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Abstract

In this paper, we propose a model for futures returns that has the potential to provide both individual investors and firms who have positions in financial and energy commodity futures a valid tail risk management tool. In doing so, we also aim to explore the commonalities between these markets and the degree of financialization of energy commodities. While empirical studies in energy markets embed either leverage or jumps in the futures return dynamics, we show that the introduction of both features improves the ability to forecast volatility as an indicator for risk for both the S&P500 and natural gas futures markets. Unlike most of the existing studies in energy derivative markets based on daily data, our empirical analysis makes use of high-frequency (tick-by-tick) data from the futures markets, aggregated to 10-minute intervals during the trading day. The intraday variation is then utilized to generate daily time series of prices, returns and realized variance. Our analysis shows that overall, the introduction of both leverage and jumps in the SVJL model provides the best forecast for risk in both a VaR and a CVaR sense for investors who have any position in natural gas futures regardless of their degree of risk aversion. In the S&P500 market, the SVJL model provides the most precise forecast of risk in a CVaR sense for risk-averse investors with any position in futures, regardless of their degree of risk aversion.

Focusing on a firm's internal risk management, the introduction of both jumps and leverage in the SVJL model would benefit speculative firms who are short natural gas futures aiming at minimizing tail risk in a VaR sense, as well as speculative firms who are long S&P500 futures and use either VaR or CVaR as financial risk management criteria while wanting to minimize the opportunity cost of capital.

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1 Introduction

In this paper, we propose a model for futures returns that has the potential to provide both individual investors and firms who have positions in financial and energy commodity futures a valid tail risk management tool. In doing so, we also aim to explore the commonalities between these markets and the degree of financialization of energy commodities. Unlike most of the existing studies in energy derivative markets based on daily data, our empirical analysis makes use of high-frequency (tick-by-tick) data from the futures markets, aggregated to 10-minute intervals during the trading day. The intraday variation is then utilized to generate daily time series of prices, returns and realized variance. We estimate stochastic volatility models using a GMM approach based on the moment conditions of the Integrated Volatility derived from high frequency data. While existing empirical studies in energy markets embed either leverage or jumps in the futures return dynamics, we show that the introduction of both features improves the ability to forecast volatility as an indicator for risk for both the S&P500 and natural gas futures markets using both the RMSE and MAE criteria. Our analysis also shows that overall, the introduction of both leverage and jumps in the SVJL model provides the best forecast for risk in both a VaR and a CVaR sense for investors who have any position in natural gas futures regardless of their degree of risk aversion. In the S&P500 market, the SVJL model provides the most precise forecast of risk in a CVaR sense for risk-averse investors with any position in futures, regardless of their degree of risk aversion.

Focusing on a firm's internal risk management, the introduction of both jumps and leverage in the SVJL model would benefit speculative firms who are short natural gas futures aiming at minimizing tail risk in a VaR sense, as well as speculative firms who are long S&P500 futures and use either VaR or CVaR as financial risk management criteria while wanting to minimize the opportunity cost of capital.

2 Literature review

Traditionally, the term *leverage effect* indicates the negative correlation between asset returns and changes in their volatility (see Ait-Sahalia et al.(2013) for an extensive literature review). The interpretation of this effect is intuitive if events that have a negative impact on financial markets would eventually cause an increase in their volatility.

As mentioned by Kristoufek (2014), the original interpretation of the leverage effect was based on Black (1976) who related decreasing expected earnings of the company to a decrease of the market value of the company which drives up the leverage ratio between debt and equity. The negative relationship between returns and volatility was therefore labelled the 'leverage effect'. As market prices are driven by many more factors besides simply expected returns, more recent literature has moved beyond this naïve interpretation. The leverage effect is simply seen as a negative relationship between returns and volatility. Negative news usually increases volatility while driving prices down, resulting in negative returns. This implies that negative shocks to the WTI and S&P500 futures markets are followed by greater volatilities than upward movements of the same magnitude (see a related study on Brent futures by Cheong (2009)). The leverage effect thus is a natural connection of the two characteristics, returns and volatility, of the traded assets. In the recent literature, the negative correlation between returns and volatility is weak but persistent. The causality goes from returns to volatility and not vice versa (Kristoufek 2014, Bollerslev et al., 2006; Bouchaud and Potters, 2001; Bouchaud et al., 2001; Pagan, 1996). Bekaert and Wu (2000) and Ait-Sahalia et al. (2013) study different possible interpretations. The latter authors find that the leverage effect in high frequency data is not statistically significant over short periods, but becomes negative and statistically significant over long periods.

