assuming that they could help resolve the challenges. In addition, she refrained from criticizing her students' low mathematical abilities and motivations as her reason for changing her practices and implementing teacher-centered ones. During the interview, she mentioned that the biggest challenge was dealing with students' achievement and negative attitudes toward school mathematics. These challenges, however, reinforced her decision to implement student-centered practices. In short, while she did not appear to have a growth mindset similar to the other teachers in this group, she already had well-established preference for student-centered practices.

Again, it is important to note that teachers varied in their perceptions of their students' struggles, their instructional practices, and their quest for further learning. Mr. Kim may have been a level above the other three teachers because he most actively sought out additional learning opportunities and was concerned not only about students' mathematics learning, but also their happiness in their daily lives. Ms. Lee and Ms. Choi were both willing to learn and improve their knowledge of mathematics education without blaming their students, and they participated in PD and asked others for help, even though their passion had not risen to the level of pursuing graduate education. While all these teachers were classified as the proceeding type, their different levels of commitment to pursue additional knowledge and their different tendencies to place blame for students' low achievement and their lack of motivation led to different types of instructional practices and beliefs.

Retreating types. Teachers who were categorized as retreating types were Mr. Yang, Ms. Woo, Mr. Sim, and Ms. Jung. Mr. Yang and Ms. Woo had the tendency to disregard the value of learning mathematics and pedagogy from PD and colleagues. They expressed that they were already well-qualified in mathematics education and confident in their teaching, so they believed that they did not need further content or pedagogical learning. These teachers emphasized their accomplishments by describing some of their students' abilities and enjoyment in their classes, and claimed that they had devised effective teaching strategies through their past teaching and learning experiences. Like proceeding type teachers, they also talked about the value of using manipulatives and students' active participation, but they also criticized student's low performance and motivations as a justification for implementing a drill and practice instructional approach.

This was most evident in Mr. Yang' explanation of how he designed lesson activities. Throughout his schooling and teaching career, Mr. Yang had never learned with others or observed others' mathematics teaching. When he was young, he gave up on learning from teachers as he preferred to study on his own. Moreover, he could not recall any mathematics-related courses during his teacher preparation program because he did not pay attention in class. After he became a teacher, he never attended mathematics PD sessions, believing that his own instructional practices were superior to what they might provide. In short, he believed that because he had overcome all difficulties by himself, the guidance by others was unnecessary.

Ms. Woo also expressed the uselessness of learning mathematics content and pedagogy from others. While she had wanted to learn more about mathematics during her K-12 schooling and had double majored in mathematics and education in college, she rarely participated in mathematics education related PD after graduation. Ms. Woo's only group-related participation was in a board-game community, which consisted mostly of non-teachers. Her negative perceptions of mathematics-related learning from others originated in her college experiences, when she was frustrated by her professors' teacher-centered practices and unengaging course content, especially, in pure mathematics courses, such as college algebra, in which her nadir experiences occurred. Similar to Mr. Yang, she believed that she had successfully overcome challenges she had experienced as a teacher without any assistance. Thus, she had never asked other teachers for help to improve their instructional practices or learn new skills.

Another difference between proceeding and retreating type teachers was that the latter blamed their students' low abilities and motivation as reasons for the failure of their instructional practices, and used them as excuses for implementing teacher-centered practices. They often referred to the impact that students' negative attitudes and lack of motivation could have on their instructional practices, but they never reflected on their own limited teaching abilities. When their student-centered practices did not go as well as they had planned, they were reluctant to reflect on their responsibility in the implementation, but instead preferred to discuss what their students did or failed to do. (Smith et al., 2005; Turner et al., 2011).

When reflecting on working with low achieving students in her first mathematics classroom, Ms. Jung attributed her failure as a teacher to her students. In the second school, where most of the students were high achieving students, she criticized her students' cram school learning as a reason for the failure of her student-centered strategies. Moreover, when asked about current difficulties with regard to classroom preparation, she criticized her co-teachers, who were not interested in learning mathematics education. Ms. Jung seemed to believe that while she had tried her best to improve her instructional practices by attending PD and pursuing a master's degree, other factors always degraded her efforts, resulting in low quality instructional practices. That is, she excused her use of teacher-centered instructional practices, which she did not consider beneficial for students, by referring to external challenges. Similarly, from his graduate program and from reading educational journals, Mr. Sim recognized the importance of flexible instructional practices. He became open to new ideas and methods and was willing to use them in his mathematics instruction to improve his students' learning and his own capacity as a teacher. However, he admitted that he used to blame students' lack of cognitive abilities for his decision to implement teacher-centered practices.

Other teachers in this group also attributed their teacher-centered practices to their students' low motivation and abilities. Ms. Woo shared her reluctance to implement student-centered instructional practices due to her students' lack of understanding of the addition of four-digit numbers (e.g., 1000+3000). Similarly, Mr. Yang referred to his students' lack of cognitive abilities as his reason for shifting from student-centered reasoning activities to teacher-centered practices. In sum, the teachers who spoke negatively about their students and disregarded the value of continued learning advocated teacher-centered practices (Warfield, Wood, & Lehman, 2005).

However, variations existed among these teachers with regard to the depth of their rejection of further learning and their criticism of their students' abilities. Ms. Jung and Mr. Sim was more open to acquiring new mathematics knowledge and talked about their continued mathematics learning experiences after college graduation. While they criticized their unmotivated students, they recognized the importance of continued learning for developing their instructional practices. These viewpoints differed from those of Ms. Woo and Ms. Yang, so some of Ms. Jung's and Mr. Sim's life stories included the characteristics of proceeding type teachers.

Conclusion

The teachers in this study shared similar negative learning experiences with teacher-centered practices, and positive experiences with student-centered practices during their K-12 and college periods. Therefore, they implemented student-centered instructional practices when working as novice teachers, but they were challenged by various factors, such as students' limited abilities and their own underdeveloped pedagogical skills (Drake et al., 2001). In this process, teachers' common characteristics, which included self-mastery, responsibility and care for students, love for mathematics, and perseverance, influenced the development of their mathematical and pedagogical beliefs and their instructional practices. In other words, all teachers in this study explicitly or implicitly expressed their wish to improve their instructional practices and support students' learning, their love for teaching mathematics, and their determination to overcome challenges. These characteristics explain why the teachers changed their beliefs and modified their instructional practices over time (Lutovac & Kaasila, 2018; Raymond, 1997).

However, it is important to note that although these common themes worked as a driving force for the teachers to overcome challenges and change in certain ways, the outcomes of their nadir experiences varied. More specifically, analyses of their life stories revealed a relationship between mathematics teachers' interpretations and ways of negotiating their experiences with failure and their current mathematical beliefs (Rousseau, 2004; Smith et al., 2005; Turner et al., 2011). Teachers' attribution of low quality mathematics instruction contributed to their perceptions of the value of their own further learning, which, in turn, influenced the construction of their current beliefs about mathematics teaching and learning (see Figure 4.2). These different pedagogical beliefs were likely to have an impact on their attitudes toward implementing student-centered or teacher-centered instructional practices.

When teachers perceived that students' abilities and motivation strongly influenced the quality of their mathematics instruction, they were unlikely to believe that they needed to acquire more knowledge about mathematics education. These teachers wanted to resolve the challenges they encountered while implementing student-centered instructional practices by changing to teacher-centered practices (Warfield et al., 2005), which was characteristic of the retreating type of teachers. On the other hand, teachers who assumed responsibility for their own low quality mathematics instruction were likely to participate in mathematics-related educational communities and programs to improve their teaching practices. These teachers actively sought to improve their implementation of student-centered practices by obtaining additional knowledge. Because proceeding type teachers were more concerned about the quality of their instructional practices, rather than individual students' motivation and abilities, they evaluated their beliefs and practices in retrospect and adjusted them to be more student-centered. These self-reflections inspired them to learn additional mathematics pedagogical knowledge, leading them to the development of student-centered beliefs. The analysis of the teachers' life stories, therefore, highlights that the ways in which the teachers interpreted their experiences with failure in implementing student-centered practices influenced their likelihood to pursue additional mathematics and pedagogical learning experiences, as well as their beliefs about mathematics teaching and learning.

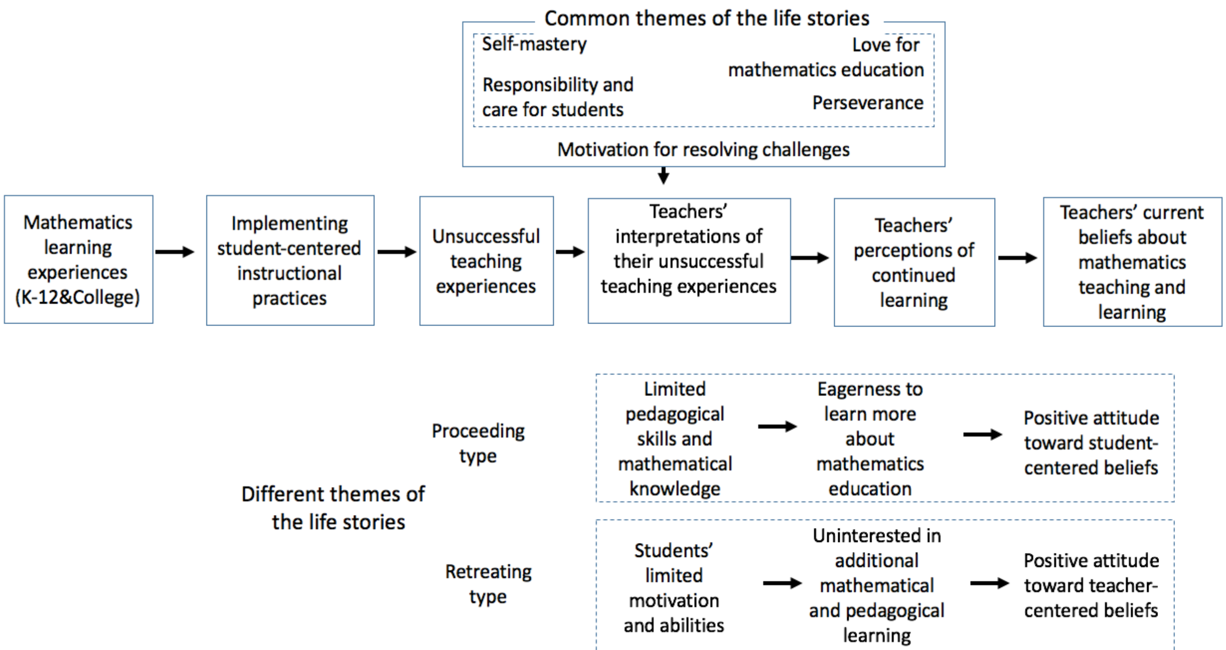


Figure 4.2 Model of the development of teachers' pedagogical beliefs

CHAPTER FIVE

TEACHERS' BELIEFS AND PRACTICES

In Chapter Four, the participating teachers framing of events and stories illustrated how their life stories influenced their beliefs about learning and teaching mathematics. The framework presented in Chapter Two functions as a tool that examines and documents eight teachers' life stories. The participants' life stories represent images of learning and teaching mathematics that are contextually embedded in events and experiences from their childhood years as learners to their early adult years as beginning teachers. The findings presented in this chapter provide information about the participating teachers' mathematical beliefs and practices, indicating how they judged their own instructional practices and came to know themselves as mathematics teachers. Again, the conceptual framework is used as an analytical tool to determine values of dialogue or discourse engaged in by the participants as they responded to contextual challenges of teaching mathematics in the elementary grades.

In this chapter, I report the analysis of teachers' mathematical beliefs and practices and the relationships between the two. I first describe each teacher's pedagogical beliefs based on the interview data addressing research question 2a), "*What are teachers' pedagogical beliefs?*" The target interview questions elicited teachers' views on a) the roles of teachers and students in the classroom, b) the meaning of high-quality mathematics instruction, c) the goals for mathematics classroom lessons, d) the frequently used teaching practices, e) and the teaching strategies used for low-performing students. By answering these target questions, including additional sub-questions, the participants revealed their beliefs about the best teaching practices related to classroom discourse, mathematical tasks, teaching strategies, and student engagement. To address research question 2b), "*What mathematical classroom norms, tasks, and discourses do* *participants identify?"* I report my analysis of participants' mathematics classrooms. To explore the alignment between teachers' beliefs and practices, I analyzed their instructional practices across three domains: teacher and student roles and student engagement, classroom discourse, and mathematical tasks.

The next major section of this chapter focuses on research question 2c), "*What beliefs are relevant to participants' instructional practices?"* During the post-observation interviews, participants were asked about any potential factors that influenced their instructional practices, and factors that either hindered or supported their achievement of the intended goals. In my analysis, I synthesized interview and observation data to investigate the relationships between teachers' beliefs and instructional practices. I plotted the data of each teacher's beliefs and practices onto a coordinate graph with four scales, ranging from strongly teacher-centered to strongly student-centered. The vertical and horizontal axes represented beliefs and practices, respectively, showing any alignment or misalignment between these two variables. It is important to recognize that there may be overlapping life event descriptions with the findings presented in the previous chapter because teachers' mathematical beliefs and practices are partially influenced by their life events. However, the data mentioned in this chapter were collected at a different time, with the time interval lasting for over a semester. Therefore, common descriptions between the two chapters provide consistency with the participants and the relationships among the factors of mathematical beliefs, practices, and life stories. Additionally, I analyzed various factors to provide an answer to research question 3), "*How does a theoretical model explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices*?".

Pedagogical Beliefs and Instructional Practices

Mr. Kim

Pedagogical beliefs. Mr. Kim's pedagogical beliefs reflected a student-centered approach. He believed that students should cognitively and physically participate in mathematical investigations, which would create an environment in which they could express their ideas. Even if students could not fully understand mathematical concepts and accurately express their ideas, Mr. Kim believed that with his support as a facilitator, they could acquire and clearly express mathematical knowledge. He also believed that teachers should modify the mathematics curriculum to stimulate students' intellectual curiosity and reasoning processes during investigative activities. Sometimes, he would spend over two hours on one lesson in order to provide students with problem solving experiences, and other times he would combine two lessons into a one-hour lesson to reduce the time allotted for solving meaningless drill-based problems in the textbook.

Mr. Kim's beliefs in investigation-based mathematics classroom differed from those of other teachers in that he valued investigation process in itself, not just as a tool to understand a certain mathematics concept represented in the textbook. Thus, still following the curriculum standards, he was able to design mathematical tools that allowed students to freely investigate mathematical concepts, following their own interests and understanding. These beliefs were related to his response to the question about the main goals of mathematics instruction. He stated that when he designed his approaches to mathematics instruction, he was most concerned about making students love the subject of mathematics itself. According to him, people who loved flowers study and raise at leisure. These people never exhausted themselves from spending their time and energy on flowers. He wanted his students to love mathematics in the same way. While high mathematics achievement holds its significance in students' lives for external reasons, he believed that loving mathematics outweighed such extrinsic values.

With regard to classroom discourse, he highlighted various types of interactions. He stated that teacher-dominant mathematics classroom discourse, which only allowed students to talk or speak up in response to the teacher's questions (top-down discourse), limited student participation and reasoning processes. Thus, Mr. Kim strongly stressed the importance of bottom-up discourse in which all students had equal opportunities to participate and contribute to their mathematics learning. He also explained that the more opportunities students had to express their ideas, the more mathematics knowledge they acquired. His beliefs regarding students' participation were consistent with his beliefs about teachers' roles. He believed that teachers should encourage students' active interaction and participation so that at the end of each class, they reflected on their engagement in the lesson, such as the extent to which their ideas were similar or different from that of their peers, how they made contributions, and what their acquired knowledge. In response to the question about group work, he highlighted the social and cognitive benefits of group work and the necessity of teacher guidance. He criticized the practice of simply organizing tables so students sat together and then assuming they would conduct group work. Because he viewed that students learned best by expressing, reasoning and justifying their ideas, group work should be guided to challenge students' cognition in the mathematics classroom.

Overall instructional practices. Ms. Kim emphasized three themes in his mathematics classroom: encouraging student participation and reasoning, providing tasks for mathematical investigation, and allowing students to share their ideas. He used different methods to increase students' participation and encourage their reasoning. He provided a few open ended problems for students to solve using various strategies. If students could not find the correct answers, he provided additional information or asked similar problems in order to give them more practice. Although various tasks were discussed and solved as a whole class activity, students were provided opportunities to express their ideas and challenge others' ideas. While instructional practices were not strictly organized, he was able to modify the process of tasks based on his students' understanding. This classroom provided a learning environment where children could become researchers pursuing their research interests. Interactions among students, in response to open-ended questions, provided further opportunities for them to share and evaluate a variety of problem solving strategies.

Mr. Kim did not devote a lot of time to the mathematics tasks provided in the textbook. To achieve the curriculum goals, he designed alternative tasks with which students learned the required concepts through investigation. For example, to explain what a rectangular parallelepiped is, he designed two tasks whereby the first task was an investigation of the properties of a rectangular parallelepiped with cereal boxes, and the second task require students to learn the term for the concept based on their understanding. Because they learned the term after understanding the properties of the geometric figure, the students could naturally connect the term parallelepiped to their own definition or understanding. He did not address the basic problems included in the mathematics textbook at the end of each lesson, but instead asked the students to describe what they learned and felt from that day's class. During this activity, every student had the opportunity to contribute to the discussion regardless of mathematical abilities. These tasks and interactions provided students with the opportunity to become a member of the mathematics learning community.

Another characteristic of his teaching was that, during the class period, he wrote students' names and explanations on the chalkboard when they provided answers, whether they were right or wrong, so all students could remember who said what. Then, he went through each student's explanation and asked the students to raise their hands if they consented with that person's idea. He almost never directly provided solutions to problems; instead, students were able to spend a lot of time discussing potential solutions to problems or best strategies. When he called on a student to explain his/her strategies, he also encouraged the other students to focus on the student who is speaking, saying, for example, "Let's listen to what Ji-ho thinks. Please direct your attention to his explanation." After Ji-ho's explanation, Mr. Kim asked his other students to restate his ideas, "What did Ji-ho say. Can anybody explain his strategies?" Through this process, students would spend a long time investigating a single task and engaging in whole group discussions.

To facilitate students' reasoning when he introducing a new concept, he regularly prompted students to refer to their prior learning experiences and prior knowledge. For example, to introduce the properties of a rectangular parallelepiped, he asked students to describe the properties of a quadrilateral and a square. Because a rectangular parallelepiped consists of six quadrilaterals, these retrospections of related knowledge helped students use their understanding of two-dimensional shapes to easily understand these new mathematics concepts. Additionally, students were encouraged to recognize the importance of mathematical definitions, which could provide a basis for further mathematical investigation. In sum, his instructional practices were primarily student-centered.

Classroom example. Mr. Kim was teaching second graders to make various figures (e.g., dogs and houses) using Tangram puzzles. While this was the first time Tangrams were introduced in the textbook, students already had experiences using them in previous lessons, where they learned the properties of a triangle and a quadrilateral, such as the number of sides and angles. Therefore, he designed more advanced tasks with Tangrams, in addition to the basic activities presented in the textbook. While a mathematics lesson usually took about 40 minutes to complete, this particular lesson lasted about 20 minutes longer. Students were sitting on their chairs, and they had no textbooks and papers; only the Tangrams puzzles were on their tables (see Figure 5.1 left).

Initiation. The teacher asked the students to recall the number of individual pieces of Tangram puzzle and then asked them to assign a number to each piece, from largest to smallest. For example, they gave the smallest triangle the number seven and the largest the number one (see Figure 5.1 right). He further explained that by giving each shape a separate number, students could easily communicate with each other, especially when referring to the different sized triangles, which would avoid confusion. Then, they sang a song with rhythmical movements representing various two-dimensional shapes (e.g., triangles and squares) with their hands. The lyrics of the song were as follows:

> *Tangram, Tangram, what is your property? Square and triangle please come together to make other figures.*

They sang the same song several times, while changing I rhythm. Mr. Kim then turned this dance and song into a problem: “You guys have made triangles and squares through dancing and singing. Today, we are going to make various shapes, such as a house and a rabbit, using Tangrams.”

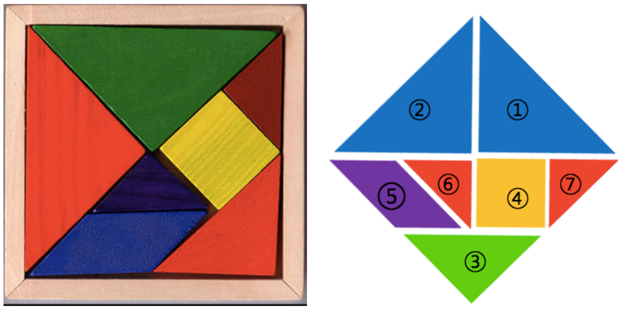


Figure 5.1 Tangram used in Mr. Kim's class (left) and individual pieces as numbered (right)

Activity one. Mr. Kim announced that for the first activity, they would find figures in the Tangram which had similar shapes. Because the Tangram consisted of figures with different sides and sizes, students categorized them in a variety of ways. Mr. Kim asked each student to use the Tangrams tool (Figure 5.1 left). Students worked alone, and then compared their answers with their peers and explained their solutions to each other. He listened to students' answers and asked them to explain their ideas. He did not cut students' explanations short, but he would either summarized students' answers in a concise manner or request additional explanations from them. In this process, students evaluated various strategies and determined the more mathematically reasonable solutions. When they found differences from or similarities to their own ideas, they raised their hands to request an opportunity to speak, thus engaging in intellectual communications. These conversations continued until all students agreed on the best solution.

Mr. Kim: I want you to find pieces that are similar shapes.

A Student: Can we find anything that we want?

Mr. Kim: Yes, you can find anything, as long as they have the same shape.

(After several minutes of individual investigations, he asked the students to raise their hands to share their strategies.)

Mr. Kim: Hyunsu raised his hand. Everybody, please stop what you've doing and listen to Hyunsu's explanation, and think about whether it is similar to yours or not.

Hyunsu: I think numbers 1 and 2 are similar because they are both big triangles and numbers 6 and 7 are also similar because they are both small triangles (see Figure 5.2).

Mr. Kim: Thanks for your explanation Hyunsu. (He writes Hyunsu' name and strategy on the board.) does anybody have any other ideas? Please raise your hand if you have an idea to share.

Mr. Kim: Yoona is speaking.

Yoona: I think numbers 1, 2, 3, 6, and 7 are similar shapes, because they are all triangles.

Mr. Kim: Great Yoona. Unlike Hyunsu, Yoona thought that 1, 2, 6, 7, and 3 are similar shapes (He writes Yoona's name and strategy on the board). Is there anybody else who has any different ideas? How about numbers 4 and 5?

Mr. Kim: Jihun is speaking.

Jihun: Teacher, my idea is similar to Yoona. However, I think that numbers 4 and 5 are different shapes.

Mr. Kim: Why do you think so? Let's listen to Jihun's explanation.

Jihun: I think the side of number 5 looks longer than the side of number 4, so their shapes look different.

Mr. Kim: Everybody agree with Jihun's idea? Raise your hand, if you agree.

