Essays in Monetary Economics

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Abstract

This dissertation consists of three essays that study macroeconomic modeling and its application with a particular focus on monetary economics.

In Chapter 1, I develop a New Keynesian model with heterogeneous workers whose wage settings are subject to downward nominal wage rigidity (DNWR) to address two puzzles of inflation dynamics: the missing deflation during the Great Recession and the excessive disinflation afterward. I demonstrate that DNWR introduces a time-varying wedge between the output gap and the marginal cost of producing one unit of output, which makes the observed Phillips curve flatter during recessions. Endogenous evolution of cross-sectional wage distribution generates various dimensions of non-linearities, while the presence of the zero lower bound (ZLB) of the nominal interest rate further reinforces the mechanism. Consequently, the model can quantitatively account for the inflation dynamics during and after the Great Recession under plausible parameter values that are consistent with micro evidence.

In Chapter 2, I study welfare-maximizing monetary policy rule in the heterogeneous agent New Keynesian model with DNWR that is developed in Chapter 1. The optimal monetary policy rule responds strongly to output to address the inefficiency generated by DNWR, while responsiveness to inflation plays a minor role in welfare. Moreover, monetary policy can improve social welfare by responding more aggressively to a contractionary shock than to an expansionary one to offset the asymmetry stemming from DNWR. In the presence of the ZLB, on the other hand, alternative policy rules such as forward guidance and price-level targeting can partly offset the adverse effects of it by committing to a future low interest rate policy. I also investigate the optimal steady-state inflation rate.

In Chapter 3, which is coauthored with Dongho Song and Jenny Tang, we propose a method of introducing theory-driven priors into the estimation of the vector autoregression (VAR). Our methodology is more flexible than existing methods in that it allows a researcher to incorporate prior beliefs on a subset of variables in theoretical models that are of key interest while remaining agnostic about other variables in the VAR. We apply to the problem of exchange rate forecasting for the British pound versus the US dollar. By imposing different combinations of priors informed by uncovered interest rate or purchasing power parity, we find that substantial gains are realized at longer forecast horizons.

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Chapter 1

Downward Nominal Wage Rigidity and Inflation Dynamics during and after the Great Recession

1.1 Introduction

During the financial crisis of 2008-2009 and its aftermath, the U.S. economy experienced little decline of inflation while suffering from severe economic downturn. The fact is known as the missing deflation puzzle. To this end, Hall (2011), in his Presidential Address to the American Economic Association, argues that the little response of inflation to the long-lasting slack after the Great Recession is inconsistent with most of economic theories. Several years later, the recovery of inflation was excessively slow despite the sluggish but steady improvement of real economic activities. The shortfall of inflation from 2 percent without many of adverse factors is expressed as a mystery by the Chair of the Federal Reserve in Yellen (2017). Constâncio (2015) labels the phenomenon as excessive disinflation.[1] These observations call into question one of the fundamental theories in modern monetary economics: the Phillips curve relationship between inflation and the level of economic activity.

In this paper, I argue that downward nominal wage rigidity (DNWR) reconciles both of the missing deflation during the Great Recession and the excessive disinflation in the subsequent years. Specifically, I introduce DNWR for individual workers into an otherwise standard New Keynesian dynamic stochastic general equilibrium (DSGE) model that embeds monopolistically competitive firms with nominal price rigidity and the Taylor (1993)-type nominal interest rate feedback rule. To the best of my knowledge, this is the first study to build a DSGE model with both nominal price rigidity and the explicit constraint on downward nominal wage adjustment for individual workers. In this setting, I demonstrate that DNWR accounts for the flattening of the observed Phillips curve during recessions. Moreover, taking into account heterogeneity of individual workers' wages enables the model to replicate many dimensions of non-linearities in the data through the endogenous evolution of cross-sectional wage distribution upon an exogenous shock. Compared to previous studies on DNWR, nominal price rigidity and DNWR have an important interaction to generate substantial persistence of real wage especially downward. Consequently, the model can quantitatively match the key moments of inflation dynamics during and after the Great Recession under plausible parameter values that are consistent

[1] Constâncio (2015) points out that the excessive disinflation is a common feature of inflation dynamics in advanced economics in recent years.

with micro evidence.

The model is motivated by two empirical facts. First, numerous studies point out that the Phillips curve relationship between inflation and the output gap was altered after the Great Recession (e.g., Stock and Watson (2010), Ball and Mazumder (2011), Coibion and Gorodnichenko (2015)). However, I find that the marginal cost representation of the Phillips curve, which is directly derived from firms' price setting behavior, remained stable in the data. The finding is robust when I relax the rational expectation assumption. Instead, I document that the relationship between the output gap and marginal cost is non-linear in the sense that marginal cost is less responsive to the output gap in recessions. These facts imply that a puzzle indeed lies in the relationship between the output gap and marginal cost rather than in the Phillips curve itself. Therefore, I focus on the relationship between them to explain the changes of the observed Phillips curve over the business cycle. Second, the rapidly growing empirical literature using micro data uncovers severe DNWR during the Great Recession and its aftermath. For instance, previous studies report that the fraction of workers with zero nominal wage changes substantially increased in the periods, which is consistent with predictions of DNWR (Daly and Hobijn (2014), Fallick et al. (2016)). I incorporate the micro evidence into a general equilibrium model to study its aggregate implications, especially on inflation dynamics.

The key mechanism of the model for generating the missing deflation is as follows. DNWR creates a time-varying wedge between real wage and the marginal rate of substitution of consumption for hours worked by impeding wage adjustment to its desired level. The wedge in turn appears in the output gap representation of the

New Keynesian Phillips Curve (NKPC) as a shift parameter, and it accounts for the flattening of the observed Phillips curve in recessions. Intuitively, the binding DNWR constraint upon a contractionary shock prevents real wage and therefore firms' marginal cost from declining. Since the forward-looking nature of the NKPC implies that inflation is expressed as infinite sum of the discounted values of the current and future marginal costs, the dampened responses of marginal cost result in little decline of inflation in recessions. On the other hand, imperfect adjustments of price variables are compensated by large contractions of real quantities including the output gap in general equilibrium. As a consequence, even though the marginal cost representation of the NKPC remains unchanged, the observed Phillips curve relationship between inflation and the output gap becomes flatter in recessions in the presence of DNWR.

I allow for heterogeneity of individual workers' wages that may or may not be subject to the DNWR constraint. By doing so, the aggregate dynamics of the model, including the degree of the aggregate wage and price stickiness, crucially depend on the evolution of cross-sectional wage distribution. To be precise, the responses of the model are asymmetric depending on the sign of an exogenous shock. A larger fraction of workers is constrained by DNWR upon a contractionary shock, whereas the constraint comes to bind for fewer workers upon an expansionary one. Hence, the aggregate wage is more rigid downward than upward and that spills over asymmetry of other variables. The aggregate dynamics are also affected by the size of an exogenous shock, because a larger shock changes the fraction of workers with or without the binding constraint more drastically. Therefore, the mechanism of the missing deflation described above is particularly strong for a large and contractionary shock such as the Great Recession.

It is noteworthy that the mechanism of the missing deflation is reinforced by the presence of the zero lower bound (ZLB) of the nominal interest rate. As is pointed out in the existing literature (e.g., Christiano et al. (2011)), the impacts of an exogenous shock are amplified under the ZLB due to the lack of offsetting monetary policy responses. However, since the room for downward wage adjustment is limited by DNWR in my model, the amplification effect of the ZLB is exclusively absorbed by further contractions of real quantities, without generating a large drop of inflation. The finding is in stark contrast to previous studies such as Gust et al. (2017), who find the responses of inflation as well as those of quantities to an exogenous shock are enlarged when the economy is at the ZLB. This effect helps to reconcile the moderate decline of inflation and the sharp fall of real quantities during the Great Recession.

On the other hand, I find the state dependency of DNWR to be the key feature of the model to address the excessive disinflation after the Great Recession. Since the DNWR constraint holds in terms of the level of wages, once workers' desired wages fall short of their actual ones upon a contractionary shock, workers never react to improvements of the state of the economy until their desired wages exceed their actual ones. Though this pent-up wage deflation mechanism is mentioned in several studies (e.g., Daly and Hobijn (2015), Constâncio (2015)), I demonstrate its formal link to the excessive disinflation, and derive quantitative outcomes in a framework of a DSGE model.

For quantifying these implications of the model, I overcome two potential challenges in numerical methods. First challenge is computing equilibrium of the model with heterogeneous agents. In this regard, since the main focus of this paper is on inflation dynamics at business cycle frequencies, I choose to solve the model under aggregate uncertainty by applying the Krusell and Smith (1998) algorithm. Although their original algorithm requires aggregate jump variables to have a closed form solution in terms of aggregate state variables, that condition is not satisfied in my New Keynesian setting with the ZLB. To address the problem, I propose a modified algorithm in which each of the aggregate- and individual-part of the economy is solved recursively with a global method.

Another important challenge is parameterization of the degree of price and wage rigidities. It is widely recognized that an estimated DSGE model often identifies much higher parameter values for the degree of price stickiness than the ones implied by micro evidence (Altig et al. (2011), Del Negro et al. (2015)), and the inflation dynamics of the model heavily relies on that parameter. Similarly, Schmitt-Grohé and Uribe (2016) discuss that the parameter governing the degree of DNWR is crucial to their quantitative results. In this regard, I calibrate the model to match various moments of the price and wage distribution in U.S. data, and find that my model endogenously generates strong persistence of inflation under the micro founded parameter values.

My quantitative results are summarized as follows. A counterfactual analysis in the calibrated model predicts that the contractionary shock that has the same magnitude as the Great Recession only leads to 2.1-2.4 percentage point decline of the year-on-year inflation rate under plausible assumptions on monetary policy rules. The quantitative result is comparable to the data during the period when the actual inflation rate in the GDP deflator declined by 2.3 percentage point from the peak to the bottom.[2] Regarding the excessive disinflation, the calibrated model suggests the recovery of inflation from a severe recession state that corresponds to the Great Recession is three times as slow as from the median state. For comparison, I show that a stylized New Keynesian model predicts a massive deflation upon the Great Recession shock and a relatively quick recovery afterward. It is notable that the only extension of my model from the stylized New Keynesian model is the presence of DNWR along with the ZLB, and my model successfully matches the key moments of the inflation dynamics during and after the Great Recession.

The remainder of this paper is organized as follows. Section 2 reviews related literature. Section 3 provides theoretical and empirical evidence to motivate my model analysis in the subsequent sections. Section 4 develops my baseline DSGE model with DNWR, and Section 5 describes an equilibrium computation method as well as calibration. Section 6 presents numerical results, which are followed by discussion in Section 7. Section 8 conducts a counterfactual analysis of the Great Recession to test for the validity of my model to account for the inflation dynamics in the periods. Section 9 is conclusion.

[2] The peak of the GDP deflator before the Great Recession is 2.5 percent as of 2007Q4, whereas the bottom is 0.2 percent as of 2009Q3.

1.2 Related literature

This paper falls into the growing literature that studies the missing deflation and the excessive disinflation after the Great Recession. There are mainly two strands of literature regarding the missing deflation puzzle. The first strand emphasizes the importance of the formation of the inflation expectations. Bernanke (2010) suggests that the credibility of modern central banks succeeded in convincing people that extremely high or low inflation would not occur, and this anchored expectation stabilized the actual inflation. In contrast, Coibion and Gorodnichenko (2015) argue that the stability of the inflation expectations is not enough to resolve the puzzle quantitatively. They instead claim that the rises of the household inflation expectations due to the surging commodity prices after 2008 prevented deflation. Bianchi and Melosi (2017) propose another mechanism that fiscal and monetary policy uncertainty, that is, a possibility of switching to a high inflation regime driven by large fiscal deficit, keeps the inflation expectations high enough. Del Negro et al. (2015) reconcile these views by estimating a medium scale DSGE model that embeds the financial friction of Bernanke et al. (1999) on the Smets and Wouters (2007) model. They conclude that the anchored expectation view is plausible if the Phillips curve is sufficiently flat, because monetary policy can have strong real effects to stabilize the inflation expectations under a flat Phillips curve.

The second strand of the literature focuses on firms' marginal cost and price markup as a potential cause of the missing deflation. Christiano et al. (2015) develop a model in which financial frictions raise firms' capital cost to hinge their marginal cost from falling in recessions. Kara and Pirzada (2016) introduce intermediate good prices in the Smets and Wouters (2007) model to take into account the rises of commodity prices in the data. On the other hand, Gilchrist et al. (2017), extending the model of consumer capital by Ravn et al. (2006), suggest that the liquidity needs during the financial crisis drove firms to raise their price markup given their nominal marginal cost at the expense of the future customer base.[3]

Regarding the excessive disinflation, one of the earliest studies to point out the puzzle is Constâncio (2015). He refers to several hypotheses to settle the puzzle including anchoring of the inflation expectations and increased international competition, though formal analysis has not yet been provided in the literature, to my knowledge. On the empirical side, Albuquerque and Baumann (2017) propose to use a short-run labor market slack measure when estimating the Phillips curve, while Bobeica and Jarocinski (2017) emphasize the importance of the distinction between global and domestic factors to determine inflation.

This paper is distinct from these studies in various important dimensions. First of all, in terms of the inflation expectations, my empirical result of the stability of the marginal cost representation of the NKPC after the Great Recession holds when I relax the rational expectation assumption by incorporating the survey based

[3]However, there is another view in the literature that financial frictions contribute to a decline of inflation. Kim (2018) finds that liquidity constrained firms committed to fire sales of their inventory to generate cash flow after the Lehman Brothers' failure. Although he also find the evidence on the inflationary effects of financial frictions proposed by Christiano et al. (2015) and Gilchrist et al. (2017), he reports that the deflationary effects of the inventory fire sales is quantitatively dominant.

inflation expectations. Moreover, I show that my model with DNWR reconciles both of the missing deflation and the excessive disinflation coherently. In contrast, if one argues that the missing deflation is driven by the rises of the inflation expectations relative to the rational expectations after the Great Recession, it is necessary to seek for another factor that prevents the recovery of inflation to address the subsequent excessive disinflation. Compared to other studies that investigate marginal cost and price markup, on the other hand, I find the quantitative importance of the wage channel to determine marginal cost. In other words, taking into account DNWR along with the ZLB explains most of the missing deflation that appears in a stylized New Keynesian model.

Another class of literature that this paper is deeply related to is that of DNWR. Here I assess the studies that explore the aggregate implications of DNWR, while numerous studies have investigated micro evidence of it.[4] A seminal work by Akerlof et al. (1996) demonstrates that the long-run wage Phillips curve is no longer vertical in the presence of DNWR. In other words, involuntary unemployment does not decay even in the long-run, since once the actual wage exceeds the desired one the discrepancy between them is not eliminated without inflation. Benigno and Ricci (2011) analytically characterize the equilibrium with DNWR under a modern setting with optimizing agents. Applying their insights, recent studies argue that the upward sloping long-run wage Phillips curve due to DNWR together with the ZLB is a cause behind the jobless recoveries after the Great Recession (Schmitt-Grohé and

[4] Discussion on the micro evidence of DNWR and its connection to this paper is provided in section 1.7.

Uribe (2017)) and secular stagnation (Eggertsson et al. (2017), Kocherlakota (2017)). Elsby (2009), on the other hand, claims that DNWR does not have significant effects on the aggregate wage growth since forward looking agents compress their wage hikes for a precautionary motive. Daly and Hobijn (2014), focusing on transition dynamics, show that DNWR bends the short-run wage Phillips curve as well, which generates the non-linear fluctuations of the unemployment rate. Schmitt-Grohé and Uribe (2016) develop a small open economy model to claim that DNWR is the fundamental cause of the high unemployment rate in the euro area in recent years.

A crucial difference of my model from the existing literature on DNWR is that I introduce DNWR into the New Keynesian setting with nominal price rigidity. It is worth pointing out that the studies on DNWR discussed above are conducted under flexible prices. More specifically, these models are developed either in the steady state with a constant inflation rate (Akerlof et al. (1996), Elsby (2009), Benigno and Ricci (2011), Schmitt-Grohé and Uribe (2017), Eggertsson et al. (2017), Kocherlakota (2017)) or under a time-varying but fully flexible inflation rate (Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016)). However, nominal price rigidity is an essential ingredient of the model for my purpose: studying inflation dynamics. In particular, nominal price rigidity is indispensable to capture the Phillips curve observed in data. I also find that the combination of nominal price rigidity and DNWR leads to substantial persistence of real wage, which replicates sluggish movements of price variables and sizable fluctuations of real quantities in data.

In this regard, several papers use a smooth asymmetric wage adjustment cost function to approximate DNWR (Kim and Ruge-Murcia (2009), Fahr and Smets

(2010), and Aruoba et al. (2017)). However, I find that the explicit DNWR constraint in my model generates a strong non-linear dynamics, and therefore the model is able to match the data under micro founded parameter values. Moreover, whereas these studies use a perturbation method to derive an approximated solution around the steady state, my global solution method captures the state-dependency of the model, which I find is essential to resolving the excessive disinflation.

On the methodological side, the model developed in this paper is classified as a heterogeneous agent model with aggregate uncertainty, which is initiated by Krusell and Smith (1998). This class of model has been used to study various dimensions of the economy in the existing literature, including asset and consumption dynamics (e.g., Krusell and Smith (1998), Krueger et al. (2016)), search and matching (e.g., Krusell et al. (2010), Nakajima (2012)), and price setting behavior (e.g., Nakamura and Steinsson (2010), Vavra (2013)). Among others, my model is closely related to Gornemann et al. (2016), McKay and Reis (2016), and Blanco (2018), who apply the equilibrium concept to the New Keynesian model with the Taylor (1993)-type monetary policy rule, but distinct from them in that I examine the heterogeneity of individual wages arising from DNWR.

1.3 Motivating evidence

This section presents motivating evidence for the model analysis in the subsequent sections. First, using a stylized New Keynesian model, I point out that the key assumption to bring about a difficulty in accounting for the inflation dynamics after the Great Recession is in the relationship between marginal cost and the output gap. Second, I provide empirical evidence to show that the assumption does not hold in the data. Specifically, I estimate two representations of the NKPC: the output gap representation and the marginal cost representation, to uncover that, whereas the farmer became flatter after the Great Recession, the latter remained stable. I also assess the empirical relationship between marginal cost and the output gap in the data.

1.3.1 Example in a stylized New Keynesian model

To see the core of the problem of a New Keynesian model in resolving the missing inflation and the excessive disinflation, I consider a stylized linear New Keynesian system that consists of the Euler equation (1.1), the NKPC (1.2), and the Taylor rule (1.3), as well as the relationship between marginal cost of the output gap (1.4):

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}]) + \epsilon_t \tag{1.1}$$

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa mc_t \tag{1.2}$$

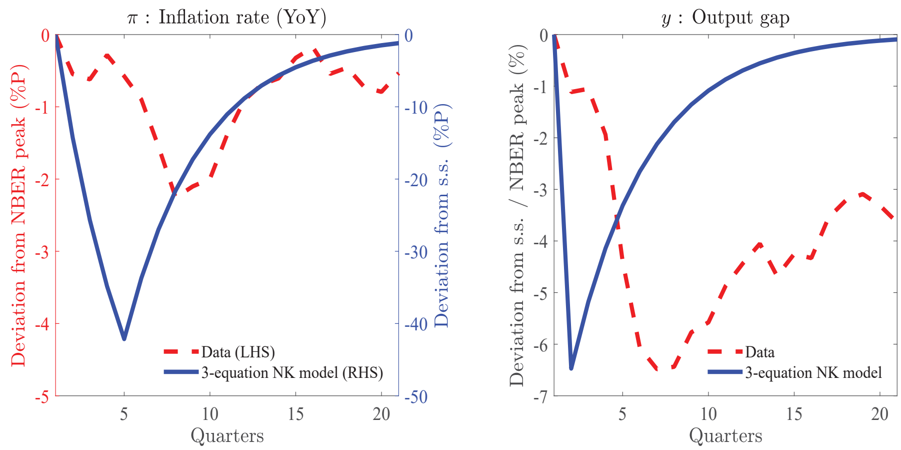
$$i_t = \delta_\pi \pi_t + \delta_y y_t \tag{1.3}$$

$$mc_t = (\sigma + \eta) y_t \tag{1.4}$$

$$where \quad \epsilon_t = \rho_\epsilon \epsilon_{t-1} + e_{\epsilon,t}, \quad e_{\epsilon,t} \sim i.i.d.N(0, \sigma_\epsilon^2)$$

y_t, π_t, i_t, and mc_t denote the output gap, the inflation rate, the nominal interest rate, and marginal cost, respectively. Each variable is in the log-deviation from the zero-inflation steady state. Loosely speaking, the Euler equation (1.1) and the Taylor rule (1.3) govern the demand side of the economy, whereas the NKPC (1.2) the supply side.

Figure 1.1: IR in the 3-equation New Keynesian model and data



Notes: The impulse responses (IR) of the stylized New Keynesian model are those to an exogenous innovation $e_{\epsilon,t}$. Size of shock is calibrated to match the drop of the output gap in data. For the series of data, the inflation rate is the year-on-year growth rate of the GDP implicit price deflator. The output gap is the one estimated by the Congressional Budget Office (CBO). Each data series is in the deviation from the business cycle peak before the Great Recession that is defined by the NBER (2007Q4).

They are connected through the relationship between marginal cost of the output gap (1.4), which is derived from the labor market equilibrium condition. The derivation of the equations follows Walsh (2010).

For simplicity, I suppose that the error term in the Euler equation ϵ_t, which follows an AR(1) process, is the only exogenous component of the economy. For the numerical exercise below, parameter values are set as follows: the discount factor $\beta = 0.995$, the relative risk aversion $\sigma = 2.00$, the inverse of the Frisch labor supply elasticity $\eta = 0.25$, the Taylor rule coefficients $\delta_\pi = 1.50, \delta_y = 0.25$, the slope of the NKPC $\kappa = 0.20$, and the persistence of the exogenous shock $\rho_\epsilon = 0.80$. Though these parameter values are in line with the literature, the choice of the parameter values is discussed in Section 1.5.

Figure 1.1 compares the impulse responses of the stylized New Keynesian model and the data after the Great Recession. It is immediate to see that the stylized New Keynesian model predicts massive deflation (-42 percentage point), whereas the actual inflation rate declined moderately (-2.3 percentage point). It is also notable that the inflation rate in the model quickly reverts toward its steady-state value, whereas the recovery of the actual inflation were so sluggish with a sizable gap from the pre-crisis rate even 20 quarters after the Great Recession.

Among others, one of the key features to bring about these undesirable results in the stylized New Keynesian model is the relationship between the output gap and marginal cost (1.4). To be precise, the households' marginal rate of substitution between consumption and hours worked is equalized to real wage in a frictionless labor market. Real wage is proportional to the firms' marginal cost of producing one unit of output, whereas the marginal rate of substitution is related to households' consumption and hours worked, and it is therefore tied with the output gap in general equilibrium. In other words, the labor market equilibrium condition determines the relationship between the output gap and marginal cost. In fact, one can derive a linear relationship between them in the stylized New Keynesian setting. Since inflation is pined down by the current and future marginal costs through the NKPC, it is quite difficult to obtain a small decline of inflation and a large drop of the output gap simultaneously as long as the linear relationship between the output gap and marginal cost is taken as given.

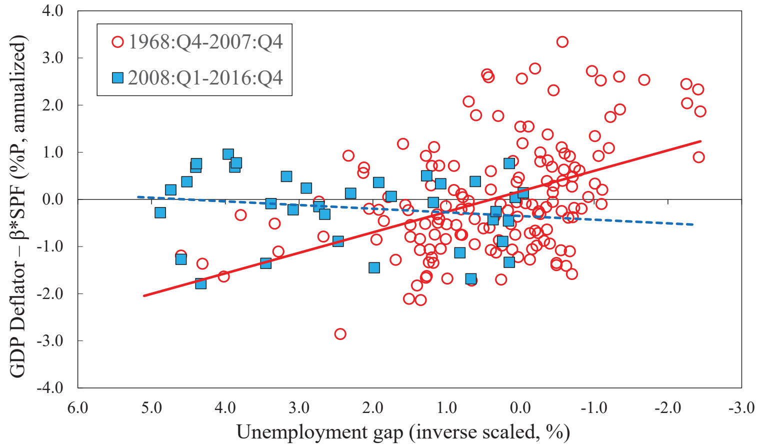
1.3.2 Empirical evidence

1.3.2.1 Estimation of the NKPC

This subsection estimates the NKPC in U.S. data. The NKPC is originally formulated from a firms' profit maximization problem under nominal price rigidity. Therefore, it relates firms' marginal cost to inflation, whereas the output gap representation is obtained as a consequence of general equilibrium. On the empirical side, early studies by Gali and Gertler (1999) and Sbordone (2002) obtain significant estimates for the slope parameter of the NKPC when they use a measure of marginal cost rather than the output gap as a regressor. Both studies conclude that employing marginal cost is preferable for the estimation of the NKPC because it purely represents the firms' optimization behavior without imposing additional assumptions on the other parts of the economy. On the other hand, many of recent studies including Ball and Mazumder (2011), Murphy (2014), and Coibion and Gorodnichenko (2015) report the flattening of the NKPC after the Great Recession by estimating the output gap representation. Therefore, it would be worthwhile to examine whether the flattening of the NKPC is also present in the marginal cost representation.

Before proceeding to a regression analysis, Figure 1.2 displays the output gap representation of the NKPC in U.S. data. The fitted lines in the figure suggest that the output gap representation became flatter in the sample after 2008Q1 in line with findings of previous studies. It implies not only that the initial decline of inflation during the Great Recession was small but also that the recovery of inflation has been slow compared to the improvements of real economic activities.

Figure 1.2: The output gap representation of the NKPC in U.S. data



Notes: The x-axis is the unemployment gap as a proxy for the output gap (x_t). The y-axis represents the inflation rate minus the inflation expectation term ($\pi_t - \beta\pi_t^e$) where the discount factor β is calibrated to be 0.995. The inflation measure is the GDP implicit price deflator. The inflation expectation is the median forecast of the SPF. The slope of the fitted lines represents the slope of the NKPC in each sub-sample.

I estimate the following specification of the NKPC:

$$\pi_t = \beta\pi_t^e + \gamma\pi_{t-1} + \kappa x_t + e_t \tag{1.5}$$

Three modifications are added to the NKPC of Equation (1.2). First, the expectation term $\mathbb{E}_t[\pi_{t+1}]$ is replaced with the survey based expectation π_t^e. It reflects empirical findings in the literature that using survey based expectation measures substantially improve the fit of a regression model (e.g., Adam and Padula (2011), Coibion and Gorodnichenko (2015), Furhrer (2017)). Second, following a convention of the literature, the lagged inflation term π_{t-1} is included to incorporate the persistent fluctuations of inflation in the data. Lastly, the error term e_t captures exogenous innovations to the current inflation such as markup shocks. Notice that, under this assumption, the OLS delivers an unbiased estimator, while the GMM estimator is often used in the literature to deal with endogeneity when the rational expectation error appears in the error term.

I use the GDP implicit price deflator as a measure of inflation. It is considered to be preferable to the Consumer Price Index (CPI) or the Personal Consumption Expenditures (PCE) inflation for the purpose of this regression analysis, because the inflation rate that appears in the NKPC need to reflect the pricing behavior of domestic firms rather than the price index faced by domestic households. The measure of the inflation expectations is the median forecast of the GDP deflator in the Survey of Professional Forecast (SPF) provided by the Federal Reserve Bank of Philadelphia. The SPF is the only survey that asks the forecast of the GDP deflator. Two cases are considered for the choice of varialbe x_t: the output gap and marginal cost. Following Coibion and Gorodnichenko (2015), I use the unemployment gap as a proxy variable for the output gap.[5] Regarding a series of marginal cost, since it is infeasible to directly observe firms' marginal cost, I follow the insights of Hall (1986) to resort to firms' optimizing behavior to estimate it. Specifically, I consider a cost minimization problem:

$$\min_{H_t, K_t} : \; P_t^H H_t + P_t^K K_t \tag{1.6}$$

$$s.t. \quad Y_t = F(H_t, K_t, Z_t) \tag{1.7}$$

where Y_t denotes output, Z_t technology, H_t labor inputs, K_t capital inputs. Production technology is given by F. Firms are assumed to be price taker in factor markets.

[5]I use the long-term unemployment gap provided by the CBO in my baseline estimation. However, I find that main results are robust to other slack measures such as the detrended output and the employment rate.

The first order condition (FOC) for the problem takes the form:

$$P_t^J = \lambda_t \frac{\partial F_t}{\partial J_t} \quad for\ J = K, H \tag{1.8}$$

where λ_t denotes the Lagrangian multiplier that represents the nominal marginal cost of producing one unit of output. The FOC can be rearranged to:

$$MC_t \equiv \frac{\lambda_t}{P_t} = \underbrace{\left(\frac{\partial log(F_t)}{\partial log(J_t)}\right)^{-1}}_{(\theta_t^J)^{-1}} \underbrace{\frac{P_t^J J_t}{P_t Y_t}}_{s_t^J} \tag{1.9}$$

with θ_t^J and s_t^J being the output elasticity with respect to input J and the expenditure share of input J, respectively. Equation (1.9) allows one to construct the real marginal cost MC_t with observable variables s_t^J and θ_t^J. I use the labor share of income for the non-farm business sector to represent the expenditure share s_t^J. To estimate the elasticity θ_t^J, on the other hand, I impose an assumption on the functional form of the production technology F. Following Basu (1996), Gagnon and Khan (2005), and Nekarda and Ramey (2013), I assume the Cobb-Douglas production function with overhead labor (CDOH) for a baseline case. Alternative specifications of the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES) are investigated in Appendix 1.10.1.

Table 1.1 presents the estimation result of the NKPC (1.5). Each coefficient without interaction is highly significant with intended sign. Regarding the changes of the slope parameter, in column (1) and (2) the coefficient of the output gap shrinks to less than a half once the observations after 2008Q1 are included. Moreover, the interaction term with the dummy variable for the post-Great Recession period is significantly negative in column (3). In contrast, the coefficient of marginal cost

Table 1.1: OLS estimation of the NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x_t	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.609***	0.570***	0.621***	0.420***	0.468***	0.441***
	(0.082)	(0.078)	(0.080)	(0.080)	(0.074)	(0.078)
π_{t-1}	0.423***	0.448***	0.410***	0.573***	0.526***	0.552***
	(0.077)	(0.075)	(0.076)	(0.081)	(0.075)	(0.080)
x_t	0.332***	0.158***	0.310***	0.236***	0.186***	0.212***
	(0.059)	(0.046)	(0.056)	(0.077)	(0.062)	(0.073)
$postGR_t \times \pi_t^e$			0.054			0.372
			(0.218)			(0.223)
$postGR_t \times \pi_{t-1}$			-0.164			-0.362
			(0.221)			(0.234)
$postGR_t \times x_t$			-0.333***			-0.0796
			(0.091)			(0.130)
Adjusted R^2	0.956	0.948	0.950	0.953	0.947	0.947
N	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The sign of the coefficient of the unemployment gap is flipped for a comparison purpose. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. The time-series spans quarterly from 1968Q4 to 2007Q4 for *Before GR*, and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The start point of the time-series corresponds to when the SPF became available.

in column (4) and (5) is quite stable after the Great Recession with one standard error of each coefficient covering the other. The interaction term with the post-Great Recession dummy in column (6) is insignificant.

For robustness check, I assess the following alternative cases: 1) alternative measures for marginal cost and the output gap, 2) the purely forward looking NKPC, 3) the rational expectation assumption, 4) rolling regression, and 5) Markov-switching model of the parameter values. I also conduct 6) dynamic panel estimation using industry level data. These alternative cases confirm the baseline result in that the coefficient of marginal cost is stable over time whereas the coefficient of the output gap drops after the Great Recession or is insignificant even before the Great Recession in some specifications.[6] The details are presented in Appendix 1.10.1.

1.3.2.2 Relationship between marginal cost and the output gap

This subsection examines the empirical relationship between marginal cost and the output gap using the industry level data of KLEMS 2017. An advantage of employing industry level data is that the intermediate share is available to measure marginal cost. To this regard, a number of studies suggest that intermediate inputs have desirable features in various dimensions (e.g., Basu (1995), Nekarda and Ramey (2013), and Bils et al. (2014)). For instance, adjustment costs for intermediates are considered to be low relative to those for capital or labor. In addition, the assumption of no overhead component seems more defensible for intermediates. Using industry level data also removes composition bias among industries.[7]

I run the following panel regression of marginal cost on measures of the output

[6]For example, I find that the coefficient of the output gap is insignificant under the rational expectation assumption regardless of sample periods. The result is indeed consistent with the existing literature. Adam and Padula (2011) report that the coefficient of the output gap is significant only when a survey based expectation measure is used instead of the rational expectation assumption.

[7]Despite these desirable properties, the survey based inflation expectations are not available for each industry. Therefore, for the estimation analysis of the NKPC in the previous subsection, I use the labor share in the aggregate data in the baseline estimation. However, for robustness check, I estimated the NKPC by using the intermediate share in the industry level data under the rational expectation assumption. The regression result confirms the baseline findings. The details are provided in Appendix 1.10.1.

gap:

$$IntShare_{i,t} = \beta_0 + \beta_1 x_{i,t} + \beta_1 D_{i,t} x_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{1.10}$$

or

$$IntShare_{i,t} = \beta_0 + \beta_1 x_t + \beta_2 D_t x_t + \alpha_i + \epsilon_{i,t} \tag{1.11}$$

where α_i and γ_t are fixed effects for industry i and time t, respectively. $IntShare_{i,t}$ denotes the intermediate share, which is a measure of marginal cost. For measures of the output gap x, I use the industry level detrended output $Output_{i,t}$ and the aggregate unemployment gap $UnempGap_t$. In order to capture potential non-linearity, I include interaction terms with dummy variables D for the post-Great Recession periods *postGR*, or the higher- and lower-quantiles of each series of x, HQT and LQT.

Table 1.2 presents the regression results. The coefficient of the detrended output is significantly positive in column (1)-(4). The finding supports the procyclicality of marginal cost (counter-cyclical markup) in line with previous studies (e.g., Basu (1995), Gali et al. (2007), and Bils et al. (2014)). Moreover, the data identifies convexity in the relation between marginal cost and the detrended output. To be precise, the interaction term with the post-Great Recession dummy is significantly negative in column (2), implying that marginal cost did not decline as much as the detrended output did after the Great Recession. Interestingly, the interaction term with the lower quartile in column (3) is negative, whereas that with the higher quartile in column (4) is positive. These observations imply that the convex relationship is present over the business cycle in general, and it appeared significantly after the Great

Table 1.2: Panel regression of the cyclicality of marginal cost to the output gap measures

	Dependent variable : $IntShare_{i,t}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Linear	post-GR	LQT	HQT	Linear	post-GR	LQT	HQT
$Output_{i,t}$	0.316^{+++}	0.371^{+++}	0.395^{+++}	0.258^{+++}				
	(0.031)	(0.035)	(0.048)	(0.043)				
$postGR_t \times Output_{i,t}$		-0.245^{***}						
		(0.076)						
$LQT_{i,t} \times Output_{i,t}$			-0.164^{**}					
			(0.078)					
$HQT_{i,t} \times Output_{i,t}$				0.129^{*}				
				(0.066)				
$UnempGap_t$					0.676^{+++}	1.159^{***}	1.118^{***}	0.242
					(0.167)	(0.362)	(0.410)	(0.223)
$postGR_t \times UnempGap_t$						-1.004^{**}		
						(0.393)		
$LQT_t \times UnempGap_t$							-0.902^{**}	
							(0.438)	
$HQT_t \times UnempGap_t$								1.074
								(1.064)
Industry FE	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	NO	NO	NO	NO
N of ind.	60	60	60	60	60	60	60	60
N of total obs.	1,800	1,800	1,800	1,800	1,800	1,800	1,800	1,800

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: Standard errors are clustered within each industry and reported in parentheses. The intermediate share and the detrended output are taken log and detrended by the Hamilton filter. The sign of the coefficient of the unemployment gap is flipped for a comparison purpose. $postGR_t$ is a dummy variable that takes 1 after year 2008. $LQT_{i,t}$ and $HQT_{i,t}$ is a dummy variable that takes 1 if the observation is in the lower and higher 25 percentile of the sample, respectively. Sample is annual data from 1985 to 2014 for 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing. In column 5-8, dummy variables for year 2008 and 2009 are included to control for the large variations during the financial crisis.

Recession. These results are supported with respect to the aggregate unemployment gap in column (5)-(8) as well.

In sum, the empirical evidence in this section suggests that the marginal cost representation of the NKPC remained stable after the Great Recession, while I confirm the flattening of the output gap representation in line with the existing literature. I also find that marginal cost has a convex relationship with the output gap, and the convexity significantly appeared after the Great Recession. I take these findings as suggestive. In particular, I do not claim that the cyclicality of markup represents causality. However, the evidence cast a doubt on one of the key features in a stylized New Keynesian model that the output gap and marginal cost has a linear relationship. In the next section, therefore, I develop a model in which labor market friction arising from DNWR creates a wedge between them. The model addresses non-linearity in the observed Phillips curve relationship between inflation and the output gap, while keeping the marginal cost representation of the NKPC unchanged.

1.4 Model

This section develops a DSGE model that embeds the DNWR constraint for individual workers. Other parts of the economy share many features of a standard New Keynesian model in the literature such as the one by Erceg et al. (2000), Ireland (2004), and Christiano et al. (2005). The economy consists of monopolistically competitive firms that set their prices with the quadratic adjustment cost *á la* Rotemberg (1982), households who make saving-consumption decision and supply differentiated labor service to the production sector, and the central bank that follows the Taylor (1993)-type nominal interest rate policy to stabilize inflation and output.