In recent years, the commodity price literature has shown that there is evidence of leverage effects in various energy markets. Chan and Grant (2016), considering lower frequency (weekly) commodity returns conclude that stochastic volatility (SV) models with a moving average component are able to replicate the main features of the data more efficiently than GARCH models. At the same time, they find a significant negative leverage effect in crude oil spot markets. Kristoufek (2014) focuses on the leverage effect in commodity futures markets and provides an extensive literature review in this area.

As a measure of market risk, VaR has been widely developed since its introduction in RiskMetrics by JP Morgan in 1994. It is defined as the maximum potential loss of an underlying asset at a specific probability level over a certain horizon. Despite its popularity, an obvious and distinctive limitation of the VaR approach is that it only specifies the maximum one can lose at a given risk level, but provides no indication for how much more than VaR one can lose if extreme tail events happen. A good alternative is conditional Value-at-Risk (CVaR), which is a coherent risk measure and retains the benefits of VaR in terms of the capability to define quantiles of the loss distribution.

Fan et al. (2008) estimate VaR for crude oil prices using a GED-GARCH approach with daily WTI and Brent prices from 1987 to 2006. They find that this type of model specification does as well as the standard normal distribution at a 95% confidence level. They also test and find evidence for asymmetric leverage effects without modelling them directly. Youssef et al. (2015) evaluate VaR and CVaR for crude oil and gasoline markets using a long memory GARCH-EVT approach. Their findings and backtesting exercise show that crude oil markets are characterized by asymmetry, fat tails and long memory. In the commodity price literature, Kristoufek (2014) and Nomikos and Andriosopoulos (2012), using daily data, find an *inverse leverage effect*, or positive correlation coefficient, in the natural gas market. Larsson and Nossman (2011) find evidence for stochastic volatility and jumps in both the returns and volatility of daily spot prices of WTI crude oil from 1989 to 2009. See Kristoufek (2014) for an extensive literature review on the leverage effect in commodity markets.

The inverse leverage effect arises because positive shocks to natural gas prices have a much more pronounced effect on futures dynamics than negative shocks. As pointed out by Benth and Vos (2013), an *inverse leverage effect* occurs in energy markets when the volatility tends to increase with the level of power prices because of the *negative relationship* *between inventories and prices*: the smaller the inventories available for that specific natural gas, the higher its price volatility (see also Deaton and Leroque, 1992).

Schwartz (1997), Schwartz and Smith (2000), and Casassus and Collin-Dufresne (2005) propose multi-factor models for energy prices where returns are only affected by Gaussian shocks, but they constrain volatility to be constant. Pindyck (2004) examines the volatility of energy spot and futures prices, estimating the standard deviation of their first differences.

Mason and Wilmot (2014) investigate the potential presence of jumps in two key daily natural gas prices: the spot price at the Henry Hub in the US, and the spot price for natural gas at the National Balancing Point in the UK. They find compelling empirical evidence for the importance of jumps in both markets, though jumps appear to be more important in the UK. They fit the data using a GARCH(1,1) jump diffusion process where volatility is time-varying and show that the best fit for natural gas futures is a model with both stochastic volatility and leverage.

We contribute to the current debate by testing for the existence of the leverage effect and the presence of jumps in the context of a near-continuous observation of the processes with the ability to study their volatility in great detail by using high frequency futures returns in the S&P500, natural gas and crude oil markets and by studying the impact of the leverage effect on measures of risk such as VaR and CVaR.

In terms of tail risk management, in the crude oil spot market, it has been shown (see Chen, Zerilli and Baum (2019)) that the introduction of the leverage effect in the traditional stochastic volatility (SV) model with normally distributed errors is capable of adequately estimating risk in a VaR and CVaR sense for conservative oil suppliers in both the WTI and Brent spot markets, while it tends to overestimate risk for more speculative oil suppliers.