Sumi: Teacher, I think Jihun's ideas are incorrect. They both have four sides and points. Thus, the shape of numbers 1, 2, 3, 6, 7 are the same and the shapes of numbers 4 and 5 are the same.

Taehun: Sumi said the sizes of the figures are not important when we are looking for figures with the same shapes. Do you guys agree with her?

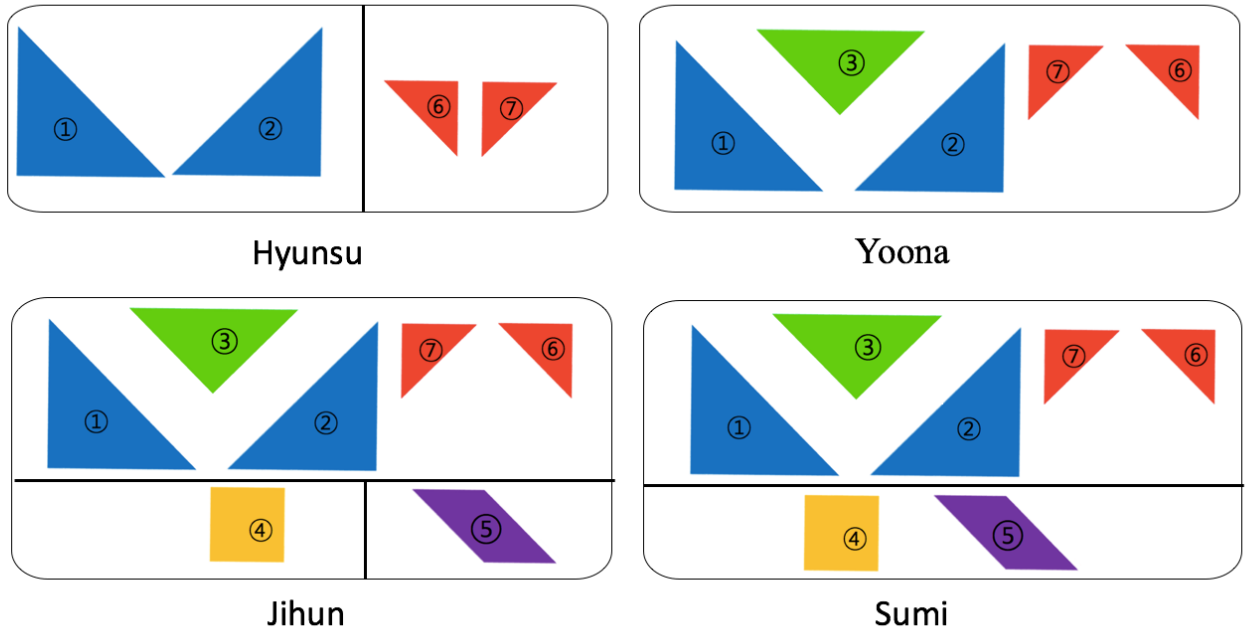


Figure 5.2 Students' responses to finding pieces with the same shape

After additional discussion, Mr. Kim asked students to clap their hands to praise Sumi's presentation and then introduced the solution to the first task, explaining Sumi's answer was correct because two-dimensional shapes should be categorized based on the number of their sides, not size or length.

Activity two. For a second activity, Mr. Kim asked his students to categorize the seven pieces of Tangrams based on their size. When he saw that students were confused about the goal of the activity, he divided it into several stages. He first encouraged students to compare only numbers 1 and 2 with the rest. When they found that the sizes of number 1 and 2 were the largest, he asked them to find the second biggest figure. All the students matched different Tangram pieces with each other to compare sizes and came up with different answers to the question. When they did not reach a consensus, he took a vote of their different answers and then suggested that they first find the smallest figure, which might be easier to determine.

Mr. Kim: Ok, you guys have different answers. Someone thinks number 3 is the second biggest one and others think number 4 or 5. Thus, I want to know your ideas. Please raise your hand, if you think number 3 is biggest. Ok, eleven students agree with number 3. How about number 4? Raise your hand. Ok, eight students. How about 5? Ok, seven students. Then, number 3 is selected as the second biggest figure. However, some students still believe that number 4 or 5 are correct. I don't want to give you an answer now. Instead, we will find the smallest figure first, and then go back to this activity again. Do you understand? I want you to compare the size of number 6 and 7.

All the students held matched 6 and 7 with each other. When they found their sizes were the same, he asked students to order the sizes of numbers 3, 4, and 5 by comparing them with numbers 6 and 7 and explain what they did. Then he called on a student who had said that number 3 was the second biggest piece to see if the student still believed that answer was correct. Because the students had discovered that they could use numbers 6 and 7 to make either numbers 3, 4, or 5, nobody argued that his/her previous answer was correct. Taeyun raised her hand said "Hey, teacher, I found that the sizes of numbers 3, 4, 5 are the same, because they all could be made by using the same small pieces." Taeyun manipulated numbers 6 and 7 several times to demonstrate how she could use them to construct numbers 3, 4, and 5, respectively (see Figure 5.3). All the students listened to Taeyun and watched her demonstration. One student admitted that because the side of number 5 looked longer than others, he assumed that number 5 would be the second biggest figure. Mr. Kim asked the students whether anyone had something different to share. When all the students agreed with Taeyun's idea, he asked them to arrange the Tangram pieces from the largest to the smallest. Individually, the students arrange the pieces from numbers 1 and 2 to numbers 3, 4, and 5 to numbers 6 and 7. Then, Mr. Kim suggested that students call the answer "Taeyun's strategy" to give her credit for her discovery.

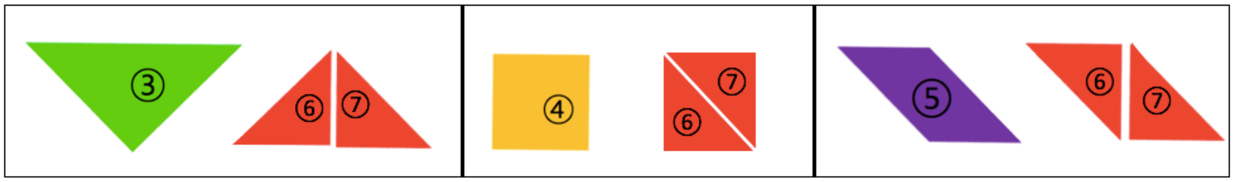


Figure 5.3 Taeyun's demonstration of manipulating Tangram pieces

Activity three. Mr. Kim gave the students an incomplete house (see Figure 5.4) and asked them to complete it. Interestingly, he did not allow them to manipulate the Tangram pieces, but r suggested that they first solve the problem using their own thinking. Students then quietly and individually narrated their strategies while drawing a house. When they were sure of their strategies, they were allowed to use the Tangram pieces to check their answers. When all the students had found the solution, Mr. Kim called on a student who had not previously contributed to the discussion to show her solution. Next, he gave students sheets of paper, and asked them to construct an original figure, such as a rabbit or dog, again without actually using the Tangram pieces. When students completed the activity, he again called on students who had not presented their ideas, and he encouraged them to explain their construction. To wrap up the lesson, he asked students what they had learned and how they felt about it. Additionally, Mr. Kim praised all the students for their participation, saying,

> I know manipulating Tangrams is not easy. However, I believe it is really a valuable activity and all you guys have accomplished great work. You actively participated in the activities and constructed your own original figures. Yangju made an eagle and Suji made a car with Tangram pieces. I really appreciate your participation. I love you guys and next time, we will study pentagons and hexagons.

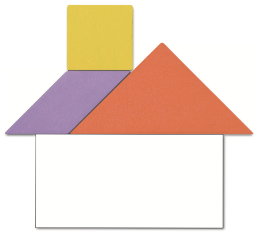


Figure 5.4 Activity three of Kim's class

Roles of teachers and students and student engagement. As they investigated the properties of the seven Tangram pieces, many students were debating about their shapes and sizes. As they engaged in continuous discussion while Mr. Kim guided their reasoning processes and encouraged them to challenge each other's ideas, the students acquired an accurate mathematical understanding. When he selected students to present their ideas, he refrained from judging whether their responses were right or wrong. Instead, he requested other students to contribute their reasoning to reach a reasonable solution, and in this way, students learned about the characteristics of both two-dimensional shapes and logical arguments. They justified their hypotheses with evidence. For example, in the second activity, some students claimed that number 5 (rhombus) was bigger than numbers 3 and 4 because the skewed side looked bigger. However, a student challenged this reasoning and demonstrated her argument by showing that she could form numbers 3, 4, and 5 of Tangram using the same two smaller pieces, which meant that the three middle pieces were same size. While students had learned about triangles and rectangles in previous lessons, they had not yet learned how to compare their sizes. Through discussions and investigations, students discovered a way to compare sizes of different shapes, and thus accepted the hypothesis suggested by their peer.

Classroom discourse. In Mr. Kim's mathematics classroom, students were offered a variety of opportunities to engage in classroom discourse, drawing on their previous knowledge and responding to their peers' arguments. Students started out presenting their ideas, which made sense to them, and were often persuaded to adopt different ideas. While most discourse was in the form of a whole-class conversation, the teacher did not provide direct explanations. Instead, the students led and dominated the conversations, responding to the teacher's questions and each other's presentation, while supporting their claims with examples. The duration of a discourse topic depended on students' reasoning processes. For example, when students were confused while figuring out the comparative sizes of some shapes of the Tangrams, Mr. Kim provided sufficient time for them to reach an agreement. Most of the classroom talk was consisted of mathematical reasoning and arguments, not answers to simple Yes/No types of questions. Students' argumentation spawned additional questions, which helped others to find answers. Mr. Kim rarely implemented procedure-based discourse, and he usually encouraged his students to give more than just the correct answer, by asking questions, such as "Why do you think so?" He focused on conceptual knowledge rather than procedural knowledge, expecting them to justify their reasoning through discussion. In this way, he did not label students' erroneous solutions as wrong but gave them a chance to change their ideas. Additionally, he paid attention to which students were presenting their ideas. He wrote students' names on the board when they contributed to the discussion, so he could identify which students were excluded from the discourse, thus inviting them to join the conversation. In this way, he provided extended learning opportunities to all students.

Mathematics tasks. The tasks that Mr. Kim designed for his students gave them experiences with multiple reasoning strategies. The different ideas presented by students illustrated how they were able to use their previous knowledge of two-dimensional shapes to do the tasks. All of the activities involved using Tangrams as a tool. The practice of labeling the pieces with numbers facilitated mathematical communication, as students carried out and discussed their investigations. These designations provided a neutral foundation for students to identify their answers as correct or wrong. For example, Hyunsu used only four of the seven pieces for his figure, which made other students think about using other pieces for their solutions. Similarly, the smallest pieces (numbers 6 and 7) provided support for students to compare the sizes of numbers 3, 4, and 5 and reach a conclusion that was different from their initial intuitive thinking. While using the Tangrams, did not, in itself, require step-by-step procedures, Mr. Kim intentionally provided activities one and two as a way for students to clearly understand various properties of Tangram pieces. Students were exposed to unfamiliar and challenging tasks, such as comparing the sizes of differently shaped pieces. These activities helped students think and reason about mathematical concepts. In activity three, the engagement in high-level cognitive activities enabled them to draw two-dimensional shapes and solve more complex problems without using concrete objects.

Ms. Lee

Pedagogical beliefs. Ms. Lee believed that her students learned mathematics best by playing games and engaging in activities where they can easily express their ideas. Because some of her first graders from low-SES backgrounds did not understand Korean, she assumed that they could not learn mathematics by simply reading the mathematics textbook. Thus, she generally modified tasks in the textbook to be more like games. In this game-based classroom, she viewed students as active learners and teachers as facilitators. That is, the teachers' job is to provide students with opportunities to think about mathematics problems and evaluate their peers' ideas. In order to increase student participation, she would also read storybooks, and then ask her students to devise mathematics problems or think about mathematics-related situations.

When she used storybooks and games, she was concerned about whether all students could have equal learning opportunities regardless of their abilities. In particular, she disliked the games where the student who could answer most quickly would get a point, and while the other students could not win any points. She preferred games and activities in which all students had the same opportunities to be engaged, such as making up mathematics problems after listening to a storybook. These beliefs about mathematical tasks were connected to her beliefs about classroom discourse. While, as the teacher, she wanted to avoid constantly calling on the same student to provide a quick answer to the question. Rather, she asked many similar questions so that all students could be engaged in her class. Thus, in discussing activities related to storybook reading, Ms. Lee emphasized how important it was to connect mathematical activities to equal learning opportunities.

She also emphasized the importance for students to have opportunities to express both their mathematical ideas and non-mathematical ideas and feelings. At the end of each class, she always asked students, "What did you learn? How do you feel about today's class?" The purpose of these questions was to have all students, including the students who were marginalized because of their lack of mathematical understanding, to feel like they were part of the mathematics learning community. Even when some students joked in response to those questions, such as saying "it just a boring class," she thought that having opportunities to say anything and reflect on their activities and learning might help them develop an interest in mathematics learning in the future. Her emphasis on equality was also reflected her beliefs about evaluation. She rarely evaluated students' answers as simply right or wrong, and she accepted most of the responses.

However, while she saw her job as a facilitator, she acknowledged the lack of challenging tasks and questions to ensure all students' participation. As a result, she said students' responses were at a low cognitive level. She was concerned that providing more challenging problems would exclude low-achievers and decrease overall student participation. She was also worried about the accuracy of students' mathematical understanding, which she knew was neglected because she focused primarily on student participation. In sum, she had a moderate level of student-centered beliefs.

Overall instructional practices. Ms. Lee emphasized three themes in her first grade mathematics classroom: implementing game-based activities, using storytelling methods, and giving equal learning opportunities. Implementing game-based activities promoted student enjoyment while learning mathematics concepts and procedures. She modified mathematics tasks in the textbook, or designed new tasks, to engage students in game-based activities, which were implemented as whole-class activities for all students to participate in mathematics learning. To maintain their interest, she introduced different types of games. With the repeated game-based activities, students gradually solidified their understanding of mathematical concepts and procedures. However, all of the games were simple and at low cognitive levels, enabling students to easily arrive at the solution, therefore, students' actual cognitive involvement was superficial and utterances were at a low cognitive level.

Also, to motivate students, she emphasized story-telling and the stories were not stories in students' textbooks. She personally selected stories from commercial children books, such as folk tales book. Therefore, most of the content was not related to mathematics itself. However, she encouraged students to create mathematical problems and to think about the lesson objectives in the context of the stories. With these methods, she was more concerned about her students' positive attitudes than about mathematical accuracy. For example, after reading a story illustrating a "Wolf and shepherd" (see Figure 5.5), she asked students to make an addition equation. When one student said that one (tree) + one (shepherd) was equal to two, Ms. Lee complimented her answer without noting that in mathematics two quantities with different units cannot be added together. Ms. Lee consistently disregards for mathematical accuracy in her other classes.

Figure 5.5 Sample storybook page used in Ms. Lee's class

The last theme in her mathematics classroom was providing equal learning opportunities for all students. While most students' answers were simple and at low cognitive levels, these discourse patterns helped many students express their ideas and strategies for problems. When a student gave his/her solution to a problem, other students were expected to listen and give similar answers to continue their discourse. They did not compare different strategies or use concrete objects to seek out more accurate mathematical reasoning, unlike the students in Mr. Kim's class. Because most of the students' answers were obvious and clear, such as one + one equals two, high level cognitive discussions, which might have increased student reasoning and problem-solving abilities, did not occur.

The three themes of Ms. Lee's class (implementing game-based activities, using storytelling methods, and giving equal learning opportunities) both supported and constrained students' mathematics learning. On the one hand, students were asked to learn mathematics using simple games that were interesting to them. They were encouraged to present their ideas freely and the teacher did not ask them to justify their reasoning: thus, all students could enjoy engaging in mathematics learning without the fear of failure. Instead, students' mathematical investigations were limited by the types of activities. Ms. Lee led and dominated classroom discourse and wasted time, as she talked about non-mathematical topics. Also, students were asked to solve similar simple problems repeatedly, while the types of games were different. Because the tasks were easy, all students could quickly find answers without having to discuss the problems with their peers. In sum, her instructional practices were moderately teacher-centered.

Classroom example. Ms. Lee's first grade students had been learning addition equations for the past few days. Students were expected to understand both spoken and written representations of quantity. For the final lesson on addition, she prepared several games to solidify their understanding of addition equations. In this class, all activities, but one, involved the entire class. The teacher asked students to come to the center for the first and last activities. The textbook and writing materials were not used in the class.

Activity one. Ms. Lee projected scanned images of previously assigned textbook pages on the board. She asked them to find numbers in the image and then to make addition equations using the figures (quantities) in the textbook. After a student had given an accurate equation, she showed another image. She sometimes called on specific students who raised their hands, but other times she requested the whole class to respond in unison. In these IREs (Teachers' Initiation-Students' Response-Teachers' Evaluation), she did not provide concrete feedback or solicit justification, but rather moved on to the next images. Ms. Lee did not wait for more than a minute to ask a student with a raised hand to present his ideas. When the student had given a correct equation, Ms. Lee showed the next image without asking students to find other equations. Students also interpreted moving on as a signal of correct answers. In this process, only Ms. Lee evaluated the accuracy of their ideas.

Ms. Lee: Ok, Doyun speaking.

Doyun: There are four seeds and four leafs.

Ms. Lee: Good job, then can you guys make an addition equation by using seeds and leafs?

Students: Four plus four is equal eight.

Ms. Lee: Let's look at another image. Can you remember this story? (Because she did not call on students by name, each student is identified as "Student")

Student: Yes. It was a really funny and weird story.

Ms. Lee: Can you find numbers on the textbook page?

Student: Five people

Student: And one monster

Ms. Lee: There are five people and one monster in the story. Can you make a problem?

Students: Yes, five plus one is equal six

Ms. Lee: You did a great job.

Activity two. Ms. Lee introduced a game activity called "Finding a Friend." She distributed nine cards containing addition equations for two numbers with single-digit sums to each student. Ms. Lee said,

> "First, you select one card from the nine cards. When I play the music, you should stand up and walk around the classroom, while holding the selected card on your chest. When the music stops, you should find that another student is standing in front of you. Then you

compare the answers to the equations on the two cards. If the sums on the two cards are the same, you both receive one point (see Figure 5.6)

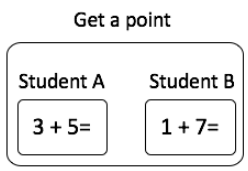


Figure 5.6 Sample cards used in Lee's class

This type of problem was not unfamiliar to the students because they had already made addition equations in the previous activity. There was no discussion of the answers to the equations, and each student had only one answer. The differences from the first activity and the second activity were that students personally selected their own equations and their answers were evaluated by their peers, and not by the teacher. Additionally, all students were constantly engaged in the mathematics learning activity, simultaneously. Within this activity, Ms. Lee did not intervene. She just stood in front of the board and observed the students as they engaged in the activity.

Activities three and four. For activity three, Ms. Lee asked the students to work in groups to play a board game that is relevant to the content in the textbook. The game had several problems that involved reading, writing, and making addition equations. Students rolled a set of die, moved their marker according to the number on the die, and then solved the problem written on the spot where the marker ended. After they had been playing about 10 minutes, she asked the students to come to the center, and she introduced the fourth activity, which had a similar format to activity three. She provided a big board and two large sponge dices. The board itself did not have any mathematics problems, but each group rolled the two together and used the sum of the two number from the dice to determine their next move. Finally, she asked the students to share their new knowledge and their feelings about the activity. She said,

Ms. Lee: Okay, what do you think about today’s class? Anybody want to share their thoughts?

Student: Our group won the game last class. Today, we also won the game.

Ms. Lee: Are you happy because you won the game? Are there any other thoughts? Yemin?

Yemin: It was really fun.

Ms. Lee: Was today's class fun? Thank you. How about Jihyun?

Jihyun: I learned a lot about addition equations.

Ms. Lee: We practiced many addition equations, right? Jihwan?

Jihwan: It was so much fun.

Ms. Lee used students' responses as a method of evaluation of her class, regardless of whether it was related to mathematical content. Students expressed their thoughts and feelings. A number of students expressed their enjoyment of the various games in her class. Some students seemed to understand the mathematical purpose of the games, but most students' reflections were limited to the activities themselves. Perhaps, they were sure that any kinds of answer, even a non-mathematical reflection would be accepted by the teacher, who did not hold to students to the expectation that they would only talk about their mathematical learning.

Roles of teachers and students and student engagement. Ms. Lee played two roles in her mathematics classroom: guiding mathematical activities and helping student to engage in the activities. She introduced the goal of each activity and managed students' participation by communicating with them. She selected the tasks, provided students with equal learning opportunities, and maintained students' attention. Although not all of the students were able to present their ideas, most students had opportunities to speak and felt included in the learning community. Nobody rejected their peers' ideas and every student valued and listed to their classmates' thoughts and ideas. To make sure that the students in her class had equal opportunities to participate in class activities, she usually presented them with basic problems to which they could easily find the answers without making mistakes. However, she was concerned about students' participation at the expense of mathematical accuracy. When students gave incorrect answers, she just accepted them and moved on to the next problem. Also, she rarely used her students' ideas to modify and improve her lesson plans and activities. Students were only allowed to investigate the tasks within limitations that Ms. Lee set. Given that the games had strict roles, the students engaged in all activities as passive-investigators.

Classroom discourse. Ms. Lee's classroom discourse involved students acquiring mathematical concepts by repeatedly articulating the ideas they derived from the stories and games. As indicated in the first activity, students were expected to verbally express mathematical concepts. Because the teacher presented similar types of problems, however, students needed to be mindful of the types of responses the teacher accepted to properly answer the problem themselves. This resulted in students simply repeating their peer answers to the problems. For example, after one student answered a question with "Four plus four is equal to eight," another student answered the next question with "Five plus one is equal to six." In addition, the students' utterances were at low cognitive levels. The teacher also accepted the answer without engaging the students in any discussion. As a result, students were not to contribute to the development of peers' mathematical understanding because they did not explain their reasoning. While most students were able to engage in such classroom discourse in whole-class conversations, Ms. Lee dominated the process of class discourse, which was aligned with the IRE protocol.

Mathematics tasks. At the beginning of her lessons, Ms. Lee showed interesting images from storybooks to increase students' learning motivation. Students formed various addition equations with the figures given in the textbook. This task allowed her students to connect mathematical understanding to an imaginary context, which may have helped them use addition equations to solve real life problems. The second task was to practice verbally expressing addition equations through the “Find a Friend” game. The third and fourth tasks involved various modes of expressions: writing, reading, and formulating addition equations. Because the board games asked students to express equations in a variety of ways, they connected various expressions to solidify their previous understanding. However, Ms. Lee consistently engaged students in many similar low cognitive demand tasks in which the goal was acquiring procedural knowledge.