1.4.1 Households

In the economy, there is a continuum of households indexed by j on the unit interval, each of whom supplies a differentiated labor service to the production sector. The aggregate labor supply has the Dixit-Stiglitz form:

$$H_t \equiv \left(\int_0^1 h_t(j)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}} \tag{1.12}$$

where θ_w represents the labor demand elasticity. The user of labor service minimizes the cost of using certain amount of composite labor inputs, taking each labor service's wage as given. The FOC for the cost minimization problem leads to the individual labor demand function:

$$h_t(j) = \left(\frac{w_t(j)}{W_t} \right)^{-\theta_w} H_t \tag{1.13}$$

where the aggregate wage index W_t is defined as

$$W_t \equiv \left(\int_0^1 w_t(j)^{1-\theta_w} dj \right)^{\frac{1}{1-\theta_w}} . \tag{1.14}$$

The utility function of each household j is assumed to be additive separable in CRRA utility from consumption $c_t(j)$ and CRRA disutility from hours worked $h_t(j)$ with parameter σ and η respectively. The disutility from labor is subject to an uninsurable idiosyncratic shock $\chi_t(j)$, which follows an *i.i.d.* log-normal distribution. The time-varying discount factor β_t is exogenous and common for each household. It captures exogenous changes of households' preference. The expected lifetime utility is given by

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty} D_{t,t+s}\left(\frac{1}{1-\sigma}c_t(j)^{1-\sigma} - \frac{1}{1+\eta}\chi_t(j)h_t(j)^{1+\eta}\right)\right] \tag{1.15}$$

$$where \;\; log(\chi_t(j)) \sim i.i.d.N(-\sigma_\chi^2/2, \sigma_\chi^2)$$

$$D_{t,t+s} = \beta_{t+s}D_{t,t+s-1}.$$

Notice that the mean of the $log(\chi_t(j))$ is adjusted such that $\mathbb{E}[\chi_t(j)] = 1$. The aggregate discount factor β_t follows an AR(1) process:

$$log(\beta_t) = (1-\rho_d)log(\bar{\beta}) + \rho_d log(\beta_{t-1}) + \epsilon_{d,t}\;,\;\; \epsilon_{d,t} \sim i.i.d.N(0, \sigma_d^2) \tag{1.16}$$

where $\bar{\beta}$ represents the unconditional mean of β_t. One can interpret a positive $\epsilon_{d,t}$ as a contractionary discount factor shock where households lose their desire to consume in the current period.

Household j's budget constraint in period t is given by

$$c_t(j) + \frac{a_t(j)}{P_t} \leq (1+\tau_w)\frac{w_t(j)}{P_t}h_t(j) + R_{t-1}\frac{a_{t-1}(j)}{P_t} + \frac{\tau_t(j)}{P_t} + \Phi_t(j) \tag{1.17}$$

where $a_t(j)$ is the amount of asset holding, $\tau_t(j)$ is the lump-sum tax, and $\Phi_t(j)$ is the share of producer's real profits distributed to household j. I assume that households do not internalize the fluctuations of $\tau_t(j)$ nor $\Phi_t(j)$. P_t, R_{t-1}, and τ_w denote the aggregate price index, the gross nominal interest rate, and the labor subsidy, respectively.

Household's nominal wage might be subject to the DNWR constraint. I assume that $1-\alpha$ fraction of households is not allowed to reduce their nominal wages in each period with $0 < \alpha < 1$, whereas the remaining α fraction of them is free to change their wages without the constraint:

$$w_t(j) \geq w_{t-1}(j) \quad \textit{with prob. } 1-\alpha. \tag{1.18}$$

This assumption reflects a well known empirical fact that nominal wage reduction is rare to occur. For instance, Baratteiri et al. (2014), who study the frequency of individual nominal wage changes in the Survey of Income and Program Participation (SIPP) during 1996-1999, report that nominal wage reduction only corresponds to 12.3 percent of all the non-zero nominal wage changes after correcting for measurement errors. Each household j maximizes the expected lifetime utility (1.15) by choosing her consumption $c_t(j)$, asset holding $a_t(j)$, and nominal wage $w_t(j)$ subject to her budget constraint (1.17), individual labor demand (1.13), and the DNWR constraint (1.18) if she is subject to it. The FOCs for the problem take the form:

$$\mathbb{E}_t\left[\beta_t\left(\frac{c_{t+1}(j)}{c_t(j)}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right] = 1 \tag{1.19}$$

$$\psi_t(j) = \left(\frac{w_t(j)}{P_t} - \frac{\bar{\mu}_w}{1+\tau_w}mrs_t(j)\right)\left(\frac{1+\tau_w}{\bar{\mu}_w}c_t(j)^{-\sigma}\frac{\theta_w h_t(j)}{w_t(j)}\right) + \beta_t\mathbb{E}_t[\psi_{t+1}(j)] \tag{1.20}$$

$$\textit{where} \quad mrs_t(j) \equiv \frac{\chi_t(j)h_t(j)^{\eta}}{c_t(j)^{-\sigma}}$$

with $\bar{\mu}_w \equiv \theta_w/(\theta_w - 1)$ being the steady-state wage markup and $\Pi_t \equiv P_t/P_{t-1}$ being the gross price inflation rate. $\bar{\mu}_w$ is the steady state wage markup stemming from the monopolistic power of each household for her differentiated labor service, and $mrs_t(j)$ is the marginal rate of substitution of household j. $\psi_t(j)$ denotes the Lagrange multiplier for the DNWR constraint, which represents the shadow value of easing the DNWR constraint by one unit. The complementary slackness conditions for the

DNWR constraint (1.18) are given by

$$\psi_t(j) \geq 0 \tag{1.21}$$

$$\psi_t(j)(w_t(j) - w_{t-1}(j)) = 0. \tag{1.22}$$

In the following analysis, I impose two additional assumptions to focus on my main points while keeping the model tractable. First, I assume that each household has an access to a complete insurance market for consumption, though she is still subject to an uninsurable idiosyncratic labor disutility shock. The assumption together with the additive separable utility function[8] guarantees that consumption is identical across households:

$$c_t(j) = C_t. \tag{1.23}$$

Second, following Erceg et al. (2000), I assume that the labor subsidy is set to remove the steady state wage markup:

$$1 + \tau_w = \bar{\mu}_w. \tag{1.24}$$

Due to these assumptions, the FOCs (1.19) and (1.20) can be simplified to

$$\mathbb{E}_t \left[\beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] = 1 \tag{1.25}$$

$$\psi_t(j) = \left(\frac{w_t(j)}{P_t} - mrs_t(j) \right) \left(C_t^{-\sigma} \frac{\theta_w h_t(j)}{w_t(j)} \right) + \beta_t \mathbb{E}_t[\psi_{t+1}(j)]. \tag{1.26}$$

[8]The assumption is extensively used in the literature. On the other hand, several studies investigate non-separable preferences (King et al. (1988), Hall (2009), Christiano et al. (2011), etc.) and Basu and Kimball (2002) provide empirical support for them. In this regard, Guerrón-Quintana (2008) shows that non-separability makes consumption more responsive to wage. His result indicates that non-separable preferences reinforce my results in that they help the model reconcile a large drop of real quantities and a small decline of price variables after the Great Recession.

The optimality conditions for individual wage setting are characterized by (1.21), (1.22), and (1.26). If it were not for the DNWR constraint, then, $\psi_t(j) = 0$ would hold for all j and t. In that case, the optimality conditions are reduced to

$$\frac{w_t^f(j)}{P_t^f} = mrs_t^f(j) \tag{1.27}$$

where x_t^f denotes the variable x_t under flexible prices and wages. Equation (1.27) formulates an optimality condition under flexible wages to equalize real wage to the marginal rate of substitution. On the other hand, in the presence of the DNWR constraint, the complementary slackness conditions for the constraint imply that either of the following has to hold true:

$$w_t(j) = w_{t-1}(j) \tag{1.28}$$

or

$$\psi_t(j) = 0. \tag{1.29}$$

The former condition corresponds to the case where the DNWR constraint binds in the current period, whereas the latter the case where it does not. The latter condition is rearranged by using (1.26):

$$\frac{w_t(j)}{P_t} = mrs_t(j) - \beta_t \mathbb{E}_t[\psi_{t+1}(j)] \left(C_t^{-\sigma} \frac{\theta_w h_t(j)}{w_t(j)} \right)^{-1}. \tag{1.30}$$

The first term in the RHS of (1.30) coincides with the optimal wage under flexible wages, whereas the second term represents the reserved wage hike due to the likelihood of the DNWR constraint binding in the future periods. It is worth pointing that the optimal wages implied by (1.30) are weakly lower than their marginal rate of substitution since $\psi_t(j)$ is non-negative by (1.21). In other words, DNWR endogenously generates upward wage stickiness. Intuitively, a household internalizes the possibilities that her DNWR constraint might bind in the future periods, and she therefore desires to hold some buffer to prevent the future constraint from binding even if the constraint does not bind in the current period. This property is in line with the finding of the previous studies such as Elsby (2009), Benigno and Ricci (2011), and Daly and Hobijn (2014). To summarize the conditions above, the optimal wage of household j who are subject to the DNWR constraint follows the rule:

$$w_t(j) = max\left\{w_t^d(j), w_{t-1}(j)\right\} \tag{1.31}$$

where the desired wage $w_t^d(j)$ satisfies the condition (1.30).

1.4.2 Firms

There is a continuum of monopolistically competitive firms indexed by i on the unit interval, each of which produces a differentiated good. The production technology available for the firm producing good i is given by

$$y_t(i) = Z_t h_t(i) \tag{1.32}$$

$$where \quad h_t(i) = \left(\int_0^1 h_t(i,j)^{\frac{\theta_w - 1}{\theta_w}} dj\right)^{\frac{\theta_w}{\theta_w - 1}}. \tag{1.33}$$

Technology Z_t is exogenous and common for each firm. Firm i uses the composite labor inputs $h_t(i)$, where $h(i,j)$ denotes the labor service supplied by household j and used in firm i.

The cost minimization problem to determine the labor inputs $h_t(i)$ is given by

$$\min_{h_t(i)} : \frac{W_t}{P_t} h_t(i) \quad s.t. \quad (1.32). \tag{1.34}$$

The FOC takes the form:

$$mc_t(i) = \frac{W_t}{Z_t P_t} \equiv MC_t \tag{1.35}$$

where MC_t is the real marginal cost of producing one unit of output. (1.35) implies that the marginal cost is identical across firms. This is because each firm has the identical labor demand elasticity and takes labor service's wages as given.

I next formulate the firms' profit maximization problem. Firms face the quadratic price adjustment cost formulated by Rotemberg (1982) with parameter ϕ_p governing the degree of price stickiness. A firm chooses its price to maximize the expected profit subject to the individual good demand function that is analogous to the individual labor demand function:

$$\max_{p_t(i)} : \mathbb{E}_t \left[\sum_{s=0}^{\infty} SDF_{t,t+s} \Phi_{t+s}(i) \right] \tag{1.36}$$

$$where \quad SDF_{t,t+s} = D_{t,t+s} \left(\frac{C_{t+s}}{C_t} \right)^{-\sigma}$$

$$\Phi_t(i) = (1+\tau_p) \frac{p_t(i)}{P_t} y_t(i) - MC_t y_t(i) - \frac{\phi_p}{2} \left(\Pi_t(i) - \Pi^* \right)^2 C_t$$

$$s.t. \quad y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t. \tag{1.37}$$

$\Pi_t(i) \equiv P_t(i)/P_{t-1}(i)$ and $\Phi_t(i)$ are the gross price changes and the real profits of firm i, while $SDF_{t,t+s}$ is the stochastic discount factor between t and $t+s$. τ_p denotes the production subsidy. As in the households' problem, I assume that the production subsidy removes the steady state price markup that arises from firms' monopolistic power:

$$1 + \tau_p = \bar{\mu}_p \tag{1.38}$$

where $\bar{\mu}_p \equiv \theta_p/(\theta_p - 1)$. I focus on the symmetric equilibrium where each firm sets identical price. The FOC for the profit maximization problem yields the NKPC:

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \kappa \left(MC_t - 1 \right) \tag{1.39}$$

where MC_t is defined in (1.35). $\kappa \equiv \theta_p/\phi_p$ represents the slope of the NKPC. The aggregate production function in the symmetric equilibrium is given by

$$Y_t = Z_t H_t. \tag{1.40}$$

1.4.3 Central bank and government

In the baseline model, I assume a Taylor (1993)-type monetary policy rule where the central bank sets the gross nominal interest rate R_t to stabilize the gross inflation rate Π_t and output Y_t around their target level Π^* and Y^*:

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{1.41}$$

where $R^* = \Pi^*/\beta$. Note that I introduce the ZLB to conduct a counterfactual analysis for the Great Recession in Section 1.8.

In this economy, the government is passive in the sense that it levies lump-sum tax on households and distributes it as production and labor subsidies to firms and households with a balanced budget:

$$\int_0^1 \tau_t(j)dj = \tau_w \int_0^1 w_t(j)h_t(j)dj + \tau_p \int_0^1 p_t(i)y_t(i)di. \tag{1.42}$$

1.4.4 Market clearing

Market clearing conditions are given as follows.

$$A_t \equiv \int_0^1 a_t(j)dj = 0 \tag{1.43}$$

$$Y_t = C_t + \frac{\phi_p}{2}(\Pi_t - \Pi^*)^2 C_t \tag{1.44}$$

$$H_t = \int_0^1 h_t(i)di. \tag{1.45}$$

The asset market clearing (1.43) is trivial because consumption is identical across households and it is therefore not necessary to keep track of individual asset holdings to characterize equilibrium. The goods market clearing (1.44) should hold in the aggregate level since I restrict the attention to the symmetric equilibrium. It should be noticed that the labor market clearing (1.45) is automatically satisfied in the symmetric equilibrium as well.

1.4.5 Equilibrium

Definition. *A recursive competitive equilibrium is a household's policy function for individual real wages $\tilde{w} = h(\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z)$, a policy function for a set of aggregate jump variables $X \equiv \{C, Y, H, \Pi, R\} = f(g_{-1}, \beta, Z)$, and a law of motion Γ for cross-sectional density of individual real wages g, such that*

(i) a household's policy function h solves a recursive wage setting problem,

$$
\begin{aligned}
V^{dnwr}\Big(\tilde{w}_{-1}, \chi; g_{-1}, \beta, Z\Big) = \max_{\tilde{w}} :\ & -\frac{1}{1+\eta} e^{\chi} h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) \\
& + \beta \mathbb{E}\Big[V\Big(\tilde{w}, \chi'; g, \beta', Z'\Big)|g_{-1}, \beta, Z\Big] \\
s.t. \quad & h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H \\
& \tilde{w} \geq \tilde{w}_{-1}/\Pi \\
V^{no}\Big(\chi; g_{-1}, \beta, Z\Big) = \max_{\tilde{w}} :\ & -\frac{1}{1+\eta} e^{\chi} h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) \\
& + \beta \mathbb{E}\Big[V\Big(\tilde{w}, \chi'; g, \beta', Z'\Big)|g_{-1}, \beta, Z\Big] \\
s.t. \quad & h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H
\end{aligned}
$$

$$
\begin{aligned}
where \quad \mathbb{E}\Big[V\Big(\tilde{w}, \chi'; g, \beta', Z'\Big)|g_{-1}, \beta, Z\Big] = & (1-\alpha)\mathbb{E}\Big[V^{dnwr}\Big(\tilde{w}, \chi'; g, \beta', Z'\Big)|g_{-1}, \beta, Z\Big] \\
& + \alpha \mathbb{E}\Big[V^{no}\Big(\chi'; g, \beta', Z'\Big)|g_{-1}, \beta, Z\Big]
\end{aligned}
$$

with $\tilde{W}$ being the aggregate real wage generated by the cross-sectional density g and C, H, and Π being consistent with the aggregate policy function f.

(ii) an aggregate policy function f solve the Euler equation (1.25), the NKPC (1.39), the monetary policy rule (1.41), the production function (1.40), and the market clearing conditions (1.44),

(iii) a law of motion Γ is generated by g, that is, the cross-sectional density g satisfies a recursive rule:

$$
g = \Gamma(g_{-1}, \beta, Z).
$$

1.4.6 Analytical example

I briefly discuss the key mechanism of the model before proceeding to a numerical solution to it. For the example below, I impose several additional assumptions to analytically characterize equilibrium. To be precise, I consider log-utility from consumption and linear-disutility from labor, i.e. $\sigma = 1$ and $\eta = 0$. I also assume that there are no idiosyncratic disutility shocks.

Under flexible prices and wages, the labor market equilibrium condition requires the marginal product of labor MPL and the marginal rate of substitution between consumption and hours worked MRS to be equalized with each other through real wage:

$$MPL_t^f = \frac{W_t^f}{P_t^f} = MRS_t^f \tag{1.46}$$

$$\begin{aligned} where \quad MPL_t^f &= Z_t \\ MRS_t^f &= Y_t^f \end{aligned}$$

where Y_t^f denotes the output under flexible prices and wages. Notice that j notation for each household is dropped in (1.46) since the marginal rate of substitution does not have idiosyncratic components in this example. In addition, wage and price markups do not appear in (1.46) because both markups are constant under flexible prices and wages and the steady state markups are canceled out with the labor and production subsidy. In the presence of DNWR and price stickiness, on the other hand, markups are no longer constant. The labor market equilibrium condition takes the form:

$$MC_t MPL_t = \frac{W_t}{P_t} = \mu_{w,t} MRS_t \tag{1.47}$$

$$where \quad MPL_t = Z_t$$

$$MRS_t = Y_t.$$

The aggregate wage markup $\mu_{w,t}$ summarizes the wedge between the real wage and the marginal rate of substitution due to the DNWR constraint. On the other hand, $MC_t(\equiv 1/\mu_{p,t})$ captures the fluctuations in the real marginal cost that results from imperfect price adjustment due to the nominal price rigidity. By combining (1.46) and (1.47) and taking logarithm of both sides, the relationship between the output gap and marginal cost is given by

$$\hat{MC}_t = (\hat{Y}_t - \hat{Y}_t^f) + \hat{\mu}_{w,t} \tag{1.48}$$

where I define $\hat{x}_t \equiv log(x_t)$. The first term in the RHS of (1.48) is the definition of the output gap, whereas the second term is the wage markup arising from DNWR.

I next examine the effect of the wage markup due to DNWR on price inflation. It is immediate to see the wage markup appears in the output gap representation of the NKPC by substituting (1.48) into the linearized version of (1.39):

$$\begin{aligned} \pi_t &= \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa \hat{MC}_t \\ &= \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa(\hat{Y}_t - \hat{Y}_t^f) + \kappa \hat{\mu}_{w,t} \end{aligned} \tag{1.49}$$

where $\pi_t \equiv log(\Pi_t)$. Equation (1.48) and (1.49) suggest that DNWR creates a time-varying wedge between the output gap and marginal cost and it works as a shift parameter of the output gap representation of the NKPC. In recessions when the wedge increases due to the binding DNWR constraint, the output representation of the NKPC shifts up, which makes the observed relationship between inflation and the output gap flatter. To quantify these implications, I solve the model numerically in the next section.

1.5 Numerical Method and Calibration

This section presents an equilibrium computation method to solve the model developed in Section 1.4, and explains my calibration strategy.

1.5.1 Modified Krusell-Smith algorithm

I present an equilibrium computation method to solve the model numerically. I choose to solve the model in a dynamic setting because my main focus is on inflation dynamics at business cycle frequencies. To this end, a perturbation method, which is widely used to solve DSGE models in the literature, cannot be applied to my model. The first reason is non-differentiability. Since the occasionally binding DNWR constraint makes the individual policy function kinked, the function is no longer differentiable. The presence of the ZLB is another source of non-differentiability. Second, the DNWR constraint introduces heterogeneity of wages among households. This is due to the state-dependent nature of the DNWR constraint. Unlike a time-dependent constraint such as the staggered contract of Calvo (1983), whether the DNWR constraint binds or not crucially depends on the previous period's wages.

Therefore, in order to characterize equilibrium, it is necessary to keep track of the history of individual wages, i.e. cross-sectional wage distribution.[9]

For these reasons, I apply the Krusell-Smith algorithm to the model. Since cross-sectional distribution is an infinite dimensional object, it is in practice impossible to track all the information in it. In this regard, Krusell and Smith (1998) propose an approximated equilibrium where each agent perceives the evolution of aggregate state variables as being a function of a small number of moments of cross-sectional distribution. Adopting their insight, I assume that the aggregate endogenous state variable, real wage $\tilde{W}_t$, is governed by the following aggregate law of motion (ALM):

$$\tilde{W}_t = \Gamma(\tilde{W}_{t-1}, \beta_t, Z_t). \tag{1.50}$$

An important challenge is that, even though the original Krusell-Smith algorithm requires aggregate jump variables to have a closed form solution in terms of aggregate state variables, that condition does not hold in the New Keynesian setting. Therefore, I propose a modified algorithm. Specifically, given a guess for the ALM of the aggregate state variable (1.50), I first solve for the aggregate jump variables, consumption C_t, hours worked H_t, price inflation Π_t, and nominal interest rate R_t, as general equilibrium outcomes of the aggregate part of the economy. Notice that the aggregate part of the economy consists of the 3-equation New Keynesian system, that is, the Euler equation (1.25), the NKPC (1.39), and the Taylor rule (1.41), as well as the production function (1.40) and the market clearing condition (1.44). Importantly, it is independent of individual workers' behavior conditional on the aggregate real

[9]Recent studies propose several computation methods to solve a model with heterogeneity. Discussion on the selection of computation methods is provided in Appendix 1.10.2

wage. Then, given all the aggregate variables, an individual variable, each worker's real wage $\tilde{w}_t(j)$, can be obtained as a solution to the individual wage setting problem. Finally, I numerically integrate the individual variables to recover the aggregate state variable and update the initial guess for the ALM. To address non-linearity stemming from DNWR and the ZLB, I used a global method in each step. The details of the computation algorithm are provided in Appendix 1.10.2.

1.5.2 Calibration

Due to the complexity of my model, I follow a calibration strategy to set parameter values. The time frequency is quarterly. The externally fixed parameters are listed in Panel (A) of Table 2.1. The choice of the discount factor β and the target inflation rate Π^* corresponds to the annual real interest rate of 2 percent and the annual price inflation rate of 2 percent, respectively. The relative risk aversion of households σ is set at 2.0 and the inverse of the Frisch labor supply elasticity η is at 0.25, which is in line with the existing literature. The values of θ_w and θ_p imply the steady state markup is 12.5 percent. I follow Fernández-Villaverde et al. (2015) to set $\delta_\pi = 1.50$ and $\delta_y = 0.25$. The value of the degree of price stickiness ϕ_p is calibrated according to the frequency of individual price changes reported by Nakamura and Steinsson (2008). They find that the median frequency excluding temporary sales is 11-13 percent per month, which implies the slope of the NKPC is around 0.20 and the corresponding parameter value is $\phi_p = 45.0$ in my model. Notice that the parameter value implies that 1 (5) percentage point deviation of inflation from its trend generates

Table 1.3: Calibration

Panel (A): Fixed parameters

Description	Symbol	Value	Target/Source
Average discount factor	$\bar{\beta}$	0.995	S.S. real interest rate = 2.0% (annual)
Relative risk aversion	σ	2.00	IES = 0.5
Inverse of Frisch labor supply elasticity	η	0.25	King and Rebelo (1999)
Labor demand elasticity	θ_w	9.00	S.S. wage markup = 12.5%
Goods demand elasticity	θ_p	9.00	S.S. price markup = 12.5%
Price adjustment cost	ϕ_p	45.0	Slope of NKPC = 0.20
(Corresponding Calvo parameter)	-	(0.64)	Nakamura and Steinsson (2008)
Coefficient of inflation in the Taylor rule	δ_π	1.50	Fernández-Villaverde et al. (2015)
Coefficient of output in the Taylor rule	δ_y	0.25	Same as above
Target inflation rate	Π^*	1.005	S.S. inflation rate = 2.0% (annual)
Target output	Y^*	1.000	Externally fixed

Panel (B): Parameters for cross-sectional wage distribution

Parameter	Symbol	Value	Target/Source
Fraction of workers without being subject to the DNWR constraint	α	0.0610	Frequency of wage changes=0.266; Barattieri et al. (2014)
S.D. of idiosyncratic labor disutility shock	σ_χ	0.1540	S.D. of wage changes (annual)=0.108; Fallick et al. (2016)

Notes: Barattieri et al. (2014) identify the fraction of workers with non-zero wage changes to be between 0.211 and 0.266 depending on the assumptions they use in their estimation. I use the most conservative value 0.266 in terms of generating wage stickiness, since the model exclude any other possibilities to generate wage rigidity than DNWR.

Panel (C): Parameters for aggregate exogenous processes

Parameter	Symbol	Value	Target/Source
AR(1) coefficient of discount factor	ρ_β	0.865	First-order autocorr. of output=0.85
S.D. of innovations to discount factor	σ_β	0.00562	S.D. of output=1.55%

Notes: Targets are the HP-filtered real GDP from 1955Q1 to 2007Q4. The end point of the sample is determined to exclude the ZLB periods.

0.225 (5.625) percent loss of consumption.[10] Although previous studies in the New Keynesian literature tend to use higher values for the price stickiness parameter to reproduce the persistence of the actual inflation, I investigate whether the model can account for the data under the parameterization that is consistent with micro

[10]The consumption loss is calculated as follows. $\frac{\phi_p}{2}(\Pi_t - \Pi^*)^2 * 100 = 45/2 * 0.01^2 * 100 = 0.225(\%)$

evidence.

The parameters regarding cross-sectional wage distribution are calibrated to match the empirical distribution in U.S. data. I choose parameter values to minimize the quadratic distance between the moments of the stationary distribution of individual wage changes in the model and the target moments in data by using a grid search method. The target moments and the calibrated parameter values are listed in Panel (B) of Table 2.1.

In the following, I focus on the consequence of exogenous variations of discount factor β_t, while keeping technology constant at the unity, i.e., $Z_t = \bar{Z} = 1$.[11] This is because a plenty of evidence in the literature suggests that a demand side shock, in particular, a wedge in the intertemporal substitution, is the key determinant of the severe contractions during the Great Recession. Many of previous studies reach that conclusion by a reduced from regression analysis (Hall (2011)) and an estimation analysis of a structural model (Justiniano et al. (2011), Christiano et al. (2014), Gust et al. (2017), etc.). To this end, as Justiniano et al. (2011) point out, the time-varying discount factor is a parsimonious way to represent the shock. For parameterization, the AR(1) coefficient of the discount factor ρ_β and the standard deviation of innovations to it σ_β are calibrated to match the persistence and the standard deviation of the real GDP in the post-war U.S. data. The calibrated parameters are listed in Panel (C) of Table 2.1.

[11]For a curious reader, the effect of a technology shock is investigated in Appendix 1.10.4.

1.6 Numerical Results

1.6.1 Unconditional moments

Before proceeding to my main analysis, this subsection presents the unconditional cross-sectional and time-series moments of the model to check the validity of it.

1.6.1.1 Stationary distribution

Figure 1.3 displays the stationary distribution of non-zero wage changes in the calibrated model. The definition of the stationary equilibrium is provided in Appendix 1.10.3. Table 1.4 reports the cross-sectional moments. The model replicates key features of the empirical distribution including: 1) a large spike at zero (not shown in the figure), 2) much less individuals with nominal wage reductions than increases, 3) a discrete difference in the density between the positive and negative sides around zero, and 4) higher mean than median. It should be noted that I do not target the asymmetry of the empirical distribution when calibrating parameters. Instead, these properties arise as a consequence of the specifications of the model.

1.6.1.2 Time-series moments

Table 1.5 compares the time-series moments of the model with the data. For this analysis, I simulate the model for 51,000 periods and discarded the initial 1,000 observations to calculate the moments. The table also reports the moments in a model without DNWR, which coincides with a stylized New Keynesian model presented in Section 1.3. The baseline model with DNWR does fairly well in matching the time-

Figure 1.3: Stationary distribution of non-zero wage changes

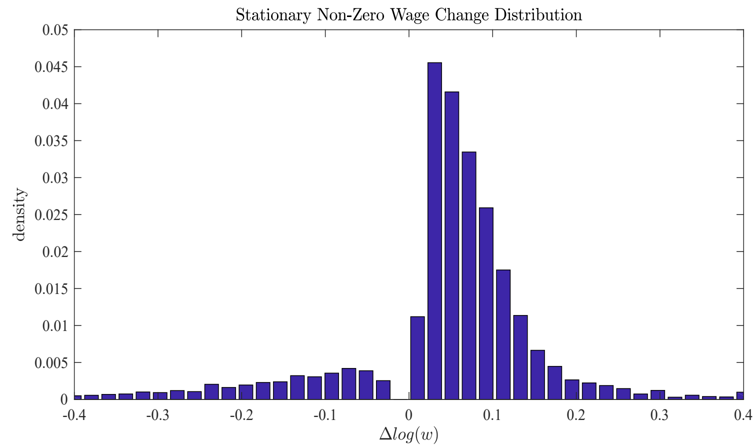


Table 1.4: Cross-sectional moments

Description	Model	Data	Source
Targeted moment:			
Fraction of workers with non-zero wage changes	0.267	0.266	Barattieri et al. (2014)
S.D. of individual wage changes (annual)	0.108	0.108	Fallick et al. (2016)
Untargeted moment:			
Fraction of wage cuts out of non-zero wage changes	0.147	0.123	Barattieri et al. (2014)
		0.22, 0.32	Author's calculation based on Elsby et al. (2016)
Mean of wage changes (annual)	0.019	0.033	Fallick et al. (2016)
Median of wage changes (annual)	0.0098	0.028	Fallick et al. (2016)

Notes: The data source and the sample period of each paper is as follows; Barattieri et al. (2014): SIPP, 1996-1999. Fallick et al. (2016): ECI, 1982-2014. Elsby et al. (2016): CPS, 1980-2012. For the fraction of wage cuts out of non-zero wage changes, 0.22 is for hourly paid workers and 0.32 is for non-hourly paid workers. Data frequency is quarterly, unless otherwise noted. The higher order moments of the distribution are not reported since I find that they are sometimes sensitive to small changes in parameter values. It might be because a large mass of workers is at the zero wage change. To this regard, Fallick et al. (2016) report the skewness of the distribution in the data takes positive and negative values in each year without a clear pattern.

series moments of the data in a number of dimensions including: 1) low standard deviation of price inflation and wage growth relative to that of output and hours worked, 2) positive skewness of price inflation, wage growth, and real wage, 3) neg-

Table 1.5: Time-series moments

	Data	Model	
	55Q1-07Q4	Baseline	w/o DNWR
$\sigma(Y)$	1.55	1.55	0.44
$\sigma(H)$	1.82	1.55	0.44
$\sigma(\pi^p)$	0.58	0.60	1.40
$\sigma(\pi^w)$	0.74	0.66	1.62
$\sigma(W/P)$	0.84	0.46	0.99
$\sigma(i)$	0.82	1.26	2.23
$\rho(Y)$	0.85	0.85	0.86
$\rho(H)$	0.90	0.85	0.86
$\rho(\pi^p)$	0.86	0.90	0.87
$\rho(\pi^w)$	0.40	0.81	0.64
$\rho(W/P)$	0.78	0.94	0.86
$\rho(i)$	0.95	0.89	0.87
$Sk(Y)$	-0.49 (-0.59)	-0.38 (-0.11)	0.34 (0.06)
$Sk(H)$	-0.28 (-0.59)	-0.38 (-0.11)	0.34 (0.06)
$Sk(\pi^p)$	1.25 (0.25)	1.50 (0.36)	-0.07 (-0.01)
$Sk(\pi^w)$	0.23 (0.02)	1.75 (0.38)	0.04 (-0.01)
$Sk(W/P)$	0.53 (-0.77)	2.26 (-0.11)	0.36 (0.07)
$Sk(i)$	1.17 (0.10)	1.01 (0.23)	-0.02 (-0.00)

Notes: The standard deviation σ, the first order autocorrelation ρ, and the skewness Sk are reported. For the skewness, as well as the standard definition Sk_1, an alternative skewness measure Sk_2 is reported in parentheses. Sk_2 is defined as $Sk_2 = (\mu - Q)/\sigma$ with the mean μ and the median Q, and bounded between -1 and 1. Kim and White (2003) argue that Sk_2 is robust to outliers. Regarding the moments of data, Y is the real GDP, H is the total hours in the non-farm business sector, π^p is the GDP implicit price deflator, π^w is the compensation per hour in the non-farm business sector, and i is the effective federal funds rate. Y and H are taken log and detrended by the HP-filter. π^p and π^w are the quarter-on-quarter growth rate. i is the annual rate divided by 4 (quarterly rate). Sample period is from 1955Q1 to 2007Q4. The end point of the sample is determined to exclude the ZLB periods. For computing the moments of the models, we simulate the economy for 51,000 periods and discard the initial 1,000 observations. The model without DNWR is solved by a policy function iteration and simulated under the same parameter values as the baseline model.

ative skewness of output and hours. On the other hand, the model without DNWR fails to match them. In particular, it generates positive skewness of output and hours worked due to the concavity of utility function.

1.6.2 Generalized impulse responses

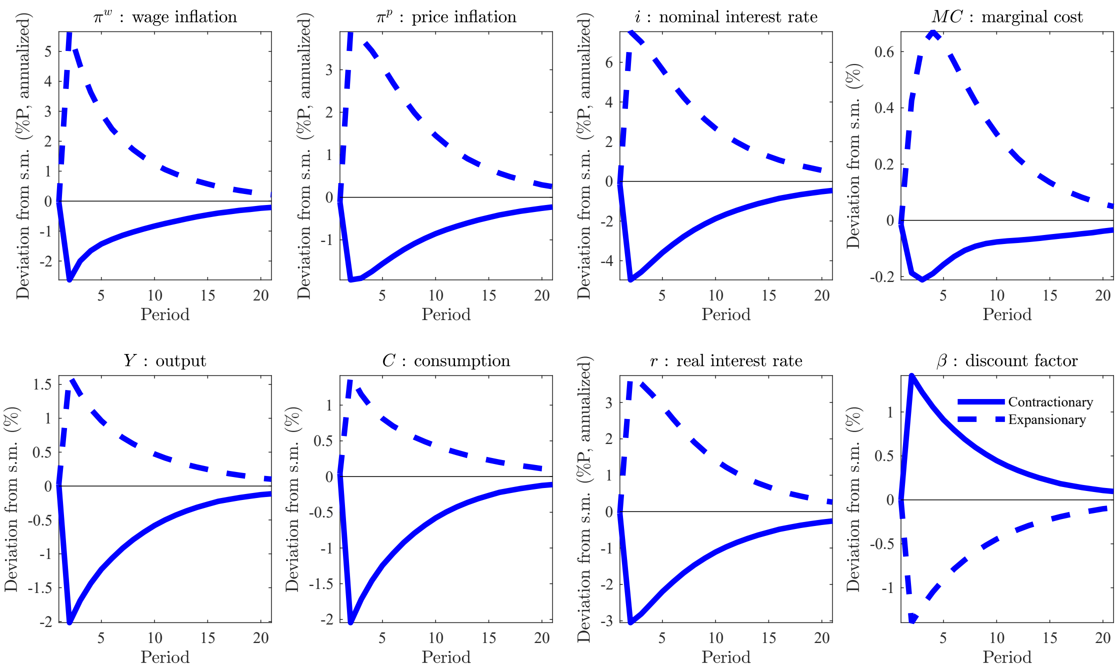
Figure 1.4 presents the generalized impulse responses (GIR) to a 2 S.D. discount factor shock. The construction of the GIR is provided in Appendix 1.10.2. A discount factor shock, as a demand side shock, generates comovements among quantity and price variables. More importantly, the responses of wage growth, price inflation, and marginal cost display strong asymmetry to contractionary and expansionary shocks. They are much more sluggish downward than upward in the presence of DNWR. On the other hand, the responses of output and consumption are larger to a contractionary shock. It implies that real quantities are adjusted instead of price variables as a consequence of general equilibrium. I find that the asymmetry of real quantities are relatively smaller than that of price variables, because the concavity of utility function makes households resist a decline of consumption more strongly than they appreciate an increase of it, and that effect partly offsets the asymmetry arising from DNWR. Moreover, the half-lives of the price variables are longer upon a contractionary shock. For example, the half-lives of price inflation are 10 quarters for a contractionary shock whereas they are 8 quarters for an expansionary one. Those of wage growth are 6 and 4 quarters, respectively. There is not clear asymmetry in the half-lives of quantity variables.

To quantify the degree of asymmetry, I introduce the following measure:

$$Asym(y, k, \epsilon_0) = \sum_{t=1}^{k} |GIR(y, t, \epsilon_0)| / \sum_{t=1}^{k} |GIR(y, t, -\epsilon_0)| \tag{1.51}$$

where y is the target variable and ϵ_0 is the initial exogenous shock with $\epsilon_0 > 0$. k is the time-horizon, which is set at 4. The measure compares relative size of the

Figure 1.4: GIR to a 2 S.D. discount factor shock



Notes: The x-axes represent the time horizon after the initial shock. The y-axes are in terms of the deviation from the stochastic mean (s.m.). The GIR are the conditional expectation on the initial shock. The construction of the GIR is provided in Appendix 1.10.2.

responses to a contractionary shock. Table 1.6 shows that the degree of asymmetry is increasing in the size of shock. Consider the responses of price inflation, for example. The response to a 2.5 S.D. contractionary shock is smaller by 50 percent than that to an expansionary shock with the same magnitude. In contrast, the difference is only 17 percent for a 0.5 S.D. shock.

1.6.3 Simulated Phillips curve

In Figure 1.5, I simulate the model economy to plot the two representations of the Phillips curve. The output gap representation of the NKPC, shown in the left panel, becomes flatter when the output gap takes negative values, and the quadratic fitted

Table 1.6: Degree of asymmetry to different size of shocks

	$Asym(\cdot, k=4, \epsilon_0)$							
ϵ_0	π^w	π^p	MC	Y	H	i	r	β
0.5 S.D.	0.80	0.83	0.62	1.05	1.05	0.89	0.94	1.00
1.0 S.D.	0.65	0.70	0.40	1.11	1.11	0.80	0.89	1.00
1.5 S.D.	0.56	0.62	0.34	1.16	1.16	0.74	0.86	1.00
2.0 S.D.	0.49	0.57	0.35	1.20	1.20	0.70	0.83	1.00
2.5 S.D.	0.46	0.50	0.31	1.24	1.24	0.64	0.78	1.00

Notes: The table reports the asymmetry measure defined in (1.51). Higher values indicate that the GIR is larger upon a contractionary shock than to an expansionary one.