While Baum and Zerilli (2016) found evidence for *jumps* in the *crude oil futures market*, this paper represents a generalization and a step forward compared to those results as it finds evidence for jumps and leverage in the S&P500 and natural gas futures markets. It also examines the impact of leverage on risk (in a VaR/CVaR sense) in the three futures markets considered. Compared to Chen, Zerilli and Baum (2018) which presented evidence for *leverage* in the *crude oil spot market* using daily data by estimating the SV models using MCMC techniques, this new paper analyses evidence for both *jumps and leverage* in the *S&P500 and natural gas futures markets and evidence for leverage in the WTI crude oil futures market* using a GMM approach based on the moment conditions of the Integrated Volatility derived from high frequency data. Our paper also examines the impact of jumps and leverage on tail risk management for both individual investors and firms who are focused on managing risk in a VaR/CVaR sense while minimizing their cost of capital.

3 Data

The raw data used in this study are 10-minute aggregations[1] of natural gas, crude oil and S&P500 futures contract transactions-level data provided by TickData, Inc. Industry analysts have noted that to avoid market disruptions, major participants in the futures market roll over their positions from the near contract to the next-near contract over several days before the near contract's expiration date. A continuous price series over contracts, which expire monthly, is created by hypothetically rolling over a position from the near contract to the next-near contract three days prior to expiration of the near contract.

The time series of daily futures returns and the corresponding Realized Variance for these markets are given in Figs. 1 to 6.

S&P500 futures are traded on the CME Group's NYMEX exchange. According to the exchange, S&P 500 futures and options offer a capital-efficient means to manage exposure to the leading large-cap companies of the U.S. stock market. Based on the underlying Standard & Poor's 500 stock index, which is made up of 500 individual stocks representing the market capitalizations of large companies, the S&P 500 Index is a leading indicator of large-cap U.S. equities. S&P500 futures trade in units of $250 x S&P 500 Index.

[1] Jiang and Oomen (2007) apply the GMM method to estimate a SVJ model find similar results when using 10-minute and 5-minute aggregated data. Other research performed with these tick-level data aggregations for crude oil and natural gas have concluded that the choice of 10-minute, 15-minute and 20-minute intervals has minor effects on their findings: e.g., Wolfe and Rosenman (2014).

Henry Hub Natural Gas (NG) futures, traded on the CME Group's NYMEX exchange, allow market participants significant hedging activity to manage risk in the highly volatile natural gas price, which is driven by weather-related demand. According to the exchange, the NG contract is the third-largest physical commodity futures contract in the world by volume.

The futures price is widely used as a national benchmark price for natural gas, which continues to grow as a global and U.S. energy source. Natural gas futures trade in units of 10,000 million British thermal units (mmBtu), which is approximately 10,000,000 cubic feet of gas. Futures prices are quoted in US dollars and cents, with a minimum price increment of $0.001 per mmBtu. At present, 118 consecutive months' contracts may be traded.

Light, sweet crude oil (West Texas Intermediate) began futures trading on the New York Mercantile Exchange (NYMEX) in 1983 and is the most heavily traded commodity future. Crude oil futures trade in units of 1,000 U.S. barrels (42,000 gallons), with contracts dated for 30 consecutive months plus long-dated futures initially listed 36, 48, 60, 72, and 84 months prior to delivery. Additionally, trading can be executed at an average differential to the previous day's settlement prices for periods of two to 30 consecutive months in a single transaction. Crude Oil Futures (CL) are quoted in dollars and cents per barrel.

3.1 Descriptive statistics

In this section we provide a detailed empirical characterization of futures returns and their variance. More specifically, we are interested in considering whether the data are normally distributed, behave in a white noise fashion and have a unit root.

Table 1 provides descriptive statistics for the futures contract returns and their realized variance. Both series exhibit excess kurtosis, while the realized variance series have large skewness coefficients. The Kolmogorov–Smirnov test (Table 2) for normality rejects its null for both series, while the Shapiro–Francia test for normality concurs with those judgements. The Box–Pierce portmanteau (or Q) test for white noise rejects its null for all the series with exception of the natural gas futures returns. Using the Augmented Dickey–Fuller and Phillips-Perron tests, the null hypothesis of a unit root is rejected for all the futures daily returns and corresponding realized variances.