Mr. Yang

Pedagogical beliefs. During the interview, Mr. Yang continuously pointed out the benefits of using repetition when learning mathematics, which he believed helped students retain mathematics concepts for a long time and was the most effective way to learn mathematics, especially for low-achievers. When his students did not understand mathematics concepts, he assumed that they had not practiced enough repetition, so he explained the concepts repeatedly and asked them to solve many similar problems. Because he equated mathematics learning with repeated practice, he introduced mathematics concepts and procedures directly at the beginning of class, and then had students work on many similar problems related to what he had just taught. Sometimes he would also implement reasoning-based instruction. However, he thought it would take too long for most of the students to discover correct mathematical concepts; therefore, he decided that this was not an effective strategy for acquiring mathematical knowledge. Instead, he described his reasoning activities as another type of repetition, which could solidify students’ understanding of mathematical concepts. Overall, he viewed mathematics as a collection of rules and algorithms that had to be memorized.

His beliefs about the types of mathematical tasks and students' abilities were aligned with his beliefs about the roles of the teacher and students. He stated that the teacher's role in student learning was comparable to the role of a manager. When students could not focus on their learning, teachers should employ external pressure to encourage them to study more. Whatever teaching methods used, even drill and practice, he believed that teachers can make sure that all students understand mathematics concepts. In this environment, the students' role was to follow their teacher's guidance and accomplish the tasks presented to them. He described his best students as those who strictly complied with his instruction without question, and he only allowed those who had acquired their knowledge in this way to investigate more complex problems. Until they reached this point, he wanted his students to follow his directions unconditionally.

In terms of classroom discourse, he did not value student-student interactions because he believed that communicating with others would take time away from individual memorization of concepts and repetition of procedures. Rather than allow time for discussion, he wanted to give his students as much time as possible for personal practice under his direction. Although he mentioned that he allowed student-student interactions for reasoning based activities, he constantly referred to teacher-centered instructional practices during his interview, and in describing the discourse patterns of reasoning activities, he said that he asked students to come to his desk and explain their reasoning processes. In this procedure, he was practicing teacher-student interaction, as he was the only person who evaluated his students' ideas and provided feedback. Student-student interactions were limited to group tables, and were not extended into whole class discussion. It is evident that he had strong teacher-centered beliefs.

Overall instructional practices. The principal features of Mr. Yang's instruction were giving direct explanations and providing plenty of practice for his students. He explained mathematics concepts and procedures in an explicit and concise manner. He did not expect his students to present new strategies and discuss different solutions, and he was the only person in the classroom who provided mathematical knowledge, which for the most part, concurred with the contents of the textbook. He did not design new tasks in accordance with students' abilities, preferably, he wanted all students to have an accurate understanding of the mathematics concepts in the textbook, irrespective of their abilities. He first read mathematical concepts out of the textbook aloud, and then he demonstrated how to solve the problems. Using a document camera, he highlighted his textbook and asked students to highlight their textbooks in the same way. He also used the document camera to show students how he solved problems and encouraged them to follow the same procedures. Sometimes, students would use manipulative tools but not in an exploratory way. In addition, the students used the tools to model and practice procedures demonstrated by the teacher, which did not require them to use investigative methods.

In Mr. Yang's classroom, a repeated practice of procedures was viewed as a primary means by which students learned mathematics. His students, therefore, spent most of their time solving many similar problems, first those in the mathematics textbook, and then those in the student manual that provided homework exercises related to the textbook content. While students worked individually to solve problems, the teacher provide support to struggling students. He stood next to them and demonstrated how to solve the problems, while the other students were quietly solving problems without disturbing anyone. They completed most of their work individually and did not have the opportunities to present their solutions through a group discussion or a whole class activity. He called out numbers of problems while providing an explicit explanation, whereby, a few students provided the correct answers. Thus, the students' understanding was primarily based on the teacher's explanation and their independent problem-solving, and not collaborative activities and discussion. Indeed, most of these problems could be solved with routinized steps Mr. Yang suggested for his students, so further discussion was not required to figure out answers.

Mr. Yang communicated with students by explaining new concepts and asking for answers to related problems. However, only a few students, who had already had learned the concepts, had the opportunity to express their ideas. He rarely called on his students, by name, to participate. In Mr. Yang's class anyone who could rapidly think of the correct answer would be heard, while the rest remained silent. After receiving the answer he sought, he moved to the next explanation without soliciting others' ideas. Generally, only one answer was allowed per problem, and because the same students always provided the class with the correct answers, most of students were excluded from the process. Also, he only evaluated students' solutions to practice problems, never probing their thinking. Therefore, classroom interaction was a simplified process of procedurally delivering mathematical knowledge and having students solidify his explanations with practice problems with no expectation of either independent or collaborative learning among the students. In sum, he had employed primarily teacher-centered instructional practices.

Classroom example. During this unit, students were learning about the concepts of *ratio and rates*. I observed a lesson about the rate of speed. While the textbook included the steps of initiation, three activities, and sample problems for this lesson, Mr. Yang combined them into two activities. The first activity involved initiation, and the second activity consisted of all other activates and sample problems. His strategy, in this modification, was to have students learn the mathematical concepts and procedures through the first activity, and then practice applying their understanding through the second group of activities. For the first activity, he explained the mathematics concepts to the whole class, and for the second activities, students worked individually and then checked their answers through a whole-class discussion, in which he used the document projector to show students what he wrote in his textbook.

Activity one. Mr. Yang began the lesson by asking students to the recall mathematics concepts from the previous lesson, which comprised of understanding rates and conversing rates. He did not call on students by their name, and the whole class answered his questions in unison. After asking several questions, he showed the image of a textbook page illustrating the railroad route between two cities and encouraged students to think about the mathematical situation (see Figure 5.7.). He also expected his students to accurately understand the figures and text in the projected images by pointing out these elements with a red pen. When some students could not understand the key mathematical concepts, Mr. Yang used a sample problem, which had been taken from the previous lesson, to guide them.

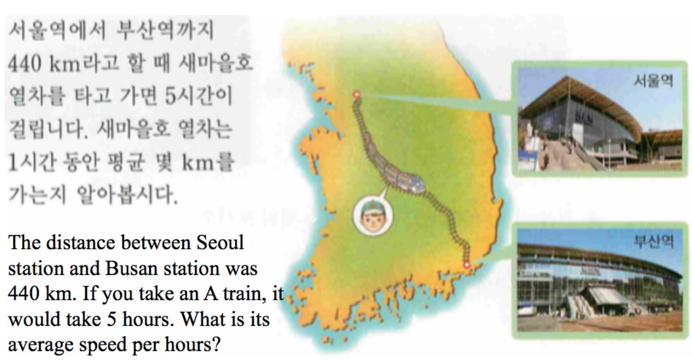


Figure 5.7 Textbook image used in Mr. Yang's first activity

Mr. Yang: Where is it? (Pointing to the Seoul station on the textbook page.)

Students: Seoul station.

Mr. Yang: Where is it? (Pointing to the Busan station on the textbook page)

Students: Busan station.

Mr. Yang: Is the distance between the two stations close or far?

Students: Very far.

Mr. Yang: By taking the A train, it would take five hours. How many kilometers does the train go in one hour? What is its average speed? Okay. Today, we will study how to present distance and time as one rate. I mean quotients.

Afterwards, he read each sentence in the textbook out loud while guiding the students' investigation with such questions, such as "What is a rate unit? What is the number?" After students responded to his questions, he read the mathematical definition in the textbook to the class, and explained the meaning of reciprocal rates, including hours per kilometer (km/h) and minutes per meter (m/m). He asked students to solve the problem of how many kilometers the train goes in one hour? and said, "If you understand the concept correctly, you guys can solve this problem in a minute". When a student answered correctly, he very briefly explained the solution.

Mr. Yang: What is the A train's average speed?

A student: 88 km per hour.

Mr. Yang: Right, 440 divided by 5 is equal to 88. So, the answer is 88 km per hour (He solved the problem with a red pen and showed it to students using a document projector.)

Activity two. Having spent about 15 minutes for the first activity, he explained the second problem in the textbook, which was to find the average speed of another train, which would take five hours and thirty minutes to cover the same distance. After combining two activities and a sample problem in the textbook into one activity, he wanted his students to solve the rest of the problems by themselves. He said,

> You know, the problem format of activities two and three, and the sample problems are quite similar. You just divide the distance by the time. You won't get an incorrect solution to the problems, if you pay attention to the numbers and the rate unit to determine the rate. When you write answers, please do not forget to use a dash, such as 60-meter. I will move around the classroom, so raise your hands if you have questions.

He walked around the classroom to support his students' individual problem-solving efforts. Most students worked alone and a few students occasionally requested help from their peers. In the post-observation interviews, he stated that he usually stood next to the struggling students to support their learning, because the other students can solve the problems independently without his support. He asked students to solve problems in the student handbook after completing the problems in the textbook. Mr. Yang then explained the answers to the problems, as he had done in activity one. He read some sentences in the textbook to the class, such as "What is the average speed of the B train?" and a student quickly presented the answer. Then, he demonstrated how to solve other problems, using his red pen, with the document projector.

Roles of teachers and students and student engagement. Mr. Yang's primary role in the classroom was to deliver mathematical knowledge correctly. He clearly explained mathematical concepts and procedures, using the textbook religiously. He showed the problem solving process in writing, on a textbook page projected for the whole class, expecting students to follow his procedures. He never changed his lesson plans and completed all of the tasks he had prepared for the lesson. He provided clear explanations when correcting students' misconceptions. In addition, he was not concerned about students actively participating in mathematical discussions because he did not call on students by name, and the same students continued to share their ideas and solutions. Mr. Yang used the same mathematical formula to solve similar problems and linked the formula to the previous lesson so that students could reinforce their procedural knowledge. In Mr. Yang's classroom, most of the students had no ownership. They could not determine the types and pace of tasks, and were only evaluated by the teacher. Their participation in the classroom was very limited, and the only students who could contribute to the learning process were the ones who knew the correct answer. Students were required to be quiet during the problem-solving activities and simply followed the procedures demonstrated by the teacher. They were encouraged to progress at the same rate and to complete the same tasks using only their writing utensils.

Classroom discourse. Most of the time, Mr. Yang dominated the classroom discourse and provided easy problems and simple questions to his students. While he did not frequently use Yes/No questions, all of his questions could be answered with two or three words. Additionally, because these questions only had specific and objective answers, there were few opportunities to expand on the solutions given by students. He asked questions in a whole-class setting and did not keep track of who spoke, but he expressed satisfaction when the same students rapidly provided correct answers, so he did not hear from his other students. When students struggled to comprehend his questions, he rephrased them in ways that would be easier to understand. Students rarely posed questions, and instead, focused on responding to his questions. They neither talked about other concepts or ideas beyond those in the mathematics textbook, nor evaluated their peers' ideas and solutions. Because most of the classroom discourse took place between the teacher and a few students, most of the students were marginalized in the learning environment, which emphasized the development of procedural knowledge and computation skills and impeded the progression in conceptual knowledge and problem-solving abilities.

Mathematics tasks. All of the tasks given in Mr. Yang's class were drawn from the mathematics textbook and the student homework book. He used figures and texts from the textbook for students in order to achieve lesson goals. He strictly followed the sequence and level of textbook, and he used the exact tasks given in the textbook in order to guide his instructional practices. Because the textbook did not introduce any tools besides different types of problems which might support students' mathematical thinking, he did not use any other tools in class. The problems in the textbook were low cognitive demand tasks, enabling students to arrive at solutions without difficulty. The sequence of the tasks, first learning the mathematical concepts and formulas, then applying them to solve similar or somewhat advanced problems, also helped students acquire procedural knowledge. For example, the answer to the first problem was a natural number, and the second problem had one-digit and two-digit decimal numbers. Therefore, most of the tasks provided students without the opportunity to investigate mathematical concepts as they might have done if engaged in pursuing challenging questions.

Ms. Choi

Pedagogical beliefs. Ms. Choi seemed to vacillate between teacher-centered and student-centered mathematical beliefs. On the one hand, she equated her students' role with that of researchers, which involved conducting individual inquiries and discovering mathematics concepts with the teachers' guidance and classroom materials (textbook and tools). She stated that she expected students to express mathematical ideas with both verbal and non-verbal (e.g., visual representations) modes. Because these students were not familiar with the origin of these concepts and equation, they could not fully understand these concepts without their own personal investigations. Oftentimes, she assessed student learning by encouraging them to explain their ideas to their peers and evaluate the ideas by using comparison. When students engaged in problem solving activities and provided incorrect answers, she invited students to support their peers. She was reluctant to directly evaluate students or intervene in their investigation and reasoning activities, and she sought to foster a knowledge-sharing community.

On the other hand, some of her pedagogical beliefs were more teacher-centered. While she incorporated student-centered practices into her instruction, she still held strong beliefs that students learned best from direct instruction. During her interview, she stated, “I believe in the value of student-centered activities, but these activities are not effective in terms of achievement. I cannot properly manage my students during discussions and manipulations. Some students constantly make disrupting noise and focus too much on the activities themselves, and not on acquiring mathematical understanding.” Moreover, she believed that students should learn mathematics concepts as a universal language; otherwise, they could easily forget newly learned concepts and skills. That is, she viewed memorization as a useful learning strategy that enables students to retain learned concepts and skills for a long time. Despite the challenges she encountered implementing student-centered practices, however, she still wanted to use them, believing that when students learned with understanding through various means of expression, they could apply this knowledge to other situations.

For classroom discourse, she emphasized student-student interactions, rather than teacher-student interactions. She believed that students should be provided opportunities to express their ideas. In order to increase student discourse, therefore, she intentionally called on low-achievers, who were likely to provide incorrect answers when she requested students’ answers to problems. Even though those students’ answers were incorrect, their ideas provided foundations to initiate further discussion, therefore, leading to increased student participation and reasoning. She would frequently modify activities in the mathematics textbook to be more practical in classroom discourse activities that originally seemed to represent individual problem solving activities, which might limit students' interactions. She tried to design new tasks which involved student engagement and discussions. In sum, she had moderate student-centered beliefs.

Overall instructional practices. The three themes in Ms. Choi's class were providing opportunities to speak, enhancing students' interest, and giving students authority. In her class, many students were involved in the whole class discussions. They were provided with many opportunities to present their ideas and solutions. Before the class, she modified the tasks and problems from the mathematics textbook because they were relatively straightforward and students' participation would be limited. For example, one of tasks in the textbook asked students to write down the difference between 4:6 and 6:4. However, she provided other related problems and asked them to verbally express the differences as a group in a whole-class activity. Sometimes, she would ask her students to come to the front of the classroom to explain their ideas and have other students respond to them. She also skipped other pre-planned activities during the class in order to give all students who raised their hands the opportunity to express their ideas.

Ms. Choi second theme of her class was increasing students' interest, which was aligned with the first theme. For example, when she perceived that the tasks in the textbook might be boring to the students, she designed new tasks to grab their attention and increase their motivation. Some activities were game-based, and others were related to real-life contexts. For example, students were given newspaper articles containing survey data, such as the types of entertainers people would select as vacation partners; then she asked them to make a circle graph representing the information. Originally, the survey data presented in the textbook was the amount of water used in a household, but she assumed that this topic would not excite students' curiosity. Students talked about the newspaper article and engaged in mathematical discussions about how to represent the data with a circle graph. For this goal, she felt that she did not need to cover all of the tasks from textbook and follow its sequence.

Ms. Choi also gave her students the authority and responsibility for their own learning. She adapted tasks and provided students with several options, so they could choose based on their own preferences. To give a clear example, students were given the option to solve problems either with other students or independently. Additionally, she did not ask students to solve all the problem on a worksheet, but rather let them select problems based on their abilities. Regarding the assessment of her students' work, Ms. Choi asked them to check their answers with their peers and get their sign-off as an indication that their answers were correct before she checked their answers. Similarly, students were able to respond to their peers' ideas during the whole class discussion. When a student presented an incorrect answer, she would say, "Is there anybody who wants to help her?" or "Do you guys have any different ideas?" These classroom norms helped student actively interact with their peers and provided opportunities for them to be mathematics investigators. In sum, Ms. Choi had strongly student-centered instructional practices.

Classroom example. Ms. Choi's class example was a lesson on *rate and ratio* units, in which students were expected to make a circle graph by analyzing a table of data. For this class, she designed a new task, which consisted of two activities: making circle graphs individually, and sharing graphs with others as a whole class activity. Instead of using the textbook, students were given a handout with a newspaper article about vacations. Students remained sitting in their seats throughout the class except when the presenters came to the front of the classroom to explain their circle graphs.

Activity one. To initiate the activity, Ms. Choi asked her students to recall the mathematics concepts they learned previously and describe the components of a circle graph and the procedures for drawing one. Next, she asked her students non-mathematical questions about the newspaper article they read to motivate and increase their interest in the learning task. She did not explicitly tell students what they were going to learn, instead, she provided questions to encourage students to think about the goal of the activity. After the class had had a brief conversation about the topic, she explained the lesson's goal.

Ms. Choi: Today, I prepared a newspaper article about vacations. What do you imagine when you hear the word vacation?

Student: Food

Student: Camping (Additional comments from students).

Ms. Choi: Yes, you guys have a lot of ideas about vacations. So, if you could select one entertainer as a vacation partner, who would you select?

Student: Taehyun Cha (a Korean entertainer) (Additional comments by students)

Ms. Choi: What place do you want to go to for a vacation?

Student: Hawaii (Additional comments)

Ms. Choi: Yes, we can think of a lot of things about vacations. Now, I will give you the newspaper article that contains data from four surveys analyzing 500 people's responses to questions about vacations: what entertainer would you choose as a vacation partner, what vacation places will you visit, what vacation theme will you choose, and what kind of food you want to eat on your vacation. I want you to select the data for any one of these to make a circle graph. If you have enough time, you can make more than one circle graph.

Students selected from one of four survey data and explained to their partner why they selected it. Using different color pens, students worked independently to make their graphs (see Figure 5.8). Ms. Choi walked around the classroom and assisted those students who needed help constructing their graphs. In this activity, students had the authority and responsibility to work as mathematics investigators to solve the real life problems.

Name of survey data:

Color						Sum
Count						
Percentage (%)						

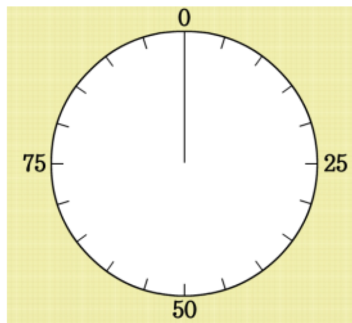


Figure 5.8 Students' handout in Ms. Choi's class

Activity two. In the second activity, Ms. Choi did not assign a new task to her students to complete. Instead, she asked her students to show which survey data they represented on their graphs. She called students by their name and asked them to use the document projector to share their work with the class. The students were expected to interpret data presented by their peers; they considered the different interpretations of the data represented in the circle graphs. Some students focused on the ranking of the data as a list, while others were concerned about the most or least selected item.

Ms. Choi: Ok. Are there any students who made a circle graph that represent the vacation theme? (Students raised their hands.) Haewon, can you please bring your handout to the front of the class? Let's look at it with the projector. This is

Haewon's circle graph. Can you interpret the graph? What does she want to represent? (Students raised their hands.) Junhi is speaking.

Junhi: In terms of the theme about vacation, 53% responded that they wanted to have a vacation where they ate good food, 11.3% preferred a vacation where they participated in activities, and 6.7% preferred to relax during their vacation.

Ms. Choi: Great job. Junhi interpreted the graph using percentages. Are there other ways to interpret the graph? Dayung is speaking.

Dayung: More than half of the people wanted to have a vacation where they could eat food, and it was the most popular vacation theme.

Ms. Cho: Dayung focused on the most popular theme. Other ideas? Honggi is speaking.

Honggi: The response to the first item is much greater than that of the second item.

Mr. Cho: Thank you Honggi. Will someone provide a clearer explanation of Honggi's idea?

Junghun: Eating food received about five times as many votes, than participating in other activities.

Ms. Choi: That is also a good way to interpret a circle graph. So, did anyone make a circle graph about places to go on a vacation?

Following a similar procedure, Ms. Choi had her students discuss the four types of survey data represented with their circle graphs. As a closing activity, she asked students to describe what they had learned from the lesson. The objective for this activity was for the students to represent the survey data using circle graphs and for them to interpret their peers' circle graphs in order to illustrate their understanding of survey data. In this way, her students took on the roles of both problem-solvers and problem-makers. The students solved problems with data provided by their teacher and constructed problems to share ideas with their peers, which included a verbal explanation.

Roles of teachers and students and student engagement. The students in Ms. Choi's classroom were given the authority and responsibility for their mathematics learning. They selected the tasks to complete or the data to summarize in graphs and evaluated their peers' solutions. They were encouraged to present their ideas and engage in the learning community by participating in classroom discussions. Ms. Choi abstained from evaluating her students' ideas and she encouraged them to challenge their peers' ideas as they engaged in classroom discussions. Also, she provided relatively high-cognitive problems, such as making a circle graph. As a result, students' products become a source of additional mathematical problems and knowledge. Through their engagement in classroom discussions, the students learned how to make and analyze circle graphs to reason, justify, and offer proofs about their mathematical ideas from different perspectives. Additionally, Ms. Choi, as a facilitator, guided her students' discussions, as well as gave them sufficient time to think about questions and ideas presented by their teacher and their peers'

Classroom discourse. Ms**.** Choi promoted different types of classroom discourse. She asked her students to explain their strategies to their partners and to engage in a whole class (student-student) discourse. Also, she encouraged students to share their ideas with her (teacher-student discourse). Classroom authority and responsibility were widely distributed, and the students were free to engaged in classroom discourse. Although the teacher initiated much of the discourse through questions, Ms. Choi did not provide answers to her questions (see Honggi's case). Instead, she elicited her students to respond to their peers' ideas. In this way, one student's comments generated new investigations and discussions in which they explored errors and offer mathematical justifications. Ms. Choi provided low cognitive questions at the beginning and ending of the lesson and high cognitive questions and tasks during the activity, so all of her students could contribute to and engage in the learning process.

Mathematics tasks. The students in Ms. Choi's classroom analyzed real data collected from newspapers articles to learn mathematical procedures and concepts. The four types of survey data helped her students to think about their own ideas and solutions and also to challenge their peers' solutions and ideas. Because the circle graphs contained various kinds of information, students were able to focus on different aspects of the graphs and provide different interpretations. This element was illustrated in responses to Haewon's circle graph**.** Making a circle graph involved procedural knowledge, therefore the students needed to follow certain steps to complete it accurately. However, Ms. Choi also had her students think conceptually about the graphs by providing an additional task that encouraged the students to analyze and interpret their peers' product. All these tasks helped her students develop mathematical understanding and served as a basis for additional mathematical investigations and communication about solutions to circle graphs. When the students presented their graphs, the task as presented on the handout provided clues for figuring out right or wrong answers. In the same way, the circle graphs helped the students to become mathematical investigators.

Ms. Ko

Pedagogical beliefs. Ms. Ko's beliefs about mathematics learning and teaching can be characterized as strongly student-centered. Because she was mainly concerned about students' mathematical understanding and authentic thinking, she often asked students to make a new problem with concepts from the textbook and have them explain their reasoning to their peers. Additionally, she wanted to provide materials and mathematical tools to support students' reasoning. Her goals for mathematics teaching were well represented by how she adapted mathematics textbooks. While she did not design original tasks, she believed that modifying these tasks to support students' investigations would be effective.