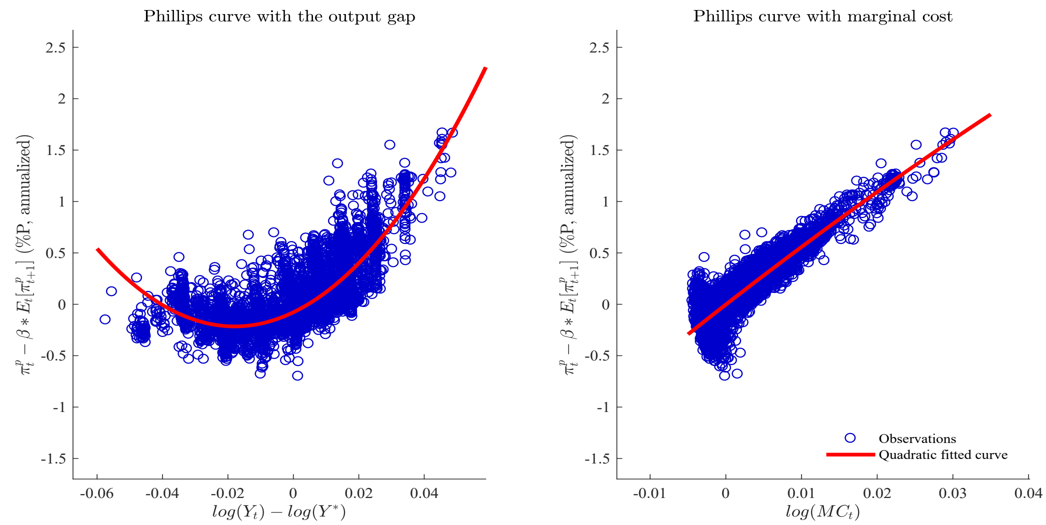
curve exhibits strong convexity. This is exactly because the binding DNWR constraint creates a wedge between the output gap and marginal cost and the wedge shifts up the NKPC in recessions. Since the effect is increasing in the size of shocks, the Phillips curve looks downward sloping on its left end. On the other hand, the marginal cost representation of the Phillips curve, displayed in the right panel, stays almost linear. It reflects the fact that the firms' price setting behavior given a level of marginal cost does not change over the business cycle in the model, though small deviations can emerge due to the fluctuations of the stochastic discount factor.

1.7 Discussion and relation to literature

This section provides discussion on the key mechanism of the model and its relation to the existing literature.

Shifts of cross-sectional wage distribution. A key feature to understand the non-linear dynamics of the model is the endogenous shifts of cross-sectional distribution.

Figure 1.5: Simulated Phillips curve

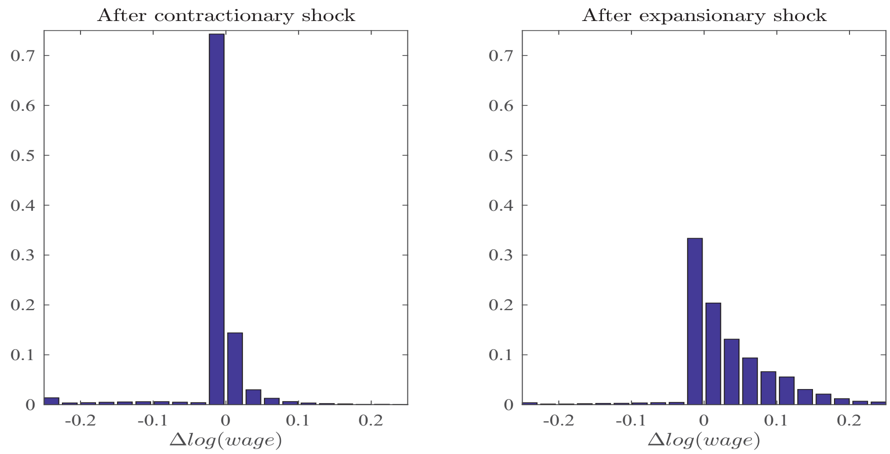


Notes: Each panel displays scatter plots of the simulated data and the quadratic fitted curves of it. Since I discretize the state space with the relatively small number of exogenous states, I use interpolation between observations to generate a smooth transition path between states. Specifically, I generated a time-series of data for 5,000 periods on the discretized state space and interpolated at the middle point of exogenous states using the linear interpolation. The total number of observations is therefore 9,999 in each panel.

Figure 1.6 shows the cross-sectional distribution of individual wage changes when the economy is hit by contractionary and expansionary discount factor shocks. The size of spike at zero wage changes indicates that a larger fraction of workers is stuck at the DNWR constraint upon a contractionary shock. The observation is consistent with the micro evidence of Daly and Hobijn (2014) and Fallick et al. (2016), who document that the fraction of workers with zero wage changes substantially increased after the Great Recession. That leads to a stronger downward sluggishness of the aggregate wage than upward.

Another important feature is the role of idiosyncratic shocks. The calibrated parameter values identify the standard deviation of idiosyncratic shocks is much larger than that of aggregate shocks. Consequently, non-trivial fraction of workers experi-

Figure 1.6: Cross-sectional distribution of individual wage changes upon aggregate shocks

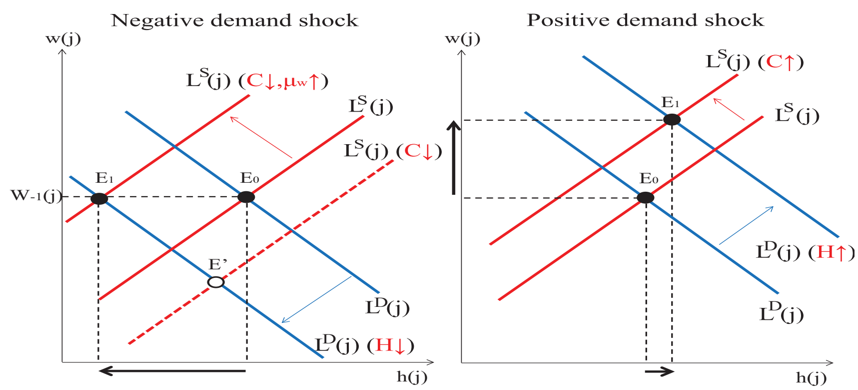


ences wage increases even after a contractionary aggregate shock because the effects of their positive idiosyncratic shock exceed the contraction of the aggregate economy. On the other hand, downward wage adjustment is truncated at zero as long as workers are subject to the DNWR constraint. Thus, the Jensen's inequality implies that the average wage change becomes higher than the case without idiosyncratic shocks. That further impedes the adjustment of the aggregate wage upon a contractionary shock.[12]

Wage markup. The wage markup in my model is closely related to that of Erceg et al. (2000), Christiano et al. (2005), and many others who incorporate the stag-

[12]It is worth pointing out that the importance of idiosyncratic shocks is also emphasized in the literature of price stickiness . For instance, Nakamura and Steinsson (2010) argue that, in their calibrated menu cost model, idiosyncratic shocks are large enough so that firms react to idiosyncratic shocks rather than aggregate shocks, which results in a substantial degree of aggregate price stickiness.

Figure 1.7: Asymmetry of wage markup fluctuations



gered contract of Calvo (1983) into wage settings. In their models, the wage rigidity introduces a time-varying wage wedge between real wage and the marginal rate of substitution between consumption and hours worked as well. However, my model is distinguished from theirs in several important dimensions. First of all, the fluctuations of the wage markup in my model depend on the sign of an exogenous shock, which leads to significant asymmetric dynamics in booms and recessions. Intuition is obtained by individual labor market equilibrium shown in Figure 1.7. In the left panel, the negative income effect reduces the marginal rate of substitution upon a contractionary shock. However, since nominal wage reductions are prevented by the DNWR constraint, the wage markup should increase so that the labor supply curve shifts up to meet the labor demand curve.[13] In the right panel, in contrast, the wage markup does not respond to an expansionary shock as long as the DNWR constraint does not bind.

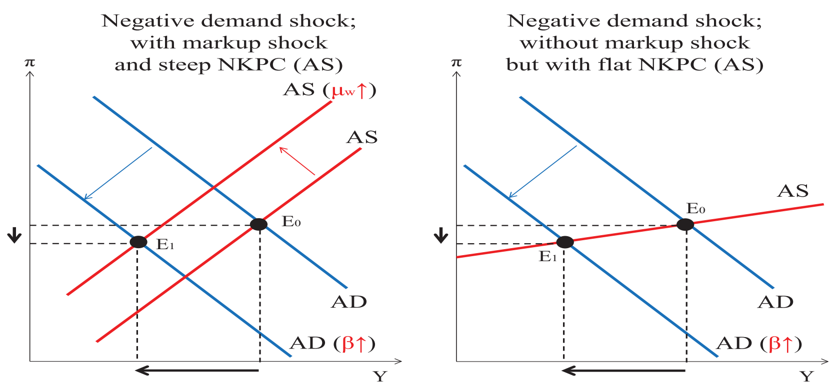
[13] I suppose that the price level is constant upon a shock in this partial equilibrium analysis.

The fluctuations of the wage markup also depend on the size of an exogenous shock. In this regard, an exogenous shock has two effects on the wage markup. The direct effect is that an exogenous shock affects the individual wage markups for the workers whose DNWR constraint is already binding. In addition, an exogenous shock changes the portion of workers with and without the binding constraint by changing their desired wages. Therefore, the total effect is increasing in the size of shocks. In contrast, the staggered contract of Calvo (1983), in which a constant fraction of workers faces with the constraint in each period, lacks the second effect and the model therefore does not generate significant non-linearity.

Markup shock to the NKPC. King and Watson (2012) point out that a medium scale DSGE model often requires sizable and frequent exogenous markup shocks to the NKPC to account for the actual inflation. In line with their finding, Del Negro et al. (2015) argue that a large positive markup shock should be in company with a negative demand shock to answer the missing deflation puzzle under a relatively steep NKPC. They instead propose that a sufficiently flat Phillips curve as in the right panel of Figure 1.8 can address the puzzle. To this end, my model generates a rise in the wage markup endogenously upon a negative demand shock through DNWR. The rise of the wage markup shifts up the NKPC (the AS curve), as a consequence of which the decline of inflation is moderate despite a large shift of the AD curve as shown in the left panel of Figure 1.8.

Implication to anchoring inflation expectations. Several studies emphasize

Figure 1.8: Demand and markup shocks with different slope of the NKPC



Notes: This figure corresponds to FIGURE 5 of Del Negro et al. (2015).

the fact that the inflation expectation was stable during and after the Great Recession to address the missing deflation puzzle. Some of them attribute it to the departure from the full information and rational expectation (FIRE) model (Coibion and Gorodnichenko (2015)) or discrete regime changes of the economy (Bianchi and Melosi (2017)). On the other hand, I argue that the stable inflation expectations are consistent with my model although I stick to a FIRE model without regime switching. To see this point, iterating the linearized version of the NKPC (1.39) forward yields

$$\mathbb{E}_t[\pi_{t+1}] = \kappa \mathbb{E}_t \left[\sum_{s=0}^{\infty} D_{t+s+1} \hat{MC}_{t+s+1} \right]. \tag{1.52}$$

(1.52) implies that the inflation expectation $\mathbb{E}_t[\pi_{t+1}]$ is the infinite sum of the discounted values of the future marginal costs. Therefore, the model potentially address the stable inflation expectations as long as the future marginal costs are sufficiently stabilized. I show that the model indeed predicts a moderate decline of the inflation expectations that matches the survey based inflation expectations in the data after the Great Recession in a counterfactual analysis in Section 1.8.

Comparison with different specifications of wage adjustment. In Appendix 1.10.5, I compare the dynamics across models with different specifications of wage adjustment. Specifically, I solve and calibrate a flexible wage model and a quadratic wage adjustment cost model as well as the baseline model with DNWR. Notice that the quadratic wage adjustment cost model coincides with the Calvo-type staggered contract model in the first order, which is widely used in the existing New Keynesian literature. I calibrate the parameter for wage stickiness according to the micro evidence reported by Barattieri et al. (2014). Other parts of the models than wage adjustment are identical to the baseline model. Figure 1.17 compares the GIR in different models. In the flexible wage model, wage growth, marginal cost, and price inflation respond strongly to a discount factor shock. Since the effects of an exogenous shock are absorbed by adjustments of price variables, real quantities such as output and consumption do not react a lot. On the other hand, the quadratic wage adjustment model generates moderate responses in price variables and sizable responses of quantity variables. However, there are several important differences from the model with DNWR. First of all, the quadratic adjustment cost model does not bring about significant asymmetry since the wage adjustment cost is symmetric by construction.[14] As a result, the inflation responses to a 2 S.D. contractionary discount factor shock

[14] Since the model is solved by a global method, non-linearity can arise from other parts of the model than the wage adjustment such as the curvature of the utility function. However, I find that such non-linearity is quantitatively small.

is almost twice as large as those in the model with DNWR, while the responses to an expansionary shock with the same magnitude is slightly smaller. Second, the propagation of an exogenous shock is not as stringent as the model with DNWR. The half-lives of wage growth and price inflation to a 2 S.D. contractionary shock are 2 and 7 quarters in the quadratic wage adjustment cost model, whereas they are 6 and 10 quarters in the model with DNWR. The difference reflects the state-dependency of DNWR. To be precise, after a contractionary shock, workers does not react to improvements of the state of the economy at all as long as the DNWR constraint binds, whereas the quadratic adjustment cost model allows for gradual responses in each period. Further discussion on the state-dependency of the model is provided in Section 1.8.

I also compare the baseline model with a model that embeds asymmetric wage adjustment costs, because the class of model potentially generates non-linearity in aggregate dynamics. In the literature, Kim and Ruge-Murcia (2009) and Fahr and Smets (2010) propose to use an asymmetric wage adjustment cost function to approximate DNWR. More recently, Aruoba et al. (2017) augment the model to include both of asymmetric wage and price adjustment cost to find that the model can capture the non-linearity in the data well. I calibrate an asymmetric wage and price adjustment cost model based on the estimated parameters of Aruoba et al. (2017).[15] Figure 1.19 compares the GIR of key variables to different sign and size of discount factor shocks. Interestingly, the non-linearity of price inflation and output are quite

[15] I use the posterior mean for the sample of 1960Q1-2007Q4 reported by Aruoba et al. (2017). The details are provided in Appendix 1.10.5.

similar in the two models. However, it is worth pointing out that my model with DNWR is calibrated consistently with the frequency of individual price changes in micro data, whereas Aruoba et al. (2017) identify a much higher parameter value for the degree of price stickiness (flatter Phillips curve). Moreover, the responses of wage growth and real wage display much stronger non-linearity in the model with DNWR. This is because the non-linearity of the model with DNWR stems from asymmetric individual wage adjustments, whereas Aruoba et al. (2017)'s estimates indicate strong asymmetry in price adjustment rather than wage adjustment. Indeed, I find, in the counterfactual analysis in Section 1.8, that the model with DNWR matches the moderate decline of wage growth and real wage after the Great Recession fairly well. However, further investigation on the comparison of different models is left for future research.

Connection to the literature of the micro evidence on DNWR. A crucial assumption of my model is the DNWR constraint for individual wage settings. Hence, I quickly review the literature on the micro evidence of DNWR to assess the validity of the assumption. More comprehensive literature review is found in Basu and House (2016). McLaughlin (1994) is one of the earliest studies to test for the presence of DNWR using individual wage data in the U.S. Though his result was not favorable for DNWR, subsequent studies found evidence of it. For instance, Card and Hyslop (1997), using the individual wage data in the CPS, find a large spike at zero in wage change distribution. Lebow et al. (1995) report asymmetry of wage change distribution arising from DNWR, and Kahn (1997) propose a formal test to verify the asymmetry of distribution. To elaborate their findings, Gottschalk (2005) corrects for a measurement error problem in self-reported data by applying an econometric strategy that detects structural breaks. Barattieri et al. (2014) employ his technique to find that nominal wage reductions correspond only to 12.3 percent of non-zero nominal wage changes in the SIPP during 1996-1999 after correcting for measurement errors.

One might be concerned about the possibility that benefits such as bonus, pensions, and other supplementary payments are used to adjust the total compensation of workers even if wages are downward rigid. However, evidence is mixed in the literature. Kurmann and McEntarfer (2017) report that, using administrative worker-firm linked data in Washington state, the spike at zero of the changes in the average hourly compensation (sum of wages and benefits) is much smaller than that of wages only, and declines of compensation are not rare in individual data. They claim that their results are less affected by measurement errors than the studies using self-reported data since they use administrative data. On the other hand, Lebow et al. (2003), computing changes of wages and benefits separately in the individual data of the ECI, document that, although benefits change more frequently than wages, the changes of benefits are not systematically related to wage changes. Based on these observations, they conclude that the hypothesis that benefits are used to offset nominal wage rigidities is not supported in the data.[16] Moreover, Gu and Prasad (2018) find that

[16] It should be noted that the unit of observation of the ECI is jobs instead of workers. How much their empirical results are affected by the difference in the unit of observations would be a subject of future research.

benefits come to more rigid over time, due to the increases of quasi-fixed benefits such as health insurance and defined contributions (IRAs, 401k, etc.). They report that the increased rigidity of benefits made the total compensation countercyclical especially after the Great Recession.

Another interesting finding in the empirical literature is international differences in the degree of DNWR, though this paper exclusively focuses on the inflation dynamics in the U.S.[17] For instance, Smith (2000) investigates the weekly average compensation in the U.K. during 1991-1996 to report that only 1 percent of workers are constrained by DNWR after correcting measurement errors and long-term contracts. Elsby et al. (2016) report that the degree of DNWR in the U.K. is weaker than the U.S. using a longer time-series of data and argue that the downward flexibility of nominal wages in the U.K. resulted in the relatively rapid adjustment of real wage after the Great Recession. On the other hand, a sequence of studies by Kuroda and Yamamoto (2003) and Kuroda and Yamamoto (2014) documents that, after the financial crisis in the late 1990s of Japan, DNWR disappeared from the individual wage data in Japan although it was present until the mid 1990s. The consequence of these cross-country variations in the degree of DNWR for the inflation dynamics of each country would be an interesting research question, although it is beyond the scope of this paper.

[17] In terms of wage rigidities in general, Dickens et al. (2007) find, in the International Wage Flexibility Project, the degree of nominal and real wage rigidity significantly varies across countries.

1.8 Modeling the Great Recession

This section adds several extensions to the baseline model to investigate whether the model can account for the inflation dynamics during and after the Great Recession.

1.8.1 ZLB

A number of studies argue that the ZLB of the nominal interest rate is an essential element to understand the Great Recession (Christiano et al. (2015), Basu and Bundick (2017), Aruoba et al. (2018), etc). In this subsection, therefore, I introduce the ZLB into the baseline model to explore its implications.

I assume that the central bank follows the Taylor (1993) rule with the ZLB:

$$R_t^d = R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{1.53}$$

$$R_t = max\{\ R_t^d\ ,\ 1\ \} \tag{1.54}$$

Although most of the previous studies that investigate the role of the ZLB after the Great Recession assume the standard Taylor rule in (2.25),[18] the Federal Reserve announced a commitment to keep the low interest rate policy when they faced the ZLB. To take into account the effect of this type of forward guidance, I also consider the history-dependent rule proposed by Reifschneider and Williams (2000):

$$R_t^d = R^* \left(\frac{R_{t-1}^d}{R_{t-1}}\right) \left(\frac{\Pi_t}{\Pi^*}\right)^{\delta_\pi} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{1.55}$$

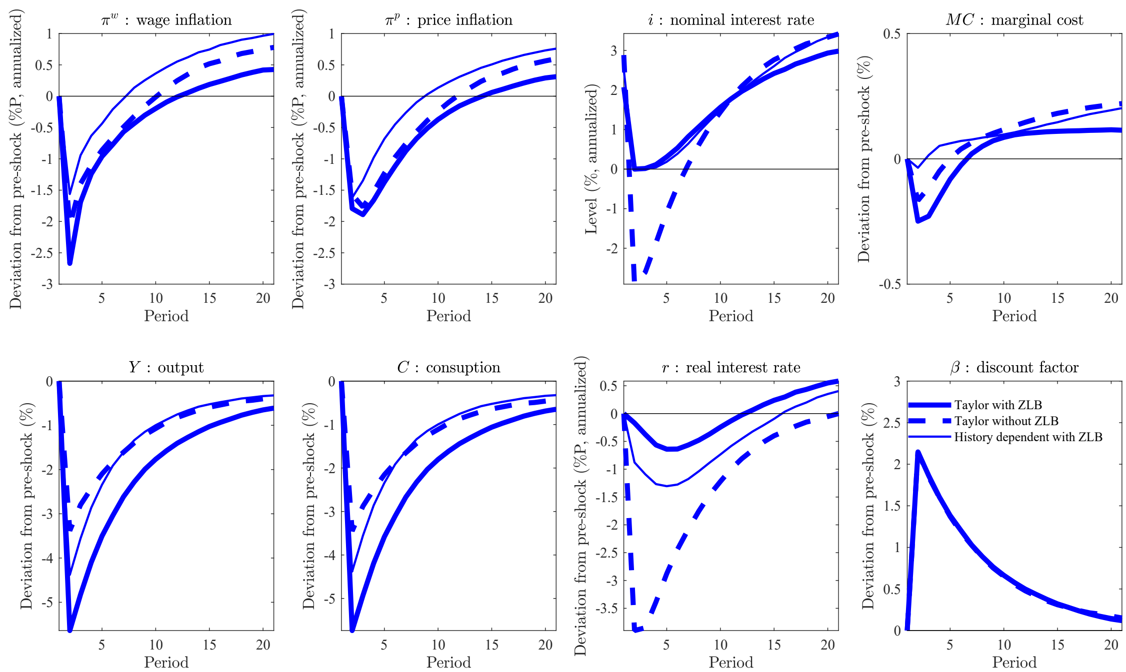
$$R_t = max\{\ R_t^d\ ,\ 1\ \} \tag{1.56}$$

[18] Few exceptions include Basu and Bundick (2015) and Katagiri (2016).

Under the history-dependent rule, the central bank keeps track of the past interest rate gap, that is, the difference between the desired interest rate implied by (1.55) R^d_{t-1} and the actual rate R_{t-1}. Once the nominal interest rate is constrained at the ZLB, the central bank continues the low interest rate policy until the gap is cleared, even if the interest rate implied by the standard Taylor rule becomes positive. Though several ways to implement forward guidance in a general equilibrium model has been studied in the literature (Eggertsson and Woodford (2003), Del Negro et al. (2012), McKay et al. (2016a), for example), the history-dependent rule is distinguished from others in that it is fully embedded in the rational expectation equilibrium as a monetary policy rule. Basu and Bundick (2015) argue that the history-dependent rule has desirable properties to remove the contractionary bias of the ZLB, that is, the bias that the central bank charges higher interest rate than the desired one on average over business cycles in the presence of the ZLB.

Figure 1.9 displays the GIR under different monetary policy rules with and without the ZLB. Several aspects of the results are noteworthy. First, in line with previous studies, the ZLB has an amplification effect to a demand shock. Second, the amplification effect in the model works much more strongly for real quantities than price variables. For instance, the responses of price inflation are larger by around 20 percent in the presence of the ZLB, while the output responses are amplified by more than 50 percent. Economic intuition behind the results is as follows. At the ZLB, a contractionary demand shock raises real interest rate due to the lack of offsetting monetary policy responses, and it reduces consumption through the Euler equation. In a frictionless labor market, the decreased marginal rate of substitution due to the

Figure 1.9: GIR with and without the ZLB



Notes: Each panel shows the GIR to a 3 S.D. contractionary discount factor shock. The x-axes represent the time horizon after the initial shock. The y-axes are in terms of the deviation from the value before the shock except for the nominal interest rate, which is shown in the level. Models with different monetary policy rules are solved under the same parameter values.

negative income effect leads to a decline of real wage, which in turn reduces inflation through the NKPC. However, in the presence of DNWR, the decline of real wage is hindered by the binding DNWR constraint. On the other hand, the dampened response of real wage is compensated by a further contraction of hours worked as a consequence of the labor market equilibrium. Third, the history-dependent rule partly offset the amplification effect of the ZLB because the commitment to the future low interest rates affects the current consumption through the forward looking nature of the Euler equation.

1.8.2 Counterfactual for the Great Recession

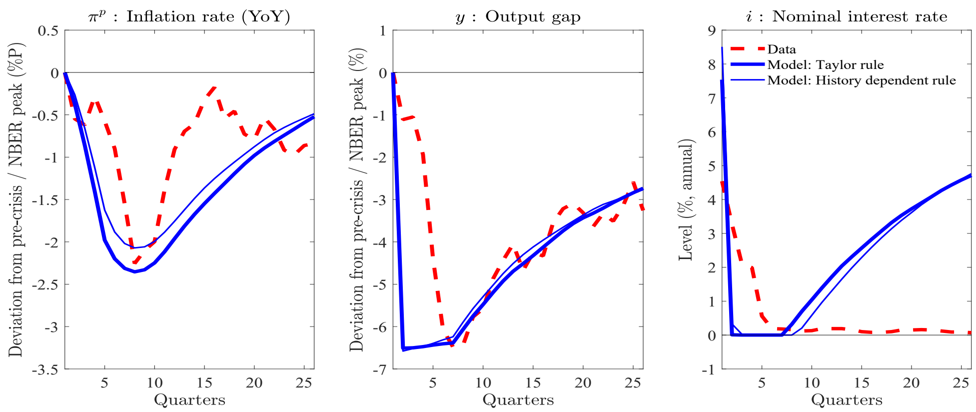
This subsection conducts a counterfactual analysis of the Great Recession. To calibrate the model to U.S. data during the Great Recession, I adopt the concept of "severe recession" proposed by Krueger et al. (2016). They define severe recession as the periods in which unemployment rate hits 9 percent at least in one quarter and stays above 7 percent. In the post-war U.S. economy, 1980Q2-1986Q2 and 2009Q1-2013Q2 satisfy these criteria. I employ their calibrated values for state transition probabilities:

$$P_\beta = \begin{bmatrix} P_{L\to L} & P_{L\to H} \\ P_{H\to L} & P_{H\to H} \end{bmatrix} = \begin{bmatrix} 0.9910 & 0.0090 \\ 0.0455 & 0.9545 \end{bmatrix} \tag{1.57}$$

where the high discount factor state denoted by H corresponds to the severe recession. Notice that the expected duration of the severe recession is much shorter (around 22Q) than that of the normal state (around 111Q).

Figure 1.10 and Figure 1.11 display the counterfactual to the contractionary shock that replicates the Great Recession. The size of the shock is calibrated to match the drop of the output gap in the data in each model. In Figure 1.10, the calibrated Great Recession shock only leads to 2.4 percentage point decline of the year-on-year inflation rate under the standard Taylor rule with the ZLB. This quantitative result is comparable with the U.S. economy during the Great Recession, in which the actual inflation rate in the GDP implicit price deflator declined by 2.3 percentage point from the peak to the bottom (2007Q4:2.5 percent→2009Q3:0.2 percent). The model also replicates the sluggish recovery of inflation in the data. In Appendix

Figure 1.10: Counterfactual for the Great Recession (1)

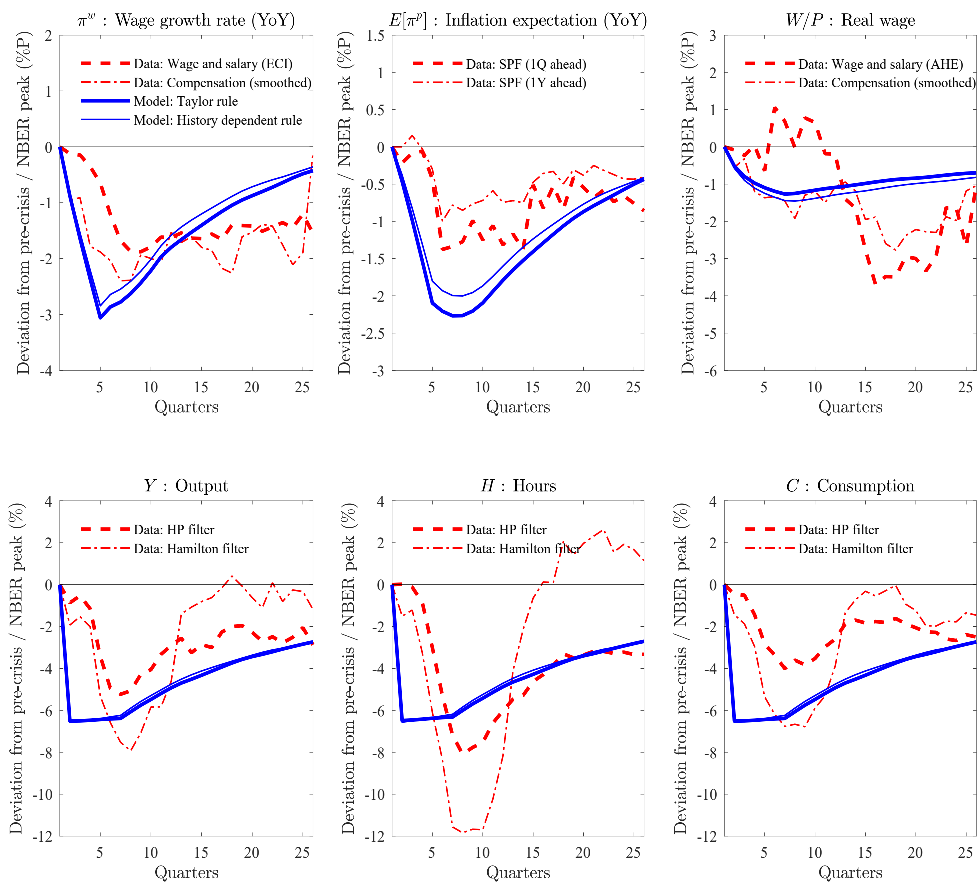


Notes: Each panel shows the GIR to exogenous 6 quarter consecutive severe recession shock after 24 quarters of boom periods. The length of shocks and boom periods correspond to those of the Great Recession (expansion: 2002Q1-2007Q4, contraction: 2008Q1-2009Q2). The nominal interest rate in the data is the effective federal funds rate. The definition of other variables is the same as Figure 1.1.

1.10.5, I compare the counterfactual in the models with different specifications of wage adjustment. Each model is calibrated with the same parameter values as the baseline model except for wage rigidities, and solved under the standard Taylor rule with the ZLB. The results shown in Figure 1.20 suggest that the flexible wage model leads to around -35 percent of deflation, while the quadratic adjustment cost leads to -10 percent, both of which are far from the actual data.

Interestingly, the magnitude of the decline of inflation under the history-dependent rule is 2.1 percentage point. It indicates that, once the magnitude of the output gap drop is taken as given, the history-dependent rule and the standard Taylor rules yield similar responses of inflation. The result is indeed reasonable because the relative responses of inflation to the output gap are governed by the NKPC and the relationship between marginal cost and the output gap, while the monetary policy rule affects the demand side of the economy through the consumption Euler equation.

Figure 1.11: Counterfactual for the Great Recession (2)



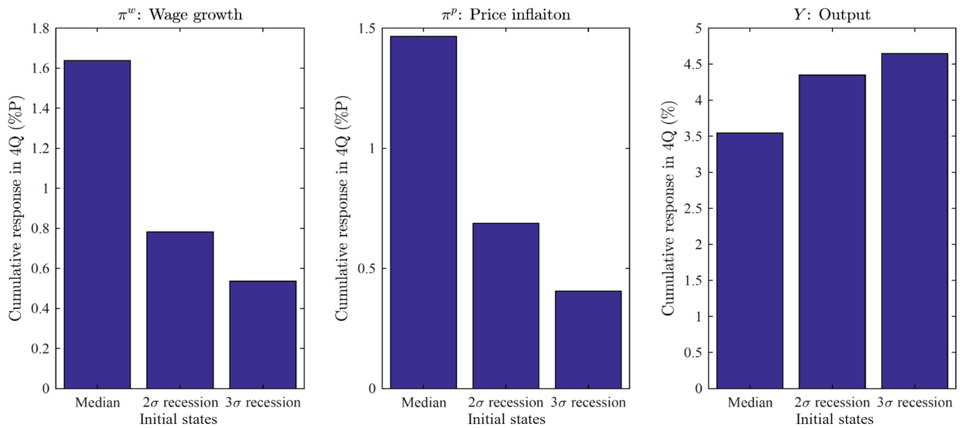
Notes: The definition of GIR is the same as Figure 1.10. Data is in the deviation from the business cycle peak before the Great Recession defined by the NBER (2007Q4). In the top panels, wage growth π^w is the wage and salary in the Employment Cost Index and the compensation per hour in the non-farm business sector. The compensation series is smoothed as five quarters centered moving average. Inflation expectation $E[\pi^p]$ is the median forecast for one quarter and four quarters ahead GDP deflator in the Survey of Professional Forecast. The one quarter ahead forecast is annualized. For real wage W/P, price index is the GDP implicit price deflator. Wage is the average hourly earnings of production and non-supervisory employees in the private sector and the compensation per hour in the non-farm business sector. Real wage series is detrended by the HP filter. In the bottom panels, output Y is the real GDP, hours H is the total hours in the non-farm business sector, consumption C is the real personal consumption expenditures. Each series is taken log and detrended by the HP filter and the Hamilton filter.

In Figure 1.11, the model replicates the dynamics of other variables fairly well. In particular, the data suggests that both of wage and compensation declined moderately after the Great Recession and the model captures these movements, with the difference in the magnitude of the responses from the data being less than 1 percentage point. It is also notable that the model is consistent with the stable inflation expectation in the data. This is because the dampened responses of marginal cost due to DNWR prevent the inflation expectations from declining. As for real quantities, though the model cannot perfectly replicate the relatively larger drop of hours worked and the smaller decline of consumption in the data, they are presumably because I abstract away capital investment, which is not the main focus of this paper. Rather, it is worth noting that the model does not have a number of ingredients that previous studies argue are important to account for the missing deflation such as high degree of price stickiness, exogenous shocks to the inflation expectations, and financial frictions. Instead, the only extension from the stylized New Keynesian model presented in Section 1.3 is the presence of the DNWR constraint for individual workers and the ZLB, but still the model succeeds in accounting for the key moments regarding the missing deflation puzzle.

1.8.3 Implication to the excessive disinflation

I next investigate the implications of the model to the excessive disinflation after the Great Recession. Figure 1.12 shows the GIR to a 1 S.D. expansionary shock from different initial states. The model displays significantly divergent responses in each initial state. For example, starting from a 3 S.D. recession state, which corresponds to the severe recession state, the positive responses of wage growth and price inflation are roughly three times smaller than those from the median state.

Figure 1.12: State dependency of GIR



Notes: Each bar shows the cumulative responses in the four quarters after a 1 S.D. expansionary discount factor shock from different initial states. To draw each initial state, I simulate the model economy randomly for ten quarters after the median state, 2 S.D. recession state, and 3 S.D. recession state, respectively.

This is due to the state-dependent nature of DNWR. Upon a severe recession shock, workers' desired wages decline due to the negative income effect and fall short of their actual wages since the DNWR constraint binds. In a recovery phase, even when their desired wages start to rise as the state of the economy improves, workers never raise their actual wages as long as the DNWR constraint binds. This mechanism delays the recovery of wage growth, which in turn leads to a slow recovery of inflation through sluggish rises of marginal cost. On the other hand, the recovery of output is relatively fast from a severe recession, but the quantitative result suggests that the differences of the output responses are not as significant as wage growth nor price inflation.

Lastly, Figure 1.13 exhibits the distribution of price inflation in time-series simulation of the model and the data. I estimate the kernel density to smooth jagged observations In the model, the distribution of inflation is positive skewed because of the asymmetric effect of DNWR. Consequently, the median inflation rate (1.21 percent) is substantially lower than the 2 percent of the calibrated target rate in the Taylor rule, while the mean inflation rate (2.07 percent) is almost around the target. In other words, lower inflation rates than the target level are more likely to realize in each period even if the target rate is achieved in the mean. This finding might be counterintuitive, but is indeed consistent with a wide class of the Taylor-type monetary policy rule. To see this point, taking the the unconditional expectation of the Taylor rule (1.41) leads to:

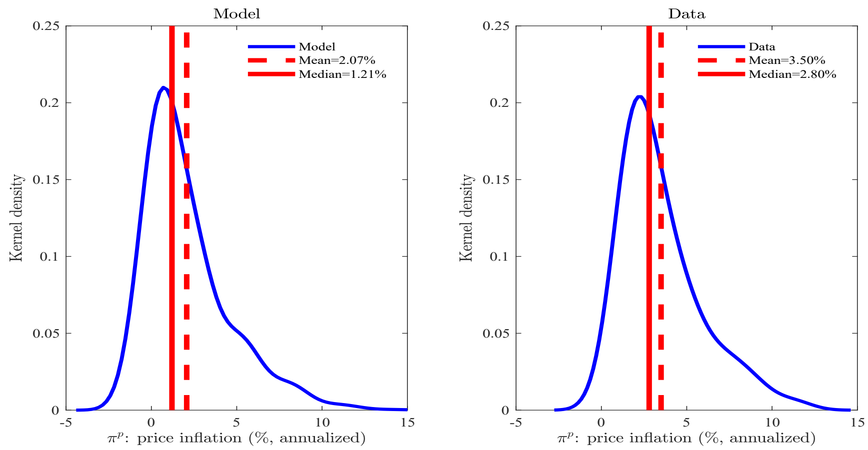
$$log(\mathbb{E}[R_t]) - log(R^*) \cong \delta_\pi(log(\mathbb{E}[\Pi_t]) - log(\Pi^*)) + \delta_y(log(\mathbb{E}[Y_t]) - log(Y^*)) \quad (1.58)$$

Notice that the equation does not strictly hold because I ignore the Jensen's inequality terms. Equation (1.58) implies that inflation is stabilized in the mean under the Taylor rule, although the equation does not guarantee the stabilization of the median inflation to the target rate.