Table 1: **Descriptive Statistics and P-values for the daily futures returns and realised variance for daily futures returns and realized variance 2001-2016**

	mean	se mean	min	max	stdev	skew	kurt
fut ret SP500	0.00	0.00	−1.44	1.35	0.12	0.08	21.14
fut ret NG	0.00	0.01	−2.38	2.42	0.32	0.36	8.71
fut ret WTI	0.00	0.00	−1.64	1.73	0.23	−0.09	7.33
fut rv SP500	0.00	0.00	0.00	0.32	0.01	12.75	258.67
fut rv NG	0.06	0.00	0.00	5.86	0.19	19.15	495.16
fut rv WTI	0.03	0.00	0.00	2.68	0.08	20.95	648.12

Table 2: **Test Statistics and P-values for the daily futures returns and realised variance for daily futures returns and realised variance 2001-2016**

	KSmirnov	p-val	SFrancia	p-val	Q	p-val	DFuller	p-val	PPerron	p-val
fut ret SP500	0.087	0.000	14.078	0.000	156.575	0.000	-64.543	0.000	-64.896	0.000
fut ret NG	0.056	0.000	11.640	0.000	42.208	0.376	-62.209	0.000	-62.221	0.000
fut ret WTI	0.046	0.000	11.097	0.000	79.113	0.000	-62.802	0.000	-62.895	0.000
fut rv SP500	0.354	0.000	18.339	0.000	1.4e+04	0.000	-38.290	0.000	-44.938	0.000
fut rv NG	0.385	0.000	18.618	0.000	766.446	0.000	-55.137	0.000	-57.911	0.000
fut rv WTI	0.360	0.000	18.458	0.000	1495.542	0.000	-52.641	0.000	-55.332	0.000

4 Estimation method

Following Bollerslev and Zhou (2002), who use continuously observed futures prices, we build a conditional moment estimator for stochastic volatility models based on matching the sample moments of *Realized Variance* with population moments of the *Integrated Variance* (see Appendix I for details). In this paper, realized variance is computed as the sum of high-frequency (10-minute interval) intraday squared returns. The returns on futures at time t over the interval $[t-k, t]$ can be decomposed as

$$r(t,k) = \ln F_t - \ln F_{t-k} \tag{1}$$

The *Realized Variance* from the sample is defined as:

$$RV(t,k,n) = \sum_{j=1}^{n \cdot k} r\left(t - k + \frac{j}{n}, \frac{1}{n}\right)^2 \tag{2}$$

In this section, we introduce the four models to be estimated and compared in terms of ability to fit the data, risk measurement and out of sample performance: the Stochastic Volatility model with both jumps and leverage ($SVJL$), the Stochastic Volatility model with jumps (SVJ), the Stochastic Volatility model with leverage (SVL) and the Stochastic Volatility model (SV).

4.1 Stochastic Volatility model with jumps and leverage (SVJL)

In the most comprehensive model, we model futures returns so as to account for a significant interaction with volatility (leverage effect) and for sudden and substantial shocks (jumps). This model is an extension of the Heston (1993) stochastic volatility model with the addition of jumps:[2]

[2] As in Bollerslev and Zhou (2002), the drift of the log price has been set equal to zero because in the series here considered (see details in the Descriptive Statistics tables) the drift is not statistically significant. In a risk neutral setting, from a theoretical point of view, for futures returns the drift is $r - r = 0$. In order to apply this estimation method to other series for which the drift is nontrivial, a drift coefficient could be included in the model.

$$
\begin{aligned}
dp_t &= d\ln(F_t) \\
&= \sqrt{V_t}dW_{1t} + xdPoisson\left(\lambda t\right) && (3) \\
dV_t &= \kappa\left(\theta - V_t\right)dt + \sigma\sqrt{V_t}dW_{2t} && (4)
\end{aligned}
$$

$$
E\left(dW_{1t}dW_{2t}\right) = \rho dt \qquad (5)
$$

$$
x \sim N\left(0, \sigma_x^2\right)
$$

In the original Heston model, there are two Wiener processes, dW_{1t} and dW_{2t}, driving the evolution of returns and volatility and three parameters κ, θ and σ. In this extended model, the two Wiener processes are augmented by a Poisson process that captures jumps in returns. This gives rise to two additional parameters, λ and σ_x, governing the effects of the jump process.

λ is the indicator of the frequency of the jumps: it tells us, on average, how many times we have extreme events (jumps in this case for us are *extreme events*) within the sample. It is the parameter of the Poisson counting process that takes values:

$$
\begin{cases} 1 & \text{when an extreme event happens} \\ 0 & \text{otherwise} \end{cases}
$$

We also allow for leverage effect: $\rho \neq 0$ as a sixth parameter.