Her beliefs about ideal classroom practices were related to her goals for her students' mathematics learning. She viewed the purpose of mathematics instruction is to help students understand the function of mathematics in their lives, which was what made the concept itself meaningful. Through cognitive-based activities, she believed, students could develop an interest in mathematics investigations to enhance their reasoning abilities. In response to the question about the characteristics of good mathematics students, she stated that they were students who devise different methods, express their ideas without fear of being wrong, and strive to seek out answers. Additionally, she believes that teachers should not present easy problems to students with the intention that they will be encouraged to learn more mathematics, because constantly solving simple problems would actually decrease students' interest in tackling more challenging problems. She also contended that while some differences might exist with regard to studying styles and preferences, all students have similar abilities. Namely, she assumed that while some students dislike studying mathematics, they had the potential to acquire new knowledge and contribute to others' mathematics learning, which was why she expected all students to explain their ideas to peers and be evaluated by them. These pedagogical beliefs were well represented by her example of common instructional practices to have each student write his/her ideas on a large sheet of paper for other group members to evaluate it using post-its.

Ms. Ko's beliefs about classroom discourse were consistent with her beliefs about best instructional practices. She explained that student-student discourse in the mathematics classroom should be encouraged; otherwise, some students could be excluded from mathematics learning communities. Although students' oral interactions would increase the noise level in the classroom, she viewed such interactions as essential to their mathematical understanding. Similarly, she was not likely to use teacher-directive methods in order to dominate classroom discourse or to evaluate students' incorrect answers with yes/no responses. Instead, she said she preferred to use why/how questions to sustain students' mathematics reasoning processes. In short, she had strongly student-centered beliefs.

Overall instructional practices. The three features of Ms. Ko's third grade mathematics classroom were manipulating concrete objects and mathematical tools, engaging in mathematical discussions, and solving complex problems. In lesson to support her students' mathematical learning, she gave them tools to work with in addition to problems provided in the textbook. In one lesson about the basic concept of fractions, she gave her students a pair of scissors, sheets of paper, and slices of bread to help them develop their understanding of dividing fractions into equal parts or pieces. For another lesson, in which the students made various shapes with a compass, she also gave her students geometry tools, such as protractors and rulers. Aligned with these instructional practices, she modified tasks from the textbook. For example, the activity in which the students cut pieces of paper and slices of bread was derived from a textbook lesson about how to divide a pizza into two and four equal parts. These experiences contributed to the development of students' mathematical understanding and led to additional discussions, whereby, the students were expected to reflect on their activities and ideas and describe to their peers how they used the tools and objects to figure out solutions.

The second feature of Ms. Ko's lesson was engaging students in mathematical discussions. Students discussed the different strategies they used to solve complex tasks with various types of manipulative tools. She always called her students by their name and encouraged them to use their language skills to articulate their solutions. Students were expected to describe verbally their strategies, so that their peers could understand them. She repeatedly asked questions, such as “Do you have other ideas?” to elicit students’ participation. All of the different strategies articulated by students were used to promote mathematical understanding of new procedures and concepts. Students learned from the teacher, their peers’ contributions, and their own reflections. In order to facilitate student discussions, Ms. Ko did not follow pre-determined procedures, but she flexibly managed the class time and the type of classroom discourse. She did not push students to move on to the next activities. Rather, she provided additional time when students wanted to continue a discussion to present their ideas, although it meant skipping a pre-planned activity.

The last theme of Ms. Ko’s lesson was investigating complex problems. Typically, Korean elementary mathematics textbooks provide three activities, including initiation and sample questions. Therefore, students were expected to spent about 10 minutes investigating one activity. However, she would prepare only two or three complex activities because she wanted her students to investigate challenging problems without wasting time on solving practice problems. The complex tasks offered students opportunities to become mathematics investigators of which they used tools and applied different strategies and ideas to solve the problems that they would share with their peers. Investigating complex problems encouraged students to engage more deeply in the mathematics learning process. Because they had not solved such problems previously and were learning new strategies, students did not feel pressured to provide correct answers. Also, this approach limited high performers from dominating classroom discourse and activities. In sum, Ms. Ko’s instructional practices was strongly student-centered.

Classroom example. Ms. Ko's first lesson was a unit about fractions and decimal numbers. The classroom tables were arranged for a group activity. The lesson's topic was *dividing equally*. Ms. Ko prepared two activities. Working in a group, the first activity required her students to divide slices of bread into two and four equal parts, and for the second activity, the students worked independently to divide sheets of paper into four equal parts. For the first activity, she asked the group members first to discuss how to cut the bread into equal parts, and for the second activity, she encouraged her students to find as many strategies as possible to show how to divide the sheets of paper.

Activity one. Before introducing the first activity, Ms. Ko posed two mathematical problems involving fractions and asked students to think about the answers. The first situation was dividing six slices of bread equally among three people (6÷3=2). Because students had learned about division and multiplication in the previous units, they could easily find the answer. The second problem was how to divide one slice of bread equally among three people (1÷3=?). Although this situation called for division, the answer was not a whole number, thus, the students had difficulty finding the solution. Ms. Ko also asked the students to share their experiences where they had divided one quantity into several pieces of less value. Then, she introduced the first activity, in which the groups divided a real slice of bread into two and four equal parts.

Ms. Ko: One day, Taeho's mother give him six slices of bread and asked him to share them equally with two of his friends. Thus, how many slices of bread can Taeho eat?

Students: Two

Ms. Ko: Then, can you represent the story with a mathematics equation? Please raise your hands, if you can represent it. Sungjin is speaking.

Sungjin: One

Ms. Ko: Do you think that six divided by three is equal to one? Do you guys all agree with him?

Students: No.

Ms. Ko: How about sharing your idea? Jiwoo is speaking.

Jiwoo: Six divided by three is equal to two.

Ms. Ko: Do you guys all agree with her? (Waiting several seconds) Right, Jiwoo's answer is correct. Then, what if Taeho's mother give him three slices of bread? How many slices can Taeho eat?

Students: One.

Ms. Ko: Sadly, this time Taeho's mother give him only one slice of bread. How many slices can Taeho eat?

Student: Tear the bread into three pieces.

Ms. Ko: Wow, that's a good idea. I was wondering if you guys had similar experiences dividing one quantity into several pieces? Raise your hands, if you want to share your experiences. Jieun is speaking

Jieun: My friend, Suyoung, had a jelly sandwich. She gave me half of it by dividing it into two equal pieces.

(Several students shared their experiences.)

Ms. Ko: As we discussed, sometimes we should divide one quantity into several pieces. For today's activity, I want you to investigate how to solve the problem. The first activity is equally dividing a slice of bread into equal parts or pieces.

Ms. Ko gave each group a slice of bread and a ruler and asked the students to divide it into equal parts. She encouraged the students to first discuss how to sliced the bread. After the students had completed their investigation, Ms. Ko asked them to explain how they sliced the bread slice. Students reported using different strategies, including slicing the bread horizontally, vertically, and diagonally into equal parts (see Figure 5.9.). She also asked the students to explain how they made sure that they sliced the bread into equal parts. Some students said that they folded the two halves pieces together, and others said they used their ruler to measure the length of the sides.

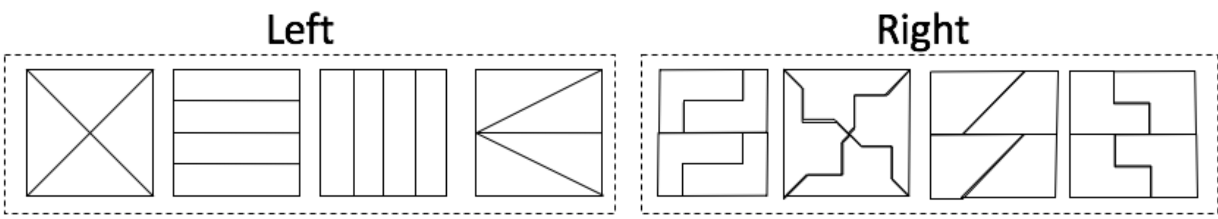


Figure 5.9 Different strategies for activity one (left) and activity two (right)

Activity two. Ms. Ko's second activity was an independent one. The students received several sheets of origami paper and were asked to find various ways to cut the paper into equal parts. But before they did that, she first asked her students to explain their strategies for cutting the paper into equal parts to their group members. For Ms. Ko, having students explain their strategies to their peers and receiving feedback from them would limit their errors. She also gave a display board to each student and asked the students to paste their products on their boards, which would be used for a whole class discussion. Ms. Ko said, "I hope you will cut the paper in various ways. However, if you think your friends' solutions are great and you do not have any ideas of your own, you may follow their ideas. Learning from your peers is also a good learning experience." About 15 minutes later during the lesson, she asked the students to wrap up the activity. Then, she asked students to bring their display board to the front of the classroom and to explain their solutions. Compared to activity one, students provided a wider variety of problem solving strategies (see Figure 5.9).

Roles of teachers and students and student engagement. For this lesson, students investigated challenging tasks and compared various strategies. While Ms. Ko had the authority because she presented the tasks and sometimes, evaluated students' answers, she distributed some of the authority and responsibility to students as well. Therefore, many issues during the lessons were resolved through students' discussions. Students explained their ideas to their group members and to their peers during the whole class activity. As members of a mathematics learning community, the students learned from their peers. They were encouraged to find various strategies and describe them verbally. Ms. Ko viewed this process as a way for her students to develop their confidence in learning mathematics. She also helped the students describe their thinking and ensured that their explanations were clear for the other students, asking, "Do you guys all agree with him/her?" She introduced to her students mathematics problems in familiar contexts to help them apply their prior knowledge and experiences to current mathematical topics.

Classroom discourse. Whole class conversations were initiated by Ms. Ko. She provided accessible mathematical contexts with open-ended questions to encourage students to investigate mathematical problems, which led to mathematical understanding. She provided indirect feedback to her students' answers and asked questions to elicited alternative strategies from other students, such as by asking, "How about your idea?" She also implemented group discussions, encouraging the students to explain their ideas to their peers. Such discussion facilitated student engagement in the mathematical learning community in which they collaboratively solved challenging problems and shared their ideas with their peers, as they challenged and justified various ideas to refine mathematical solutions and concepts. As a result, the students had some of the authority and responsibility in the classroom discourse and they produced more diverse ideas during the second activity than they did during the first activity. Similar to Mr. Kim, Ms. Ko called on students by their name and asked them to explain their ideas to their peers. At the same time, other students were expected to listen carefully and evaluate their peers' solutions.

Mathematics tasks. The classroom activities consisted of two parts, working with group members to divide a slice of bread into equal parts and working independently to cut sheets of origami paper into equal parts. Consequently, the students investigated one warm-up task and one challenging task related to dividing a square into four equal parts. Because the tasks had several answers, students could approach them from various entry points. Students freely manipulated concrete objects and used them as ways to justify their arguments. However, the challenging task was not limited to personal investigations. During the activity, students were encouraged to connect verbal and physical representations and share their investigations with their peers. They explained and compared their multiple solutions. Through this task, students learned about the concept of fractions, but they also learned how to reason mathematically and how to effectively engage in classroom discussions. As explained by Ms. Ko, the students had had similar experiences related to the current task. Previously, on different occasions, they had divided candy, pizza, and jellies into equal parts. The connections between mathematical tasks and daily practices might have helped her students to respond to challenging problems and offer various solutions to those problems.

Ms. Woo

Pedagogical beliefs. Ms. Woo believed that a mathematics teacher should be concerned about topics and activities to increase students' interest and engage them in mathematics learning. In order to achieve this goal, she used contextual situations and board games related to mathematics lessons. For instance, she designed tasks based on “Harry Potter” to teach multiplication and division of decimal numbers, asking them, “if Harry Potter drinks three 1.5L bottles of water, how much water does he drink?” Interestingly, although she provided additional tasks following each new story, she declared that she also used all the textbook tasks in her instruction. When I asked how she managed the time for the stories, and new tasks and all the problems in the textbook, she replied that because her students were smart, they could solve all of the mathematics problems in a textbook in less than twenty minutes. When she did not use story-telling methods, she prepared additional handouts to engage them in the remaining forty minutes. She considered that her best instructional practices involved using story-telling and games to introduce new concepts and providing additional problems to solidify acquired concepts.

In terms of group organization, she preferred to apply individual and whole-class activities as opposed to group activities. During an interview, she stated, “Well, I am implementing group activities once or twice out of ten mathematics lessons. I don’t want to waste time. Also, you know, students’ abilities are quite varied. Due to the achievement gap, some students did not benefit from the group activities.” Her negative beliefs about group activities were reflected in her lesson plan. While her school administrator asked teachers to design lesson plans to increase students’ collaboration, such as respecting peers’ opinions, communicating with others, and caring about discrimination among peers, she was not concerned about such issues, as she believed that mathematics learning was a relatively personal domain. Instead, she designed lesson plans with new stories to stimulate each student’s cognitive curiosity and individual engagement in problem solving activities, which she considered more meaningful than group work.

To understand her beliefs more clearly, I asked her how she evaluated students' mathematical understanding. She responded that she explains correct answers on the blackboard and then asked students to evaluate their own answers. She would also collect all students' test sheets and evaluate them primarily to check their understanding. Ms. Woo believes that when she provided students with correct answers, they would understand mathematical concepts accurately and gain more thorough explanations than they could get from their peers. As shown in her classroom activity patterns, she believed that mathematics learning was a personal domain, and most students could not evaluate or contribute to others' learning by justifying their ideas and critiquing others' ideas. Given her beliefs about classroom discourse patterns and practices, despite her use of games based activities and story-telling methods, her overall pedagogical beliefs were classified as moderately teacher-centered.

Overall instructional practices. The three themes of Ms. Woo's classroom instruction were disregarding students' participation, directly criticizing students' low performance, and modifying the tasks in the textbook. In her classroom, the authority was not distributed. She dominated the classroom discourse and required her students to be quiet. She determined what problems students would solve and directed the students to follow her guidance. Students' opportunities to participate in the lesson were very limited. She allowed only one student to respond to each of her questions and students were not allowed to initiate discourse, unless they wanted Ms. Woo to clarify her instructions. During two observations of Ms. Woo's teaching, there were not any group or manipulative activities, which could have improved students' participation and reasoning abilities. The students were expected just to listen to the teacher's explanations and take notes to acquire the procedural knowledge. Also, she did not have them investigate any challenging problems. When a task in the textbook was challenging, she directly gave them the solution and asked them to copy it.

The second theme of Ms. Woo's mathematics classroom was criticizing students' low performance. The students were expected to provide only correct answers, and when they provided incorrect answers during whole class discourse, she immediately discredited their answers and blamed it on their lack of mathematical abilities, for example, saying, "Third graders would do better than you." As a result, only students who already knew the answers to problems could engage in classroom discourse. When these students provided correct answers, the teacher just restated their answers without mentioning their names or praising their contribution as other teachers might do. Moreover, if the students were struggling to understand the solution to a problem, she assumed the difficulty was caused by their low cognitive abilities, and not her instructional practices. Her students were afraid to talk to her, so they whispered to their partners when they needed help.

Ms. Woo's last theme was modifying tasks in the textbook, which was aligned with the first and second themes. Because she disregarded students' participation and abilities, she intentionally excluded challenging tasks or modified them to be easier for her students. Thus, Ms. Woo introduced only tasks that she determined to be easy for her to demonstrated and that were easy for her students to solve. When a task contained procedural problems that students could answers, she would skip all of the interim questions and just asked the students, "What is the answer?" She introduced interesting stories, such as examples from the Harry Porter series, and provided problems that she designed to connect the story and that days' mathematical goals. However, this additional activity was possible only at the expense of the mathematics problems and tasks in the textbook. That is, she hurried through the content of the textbook problems and tasks by modifying lessons and then she introduced her additional activities during the class time that remained. Overall her mathematics classroom practices were strongly traditional.

Classroom example. The title of example lesson was "*Making problems*" from the *Various Problems* unit. Ms. Woo combined two lessons into one. Originally, the first and second activities were related to constructing problems related to dividing fractions and geometry, respectively. The lesson consisted of six activities and two sample problems. However, she omitted two activities and one sample problem. Throughout the class, students sat in their seats and worked independently without using any tools or objects.

Activity one. In the first activity, Ms. Woo's students were supposed to solve a problem using their understanding of dividing fractions. She first introduced a mathematical story problem from the textbook (see figure 5.10) and instructed the students spend a few minutes to find the solution. When the students did not provide the right answer, Ms. Woo explained the solution to the problem and criticized her students' low participation and abilities.

Translation

Frame 1: I have17 cows and want the first son to receive one-half of the cows and the second and third sons to receive one-third and one-ninth of the cows, respectively.

Frame 3: I have no idea. 17 is not divisible by 2, 3, and 9; We can't divide the cows for distribution.

Frame 4: I will lend my cow to you. Please return it to me after solving the problem.

Figure 5.10 Activity one of the textbook used in Ms. Woo's class

Ms. Woo: Let's think about the problem. A father has 17 cows and wants to give them to his three sons in his last will. The first son will receive one-half of the cows and the second and third sons will receive one-third and one-ninth of the cows, respectively. However, the problem is that 17 is not divided by 2, 3, and 9. At the time, one man suggested, "You can borrow a cow from me and then give it back to me later, after solving the problem." So, do you guys understand the problem? Do you have any ideas how to solve it?

(None of the students provide any mathematical solutions. Instead, they are murmuring to each other.)

Ms. Woo: (After few minute, she summarizes the problem again). You know 17 is not divisible by 2, 3, or 9. Because the father wanted to distribute the cows, the quotient should be a natural number. You know, you cannot divide one cow into several pieces. That is, we should distribute the 17 cows without cutting them up. So let's think about a common multiple for 2, 3, and 9. What is the common multiple?

Student: A number which is a multiple of more than two numbers.

Ms. Woo: Right. You should have learned it in the fifth grade. Is the common multiple bigger or smaller than 2, 3, and 9?

Student: Bigger.

Ms. Woo: Right. Then the least common multiple of the three numbers is 18, which is the sum of the 17 original cows plus the borrowed cow. Let's think, how many cows would the first son get? What is 18 divided by two? Nine cows. The first son gets nine cows. How about the second son?

Student: One-third.

Ms. Woo: One third of 18 is equal to six. So, the second son gets six cows.

She continued to dominate with these directive explanations. In the text, the second activity involved thinking about how to distribute 17 cows without borrowing an extra cow. However, she skipped the activity and said, "I think the second activity is too challenging, so we are going to skip it."

Activity two. The original activity in textbook involved creating a new problem by changing conditions of activity one, such as changing number of cows or of sons. However, Ms. Woo did not ask her students to construct new problems but rather she provided a similar problem and asked them to solve it. She also provided the solution to the problem without providing sufficient time for the students to explore the problem.

Ms. Woo: Now, we have 23 cows and the first, second, and third sons will receive one-half, one-third, and one-eighth of 23 cows. What is the least common multiple of the three numbers?

Student: 24

Ms. Woo: Right, 24 is the least common multiple of three numbers. Then, the first son will get 12 cows and the second and third sons will get eight and three cows.

Activity three. The third activity was calculating the number of different shapes of street tiles needed, such as 25 hexagon-shaped street tiles, to cover an area (see Figure 5.11). For example, one of the questions was "How many right triangles do they need to cover the same street?" She briefly read the question in the textbook and asked the students spend a few minutes to solve the problem independently. When she found that no one had solved the problem, she criticized their lack of effort and mathematical abilities. Then, she explained how to solve the problem procedurally.

Translation

Jihyun and Taeho saw a lot of tiles on the street. They wondered how to cover the street by using different shapes of tiles.

Figure 5.11 Activity three in the textbook used in Ms. Woo's classroom

Ms. Woo: Not yet? Is there anyone who can solve the problem? Why can't anyone find the answer? I believe that none of you guys made an effort to solve the problem. I know some of you are not smart. That's not the problem. Because if you do your best to solve it, you might find the solution. However, some of you are not studying hard. That's the reason why nobody can find answers.

The next activity in the textbook was creating similar problems by changing the conditions similar to the above activity. While she gave the students several minutes to construct their own problems, she did not check their answers and just gave her own alternative problem due to the lack of sufficient time.

Roles of teachers and students and student engagement. Ms. Woo's instructional practices were authoritarian. She read problems from the textbook and asked her students to solve them without providing them sufficient times or allowing them to ask questions. She only evaluated the students' solutions, and interactions among students were limited. They were expected to solve problems following her instruction and primarily use pencil and paper, with the exclusion of mathematical tools. She rarely called her students by names and she was not concerned about who spoke during the lesson. She never implemented group work. The purpose of her questions was not to check students' mathematical understanding but to proceed quickly to her explanations. She only wanted the students to present correct answers, and when students gave incorrect answers, they were publicly criticized. Ms. Woo did not want her students to be active mathematics investigators because they were not allowed compare different strategies or share their ideas with their peers; than students were passive followers of Ms. Woo's instruction.

Classroom discourse. Ms. Woo dominated classroom discussion with teacher-directed discourse without giving the students opportunities to initiate their own discourse. Because she did not call her students by their names or give them time for investigations, most of the student conversations were dominated by the same "smart" students who could quickly give the right answers. She also was not cognizant of the other students' lack of participation. Thus, the discourse pattern in her classroom was a question-answer approach. She rarely asked how or why questions, which might have stimulated classroom discussions and increased students' mathematical understanding. Rather, she provided very simple questions, such as "Is the common multiple bigger or smaller than 2, 3, and 9?" expecting brief answers from the students, such as "bigger." Sometimes student-student discourses were implemented quietly as students asked each other questions and shared their strategies. Because she did not officially implement such discourses, however, students felt that their discussions should not be audible to other students or to their teacher.

Mathematics tasks. All of the tasks presented in Ms. Woo's classroom could be solved without using representations. While some tasks in the textbook required her students to use representations and tools for investigations, she modified those tasks so the students could solve the problems using only pencil and paper. Similarly, she skipped tasks that asked students to discuss their strategies and ideas with their peers or to construct new problems; then she would modify them to limit the students' active participation in the lesson. She wanted her students to solve only simple problems independently, therefore, challenging tasks were changed to simple ones. Because she explained specific procedures and provided only simple tasks, she was able to address many tasks in one lesson. However, the students did not have any opportunities to investigate challenging mathematics problems and discuss them with their peers.

Mr. Sim

Pedagogical beliefs. Mr. Sim felt responsible for creating a classroom environment that stimulated students' interest and curiosity. However, he admitted that from some students' perspectives, mathematics itself might not be interesting. Therefore, he often offered manipulatives and showed video clips related to lesson goals. In addition, he sometimes acted like a performer by changing his accent and actions, which could capture students' attention, even if the subject matter did not. However, unlike teacher Woo, who relied on non-mathematical tools (e.g., Harry Potter) to increase students' interest, Mr. Sim designed mathematics tasks that were connected to students' prior knowledge. Because he believed that students would be excited to learn that new tasks could be solved by using their previous knowledge and discover the relationships between past and present learning, he felt an obligation to consider students' cognitive abilities and prior learning experiences when designing mathematics tasks.