1.9 Conclusion

In this paper, I introduce DNWR for individual workers into an otherwise standard New Keynesian DSGE model. DNWR accounts for the flattening of the observed Phillips curve relationship between inflation and the output gap, while keeping the marginal cost representation of the NKPC unchanged. The endogenous evolution of cross-sectional wage distribution generates non-linear dynamics in many dimensions, including the sign-, the size-, and the state-dependency of the consequence of an exogenous shock. Consequently, the calibrated model successfully matches the key

Figure 1.13: Distribution of inflation



Notes: The kernel density estimator is computed from simulated and actual data. In the model, I simulate the model economy for 51,000 periods and discard the initial 1,000 observations. Data is the quarter on quarter growth rate of the GDP implicit price deflator. Sample period is from 1955Q1 to 2007Q4. The end point of the sample period is determined to exclude the ZLB periods.

moments of the inflation dynamics during and after the Great Recession, which are often referred to as the missing deflation and the excessive disinflation.

A number of extensions are possible for future research. First, assessing other dimensions of aggregate dynamics through the lens of my model is a natural extension. For instance, incorporating unemployment is one promising option. Studying the interaction between the heterogeneity of labor and that of consumption would be also interesting. In this regard, though recent papers such as Hall (2017) identify that the movements of discount factor are essential to explain the aggregate dynamics after the Great Recession, dealing with heterogeneity might help one to reconcile the large fluctuations of discount factor. Second, it would be worthwhile exploring optimal monetary policy in the economy with DNWR. Although previous studies investigate optimal policy in a representative agent framework (e.g., Kim and Ruge-Murcia (2009) and Coibion et al. (2012)), taking into account heterogeneity might deliver rich policy implications. Moreover, though the state of the economy is characterized by cross-sectional distribution in a heterogeneous agent setting, it is almost impossible in practice for the central bank to keep track of the distribution in a timely manner. Therefore, how to approximate the optimal policy as an implementable policy rule would be a valuable question. Lastly, on the empirical side, my model yields a number of testable implications. In particular, it would be beneficial to explore how much the model can account for the evolution of cross-sectional wage distribution after the Great Recession in more detail.

1.10 Appendix

1.10.1 Empirical Evidence

1.10.1.1 Construction of marginal cost

Specification of production function. For a baseline case, I assume the Cobb-Douglas production function with overhead labor (CDOH):

$$Y_t = F(H_t, K_t, Z_t) = \{Z_t(H_t - \bar{H})\}^{\alpha} K_t^{1-\alpha} \tag{1.59}$$

where Y_t denotes output, Z_t labor augmenting technology, H_t labor inputs, K_t capital inputs. $\bar{H}$ is the overhead component of labor inputs that is not directly linked to value added production. The first order condition (FOC) for labor inputs implies

$$\theta_t^H \equiv \frac{\partial log(F_t)}{\partial log(H_t)} = \alpha \frac{H_t}{H_t - \bar{H}} \tag{1.60}$$

Using (2.8), the FOC is rearranged to the specification of marginal cost:

$$MC_t^{CDOH} = \frac{1}{\alpha}\left(1 - \frac{\bar{H}}{H_t}\right) s_t^H \tag{1.61}$$

with $s_t^H = \frac{W_t H_t}{P_t Y_t}$ being the labor share. The specification leads to

$$\hat{MC}_t^{CDOH} = \frac{\bar{H}/H^{ss}}{1 - \bar{H}/H^{ss}} \hat{H}_t + \hat{s}_t^H \tag{1.62}$$

where $\hat{x}$ denotes the log-deviation from the steady state. For parameterization, I borrow the estimated value of Basu (1996) to calibrate $\bar{H}/H^{ss} = 0.288$. The value is in line with other estimates in literature such as 0.20 of Ramey (1991) and 0.14 of Bartelsman et al. (2013). Bartelsman et al. (2013) point out as a reference that in the U.S. manufacturing industries non-production workers compose of roughly 30 percent of total employment and managers of 10 percent.

For robustness check, I consider alternative specifications: the Cobb-Douglas production function (CD) and a production function with constant elasticity of substitution (CES). The Cobb-Douglas production function is given by:

$$F(H_t, K_t, Z_t) = (Z_t H_t)^{\alpha} K_t^{1-\alpha} \tag{1.63}$$

The FOC for labor inputs formulates marginal cost to be proportional to the labor share:

$$MC_t^{CD} = \frac{1}{\alpha} s_t^H \tag{1.64}$$

and

$$\hat{MC}_t^{CD} = \hat{s}_t^H \tag{1.65}$$

Under a CES production function,

$$F(H_t, K_t, Z_t) = \left\{\alpha\,(Z_t H_t)^{\frac{\nu-1}{\nu}} + (1-\alpha)K_t^{\frac{\nu-1}{\nu}}\right\}^{\frac{\nu}{\nu-1}} \tag{1.66}$$

we obtain

$$MC_t^{CES} = \frac{1}{\alpha}\left(\frac{Y_t}{Z_t H_t}\right)^{\frac{\nu-1}{\nu}} s_t^H \tag{1.67}$$

and

$$\hat{MC}_t^{CES} = \frac{\nu-1}{\nu}(\hat{Y}_t - \hat{Z}_t - \hat{H}_t) + \hat{s}_t^H \tag{1.68}$$

where ν represents the elasticity of substitution between labor and capital inputs. I follow Gali et al. (2007) to calibrate $\nu = 0.5$. For the series of Z_t, I use the utilization-adjusted quarterly-TFP for the U.S. business sector constructed based on Fernald (2014).

Detrending. An important issue when using the series of the labor share in U.S. data is detrending, because the data displays a low frequent downward trend. Though there is substantial debate regarding the causes behind the trend, many of existing studies attribute it to structural changes of the economy such as offshoring of manufacturing industries and declining relative price of investment goods due to advances in information technology, or others point out mismeasurement of data (Elsby et al. (2013), Karabarbounis and Neiman (2013), etc). Since the main focus of this paper is on business cycle fluctuations related to inflation dynamics, I use a filtering method to extract cyclical components of the labor share. Similar methods are employed by Mavroeidis et al. (2014) when they estimate the NKPC. Specifically, each series of marginal cost is detrended by the Hamilton filter. Hamilton (2017) argues that the Hamilton filter has desirable time-series properties compared to the HP-filter, which is widely used in business cycle analysis. In particular, the one-sided method of the Hamilton filter addresses the end of sample problem of the HP-filter.

1.10.1.2 Robustness checks for the estimation of the NKPC

1.10.1.3 Robustness check (1): alternative measures for marginal cost and the output gap

Table 1.7 reports the results of the OLS estimation of the NKPC (1.5) with alternative measures for marginal cost: marginal cost based on the Cobb-Douglas production function (CD) in column 1-4 and a production function with constant elasticity of substitution (CES) in column 5-8. The estimation results display quite similar patterns to the baseline specification of the Cobb-Douglas production function with overhead labor (CDOH) in Table 1.1. In other words, the coefficients of marginal cost remain stable after the Great Recession, and the interaction terms with the post Great Recession dummy are not statistically significant.

Table 1.7: OLS estimation with alternative measures of marginal cost

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Marginal cost (CD)			Marginal cost (CES)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.447***	0.485***	0.465***	0.419***	0.467***	0.441***
	(0.079)	(0.074)	(0.078)	(0.080)	(0.074)	(0.079)
π_{t-1}	0.548***	0.510***	0.529***	0.575***	0.527***	0.552***
	(0.080)	(0.074)	(0.079)	(0.082)	(0.075)	(0.081)
x_t	0.208**	0.167**	0.193**	0.170***	0.136***	0.148***
	(0.082)	(0.067)	(0.078)	(0.056)	(0.045)	(0.053)
$postGR_t \times \pi_t^e$			0.290			0.439*
			(0.219)			(0.226)
$postGR_t \times \pi_{t-1}$			-0.308			-0.404*
			(0.232)			(0.234)
$postGR_t \times x_t$			-0.120			-0.130
			(0.155)			(0.098)
Adjusted R^2	0.952	0.946	0.946	0.935	0.931	0.930
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

I next explore alternative measures for the output gap. Following the literature, I use the detrended output. Specifically, I construct the detrended output by removing a quadratic trend from the log output.[19] I also investigate the employment rate, which is defined as the ratio of employment out of the population older than 16 years. The employment rate potentially addresses issues regarding the changes in the labor force participation after the Great Recession. Table 1.8 reports the regression results. In line with my baseline estimation, the interaction terms of the output gap measures with the post-Great Recession dummy variable are significantly negative. The results

[19]Though many of previous studies (e.g., Gali and Gertler (1999), Adam and Padula (2011)) only take into account a linear trend, I find the quadratic term is significant at the one percent level.

indicate that the flattening of the output gap representation of the NKPC holds with alternative measures as well.

Table 1.8: OLS estimation with alternative measures for the output gap

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Detrended output			Employment rate		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	0.555***	0.583***	0.580***	0.462***	0.497***	0.482***
	(0.085)	(0.078)	(0.083)	(0.080)	(0.074)	(0.078)
π_{t-1}	0.447***	0.421***	0.424***	0.532***	0.496***	0.511***
	(0.084)	(0.077)	(0.082)	(0.080)	(0.075)	(0.078)
x_t	0.0948***	0.0861***	0.0920***	0.317**	0.232***	0.308**
	(0.021)	(0.019)	(0.020)	(0.138)	(0.119)	(0.138)
$postGR_t \times \pi_t^e$			0.132			0.216
			(0.210)			(0.215)
$postGR_t \times \pi_{t-1}$			-0.185			-0.254
			(0.230)			(0.221)
$postGR_t \times x_t$			-0.105*			-0.418*
			(0.056)			(0.222)
Adjusted R^2	0.954	0.949	0.948	0.951	0.945	0.945
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for *Before GR* and from 1968Q4 to 2016Q4 for *Full sample*, respectively. The starting period corresponds to the period when the SPF became available.

1.10.1.4 Robustness check (2): the purely forward looking NKPC

I estimate the following purely forward looking NKPC:

$$\pi_t = \beta\pi_t^e + \kappa x_t + e_t \tag{1.69}$$

Table 1.9 reports the results of the OLS estimation of the purely forward looking NKPC (1.69). Similar to the hybrid NKPC in Table 1.1, the coefficient of the marginal cost is stable after the Great Recession, and the interaction term of the marginal cost and the dummy variable for the post-Great Recession period is not significant.

Table 1.9: OLS estimation of the purely forward looking NKPC

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_t^e	1.048***	1.029***	1.045***	0.998***	0.996***	0.996***
	(0.023)	(0.024)	(0.023)	(0.024)	(0.023)	(0.023)
x_t	0.458***	0.229**	0.425***	0.162*	0.140**	0.138*
	(0.063)	(0.052)	(0.060)	(0.083)	(0.066)	(0.080)
$postGR_t \times \pi_t^e$			-0.126			0.00500
			(0.137)			(0.097)
$postGR_t \times x_t$			-0.444***			0.0112
			(0.099)			(0.128)
Adjusted R^2	0.947	0.936	0.941	0.934	0.930	0.929
N of obs.	157	193	193	157	193	193

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: Dependent variable is the current inflation rate π_t. Heteroscedasticity corrected standard errors are reported in parentheses. The sign of the coefficient of the unemployment gap is flipped for comparison purposes. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1968Q4 to 2007Q4 for $Before\ GR$ and from 1968Q4 to 2016Q4 for $Full\ sample$, respectively. The starting period corresponds to the period when the SPF became available.

1.10.1.5 Robustness check (3): the rational expectation assumption

I follow Gali and Gertler (1999) to assume the rational expectation for seeing robustness of the baseline result in terms of assumptions on the expectation formation. Using the rational expectation assumption, the expected inflation can be replaced with the realized inflation and the rational expectation error:

$$\mathbb{E}_t[\pi_{t+1}] = \pi_{t+1} + \tilde{e}_{t+1} \tag{1.70}$$

The NKPC is rearranged to:

$$\pi_t = \beta\pi_{t+1} + \gamma\pi_{t-1} + \kappa x_t + e_{t+1} \tag{1.71}$$

with $e_{t+1} \equiv \beta\tilde{e}_{t+1}$. Notice that the OLS estimator is biased because e_{t+1} might be correlated with π_{t+1}. Adopting the insight of Gali and Gertler (1999), therefore, I use lagged variables as instruments for π_{t+1} to derive the GMM estimator. To this end, any variables at and before period t are valid instruments, because the rational expectation error e_{t+1} is orthogonal to any variable in the information set at period t. The estimation result is reported in Table 1.10. The coefficient of the unemployment gap is not significant in column 1 and 2, and weakly significant with a negative sign to in column 3. Although the result is inconsistent with the theory of the NKPC, it is in line with the findings of previous empirical studies. To be precise, Gali and Gertler (1999) and Sbordone (2002) obtain insignificant estimates for the coefficient of the output gap under the rational expectation assumption. More recently, Adam and Padula (2011) find that the coefficient of the output gap is significant only when a survey based expectation measure is used instead of the rational expectation assumption. On the other hand, I confirm the baseline result regarding the marginal cost representation of the NKPC. The coefficient of marginal cost is significantly positive and does not decline after the Great Recession.

Table 1.10: GMM estimation under the rational expectation assumption

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of x	Unemployment gap			Marginal cost (CDOH)		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.729***	0.700***	0.709***	0.704***	0.683***	0.692***
	(0.051)	(0.045)	(0.058)	(0.048)	(0.056)	(0.053)
π_{t-1}	0.261***	0.289***	0.277***	0.283***	0.304***	0.294***
	(0.051)	(0.046)	(0.058)	(0.049)	(0.056)	(0.053)
x_t	-0.0273	-0.0138	-0.0350*	0.129***	0.104***	0.111***
	(0.021)	(0.012)	(0.021)	(0.040)	(0.033)	(0.040)
$postGR_t \times \pi_{t+1}$			0.208			-0.00340
			(0.284)			(0.164)
$postGR_t \times \pi_{t-1}$			0.174			0.0632
			(0.152)			(0.166)
$postGR_t \times x_t$			0.210**			-0.150
			(0.090)			(0.147)
N of obs.	208	243	243	208	243	243

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The two step GMM with an HAC weight matrix is employed. Instruments are the first to forth lagged GDP implicit price deflator, marginal cost, labor share, output gap, wage growth rate, commodity price inflation, short-long term interest rate spread, post-Great Recession dummy. HAC corrected standard errors are reported in parentheses. $postGR_t$ is a dummy variable that takes 1 after 2008Q1. Sample period is from 1955Q1 to 2007Q4 for $Before\ GR$ and from 1955Q1 to 2016Q3 for $Full\ sample$, respectively.

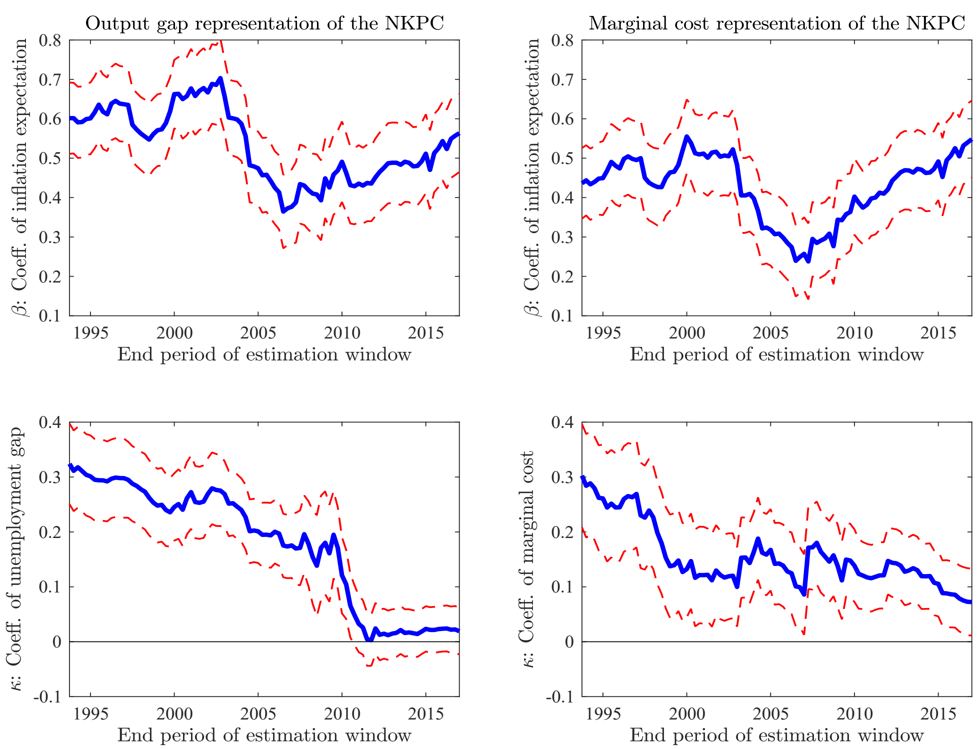
1.10.1.6 Robustness check (4): rolling OLS regression

To identify the specific timing of the changes of coefficients in the NKPC, I conduct a rolling OLS regression of the NKPC. I roll over 25 year window samples, each of which includes 100 observations. In order to save the number of parameters to estimate in a relatively small sample size, I impose a restriction $\beta + \gamma = 1$ as is often assumed in the literature such as Blanchard et al. (2015). The regression model for each rolling sample is given by:

$$\pi_t = \beta_T \pi_t^e + (1 - \beta_T)\pi_{t-1} + \kappa_T x_t + e_t, \quad for\ t \in [T-99, T] \tag{1.72}$$

Figure 1.14 shows the evolution of the estimated coefficients. The coefficient of the output gap shown in the left-bottom panel has a sharp drop around 2010 and stays around zero afterward. On the other hand, the coefficient of marginal cost in the right-bottom panel remains roughly constant around the period. The estimation result detects other interesting patterns in the time-variations of coefficients, such as a rise in the coefficients of the forward looking inflation term after mid 2000s and a slowly declining trend in the coefficient of the output gap and marginal cost in 1990s. However, they are beyond the scope of this paper.

Figure 1.14: Rolling OLS regression of the NKPC with 25 year window



Notes: Solid line is the OLS estimator, while dashed line is the 68% confidence band. The definition of each variable is the same as Table 1.1.

1.10.1.7 Robustness check (5): Markov-switching model

One might be concerned that the result of the rolling regression reflects particular observations in each sample window. The Bayesian methods, on the other hand, make full use of the entire sample to specify the timings of parameter changes. Specifically, I estimate the Markov-switching model for the coefficients and the standard deviation of the error term in the NKPC:

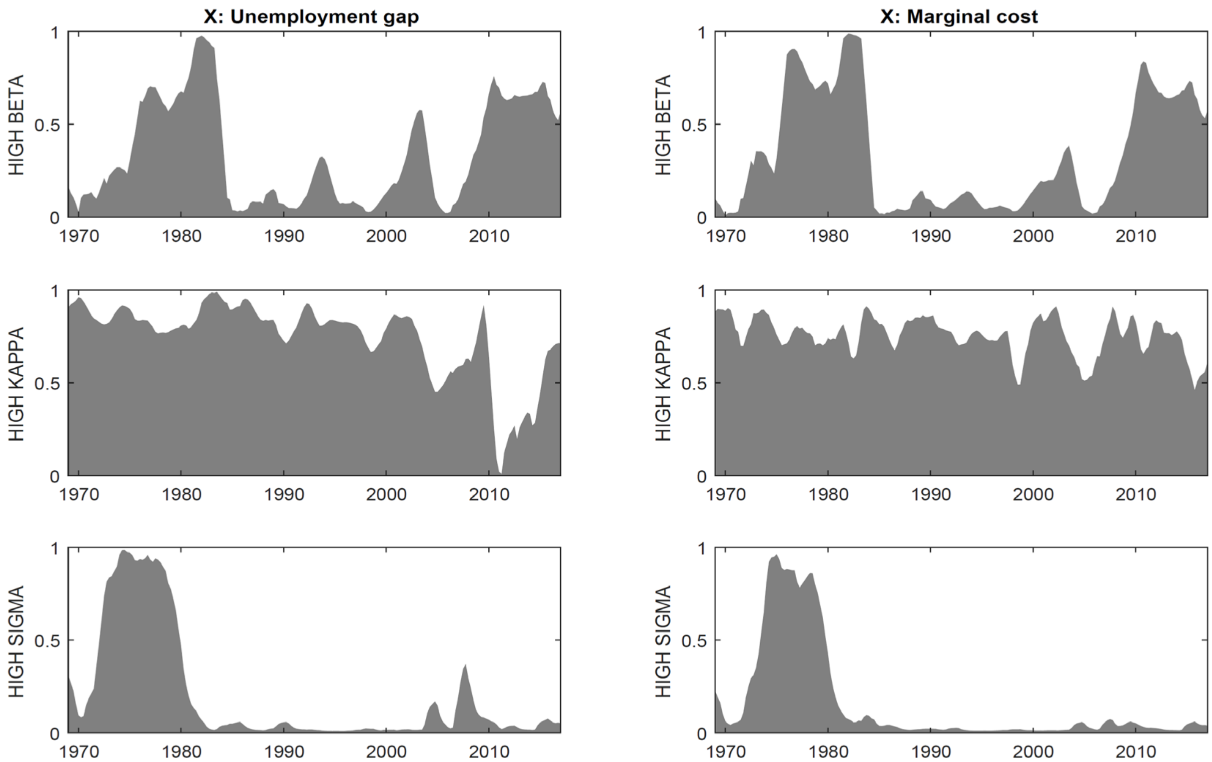
$$\pi_t = \beta(S_t)\pi_t^e + (1 - \beta(S_t))\pi_{t-1} + \kappa(S_t)x_t + e_t, \quad e_t \sim N(0, \sigma(S_t)^2) \tag{1.73}$$

I consider high and low regime for each parameter $\{\beta, \kappa, \sigma\}$. The total number of regimes is $2 \times 2 \times 2 = 8$.

$$S_t = \{\beta_H, \beta_L\} \times \{\kappa_H, \kappa_L\} \times \{\sigma_H, \sigma_L\} \tag{1.74}$$

The latent states are estimated by the Hamilton filter given a set of parameters, and the parameter values are estimated to maximize the likelihood of the model. Figure 1.15 shows the estimated probabilities of each regime. For the slope parameter of the NKPC κ, the estimated probabilities of high regime declines sharply in terms of the output gap around 2010. The coefficient of marginal cost is roughly stable over time, though volatile state probabilities indicates that the series include sizable noises. The estimation result delivers other interseting observations including a rise of the volatility σ during the Great Inflation period in 1970s and rises of the coefficient of the forward looking inflation term β around the Volcker period in the early 1980s and after the Great Recession. These observations are subject to future research, although I focus on the differences in the differences in the slope parameter in the output gap and the marginal cost representations of the NKPC.

Figure 1.15: Smoothed probabilities of high parameter regime



Notes: Smoothed probabilities are the centered 5 quarters average of the state probabilities.

1.10.1.8 Robustness check (6): dynamic panel estimation using industry level data

One concern regarding the estimation of the NKPC using the aggregate variables is that the estimated marginal cost might include considerable measurement errors. In this subsection, therefore, I investigate the intermediate share in industry level data as an alternative measure of marginal cost for assuring robustness of the analysis. To this end, it is notable that the firm's cost minimization condition can be applied to any factor inputs. Moreover, a number of studies, for example, Basu (1995), Nekarda and Ramey (2013) and Bils et al. (2014), suggest that intermediate inputs are promising in many dimensions. First, adjustment costs for intermediates are considered to be low relative to those for capital or labor. Second, the assumption of no overhead component seems more defensible for intermediates. In addition, using industry level data removes composition bias among industries. I use the KLEMS 2017 dataset to construct the intermediate share. The KLEMS 2017 dataset is annual from 1947 to 2014, covering 65 industries. I focus on 60 industries in the non-farm business sector, including 18 manufacturing and 42 non-manufacturing.

Since a measure of inflation expectations is not available for each industry, I rely on the rational expectation assumption to estimate the industry level NKPC:

$$\pi_{i,t} = \alpha_i + \beta\pi_{i,t+1} + \gamma\pi_{i,t-1} + \kappa x_{i,t} + e_{i,t+1} \tag{1.75}$$

$$where\ \mathbb{E}_t[\pi_{i,t+1}] = \pi_{i,t+1} + \tilde{e}_{i,t+1} \tag{1.76}$$

with $e_{i,t+1} = \beta\tilde{e}_{i,t+1}$. For varialbe $x_{i,t}$, I consider marginal cost measured by the intermediate share, and the detrended output as a measure of the output gap. α_i is an unobserved industry fixed effect and $e_{i,t+1}$ is the rational expectation error of industry i in period $t+1$. I employ the two step GMM model procedure to correct industry fixed effects, which is called the Arellano-Bond estimator. Since I take the first difference of (1.75) to remove industry fixed effects, valid instruments for moment conditions are one-period more lagged than those in the standard GMM estimator. More discussion is found in Arellano and Bond (1991). The estimation result is presented in Table 1.11. The coefficient of the detrended output is insignificant in each specification, which is in line with our GMM estimation with aggregate data. On the other hand, the coefficient of intermediate share is significantly positive and the interaction term is not significant in each case.The purely forward looking NKPC yields similar results to the hybrid NKPC. These results confirm the observations of the baseline estimation that the decline of the slope of the NKPC is not observed in terms of marginal cost.

Table 1.11: Dynamic panel GMM estimation of the NKPC in industry level data

	Dependent : Output price inflation π					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.358^{+++}	0.239^{+++}	0.240^{+++}	0.403^{+++}	0.263^{***}	0.264^{***}
	(0.045)	(0.069)	(0.070)	(0.075)	(0.093)	(0.096)
π_{t-1}	0.321^{+++}	0.227^{+++}	0.235^{+++}			
	(0.033)	(0.061)	(0.059)			
$Output_t$	0.0139	0.00224	0.0117	-0.0214	-0.0130	-0.0120
	(0.024)	(0.024)	(0.027)	(0.029)	(0.027)	(0.031)
$postGR_t \times Output_t$			-0.0690			-0.00946
			(0.065)			(0.076)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

	Dependent : Output price inflation π_t					
	(1)	(2)	(3)	(4)	(5)	(6)
	Hybrid NKPC			Purely forward looking NKPC		
	Before GR	Full sample		Before GR	Full sample	
π_{t+1}	0.359^{+++}	0.244^{+++}	0.244^{+++}	0.393^{+++}	0.261^{***}	0.254^{***}
	(0.045)	(0.072)	(0.073)	(0.073)	(0.093)	(0.095)
π_{t-1}	0.316^{+++}	0.222^{+++}	0.225^{+++}			
	(0.032)	(0.060)	(0.060)			
$IntShare_t$	0.0164^{**}	0.0337^{*}	0.0347^{*}	0.0259^{**}	0.0375^{*}	0.0413^{**}
	(0.008)	(0.018)	(0.020)	(0.013)	(0.019)	(0.021)
$postGR_t \times IntShare_t$			0.00270			0.00799
			(0.005)			(0.005)
AR(p) test	1	1	1	1	1	1
N of ind.	60	60	60	60	60	60
N of total obs.	3,180	3,540	3,540	3,180	3,540	3,540

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, $^{+++}$ $p < 0.001$

Notes: The two step GMM is employed. Output is taken log and detrended by the Hamilton filter. Output price inflation and intermediate price inflation are in log difference while the intermediate share is in log level. Instruments are the second to forth lagged output price inflation rate, the intermediate share, intermediate price inflation, the detrended output. Windmeijer corrected standard errors are reported in parentheses. *AR(p) test* indicates the order of AR process implied by the Arellano-Bond AR(p) test. Industry fixed effects are included. $postGR_t$ is a dummy variable that takes 1 after 2008. A dummy variable for year 2009 is included as an independent variable to control for volatile price inflation of the year, which is not reported in table. Sample period is from 1955 to 2014 for $beforeGR$ and from 1955 to 2014 for $fullsample$, respectively.

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1.10.2 Computation

1.10.2.1 Details of equilibrium computation algorithm

ALM. Following the insight of Krusell and Smith (1998), I conjecture that the aggregate law of motion (ALM) Γ is given by a quadratic form:

$$log(\tilde{W}_s) = \ B_{0,s} + B_{1,s}log(\tilde{W}_{-1}) + B_{2,s}log(\tilde{W}_{-1})^2 \ , \quad for \ \beta_t = \beta_s \tag{1.77}$$

where s denotes the exogenous state of the economy. Notice that, even though the ALM is quadratic in terms of $log(\tilde{W}_{-1})$, it can capture rich non-linear dynamics of the model because the coefficients B_s are different across states s.

Modified Krusell-Smith algorithm. I sketch the outline of the equilibrium computation. The algorithm takes the following steps in each iteration $m = 1, 2, 3...$

1. (Initial guess) Each agent uses the ALM $B^{(m)} = \{B_{0,s}^{(m)}, B_{1,s}^{(m)}\}_{s=1}^{S}$ to forecast the aggregate state variable $\tilde{W}$.

2. (Aggregate problem) Given the aggregate state variables $\{\beta, \tilde{W}\}$, the policy function for aggregate jump variables $f^{(m)}$ is obtained by solving a New Keynesian system, i.e., the Euler equation, the NKPC, and the Taylor rule, together with the production function and the market clearing condition. A policy function iteration is used for this procedure.

3. (Individual problem) Given the aggregate policy function $f^{(m)}$, households solve their wage setting problem to derive their policy function $h^{(m)}$. A value function iteration is used for this procedure.

4. (Stochastic aggregation) Given the aggregate policy function $f^{(m)}$ and the individual policy function $h^{(m)}$, I simulate the model economy with N households for T periods and discard the initial T_0 periods to obtain the series of aggregate variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$. I set $N = 10,000$, $T = 51,000$, and $T_0 = 1,000$. I confirm that the computation results do not change even if I further increase N or T.

5. (Update) Using the simulated variables $\{X_t^{(m)}\}_{t=T_0+1}^{T}$, I obtain the suggested ALM $\hat{B}$ by running the OLS of the ALM (1.77). Then, I update the coefficients $B^{(m+1)}$ according to:

$$B^{(m+1)} = \lambda \hat{B} + (1 - \lambda) B^{(m)} \tag{1.78}$$

   where λ is the weight for updating. λ is set to be 0.2.

6. Repeat from step 1 to step 5 until a criteria for convergence of B is attained.

Discretization. I discretize the AR(1) process of exogenous variables using the Rouwenhorst (1995) method. Though the Tauchen (1986) method is widely used in the literature for this purpose, as Kopecky and Suen (2010) pointed out, the Rouwenhorst (1995) method can precisely match several moments of stationary AR(1) process including the first-order autocorrelation and the unconditional variance, even when the process is highly persistent and the number of discretized states is relatively small.

State space of endogenous state variables are discretized to use a value function iteration and a policy function iteration, and the linear interpolation is employed to approximate the variables between grids when simulating the economy.

Accuracy check of the ALM. For checking accuracy of the ALM, the Den Haan (2010) statistics is employed. The statistics measures the maximum distance between the aggregate state variables computed according to the ALM $(\tilde{W})_t^{alm}$, and those derived from equilibrium conditions in the simulation $(\tilde{W})_t^{sim}$:

$$DH(B) = sup_{t \in [T_0+1, T]} \ |log(\tilde{W})_t^{sim} - log(\tilde{W})_t^{alm}| \tag{1.79}$$

The critical value is set at $DH(B) = 10^{-3}$, which means that the cumulative error of agents' prediction is smaller than 0.1% over 50,000 periods. The criteria is much more strict than R^2, because R^2 measures the average error in the one-period ahead forecast.

1.10.2.2 Discussion on other computation methods

The model developed in this paper is classified into a heterogeneous agent model with aggregate uncertainty, which starts from Krusell and Smith (1998). On the other hand, recent studies propose other computation methods to deal with a heterogeneous agent model. For instance, Reiter (2009) approximates cross-sectional distribution with finite dimensional histograms, whereas Winberry (2016) propose a method to parameterize the distribution by using a family of polynomial functions. Moreover, Ahn et al. (2017) and Kaplan et al. (2018) build a continuous time model where the evolution of the distribution is formulated in the Kolmogorov forward equation and its boundary conditions. These studies use the first order approximation around the stationary distribution in terms of aggregate dynamics to gain the efficiency of computation. However, as discussed in Ahn et al. (2017), this class of solution method cannot capture the sign- and the size-dependency of the effects of an aggregate shock as long as the aggregate dynamics is approximated in the first order. In contrast, I find that the sign- and the size dependency are crucial to accounting for the missing deflation because a large and negative shock such as the Great Recession changes the cross-sectional wage distribution severely. Moreover, the endogenous shifts of the cross-sectional distribution after the initial shocks allow the model to address the excessive disinflation in the subsequent periods. In addition, the ZLB is another reason for us to use a global solution method in terms of the aggregate dynamics, because a local method cannot be used due to the kink of the monetary policy rule.

1.10.2.3 Construction of generalized impulse responses

Definition. Following Koop et al. (1996), I define the generalized impulse responses (GIR) as follows:

$$GIR(y,t,\epsilon_0) = \mathbb{E}[y_t|\epsilon_0] - \mathbb{E}[y_t] \tag{1.80}$$

$$= \mathbb{E}\Big[\mathbb{E}[y_t|\beta_0 = \tilde{\beta}_0 e^{-\epsilon_0}, \beta_1 = \tilde{\beta}_1, \cdots, \beta_t = \tilde{\beta}_t, \omega_0 = \tilde{\omega}_0]|\epsilon_0\Big]$$

$$- \mathbb{E}\Big[\mathbb{E}[y_t|\beta_0 = \tilde{\beta}_0, \beta_1 = \tilde{\tilde{\beta}}_1, \cdots, \beta_t = \tilde{\tilde{\beta}}_t, \omega_0 = \tilde{\omega}_0]\Big] \tag{1.81}$$

where y_t, ω_t, and ϵ_t are target variables, state variables, and exogenous shocks, respectively.

Computation. Since the GIR do not have a closed form solution, a simulation based method is employed. The construction of the GIR takes the following steps:

1. Draw an initial state $\tilde{\omega}_0$ and $\tilde{\beta}_0$ randomly.

2. Draw a series of exogenous shocks $\{\tilde{\beta}_s\}_{s=1}^t$ and $\{\tilde{\tilde{\beta}}_s\}_{s=1}^t$ with or without the initial shock ϵ_0, given the initial state.

3. Simulate the economy along with the path of exogenous variables.

4. Repeat the procedure 1-3 for 10,000 times and take the mean to compute expectation.

1.10.3 Model

1.10.3.1 Definition of stationary equilibrium

Definition. *A stationary equilibrium is a household's policy function for individual real wages $\tilde{w} = h(\tilde{w}_{-1}, \chi)$, aggregate variables $X = \{\tilde{W}, C, Y, H, \Pi, R\}$, and a probability distribution $p(\tilde{w}, \chi)$, such that* *(i) a household's policy function h solves a recursive wage setting problem,*

$$V^{dnwr}\Big(\tilde{w}_{-1},\chi\Big) = \max_{\tilde{w}}: \; -\frac{1}{1+\eta}e^{\chi}h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\Big(\tilde{w},\chi'\Big)\Big] \tag{1.82}$$

$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$

$$\tilde{w} \geq \tilde{w}_{-1}/\Pi$$

$$V^{no}\Big(\chi\Big) = \max_{\tilde{w}}: \; -\frac{1}{1+\eta}e^{\chi}h^{1+\eta} + C^{-\sigma}(1+\tau_w)(\tilde{w}h) + \beta\mathbb{E}\Big[V\Big(\tilde{w},\chi'\Big)\Big] \tag{1.83}$$

$$s.t. \quad h = \Big(\tilde{w}/\tilde{W}\Big)^{-\theta_w} H$$

$$where \quad \mathbb{E}\Big[V\Big(\tilde{w},\chi'\Big)\Big] = (1-\alpha)\mathbb{E}\Big[V^{dnwr}\Big(\tilde{w},\chi'\Big)\Big] + \alpha\mathbb{E}\Big[V^{no}\Big(\chi'\Big)\Big]$$

(ii) aggregate jump variables X solve the Euler equation (1.25), the NKPC (1.39), the monetary policy rule (1.41), the production function (1.40), and the market clearing conditions (1.44), that is,

$$\tilde{W} = Z \;, \quad Y = ZH = C \;, \quad \Pi = \Pi^* \;, \quad R = \Pi^*/\bar{\beta} \tag{1.84}$$

(iii) a probability distribution p is a stationary distribution, that is,

$$p(\tilde{w}',\chi') = \int_{\chi}\int_{\tilde{w}:\tilde{w}'=h(\tilde{w},\chi')} p(\tilde{w},\chi)P(\chi'|\chi)d\tilde{w}d\chi \tag{1.85}$$

(iv) the aggregate hours H satisfies the market clearing condition,

$$H = \left(\int_{\chi}\int_{\tilde{w}} \Big\{(\tilde{w}/\tilde{W})^{-\theta_w} Hp(\tilde{w},\chi)\Big\}^{\frac{\theta_w-1}{\theta_w}} d\tilde{w}d\chi\right)^{\frac{\theta_w}{\theta_w-1}} \tag{1.86}$$

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1.10.4 Additional Results

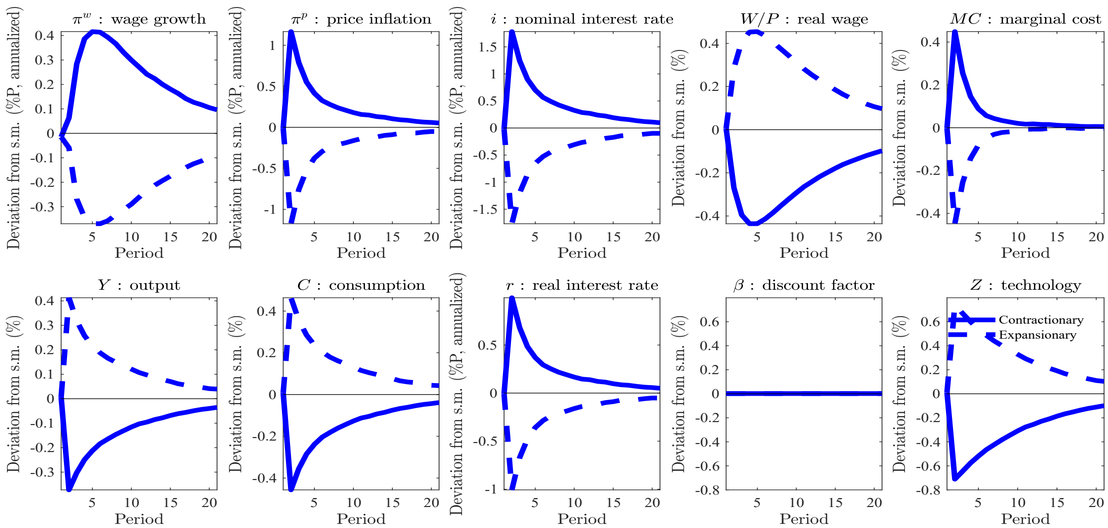
1.10.4.1 Effects of supply side shock

This subsection investigates the effects of a technology shock in the model. For this exercise, I consider that the aggregate technology Z_t follows an AR(1) process while keeping the discount factor constant:

$$log(Z_t) = \rho_z log(Z_{t-1}) + \epsilon_{z,t} \ , \ \epsilon_{z,t} \sim N(0, \sigma_z^2) \tag{1.87}$$

For parameterization, I follow Fernández-Villaverde et al. (2015) to set $\rho_z = 0.900$ and $\sigma_z = 0.0025$. Figure 1.16 shows the GIR to a 2 S.D. technology shock. Interestingly, a technology shock does not generate significant asymmetry in the GIR. In addition, the relative response of price inflation to that of output is much larger than the response to a demand shock in Figure 1.4. It might be because a technology shock directly affects firms' marginal cost through Equation (1.35) and that results in an almost symmetric and large effect on price inflation through the NKPC. This mechanism is considered to be particularly strong given the relatively low degree of price stickiness in my calibration. In literature, on the other hand, Altig et al. (2011) find moderate responses of price inflation to a neutral technology shock in the VAR analysis. They propose to take into account the firm specific capital to match the VAR responses under the micro founded degree of price stickiness. These ingredients might be a potential extension of the model.