We estimate this model over the full sample, imposing the five moment conditions implied by the model in the GMM procedure. The main moment conditions used in this section are

$$
\begin{bmatrix} e_{1J} \\ e_{2J} \\ e_{3J} \\ e_{4J} \\ e_{5J} \end{bmatrix}
$$

augmented using ten lagged counterparts (see Appendices I and III for details). As there are 15 moment conditions and 6 estimated parameters, there are 9 overidentifying restrictions that may be used to evaluate the model for each market. The Hansen's J

statistic indicates that the overidentifying restrictions are valid (for details see Appendix III). As shown in Tables 3, 4 and 5, the estimated parameters of the model are very precisely estimated and take on sensible values from an analytical perspective showing evidence for both jumps and leverage effect for both the S&P500 and the natural gas futures series. However the SVJL model is not a good fit for the WTI crude oil futures market when considering the whole sample. We can therefore conclude that both natural gas and S&P500 futures show evidence for unexpected and substantial changes in returns and they move in the same direction (inverse leverage effect) as volatility in the case of natural gas futures while they move in opposite directions (leverage effect) in the case of S&P500 futures. Inverse leverage effect derives from the fact that positive shocks to natural gas prices have a much more pronounces effect on futures dynamics than negative ones. As pointed out by Benth and Vos (2013), inverse leverage effect occurs in energy markets when the volatility tends to increase with the level of power prices because of the negative relationship between inventories and prices: the smaller the inventories available for natural gas, the higher its price volatility (see also Deaton and Leroque, 1992). Clearly inventories are less of an issue for crude oil.

Jumps in the natural gas market could be seen as a consequence of market deregulation (see Cooper, 1983) or as a result of the financialization of this market.

We now explore special cases of this general model in order to find a suitable fit for the WTI crude oil futures market.

4.2 Stochastic Volatility with Jumps model (SVJ)

This is a special case of the general model where there is no leverage ($\rho = 0$). In this case, these are the four main moment conditions, augmented using eight lagged counterparts (see Appendices I and III for details):

$$\begin{bmatrix} e_{1J} \\ e_{2J} \\ e_{3J} \\ e_{4J} \end{bmatrix}$$

As there are twelve moment conditions (for details see Appendix III) and five estimated parameters, there are seven overidentifying restrictions that may be used to evaluate the model for each market. The Hansen's J statistic indicates that the overidentifying restrictions are valid. As shown in Tables 3, 4 and 5, the three estimated parameters of the model are very precisely estimated (except λ and σ_x for the S&P500 dataset) and take on sensible values from an analytical perspective showing empirical evidence for jumps in the crude oil and natural gas futures markets.

Jumps in the natural gas and crude oil market could be seen as a consequence of market deregulation (see Cooper, 1983, Paul 1978 and Nelson 1983) or as a result of the financialization of these markets.

4.3 Stochastic Volatility with Leverage model (SVL)

In this special case, the correlation coefficient ρ between the shocks that are affecting futures returns and their volatility (*leverage effect coefficient*) is not restricted to be zero but its value is estimated from the data. There are no jumps ($\lambda = 0$ and $\sigma_x = 0$).

In this case, these are the three main moment conditions, augmented using six lagged counterparts (see Appendices I and III for details):

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

We estimate this model over the full sample, imposing the nine moment conditions implied by the data in the GMM procedure.

Additional details about the moment conditions and more specifically about the equations for the leverage effect can be found in Garcia et al. (2011). As shown in Tables 21, 25, 29 all the moment conditions are in accordance with the data and the overall Hansen's J statistic indicates that the overidentifying restrictions are valid. As shown in Tables 3, 4, 5 all estimated parameters of the model are very precisely estimated (κ is significant at 10% level for WTI) and take on sensible values from an analytical perspective.

We find that stochastic volatility models with leverage are effective in fitting the volatility of futures returns for all the three markets. More specifically, we find significant evidence of a *leverage effect* for S&P500 and crude oil markets: a negative shock to returns increases volatility in these markets. In contrast, we find evidence of *inverse leverage effect* for the natural gas market (in line with Kristoufek (2014)).