With regard to classroom discourse, he explained two types of discourse patterns which he frequently used. On the one hand, he liked to implement teacher-centered discourse when he introduced new concepts because he believed that students learned best by focusing on his explanations. Additionally, this type of discourse helped them recall their prior knowledge without any distractions. On the other hand, after students had acquired the necessary mathematics knowledge, he preferred to implement student-student discourse. With this pattern, he allowed students to solve problems in various ways and then compare their individual strategies. In the latter interview, he elaborated further on his beliefs about classroom discourse. He believed that students could engage independently in problem-solving activities after they had thoroughly comprehended mathematics concepts. Occasionally, students might find and understand the concepts through personal investigations, but such cases were not common. Of the seven lessons in one unit, therefore, he usually allowed student-student interactions in only one or two lessons.

He also clarified that in the mathematics classroom, his role was a facilitator of students' learning. While all students could not achieve the same level of understanding, with the support of his instruction, they could make some progress beyond their previous levels. Specifically, he attributed the development of students' mathematical understanding to his instructional practices, not to the interactions with their peers. Mr. Sim also seemed to be equating good student behavior with quietly listening to his instruction. Indeed, he said that good students were those who "nod their heads to express their engagement in class when I am explaining mathematical concepts in front of the blackboard." As a mathematics specialist who studied mathematics education in a master's degree program and by reading books and academic journals, he believed that students might experience success in mathematics by listening to his clear explanations, so he preferred to implement teacher-centered discourse and instruction and limit individual student engagement. In sum, because he gave less credence to students' contributions to and authority over their own learning than to his instruction, his pedagogical beliefs were classified as a moderately teacher-centered.

Overall instructional practices. The principal norms of Mr. Sim's classroom were modifying mathematics tasks, providing ample information, and conducting fast-paced instructional practices. He modified tasks from the textbook before and during the lesson. One the one hand, unlike the other teachers who taught and focused on one lesson, he introduced key mathematics ideas of one unit in one lesson and then repeatedly introduced those concepts throughout the subsequent lessons. For example, the unit on *rates and ratio* consisted of nine lessons: two on rates, two on ratios, and two on percentiles, plus three problem-solving lessons. However, he introduced the concept of rates and ratios in the first lesson of the unit and discussed the concepts repeatedly in the other lessons. Thus, students learned several mathematics concepts simultaneously, which helped them to understand how those concepts were interrelated but this process imposed a huge cognitive burden on his students. Furthermore, he modified tasks during the lesson based on the students' mathematical understanding. When he believed that students could easily solve the mathematics tasks in textbook, he provided additional impromptu questions and problems by changing the conditions of the original tasks. In the post-observation interview, he said that he provided additional impromptu problems because he wanted to check whether his students solved problems with accurate mathematics understanding or by luck.

The second feature of Mr. Sim's class was providing ample or sufficient information. He not only introduced mathematics concepts in the textbook but also discussed other related mathematical situations and concepts. For example, he talked about batting averages of baseball players, cooking recipes, and death ratios for diseases in a lesson on ratios. This additional information helped students use prior knowledge during the mathematics lesson to develop their understanding of how mathematics concepts are applied to real-life contexts. However, a drawback of this process was that introducing several concepts during one lesson might confuse the students regarding the specific learning goals of the lesson. In addition, he spent less time than the other teachers did on activities from the textbook because he used class time to talk about non-mathematical topics. With respect to mathematics learning, he provided considerable verbal information about the problem the students were expected to solve. He divided information about the mathematics problem into several parts, which required students to respond to his questions. For example, after reading a word problem, he asked questions, such as, "How many doughnuts do it starts with?" "How about the second one?" "What is the question?" "What do we have to write?" In this way, he was checking to make sure that his students clearly understood all the information in the problem by providing and requesting verbal information from them.

The last feature of Mr. Sim's classroom was fast-paced instructional practices. As discussed in the first and second features, he provided a plethora of information during the lesson by modifying mathematics tasks and devising new ones. In order to cover all of this information, which involved additional discussions and tasks, he moved quickly from one utterance to the next, so the topics of conversations changed rapidly with only brief intervals for absorption. He also gave students only a few seconds to respond to his questions. During class, he often emphasized the importance of solving problems rapidly, stating, "If you can't solve the problem in ten seconds, it means that you do not have acute mathematics skills" and then he would count aloud to ten seconds. These instructional practices influenced his classroom discourse. Only the students who responded immediately with the answers to problems were able to engage in the classroom discourse, while other students were marginalized because they did not have enough time to solve the problems. While he provided mathematical situations for student investigations and helped them connect related mathematical concepts, his overall instructional practices were strongly teacher-centered.

Classroom example. The topic of Mr. Sim's example lesson was understanding the concept of ratio in the *rate and ratio* unit. In the previous lesson, students had learned about the concept of rate and understood that today's class was the first lesson about ratio. Mr. Sim based the lesson on tasks from the mathematics textbook. However, he also introduced additional related problems. Student-student conversations were not implemented, since they were expected to communicate only with the teacher. Other than the textbook, additional tools were not used.

Activity one. Mr. Sim asked students to recall what they had learned during the previous lessons. Because he had already taught the concepts included in this lesson, the students could describe several of them. Then, he read the first activity from the textbook (see Figure 5.12) and posed several related questions, increasing the number of questions from three provided in the textbook to more than ten.

1 연수네 모둠은 도넛 20개 중에서 10개를 팔았습니다. 판매한 도넛 수는 처음에 있던 도넛 수의 몇 배인지 알아봅시다.

Translation

The students had 20 doughnuts and sold 10 of them. How many more times greater is the original number of doughnuts than that of the number of doughnuts sold?

Figure 5.12 Textbook image for activity one used in Mr. Sim's class

Mr. Sim: Can you see the doughnuts?

Students: Yes.

Mr. Sim: How many doughnuts did they have at first?

Students: 20

Mr. Sim: How many doughnuts did they sell?

Students: 10

Mr. Sim: Ok, look at the first question. Find out the rate of the original number of doughnuts to the number of those sold. What is the most important word in the question?

Students: To.

Mr. Sim: Highlight *to* in your textbook and write down the rate between them. You learned this in the last class, right? If you can't find the answer within 10 seconds, it means you do not understand it. Do you find answers?

Students: Yes

Mr. Sim: Lets' talk about several ways to expression ratios.

A student: 20 to 10

Mr. Sim: Ok. Other expressions?

Student: 20:10

Mr. Sim: Other expressions?

A student: 20 divided by 10

Mr. Sim also asked the students to describe how to represent the rate in a decimal number (0.5) as a step-by-process. Then, he wrote similar problems on the blackboard, changing the condition of the question, such as the number of original and sold doughnuts, and asked students to solve them independently. He checked their answers to these problems, as he had done in the previous exchange.

Mr. Sim: (Two numbers [20 and 15] were written on the boards) How many doughnuts did they have at first?

Students: 20

Mr. Sim: How many doughnuts did they sell?

Students: 15

Mr. Sim: What is the rate?

Students: 20:15

Mr. Sim: What are other ways to express the ratio?

After checking answers of the additional questions, he asked students to read the definition and several expressions for unit rate and ratio from the textbook. He also asked them to highlight some of the words with colored pencils.

Activity two. The second activity was finding ratios for two pictures (see Figure 5.13). He read the problems for activity two and checked the information given in the textbook, such as the lengths of sides. He also asked students to highlight words that seemed important. Then, he asked students to think about the ratio between the two measurements and gave the answer within in five seconds.

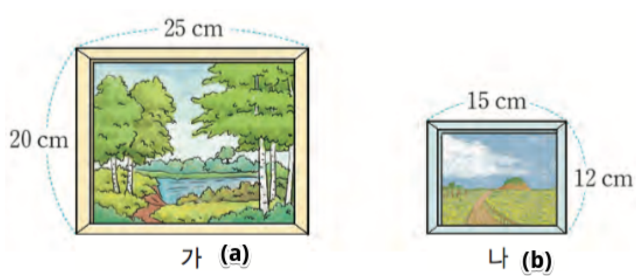


Figure 5.13 Textbook image for activity two used in Mr. Sim's class

Mr. Sim: What is the ratio of the height to the width of pictures a and b, respectively? I will give you five seconds to think about the answers. Five, four, three, two, and one. Are you ready to write down your answers?

Students: Yes.

Mr. Sim: Okay, can you see the white space below the images? Write down your answer there, make sure that you write the ratio of the height to the width

He checked students' answers by using step-by-step questions. Then, he asked students to represent the ratios as decimal numbers and fractions. In the textbook, the last question of activity two asked students to discuss what they had learned from the first and second activities. However, he skipped this question and gave them similar mathematics questions by changing the lengths in activity two, such as the ratio of 15 to 10. He also explained the batting rates of baseball players to support students understanding the concept of ratio. Despite his intention, however, some the students did not connect the concept of ratio to the example of batting rates. They just talked about famous Korean MLB players and other Korean sportsmen without thinking about the mathematical concepts.

Activity three. The sequence of the last activity was similar to that of activity two. Mr. Sim first read a problem from the textbook and asked students to highlight words that seemed important to solving the problem, which entailed finding the ratio between the sizes of two rooms that served a different number of people. Then, he gave students time to work independently on a problem-solving activity, and following his direction, Mr. Sim concluded the lesson by having the students check their answers together.

Roles of teachers and students and student engagement. Mr. Sim directed all classroom activities. He explained mathematics concepts directly, asked his students to repeat his explanations, and gave the students several similar problems related to his explanations. The students were expected to solve the problems independently following the procedures he taught them. While he provided real-life situations to support students' learning, he did not give them opportunities to discuss their own mathematics-related life experiences. They just listened to what Mr. Sim explained. Additionally, the students were expected to rapidly follow and understand his examples, such as batting rates, which were not presented in textbook. Mr. Sim evaluated students' ideas with brief comments. He also emphasized speed in solving problems and expected the students to solve each problem in less than ten seconds, even counting aloud to push students to solve it quickly. Students were never engaged in group activities and were expected to learn mathematics only from him. Because Mr. Sim was the source of all of the information presented during the lesson, students functioned as passive listeners or learners in his classroom.

Classroom discourse. In Mr. Sim's mathematics classroom, whole class discourse was the most primary source of students' learning. They never manipulated tools or materials or engaged in small group discourse. All mathematical information was transmitted from the teacher to the whole class, and students were not allowed to initiate their own discourse. They only responded to their teacher's questions or asked for clarification regarding his directions. Otherwise, they kept quiet during the mathematics lesson. Although his questions to students required answers beyond responding yes or no, most of them were very simple, low cognitive level demand level questions. He divided a problem into several questions based on the conditions given about the problem, such as original number of doughnuts. Thus, students responded in a few words, not full sentences. He also did not solicit additional ideas, but was satisfied with the first correct answer provided. Then, he presented another problem without asking students to explain why or how, which did not allow him the opportunity to probe students' mathematical reasoning process. Also, he rarely called students by name, and he was not concerned with who did or did not engage in the mathematics lesson or the nature of students' engagement. Thus, some students immediately responded to his questions without waiting for his more direction, while other students remained silent.

Mathematics tasks. The mathematics tasks were based on textbook activities. All problems could be solved using paper and pencil, thus, manipulations and discussions were not required. To solidify students' understanding, Mr. Sim followed the textbook sequence and provided extra tasks that were similar to the textbook problems. Consequently, the extra tasks were drill-based to give the students opportunities to practice certain skills and procedures. During the lesson, he presented three main tasks, all of which had similar structures, such as the ratio of A to B; accordingly, the students were able to solve them using the same procedures. Because each task had only one correct answer, an alternative solution was not relevant. In keeping with the first theme of fast-paced instruction, all of the tasks were solved quickly. He frequently said that quickly solving problems was very important and when students took a long time to solve a problem, it indicated that they had insufficient mathematical understanding. Thus mathematical tasks in his class was characterized by an emphasis on easy problems, drill-and-practice with similar procedures, lack of student engagement in mathematics investigations and discussions, and the teacher's appropriation of authority over mathematics learning.

Ms. Jung

Pedagogical beliefs. Ms. Jun's responses concerning pedagogical beliefs were moderately student-centered, emphasizing students' participation and activities. She believed that students learned best by doing activities and applying the new concepts to real-world environments. After learning fractions, for example, she expected her students to recognize that the concept could be applied to distribute money or dividing a cake equally. Therefore, her goal for teaching mathematics was to help students connect their acquired mathematical understanding using manipulations, to their daily lives. She also believed that parents should support their students' mathematics learning. Without their parents' support, students were not likely to apply mathematics concepts beyond the classroom. In connection with this belief, she emphasized the importance of mathematics learning communities consisting of students, teachers, and parents.

She elaborated on her pedagogical beliefs by explaining the roles she played in class, one of which was that of a facilitator. She believed that her job was to introduce new tools and concepts to support students' learning, and the students' role was to acquire mathematical concepts by participating in classroom activities. In order to achieve these goals, she believed that it was important to provide as many tools as possible. Because many students learned mathematics only by reading textbooks, they did not properly understand mathematics concepts. By manipulating various tools, however, she believed that students could learn mathematics concepts accurately and represent them in various ways. Whereas Korean mathematics lessons usually included two or three tasks, she wanted to design lessons with four or more activities.

Compared to her student-centered beliefs about teaching strategies, her beliefs about discourse practices were teacher-centered, largely because many of her activities did not leave enough time to implement student-centered discourse. She considered providing students with additional tasks and activities that are more meaningful than having them participate in discourse. Moreover, she believed that as third graders her current students were not ready to engage in high-cognitive discourse. Therefore, she allowed student-centered discourse only in lessons specifically designed for discussion and collaboration. Hence, her beliefs were moderately student-centered.

Overall instructional practices. The three themes of Ms. Jung's classroom were providing additional materials, having students solve problems with group members, and providing additional activities. Mathematics teaching and learning were centered on the use of materials, which she used to explain mathematical concepts, emphasizing the importance of using pictorial and concrete representations. She modified tasks in the textbook to achieve this goal and asked her students to use various representations to solve problems, which were not presented in the textbook. For example, she gave the students sheets of origami paper and colored pencils to visually explore fraction concepts as well as Pentominoes and Tangrams for investigating the characteristics of two-dimensional shapes. The students investigated and discuss mathematics concepts and solutions using manipulative materials. The use of materials was not limited to individual and group work. She also provided pin magnetics on the board to students in order for them to share their products with the rest of class.

Ms. Jung also emphasized group activities. She did not distribute mathematics tools to individual students, but gave tools to pairs or groups of four students to encourage them to communicate with their peers, as they used the tools to solve problems. To determined what the best solution might be for a particular problem, the students worked together to resolve conflicts about the problem. In such activities, students reasoned about and justified various mathematical ideas. Despite her emphasis on group activities, however, she did not ask students to explain what they discussed or to present any alternative ideas they had. Rather, she asked direct and simple questions that focused on the answers to problems and tasks, not on the process. Additionally, she was not interested in which groups presented their ideas. Similarly, in whole-class discussions, she posed questions and gave only a few of the students the opportunity to respond to the questions. When she heard a correct answer, she moved on to the next questions or activities. Therefore, students' group activities and discussions were not extended beyond the group level.

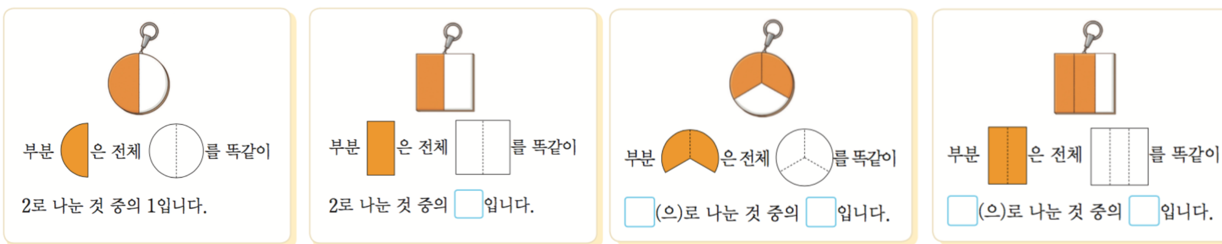
Ms. Jung provided her students with additional tasks. After solving the problems from the textbook, she asked students to solve the extra tasks in their homework book or the tasks she had personally designed. The level and type of extra tasks were similar to those in the textbook. For example, the textbook's lesson on fractions introduced the national flags of Indonesia, France, and Nigeria and asked students to represent their designs as fractions. As an additional task, she introduced other national flags, such as those of Austria, Mauritius, and Portugal (see Figure 5.14) and instructed students to also represent them as fractions. In this way, students could solidify certain mathematical concepts and procedures through repetition. Rather than enhancing student-centeredness, the introduction of extra tasks strengthened the teacher-dominant classroom culture. Because she did not have sufficient time to cover all of the tasks, she limited students' participation to whole group activities. Students' answers to her short and simple questions were usually evaluated by her, which restricted students' further investigations. In sum, her instructional practices were moderately student-centered.

Translation

Read and write the fractions represented by these flags.

Figure 5.14 National flags used in Ms. Jung's class (Top: flags in the textbook and bottom: additional flags introduced by Ms. Jung)

Classroom example. The example lesson was the second lesson covering *fractions and decimal numbers*. In the previous lesson, Ms. Jung's third graders had learned how to divide a quantity equally. The lesson goal for this lesson was for the students to understand the concept of fractions. The lesson originally consisted of two activities: (1) dividing a shape, such as a circle and square, into two or three equal parts and defining the fractional parts, and (2) determining and writing the fraction for a certain area of a national flag. However, she introduced an additional activity following each of the first and second activities. Desks were arranged for group work and color pencils and pens were placed on the desks.



Translation

Images 1 and 2: ½ means 1 out of 2 equal parts

Images 3 and 4: 2/3 means 2 out of 3 equal parts

Figure 5.15 Images for activity one in Ms. Jung's class

Note. The second and fourth images were presented on the board with large pieces of magnetized paper.

Activity one. The first activity from the textbook had four images describing the proportion of a shape represented by a fraction. Ms. jung read the activity aloud and provided questions for the first image, which divided a circle into two parts (pictorial representation). Next, she showed the second image, but larger, on the board using magnetized paper and demonstrated how a square could be divided into two equal parts by folding it (physical representation). When the students had found the answers to her questions, she asked them to focus on the third image in the textbook. After discussing the third image as a pictorial representation, she again asked the students to look at an enlarged fourth image on the board and showed them how a large sheet of paper could be folded into three equal parts (physical representation). Thus, she presented two of the four images on the board and visually demonstrated how the shapes could be divided equally (see Figure 5.15). Then she asked students to read the mathematical definition of a fraction found in the textbook.

Activity two. For the second activity, which were not presented in the textbook, she gave pairs of students outlines of images. The first group consisted of the first and second images of activity one, a circle and a square divided into two parts. Students colored one part of each image with a colored pencil to represent ½. They wrote the term for the fraction (one-half) in relation to the whole figure. Students were also asked to post their products on the board using push pin magnetics but were not asked explain their product and did not receive feedback on whether they were right or wrong. Instead, she pointed out a group's incorrect answer during a whole group discussion and provided the correct answer.

Mr. Jung: (Looking at students' products posted on the board). Ok, you used colored papers to represent fractions. What is this? (pointing to the products)

Students: One-half (1/2)

Mr. Jung: Say it again?

Students: One-half

Mr. Jung: Say it again? What is this?

Students: One-half

Mr. Jung: However, I found that some groups wrote the two first (2/1) and not one-half (1/2). Which expression is correct?

Students: The second expression is correct.

Mr. Jung: Yes, one-half is the right expression.

Student: Some students wrote the two first.

Student: Are you kidding me?

Mr. Jung: Which students made such a mistake?

Student: We did, but we changed the expression later.

Mr. Jung: (Pointing to another group's product on the blackboard). This group wrote an incorrect expression too. Please be careful when you write fraction expressions. I will check whether you make a mistake next time.

Then Ms. Jung gave students the third image for activity one, which was a circle divided into three parts. This time, however, she did not ask students to represent 2/3 following the textbook. Instead, she gave the students the choice of representing either 1/3 or 2/3.

Activities three and four. She showed national flags which were divided into two or three equal parts with different colors (see Figure 5.14). Originally, the textbook included only three flags, but she showed those images for activity three and another flags for activity four. In a whole class discussion, she asked students how to represent the colored areas of the flags as fractions, for example, "what is the fraction represented by the green area of Nigeria's national flag?" She also showed students flags which could not be divided equally such as the national flag of Portugal and asked them whether it could be represented with factions. However, she did not solicit students' mathematical reasoning but rather was likely to provide the answers to her questions.

Ms. Jung: (Showing the national flag of Portugal) Is it divided equally?

Students: No.

Ms. Jung: Can you represent it with fractions?

Students: No.

Ms. Jung: Then, how about this? (pointing out the national flag of Spain)

Students: It cannot be represented with fractions.

Ms. Jung: Yes, you are right. Neither of them can be represented as fractions because they are not divided equally.

Roles of teachers and students and student engagement. One of Ms. Jung's vital roles was to provide sufficient materials to help her students' investigations and understanding. She changed mathematics activities from the textbook, such as from pictorial to concrete representations, and introduced additional tasks. In the process, students were expected to learn mathematics concepts through manipulation and investigation. However, she did not change her pre-planned activities based on students' understanding and discussions. She wanted to complete all material demonstrations and tasks and controlled students' activities to achieve this goal. Despite the use of materials, most activities involved low-cognitive tasks. Thus, student's answers were also at a low-cognitive level. During group activities, students shared their ideas and investigated problems, but these processes did not extend beyond the groups. That is, her role was transmitting knowledge through materials, and the students' role was acquiring such knowledge using manipulatives.

Classroom discourse. Seemingly there was a variety of communications, indicating the students' engagement in mathematics learning. However, their actual cognitive involvement was superficial. They were expected to briefly respond to Ms. Jung's simple questions, and because the problem and solutions were obvious to them, students almost never challenged their peers' ideas or tried to figure out problems beyond group discourse. Additionally, students did not initiate new discourse or investigate advanced problems. Instead, Ms. Jung asked students only to describe what they did with the provided materials, so there was almost no discourse related to reasoning, providing justification, and offering proof in whole class discussion. She was satisfied when some students presented the answers she was looking for and moved on to the next questions. For example, when students said that the national flag of Portugal could not be represented as fractions, she did not solicit the reasons for their answers or other students' different ideas. She just showed them Spain's national flag. Moreover, she paid no attention to which students spoke, so the discourse was dominated by a few students.

Mathematics tasks. The mathematical tasks during Ms. Jung's lesson were not limited to traditional paper-and-pencil tasks. She provided various representations and introduced real-life tasks, such as the national flags of various countries. She followed the textbook sequence, but she also modified activities to give the students opportunities to engage in mathematical investigations. Some tasks involved a group activity, in which the students discussed the mathematical tools and solutions to problems with their peers. However, her focused was on accomplishing the task itself rather than on what students did during the problem solving process. Furthermore, the students were not encouraged to talk about the process, because they were only asked to provide the answers to low cognitive level tasks that did not require her students to engage in high cognitive investigations.