Figure 1.16: GIR to a 2 S.D. technology shock



Notes: Each series is the deviation from the stochastic mean (s.m.).

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1.10.5 Model Comparison

1.10.5.1 Flexible wage model

Wage setting. The friction less labor market equilibrium implies that real wage is equalized to the marginal rate of substitution.

$$\frac{W_t}{P_t} = \frac{H_t^{\eta}}{C_t^{-\sigma}} \tag{1.88}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{1.89}$$

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \frac{\theta_p}{\phi_p} \left(\frac{W_t}{P_t} - 1 \right) \tag{1.90}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{1.91}$$

$$Y_t = H_t = C_t + \frac{\phi_p}{2} (\Pi_t - \Pi^*)^2 C_t \tag{1.92}$$

Computation. I discretize the state space and use a policy function iteration to derive a global solution.

1.10.5.2 Quadratic wage adjustment cost model

Wage setting. Households are subject to the following budget constraint with quadratic wage adjustment cost:

$$C_t + \frac{A_t}{P_t} \leq (1 + \tau_w) \frac{W_t}{P_t} H_t - \frac{\phi_w}{2} (\Pi_t^w - \Pi^*)^2 H_t + R_{t-1} \frac{A_{t-1}}{P_t} + \frac{\Phi_t}{P_t} \tag{1.93}$$

where the notation follows the benchmark model. The FOC yields the wage Phillips curve:

$$(\Pi_t^w - \Pi^*)\Pi_t^w = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{H_{t+1}}{H_t} \right) (\Pi_{t+1}^w - \Pi^*)\Pi_{t+1}^w \right] + \frac{\theta_w}{\phi_w} \left(\frac{H_t^\eta}{C_t^{-\sigma}} - \frac{W_t}{P_t} \right) \tag{1.94}$$

Other parts of the model. The Euler equation, the NKPC, the Taylor rule, and the resource constraint.

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{1.95}$$

$$(\Pi_t - \Pi^*)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi_{t+1} - \Pi^*)\Pi_{t+1} \right] + \frac{\theta_p}{\phi_p} \left(\frac{W_t}{P_t} - 1 \right) \tag{1.96}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{1.97}$$

$$Y_t = H_t = C_t + \frac{\phi_p}{2} (\Pi_t - \Pi^*)^2 C_t + \frac{\phi_w}{2} (\Pi_t^w - \Pi^*)^2 C_t \tag{1.98}$$

Computation. The same solution method is used as the flexible wage model.

Calibration. I calibrate the parameter value for the degree of wage stickiness ϕ_w according to the micro evidence reported by Barattieri et al. (2014). They identify the frequency of individual wage changes to be 23.9% per quarter as the midpoint of the estimates under different plausible assumptions. The estimate implies the slope of the wage Phillips curve to be 0.076.

1.10.5.3 Asymmetric wage and price adjustment cost model

Model equations. The model consists of the Euler equation, the price Phillips curve, the wage Phillips curve, the Taylor rule, and the resource constraint. Derivations follow Aruoba et al. (2017).

$$1 = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right] \tag{1.99}$$

$$\Phi'_p(\Pi^p_t)\Pi_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) \Phi'_p(\Pi^p_{t+1})\Pi_{t+1} \right] + \theta_p \left(\frac{W_t}{P_t} - 1 \right) \tag{1.100}$$

$$where\ \Phi_p \equiv \phi_p \left(\frac{exp(-\psi_p(\Pi^p_t - \Pi^*)) + \psi_p(\Pi^p_t - \Pi^*) - 1}{\psi_p^2} \right)$$

$$\Phi'_w(\Pi^w_t)\Pi^w_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{H_{t+1}}{H_t} \right) \Phi'_w(\Pi^w_{t+1})\Pi^w_{t+1} \right] + \theta_w \left(\frac{H^\eta_t}{C^{-\sigma}_t} - \frac{W_t}{P_t} \right) \tag{1.101}$$

$$where\ \Phi_w \equiv \phi_w \left(\frac{exp(-\psi_w(\Pi^w_t - \Pi^*)) + \psi_w(\Pi^w_t - \Pi^*) - 1}{\psi_w^2} \right)$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{1.102}$$

$$Y_t = H_t = C_t + \Phi_p(\Pi^p_t)C_t + \Phi_w(\Pi^w_t)C_t \tag{1.103}$$

where ϕ and ψ govern the slope and the curvature of the Phillips curve, respectively.

Computation. Following Aruoba et al. (2017), I use the second order perturbation method around the steady state to solve the model.

Calibration. The parameter values for adjustment cost functions are set according to the posterior mean of Aruoba et al. (2017) for the sample of 1960Q1-2007Q4. Other parameters are identical to the baseline model.

Table 1.12: Calibrated parameter values in an asymmetric wage and price adjustment cost model

ϕ_p (ξ_p)	ψ_p	ϕ_w (ξ_w)	ψ_w
450 (0.87)	150	*28.1 (0.57)*	67.4

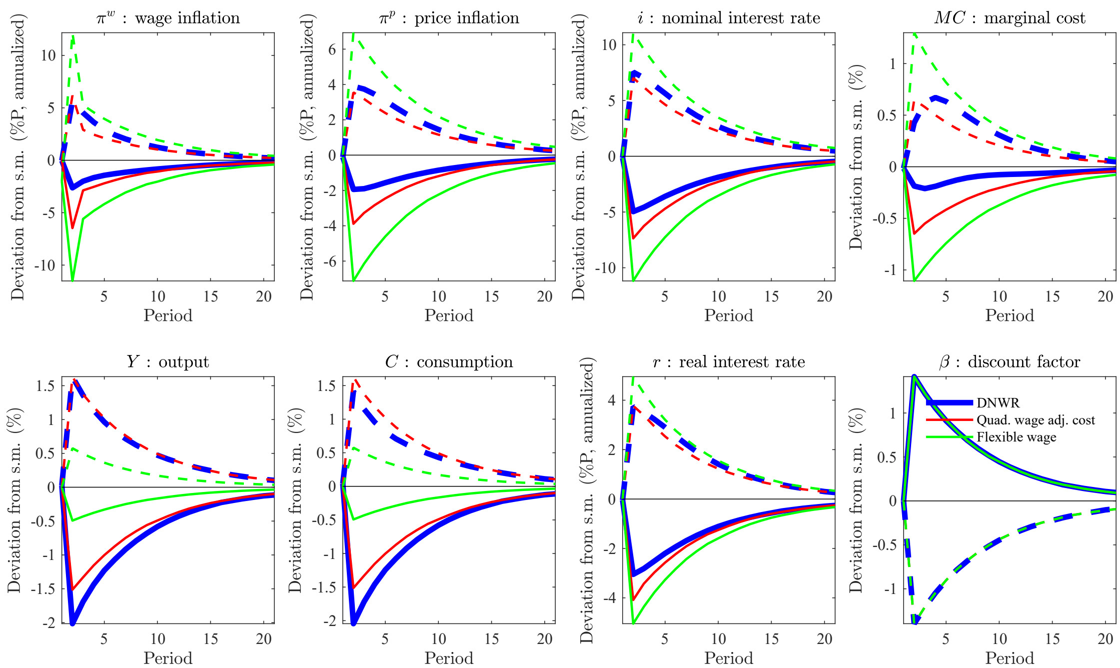
Notes: ξ is the Calvo parameter corresponding to ϕ with $\theta/\phi = (1-\xi)(1-\beta\xi)/\xi$. Italic values are the author's calculation based on Aruoba et al. (2017). For example, the posterior mean of Aruoba et al. (2017) for the slope of price Phillips curve is 0.02. In our model, $0.02 = \theta_p/\phi_p$ and $\theta_p = 9$ implies $\phi_p = 450$.

1.10.5.4 Numerical results

1.10.5.5 Comparison of GIR

Figure 1.17 presents the GIR to discount factor shocks in the flexible wage model and the quadratic wage adjustment cost model as well as our baseline model with DNWR.

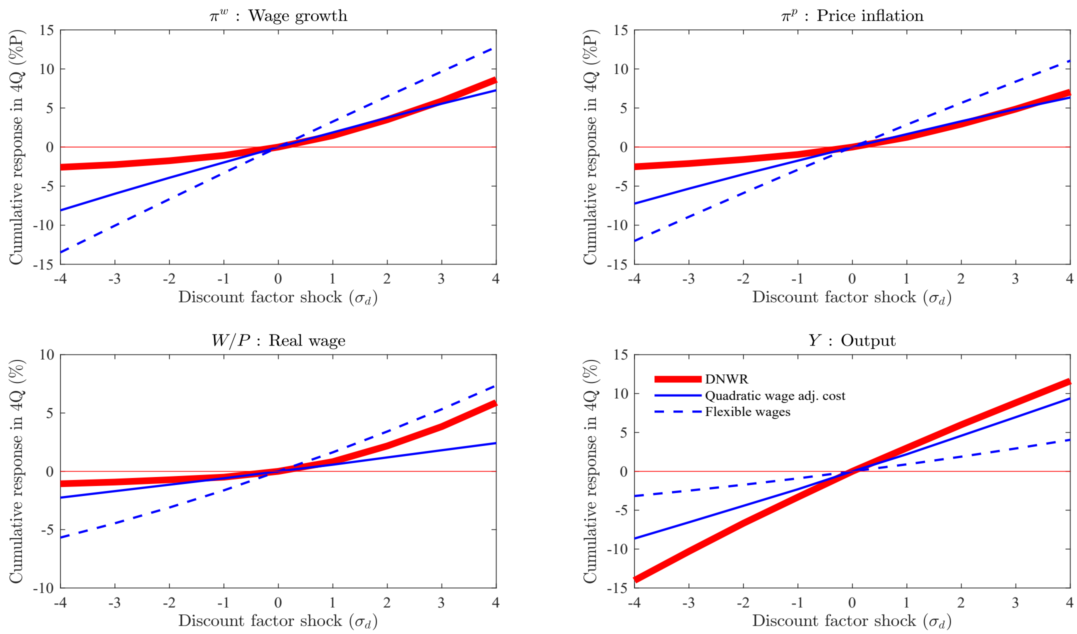
Figure 1.17: GIR in different models



1.10.5.6 Comparison of non-linearity

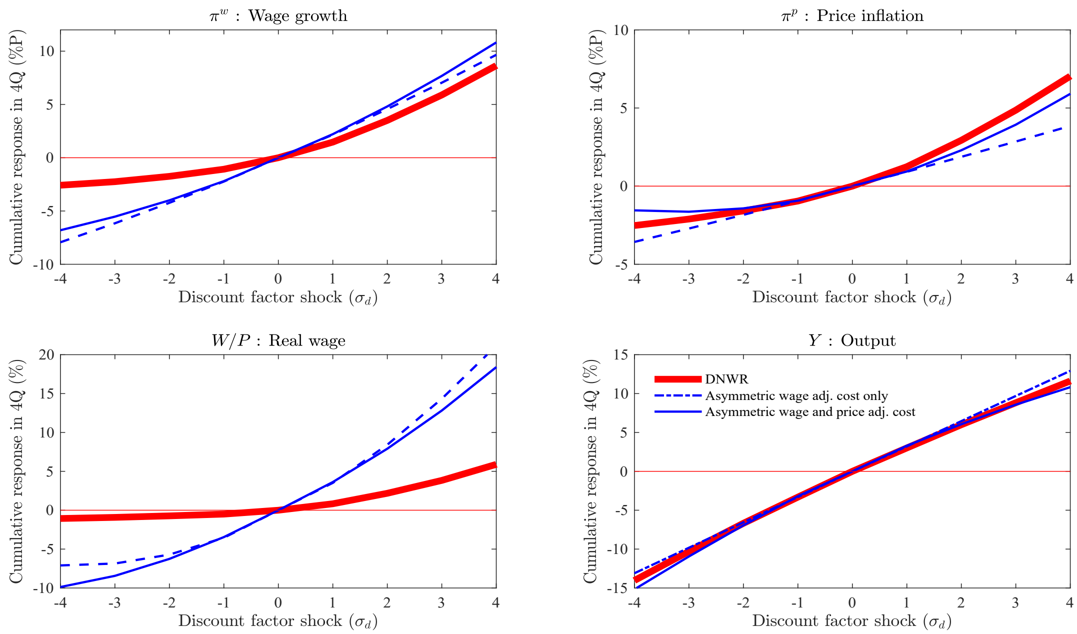
To compare the non-linearity in terms of the responses to exogenous shocks in each model, Figure 1.18 and Figure 1.19 display the cumulative responses of the selected aggregate variables in the initial 4 quarters after different size and direction of discount factor shocks.

Figure 1.18: Comparison of non-linearity (1)



Notes: Each panel displays the cumulative responses in the initial 4 quarters after discount factor shocks. For the asymmetric wage adjustment cost model, the parameter for the asymmetry of price adjustment cost ψ_p is set at zero.

Figure 1.19: Comparison of non-linearity (2)

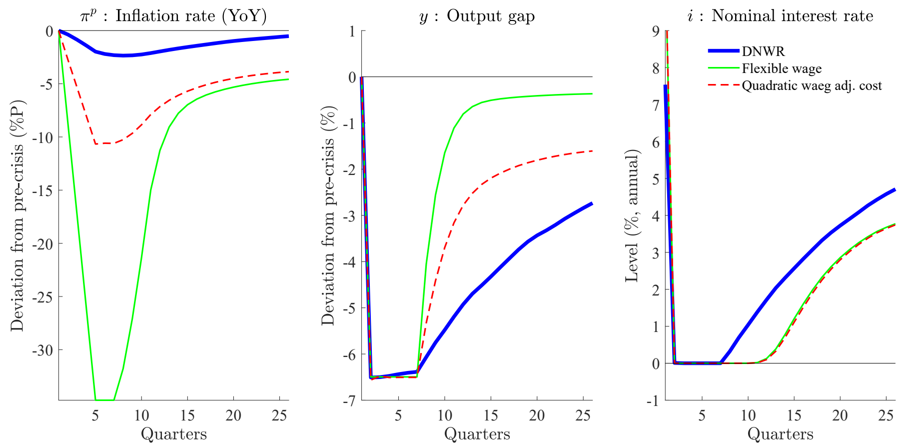


Notes: Each panel displays the cumulative responses in the initial 4 quarters after discount factor shocks. For the asymmetric wage adjustment cost model, the parameter for the asymmetry of price adjustment cost ψ_p is set at zero.

1.10.5.7 Comparison of counterfactual for the Great Recession

Figure 1.20 shows the the counterfactual analysis of the Great Recession in each model. The size of an exogenous shock is calibrated to match the drop of the output gap in the data.

Figure 1.20: Counterfactual in different models



Chapter 2

Optimal Monetary Policy Rule in a Heterogeneous Agent Model with Nominal Rigidities

2.1 Introduction

The heterogeneous agent (HA) model has attracted a lot of attention of researchers in recent years. To this end, Heathcote et al. (2009), who review the literature on HA models with incomplete markets, point out that one of the main advantages of HA models is that the models answer welfare questions in realistic situations. They argue that the welfare consequences of HA models can differ substantially from those of representative agent (RA) models since aggregate fluctuations do not necessarily have symmetric effects on each agent, and they therefore stress that explorations of HA models can deliver rich policy implications.

In the analysis of monetary policy, the New Keynesian literature contains tremendous efforts to build a stylized model that formulates the laws of motion of aggregate variables while abstracting heterogeneity among agents in both positive and normative analysis (Clarida et al. (1999), Christiano et al. (2005), etc.). However, recent studies argue that heterogeneity can have significant effects on the transmission mechanism of monetary policy. For example, Kaplan et al. (2018) find that asset distribution changes how the income and the substitution effect of monetary policy work in a New Keynesian model with incomplete markets. A natural subsequent question is the welfare consequences and the policy implications of HA models for monetary policy analysis, which is the main theme of this paper.

In this paper, I investigate the optimal monetary policy rule in a HA New Keynesian model. Specifically, I focus on the heterogeneity of individual workers' wages that arises from downward nominal wage rigidities (DNWR) and the resulting inefficient allocation in the labor market. Although there are considerable variations in the dimensions of heterogeneity in reality, I set focus on these aspect for several reasons.

First, DNWR is a robust feature in data. Numerous studies test for the presence of DNWR in individual wage data. Moreover, recent micro evidence suggests severe DNWR after the Great Recession (Daly and Hobijn (2014), Fallick et al. (2016)). Motivated by these observations, subsequent studies investigate the significance of DNWR in accounting for aggregate dynamics, including the high unemployment rate after the Great Recession (Schmitt-Grohé and Uribe (2016)) and secular stagnation (Eggertsson et al. (2017)). In this paper, on the other hand, I contribute to the literature by exploring a normative aspect of DNWR.

Second, DNWR naturally introduces heterogeneity among workers' wages that cannot be attributed to aggregate variables. In this regard, it is worth pointing out that most studies in the New Keynesian literature that investigate nominal wage (price) rigidities assume monopolistic competitors that determine their individual wages (prices), and these models therefore inherently embed heterogeneity among agents. However, in the staggered contract model of Calvo (1983), which is widely used in the literature, for example, aggregate variables are sufficient statistics to measure social welfare to the second-order, because it is possible to summarize cross-sectional variations in the variance of aggregate inflation (Rotemberg and Woodford (1997), Erceg et al. (2000), etc.).[1] In contrast, I show that, once I take DNWR into account, due to the state-dependent nature of the constraint, the cross-sectional distribution of wages matters for social welfare, even being conditional on aggregate variables.

Third, previous studies do not address the welfare consequences of the heterogeneous individual wages and the optimal monetary policy in the setting well. In fact, there are only a few studies that examine optimal monetary policy in HA models, and these studies focus on heterogeneous asset holdings (Lippi et al. (2015), Nuño and Thomas (2017)) or heterogeneous firm productivity (Adam and Weber (2017), Blanco (2018)). It is also noteworthy that the existing literature on the optimal monetary policy in a model with DNWR uses RA models (Kim and Ruge-Murcia (2009),

[1]The adjustment cost model, such as Rotemberg (1982), is another popular model in the literature. In that class of model, it is a convention to focus on the symmetric equilibrium where each agent behaves identically.

Coibion et al. (2012), Carlsson and Westermark (2016)).

For the analysis in this paper, I use the New Keynesian dynamic stochastic general equilibrium (DSGE) model developed in Chapter 1.The model embeds heterogeneous workers whose wages might be subject to the DNWR constraint. The other parts of the model shares many features with the existing New Keynesian literature, such as monopolistically competitive firms that set their prices with quadratic adjustment cost *á la* Rotemberg (1982), and households who share consumption across agents to make the saving-consumption decision. To analyze the realistic policy trade-off, I consider both demand- and supply-side shocks as sources of aggregate fluctuations, though Mineyama (2018) focuses on the demand side shock to study the Great Recession. To solve the model, I apply a modified version of the Krusell-Smith algorithm, and calibrate the model to match various moments of U.S. macro and micro data.

In terms of monetary policy, I adopt the idea of simple and implementable monetary policy rules, as Schmitt-Grohé and Uribe (2007) suggest. To be precise, I restrict the attention to simple monetary policy rules in which the central bank sets the nominal interest rate in response to observable macroeconomic variables. One prominent example is the Taylor (1993) rule, i.e., an interest rate feedback rule that reacts to measures of inflation and output. On the other hand, implementability requires monetary policy rules to deliver the determinacy of the rational expectation equilibrium. Assuming such a stylized monetary policy rule, I explore the optimal responsiveness to each macroeconomic variable to maximize social welfare.

In this regard, another possible subject for study is the solution to the Ramsey problem, which delivers optimal monetary policy as a function of the state variables of the model. In my HA setting, however, state variables include the cross-sectional distribution of wages, which is nearly impossible for the central bank to observe in a timely manner in practice. On the other hand, the only information necessary to conduct monetary policy under simple and implementable rules is aggregate variables. Therefore, such policy rules are justifiable as a feasible solution to an economic problem that the central bank faces in reality. Moreover, Schmitt-Grohé and Uribe (2007) argue that simple policy rules have an advantage in that they can be easily explained to the public.

I also consider the zero lower bound (ZLB) of the nominal interest rate. The literature contains extensive studies on the welfare cost of the ZLB and remedies for it. However, as Coibion et al. (2012) discuss, the welfare implications of the ZLB might differ considerably depending on the setting of each model. To this end, I revisit the consequences of the ZLB in my model with a particular focus on how the HA setting affects the quantitative results.

My main findings are as follows. First, the welfare cost of DNWR in the HA model is much larger than in the RA model. Under the calibrated Taylor rule that replicates the Federal Reserve's actual policy responses, the consumption-equivalent welfare loss from the undistorted economy is -2.3 percent in the HA model, whereas it is -0.4 percent once heterogeneity is removed. The result is in stark contrast to previous studies that use RA models to find a moderate welfare cost of DNWR (Kim and Ruge-Murcia (2009), Coibion et al. (2012), Carlsson and Westermark (2016)). The key reason the HA model generates a large welfare cost is that individual workers face much larger uncertainty about their desired wages due to idiosyncratic variations compared to that stemming from aggregate fluctuations. Since the disutility of inefficient wage allocation is convex in the size of the deviations of their wages from their desired levels, the average cross-sectional welfare cost of DNWR is much larger than that for a hypothetical RA.

Second, in a class of the Taylor rule, the optimal monetary policy rule responds strongly to output, whereas responsiveness to inflation plays a minor role in welfare. To obtain the intuition behind this result, it is helpful to see that, in a stylized New Keynesian model without wage rigidity, the strict inflation targeting rule restores efficient allocation by closing the price markup due to nominal price rigidity. In my model, on the other hand, since DNWR creates another markup; that is, the wage markup between real wages and the marginal rate of substitution, the central bank faces a trade-off in closing two markups. Loosely put, responsiveness to output stabilizes the fluctuations of the marginal rate of substitution, and therefore helps to offsetting the wage markup. Quantitatively, I find that social welfare is sensitive to responsiveness to output rather than to inflation, since the welfare loss associated with the wage markup is larger.

Third, monetary policy can improve social welfare by responding more aggressively to a contractionary shock than to an expansionary one. This is due to the asymmetric distortion generated by DNWR. For instance, upon a contractionary demand shock, a larger fraction of workers is constrained by DNWR since their desired wages fall due to the negative income effect. As a consequence, the wage markup sharply increases. On the other hand, a decline in the wage markup is moderate upon an expansionary demand shock because the DNWR constraint does not bind for many workers. To offset the asymmetric fluctuations of the wage markup, monetary policy needs to react more strongly to a contractionary shock.

Fourth, regarding the ZLB, I find that the ZLB enlarges the welfare loss by -0.4 percentage point under the Taylor rule since the lack of monetary policy responses to an exogenous shock at the ZLB amplifies the fluctuations of the economy. In this regard, alternative policy rules, such as forward guidance and price-level targeting, can partly offset the adverse effects of the ZLB by committing to a future low interest rate policy when the economy is constrained at the ZLB.

Fifth, my model has a sharp prediction about the optimal steady-state inflation rate. Positive steady-state inflation benefits the economy because DNWR and the ZLB are less likely to bind under higher inflation, whereas higher inflation generates larger cost of price adjustments. The optimal rate is determined as a consequence of trade-off between the benefit and the cost of inflation. Quantitatively, I find that the optimal rate considerably differs depending on the model specification, including the assumptions about monetary policy rules. However, given each specification, the optimal rate in the HA model is higher than in the RA model by from 0.5 to 5.5 percentage points in the annual rate. The result suggests that the implications of previous studies that uses RA models need to be reconsidered.

2.2 Related literature

There is a long line of literature examining optimal monetary policy in a RA framework with nominal rigidities.[2] Clarida et al. (1999) and Woodford (2003) show that an inflation stabilization policy is welfare-maximizing in an economy with nominal price rigidity. Subsequent studies extend the model in many dimensions, such as wage rigidity (Erceg et al. (2000)), the ZLB (Kato and Nishiyama (2005)), information frictions (Ball et al. (2005)), financial frictions (Cúrdia and Woodford (2016)), house prices (Notarpietro and Siviero (2015)), and so on. On the methodological side, Khan et al. (2003) formulate a method to characterize optimal monetary policy as a solution to the Ramsey problem.

Optimal monetary policy in HA models, on the other hand, is a relatively new research field. Several studies investigate optimal monetary policy in stylized two-sector models (Aoki (2001), Menna and Tirelli (2017), etc.). Recent studies explore models with a continuum of heterogeneous agents. For instance, Nuño and Thomas (2017) solve the Ramsey problem in an incomplete market model with heterogeneous nominal asset and debt holdings. Adam and Weber (2017) and Blanco (2018), on

[2] In a model with money, on the other hand, Friedman (1969) formulates the so-called Friedman rule. He argues that setting nominal interest rates at zero is welfare-maximizing since it minimizes the opportunity cost of holding real money balances as long as the cost of the money supply is zero. The rule is examined by subsequent studies in different settings, such as under distortionary taxes (Chari and Kehoe (1999)) and monopolistic competition (Ireland (2003)). In the HA setting, a recent study by Lippi et al. (2015) build a model with heterogeneous money holdings and studies the distributional effects of money injection.

the other hand, study the consequences of firms' heterogeneous productivity on the optimal steady-state inflation rate. This paper is distinct from these studies in that I investigate the heterogeneity in individual wages and that I focus on simple and implementable monetary policy rules.

Regarding the optimal stead-state inflation rate, a number of studies explore the benefits of positive steady-state inflation with respect to DNWR (Kim and Ruge-Murcia (2009), Carlsson and Westermark (2016)), the ZLB (Schmitt-Grohé and Uribe (2010), Coibion et al. (2012)), shifts of the natural interest rate (Andrade et al. (2018)), different trends in consumption and investment goods prices (Ikeda (2015)), a life-cycle model with capital accumulation (Oda (2016)), and so on. On the other hand, previous studies point out that non-zero steady-state inflation generates sizable welfare loss through nominal price rigidities. In this regard, Ascari et al. (2015) claim that shifting the steady-state inflation from 2 to 4 percent decreases social welfare by around 4 percent due to the increased price dispersion in the staggered contract model of Calvo (1983). As a consequence, most studies conclude that a moderate positive rate, if any, is optimal in the different specifications of the model mentioned above. As for DNWR, Kim and Ruge-Murcia (2009) find that the optimal inflation rate in the Ramsey policy is 0.35 percent in the annual rate, while Carlsson and Westermark (2016) suggest that it is 1.16 percent. Though both studies use a RA model, I investigate how the result changes in the presence of heterogeneity. Indeed, I find that the optimal steady-state inflation is substantially higher once I take into account heterogeneous wages among workers. The finding is related to recent studies by Adam and Weber (2017) and Blanco (2018), who study the heterogeneity on the firm side.

This paper is also related to the growing literature on the link between monetary policy and heterogeneity in a broad sense. On the one hand, previous studies investigate how heterogeneity changes monetary policy transmission (Kaplan et al. (2018), Gornemann et al. (2016), Wong (2018), etc.). For example, Kaplan et al. (2018) develop a HA New Keynesian model with incomplete markets to demonstrate that, in the presence of liquidity constrained households, a monetary policy shock generates real effects through the income effect due to the increase in aggregate income rather than the substitution effect by interest rate changes. On the other hand, some studies examine the distributional effects of monetary policy (Coibion et al. (2017), Beraja et al. (2017), etc.). A pioneering work by Coibion et al. (2017) finds that a contractionary monetary policy shock increases the inequality of income and consumption across households. Beraja et al. (2017) argue that the first round of the Federal ReserveâĂŹs large-scale asset purchase program (QE1) enlarged regional consumption heterogeneity since the regions with more depressed housing prices are less likely to refinance their mortgage loans to benefit from the interest rate cuts.

2.3 Model and Numerical Method

2.3.1 Equilibrium equations

I use the New Keynesian DSGE model developed in Chapter 1.The model embeds individual households who supply differentiated labor service to the production sector and whose nominal wages might be subject to the DNWR constraint. Other parts of the economy share many features of a standard New Keynesian model in the literature, such as the one by Erceg et al. (2000), Ireland (2004), and Christiano et al. (2005). The economy consists of monopolistically competitive firms that set their prices with quadratic adjustment cost *á la* Rotemberg (1982), households who share consumption across agents to make saving-consumption decision, and the central bank that determines the nominal interest rate. The steady-state distortion arising from the monopolistic power of households and firms is eliminated by distortionary labor and production subsidies that are financed by lump-sum tax. There are two sources of aggregate fluctuations: discount factor shocks and technology shocks.

Equilibrium equations are presented as follows. The aggregate economy is formulated as a 3-equation New Keynesian system that consists of the consumption Euler equation (2.1), the New Keynesian Phillips curve (NKPC) (2.2), and the monetary policy rule (2.3), together with the resource constraint (2.4) and the production function (2.5). In a baseline case, I assume the Taylor (1993)-type interest rate feedback rule that responds to inflation and output. More discussion about the specification of the monetary policy rule will be provided shortly. The key ingredient of the model is individual wages that are determined by (2.6)-(2.10). Individual workers are monopolistic competitors to determine their wages given the individual labor demand function. I assume that $1-\alpha$ fraction of workers is subject to the DNWR constraint in each period, whereas the remaining α fraction can change their wages without the constraint. Exogenous processes are governed by the laws of motion in (2.11)-(2.13). Derivation of each equation is presented in Section 1.4.

Aggregate economy.

$$\beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi^p_{t+1}} \right] = 1 \qquad (\textit{Euler equation}) \tag{2.1}$$

$$(\Pi^p_t - \bar{\Pi})\Pi^p_t = \beta_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t} \right) (\Pi^p_{t+1} - \bar{\Pi})\Pi^p_{t+1} \right] + \frac{\theta_p}{\phi_p} \left(\frac{\tilde{W}_t}{Z_t} - 1 \right) \quad (NKPC) \tag{2.2}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\delta_\pi} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \quad (\textit{baseline monetary policy rule}) \tag{2.3}$$

$$Y_t = C_t + \frac{\phi_p}{2}(\Pi^p_t - \bar{\Pi})^2 C_t \qquad (\textit{resource constraint}) \tag{2.4}$$

$$Y_t = Z_t H_t \qquad (\textit{production function}) \tag{2.5}$$

where β_t: discount factor, Z_t: technology, C_t: consumption, Y_t: output, H_t: hours worked, R_t: gross nominal interest rate, $\tilde{W}_t$: real wage index, Π^p_t: gross price inflation rate, $\bar{\Pi}$: reference rate for price adjustment costs, Π^*: target inflation, and Y^*: target output.

Individual wage setting.

$$\psi_t(j) = (\tilde{w}_t(j) - mrs_t(j))\left(C_t^{-\sigma}\frac{\theta_w h_t(j)}{\tilde{w}_t(j)}\right) + \beta_t\mathbb{E}_t[\psi_{t+1}(j)] \tag{2.6}$$

$$\psi_t(j)\left(\tilde{w}_t(j) - \frac{\tilde{w}_{t-1}(j)}{\Pi_t^p}\right) = 0\ , \quad \psi_t(j) \geq 0 \quad w.pr.\ 1-\alpha \quad (wage\ setting\ with\ DNWR) \tag{2.7}$$

or

$$\tilde{w}_t(j) = mrs_t(j) - \beta_t\mathbb{E}_t[\psi_{t+1}(j)]\left(C_t^{-\sigma}\frac{\theta_w h_t(j)}{\tilde{w}_t(j)}\right)^{-1} \quad w.pr.\ \alpha\ (wage\ setting\ without\ DNWR) \tag{2.8}$$

$$where \quad mrs_t(j) \equiv \frac{\chi_t(j)h_t(j)^\eta}{C_t^{-\sigma}} \quad (marginal\ rate\ of\ substitution) \tag{2.9}$$

$$h_t(j) = \left(\frac{\tilde{w}_t(j)}{\tilde{W}_t}\right)^{-\theta_w} H_t \qquad (individual\ labor\ demand) \tag{2.10}$$

with $\chi_t(j)$: idiosyncratic labor disutility, $\tilde{w}_t(j)$: real wage, h_t: hours worked, $\psi_t(j)$: shadow value of the DNWR constraint of household j.

Exogenous processes.

$$log(\beta_t) = (1-\rho_\beta)log(\bar{\beta}) + \rho_\beta log(\beta_{t-1}) + \epsilon_{\beta,t}\ , \quad \epsilon_{\beta,t} \sim i.i.d.N(0,\sigma_\beta^2) \tag{2.11}$$

$$log(Z_t) = \rho_z log(Z_{t-1}) + \epsilon_{z,t}\ , \quad \epsilon_{z,t} \sim i.i.d.N(0,\sigma_z^2) \tag{2.12}$$

$$log(\chi_t(j)) \sim i.i.d.N(-\sigma_\chi^2/2, \sigma_\chi^2). \tag{2.13}$$

2.3.2 Social welfare

I define the social welfare as the unconditional expectation of average household utility:

$$SW \equiv \mathbb{E}\left[\frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\eta}\int_0^1 \chi_t(j)h_t(j)^{1+\eta}dj\right]. \tag{2.14}$$

To obtain the intuition behind the welfare loss of the economy, I derive the second order approximation of the social welfare around the deterministic steady state. Proof is provided in Appendix 2.8.1.

$$\begin{aligned} SW - SW^f \approx & \underbrace{-\frac{1}{2}(\sigma+\eta)\mathbb{V}ar\left(\hat{Y}_t^{gap}\right) - \frac{\phi_p}{2}\mathbb{V}ar\left(log(\Pi_t^p)\right)}_{Aggregate\ variance} \\ & \underbrace{-\frac{1}{2}\left(\eta+\frac{1}{\theta_w}\right)\mathbb{E}\left[\mathbb{V}ar_j\left(\hat{h}_t(j)\right) - \mathbb{V}ar_j\left(\hat{h}_t^f(j)\right)\right]}_{Cross-sectional\ variance} \\ & \underbrace{-\mathbb{E}\left[\mathbb{C}ov_j\left(\hat{\chi}_t(j), \hat{h}_t(j)\right) - \mathbb{C}ov_j\left(\hat{\chi}_t(j), \hat{h}_t^f(j)\right)\right]}_{Cross-sectional\ covariance} \end{aligned} \tag{2.15}$$

where the deterministic steady state is the economy under flexible prices and wages without any exogenous shocks. I define $\hat{a} \equiv log(a) - log(\bar{a})$ with $\bar{a}$ being the value of a in the deterministic steady state, whereas a^f denotes the value of a in economy under the flexible prices and wages. $\mathbb{V}ar_j(\cdot)$ and $\mathbb{C}ov_j(\cdot,\cdot)$ represent the cross-sectional variance and covariance, respectively.

Equation (2.15) gives three main sources of the welfare loss of this economy; 1) the variance of the output gap and price inflation, and 2) the inefficient cross-sectional variance of hours worked, and 3) the inefficient cross-sectional covariance between hours worked and labor disutility shocks. Variations of the output gap and price inflation are sources of welfare loss as are in a standard New Keynesian model with nominal price rigidity. The magnitude depends on the curvature of utility function governed by σ and η, and the cost of price adjustment ϕ_p. Regarding cross-sectional moments, larger variance of hours worked reduces social welfare due to the convexity of the disutility of labor and the production inefficiency stemming from the concavity of the labor aggregates. In fact, if the disutility of labor is linear $\eta = 0$ and each labor service is the perfect substitute $\theta_w = \infty$, then, the cross-sectional variance term vanishes. The other key cross-sectional moment is the covariance between hours worked and labor disutility shocks. Intuitively, workers with high (low) disutility are willing to set higher (lower) wages to decrease (increase) their hours worked through the individual labor demand function. Disability of such adjustment due to DNWR is a source of welfare loss.

It is worth pointing out that the cross-sectional moments in (2.15) cannot be summarized to aggregate variables in the presence of DNWR, while Erceg et al. (2000) demonstrate that, in the Calvo-type staggered wage model, it is represented by the variance of the aggregate wage growth. To see this point, I begin with the fact that the variance of hours worked is linked to that of wages through individual labor demand function:

$$\mathbb{V}ar_j(\hat{h}_t(j)) = \theta_w^2 \mathbb{V}ar_j(log(w_t(j))). \tag{2.16}$$

Moreover, in the Calvo model without idiosyncratic shocks, it can be shown that:

$$\mathbb{E}\left[\mathbb{V}ar_j(log(w_t^{Calvo}(j)))\right] = \frac{\xi_w}{(1-\xi_w)^2}\mathbb{V}ar\left(log(\Pi_t^{w,Calvo})\right) \tag{2.17}$$

where a^{Calvo} denotes the variable a in the Calvo model. ξ_w denotes the fraction of workers without wage changes in each period. In Appendix 2.8.1, I demonstrate that the key assumptions to derive the result above are; 1) the fraction of workers with and without wage changes is constant over time and 2) the reset wage is identical across workers. In my model, on the other hand, the first assumption does not hold due to DNWR, whereas the second is violated by the presence of idiosyncratic shocks. To focus on the effects of DNWR, I suppose that there are no idiosyncratic shocks, i.e. $\chi_t(j) = \bar{\chi}$. Now that the second assumption is satisfied. However, the first assumption still does not hold and I obtain the following expression:

$$\mathbb{E}\Big[\mathbb{V}ar_j(log(w_t(j)))\Big] = \frac{1}{1-\mathbb{E}[\xi_{w,t}]}\mathbb{E}\left[\frac{\xi_{w,t}}{1-\xi_{w,t}}\right]\mathbb{V}ar\Big(log(\Pi_t^w)\Big)$$
$$+\frac{1}{1-\mathbb{E}[\xi_{w,t}]}\left\{\mathbb{C}ov\Big(\xi_{w,t},\mathbb{V}ar_j(log(w_{t-1}(j)))\Big)+\mathbb{C}ov\left(\frac{\xi_{w,t}}{1-\xi_{w,t}},(log(\Pi_t^w))^2\right)+\mathbb{E}\Big[C\Big]\right\} \tag{2.18}$$

where C includes several cross-sectional moments. Details are provided in Appendix 2.8.1. Several differences between (2.17) and (2.18) are noteworthy. First of all, the fraction of workers without wage changes $\xi_{w,t}$ in (2.18) is time-varying in the presence of DNWR. Therefore, the coefficient of the variance of the aggregate wage growth in (2.18) does not necessarily coincide with that of the Calvo model in (2.17). More importantly, even being conditional on the aggregate wage growth, cross-sectional moments that appear in the second term of the RHS in (2.18) matter for social welfare. For example, the fraction of workers with and without wage changes depends on the cross-sectional wage distribution in the previous period and the current wage growth. Indeed, the sign of the covariance terms in (2.18) is ambiguous.

2.3.3 Numerical method

I use the modified Krusell-Smith algorithm developed in Chapter 1 to solve the model numerically. The details are presented in Appendix 1.10.2.

2.3.4 Calibration

I calibrate the model to U.S. macro and micro data. The time frequency is quarterly. The externally fixed parameters are listed in Panel (A) of Table 2.1. The value of the average discount factor $\bar{\beta}$ and that of the target inflation rate Π^* in the Taylor rule (2.3) correspond to the annual real interest rate of 2 percent and the annual price inflation rate of 2 percent, respectively. I set the reference inflation rate for price adjustment costs at $\bar{\Pi} = 1$ to take into account the welfare cost of the steady-state inflation. I fix the target output in (2.3) at $Y^* = 1$. The relative risk aversion of households σ is set at 2 and the inverse of the Frisch labor supply elasticity η is at 0.25, which are in line with the literature. I follow Coibion et al. (2012) to choose the elasticity of substitution across individual goods $\theta_p = 7$. The value implies the steady-state price markup is 17 percent, which is broadly consistent with the empirical literature such as Basu and Fernald (1997). I use the same value for the elasticity of substitution across individual labor service θ_w. The value of the degree of price stickiness ϕ_p is calibrated according to the frequency of individual price changes reported by Nakamura and Steinsson (2008). They find that the median frequency excluding temporary sales is 11-13 percent per month, which implies the slope of the

NKPC is around 0.20 and the corresponding parameter value is $\phi_p = 35$ in my model.[3] The parameter value implies that the adjustment cost of 1 (5) percentage price change is 0.175 (4.375) percent of consumption.[4] Notice that the nominal price rigidity generates welfare loss through changing the dynamics of the model as a consequence of general equilibrium as well as the direct cost of price adjustments. In this regard, I demonstrate that, in Section 2.4.1, the welfare loss of the calibrated model without DNWR is in line with that of corresponding models in the existing literature.

The parameters regarding cross-sectional wage distribution, that is, the fraction of workers without being subject to the DNWR α and the standard deviation of labor disutility shocks σ_χ, are calibrated to match the empirical distribution in U.S. data. Specifically, I choose the parameter values to minimize the quadratic distance between the moments of the stationary distribution of individual wage changes in the model and the target moments in data by using a grid search method. The target moments and the calibrated parameter values are listed in Panel (B) of Table 2.1. The definition of stationary equilibrium is provided in Appendix 1.10.5.

For parameterization of aggregate exogenous processes, the AR(1) coefficient of technology ρ_z and the standard deviation of innovations to it σ_z are set according to Fernández-Villaverde et al. (2015). For those of discount factor shocks, I first set the policy parameters in the Taylor rule (2.3) at $\delta_p = 1.50$ and $\delta_y = 0.25$ following

[3]The slope of the NKPC in my model is given by θ_p/ϕ_p in (2.2). $\theta_p/\phi_p = 0.20$ leads to $\phi_p = 7/0.2 = 35$.

[4]The consumption loss of 1 percent inflation is calculated as follows. Then, $\frac{\phi_p}{2}(\Pi_t - \bar{\Pi})^2 * 100 = 35/2 * 0.01^2 * 100 = 0.175(\%)$.

Table 2.1: Baseline calibration

Panel (A): Fixed parameters

Description	Symbol	Value	Target/Source
Average discount factor	$\bar{\beta}$	0.995	S.S. real interest rate = 2.0% (annual)
Relative risk aversion	σ	2.00	IES = 0.5
Inverse of Frisch labor supply elasticity	η	0.25	King and Rebelo (1999)
Labor demand elasticity	θ_w	7.00	S.S. markup = 16.7%
Goods demand elasticity	θ_p	7.00	Coibion et al. (2012)
Price adjustment cost	ϕ_p	35.0	Slope of NKPC = 0.20
(Corresponding Calvo parameter)	-	(0.64)	Nakamura and Steinsson (2008)
Target inflation rate	Π^*	1.005	S.S. inflation rate = 2.0% (annual)
Target output	Y^*	1.000	Externally fixed

Panel (B): Parameters for cross-sectional wage distribution

Parameter	Symbol	Value	Target/Source
Fraction of workers without being subject to the DNWR constraint	α	0.0600	Frequency of wage changes=0.266; Barattieri et al. (2014)
S.D. of idiosyncratic labor disutility shock	σ_χ	0.1360	S.D. of wage changes (annual)=0.108; Fallick et al. (2016)

Panel (C): Parameters for aggregate exogenous processes

Parameter	Symbol	Value	Target/Source
AR(1) coefficient of technology	ρ_z	0.900	Fernández-Villaverde et al. (2015)
S.D. of innovations to technology	σ_z	0.0025	same as above
AR(1) coefficient of discount factor	ρ_β	0.865	First-order autocorr. of output=0.85
S.D. of innovations to discount factor	σ_β	0.0053	S.D. of output=1.55%

Notes: Targets are the HP-filtered real GDP from 1955Q1 to 2007Q4. For calibration, policy parameters are set at $\delta_p = 1.50$ and $\delta_y = 0.25$ following Fernández-Villaverde et al. (2015).

Fernández-Villaverde et al. (2015). These parameter values are intended to replicate the Federal Reserve's actual policy reactions. Then, the parameters ρ_β and σ_β are calibrated to match the persistence and the standard deviation of the real GDP in the post-war U.S. data. The calibrated parameters are listed in Panel (C) of Table 2.1.

2.4 Numerical Results

2.4.1 Social welfare under calibrated Taylor rule

This subsection computes the social welfare under the calibrate Taylor rule. Since the social welfare of the economy (2.14) does not have the exact closed form solution, I numerically compute it by simulating the economy with a large number of households for a long period of time and taking the unconditional mean.

For this analysis, I assume the following specification for the monetary policy rule:

$$R_t = R^* \left(\frac{\Pi_t^p}{\Pi^*} \right)^{\delta_p} \left(\frac{Y_t}{Y^*} \right)^{\delta_y} \tag{2.19}$$

where $R^* \equiv \Pi^*/\bar{\beta}$ is the target gross nominal interest rate. The policy rule (2.19) corresponds to the Taylor (1993) rule, in which the central bank sets the nominal interest rate in response to inflation and output. I calibrate the policy parameters $\delta_p = 1.50$ and $\delta_y = 0.25$. These parameter values are standard ones in the literature to replicate the Federal Reserve's actual policy responses.

Table 2.2 reports the social welfare and its relevant moments under the calibrated Taylor rule. The consumption-equivalent welfare loss from the undistorted economy, which is the one under flexible prices and wages, is -2.31 percent in my baseline model in column (1). To see the contribution of aggregate and idiosyncratic shocks, I mute one of them in column (2) and (3). When aggregate shocks are muted in column (2), the economy still bears fairly large welfare loss of -1.83 percent. On the other hand, in column (3), once idiosyncratic labor disutility shocks are muted, the welfare loss is reduced to -0.36 percent. Notice that the model without idiosyncratic shocks roughly correspond to a RA model with DNWR in the literature, although the model still has cross-sectional dispersion due to the random probability of being subject to the DNWR constraint. These results suggest that, even without the welfare loss from the variations of aggregate variables, idiosyncratic labor disutility shocks to individual workers lead to considerable inefficiency in the labor market since DNWR prevents the adjustment of individual wages.

The economy without DNWR in column (4) corresponds to the 3-equation New Keynesian model that consists of the Euler equation, the NKPC, and the Taylor rule. Although idiosyncratic labor disutility shocks still leads to cross-sectional dispersion, they do not generate welfare loss because individual labor is efficiently allocated. The magnitude of welfare loss, -0.46 percent, in the economy is in line with previous studies. For example, Nakamura et al. (Forthcoming) find that the welfare loss in their calibrated models ranges from -0.4 percent in the menu cost model to -1.0 percent in the Calvo model. Interestingly, comparing column (3) and (4), adding DNWR without idiosyncratic shocks to the stylized New Keynesian model is welfare improving. Although the result might be counterintuitive, it can be the case because DNWR reduces the variance of inflation by adding persistence of firms' marginal cost. It is indeed consistent with the findings of previous studies such as Carlsson and Westermark (2008) and Coibion et al. (2012) who find that DNWR is welfare improving in RA models.[5]

[5]Coibion et al. (2012), who build a RA model with the ZLB and DNWR, find that DNWR improves social welfare through reducing the likelihood of hitting the ZLB as well as decreasing the inflation variations.

Table 2.2: Social welfare under calibrated Taylor rule

	(1)	(2)	(3)	(4)
		with DNWR		w/o DNWR
	Baseline	w/o aggregate shocks	w/o idiosyncratic shocks	
Social welfare:				
C.E. (%)	-2.31	-1.83	-0.36	-0.46
$\sigma(Y^{gap})$	1.55	0.00	1.39	0.58
$\sigma(\Pi^p)$	0.61	0.00	0.90	1.33
$\sigma(h_j)$	0.12	0.11	0.08	0.35
$\rho(h_j, \chi_j)$	-0.37	-0.50	-	-1.00

Notes: $C.E.$ denotes the consumption-equivalent welfare loss from the economy under flexible prices and wages. $\mu(\cdot)$, $\sigma(\cdot)$, and $\rho(\cdot,\cdot)$ represent the mean, the standard deviation, and the correlation, respectively. In column (2), aggregate shocks are set constant, i.e., $\beta_t = \bar{\beta}$ and $Z_t = 1$. The economy coincides with the stationary equilibrium. In column (3), idiosyncratic shocks are muted, i.e., $\chi_t(j) = 1$. In column (4), DNWR is removed by setting $\alpha = 1$.

2.4.2 Optimal Taylor rule

This subsection turns to investigation of optimal monetary policy rule. I choose the policy parameters δ_p and δ_y in (2.19) to maximize social welfare by adopting the grid search method used by Schmitt-Grohé and Uribe (2007). Specifically, I set the equally spaced grid points in the parameter space $\delta_p \in [0.0, 3.5]$ and $\delta_y \in [0.0, 3.5]$ with the width of each interval 0.25. The range of the parameter space is slightly wider than the one used by Schmitt-Grohé and Uribe (2007) to guarantee an interior solution in the baseline model.[6] I search for the maximum of the social welfare (2.14) on the grids points. For this analysis, the target inflation rate is fixed at $\Pi^* = 1.005$, which indicates 2 percent steady-state inflation in the annual rate.

Table 2.3 presents the optimized parameter values and the welfare measures. The table also reports the outcomes of the calibrated Taylor rule in column (1) for a

[6]Schmitt-Grohé and Uribe (2007) restrict the parameter space within [0.0,3.0].

Table 2.3: Optimal Taylor rule

	(1)	(2)	(3)	(4)
	Calibration	Optimized rule		
		Π^p targeting	Y targeting	Hybrid targeting
δ_p	1.50	1.50	-	1.50
δ_y	0.25	-	2.75	3.00
Social welfare:				
C.E. (%)	-2.31	-2.72	-2.12	-2.11
$\sigma(Y^{gap})$	1.54	2.45	0.58	0.42
$\sigma(\Pi^p)$	0.61	0.78	0.35	0.29
$\sigma(h_j)$	0.12	0.17	0.11	0.11
$\rho(h_j, \chi_j)$	-0.37	-0.22	-0.46	-0.48

Notes: For each specification in column (2)-(4), the parameter values of δ_p and δ_y are chosen to maximize social welfare on the equally spaced grid points in the parameter space of $\delta_p \in [0.0, 3.5]$ and $\delta_y \in [0.0, 3.5]$ with the width of each interval 0.25. *C.E.* denotes the consumption-equivalent welfare loss from the economy under flexible prices and wages. $\sigma(\cdot)$ and $\rho(\cdot, \cdot)$ represent the standard deviation, and the correlation, respectively.

comparison purpose. To begin with, in column (2) and (3), I focus on single variable targeting rules by setting one of the parameters δ_p and δ_y at zero. In column (2), the optimized inflation targeting rule deteriorates social welfare compared to the calibrated Taylor rule in column (1). Without responsiveness to output, the inflation targeting rule generates a large variations of the output gap, and that also leads to an inefficient allocation of labor. In column (3), on the other hand, the output targeting rule improves social welfare from the calibrated rule. The strong responsiveness to output decreases the variance of both the output gap and inflation. The stabilized aggregate variables improves the cross-sectional efficiency in the labor market as well. When I allow for responsiveness to both of inflation and output in column (4), the hybrid targeting rule does not improve a lot from output targeting rule. Each welfare relevant moment is fairly similar in column (3) and (4).

Figure 2.1 displays the optimal and implementable policies on the parameter space.

In Panel (A), the filled circle denotes the optimal pair of the parameters (δ_p, δ_y), while unfilled ones indicate the ”nearly-optimal” pairs that are sub-optimal but whose welfare loss is close to that of the optimal one.[7] The figure suggests that, as long as responsiveness to output is high enough, different responsiveness to inflation does not change social welfare significantly. Regarding implementability, the implementable policies that deliver the determinacy of equilibrium are shown with crosses in the figure. When responsiveness to inflation and output is not strong enough, the economy suffers from indeterminacy of equilibrium, as is the case in a simple New Keynesian model without DNWR. Moreover, too strong responsiveness to inflation without reacting to output can lead to indeterminacy as well. Intuitively, in my model, the relationship between real quantities and price variables is loosened since DNWR creates a wedge between them. In particular, upon a negative technology shock, strong responsiveness to inflation reduces consumption by raising the real interest rate. However, the policy does not necessarily stabilize inflation since real wage, and therefore marginal cost, does not decline in response to the drop of real quantities in the presence of DNWR. Consequently, such strong responsiveness to inflation can make the economy unstable.[8]

For comparison, Panel (B) shows the results in the model without idiosyncratic

[7] I define the nearly-optimal policy as the pairs of the parameters whose welfare loss is within 0.03 percentage point of that of the optimal policy.

[8] In contrast, in a simple New Keynesian model without DNWR, the labor market equilibrium formulates a one-to-one relationship between the output gap and marginal cost. As long as the relationship holds, responsiveness to inflation stabilizes both of real quantities and price variables, and delivers the determinacy of equilibrium.

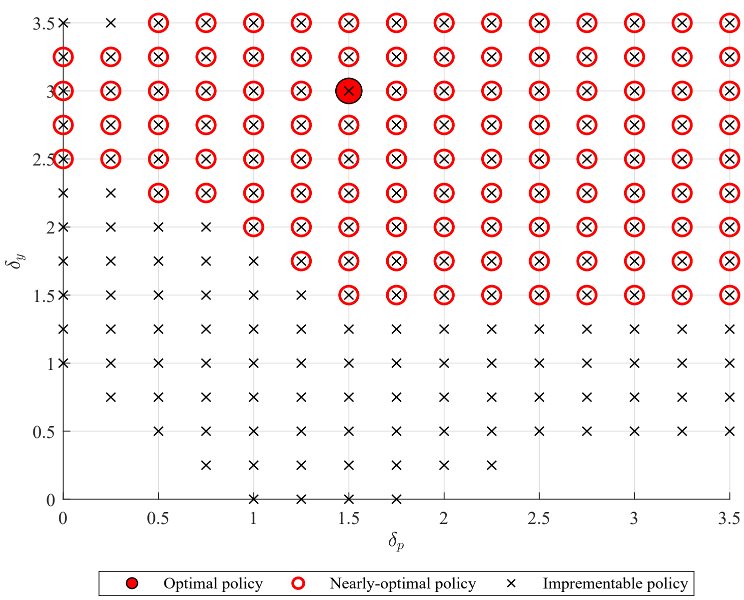
shocks. The figure shares much areas of the implementable policy and the nearly-optimal policy with those in Panel (A). However, without idiosyncratic shocks, the output targeting rule is strictly suboptimal. It is also worth pointing out that the optimal point without idiosyncratic shocks in Panel (B) is strictly sub-optimal in the baseline model in Panel (A). These results imply that the optimal monetary policy analysis without taking into account idiosyncratic shocks can be misleading because the analysis understates the inefficient cross-sectional allocation in the labor market.

Panel (C) presents those for the model without DNWR. Since the welfare loss in the model only stems from the nominal price rigidity, strong responsiveness to inflation improves social welfare. The result is consistent with the findings of previous studies showing that an inflation stabilization policy is welfare-maximizing in a model with nominal price rigidities, such as Clarida et al. (1999) and Schmitt-Grohé and Uribe (2007).[9]

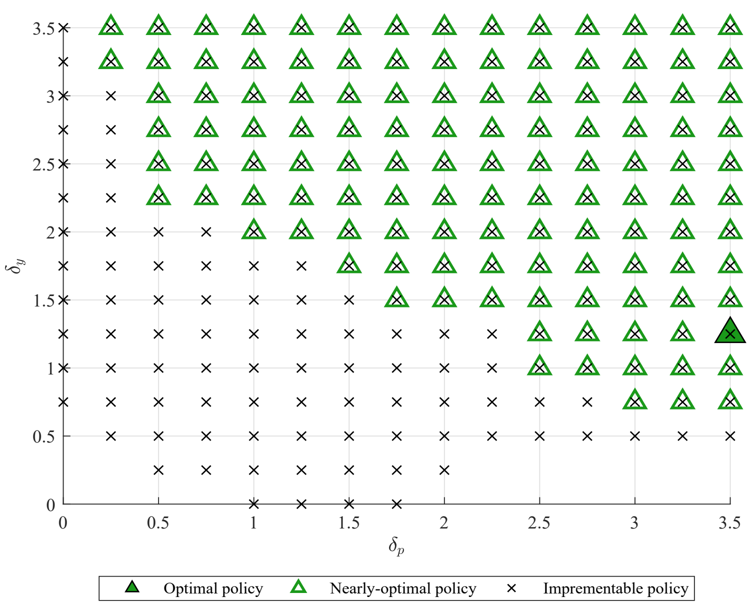
[9] I find that, even without DNWR, small responsiveness to output is welfare improving. Indeed, the optimal response is $\delta_p = 3.50$ and $\delta_y = 0.25$ in Panel (B). In this regard, since the model is solved globally and the social welfare is computed without the Taylor approximation, the strict inflation targeting does not necessarily maximize social welfare. In the numerical analysis, I find that under a non-zero steady-state inflation rate, both inflation and the output gap tend to become more volatile than in the economy around the zero steady-state inflation, and responsiveness to output helps to stabilize them.

Figure 2.1: Optimal Taylor rule

Panel (A): Baseline model with DNWR

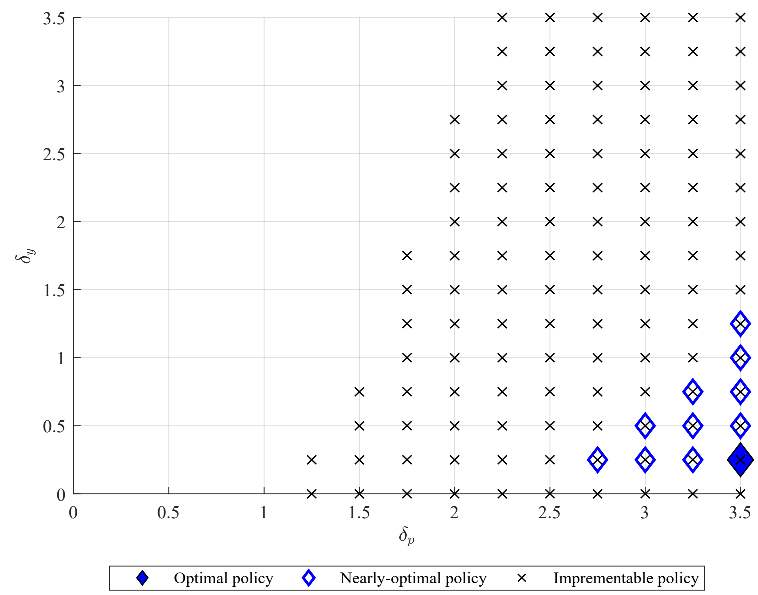


Panel (B): Model without idiosyncratic shock



Notes: The optimal policy is the pair of the parameters (δ_p, δ_y) that delivers the highest social welfare in the parameter space. The nearly-optimal policy is the ones whose welfare loss is within 0.03 percentage point of that of the optimal policy. The implementable policy is the ones that delivers the determinacy of equilibrium.

Panel (C): Model without DNWR



To understand the mechanism behind the results above, it is important to notice that the baseline model has two sources of welfare loss. The first source is nominal price rigidity, that is, the price adjustment cost of firms. It introduces the price markup μ^p between firms' real marginal costs and their relative prices:

$$\frac{p_t(i)}{P_t} = \mu_t^p(i) MC_t(i) \tag{2.20}$$

where i denotes each firm. On the other hand, DNWR is the second source of welfare loss. It creates the wage markup μ^w between workers' real wages and their marginal rate of substitution:

$$\frac{w_t(j)}{P_t} = \mu_t^w(j) MRS_t(j). \tag{2.21}$$

where j denotes each worker. As Erceg et al. (2000) discuss, since the central bank has only one policy instrument, that is, the nominal interest rate, they face trade-off in closing two markups.[10] Loosely speaking, the inflation targeting rule reduces the fluctuations of the price markup by stabilizing inflation. On the other hand, the output targeting rule offsets the fluctuations of output, and that in turn stabilizes the marginal rate of substitution, because it is determined by consumption and hours worked, which are linked to output in general equilibrium. Hence, the output targeting rule helps to reduce the fluctuations of the wage markup. Quantitatively, since DNWR together with idiosyncratic shocks generates sizable welfare loss through the fluctuations of the wage markup, the optimal monetary policy rule needs to put a sufficiently large weight on output rather than on inflation.

2.4.3 Alternative policy rules

This subsection explores alternative policy rules to the Taylor rule investigated in the previous subsection. Specifically, I investigate the performance of asymmetric Taylor rule and wage growth and employment targeting rule.

2.4.3.1 Asymmetric Taylor rule

I consider the following asymmetric Taylor rule:

$$R_t = R^* \left(\frac{\Pi_t^p}{\Pi^*} \right)^{\delta_{p,t}} \left(\frac{Y_t}{Y^*} \right)^{\delta_{y,t}} \tag{2.22}$$

$$\begin{aligned} \textit{where} \quad \delta_{p,t} &\equiv \mathbf{1}_{\Pi_t^p \geq \Pi^*} \ \delta_p^+ + \left(1 - \mathbf{1}_{\Pi_t^p \geq \Pi^*}\right) \ \delta_p^- \\ \delta_{y,t} &\equiv \mathbf{1}_{Y_t \geq Y^*} \ \delta_y^+ + \left(1 - \mathbf{1}_{Y_t \geq Y^*}\right) \ \delta_y^- . \end{aligned}$$

[10] Another way of interpreting this result is that the "divine coincidence" discussed by Blanchard and Gali (2005) does not hold in the presence of the wage markup.

Each coefficient $\delta_{p,t}$ and $\delta_{y,t}$ can take two different values depending on whether the target variable is above or below the target level. The specification is based on the conjecture that monetary policy rule might improve social welfare by responding differently upward and downward since DNWR is an asymmetric constraint.

Table 2.4 reports the optimized parameter values and the resulting welfare measures. The optimized responsiveness are larger to negative deviations of target variables from their target levels than to positive ones in each specification. This result reflects the fact that DNWR binds for more workers upon a contractionary shock than upon an expansionary one, which leads to a larger welfare loss. To address the asymmetry, monetary policy rule needs to respond more aggressively to negative deviations than to positive ones. In fact, the asymmetric Taylor rule delivers higher social welfare than the baseline Taylor rule in Section 2.4.2. For example, in the hybrid targeting rule, the gain of adopting the asymmetric rule is about 0.1 percent in terms of the consumption-equivalent welfare loss.

2.4.3.2 Wage growth and employment targeting rule

I next investigate a targeting rule that responds to wage growth and employment. Since one of the main distortion of the economy arises from wage rigidity, it is natural for monetary policy rule to respond to measures of the labor market. In the real world, the Federal Reserve has their mandate of achieving maximum sustainable employment as well as price stability. Although my model does not explicitly distinguish the employment and unemployment status of workers, since individual hours worked is determined through individual labor demand function, hours worked H_t can be seen

Table 2.4: Optimal asymmetric Taylor rule

	(1)	(2)	(3)
	Π^p targeting	Y targeting	Hybrid targeting
δ_p^+	1.25	-	1.50
δ_p^-	2.50	-	3.00
δ_y^+	-	2.25	0.75
δ_y^-	-	3.00	3.50
Social welfare:			
C.E. (%)	-2.30	-2.11	-2.03
$\sigma(Y^{gap})$	2.60	0.62	0.60
$\sigma(\Pi^p)$	0.69	0.39	0.40
$\sigma(h(j))$	0.13	0.11	0.12
$\rho(\chi(j), h(j))$	-0.46	-0.49	-0.54

Notes: In column (1) and (2), the parameter values are chosen as the global maximum in the parameter space [0.0,3.5] by using the grid search method as in Section 2.4.2. In column (3), on the other hand, due to the high dimensionality of the parameter space, I search for a local maximum around the optimized parameter values in the symmetric targeting rule in Section 2.4.2. *C.E.* denotes the consumption-equivalent welfare loss compared from the economy under flexible prices and wages. $\sigma(\cdot)$ and $\rho(\cdot,\cdot)$ represent the standard deviation and the correlation, respectively.

as a measure of employment. Wage growth Π_t^w is another relevant measure since the tightness of DNWR crucially depends on it. To implement the idea, I define the wage growth and employment targeting rule as follows:

$$R_t = R^* \left(\frac{\Pi_t^w}{\Pi^*}\right)^{\delta_w} \left(\frac{H_t}{H^*}\right)^{\delta_h} \tag{2.23}$$

where δ_w and δ_h govern the responsiveness to each variable. I fix at $H^* = 1$.

Table 2.5 shows the results. Interestingly, the wage growth and employment targeting rule delivers quite similar outcomes to the baseline Taylor rule in Section 2.4.2. To be precise, in single variable targeting rules, the wage growth targeting in column (2) deteriorates social welfare compared to the calibrated rule, and the employment targeting rule in column (3) improves it. The hybrid targeting rule in column (4) has little improvement from the employment targeting rule. Moreover, the magnitude of the welfare loss in the optimized rules is within the range of 0.1 percent from that

Table 2.5: Optimal wage growth and employment targeting rule

	(1)	(2)	(3)	(4)
	Calibration	Optimized rule		
Parameters:		Π^w targeting	H targeting	Hybrid targeting
δ_w	1.50	1.50	-	2.50
δ_h	0.25	-	2.25	2.25
Social welfare:				
C.E. (%)	-2.58	-2.65	-2.15	-2.10
$\sigma(Y^{gap})$	1.52	2.58	0.54	0.38
$\sigma(\Pi^p)$	0.89	0.89	0.23	0.19
$\sigma(h_j)$	0.16	0.16	0.10	0.11
$\rho(h_j, \chi_j)$	-0.24	-0.26	-0.44	-0.48

Notes: For each specification, the parameter values are chosen to maximize social welfare on the equally spaced grid points in the parameter space [0.0,3.5] with the width of each interval 0.25. *C.E.* denotes the consumption-equivalent welfare loss from the economy under flexible prices and wages. $\sigma(\cdot)$ and $\rho(\cdot,\cdot)$ represent the standard deviation and the correlation, respectively.

in the baseline Taylor rule. In this regard, in my model, hours worked co-moves with output except for the fluctuations of technology through the production function, whereas wage growth is defined as the product of price inflation and real wage growth. Therefore, responsiveness to employment roughly corresponds to the output targeting, while responsiveness to wage growth resembles the inflation targeting. These findings imply that, even though a large part of the distortion of the economy arises from the labor market friction, that is, DNWR, responding to measures of the labor market does not deliver significant gains as long as each aggregate measure is tightly linked in general equilibrium.

2.5 ZLB

2.5.1 Policy rules with ZLB

This section introduces the ZLB of the nominal interest rate into the baseline model. In the literature, particularly after the Great Recession when most of the central banks in advanced economies experienced or were threatened by it, the ZLB is regarded as one of essential elements to consider in monetary policy analysis. The constraint simply suggests that the nominal net (gross) interest rate cannot be below zero (one):

$$R_t = \max\left\{R_t^d, 1\right\} \tag{2.24}$$

where R_t^d denotes the desired interest rate that is determined by monetary policy rules.

For monetary policy rules with the ZLB, I consider several variations of alternative rules suggested in the literature as well as the standard Taylor rule.

Standard Taylor rule. The Standard Taylor rule is given by:

$$R_t^d = R^* \left(\frac{\Pi_t^p}{\Pi^*}\right)^{\delta_p} \left(\frac{Y_t}{Y^*}\right)^{\delta_y}. \tag{2.25}$$

History-dependent rule. The Federal Reserve announced a commitment to keep the low interest rate policy when they faced the ZLB after the Great Recession. To take into account the effect of this type of forward guidance, I consider the history-dependent rule proposed by Reifschneider and Williams (2000):

$$R_t^d = R^* \left(\frac{R_{t-1}^d}{R_{t-1}}\right) \left(\frac{\Pi_t^p}{\Pi^*}\right)^{\delta_p} \left(\frac{Y_t}{Y^*}\right)^{\delta_y}. \tag{2.26}$$

Under the history-dependent rule, the central bank keeps track of the past interest rate gap, that is, the difference between the desired interest rate implied by (2.26) R_{t-1}^d and the actual rate R_{t-1}. Therefore, once the nominal interest rate is constrained at the ZLB, the central bank continues a low interest rate policy until the gap is cleared, even if the interest rate implied by the standard Taylor rule becomes positive.

Price-level targeting rule. I also consider the price-level targeting rule. Vestin (2006) shows that the price-level targeting rule generates desirable history dependence, which provides a commitment device for the central bank.[11] Though the main interest of his study is in the monetary policy analysis under discretion, his insight can also be applied to the ZLB. I use the following specification of the rule:

$$R_t^d = R^* \left(\frac{P_t}{P^*}\right)^{\delta_{pl}} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} \tag{2.27}$$

where δ_{pl} governs responsiveness to price level. I normalize P^* such that P_t/P^* represents the deviation of price level from the deterministic path due to the steady-state inflation. It is noteworthy that the nominal GDP targeting rule, which is proposed by Jensen (2002) and recently studied by Garín et al. (2016) and Billi (2017), is a special case of the rule (2.27) where responsiveness to price level and output is set

[11]There are several earlier studies that refer to the price-level targeting rule, such as Wolman (1999) and Blinder (2000).

equal, i.e., $\delta_{pl} = \delta_y$.

Temporary price-level targeting rule. A recent study by Bernanke (2017) proposes the temporary price-level targeting rule. A possible disadvantage of the price-level targeting is that the strong commitment to price level can generate large welfare loss upon a negative supply-side shock, and he argues that the central bank can address the problem by restricting its usage during the ZLB periods. I specify the rule in the following way:

$$R_t^d = \begin{cases} R^* \left(\frac{\Pi_t^p}{\Pi^*}\right)^{\delta_p} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} & if\ R_{t-1} > 1 \\ R^* \left(\frac{P_t}{P^*}\right)^{\delta_{pl}} \left(\frac{Y_t}{Y^*}\right)^{\delta_y} & if\ R_{t-1} = 1. \end{cases} \tag{2.28}$$

Under the temporary price-level targeting rule, as long as the economy is out of the ZLB, the central bank implements the standard Taylor rule. On the other hand, once the economy hits the ZLB, the central bank switches the policy rule to the price-level targeting until the economy exits from it.

2.5.2 Numerical results with the ZLB

I choose the parameter values in each monetary policy rule to maximize social welfare in the same way as in the previous section. Table 2.6 summarizes the optimized parameter values and the welfare measures. The table also reports the mean consumption and the frequency of staying at the ZLB.

Several points are noteworthy in the table. First, under the optimized Taylor rule, the welfare loss in the economy with the ZLB shown in column (2) is larger by -0.38 percent point than the one without the ZLB in column (1). At the ZLB, the effects of an exogenous shock are amplified because of the lack of offsetting monetary policy responses, as is pointed out in the existing literature. My numerical results suggest that the variance of the output gap is 5.6 times larger in the presence of the ZLB, while that of inflation is twice. The ZLB also leads to larger inefficient cross-sectional allocation in the labor market because larger aggregate fluctuations make workers' desired wages more volatile. In addition, it is worth pointing out that the mean consumption is lower by around -0.25 percent in the presence of the ZLB. The binding ZLB implies that the actual nominal interest rate is higher than the level implied by the Taylor rule. Consequently, the monetary policy rule is more contractionary on average, and the higher real interest rate reduces consumption through the Euler equation. Basu and Bundick (2015) call the property as the contractionary bias of the ZLB. Since larger responsiveness to inflation and output enlarges the bias, the optimized parameters with the ZLB are much smaller than those in the rule without it.

Second, the alternative policy rules with the ZLB shown in column (3)-(5) can reduce the adverse effects of the ZLB to achieve the social welfare that is close to that in the economy without the ZLB. This is because the alternative policy rules commit to a future low interest rate policy when the economy hits the ZLB. The commitment reduces the future real interest rates, which affects the current variables through the forward looking nature of the Euler equation.[12] Due to the commitment,

[12]Since the model is solved globally, the precautionary saving of households discounts the effects of the future interest rate changes to some extent. Therefore, the alternative policy rules fail to fully recover the social welfare in the economy without the ZLB. Related discussion is found in the

the frequency of staying at the ZLB is substantially higher in the alternative rules than the Taylor rule.

Third, among the alternative policy rules, the price-level and the temporary price-level targeting rules in column (4) and (5) performs slightly better than the history-dependent rule in column (3). In this regard, both of the history-dependent rule and the price-level targeting rule remove the contractionary bias by fully responding to the fluctuations of the target variables in an unconditional sense. Therefore, both rules almost restore the mean consumption in the Taylor rule without the ZLB. However, the price-level targeting generates more persistence of the nominal interest rate, which helps to reduce the volatility of the economy than the history-dependent rule.[13] On the other hand, I did not find significant differences of welfare outcomes between the price-level targeting rule and the temporary price-level targeting rule.

2.6 Optimal steady-state inflation

This section investigates the optimal steady-state inflation rate. In my model, positive steady-state inflation can benefit the economy because DNWR and the ZLB are less likely to bind under higher inflation. On the other hand, higher inflation generates a larger cost of price adjustments. The optimal rate is determined as a consequence of trade-off between the benefit and cost of inflation.

literature on the forward guidance puzzle, such as Del Negro et al. (2012), McKay et al. (2016b) and Gabaix (2016).

[13] A possible modification to the history-dependent rule is introducing the lagged interest rate term to strengthen the inertia of the nominal interest rate. That is a subject to the future research.

Table 2.6: Optimal monetary policy rules with the ZLB

	(1)	(2)	(3)	(4)	(5)
	w/o ZLB	with ZLB			
	Standard Taylor	Standard Taylor	Alternative rules		
			History-dependent	Price-level targeting	Temporary price-level
δ_p	3.00	1.50	2.50	-	1.50
δ_{pl}	-	-	-	1.00	1.00
δ_y	2.75	0.25	0.25	1.25	1.25
Social welfare: C.E. (%)	-2.11	-2.49	-2.24	-2.14	-2.13
$\sigma(Y^{gap})$	0.37	2.09	1.41	0.76	0.84
$\sigma(\Pi^p)$	0.24	0.46	0.35	0.16	0.21
$\sigma(h_j)$	0.11	0.14	0.11	0.10	0.11
$\rho(h_j, \chi_j)$	-0.47	-0.26	-0.40	-0.46	-0.46
$\mu(C)$	0.9993	0.9968	0.9991	0.9990	0.9988
$Prob.(ZLB)$	0.00	0.139	0.299	0.311	0.321
$Dura.(ZLB)$	-	5.4	8.5	6.7	6.9

Notes: For each specification, the parameter values are chosen to maximize social welfare on the equally spaced grid points in the parameter space [0.0,3.5] with the width of each interval 0.25. $C.E.$ denotes the consumption-equivalent welfare losses compared from the economy under flexible prices and wages. $\mu(\cdot)$, $\sigma(\cdot)$, and $\rho(\cdot,\cdot)$ represent the mean, the standard deviation, and the correlation, respectively. $Prob.(ZLB)$ is the unconditional probabilities of staying at the ZLB, whereas $Dura.(ZLB)$ is the average duration of each ZLB episode.