Synthesis of the Eight Teachers' Cases

In this section, I synthesize the eight teachers' cases to discuss the relationships between the eight Korean elementary mathematics teachers' pedagogical beliefs and their instructional practices, focusing on their alignments and misalignments, as well as the influence of their related mathematics life events on their beliefs and practices. First, I classify the eight teachers based on the coordinate system explained in the method section of Chapter Three. Then, I explicate factors influencing the relationships between their beliefs and their practices.

Classification of the Eight Cases

Using the coordinate system, I classified the teachers' pedagogical beliefs and instructional practices into four levels, strongly teacher-centered, moderately teacher-centered, moderately student-centered, and strongly student-centered (see Table 5.1). Specifically, teachers in Quadrant 1 had both student-centered beliefs and practices; teachers in Quadrant 2 had teacher-centered beliefs and student-centered practices; teachers in Quadrant 3 had both teacher-centered beliefs and practices; and teachers in Quadrant 4 had student-centered beliefs and teacher-centered practices. I originally assumed that teachers in Quadrants 1 and 3 would exhibit alignment between beliefs and practices and those in Quadrants 2 and 4 would exhibit misalignment. However, my analysis of teachers' pedagogical beliefs and teaching practices revealed moderate alignment for teachers in Quadrants 1 and 3 (see Figure 5.16). The mathematical beliefs and instructional practices of four of the teachers (Mr. Kim, Mr. Yang, Ms. Ko, and Ms. Jung) were aligned; those of the three teachers (Ms. Choi, Ms. Woo, and Mr. Sim) were moderately aligned, and those of one (Ms. Lee) were misaligned. These findings were consistent with those of previous studies (Raymond, 1997; Thompson, 1992) showing different levels of agreement between teachers' beliefs and practices.

Therefore, I classified the relationship between mathematics teachers' pedagogical beliefs and instructional practices into three levels: Alignment, Moderate alignment, and Misalignment (see Table 5.1). The Alignment level refers to consistency between teachers' pedagogical beliefs and instructional practices, suggesting that their instruction was guided primarily by their beliefs. However, the other two levels indicated that teachers' practices were influenced by not only their pedagogical beliefs but also other factors. Analyzing those factors provide an answer to research question 3), "*How does a theoretical model explain the relationship among the Korean* *elementary teachers' life stories, the development of their beliefs, and their instructional practices*?" Thus, in the following section, I present my analysis of how and why the teachers' pedagogical beliefs were consistent or inconsistent with their instructional practices. Based on these findings, I then present a conceptual framework to illustrate the relationship among related mathematics life stories, pedagogical beliefs, and instructional practices.

Table 5.1

Classification of the Eight Cases

Name	Life story type	Pedagogical beliefs	Instructional practices	Classification
Kim	Proceeding	Strongly SC [a]	Strongly SC	Alignment
Lee	Proceeding	Moderately SC	Moderately TC	Misalignment
Yang	Retreating	Strongly TC [b]	Strongly TC	Alignment
Choi	Proceeding	Moderately SC	Strongly SC	Moderate Alignment
Ko	Proceeding	Strongly SC	Strongly SC	Alignment
Woo	Retreating	Moderately TC	Strongly TC	Moderate Alignment
Sim	Retreating	Moderately TC	Strongly TC	Moderate Alignment
Jung	Retreating	Moderately SC	Moderately SC	Alignment

Note. [a] SC: Student-centered, [b] TC: Teacher-Centered

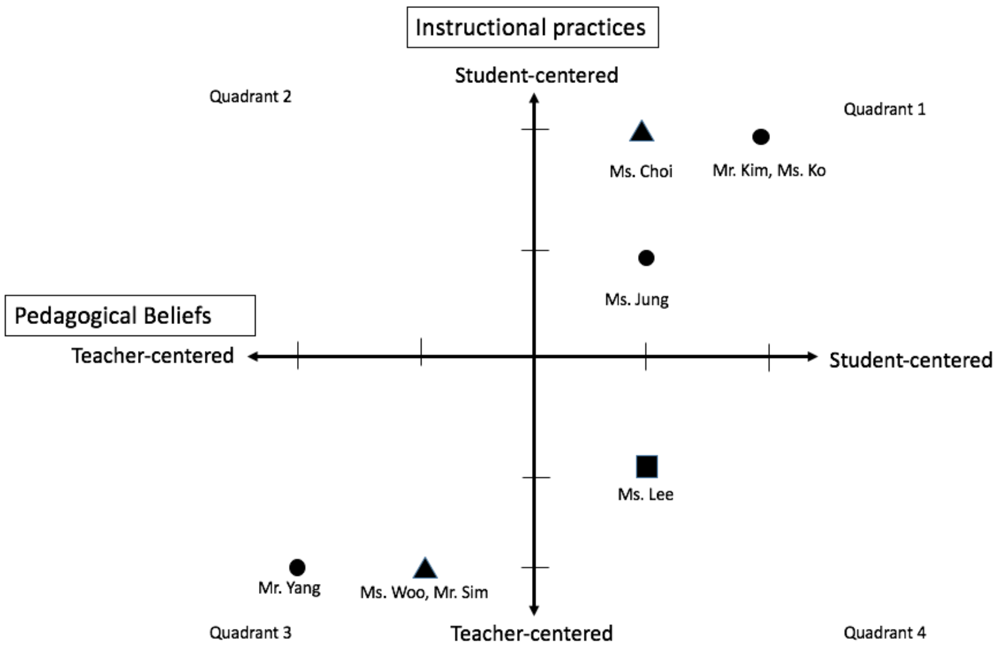


Figure 5.16 The relationships between teachers' pedagogical beliefs and instructional practices

Note. Circle, triangle, and square refer to alignment, moderate alignment, and misalignment, respectively.

Alignment Cases

Mr. Kim and Ms. Ko. Mr. Kim and Ms. Ko both had strong student-centered pedagogical beliefs and instructional practices. They believed that students should be active investigators, while teachers supported their learning as facilitators. In their classes, they emphasized students' investigations and modified their textbooks to provide these opportunities. Their students solved cognitively challenging tasks and justified their reasoning in discussions with their peers. These teachers did not have students work on meaningless drill-based problems. As proceeding-type teachers, they evaluated their beliefs and practices from students' perspective and changed them to be more student-centered. Mr. Kim pursued his own additional mathematics learning and listened to students' voices to resolve challenges that arose in his mathematics classroom. While his own K-12 mathematics learning experiences were not quite related to student-centered instructional practices, the mathematical and pedagogical knowledge he acquired from graduate courses and the practical knowledge (Cochran-Smith & Lytle, 1999) he gained in real-life contexts served as a catalyst for developing student-centered beliefs and practices (Ebby, 2000).

Unlike Mr. Kim, Ms. Ko did not attend graduate school to study mathematics education. However, her positive mathematics learning experiences with her eighth grade teacher influenced her development of student-centered beliefs and practices, and most of her current instructional practices were aligned with experiences she had with her eighth grade mathematics teacher's instructional approaches, which included having students manipulate concrete objects and mathematical tools, engage in mathematical discussions, and solve complex problems. That is, despite her lack of additional mathematics learning experiences, she was able to maintain student-centered beliefs and practices, largely because of the positive influence of a previous teacher (Drake & Sherin, 2006; Foote & Gau Bartell, 2011).

Mr. Yang. Mr. Yang had both strong teacher-centered beliefs and instructional practices. He believed in using repetition and external pressure to encourage students to study mathematics more. As a retreating type of teacher, he did not believe that the students' abilities and inner motivations were important, nor did he perceive the need to acquire further mathematical knowledge for himself. He blamed the students' low abilities on their lack of effort, which justified his implementation of teacher-centered practices. In the classroom, he assumed the role of manager of student learning. He provided direct explanations and gave students sufficient time to practice applying the concepts or procedures he taught. He was highly print-oriented, whereby he instructed his students to highlight certain words in their textbooks and taught problem-solving strategies that required his students to only use pencil and paper. Students in his class were asked to remain quiet and follow the procedures exactly as he demonstrated them. Interestingly, his instructional practices were similar to those of his previous school teachers. Even though in his life story interview, he criticized them for dominating classroom discourse and activities and requiring students to take notes according to the teacher's instruction.

Similar to Mr. Kim, Mr. Yang had had generally negative mathematics learning experiences throughout his K-12 schooling and in college, but his pedagogical beliefs and instructional practices were very different from those of Mr. Kim, which might be related to Mr. Yang's disinterest in pursuing additional mathematics or pedagogical learning (Ball, Thames, & Phelps, 2008; Ebby, 2000). Teachers' own learning experiences influence not only their mathematical knowledge but also their pedagogical beliefs and instructional practices, so acquiring knowledge about the development of students' mathematical thinking in graduate study or professional development (PD) programs may increases the likelihood that they will develop student-centered beliefs and instructional practices (Fennema et al., 1996; Polly, Neale, & Pugalee, 2014). However, Mr. Yang had not attended any mathematics-related PD during his teaching career and disregarded the importance of learning from teacher education programs. Therefore, Mr. Kim just replicates his previous teachers' instructional practices because it is likely that he has very limited ideas about student-centered beliefs and practices that are currently being emphasized in mathematics education.

Ms. Jung. She had moderate student-centered beliefs and practices. She emphasized students' participation and manipulative use to acquire conceptual understanding. However, she believed that providing experiences manipulating concrete objects was more important than having students share their ideas and solutions through discussions. During the class, she modified tasks in the textbook and provided additional activities to allow students to investigate mathematics concepts. However, she dominated classroom discourse and did not provide sufficient time for discussions because she was concerned about having enough time to introduce the additional tasks for her students to complete.

Her mathematics learning experiences from K-12 and in college had been generally negative. She praised only her private tutor's instructional practices and criticized those practices of her mathematics teachers. She also did not recall any positive experiences in mathematics courses, during her teacher education program. Moreover, as a retreating type teacher, and during the interviews, she criticized her students' lack of mathematical abilities, and highlighted them as the reason she did not implement whole class discussions. Despite these limitations, she was opened to acquiring new mathematical knowledge and was eager to improve her instructional practices. She participated in mathematics-related PD and enrolled in a master's degree program in mathematics education. These learning experiences had led her to recognize the importance of using real-life contexts and providing students with opportunities to investigate mathematics situations (Fennema et al., 1996). As a consequence, she had moderate levels of both student-centered beliefs and practices.

Moderate Alignment Cases

Ms. Choi. Ms. Choi's beliefs about mathematics teaching and learning were moderately student-centered, while her instructional practices were strongly student-centered. This moderate alignment resulted from her uncertainty about the effects of student-centered practices. Although she employed student-centered practices, she expressed the belief that teacher-centered practices were more effective for student achievement and that she could easily manage students' participation in teacher-centered classroom environments. Nevertheless, she still chose to implement student-centered practices, believing that they would help students develop conceptual understanding. In sum, her instructional practices were influenced not only by her pedagogical beliefs, but also by her concerns about student achievement and classroom management.

Among the teachers that participated in this study, only Ms. Choi expressed concern about student achievement and classroom management. The other teachers had already developed teaching strategies to effectively manage students' participation and improve their achievement over time (Pajares, 1992). As the only novice teacher, Ms. Choi had not had sufficient teaching experiences. Therefore, it made sense that she still struggled with developing her own pedagogical beliefs and instructional practices (Ambrose, 2004; Bandura, 1977; Raymond, 1997). The struggle may undoubtedly be described in terms of her self-efficacy in teaching mathematics. During the interview, she mentioned that while she knew the value of student-centered practices, she lacked confidence when implementing them. This lack of self-efficacy beliefs resulted in moderate alignment between her pedagogical beliefs and instructional practices.

Ms. Woo. Ms. Woo had moderate teacher-centered beliefs and strong teacher-centered instructional practices. She mentioned that the goals of her mathematics classroom were to enhance her students' cognitive interest and increase their engagement. To achieve these goals, she used board games and provides contextual situations for math problems. She also admitted that she rarely implemented group activities because she preferred individual and whole-class activities. As observed during her mathematics lesson, her stated beliefs were fairly consistent with her actual teaching practices. During the two observations, she never implemented group activities and provided additional contextual situations by devising new tasks for her students. However, those activities seemed to oppose to her teaching goals of increasing students' interest and engagement. She dominated classroom discourse and provided explicit mathematical information. The students listened carefully to her instructions in order to follow them without questions, because she, and not the textbook, was the sole source of information. However, if the students were not mindful of what Ms. Woo said, they would have limited understanding of the game rules and stories, which interfered with them engaging in the extra activities. In short, the stories and games might have been interesting tools from Ms. Woo's perspective, but this was not the case for the students. Their role in the classroom was to be powerless listeners. As a result, her instructional practices were only moderately aligned with her pedagogical beliefs.

This inconsistency might have been related to her lack of mathematical knowledge. As suggested by Richardson (1996), when teachers lack sufficient mathematical knowledge, their beliefs are likely to be inconsistent with their actual instructional practices because they do not know how to teach mathematics in accordance with their pedagogical beliefs. Ms. Woo had stopped studying mathematics education after graduating from her teacher education program. In addition, as a retreating type of teacher, she did not pursue opportunities to acquire new mathematical knowledge. She disregarded the importance of continuous learning about mathematics education and just mirrored her eighth-grade mathematics teacher's instructional practices with which she had had positive learning experiences. The problem was that her eighth-grade teacher implemented teacher-centered instructional practices.

During the interview, for example, Ms. Woo talked about what the teacher said, but she could not recall what she herself had done as a mathematics investigator (e.g., discussions and manipulations). The teacher had focused on transmitting knowledge with interesting stories, without sharing any of the authority with her students. Thus, because she had not strived to improve her instructional practices and acquire new knowledge, Ms. Woo's mathematics education learning did not extend beyond her own eighth-grade learning experiences. Therefore, she simply copied her previous teachers' instructional practices as ways to implement her beliefs (Kaasila, 2007a).

Mr. Sim. As a mathematics specialist, Mr. Sim believed that his students would learn best by listening to his explanations, which would stimulate their interest and curiosity, not by engaging in their own investigations and discussions, so he limited student-student discourse and engagement. Therefore, I classified his pedagogical beliefs as moderately teacher-centered because he gave little credence to students' abilities and authority, while he tried to provide rich contextual information to support their learning. Overall, his instructional practices were consistent with his stated beliefs. He provided ample information with fast-paced, teacher-dominated classroom discourse, while controlling students' activities. He not only explained the mathematics concepts and procedures in the textbook but he also presented real-life problems related to them. However, these instructional practices were not connected to his expressed goal of having a mathematics classroom in which students learned mathematics with interest and curiosity.

This moderate alignment can be explained in two ways. First, Mr. Sim had rarely evaluated his instructional practices from his students' perspective. As discussed in the analysis of his life story presented in Chapter Four, his instructional practices were generally influenced by his independent learning in graduate school and from reading journals. Other teachers changed their instructional practices when they proved disconnected from students' perspectives, so they gained an understanding of their students' abilities and preferences, as well as what and how they should teach (Drake & Sherin, 2006; Foote & Gau Bartell, 2011). However, Mr. Sim did not take advantage of opportunities to reflect on and evaluate his instructional practices from his students' perspectives. Rather, he just assumed that providing ample information would engage their intellectual curiosity and increase their mathematical understanding.

This interpretation is especially supported by his depiction of students' abilities. He did not understand that some students experienced challenges in learning mathematics, but he believed that they could effortlessly solve problems because the problems were easy and his explanations were very clear. As indicated in his life stories, Mr. Sim had achieved success in mathematics from his personal efforts, so he was likely to believe that his students could do the same. That is, he evaluated student ability and efficacy based on his, not the students,' life experiences (Hauk, 2005). As a result, his beliefs and instructional practices were consistent only from his perspective. The second reason for this interpretation is related to his strong self-confidence in his mathematics teaching (Phelps, 2010), which was reinforced from being praised by other teachers and receiving several awards. Such recognition made him oblivious to his current students' perspectives. Connected with the first reason, he thought that his instructional practices were excellent and could not fail to arouse their intellectual curiosity from them. This overconfidence in his teaching approach made him implement strongly teacher-centered instructional practices (Vancouver, Thompson, Tischner, & Putka, 2002) that were moderately aligned with his beliefs.

A Misalignment Case

Ms. Lee's pedagogical beliefs and instructional practices were inconsistent. While she had moderately student centered beliefs, her instructional practices were moderately teacher centered. During her interviews, she emphasized her role as a facilitator, providing students with opportunities to think about mathematics problems and develop conceptual understanding. Superficially, her instructional practices seemed aligned with her stated mathematical beliefs.

She implemented game-based activities and storytelling methods, and she gave students opportunities to share their ideas in the classroom. However, most of these practices did not contribute to the development of students' actual mathematical understanding.

All classroom discourse and games were at low cognitive levels, as Ms. Lee gave no consideration to mathematical accuracy. Moreover, she led and dominated classroom discourse by explaining the game rules and introducing storybook problems and tasks. Her students were asked to solve many similar simple problems drawn from storytelling and textbooks and were seldom given challenging tasks. As a result, the students were not supported in connecting these types of mathematics activities with mathematical understanding, nonetheless they regarded them as opportunities to play, and not as times to learn mathematics. Despite her desire to support students' learning, Ms. Lee did not have enough knowledge about how to properly organize her mathematics classroom to achieve her goals. Therefore, her instructional practices constrained students' mathematical learning and led to a misalignment with her beliefs.

Again, this misalignment might be caused by her lack of both mathematics content and mathematics pedagogical knowledge (Ball et al., 2008; Tchoshanov, 2011). Ms. Lee was the only teacher participant who had not graduated from a teacher education program for elementary pre-service teachers. Her major in college had been Korean language and literature for secondary students. Therefore, except for several PD sessions during a short period of time, she had never acquired the necessary knowledge needed for teaching mathematics. Although learning from mentor teachers and PD helped her change from teacher-centered to student-centered pedagogical beliefs, her mathematics classroom was more teacher-centered than she believed it to be. Because she had not acquired knowledge for teaching mathematics in her teacher education program (Tirosh, 2000), many features of her mathematics instructional practices were drawn from her previous teachers' implementation of teacher-centered practices. While Ms. Lee expressed an aversion to these practices, she unintentionally reverted to them as a consequence of her lack of mathematical knowledge. These findings concur with Shechtman, Roschelle, Haertel, and Knudsen's (2010) argument that "short-term content knowledge gains in teacher workshops may not persist in classroom instruction" (p. 317). In sum, although from her perspective, Ms. Lee's instructional practices (enacted beliefs) concurred with her beliefs (stated beliefs), in reality, her insufficient mathematical knowledge produced misalignment that she did not recognize, so she was unwittingly implementing teacher-centered practices.

Factors Influencing Mathematics Teachers' Pedagogical Beliefs and Instructional Practices

In Chapter Two, I presented a conceptual framework based on sociocultural theory (see Figure 2.2), proposing that mathematics teachers' pedagogical beliefs are influenced by their mathematics-related life experiences and school characteristics. In turn, these beliefs affect their instructional practices, which are represented in the roles of teachers and students, classroom discourse, mathematical tasks, and student engagement. As indicated in the cases showing moderate alignment and misalignment, the relationships among mathematics-related life events, pedagogical beliefs, and instructional practices, found in this study, were more complicated. The life events affected the development of the mathematics teachers' self-efficacy and mathematical knowledge, and these factors directly and indirectly influenced their instructional practices, which explained the existence of moderate alignment and misalignment for these participants. The factors associated with moderate alignment and misalignment were different among these teachers. Ms. Lee's and Ms. Woo's cases were strongly influenced by their lack of mathematical knowledge, while Mr. Sim's and Ms. Choi's cases were affected their inflated and low self-efficacy for teaching mathematics, respectively. However, all these factors were related to their mathematics-related life events.

Mathematics-related life events. Previous researchers investigating the relationship between mathematics teachers' beliefs and practices (Alba, 2001; Barkatsas & Malone, 2005; Handal, 2003; Raymond, 1997) have argued that the incongruences between beliefs and practices were due to (a) the need to prepare students for standardized tests, (b) the lack of resources and time, (c) pressure from school administrators to use a specific pedagogy, (d) classroom management issues, (e) insufficient student effort, and (f) teachers' limited mathematical knowledge. Except for a few studies (e.g., Alba, 2001), however, most of the studies attributed the discrepancy to the first five factors. Therefore, previous researchers have generally concluded that external factors hindered teachers' implementation of their beliefs despite their abilities and endeavors. However, in this study, none of the teachers, except Ms. Choi, blamed such factors for the misalignment or moderate alignment between their beliefs and practices.

Regarding the external factors, there was not any standardized testing in Korean elementary schools. All tests were developed and assessed by the classroom teachers. The teachers were provided sufficient resources and had the authority to decide what and how to teach, which included how to manage their classroom and time. Additionally, all of the teachers were required to rotate among the schools within in the same large city every six years, so the influences of particular school principals and school contexts were not persistent. As indicated in Ms. Woo's case, for example, school administrators could suggest specific pedagogies, but teachers had the power to decide whether or not to use them. Some teachers, in this study, criticized their students' lack of mathematical abilities and efforts. However, those students' characteristics influenced both their beliefs and their instructional practices, so this factor may not be used to explain any inconsistencies. As discussed in the teachers' life stories, they had already experienced challenging external factors that positively and negatively influenced their beliefs, which tended to be matched to their practices.

Hence, the teachers' current beliefs and instructional practices were outcomes of their mathematics-related life events as revealed in their retrospections. Some the teachers formed teacher-centered beliefs, while other teachers formed student-centered beliefs, and to resolve challenging events, both selected instructional practices that were at least moderately congruent with their beliefs. Drawing on their mathematical knowledge acquired in their teacher education programs, they resolved challenges in their teaching and developed certain types of pedagogical beliefs and instructional practices, whether they were student-centered or not student-centered. For example, Mr. Yang rationalized his challenging moments while implementing investigation-based instruction by defending his teacher-centered beliefs and practices. Also, Mr. Kim explained his commitment to student-centered practices by referring to his previous students' questions about the meanings of basic mathematics concepts. It was not the purpose of this study to demonstrate the superiority or inferiority of either teacher-centered or student-centered instructional beliefs and practices. Rather, this study explored why some teachers mathematical beliefs are aligned or misaligned with their instructional practices through life events.

Self-efficacy beliefs for teaching mathematics. Researchers have found positive relationships between teachers' self-efficacy beliefs and implementation of student-centered instructional practices (Ghaith & Yaghi, 1997; Nie, Tan, Liau, Lau, & Chua, 2013). Because teachers' self-efficacy beliefs were related to their judgment regarding their capabilities to accomplish their educational goals in their classrooms despite challenging students and contexts, those with high levels of self-efficacy beliefs were likely to have highly effective instructional strategies, student engagement, and classroom management skills (Charalambous & Philippou, 2010; De Mesquita & Drake, 1994; Smith, 1996; Tschannen-Moran & Hoy, 2001). From this perspective, Henson, Kogan, and Vacha-Haase (2001) argued that teachers' sense of their teaching efficacy was one of the most important attributes of effective teachers.