For the numerical analysis below, I change the target inflation rate in monetary policy rules Π^* from 1.000 (0 percent inflation in the annual rate) to 1.025 (10 percent) with an interval 0.00125 (0.5 percent) to search for the welfare-maximizing one. I consider three monetary policy rules; 1) the Taylor rule without the ZLB (2.19), 2) the Taylor rule with the ZLB (2.25), and 3) the history-dependent rule with the ZLB (2.26). The policy parameters in each policy rule are fixed at $\delta_p = 2.50$ and $\delta_y = 1.50$. These parameter values roughly correspond to those used by Coibion et al. (2012). The reason for using the higher values than the calibration in Section 2.4.1 is to guarantee the determinacy of equilibrium under high steady-state inflation.

Figure 2.2 displays the optimal steady-state inflation rate as a function of the parameter governing the cost of price adjustment ϕ_p. The baseline calibration $\phi_p = 35$ corresponds to the left end of each figure. The figure reports the results for the RA model with DNWR as well as the baseline specification of the HA model.

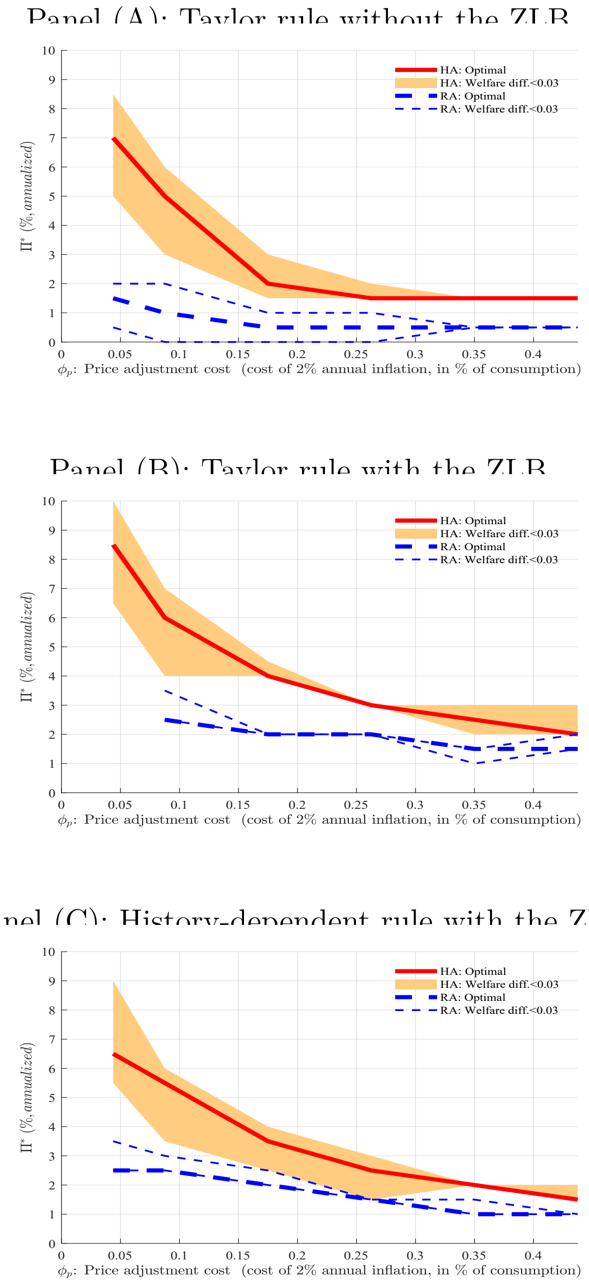
The first thing to note is that the optimal steady-state inflation rate differs considerably depending on the model specification. For example, under the Taylor rule without the ZLB in Panel (A), the optimal rate ranges from 1.5 percent to 7.0 percent for different parameter values of the price adjustment cost. In this regard, though I determine the parameter ϕ_p to match the slope of the NKPC implied by micro data of individual price changes in the baseline calibration, it is widely known that an estimated DSGE model often identifies a higher value for the parameter.[14] In addition, the optimal rate also depends on the assumptions about the monetary policy rule. Under the Taylor rule in Panel (A) and (B), when the ZLB is taken into account, the optimal rate increases by 0.5-1.5 percentage point, because the ZLB is another source to generate the benefit of inflation. On the other hand, once the history-dependent rule is implemented in Panel (C), the optimal rate is reduced to around the same range as that under the Taylor rule without the ZLB.

Though the optimal steady-state inflation rate depends on the model specification, in each specification, taking into account heterogeneity increases the optimal rate. To

[14] For instance, Del Negro et al. (2015), who estimate the medium-scale DSGE model with financial frictions, obtain 0.868 as the posterior mean of the Calvo parameter (the average duration of price changes is 7.6 quarters). The value corresponds to $\phi_p = 350$ in my model, which is shown on the right end of Figure 2.2.

be precise, the optimal rate in the HA model is higher than that in the RA model, by from 0.5 to 5.5 percentage point under the Taylor rule without the ZLB, from 0.5 to 3.5 percentage point under the Taylor rule with the ZLB, and 0.5 to 4.0 percentage point under the history-dependent rule. Since the HA model generates the sizable welfare loss associated with the cross-sectional inefficiency in the labor market, the benefit of higher inflation is larger in the HA model than in the RA model. The finding suggests that the policy implications of the existing literature that ignores the effects of HA, such as Kim and Ruge-Murcia (2009), Coibion et al. (2012), and Carlsson and Westermark (2016), need to be reconsidered.

Figure 2.2: Optimal steady-state inflation rate



Notes: For each specification, the steady-state inflation rate Π^* is chosen to maximize social welfare on the equally spaced grid points in the parameter space [1.000,1.025] ([0.0,2.5] percent inflation in the annual rate) with the width of each interval 0.0125 (0.5 percent). The parameters of responsiveness are fixed at $\delta_p = 2.5$ and $\delta_y = 1.5$. The x-axis is in terms of the adjustment cost of 2 percent inflation in the annual rate. The corresponding parameter values of ϕ_p ranges from 35 to 350. The shaded area and the thin dashed lines indicate the nearly-optimal rate whose welfare loss is within 0.03 percentage point of that of the optimal rate. In Panel (B), the optimal rate in the RA model when $\phi_p = 35$ (on the left end) is not shown, because the model does not deliver the determinacy of equilibrium under most cases of different steady-state inflation rates.

2.7 Conclusion

In this paper, I study the optimal simple and implementable monetary policy rule in a heterogeneous agent model with DNWR and the ZLB. In the calibrated model, DNWR generates sizable welfare loss through cross-sectional inefficiency in the labor market as well as aggregate fluctuations. As a consequence, the model delivers rich policy implications for both the optimal responsiveness to the fluctuations of aggregate variables and the optimal steady-state inflation rate.

The methodology that is used in this paper is fairly general to leave many possible extensions for future works. First of all, studying other dimensions of heterogeneity is a natural extension. For example, Kaplan et al. (2018) argue that heterogeneity of asset holdings with incomplete markets significantly changes the transmission mechanism of monetary policy. Exploring optimal monetary policy rule in the model would be of great interest. Moreover, recent studies point out that the cost of inflation differs considerably depending on the specification of the firm side of the model, though I use the stylized Rotemberg (1982) price adjustment cost to keep the tractability of the model. In this regard, Burstein and Hellwig (2008) find that the menu cost model generates much smaller welfare loss of inflation than that of the Calvo model due to the selection effect of price changes.[15] It would be worth investigating how the specification of firms affects optimal monetary policy. Lastly, though I focus on monetary policy in this paper, there are other policy alternatives to address the distortion of

[15] Alvarez et al. (2011) and Nakamura et al. (Forthcoming) find empirical evidence to favor the menu cost model than the Calvo model.

DNWR and the ZLB. In particular, since cross-sectional inefficiency is a main source of welfare loss in the economy, a promising candidate would be the redistribution through tax and transfer policies.

2.8 Appendix

2.8.1 Social welfare

2.8.1.1 Derivation of the second order approximation of social welfare

The approach adopted in this subsection largely follows Rotemberg and Woodford (1997) and Erceg et al. (2000). I take the second order Taylor expansion of the current social welfare around the deterministic steady state:

$$
\begin{aligned}
SW_t &\equiv \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} \int_0^1 \chi_t(j) h_t(j)^{1+\eta} dj && (2.29) \\
&\approx \bar{SW} + \bar{C}^{1-\sigma} \left(\frac{dC_t}{\bar{C}} \right) - \frac{1}{2} \sigma \bar{C}^{1-\sigma} \left(\frac{dC_t}{\bar{C}} \right)^2 \\
&\quad - \bar{\chi} \bar{h}^{1+\eta} \int_0^1 \left(\frac{dh_t(j)}{\bar{h}} \right) dj - \frac{1}{2} \eta \bar{\chi} \bar{h}^{1+\eta} \int_0^1 \left(\frac{dh_t(j)}{\bar{h}} \right)^2 dj \\
&\quad - \frac{1}{1+\eta} \bar{\chi} \bar{h}^{1+\eta} \int_0^1 \left(\frac{d\chi_t(j)}{\bar{\chi}} \right) dj \\
&\quad - \bar{\chi} \bar{h}^{1+\eta} \int_0^1 \left(\frac{d\chi_t(j)}{\bar{\chi}} \right) \left(\frac{dh_t(j)}{\bar{h}} \right) dj && (2.30) \\
&\quad with \quad \frac{da_t}{\bar{a}} \equiv \frac{a_t - \bar{a}}{\bar{a}}
\end{aligned}
$$

where I define the deterministic steady state as the economy under flexible prices and wages without any exogenous shocks. $\bar{a}$ is the value of a in the deterministic steady state.

Following Erceg et al. (2000), I use two approximations:

$$\frac{da_t}{\bar{a}} \equiv \frac{a_t - \bar{a}}{\bar{a}} \approx \hat{a}_t + \frac{1}{2}\hat{a}_t^2 \tag{2.31}$$
$$where \quad \hat{a}_t \equiv log(a_t) - log(\bar{a})$$

and, if $a_t = [\int_0^1 a_t(j)^{\varphi} dj]^{1/\varphi}$, then

$$\hat{a}_t \approx \mathbb{E}_j[\hat{a}_t(j)] + \frac{1}{2}\varphi \mathbb{V}ar_j(\hat{a}_t(j)) \tag{2.32}$$

where $\mathbb{E}_j[\cdot]$ and $\mathbb{V}ar_j(\cdot)$ are the expectation and the variance across j.

In the following, I assume:

$$\bar{Z} = \bar{\chi} = 1 \quad and \quad \bar{\Pi}^p = \bar{\Pi} \tag{2.33}$$

That leads to:

$$\bar{Y} = \bar{C} = \bar{H} = \bar{h} = 1 \tag{2.34}$$

Using (2.31), (2.33), and (2.34), (2.30) can be rearranged to:

$$\begin{aligned}
SW_t \approx\ & S\bar{W} + \left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right) - \frac{1}{2}\sigma\left(\hat{C}_t + \frac{1}{2}\hat{C}_t^2\right)^2 \\
& - \int_0^1 \left(\hat{h}_t(j) + \frac{1}{2}\hat{h}_t(j)^2\right) dj - \frac{1}{2}\eta \int_0^1 \left(\hat{h}_t(j) + \frac{1}{2}\hat{h}_t(j)^2\right)^2 dj \\
& - \frac{1}{1+\eta}\int_0^1 \left(\hat{\chi}_t(j) + \frac{1}{2}\hat{\chi}_t(j)^2\right) dj \\
& - \int_0^1 \left(\hat{\chi}_t(j) + \frac{1}{2}\hat{\chi}_t(j)^2\right)\left(\hat{h}_t(j) + \frac{1}{2}\hat{h}_t(j)^2\right) dj \qquad (2.35) \\
\approx\ & S\bar{W} + \hat{C}_t - \frac{1}{2}(\sigma - 1)\hat{C}_t^2 \\
& - \int_0^1 \hat{h}_t(j) dj - \frac{1}{2}(1+\eta)\int_0^1 \hat{h}_t(j)^2 dj \\
& - \frac{1}{1+\eta}\int_0^1 \left(\hat{\chi}_t(j) + \frac{1}{2}\hat{\chi}_t(j)^2\right) dj \\
& - \int_0^1 \hat{\chi}_t(j)\hat{h}_t(j) dj \qquad (2.36)
\end{aligned}$$

Notice that, from (2.35) to (2.36), I ignore the third and higher order terms since I focus on the second-order approximation.

By (2.31), the aggregation of labor service leads to:

$$\hat{H}_t \approx \mathbb{E}_j[h_t(j)] + \frac{1}{2}\frac{\theta_w - 1}{\theta_w}\mathbb{V}ar_j(h_t(j)) \tag{2.37}$$

On the other hand, the resource constraint and the production function lead to:

$$\hat{Y}_t \approx \hat{C}_t + \frac{\phi_p}{2}(log(\Pi_t^p) - log(\bar{\Pi}))^2 \tag{2.38}$$

$$\hat{Y}_t^2 \approx \hat{C}_t^2 \tag{2.39}$$

and

$$\hat{Y}_t \approx \hat{Z}_t + \hat{H}_t \tag{2.40}$$

$$\hat{Y}_t^2 \approx \hat{Z}_t^2 + 2\hat{Z}_t\hat{H}_t + \hat{H}_t^2 \tag{2.41}$$

By substituting (2.37)-(2.41) into (2.36) and taking difference from the social welfare in the economy under flexible prices and wages, I obtain:

$$\begin{aligned} SW_t - SW_t^f \approx & -\frac{1}{2}(\sigma+\eta)\Big((\hat{Y}_t)^2 - (\hat{Y}_t^f)^2\Big) + (1+\eta)\hat{Z}_t(\hat{Y}_t - \hat{Y}_t^f) \\ & - \frac{\phi_p}{2}\Big(log(\Pi_t^p) - log(\Pi^*)\Big)^2 \\ & - \frac{1}{2}\left(\eta + \frac{1}{\theta_w}\right)\Big(\mathbb{V}ar_j(\hat{h}_t(j)) - \mathbb{V}ar_j(\hat{h}_t^f(j))\Big) \\ & - \Big(\mathbb{C}ov_j(\chi_t(j), \hat{h}_t(j)) - \mathbb{C}ov_j(\chi_t(j), \hat{h}_t^f(j))\Big) \end{aligned} \tag{2.42}$$

Under flexible prices and wages, it can be shown that:

$$\hat{Y}_t^f = \frac{1+\eta}{\sigma+\eta}\hat{Z}_t \tag{2.43}$$

Then, the first two terms in the RHS of (2.42) can be summarized as:

$$-\frac{1}{2}(\sigma+\eta)\Big(\hat{Y}_t - \hat{Y}_t^f\Big)^2$$

Notice that the term corresponds to the variance of the output gap since $\hat{Y}_t^{gap} = \hat{Y}_t - \hat{Y}_t^f$.

By taking unconditional expectation with respect to t, I finally obtain (2.15).

$$\begin{aligned} SW - SW^f \approx & -\frac{1}{2}(\sigma+\eta)\mathbb{V}ar\Big(\hat{Y}_t^{gap}\Big) - \frac{\phi_p}{2}\mathbb{V}ar\Big(log(\Pi_t^p)\Big) \\ & -\frac{1}{2}\left(\eta+\frac{1}{\theta_w}\right)\mathbb{E}\Big[\mathbb{V}ar_j\Big(\hat{h}_t(j)\Big) - \mathbb{V}ar_j\Big(\hat{h}_t^f(j)\Big)\Big] \\ & -\mathbb{E}\Big[\mathbb{C}ov_j\Big(\hat{\chi}_t(j),\hat{h}_t(j)\Big) - \mathbb{C}ov_j\Big(\hat{\chi}_t(j),\hat{h}_t^f(j)\Big)\Big] \end{aligned} \tag{2.44}$$

where I assume that $\mathbb{E}[\hat{Y}_t^{gap}]$ and $\mathbb{E}[log(\Pi^p)_t - log(\bar{\Pi})]$ are of second-order based on the insights of Erceg et al. (2000), and therefore $(\mathbb{E}[\hat{Y}_t^{gap}])^2$ and $(\mathbb{E}[log(\Pi^p)_t - log(\bar{\Pi})])^2$ are neglected in the second-order approximation.

2.8.1.2 Relation to the staggered contract model of Calvo (1983)

This subsection provides the intuition behind the fact that cross-sectional wage distribution matters for social welfare in the presence of DNWR even being conditional on aggregate variables.

To see differences of the welfare implication of DNWR from the Calvo model, it is convenient to begin with the relationship between cross-sectional dispersion of hours worked and wages. Taking the logarithm of the individual labor demand function (1.13) leads to:

$$\hat{h}_t(j) = -\theta_w\Big(log(w_t(j)) - log(W_t)\Big) + \hat{H}_t \tag{2.45}$$

Taking cross-sectional moments of (2.45), I obtain:

$$\mathbb{V}ar_j(\hat{h}_t(j)) = \theta_w^2 \mathbb{V}ar_j(log(w_t(j))) \tag{2.46}$$

$$\mathbb{C}ov_j(\hat{\chi}_t(j), \hat{h}_t(j)) = -\theta_w \mathbb{C}ov_j(\hat{\chi}_t(j), log(w_t(j))) \tag{2.47}$$

(2.46) and (2.47) determine the tight link between the cross-sectional dispersion of hours worked and that of wages.

Calvo model. In the Calvo model without idiosyncratic shocks, which is a standard setting in the literature including Erceg et al. (2000) and Christiano et al. (2005), the cross-sectional variance of wages is expressed as:

$$\begin{aligned}\mathbb{V}ar_j(log(w_t^{Calvo}(j))) = \xi_w \mathbb{E}_j\Big[log(w_{t-1}^{Calvo}(j)) - \mathbb{E}_j[log(w_t^{Calvo}(j))]\Big]^2 \\ + (1-\xi_w)\Big(log(w_t^{d,Calvo}) - \mathbb{E}_j[log(w_t^{Calvo}(j))]\Big)^2\end{aligned} \tag{2.48}$$

where ξ_w is the fraction of workers without wage changes and $w_t^{d,Calvo}$ is the reset wage set by workers who change their wages. It should be noted that ξ_w is constant over time by construction. Using the following relationship

$$\mathbb{E}_j\Big[log(w_{t-1}^{Calvo}(j)) - \mathbb{E}_j[log(w_t^{Calvo}(j))]\Big]^2 = \mathbb{V}ar(log(w_t^{Calvo}(j))) + (log(\Pi_t^{w,Calvo}))^2 \tag{2.49}$$

and

$$log(w_t^{d,Calvo}) - \mathbb{E}_j[log(w_t^{Calvo}(j))] = \frac{\xi_w}{1-\xi_w} log(\Pi_t^{w,Calvo}) \tag{2.50}$$

I obtain a recursive expression of the cross-sectional variance of wages:

$$\mathbb{V}ar_j(log(w_t^{Calvo}(j))) = \xi_w \mathbb{V}ar_j(log(w_{t-1}^{Calvo}(j))) + \frac{\xi_w}{1-\xi_w} log(\Pi_t^{w,Calvo})^2 \tag{2.51}$$

Finally, by taking the unconditional expectation with respect to t, the cross-sectional variance of wages is represented by the variance of aggregate wage growth rate:

$$\mathbb{E}\Big[\mathbb{V}ar_j(log(w_t^{Calvo}(j)))\Big] = \frac{\xi_w}{(1-\xi_w)^2}\mathbb{V}ar\Big(log(\Pi_t^{w,Calvo})\Big) \tag{2.52}$$

DNWR. Essential assumptions to obtain the result above are: 1) the fraction of workers without wage changes ξ_w is constant over time, and 2) the reset wage $w_t^{d,Calvo}$ is identical across workers. However, neither of them holds in the presence of DNWR and idiosyncratic shocks. To focus on the effect of DNWR, in the following, I assume that there are no idiosyncratic shocks, i.e. $\chi_t(j) = \bar{\chi}$. Then,

$$\mathbb{V}ar_j(log(w_t(j))) = \mathbb{E}_j\Big[I_t(j)(log(w_{t-1}(j)) - log(W_t))^2\Big] + \mathbb{E}_j\Big[(1-I_t(j))(log(w_t^d) - log(W_t))^2\Big] \tag{2.53}$$

where $I_t(j)$ is an indicator function that takes one if worker j does not change her wage from the previous period. The equation is rearranged to:

$$\begin{aligned}\mathbb{V}ar_j(log(w_t(j))) &= \xi_{w,t}\mathbb{E}_j\Big[(log(w_{t-1}(j)) - log(W_t))^2\Big] + \mathbb{C}ov_j\Big(I_t(j), (log(w_{t-1}(j)) - log(W_t))^2\Big) \\ &\quad + (1-\xi_{w,t})(log(w_t^d) - log(W_t))^2 \qquad (2.54) \\ &where \quad \xi_{w,t} \equiv \mathbb{E}_j\Big[I_t(j)\Big]\end{aligned}$$

Notice that the covariance term appears since $\xi_{w,t}$ is time-varying.

Each term in the RHS of (2.54) can be rewritten as follows.

$$\begin{aligned}\mathbb{E}_j\Big[(log(w_{t-1}(j)) - log(W_t))^2\Big] &= \mathbb{E}_j\Big[((log(w_{t-1}(j)) - log(W_{t-1})) - (log(W_t) - log(W_{t-1})))^2\Big] \\ &= \mathbb{V}ar_j\Big(log(w_{t-1}(j))\Big) + \Big(log(\Pi_t^w)\Big)^2 \qquad (2.55)\end{aligned}$$

$$
\begin{aligned}
&\mathbb{C}ov_j\Big(I_t(j), (log(w_{t-1}(j)) - log(W_t))^2\Big)\\
&\quad = \mathbb{C}ov_j\Big(I_t(j), ((log(w_{t-1}(j)) - log(W_{t-1})) - (log(W_t) - log(W_{t-1})))^2\Big)\\
&\quad = \mathbb{C}ov_j\Big(I_t(j), (log(w_{t-1}(j)) - log(W_{t-1}))^2\Big) - 2\mathbb{C}ov_j\Big(I_t(j), log(w_{t-1}(j))\Big)log(\Pi_t^w)
\end{aligned}
\tag{2.56}
$$

where $log(\Pi_t^w) \equiv log(W_t) - log(W_{t-1})$, and

$$
\begin{aligned}
log(w_t^d) - log(W_t) &= log(w_t^d) - \int_0^1 \Big\{(1 - I_t(j))log(w_t^d) + I_t(j)log(w_{t-1}(j))\Big\}dj\\
&= log(w_t^d) - (1 - \xi_{w,t})log(w_t^d) - \xi_{w,t}log(W_{t-1}) - \mathbb{C}ov\Big(I_t(j), log(w_{t-1}(j))\Big)\\
&= \xi_{w,t}(log(w_t^d) - log(W_t)) + \xi_{w,t}log(\Pi_t^w) - \mathbb{C}ov\Big(I_t(j), log(w_{t-1}(j))\Big)\\
\Leftrightarrow log(w_t^d) - log(W_t) &= \frac{\xi_{w,t}}{1 - \xi_{w,t}}log(\Pi_t^w) - \frac{1}{1 - \xi_{w,t}}\mathbb{C}ov\Big(I_t(j), log(w_{t-1}(j))\Big) \qquad (2.57)
\end{aligned}
$$

By substituting (2.55)-(2.57) into (2.54) and rearranging it, I obtain

$$
\begin{aligned}
\mathbb{V}ar_j\Big(log(w_t(j))\Big) &= \xi_{w,t}\mathbb{V}ar_j\Big(log(w_{t-1}(j))\Big) + \frac{\xi_{w,t}}{1 - \xi_{w,t}}(log(\Pi_t^w))^2 + C \qquad (2.58)\\
where \quad C &\equiv -\frac{2}{1 - \xi_{w,t}}\mathbb{C}ov_j\Big(I_t(j), log(w_{t-1}(j))\Big)log(\Pi_t^w)\\
&\quad + \mathbb{C}ov_j\Big(I_t(j), (log(w_{t-1}(j)) - log(W_{t-1}))^2\Big)\\
&\quad + \frac{1}{1 - \xi_{w,t}}\mathbb{C}ov\Big(I_t(j), log(w_{t-1}(j))\Big)^2
\end{aligned}
$$

By taking the unconditional expectation with respect to t and rearrange it, I finally obtain (2.18).

$$
\begin{aligned}
&\mathbb{E}\Big[\mathbb{V}ar_j\Big(log(w_t(j))\Big)\Big] = \mathbb{E}\big[\xi_{w,t}\big]\mathbb{E}\Big[\mathbb{V}ar_j\Big(log(w_{t-1}(j))\Big)\Big] + \mathbb{C}ov\Big(\xi_{w,t}, \mathbb{V}ar_j\Big(log(w_{t-1}(j))\Big)\Big) \\
&\qquad + \mathbb{E}\left[\frac{\xi_{w,t}}{1-\xi_{w,t}}\right]\mathbb{E}\Big[(log(\Pi_t^w))^2\Big] + \mathbb{C}ov\Big(\frac{\xi_{w,t}}{1-\xi_{w,t}}, (log(\Pi_t^w))^2\Big) + \mathbb{E}\big[C\big] \\
&\Leftrightarrow \mathbb{E}\Big[\mathbb{V}ar_j(log(w_t(j)))\Big] = \frac{1}{1-\mathbb{E}[\xi_{w,t}]}\mathbb{E}\left[\frac{\xi_{w,t}}{1-\xi_{w,t}}\right]\mathbb{V}ar\Big(log(\Pi_t^w)\Big) \\
&\qquad + \frac{1}{1-\mathbb{E}[\xi_{w,t}]}\Big\{\mathbb{C}ov\Big(\xi_{w,t}, \mathbb{V}ar_j\Big(log(w_{t-1}(j))\Big)\Big) + \mathbb{C}ov\left(\frac{\xi_{w,t}}{1-\xi_{w,t}}, (log(\Pi_t^w))^2\right) + \mathbb{E}\big[C\big]\Big\}
\end{aligned}
\tag{2.59}
$$

Chapter 3

Dynamic Priors for VAR

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3.1 Introduction

This paper proposes a method of introducing theory-driven priors into the estimation of vector autoregressions (VARs) that is more flexible than existing methods. VARs are an important tool for empirical macroeconomists because of their flexibility in capturing dynamic relationships across economic variables (see Sims (1980)). The literature offers several approaches to formulating priors for the coefficients of VARs which can lead to more precise estimates and greater forecasting performance. A widely used prior in the literature is the so-called Minnesota prior (MN), which dates

[1]The views expressed in this paper are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Boston or the Federal Reserve System.

back to Litterman (1980). The idea behind the Minnesota prior is to impose prior values that imply random-walk behavior for each model variable. Doan et al. (1984), Sims (1993), Kadiyala and Karlsson (1993), Kadiyala and Karlsson (1997), and Sims and Zha (1998) extend the idea of the Minnesota prior and additionally include priors for cross-variable restrictions such as the sums-of-coefficient prior or the co-persistence priors.[2]

A more recent literature has advocated the use of structural economic models in formulating priors for the coefficients of VARs. For example, the DSGE-VAR literature of DeJong et al. (1993), Ingram and Whiteman (1994), Del Negro and Schorfheide (2004) and Del Negro et al. (2007) introduce a prior that pulls the likelihood estimate of the VAR parameters toward the restrictions implied from a Dynamic Stochastic General Equilibrium (DSGE) model.[3]

Building on the insights of the DSGE-VAR literature, we generalize the methodology to allow more flexibility. More specifically, we allow the researcher to rely on a wide class of theoretical models for predictions on a subset of variables that are of

[2]Kadiyala and Karlsson (1997) consider general prior distributions that incorporate cross-variable dependence and unknown residual variance-covariance matrix. They also discuss the benefit of using other prior distributions such as a truncated normal distribution to account for the behavior of the data. Sims and Zha (1998) formulate a tractable method to incorporate prior beliefs into structural VAR models by augmenting the Minnesota prior of Litterman (1980).

[3]The magnitude of deviations is governed by a hyperparameter which captures the degree of misspecification of the DSGE model. In the two polar cases where the hyperparameter goes to infinity or is zero, the cross-variable restrictions of the DSGE model are strictly enforced or completely ignored, respectively.

key interest while remaining agnostic about other variables in the VAR. Practically speaking, one could be more interested in the dynamics of a subset of economic variables, e.g., output or inflation, than other variables in the VAR system. In addition, it may be difficult to find a benchmark theoretical model that provides a complete set of restrictions on all cross-equation dynamics. This is going to be increasingly relevant when the number of variables included in the VAR is large. These are practically important concerns when estimating VARs. Another consideration is that a researcher may have more faith in a subset of the predictions of a theoretical model. Our method allows the researcher to introduce theory-based priors for only a subset of the VAR dynamics while the remaining dynamics can be shrinked toward, for example, random-walk behavior, which is known to achieve great forecasting performance.

In this sense, we are very much related to the work of Giannone et al. (2016) who propose a prior that disciplines only the long-run behavior of VARs based on economic theory. However, they employ a commonly used Minnesota prior for the coefficients of a VAR which does not allow the variable's dynamics to be influenced by other variables' lagged terms. While one can introduce, for example, the co-persistence prior, proposed by Sims (1993), to induce correlation among the coefficients on other variables' lags, it can only be implemented in a non-theory-driven way. We demonstrate that a theory-driven elicitation of the prior for VAR coefficients can be easily implemented while the same prior can also provide guidance to the long-run dynamics of the VAR in a consistent manner. This can be important if the researcher is interested in both short- and long-horizon forecasts.

It is important to emphasize that our framework can flexibly incorporate theoryled dynamic restrictions across variables not just through the first lagged terms, but through distant lagged terms as well. Since our priors preserve conjugacy and can be easily implemented with dummy observations, inference can be achieved by the direct sampling. The selection of hyperparameters, which control the variance of the prior distribution, is based on a data-driven way of maximizing the marginal likelihood which is obtained in closed form.

We apply our methodology to the problem of exchange rate forecasting. We choose this application for two reasons. First, exchange rates are notoriously difficult to forecast (see Cheung et al. (2005) and Rogoff and Stavrakeva (2008)). Second, if a researcher is mainly interested in forecasting an exchange rate, it may be desirable to elicit priors only from theoretical relationships that exist in a wide range of open-economy macroeconomic models—such as those between exchange rates and nominal interest rates (as in uncovered interest rate parity (UIP)) or price levels (as in purchasing power parity (PPP))—while remaining agnostic about other predictions of these models. For example, a researcher who wishes to tilt an estimated VAR toward some version of a UIP relationship may not wish to take a theory-implied stand on the behavior of nominal interest rates.

We use our econometric framework to generate exchange rate forecasts for the British pound versus the US dollar (the GBP/USD pair) using different combinations of priors on short- and/or long-run dynamics informed by UIP and/or PPP. We compare the forecasting ability of our VAR relative to a naïve random walk as well as a VAR estimated using a standard Minnesota prior. We find that most of the forecasting gains of our priors are realized at longer forecast horizons. This is a result that is distinct from, but consistent with, the literature showing that UIP and PPP relationships are less-often rejected in direct regressions of long-horizon exchange rate changes (see Chinn and Meredith (2004) and Rogoff (1996)). Furthermore, we find some interesting variation over time in the extent to which data is consistent with priors based on these relationships. More specifically, the data favors UIP-based priors more in samples that include more recent data.

3.2 Modeling Framework

3.2.1 A vector autoregressive model

We consider a vector autoregressive model

$$y_t = \Phi_1 y_{t-1} + ... + \Phi_p y_{t-p} + \Phi_0 + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma), \tag{3.1}$$

where y_t is an $n \times 1$ vector of observables, Φ_js are parameters, $j \in \{0, 1, ..., p\}$, and ε_t is an $n \times 1$ vector of one-step-ahead forecast error shocks which does not have a specific economic interpretation.

3.2.2 Prior elicitation

An important challenge in practice with VARs is to cope with the dimensionality of the parameter matrix Φ. Informative prior distributions can often mitigate the curse of dimensionality. We introduce priors that preserve the stationarity of the linear combinations of the model variables, y_t. Define

$$z_t \equiv H_0 y_t - H_1 y_{t-1} \cdots - H_p y_{t-p} \tag{3.2}$$

where z_t is an $n \times 1$ stationary vector, and $H_0, H_1, ..., H_p$ are $n \times n$ invertible matrices that determine dynamic relationship among model variables. For ease of illustration, we work with non-zero H_0 and H_1 only.

Note that (3.2) conveniently characterizes the existing priors in general form. For example, consider the values of $H_0 = I_n$ and $H_1 = I_n + \Lambda H^*$. The discussion on H^* will be provided shortly. The stationary component becomes

$$z_t = \Delta y_t - \Lambda H^* y_{t-1}. \tag{3.3}$$

A widely used prior in the VAR literature is the so-called Minnesota prior (MN), which dates back to Litterman (1980). The Minnesota prior is a special case of (3.3) in which $\Lambda = 0$ is assumed. The idea behind the Minnesota prior is to impose prior value that implies a random-walk behavior for each component of the model variables. Doan et al. (1984), Sims (1993), and Sims and Zha (1998) extend the idea of the Minnesota prior and additionally incorporates the possibility of cointegration among model variables through the sums-of-coefficient prior. Their idea is to set $H^* = I_n$ and shrink Λ, which governs the degree of error correction, toward zero. To the extent that data favor cointegration relationship within model variables, some columns of Λ will be shrunk less to zero.

A more recent paper by Giannone et al. (2016) proposes a prior, which is called the prior for the long run (LR), that disciplines the long-run behavior of the model. Giannone et al. (2016) essentially rely on the setting of (3.3) in which $H^* = H^{LR}$ based on the long-run predictions of economic theory.[4] They formulate a prior on

[4]One of the examples provided in Giannone et al. (2016) suggests that output and investment

Λ conditional on H^* that combines data in a way that distinguishes the stationary combinations from the non-stationary ones.

We adopt the main insight of Giannone et al. (2016) that economic theory should play a central role for the elicitation of priors. That is, we assume that the long-run predictions of economic theory can guide us on the selection matrices. We work with the general form of (3.2) because it can richly accommodate both intra- and inter-temporal restrictions among variables. In the context of exchange rate forecasting, the optimal forecast of the change in the exchange rate between time t and $t+1$ is the interest differential between the home and foreign country at time t if the uncovered interest parity (UIP) condition held. Alternatively, if the purchasing power parity (PPP) condition held, then the optimal forecast of the change in the exchange rate between time t and $t+1$ is the inflation differential between the home and foreign country at time $t+1$. These priors on parities can be easily accommodated in our setting by including the level of exchange rate and interest rates (inflations) for the home and foreign country in y_t and specifying the selection matrix H_0 and H_1 in (3.2) in a way that imposes the UIP (PPP) condition.[5] The specific forms of H_0 and H_1

are likely to share a common trend, while the log-investment-to-output ratio is expected to be stationary. In their bivariate VAR example, the observation vector is defined by $y_t = [gdp_t, i_t]'$ and the corresponding H^{LR} matrix is set to

$$H = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

[5]Technically speaking, the UIP condition can be accommodated in (3.3) then it no longer preserves the error correction interpretation. However, to accommodate the PPP condition we ought

are provided in Section 3.3.

We now show how to formulate prior beliefs of (3.2) within the benchmark VAR model (3.1). Our exposition follows the version Del Negro and Schorfheide (2011) which is based on Sims and Zha (1998). We re-express (3.2) by

$$\begin{aligned} H_0 y_t &= H_1 y_{t-1} + z_t \qquad (3.4) \\ y_t &= H y_{t-1} + r_t. \end{aligned}$$

The second line is from the invertibility of H_0 and $H = H_0^{-1} H_1$ and $r_t = H_0^{-1} z_t$. We re-arrange the VAR model (3.1) below

$$\begin{aligned} y_t &= \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + ... + \Phi_p y_{t-p} + \Phi_0 + \varepsilon_t \qquad (3.5) \\ &= \tilde{\Phi}_1 \tilde{y}_{t-1} + \Phi_2 y_{t-2} + ... + \Phi_p y_{t-p} + \Phi_0 + \varepsilon_t \end{aligned}$$

where $\tilde{\Phi}_1 = \Phi_1 H^{-1}$ and $\tilde{y}_{t-1} = H y_{t-1}$. Our goal is to shrink the VAR coefficients in (3.5) towards the coefficients that reflect our prior beliefs in (3.4). This amounts to shrinking $\tilde{\Phi}_1$ towards an identity matrix[6]

$$vec(\tilde{\Phi}_1)|H, \Sigma \sim N\left(vec(I_n), diag\left(\frac{1}{\theta_i^2 \underline{s}_i^2}\right) \otimes \Sigma\right) \qquad (3.6)$$

θ_i is a hyperparameter, which corresponds to the i-th column, that controls the tightness of the prior and $\underline{s}_i$ is the pre-sample standard deviation of the i-th element of y_t.

While our prior beliefs in (3.4) formulate the relationship in two periods, we can extend to multi-periods easily. We can re-write the VAR model (3.1) in the following

to work with a non-identity matrix H_0 and rely on the specification (3.2).

[6]Note that $vec(\Phi_1)|H, \Sigma \sim N\big(vec(H), (H' diag\left(1/(\theta_i^2 \underline{s}_i^2)\right) H) \otimes \Sigma\big)$ and Φ_1 no longer shrinks toward an identity matrix.

way

$$y_t = \tilde{\Phi}_1 \tilde{y}_{t-1} + \tilde{\Phi}_2 \tilde{y}_{t-2} + ... + \tilde{\Phi}_p \tilde{y}_{t-p} + \Phi_0 + \varepsilon_t \tag{3.7}$$

where $\tilde{\Phi}_j = \Phi_j H^{-1}$ and $\tilde{y}_{t-j} = H y_{t-j}$ for $j \in \{2, ..., p\}$. We consider priors that imply $\tilde{\Phi}_j$ shrinks towards zero

$$vec(\tilde{\Phi}_j)|H, \Sigma \sim N\left(0, diag\left(\frac{1}{\theta_i^2 \underline{s}_i^2 j^\mu}\right) \otimes \Sigma\right) \tag{3.8}$$

where μ is a hyperparameter that governs the shrinkage of distant lagged terms. Notice that the variance-covariance structure of Φ_j is adjusted by H such that each column of $\tilde{\Phi}_j$ shrinks towards the prior mean proportionally to that of $\tilde{\Phi}_1$.[7] In other words, we regard $\tilde{y}_{t-j}$ (instead of y_{t-j}) as a relevant variable of the model which is consistent with our prior beliefs (3.4).