Bandura (1977, 1997) argued that self-efficacy beliefs originate from four sources: performance accomplishments (enactive experiences), vicarious experiences, verbal persuasion, and emotional arousal. Although all four sources are important, actual teaching experiences in mathematics classroom are considered the most important for the development of mathematics teachers' self-efficacy beliefs (Charalambous, Philippou, & Kyriakides, 2008). Over the course of their teaching experiences, the teachers could experiment with different teaching strategies and mitigate their concerns and conflicts with regard to teaching mathematics, which contributed to the development of their self-efficacy for teaching mathematics. At the time of this study, it was Ms. Choi's first full year of teaching mathematics. Thus, she had not yet experienced many mathematics-related life events or had many opportunities to acquire the practical knowledge needed (Cochran-Smith & Lytle, 1999), making it more difficult for her to establish instructional practices linked to her pedagogical beliefs and had not engaged in activities that served as a catalyst for modifying her beliefs to fulfill her instructional goals. As a result, her beliefs and instructional practices were only moderately aligned.

Despite the positive relationships generally found between teachers' self-efficacy beliefs and their student-centered instructional practices, a few studies have documented some negative influences of high self-efficacy in cases in which it fostered satisfaction with established teaching performance and thus hindered further growth (Stone, 1994; Vancouver, Thompson, & Williams, 2001). Bandura and Jourden (1991) warned about the negative effects of high self-efficacy, stating, “Complacent self-assurance creates little incentive to expend the increased effort needed to attain high levels of performance” (p. 949). That is, when teachers have achieved a sense of superiority and are sure that they can easily accomplish certain goals, their motivation for further development is likely to decline. Due to several years of successful experiences as an elementary school teacher, Mr. Sim overestimated his abilities and the effectiveness of his instructional practices. He pushed students to follow his teaching strategies without attending to their voices and perceiving their challenges. As a result, his instructional practices did not change in accordance with his students’ performance but remained strongly teacher-centered.

The levels of teachers’ self-efficacy beliefs for teaching mathematics affected their pedagogical beliefs and instructional practices. Ms. Choi’s low level of self-efficacy beliefs made her question the value of student-centered instructional practices, and Mr. Sim’s high level of self-efficacy beliefs made him champion teacher-centered instructional practices. Therefore, this study argues that teachers’ over- or under-estimated levels of self- efficacy beliefs for teaching mathematics led them to support teacher-centered beliefs and implement teacher-centered instructional practices.

Mathematical knowledge. Teachers’ mathematical knowledge for teaching has received a great deal of attention from researchers (Wilkins, 2008), who have found that mathematics teachers’ knowledge plays an important role in their instructional practices (Kim & Albert, 2015). Following Shulman’s (1986, 1987) categorization, teachers’ knowledge is generally divided into domain-specific content knowledge and pedagogical content knowledge. Mathematics content knowledge refers to teachers’ knowledge of and problem-solving abilities in mathematics as a discipline, and mathematics pedagogical content knowledge refers to teachers' knowledge of strategies for teaching mathematics and of children's mathematical understanding (Ball et al., 2008). Given that K-12 mathematics instruction is focused on students' learning of mathematics content, pedagogical content knowledge is initially acquired in teacher education programs and further developed through mathematics-related PD.

Preservice and novice teachers who received high scores and positive feedback from their own K-12 teachers who implemented teacher-centered instructional practices are likely to advocate for teacher-centered beliefs and instructional practices (Kaasila, 2007a). However, these are likely to start to change to student-centered beliefs and instructional practices through learning in teacher preparation courses and from other teachers (Lutovac & Kaasila, 2018). Unfortunately, this was not the case for Mr. Yang and Ms. Woo. They disregarded the value of continued mathematics learning from teacher education programs and other teachers, considering them useless. As a result, they espoused teacher-centered beliefs and implemented teacher-centered instructional practices.

The inconsistency between Ms. Lee's beliefs and practices could also be explained by her lack of mathematical knowledge. Unlike the other teachers in this study, Ms. Lee had not undergone preparation for elementary teaching. Therefore, she had not acquired the mathematical content and pedagogical knowledge and preservice practicum experiences with model and mentor teachers that could help her resolve challenges in mathematics teaching to develop consistency between beliefs and practices. All the other teachers' experiences in their teacher education programs directly and indirectly influenced their mathematical knowledge, which, in turn, influenced how they constructed and implemented their beliefs in their mathematics classrooms. Therefore, this study also confirms that the teachers' experiences and learning in their teacher education programs influenced both their pedagogical beliefs and their instructional practices, leading to alignment between them.

A Theoretical Model Explaining the Relationships among Teachers' Life Stories, Pedagogical Beliefs, and Instructional Practices

Based on these findings, this study supports the conclusion that teachers' various levels of consistency between beliefs and practices can be explained by their different life stories. As described in Chapter Four, each teacher's unique life experiences and interpretation of them influenced his/her current beliefs and practices at different levels. Specifically, mathematics-related life events aligned with teachers' attributions of their low-quality instructional practices and contributed to their perceptions of the value of their own further learning, which, in turn, influenced the development of their current pedagogical beliefs. All of the proceeding type teachers had student-centered beliefs, and all but one (Ms. Jung) retreating type of teachers had teacher-centered beliefs. These constructed beliefs, sequentially, influenced the development of their current instructional practices. Except for Ms. Lee, all of the teachers' pedagogical beliefs and instructional beliefs were aligned at least at moderate levels; all of them were located in Quadrant 1 or 3. However, teachers' mathematical knowledge and self-efficacy beliefs for mathematics teaching influenced their pedagogical beliefs and actual instructional practices. The lack of mathematical knowledge and inappropriate levels of self-efficacy for mathematics teaching steered the teachers to implement teacher-centered instructional practices. The theoretical model explaining the relationships among life stories, pedagogical beliefs, and instructional practices based on these findings is shown in Figure 5.17. This model can be used to explain the consistencies and inconsistencies between teachers' beliefs and practices and the factors that affected the relationships.

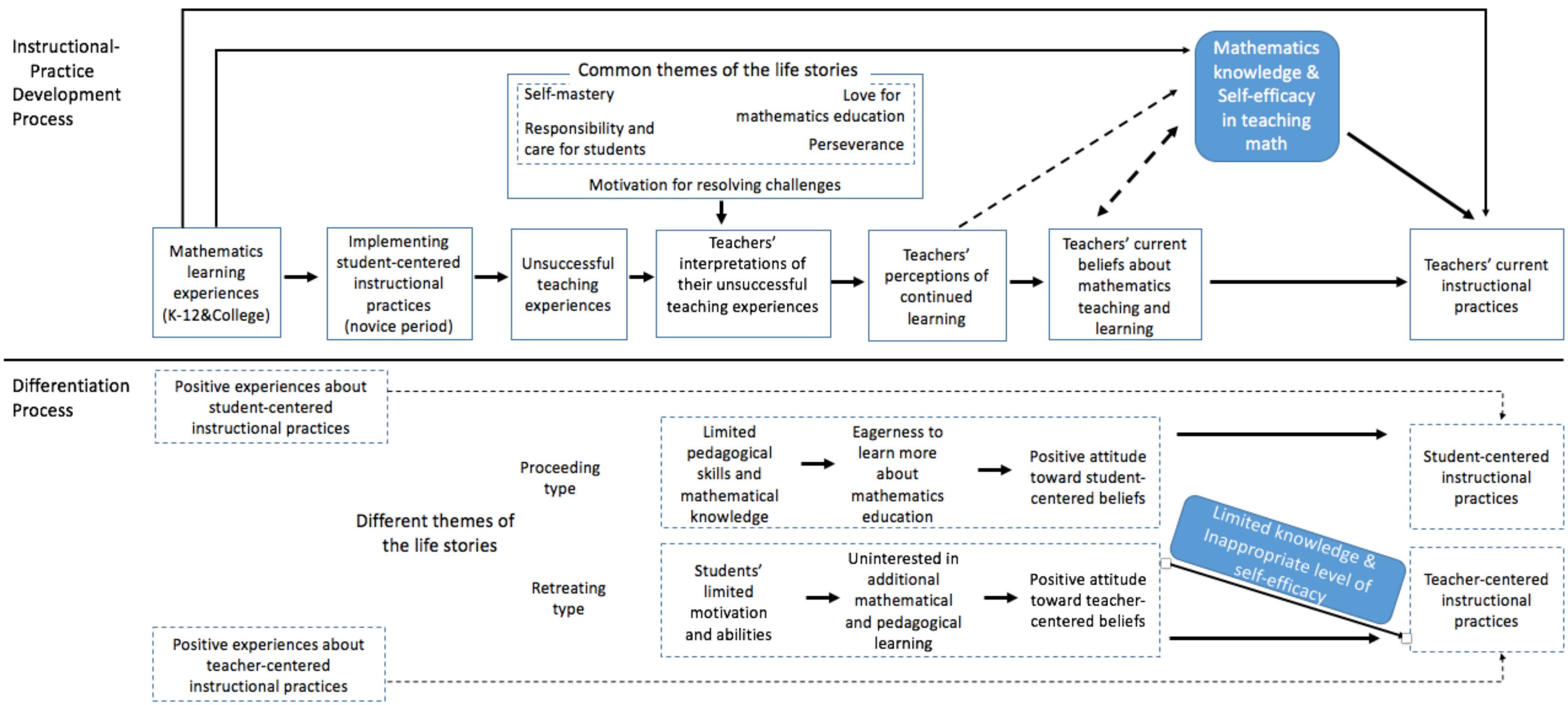


Figure 5.17 The theoretical model explaining relationships among teachers' life stories, beliefs, and instructional practice

CHAPTER SIX

SUMMARY, CONCLUSION, AND IMPLICATIONS

The previous two chapters described the findings of this qualitative case study. Chapter Four focused on the common and different themes in the participating teachers' life stories, as well as relationships between their mathematics-related life events and their pedagogical beliefs. Chapter Five introduced the relationships among the teachers' life stories, pedagogical beliefs, and instructional practices, as well as between their mathematical knowledge and self-efficacy for teaching mathematics. This chapter includes the summary of the research, a discussion of the findings, and the conclusions, implications, and limitations of the study. The summary of the study presents the importance of the study and research questions. The discussion of findings highlights the relationships among mathematics teachers' mathematics-related life stories, their pedagogical beliefs, and their instructional practices. The findings also help explain the alignments and misalignments between their beliefs and practices. The implications describe how the findings of this study could be used to support mathematics teachers, mathematics teacher educators, and other education stakeholders. Finally, the limitations of the study and recommendations for future research are discussed building upon the findings, and implications of the study.

Summary of the Study

The purpose of this qualitative case study was to explore how eight Korean elementary teachers' sociocultural life stories shaped their mathematical beliefs and practices and to determine why their beliefs and practices were aligned and misaligned. Vygotsky's (1986, 1978) sociocultural theory provided a theoretical framework with which to conceptualize mathematics teachers' beliefs and practices as an outcome of their interpretation of life events. An assumption of this conceptual framework was that personal mathematics-related life stories and contextual factors, such as school and students' characteristics, influence teachers' pedagogical beliefs, which, in turn, affect their current instructional practices, including teacher and student roles, classroom discourse, mathematical tasks, and student engagement.

Eight Korean elementary mathematics teachers from four schools participated in this study. The overarching research question was: "How does a theoretical model based on Vygotsky's sociocultural theory explain the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?" In order to address this question, three research questions were formulated to identify the relationships among the three elements.

1) How do Korean elementary teachers' sociocultural life stories influence their mathematical beliefs?

2) What is the relationship between Korean elementary teachers' mathematical beliefs and their instructional practices?

3) What is the relationship among the Korean elementary teachers' life stories, the development of their beliefs, and their instructional practices?

I approached this study with several assumptions based on the review of relevant research. The first three assumptions were that 1) teachers' past mathematics learning experiences positively and negatively influence their pedagogical beliefs; 2) teachers' current sociocultural context affects their pedagogical beliefs; and 3) teachers' past mathematics learning experiences, current social cultural context, and pedagogical beliefs influence their current instructional practices. I further assumed that the incongruences between teachers' pedagogical beliefs and instructional practices are caused by limitations of contextual factors. While evidence supporting these assumptions was found in this study, I did not anticipate the influence of teachers' mathematical knowledge and self-efficacy beliefs on teaching mathematics in my initial assumptions. Additionally, this study found that the influence of contextual factors on experienced teachers' beliefs and practices was minimal.

Importance of the Study

Teachers' mathematical beliefs have been regarded as one of the most important factors influencing their instructional practices (Philipp, 2007). Because teachers interpret and implement curriculum based on their beliefs (Stein, Remillard, & Smith, 2007), researchers have assumed that these beliefs might serve as an explanatory source for their instructional practices (Cross, 2009; Skott, 2009). However, some studies have found that teachers' instructional practices are not always consistent with their beliefs, and that other factors limit their implementation of their espoused beliefs (Handal, 2003; Speer, 2005). Amidst these contentions about the relationships between teachers' beliefs and practices, our actual knowledge about these relationships is limited (Skott, 2009). Additionally, previous research on alignments and misalignments has focused mainly on external factors currently affecting teachers' performance, which are not likely to be influenced by their past mathematics learning and teaching experiences, and not on teachers' inner factors, (e.g., Barkatsas & Malone, 2005; Turner et al., 2011).

In this study, the influence of mathematics-related life events was analyzed to understand the relationships between pedagogical beliefs and instructional practices. This approach was taken because teachers' life stories provided richly contextualized information concerning the development of their beliefs and the current status of their knowledge (Drake, 2006). The importance of peoples' life stories in bringing out their beliefs and practices is highlighted by

McAdams (2001), who indicated that analyzing people's life stories can help researchers to understand their sociocultural histories, beliefs, practices, and future practices. Mathematics educators have also concluded that mathematics-related life stories are key elements in understanding teachers' beliefs, knowledge, and instructional practices, as well as their identity (Drake et al., 2001; Foote & Gau Bartell, 2011; Kaasila, 2007b). The study analyzed eight Korean elementary mathematics teachers' life stories, how the stories related to their current beliefs and practices, and what other factors influenced these relationships. The findings of this study provide an initial outline upon which further research can be based.

Discussion of Major Findings

The findings of this study were presented in Chapters Four and Five. Many of the findings concurred with assumptions noted in the conceptual framework. However, findings concerning the influence of teachers' mathematical knowledge and levels of self-efficacy were the unique contribution of this study to the literature on mathematics teachers' beliefs and practices.

Mathematics-Related Life Stories and Pedagogical Beliefs

While the teachers in this study shared common themes in their life events, there were variations with regard to how they responded to and resolved similar events. For some, their self-attributions of their unsuccessful teaching experiences and perceptions of continued learning led them to develop certain types of pedagogical beliefs.

Common themes in participants' life stories. The teachers reported similar negative learning experiences with teacher-centered practices, and positive experiences with student-centered practices during their K-12 and college experiences. Therefore, as novice teachers they had implemented student-centered instructional practices. However, they were challenged by various factors, such as students' limited abilities, and modified their beliefs over time. In this process, certain common characteristics helped them to continue in their jobs and overcome the challenges they faced: 1) Self-mastery, 2) Responsibility and care for students, 3) Love of teaching mathematics, and 4) Perseverance.

These themes can explain why/how the teachers wanted to change their instructional practices. Many teachers sought to achieve self-mastery by strengthening their abilities and learning new skills in order to become more powerful and effective teachers. They were intellectually curious and internally motivated to more deeply understand mathematics concepts. Although supervisors and colleagues did not explicitly ask them to change their instructional practices, the teachers themselves felt the necessity to change their instructional practices for self-improvement. The second theme, responsibility and care for students, was evident in the teachers' concerns for their students' mathematics achievement, learning experiences, and enjoyment in mathematics, which led them to develop particular teaching practices and beliefs. Although each teacher's mathematics teaching goals were different, they commonly believed that they had the power to impact their students' mathematics learning and felt responsibility for doing so. Their sense of responsibility led them to change their previous mathematical beliefs and adopt new perspectives.

The third common theme was the love of teaching mathematics. All teachers loved teaching mathematics, regardless of their majors in their teacher education programs. They desired to teach mathematics in the face of challenges because they believed that mathematics was one of the most important school subjects. The teachers also assumed that they could make an important difference in students' mathematics achievement by teaching mathematics well, which, in turn, might improve the students' lives. The last theme was perseverance. Some teachers valued negative experiences as a way to improve their instructional practices. They believed in their ability to triumph over adversities made achieving their learning goals as attainable.

Different themes in participants' life stories. Besides these common themes in the teachers' life events, there were variations with regard to how they responded to and resolved similar events. Most of the mathematics teachers' instructional practices and beliefs were substantially changed because of critical mathematics-related life events that were unique to their situations and experiences. According to how the teachers negotiated and resolved challenges, they were categorized as Proceeding or Retreating types.

Proceeding type. The teachers in this group (Mr. Kim, Ms. Lee, Ms. Choi, and Ms. Ko) viewed students' struggles with mathematics as opportunities to implement student-centered practices. They believed that teacher-centered instructional practices had led to students' faulty understanding of mathematics and low mathematics outcomes. For example, when students were unmotivated, the teachers inferred that they had not organized the mathematics classroom in ways that respected students' ideas and invited their participation during the lesson. Therefore, they adopted student-centered instructional practices to engage their students in meaningful mathematics learning. Aligned with this perspective, they rarely blamed their students' lack of abilities or motivation as an excuse for low quality instructional practices. Rather, they sought ways to improve their instructional practices to meet their students' needs. To find solutions to the challenges they faced, these teachers also took advantage of additional mathematical learning opportunities from others and the professional literature.

Retreating type. One of the characteristics of retreating type of teachers (Mr. Yang, Ms. Woo, Mr. Sim, and Ms. Jung) was that they used their students' low abilities and motivation as justifications for implementing teacher-centered instructional approach. During the interviews, they talked about their accomplishments and claimed that they already had devised effective teaching strategies. However, they rarely shared their lack of effective teaching strategies, which might have led to the failure of their instructional practices. When student-centered practices did not go well, they were reluctant to reflect on their responsibility in the implementation of them, but preferred to discuss what their students failed to do. Another characteristic of the retreating type of teachers was that some of them disregarded the value of pursuing further learning of mathematics and pedagogy from others and the professional literature.

The relationships between teachers' mathematics life stories and pedagogical beliefs. As novice teachers, all of the teachers participating in this study had implemented student-centered instructional practices because of their own negative learning experiences with teacher-centered practices. However, they were challenged by various factors and experienced nadir points. While the common themes worked as a driving force for the teachers to overcome challenges, the outcomes were not always student-centered. The teachers' self-attribution of their unsuccessful teaching experiences contributed to their perception of their own further learning, which, sequentially, influenced the construction of their current beliefs about mathematics teaching and learning. These different interpretations of similar events were likely to have an impact on their attitudes toward student-centered or teacher-centered pedagogical beliefs. More specifically, this study found that while there were some variations, the proceeding types of teachers were likely to embrace student-centered and retreating types of teachers embraced teacher-centered pedagogical beliefs.

Pedagogical Beliefs and Instructional Practices

As discussed in Chapter Five, teachers' pedagogical beliefs were generally aligned with their instructional practices. However, the teachers' knowledge and self-efficacy beliefs about teaching mathematics influenced this relationship, resulting in different levels of alignment and even misalignment.

Classification of the eight cases. The eight teachers' pedagogical beliefs and instructional practices were classified using the coordinate system. Specifically, their beliefs and practices were analyzed into one of four levels: (1) strongly student-centered, (2) moderately student-centered, (3) moderately teacher-centered, and (4) strongly teacher student-centered. Four teachers (Mr. Kim, Ms. Ko, Ms. Choi, and Ms. Jung) were in Quadrant 1, three teachers (Ms. Woo, Mr. Sim, and Ms. Yang) were in Quadrant 3, and one teacher (Ms. Lee) was in Quadrant 4. Additionally, four teachers' (Mr. Kim, Mr. Yang, Ms. Ko, and Ms. Jung) mathematical beliefs and instructional practices were aligned, those of three teachers (Ms. Choi, Ms. Woo, and Mr. Sim) were moderately aligned, and those of one teacher (Ms. Lee) were misaligned, showing different levels of agreement between beliefs and practices (Raymond, 1997; Thompson, 1992).

Alignment cases. Mr. Kim and Ms. Ko both had strongly student-centered pedagogical beliefs and instructional practices. They believed that their students should be active investigators, while the teachers assumed the roles of facilitators. In their classrooms, they emphasized students' investigations, provided cognitively challenging tasks, and asked students to justify their mathematical reasoning. While Mr. Kim's K-12 mathematics learning experiences were closely related to teacher-centered instructional practices, the mathematical and pedagogical knowledge he acquired from graduate courses (Ebby, 2000) and the practical knowledge (Cochran-Smith & Lytle, 1999) he gained in rea-life-contexts led to his development of student-centered beliefs and practices. Ms. Ko had not pursued graduate study in mathematics education. Despite of her lack of additional mathematics learning experiences, however, her positive mathematics learning experiences with her eighth grade teacher influenced her development of student-centered beliefs and practices (Drake & Sherin, 2006; Foote & Gau Bartell, 2011).

As a retreating type teacher, Mr. Yang had both strongly teacher-centered beliefs and instructional practices. He believed that using repetition benefits his student learning and he did not believe that students' abilities and inner motivations were important. In the classroom, he provided direct explanations and gave students ample practice applying the concepts or procedures he taught. While Mr. Yang, like Mr. Kim, had had generally negative mathematics learning experiences throughout his K-12 schooling and in college, he tended to replicate his previous teachers' instructional practices. Because he was not interested in pursuing additional mathematics or pedagogical learning, he might have had limited ideas about student-centered beliefs and practices currently being emphasized in mathematics education (Fennema et al., 1996; Polly et al., 2014).

Ms. Jung had moderately student-centered beliefs and practices. She believed in the importance of students' classroom participation and use of manipulative materials to acquire conceptual understanding. Though she promoted students' manipulating concrete objects rather than engage them in classroom discussions. In accordance with her beliefs, she provided activities to allow students to investigate mathematics concepts, but as the teacher she tended to dominate classroom discourse. Her own mathematics learning experiences had been negative, and, as a retreating type of teacher, she criticized her students' lack of mathematical abilities.

However, she continued to acquire new mathematical knowledge and teaching strategies through PD and graduate courses. As a result, she exhibited moderate levels of student-centered beliefs and practices.

Moderate alignment cases. Ms. Choi's pedagogical beliefs were moderately student-centered while her instructional practices were strongly student-centered. Reflecting her lack of self-efficacy in mathematics teaching, Ms. Choi expressed concern about student achievement and classroom management in student-centered classroom environments and believed that teacher-centered instructional practices would resolve those issues (Bandura, 1977; Ghaith & Yaghi, 1997; Hauk, 2005). Aligned with her lack of self-efficacy beliefs and her novice status as a mathematics teacher, she was still establishing her own pedagogical beliefs and instructional practices (Ambrose, 2004; Raymond, 1997). At this point in her career, her pedagogical beliefs were moderately consistent with her instructional practices.