To the extent that the linear combinations of the model variables affect the short-run dynamics, the corresponding VAR can have an error correction representation of the form

$$\begin{aligned} \Delta y_t &= \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{p-1} \Delta y_{t-p+1} + \Phi_0 + \varepsilon_t, \qquad (3.9) \\ &= \Lambda \tilde{y}_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{p-1} \Delta y_{t-p+1} + \Phi_0 + \varepsilon_t, \end{aligned}$$

where $\Pi = (\Phi_1 + ... + \Phi_p - I_n)$ and $\Gamma_j = -(\Phi_{j+1} + ... + \Phi_p)$ with $j \in \{1, ..., p-1\}$ and Π (Λ) captures the effect of y_{t-1} ($\tilde{y}_{t-1}$) on Δy_t with $\Pi = \Lambda H$. We are primarily interested in eliciting a prior for Π. As shown in Giannone et al. (2016), choosing a prior on Π is isomorphic to choosing a prior on Λ conditional on the selection matrix H. If any row of H contains the coefficients of a linear combination of y that is

[7]Note that $vec(\Phi_j)|H, \Sigma \sim N\big(0, (H' diag\left(1/(\theta_i^2 \underline{s}_i^2) j^\mu\right) H) \otimes \Sigma\big)$.

likely to be stationary (non-stationary), one can apply less (more) shrinkage on the elements of the corresponding column of Λ. If the corresponding column of Λ were all zero, then the error correction feature is not added to Δy_t. The sums-of-coefficient prior introduced by Doan et al. (1984), subsequently developed by Sims and Zha (1998), shrinks Π or Λ toward a zero matrix. In our setting, however, we impose the following prior distribution on Λ conditional on H

$$vec(\Lambda)|H, \Sigma \sim N\left(vec(\Lambda^{d.p.}), diag\left(\frac{1}{\kappa_i^2 \underline{r}_i^2}\right) \otimes \Sigma\right) \tag{3.10}$$

where $\Lambda^{d.p.} = I - H^{-1}$ and $\underline{r}_i$ is the pre-sample mean of the i-th element of r_t. A hyperparameter κ_i governs the shrinkage of each column $\Lambda_{\cdot i}$ toward the prior mean. This prior implies that all the variables of the VAR are forced to follow (3.4) at dogmatic prior value.[8] We provide the prior expressions for the constant term in Appendix 3.5.2.

We use dummy observations to implement priors. The use of dummy observations provides a parsimonious way of introducing plausible correlations between parameters. This insight of introducing prior information in the form of dummy observation dates back at least to Theil and Goldberger (1961). The details are provided in Appendix 3.5.2. After collecting T^* dummy observations in matrices Y^* and X^*, we now use the likelihood function (provided in Appendix 3.5.1) to relate the dummy observations to the parameters Φ and Σ. Combining the likelihood function with the improper prior $p(\Phi, \Sigma) \propto |\Sigma|^{-(n+1)/2}$, we can deduce that the product $p(Y^*|\Phi, \Sigma) \cdot |\Sigma|^{-(n+1)/2}$

[8] Note that $vec(\Pi)|H, \Sigma \sim N\big(vec(H - I), (H' diag\left(1/(\kappa_i^2 \underline{r}_i^2)\right) H) \otimes \Sigma\big)$.

can be interpreted as

$$(\Phi, \Sigma) \sim MNIW(\underline{\Phi}, (X^{*'}X^*)^{-1}, \underline{S}, T^* - k), \tag{3.11}$$

where $\underline{\Phi}$ and $\underline{S}$ are $\underline{\Phi} = (X^{*\prime}X^*)^{-1}X^{*\prime}Y^*$ and $\underline{S} = (Y^* - X^*\underline{\Phi})'(Y^* - X^*\underline{\Phi})$ and MNIW refers to the Normal-Inverted Wishart distribution. Provided that $T^* > k+n$ and $X^{*'}X^*$ is invertible, the prior distribution is proper.

3.2.3 Direct sampler

Since our prior for (Φ, Σ) belongs to the Normal-Inverted Wishart distribution family, so does the posterior and draws from this posterior can be obtained by direct Monte Carlo sampling. A detailed discussion of these computations can be found in Del Negro and Schorfheide (2011).

3.2.4 Hyperparameter selection

The empirical performance of the VAR depends on the choice of hyperparameters,

$$\Theta = \{\theta_1, \ldots, \theta_n,\ \mu,\ \nu,\ \kappa_1, \ldots, \kappa_n,\ \omega\} \tag{3.12}$$

which controls degree of shrinkage. Note that θ_i and κ_i are the key hyperparmeters that govern the degree of shrinkage of the first lagged term and the error correction term, respectively. We explain the role of ν and ω in Appendix 3.5.2. The prior is parameterized such that $\Theta \to 0$ corresponds to a flat prior for (Φ, Σ). On the other hand, as $\Theta \to \infty$, the VAR is estimated subject dogmatically to our prior restriction on the H matrix. The best forecasting performance of the VAR is likely to be achieved for values of Θ that are in between the two extremes. We select the hyperparameters Θ to maximize the marginal data density (MDD). The details are provided in Appendix 3.5.3.

3.3 Application to the Exchange Rate Forecasts

3.3.1 Evidence for UIP and PPP

Uncovered interest parity. The UIP hypothesis is that, under risk-neutrality, an investor should expect equal home-currency returns from investing in a home currency asset or a foreign currency asset (by converting home into foreign currency to make the investment and then converting the returns back). For an investment horizons of h periods, this parity relationship can be expressed in terms of log variables as

$$E_t[s_{t+h} - s_t] = i^j_{t,h} - i^k_{t,h},$$

where s_t denotes the time t nominal exchange rate in terms of units of country j's currency per unit of country k's currency and $\{i^j_{t,h}, i^k_{t,h}\}$ denote the nominal h-period asset returns denominated in each currency.

Under rational expectations, this hypothesis can be tested using the Fama (1984) regression,

$$s_{t+h} - s_t = \alpha + \beta(i^j_{t,h} - i^k_{t,h}) + error_{t+h},$$

where the null hypothesis under UIP is that $\alpha = 0$ and $\beta = 1$ while the error term reflects a rational forecast error equal to $E_t[s_{t+h}] - s_{t+h}$ that should be uncorrelated with variables observable at time t. The UIP hypothesis can be relaxed to allow a non-zero α if the investor is considered to not be risk-neutral but to require a constant risk premium on this investment.

Froot and Thaler (1990) survey evidence from this test for investment horizons under a year and find that estimates of β tend to be quite negative. Bansal and Dahlquist (2000) find more mixed results. Chinn and Meredith (2004) and Chinn and Quayyum (2012) find estimates of β that are closer to 1 for longer investment horizons.

Purchasing power parity. The PPP hypothesis states that the exchange rate between two country's currencies should be equal to the ratio of the two countries' overall price levels. That is, the purchasing power of currencies in terms of real goods should be equal across countries. In other words, PPP states that the real exchange rate between two countries should be equal to 1.[9]

Many tests of PPP are based on examining behavior of deviations from PPP or testing mean-reversion in real exchange rates. Rogoff (1996) finds 3-5 year half lives of PPP deviations. These estimates are confirmed by Murray and Papell (2005) using panel methods while Alba and Papell (2007) finds stronger evidence for PPP in countries that are more open, with lower inflation, moderate exchange rate volatility, and growth rates similar to that of the U.S.

[9]The law of one price holding for all individual goods is sufficient, but not necessary, for PPP to hold.

3.3.2 Selection of the H matrix

Uncovered interest parity. As stated above, the UIP condition under rational expectations for an investment horizon of one period implies that the realized excess return, $\Delta s_{t+1} - (i_t^j - i_t^k)$, is uncorrelated with time t variables and is mean zero. For our empirical specification, we will use a much weaker form of the UIP condition which simply states that $\Delta s_{t+1} - (i_t^j - i_t^k)$ is stationary. These relationships can be included in our setup as follows:

$$z_t^{uip} = H_0^{uip} y_t - H_1^{uip} y_{t-1} \tag{3.13}$$

where

$$y_t = \begin{bmatrix} s_t \\ i_t^j \\ i_t^k \end{bmatrix}, \quad H_0^{uip} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_1^{uip} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.14}$$

Notice that we are imposing a random-walk prior on the nominal interest rates in the second and third rows to ensure the invertability of H_0^{uip} and H_1^{uip}. We emphasize that the empirical results are not sensitive to the prior choices of the additional rows because we allow for individual hyperparameters governing the degree of shrinkage toward our prior which are selected in a data-driven way.

Purchasing power parity. The purchasing power parity (PPP) condition states that, in log terms,

$$s_t = p_t^j - p_t^k,$$

where $\{p^j, p^k\}$ are price levels in each country. We can write this in changes as

$$\Delta s_t = \pi_t^j - \pi_t^k,$$

where $\{\pi^j, \pi^k\}$ are inflation rates in each country. A prior based on this relationship that also uses agnostic random-walk priors for inflation can be included in our setup as follows:

$$z_t^{ppp} = H_0^{ppp} y_t - H_1^{ppp} y_{t-1} \tag{3.15}$$

where

$$y_t = \begin{bmatrix} s_t \\ \pi_t^j \\ \pi_t^k \end{bmatrix}, \quad H_0^{ppp} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_1^{ppp} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.16}$$

Combining UIP and PPP. We can also write a larger VAR with priors that are informed by both the UIP and PPP relationships. The UIP and PPP conditions along with the assumptions that $\Delta i^j + \Delta i^k$, $\Delta \pi^j + \Delta \pi^k$, and $\Delta i^j - \Delta \pi^j$ are stationary give

$$z_t^{uip\&ppp} = H_0^{uip\&ppp} y_t - H_1^{uip\&ppp} y_{t-1} \tag{3.17}$$

where

$$y_t = \begin{bmatrix} s_t \\ i_t^j \\ i_t^k \\ \pi_t^j \\ \pi_t^k \end{bmatrix}, \quad H_0^{uip\&ppp} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}, \quad H_1^{uip\&ppp} = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \tag{3.18}$$

The additional assumptions are required to ensure the invertibility of $H_0^{uip\&ppp}$ and $H_1^{uip\&ppp}$ matrices.

Table 3.1: Descriptive statistics of the data

	s_t	Δs_t	i_t^{uk}	i_t^{us}	π_t^{uk}	π_t^{us}
Mean	-0.665	0.012	0.072	0.040	0.026	0.036
Std.	0.246	0.304	0.047	0.029	0.029	0.030
Autocorr.(1)	0.992	0.078	0.989	0.989	0.323	0.618
Autocorr.(2)	0.982	0.038	0.977	0.978	0.332	0.484

Note: The U.K. corresponds to country j, whereas the U.S. to k in the notations above. Each series except for s_t is in the annual rate. Moments are computed based on the full length of the available sample for each series.

3.3.3 Data

We use the exchange rate of U.S. and U.K. (GBP/USD) and 1-month LIBOR rates and CPI inflation rates for the two countries. The exchange rate is expressed in log levels, the LIBOR rate is de-annualized, and CPI inflation is the log difference of the CPI level from the previous month. We use monthly data from 1984:M12 to 2016:M9 for the UIP condition, and from 1988:M2 to 2016:M9 for the PPP condition and the combination of the two conditions. The sample start dates are determined by the availability of the U.S LIBOR rates and the U.K. CPI, respectively.[10] Descriptive statistics are reported in Table 3.1.

We consider an increasing sequence of estimation-samples $Y_{-p+1:T}$, $T = T_{min}, \dots, T_{max}$, and generate the out-of-sample forecasts for periods $T+1, \dots, T+F_h$. The maximum forecast horizon F_h is set to be 120 months (10 years). We use the initial 60 months (5 years) as the pre-sample to construct the dummy observations, and start the esti-

[10]The availability of each series is as follows. The GBP/USD exchange rate: from 1955:M1 to 2016:M9, the 1 month U.K. LIBOR: from 1976:M1 to 2016:M9, the 1 month U.S. LIBOR: from 1984:M12 to 2016:M9, the U.K. CPI: from 1988:M2 to 2017:M2, and the U.S. CPI: from 1955:M2 to 2017:M2.

mation with a minimum sample length of 120 months (10 years). More specifically, in the UIP specification, the data from 1984:M12 to 1989:M11 is used as the pre-sample, and the first estimation sample spans from 1989:M12 to 1999:M11, to generate the out-of-sample forecast for 1999:M12 to 2009:M11. The lag length of the VAR system is set at $p = 6$.

3.3.4 Empirical results

For each estimation sample, we determine hyperparameters to maximize the MDD. However, we find it challenging in practice to search for the maximum over the high dimensional parameter space of hyperparamters. For ease of exposition, the set of hyperparameters is reproduced below

$$\Theta = \{\theta_1, \ldots, \theta_n,\ \mu,\ \nu,\ \kappa_1, \ldots, \kappa_n,\ \omega\} \tag{3.19}$$

where θ_i and κ_i are the key hyperparmeters that govern the shrinkage of the first lagged term and the error correction term towards our priors, respectively. We refer to Appendix 3.5.2 for the rest of hyperparameters. Notice that $n = 3$ for the UIP and the PPP and $n = 5$ for the combination of the UIP and the PPP. Therefore, the number of hyperparameters to select is 8 in the UIP and the PPP, and 12 in the combination of the UIP and the PPP.[11]

To reduce the burden of dimensionarity, we impose an additional restriction that the hyperparameters governing the tightness of the error correction representation κ_i

[11]In practice, we often fix ν. Then, the number of hyperparamters to select is reduced by 1 from the number of parameters in (3.19).

are proportional to the hyperparameters for the overall shrinkage θ_i. That is,

$$\kappa_i = \kappa\theta_i. \tag{3.20}$$

With the restriction, the number of hyperparameters is reduced to 6 in the UIP and the PPP, and 8 in the combination of the UIP and the PPP. The restriction implies that the shrinkage of each column of coefficient matrices $\tilde{\Phi}_j$ in the VAR system towards our priors is proportional to that of each column of Λ in the error correction representation. For example, if the UIP relationship (the first row of H matrices in (3.14)) holds more tightly than the random-walk of the interest rates (the second and third rows) in the VAR system, we conjecture that it is also the case in the error correction representation. More discussion on restrictions of hyperparameters is provided in Appendix 3.5.4.

Given the hyperparameters, we generate 20,000 draws from the posterior distribution of Φ and Σ using the direst sampler. We discard the first 5,000 draws and use the remaining 15,000 draws. We compare our baseline model of dynamic priors (DP) with the Minnesota prior (MN) and the naïve random walk (RW). All forecasts are evaluated based on their mean absolute forecast errors (MAFE) relative to the MAFE based on the DP. Notice that higher ratios indicate an improvement on the DP forecasts.

Results are reported in Table 3.2. For all specifications, the DP leads to a substantial MAFE reduction in the long run. Consider the UIP case. The MAFE of the RW is larger than that of the DP by 32% in the 5-year horizon. The gap increases to 75% in the 7-year horizon, and 115% in the 10-year horizon. The improvement relative to

Table 3.2: Forecasting the exchange rates of US and UK

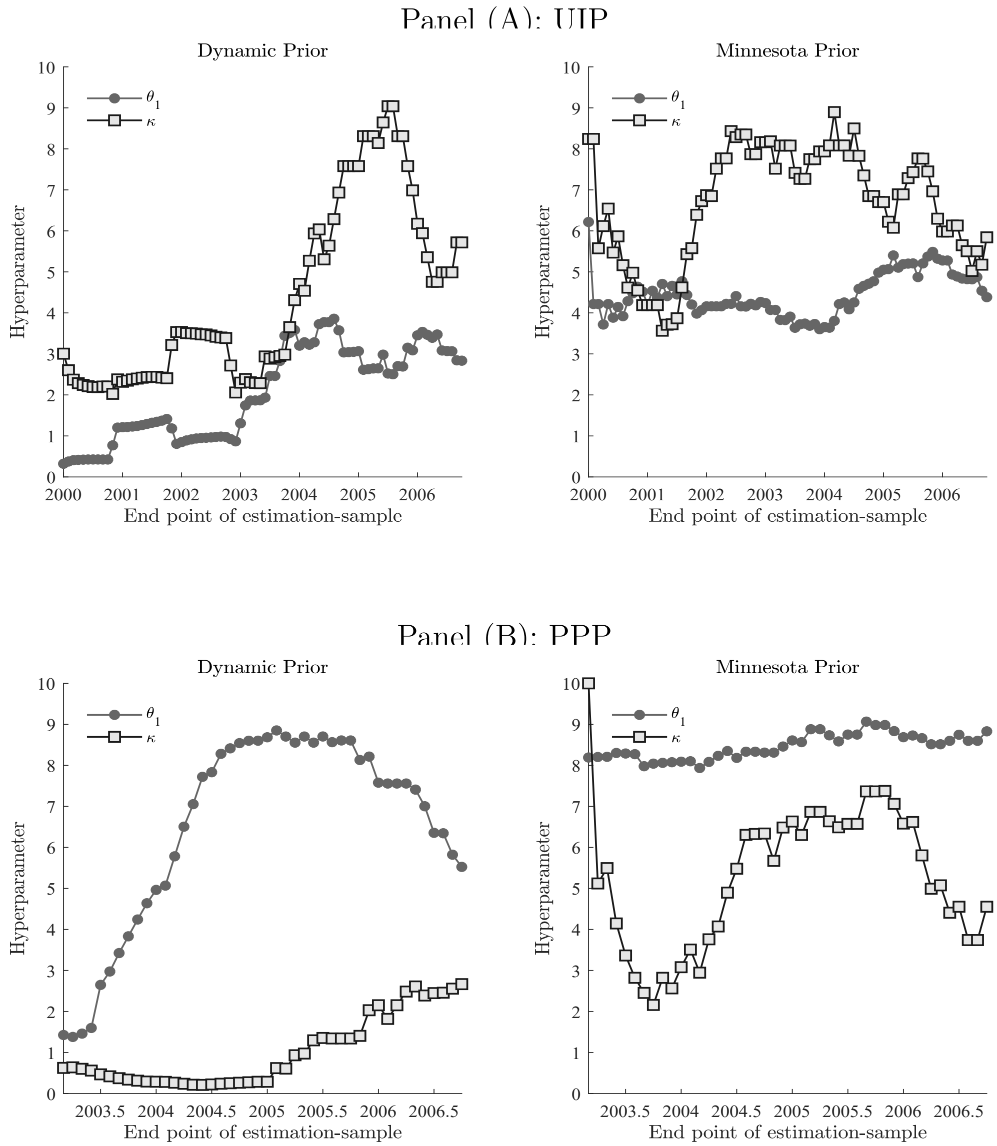
	Forecast horizon				
	1 year	3 year	5 year	7 year	10 year
	Uncovered interest parity (UIP)				
MN	0.88	0.93	1.34	1.74	2.15
RW	0.83	1.00	1.32	1.75	2.19
	Purchasing power parity (PPP)				
MN	0.96	1.03	1.45	1.63	1.62
RW	1.02	1.06	1.49	1.91	1.85
	UIP & PPP				
MN	0.93	1.00	1.18	1.26	1.28
RW	1.00	1.13	1.42	1.65	1.66

Note: All forecasts are evaluated based on their mean absolute forecast errors (MAFE) relative to the MAFE based on the dynamic priors (DP). Higher ratios indicate an improvement on the DP forecasts.

the MN is quite similar, which brings about a MAFE reduction by more than 100% in the 10-year horizon. Moreover, in the PPP case and with the combination of both UIP and PPP, the DP uniformly improves the MAFE after the 3-year horizon. The improvement on the DP also increases in the forecast horizon. In the 10-year horizon, for instance, the reduction of the MAFE is 62% relative to the MN and 85% to the RW in the PPP case, and 28% to the MN and 66% to the RW in the combination of UIP and PPP.

We next compare the forecasting performance of each specification. For this purpose, it is convenient to consider the RW as a benchmark, since the MAFE of the RW does not depend on the model specification. The MAFE relative to the DP in the UIP case is larger than that in the PPP case in the 10-year horizon, whereas that is smaller in the other horizons. The result implies that the UIP prior performs better than the PPP prior in the long run in terms of forecasting the exchange rate with

Figure 3.1: Evolution of hyperparameters



Notes: Each panel displays the 12 month backwards moving average of the selected hyperparameters. θ_1 denotes the hyperparameter governing the overall shrinkage towards the UIP and the PPP, whereas κ is the one for the tightness of the error correction representation. The x-axis is the end point of each estimation sample. Notice that the start point is identical for each estimation sample since we consider an increasing sequence of estimation samples.

the DP. Interestingly, once we combine UIP and PPP, the forecasting performance in the 7- and 10-year horizon is worse than each case of using the UIP and PPP priors individually.

Figure 3.1 reports the evolution of the selected hyperparameters in each estimation sample. We focus on the hyperparameter governing the overall shrinkage towards the UIP or PPP priors individually, θ_1, and the hyperparameter for the tightness of the error correction representation, κ. In the UIP specification in Panel (A), the hyperparameters for the DP tend to rise over time. It indicates that the more recent data supports stronger shrinkage of the estimates toward the UIP prior. On the other hand, the hyperparameters for the MN do not have a particular trend. Similar patterns are found in the PPP case shown in Panel (B), even though the number of data points is relatively smaller the due to the availability of data.

3.4 Conclusion

We propose a method of introducing theory-driven priors into the estimation of VARs. Our methodology is more flexible than existing methods in that it allows a researcher to incorporate prior beliefs on a subset of variables in theoretical models that are of key interest while remaining agnostic about other variables in the VAR. We demonstrate that our method can be easily implemented with dummy observations and inference is achieved by the standard direct sampling. We apply to the problem of exchange rate forecasting for the British pound versus the US dollar by imposing different combinations of priors informed by uncovered interest rate or purchasing power parity. Compared to the forecasting ability of a naïve random walk as well as a VAR estimated using a standard Minnesota prior, substantial gains are realized at longer forecast horizons.

3.5 Appendix

3.5.1 The Likelihood Function

We express the likelihood function. Let $k = np + 1$ and define the $k \times n$ matrix $\Phi = [\Phi_1, ..., \Phi_p, \Phi_0]'$. Using this notation, we can re-express (3.1) as

$$y_t' = x_t'\Phi + \varepsilon_t', \tag{3.21}$$

where the $k \times 1$ vector x_t is given by $x_t' = [y_{t-1}', ..., y_{t-p}', 1]$. The joint density of $Y_{1:T}$ conditional on $Y_{1-p:0}$ and parameters Φ, Σ can be expressed as

$$\begin{aligned} p(Y_{1:T}|\Phi, \Sigma, Y_{1-p:0}) &= \prod_{t=1}^{T} p(y_t|\Phi, \Sigma, Y_{1-p:t-1}) \\ &\propto |\Sigma|^{-T/2} \exp\left\{ -\frac{1}{2} tr[\Sigma^{-1}\hat{S}] \right\} \exp\left\{ -\frac{1}{2} tr[\Sigma^{-1}(\Phi - \hat{\Phi})'X'X(\Phi - \hat{\Phi})] \right\}, \end{aligned} \tag{3.22}$$

where Y is the $T \times n$ matrix with rows y_t', X is the $T \times k$ matrix with rows x_t' and

$$\hat{\Phi} = (X'X)^{-1}X'Y, \quad \hat{S} = (Y - X\hat{\Phi})'(Y - X\hat{\Phi}).$$

$\hat{\Phi}$ is the maximum-likelihood estimator (MLE) of Φ, and $\hat{S}$ is a matrix with sums of squared residuals.

3.5.2 Prior Implementation with Dummy Observations

We use dummy observations to implement priors. In turn, we will specify the rows of the matrices Y^* and X^*. To simplify the exposition, suppose that $n = 2$ and $p = 2$. Thus, (3.21) reduces to

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \Phi_0 + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma). \tag{3.23}$$

The priors are specified conditional on several hyperparameters. Using a pre-sample, let $\underline{y}$ and $\underline{s}$ be $n \times 1$ vectors of means and standard deviations. The dummy observations are interpreted as observations from the regression model

$$Y^* = X^*\Phi + \varepsilon. \tag{3.24}$$

We begin with dummy observations that generate a prior distribution for Φ_1. Define a 2×2 matrix Y^* and a 2×5 matrix X^* such that

$$Y^* = diag(\theta)diag(\underline{s}), \quad X^* = \left[\begin{array}{ccc} diag(\theta)diag(\underline{s})(H^{-1})' & 0_{2\times 2} & 0_{2\times 1} \end{array}\right] \tag{3.25}$$

where θ is a $n \times 1$ vector with elements θ_j. The dummy observations are plugged into (3.24):

$$\begin{bmatrix} \theta_1\underline{s}_1 & 0 \\ 0 & \theta_2\underline{s}_2 \end{bmatrix} = \begin{bmatrix} \theta_1\underline{s}_1 & 0 & 0 & 0 & 0 \\ 0 & \theta_2\underline{s}_2 & 0 & 0 & 0 \end{bmatrix} \tilde{\Phi} + \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \tag{3.26}$$

where $\tilde{\Phi} = [\Phi_1 H^{-1}, \Phi_2 H^{-1}, ..., \Phi_p H^{-1}, \Phi_0]' = [\tilde{\Phi}_1, \tilde{\Phi}_2, ..., \tilde{\Phi}_p, \Phi_0]'$. According to the distributional assumption, the rows of ε are normally distributed. Thus, we can rewrite the first row of (3.26) as

$$\theta_1\underline{s}_1 = \theta_1\underline{s}_1\tilde{\phi}_{11} + \varepsilon_{11}, \quad 0 = \theta_1\underline{s}_1\tilde{\phi}_{21} + \varepsilon_{12}$$

and interpret it as

$$\tilde{\phi}_{11} \sim N(1, \Sigma_{11}/(\theta_1^2\underline{s}_1^2)), \quad \tilde{\phi}_{21} \sim N(0, \Sigma_{22}/(\theta_1^2\underline{s}_1^2)).$$

$\tilde{\phi}_{ij}$ denotes the element i, j of the matrix $\tilde{\Phi}_1$, and Σ_{ij} corresponds to element i, j of Σ. The hyperparameter θ_1 controls the tightness of the prior.

The prior for Φ_2 is implemented with the dummy observations

$$Y^* = 0_{2\times 2}, \quad X^* = \begin{bmatrix} 0_{2\times 2} & diag(\theta)diag(\underline{s})2^{\mu}(H^{-1})' & 0_{2\times 1} \end{bmatrix}. \tag{3.27}$$

which imply

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \theta_1\underline{s}_1 2^{\mu} & 0 & 0 \\ 0 & 0 & 0 & \theta_2\underline{s}_2 2^{\mu} & 0 \end{bmatrix} \tilde{\Phi} + \varepsilon, \tag{3.28}$$

where the hyperparameter μ is used to scale the prior standard deviations for coefficients associated with y_{t-l} according to $l^{-\mu}$.

A prior for the covariance matrix Σ, centered at a matrix that is diagonal with elements equal to the pre-sample variance of y_t, can be obtained by stacking the observations

$$\begin{bmatrix} \underline{s}_1 & 0 \\ 0 & \underline{s}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Phi + \varepsilon \tag{3.29}$$

ν times. Note that we assume $\nu = 1$.

The remaining sets of dummy observations provide a prior for the intercept Φ_0 and will generate some *a priori* correlation between the coefficients. They favor unit roots and cointegration, which is consistent with the beliefs of many applied macroeconomists, and they tend to improve VAR forecasting performance.

The *sums-of-coefficients* dummy observations are defined by

$$Y^* = H^{-1}diag(\kappa)diag(\underline{r})diag(H\underline{y}) \tag{3.30}$$

where κ is a $n \times 1$ vector with elements κ_j. $\underline{r}$ is the pre-sample mean of r_t defined in (3.4). We re-express the error correction representation (3.9) by

$$\Gamma(L)\Delta y_t = \Lambda H y_{t-1} + \varepsilon_t \tag{3.31}$$

and plug dummy observations into (3.31):

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \kappa_1 \underline{r}_1 & 0 \\ 0 & \kappa_2 \underline{r}_2 \end{bmatrix} \Lambda' + \varepsilon. \tag{3.32}$$

According to the distributional assumption, the rows of ε are normally distributed. Thus, we can rewrite the first row as

$$0 = \lambda_{11}\kappa_1\underline{r}_1 + \varepsilon_{11}, \quad 0 = \lambda_{12}\kappa_2\underline{r}_2 + \varepsilon_{12}$$

and interpret it as

$$\lambda_{11} \sim N(0, \Sigma_{11}/\left(\kappa_1^2\underline{r}_1^2\right)), \quad \lambda_{12} \sim N(0, \Sigma_{22}/\left(\kappa_2^2\underline{r}_2^2\right)).$$

The *co-persistence dummy observations*, proposed by Sims (1993) reflect the belief that when all lagged y_t's are at the level $\underline{y}$, y_t tends to persist at that level:

$$\begin{bmatrix} \omega\underline{y}_1 & \omega\underline{y}_2 \end{bmatrix} = \begin{bmatrix} \omega\underline{y}_1 & \omega\underline{y}_2 & \omega\underline{y}_1 & \omega\underline{y}_2 & \omega \end{bmatrix} \Phi + \varepsilon. \tag{3.33}$$

The strength of these beliefs is controlled by κ and ω. These two sets of dummy observations introduce correlations in prior beliefs about all coefficients, including the intercept, in a given equation.

3.5.3 Marginal Data Density and Hyperparameters

The set of hyperparameters is $\Theta = \{\theta, \mu, \nu, \kappa, \omega\}$. The marginal data density (MDD) is obtained as

$$p_\Theta(Y_{1:T}|Y_{1-p:0}) = \int p(Y_{1:T}|\Phi, \Sigma, Y_{1-p:0})p(\Phi, \Sigma|\Theta)d(\Phi, \Sigma) \tag{3.34}$$

The MDD can be rewritten as

$$p_\Theta(Y_{1:T}|Y_{1-p:0}) = \prod_{t=1}^{T} \int p(y_t|\Phi, \Sigma, Y_{1-p:t-1}) p(\Phi, \Sigma|Y_{1-p:t-1}, \Theta) d(\Phi, \Sigma) \tag{3.35}$$

In other words, maximizing the MDD is interpreted as the maximizing the one-step-ahead prediction performance. Zellner (1971) shows that the MDD can be calculated from the normalization constants of the MNIW distribution:

$$p_\Theta(Y_{1:T}|Y_{1-p:0}) = (2\pi)^{-nT/2} \times \frac{|\bar{X}'\bar{X}|^{n/2}|\bar{S}|^{-(\bar{T}-k)/2}}{|X^{*\prime}X^*|^{n/2}|S|^{-(T^*-k)/2}} \times \frac{2^{n(\bar{T}-k)/2}\prod_{i=1}^{n}\Gamma[(\bar{T}-k+1-i)/2]}{2^{n(T^*-k)/2}\prod_{i=1}^{n}\Gamma[(T^*-k+1-i)/2]} \tag{3.36}$$

where Γ denotes the gamma function. X^* and Y^* are the dummy observations, and $\bar{X} = [X^{*\prime}, X']'$ and $\bar{Y} = [Y^{*\prime}, Y']'$ combine the dummy observations and the actual ones. The length of $\bar{X}$ and $\bar{Y}$ is given by $\bar{T} = T^* + T$. S and $\bar{S}$ are the sum of squared residuals of the MLE estimator. Notice that hyperparameters enter the MDD through the dummy observations.

3.5.4 Restrictions on hyperparamters

In the baseline empirical analysis in Section 3.3 , we impose a restriction (3.20) to reduce the burden of dimensionality when choosing hyperparameters. In this section, we investigate the forecasting performance of other restrictions on hyperparameters.

We begin with a general specification where we allow for different hyperparameters of the overall shrinkage θ_i and the tightness of the error correction representation κ_i for each row of coefficient matrices. In other words, we select the set of hyperparameters

$$\Theta = \{\theta_1, \ldots, \theta_n,\ \mu,\ \nu,\ \kappa_1, \ldots, \kappa_n,\ \omega\} \tag{3.37}$$

where $n = 3$ for the UIP and the PPP and $n = 5$ for the combination of the UIP and the PPP. Table 3.3 reports the MAFE relative to the DP. In the UIP and the PPP cases, the DP reduces the MAFE in the long run compared to the MN and the RW, as in our baseline results. However, in the combination of the UIP and the PPP, the DP leads to larger MAFE than the MN, and the improvement relative to the RW is substantially smaller than our baseline. In this regard, the curse of dimensionality might be especially severe for the combination of the UIP and the PPP, since the number of hyperparmeters is larger than the other specifications.[12]

Table 3.3: Forecasting the exchange rates of US and UK - General case

	Forecast horizon				
	1 year	3 year	5 year	7 year	10 year
	Uncovered interest parity (UIP)				
MN	0.85	1.02	1.34	1.70	2.07
RW	0.87	1.04	1.32	1.67	2.04
	Purchasing power parity (PPP)				
MN	0.78	0.80	1.13	1.41	2.00
RW	0.85	0.87	1.14	1.45	2.02
	UIP & PPP				
MN	0.95	1.04	0.90	0.82	0.85
RW	0.99	1.15	1.05	1.13	1.12

Note: The set of hyperparameters is listed in (3.37). The number of hyperparameters to select is 8 (3 for θ_i, 1 for μ, 3 for κ_i, and 1 for ω) for the UIP and the PPP, and 12 (5 for θ_i, 1 for μ, 5 for κ_i, and 1 for ω) for the combination of the UIP and the PPP. All forecasts are evaluated based on their mean absolute forecast errors (MAFE) relative to the MAFE based on the dynamic priors (DP). Higher ratios indicate an improvement on the DP forecasts.

We next consider several additional assumptions on hyperparameters to our base-

[12]For the combination of the UIP and the PPP, the total number of hyperparameters to select is 12 (5 for θ_i, 1 for μ, 5 for κ_i, and 1 for ω). For the UIP and the PPP, on the other hand, it is 8 (3 for θ_i, 1 for μ, 3 for κ_i, and 1 for ω). Notice that ν is fixed following a convention in the literature.

line restriction (3.20). First, we add a restriction that the overall shrinkage is common for each country. The set of hyperparameters for the UIP and the PPP is

$$\Theta = \{\theta_1, \theta_2,\ \mu,\ \nu,\ \kappa,\ \omega\} \tag{3.38}$$

where $\theta_2 = \theta_3$ and $\kappa_i = \kappa\theta_i$ for $i = 1, 2, 3$. For example, in the UIP case, $\theta_2 = \theta_3$ implies that the tightness of a random-walk behavior of the US nominal interest rate is identical to that of the UK. For the combination of the UIP and the PPP, on the other hand, it is

$$\Theta = \{\theta_1, \theta_2, \theta_4,\ \mu,\ \nu,\ \kappa,\ \omega\} \tag{3.39}$$

where $\theta_2 = \theta_3$, $\theta_4 = \theta_5$, and $\kappa_i = \kappa\theta_i$ for $i = 1, ..., 5$. Notice that θ_2 and θ_3 govern the tightness of a random-walk behavior of the nominal interest rate in each country, whereas θ_4 and θ_5 govern that of the inflation rate. Table 3.4 shows results. Under the additional restriction, the MAFE relative to the MN is slightly less than one, while the other specifications lead to a substantial MAFE reduction in the long run as in our baseline results.

Table 3.4: Forecasting the exchange rates of US and UK - Common overall shrinkage θ for each country, as well as the baseline restriction

	Forecast horizon				
	1 year	3 year	5 year	7 year	10 year
	Uncovered interest parity (UIP)				
MN	0.88	0.92	1.35	1.75	2.17
RW	0.83	0.98	1.31	1.73	2.18
	Purchasing power parity (PPP)				
MN	0.95	1.02	1.53	1.81	1.82
RW	1.01	1.06	1.48	1.93	1.87
	UIP & PPP				
MN	0.96	1.00	0.96	0.90	0.91
RW	1.00	1.13	1.44	1.67	1.69

Note: The set of hyperparameters is listed in (3.38) and (3.39). The number of hyperparameters to select is 5 (2 for θ_i, 1 for μ, 1 for κ, and 1 for ω) for the UIP and the PPP, and 6 (3 for θ_i, 1 for μ, 1 for κ, and 1 for ω) for the combination of the UIP and the PPP. All forecasts are evaluated based on their mean absolute forecast errors (MAFE) relative to the MAFE based on the dynamic priors (DP). Higher ratios indicate an improvement on the DP forecasts.

We also consider a restriction that the tightness of the error correction representation is identical among each variable. The set of hyperparameters are given by

$$\Theta = \{\theta_1, \dots, \theta_n,\ \mu,\ \nu,\ \kappa,\ \omega\} \tag{3.40}$$

where $n = 3$ for the UIP and the PPP and $n = 5$ for the combination of the UIP and the PPP. Table 3.5 reports results. Similar to the previous case, the DP reduces the MAFE in the long run, except for the combination of the UIP and the PPP relative to the MN.

Table 3.5: Forecasting the exchange rates of US and UK - Common shrinkage of the error correction representation κ

	Forecast horizon				
	1 year	3 year	5 year	7 year	10 year
	Uncovered interest parity (UIP)				
MN	0.87	0.90	1.24	1.61	1.99
RW	0.83	1.00	1.33	1.75	2.24
	Purchasing power parity (PPP)				
MN	0.99	0.99	1.08	1.14	1.15
RW	1.01	1.06	1.46	1.93	1.88
	UIP & PPP				
MN	0.98	0.98	0.98	0.92	0.95
RW	1.01	1.13	1.41	1.64	1.65

Note: The set of hyperparameters is listed in (3.40). The number of hyperparameters to select is 6 (3 for θ_i, 1 for μ, 1 for κ, and 1 for ω) for the UIP and the PPP, and 8 (5 for θ_i, 1 for μ, 1 for κ, and 1 for ω) for the combination of the UIP and the PPP. All forecasts are evaluated based on their mean absolute forecast errors (MAFE) relative to the MAFE based on the dynamic priors (DP). Higher ratios indicate an improvement on the DP forecasts.

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