Ms. Woo had moderately teacher-centered beliefs and strongly teacher-centered instructional practices. She wanted to provide contextual situations for mathematics learning to enhance students' cognitive interest, but she did so at the expense of group activities, discussions, and manipulations. In her mathematics classroom, she dominated all discourse and activities. Her students were expected to be passive listeners and despite her learning goals, the students did not develop any interest in learning mathematics. The inconsistency between her pedagogical beliefs and instructional practices might be attributed to her lack of mathematical knowledge (Richardson, 1996). Because she disregarded the importance of continued learning about mathematics education, she did not have formal opportunities to acquire sufficient mathematical knowledge. Therefore, she just mirrored her eighth-grade mathematics teacher's teacher-centered instructional practices with which she had had positive learning experiences, assuming that those practices would also appeal to her students and arouse their curiosity (Kaasila, 2007a).

Mr. Sim had moderate teacher-centered beliefs and strong teacher-centered instructional practices. While his teaching goal was to stimulate students' interest and curiosity, he believed that his students would learn best by listening to his explanations. During the mathematics class, he provided ample information while dominating classroom discourse. He explicitly explained mathematics concepts and procedures from the textbook and required his students to follow his methods and explanations. This moderate alignment might have resulted from his lack of opportunities to retrospectively consider his instructional practices (Drake & Sherin, 2006; Foote & Gau Bartell, 2011). Because he did not take advantage of opportunities to evaluate his instructional practices from the students' perspectives, he did not understand the challenges that some his students encountered in learning mathematics. Rather, he assumed that providing interesting stories and information would stimulate students' cognitive curiosity. This tendency to disregard students' challenges and emotions were related to his strong self-confidence in his teaching (Phelps, 2010). This overconfidence caused him to implement strongly teacher-centered instructional practices that were moderately aligned with his beliefs (Vancouver et al., 2002).

A Misalignment case. Ms. Lee was the only one teacher who was classified as a misalignment case. Although she had moderately student-centered beliefs and emphasized her role as a facilitator, her actual instructional practices were moderately teacher-centered. Most of the classroom discourse and games provided during a lesson targeted low cognitive levels. Additionally, she dominated classroom discourse and asked her students to solve similar simple problems. Despite her student-centered beliefs, she did not properly organize her mathematics instruction to achieve her stated beliefs. This misalignment might have been caused by her lack of knowledge for teaching mathematics (Ball et al., 2008; Tchoshanov, 2011). Because she had not graduated from a program for elementary preservice teachers, many features of her mathematics instructional practices were drawn from her previous teachers' implementation of teacher-centered practices. While she expressed an aversion to these previous teachers' practices, her lack of mathematical knowledge made her unintentionally revert to those practices (Shechtman et al., 2010; Tirosh, 2000).

Factors Influencing Mathematics Teachers' Pedagogical Beliefs and Instructional Practices

The relationships among the teachers' mathematics-related life events, pedagogical beliefs, and instructional practices were more complicated than anticipated in the conceptual framework. The life events influenced their self-efficacy and mathematical knowledge, and these factors directly and indirectly influenced their pedagogical beliefs and instructional practices, which explained the alignment or misalignment between them.

Mathematics-related life events. Previous researchers investigating the relationship between mathematics teachers' beliefs and practices have argued that the incongruences between them were due to external factors, such as students' abilities and test pressure (Barkatsas & Malone, 2005; Handal, 2003). However, for this study, only one teacher referred to such external factors to explain inconsistences between her beliefs and practices. The other teachers had already experienced challenging events during the past teaching years and formed certain types of pedagogical beliefs that were most likely matched to their practices. Some of the teachers formed teacher-centered beliefs and other teachers formed student-centered beliefs, and both groups of teachers selected instructional practices that were at least moderately congruent with their beliefs. The teachers justified their pedagogical beliefs and instructional practices in relation to their pivotal events in their mathematics-related life stories. Therefore, this study argues that teachers' current beliefs and instructional practices were outcomes of their mathematics-related life events as revealed in their retrospections. Thus, external factors were not supported as the reasons for inconsistencies their pedagogical beliefs and instructional practices.

Self-efficacy beliefs for teaching mathematics. Teachers' self-efficacy beliefs in teaching were likely to be developed through actual teaching experiences (Bandura, 1977; Charalambous et al., 2008). Ms. Choi had not yet experienced many mathematics-related life events or had many opportunities to acquire the practical knowledge (Cochran-Smith & Lytle, 1999), making it more difficult for her to establish instructional practices tied to her pedagogical beliefs and modify those beliefs to fulfill her instructional goals. As a result, she had low level self-efficacy beliefs regarding mathematics teaching, accordingly, her beliefs and instructional practices were only moderately aligned.

Unlike Ms. Choi, Mr. Sim overestimated his abilities and the effectiveness of his instructional practices. He was satisfied with his established teaching performance, which hindered further growth in developing instructional strategies that were more student-centered (Stone, 1994; Vancouver et al., 2001). He encouraged students to follow his teaching strategies without attending to their voices. As a result, despite his mathematics teaching goal of engaging students' interest and curiosity, his instructional practices did not change in accordance with his student performance, but they remained strongly teacher-centered. In sum, the levels of teachers' self-efficacy beliefs for teaching mathematics affected their instructional practices.

Mathematical knowledge. Mathematics teachers' knowledge plays an important role in their instructional practices (Kim & Albert, 2015). Following Shulman's (1986, 1987) categorization, Ball et al. (2008) have endorsed the importance of both domain-specific content knowledge and pedagogical content knowledge. Given that K-12 mathematics instruction is focused on students' learning of mathematics content, pedagogical content knowledge is initially acquired in teacher education programs and further developed through mathematics-related PD.

Recent trends in pedagogical knowledge encouraged teachers to develop student-centered pedagogical beliefs and instructional practices (Munter, 2014; Philipp, 2007), however, Mr. Yang and Ms. Woo disregarded the value of continued mathematics learning, especially developing pedagogical knowledge, so they espoused teacher-centered beliefs and implemented teacher-centered instructional practices. Additionally, the inconsistency between Ms. Lee's beliefs and practices might have been initiated by her limited mathematical knowledge. She had not undergone preparation for elementary teaching, so she did not have opportunities to develop appropriate pedagogical knowledge (Lutovac & Kaasila, 2018). Rather, she just followed her previous K-12 mathematics teachers' teaching practices. Therefore, this study also confirms that the teachers' experiences and learning in their teacher education programs and subsequent PD influenced their instructional practices.

A theoretical model explaining the relationships among teachers' life stories, pedagogical beliefs, and instructional practices. This study concludes that teachers' various levels of consistency between beliefs and practices can be explained by their different life stories. The teachers' unique life experiences and interpretations of them influenced their current beliefs and practices at different levels. All proceeding types of teachers had student-centered beliefs, and all but one of the retreating types of teachers had teacher-centered beliefs. However, the teachers' mathematical knowledge and self-efficacy beliefs for mathematics teaching influenced their actual instructional practices. Limited mathematical knowledge and inappropriate levels of self-efficacy for mathematics teaching led teachers to implement teacher-centered instructional practices. This model can be used to explain the consistencies and inconsistencies between teachers' beliefs and practices and factors affecting those relationships.

Conclusions and Implications

Building upon the initial conceptual framework, I investigated eight Korean elementary teachers' mathematics-related life stories, their influences on these teachers' pedagogical beliefs and instructional practices, and the extent to which these practices were aligned or misaligned in relation to their mathematical knowledge and self-confidence in teaching mathematics, as these developed through their life events. Following is a discussion of the conclusions of the study and implications for teacher educators, school administrators, and teachers.

A major conclusion of this study is that teachers' mathematics-related life experiences are one of the main factors influencing their pedagogical beliefs and instructional practices. Teachers used their past mathematics learning and teaching experiences to justify their current beliefs and practices and to explain their classroom cultures, including the roles of teachers and students, mathematical tasks, classroom discourse, and student engagement. The findings of this study were in agreement with Lortie's (1975) argument that teachers' instructional practices are strongly influenced by their own teachers' practices, transmitted through an "apprenticeship of observation" (p. 61). In addition, however, this study revealed that teachers' mathematical knowledge and self-efficacy in teaching mathematics were also important influences. This finding suggest that teacher educators and school administrators can help change teachers' unproductive teacher-centered beliefs and practices into productive student-centered approaches.

Prospective teachers' mathematics-related life experiences and how these align with their beliefs need to be explicitly addressed in teacher education programs. Preservice teachers should be challenged to explore and describe their own pedagogical beliefs by reflecting on their past mathematics learning experiences and evaluating them. Preservice teachers also should be given opportunities discuss various teaching strategies, debate whether or not these strategies support the development of students' conceptual understanding, and how these strategies are aligned or misaligned with their initial pedagogical beliefs. When preservice teachers reach the culminating experience of student teaching in their programs and are implementing strategies consistent with their beliefs or modeling the instructional strategies of their practice by their supervising teachers, teacher educators need to provide continuous support to help them sustain the student-centered beliefs and practices they have developed during their methods courses and early practicum experiences. As indicated in Ms. Jung's case, preservice teachers might work with traditional teachers who espouse teacher-centered beliefs and practices, which could negatively affect the fledgling teachers' beliefs and practices. Therefore, teacher educators need to continue to support preservice teachers' development of and commitment to student-centered beliefs and practices throughout their teacher education programs, including the critical period of student teaching.

Also, teacher education programs should be designed to ensure that preservice elementary teachers acquire sufficient mathematical knowledge and develop adequate levels of self-efficacy beliefs. Teacher educators need to consider how to provide appropriate mathematics content and pedagogical content courses to prepare preservice teachers to accomplish student-centered goals in real teaching contexts. These courses should include knowledge about students' mathematical understanding, mathematics curricula, teaching strategies, instructional problem-solving skills, and mathematics learning itself; based on the premise that to help students develop conceptual understanding of mathematics, teachers must have sufficient conceptual understanding of the mathematics they will. With the knowledge acquired in their programs, preservice teachers could analyze their instructional practices, determine whether they hinder or support students' conceptual learning, and consider strategies to deal with challenging moments they might face during student teaching.

Similarly, school administrators should provided opportunities for practicing teachers to reflect on their past mathematics learning and teaching experiences and current pedagogical beliefs and practices, as well as their mathematical knowledge and self-efficacy in teaching mathematics. Teachers who have experienced only teacher-centered mathematics teaching, have limited mathematical knowledge, and/or insufficient levels of self-efficacy beliefs are unlikely to effectively implement student-centered instructional practices. However, teachers who have had positive mathematics learning experiences in student-centered classrooms, have acquired adequate mathematical knowledge, and have achieved adequate levels of self-efficacy beliefs are more likely to implement effective student-centered practices and to maintain consistency between their beliefs and practices. Therefore, school administrators should provide opportunities for teachers at all levels of experiences to recount their mathematics teaching and learning experiences, pedagogical beliefs, and instructional practices as a way to open the door to make changes when it is necessary. Administrators also support teachers' development of student-centered beliefs and practices and acquire new teaching knowledge and skills that will help them carry out these practices in their classrooms.

School administrators also could implement mentor programs to support novice teachers as they try to apply student-centered instructional practices while dealing with all the challenges of being beginning teachers, because these challenges can influence them to implement teacher-centered practices as a way of maintaining control over classroom management. In this situation, mentor teachers with student-centered beliefs and practices can support novice teachers by sharing effective teaching skills and opening their mathematics classrooms for observations. With such support, novice teachers can maintain what they have learned and experienced in their teacher education programs and continue to develop productive pedagogical and self-efficacy beliefs and not give up on student-centered instructional practices.

Teachers' mathematics-related learning experiences play a pivotal role in constructing their pedagogical beliefs and instructional practices. To maintain student-centered beliefs, teachers should continue to retrospectively examine their past learning and teaching experiences and continue to develop their mathematical knowledge. These endeavors may help them recognize low-quality instructional practices are caused by their unproductive beliefs and limited mathematical and pedagogical knowledge and lack of confidence. Additionally, teachers should understand that student-centered beliefs and efforts can assist them in overcoming challenges they encounter in mathematics teaching and supporting students' mathematics learning. In such classroom environments, students can be independent learners and experience a great sense of accomplishment as they explore various ideas and devise creative solutions as mathematics investigators.

Limitations of this Study

Although this study has provided a number of implications, there are some limitations. First, as a qualitative case study, it provided an analysis of a small number of Korean elementary teachers using interview, observational, and field notes data. Moreover, I was the sole researcher who interpreted the data and drew conclusions. In this process, my personal bias or educational background might have influenced my interpretations (Creswell & Creswell, 2017). The findings should be interpreted as a new perspective on the influence of teachers' life stories on their instructional beliefs and practices, which should be further studied. Additionally, while the number of the participants is not uncommon in qualitative case studies, it does narrow the level of generalizations across the population of elementary Korean teachers. However, the findings and conclusions of this study about the eight participants provides researchers an opportunity to study and interpret the findings and use them to design measures that are inclusive of a larger number of participants.

The next limitation is that the relationship between teachers' pedagogical beliefs and knowledge for teaching mathematics raises some questions. In this study, a bi-directional relationship has been assumed, although it is not as strong as the influence of knowledge on instructional practices (see Figure 6.1). Because in this study, following Kagan (1992) and Philipp (2007), beliefs were defined as the convergence zone between the cognitive and affective domains, teachers' pedagogical beliefs were considered to both influence and be influenced by their mathematical knowledge. However, other researchers have totally differentiated beliefs and knowledge (e.g., Nespor, 1987). From their perspective, therefore, assumption 1-2 might be more reasonable to explain the relationship. While most current mathematics educators assume certain relationships between mathematics teachers' beliefs and knowledge, as described in Furinghetti and Pehkonen's (2002) mathematics specialist panel study, the specific directionality between them is still obscure. Do teachers' pedagogical beliefs influence their mathematical knowledge or vice versa?

The obscurity of these relationships resulted from the design of this study. I collected data only at specific time points, so I could not make sure whether teachers' beliefs, knowledge, and instructional practices developed sequentially or simultaneously. Additionally, teachers could not properly describe the complex relationships and recognize subtle changes because most of these changes happened cognitively and subjectively. While such discussion is beyond the scope of this study, it would be valuable to conduct studies that shed light about how teachers' beliefs, knowledge, and instructional practices develop over time.

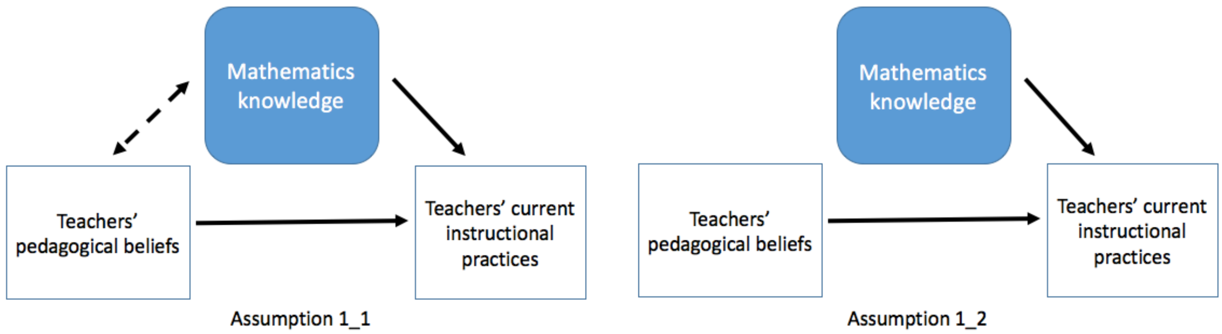


Figure 6.1. Assumptions between teachers' pedagogical beliefs and mathematical knowledge

Recommendations for Future

This study has generated a set of additional lines of inquiry. First, the structure of this study could be used to analyze the relationships between other teachers' life stories, pedagogical beliefs, and instructional practices across different international settings and levels of instruction (e.g., secondary school teachers). Second, this study analyzed only teachers' voices. For a more holistic picture, researchers could analyze administrators,' parents,' and students' interpretations of their teachers' instructional practices. The findings of this study might also be more robustly validated with quantitative research methods. For example, all types of teachers' classroom discourse could be counted and compared with those of other teachers using a t-test. Moreover, going beyond the limitations of this study, the structural relationship between teachers' life stories, pedagogical beliefs, mathematical knowledge, and instructional practices could be examined with the structural equation modeling. Last, teachers' development of their pedagogical beliefs and instructional practices could be traced longitudinally. For example, we might analyze the transition from preservice to inservice and from novice to experienced teacher status.

Closing Comments

When I was a teacher, I always wanted to know why teachers' instructional practices were different, which ultimately led me to pursue doctoral study in the U.S. From this study, I have gained a nuanced insight into how teachers' instructional practices are developed and changed by focusing on their life stories and pedagogical beliefs. However, I also believe that many factors which were not explored in this study influence teachers' instructional practices. Therefore, we should continuously conduct research that illuminates teachers' instructional practices. The outcome of those studies would be programs and interventions that enhance teachers' instructional practices and student outcomes. I admit that there are many unanswered questions about teachers' instructional practices. This dissertation is not the end of my pursuit of such questions but the beginning of life-long research.

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APPENDIX A

TEACHERS' MATHEMATICAL BELIEFS INTERVIEW PROTOCOL

A. Self-efficacy

A1. Do you believe that you teach mathematics concepts effectively? If so, explain how you know?

A2. Do you believe that you have the necessary skills and knowledge to teach mathematics? Explain how do you know?

A3. Are you able to answer most of your students' questions correctly?

A4. Do you welcome your students' questions when teaching mathematics? Why?

A5. Do you believe that when a student does better than usual in mathematics, it is often because you excreted a little extra effort?

B. Nature of Mathematics

B1. Some people believe that people need to know the correct procedure to solve mathematics problems. Do you agree? If so, why?

B2. Do you believe that the development of mathematical knowledge is related to social development?

B3. Do you believe that mathematics knowledge change and develop over time?

C. Teaching and learning mathematics

C1. Some people believe that teachers should teach the exact procedures for solving problems. Do you agree? Why?

C2. Do you believe that teachers need to make the mathematics tasks easy for students to ensure all students are not frustrated or confused? Do you agree? Why?

C3. Do you believe that time should be spent practicing simple problems before students spend time solving perplexing procedures? Do you agree? Why?

C4. Some people believe that mathematical ability is fixed and unchanged through a person's life. Do you agree? Why?

C5. Some people believe that all students should have a natural ability to be good at mathematics. Do you agree? Why?

D. Perceptions about students

D1. Describe your classroom students, regarding their mathematics abilities and motivations.

D2. What is their role in mathematics classroom? What do they usually do in mathematics classroom?

D3. Who is a good student in your mathematics classroom?

D4. What types of instructional practice do your students like best? Why? (e.g., drill practice, word problems, working together)

D5. Which 3 words would you use to describe your students? Why?

E. School Context and Culture

E1. How about your school culture?

E2. Which 3 words would you use to describe this school? Why?

E3. Do you think that your school want you to use specific teaching practices or classroom management skills?

E4. How would you describe your relationship with parents? Do they support your teaching and student learning?

APPENDIX B

TEACHER'S LIFE STORY INTERVIEW PROTOCOL

Part 1. Overall Life Stories

A. Experiences as a learner

A1. What is your schooling experience as a child growing up?

A2. Which 3 words would you use to describe you as a student in mathematics classroom?

A3. Did you like mathematics or not? Explain why did you like it or not?

A4. Briefly describe your favorite mathematics teachers, and least favorite mathematics teacher?

B. Motivation to become a teacher

B1. Why did you decide to become a mathematics teacher? (What made you decide to become an educator?)

B2. Have your upbringing or your past home and family life influenced your decision to become a mathematics teacher or your views about it?

B3. If you could have chosen another profession besides teaching, what would it be and why?

C. Experiences at teacher education program

B1. Among the student teaching (practicum), college courses, and field experiences, what was the most useful experience to you? Why?

B2. Briefly describe your favorite (memorable) mathematics-related experiences

B3. Which 3 words would you use to describe your teacher education program? Why?

B4. Do you think that mathematical knowledge you acquired from the teacher education program is useful or not?

D. Perspectives and Experiences on Teaching Mathematics:

D1. Do you like to teach mathematics, if not, why?

D2. What is your teaching philosophy and goal in mathematics?

D3. What experiences or who influence on your current mathematics teaching practice?

D4. What types of instructional practices do you like most? (e.g., drill practice, word problems, working together)

D5. Describe your teaching practices with three words. Why?

D6. What do you think of when you hear the phrase "quality mathematics lesson"?

D7. Among the following examples, what is the most challenging aspects when teaching mathematics?

(a) preparing for standardized tests, (b) lacks of resources and time, (c) pressure of school administers using a specific pedagogy, (d) classroom management, (e) insufficient student effort, (f) teachers' limited mathematical knowledge.

D8. What curriculum materials, including textbooks, do you use, when you design mathematics lesson? Do you modify curriculum materials? Why?

Part 2. Critical Life Stories
Retrieved from Drake, Spillane & Hufferd-Ackles (2001) based on McAdams (1993)

A. Critical Events: Please ready following descriptions and explain your experiences

A1. Peak experience—high point in your story about mathematics in your life

It would be a moment or episode in the story in which you experienced extremely positive emotions; like joy, excitement, great happiness, uplifting, or even deep inner peace after some mathematics experience. Tell me what happened, where it happened, who was involved, what you did, what you were thinking and feeling, what impact this experience may have had upon you, and what this experience says about who you were or who you are now as a teacher.

A2. Nadir experience—low point in your experiences with mathematics

Looking back on your life, try to remember a specific experience in which you felt extremely negative emotions about mathematics. You should consider this experience to represent one of the 'low points' in your mathematics story. What happened? When? Who was involved? What did you do? What were you thinking and feeling? What impact has the event had on you? What does the event say about who you are or who you were as a teacher?

A3. Turning point—episodes through which a person undergoes substantial change.

I am especially interested in a turning point in your understanding of mathematics. Please identify a particular episode in your life-story that you now see as a turning point. If you feel that your mathematics story contains no turning points, then describe a particular episode in your life that comes closer than any other to qualifying as a turning point.

A4. Important childhood scene—describe a memory about mathematics from your childhood that stands out in our mind as especially important or significant?

Now, describe a memory about mathematics from your childhood that stands out in your mind as especially important or significant. It may be a positive or negative memory. What happened? Who was involved? What did you do? What were you thinking and feeling? What impact has the event had on you? What does it say about who you were? Why is it important?

A5. Important adolescent scene—describe a specific event from your adolescent years that stands out as being especially important or significant with respect to mathematics

Describe a specific event from your adolescent years that stands out as being especially important or significant with respect to mathematics.

B. Life Challenge:

B1. Greatest Challenge

Looking back over your life and interactions with mathematics, please describe the single greatest challenge that you have faced. How have you faced, handled, or dealt with this challenge? Have other people assisted you in dealing with this challenge? How has this challenge had an impact on your experiences with mathematics?

B2. Influences on the Life Story: Positive and Negative

Positive—looking back over your life-story, please identify the single person, group of persons, or organization/institution that has or have had the greatest positive influence on your perspective of mathematics. Why?

Negative—please identify the single person, group of persons, or organization/institution that has or have had the greatest negative influence on your perspective of mathematics. Why?