4.4 Stochastic Volatility model (SV)

This is a special case of the general model where there are no jumps and no leverage ($\lambda = 0, \sigma_x = 0$ and $\rho = 0$).

In this case, these are the two main moment conditions, augmented using four lagged counterparts (see Appendices I and III for details):

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

As there are six moment conditions and three estimated parameters, there are three overidentifying restrictions that may be used to evaluate the model for each market. The Hansen's J statistic indicates that the overidentifying restrictions are valid. As shown in Tables 3 and 4 the three estimated parameters of the model are very precisely estimated (except κ for the WTI dataset) and take on sensible values from an analytical perspective.[3]

[3] In order to implement this estimation, we define the moment conditions and build specific t-tests on the moment conditions.

Table 3: **GMM estimates for the SV, SVJ, SVL, SVJL models for the S&P500 futures: 09/2001–06/2016**

	SV	SVJ	SVL	SVJL
κ	1.507***	0.0869***	0.0424***	0.227***
	(5.87)	(4.58)	(2.92)	(29.99)
θ	0.00398***	0.00994***	0.00649***	0.00376***
	(8.41)	(12.56)	(5.55)	(15.69)
σ	0.283***	0.338***	0.249***	0.12015172***
	(6.55)	(19.62)	(17.96)	(−54.75)
λ		0.979		0.156923006***
		(0.38)		(−10.09)
σ_x		0.0159		0.038120618***
		(0.77)		(−30.60)
ρ			−0.379***	−0.490***
			(−11.29)	(−29.11)
N	3708			

t statistics in parentheses
$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 4: **GMM estimates for the SV, SVJ, SVL, SVJL models for Natural Gas futures: 09/2001–06/2016**

	SV	SVJ	SVL	SVJL
κ	0.923**	0.772***	0.760***	0.0556*
	(2.19)	(4.11)	(3.45)	(1.75)
θ	0.0483***	0.0568***	0.0460***	0.0545***
	(4.36)	(6.15)	(5.60)	(4.97)
σ	1.139**	1.041***	0.925***	0.24293***
	(2.33)	(6.23)	(3.49)	(−3.82)
λ		0.0101***		0.04345***
		(4.03)		(−18.52)
σ_x		0.932***		0.97814
		(32.63)		(−0.53)
ρ			0.201***	0.0495**
			(4.57)	(2.14)
N	3708			

t statistics in parentheses
$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

Table 5: **GMM estimates for the SV, SVJ, SVL models for WTI futures: 09/2001–06/2016**

	SV	SVJ	SVL
κ	0.117	0.0596*	0.0963*
	(1.43)	(1.77)	(1.71)
θ	0.0247***	0.0224***	0.0242***
	(5.75)	(3.45)	(6.43)
σ	0.176**	0.131***	0.162**
	(2.04)	(2.60)	(2.50)
λ		0.0190**	
		(2.44)	
σ_x		0.439***	
		(39.24)	
ρ			−0.276***
			(−3.64)
N	3708		

t statistics in parentheses
$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

5 Robustness check for subsamples

In this section, we perform a robustness check by splitting the entire sample in two subsamples: before and after the Lehman Brothers bankruptcy in mid-September 2008. Within each subsample, the choice of the most appropriate model differs for the energy futures series, perhaps reflecting evolutionary forces in energy markets such as the widespread use of fracking and the resulting increases in natural gas supply. Given the underlying structural changes in the US energy sector, it is not surprising that a model fit over the entire period may not be the best choice over a restricted subsample.

As shown in Table 6, the SVJL model provides the best fit for the S&P500 futures market on the overall sample and on the two subsamples. For the natural gas futures market, the SVJL model provides the best fit for the overall sample and for the pre-crisis subsample, while the SVL model is the most appropriate to fit the post-crisis subsample.

For the WTI crude oil futures market, the SVL model provides the best fit for the overall sample and for the pre-crisis subsample while the SVJL model performs best for the post-crisis subsample.

Table 6: **GMM estimates for S&P500, Natural Gas and WTI futures before and after September 15, 2008 (Lehman Brothers bankruptcy)**

	S&P500	NG	WTI
Before September 15, 2008 $N = 1699$	SVJL	SVJL	SVL
κ	0.137***	0.0871***	0.276***
	(13.27)	(5.55)	(3.87)
θ	0.00331***	0.0836***	0.0328***
	(16.47)	(11.69)	(8.33)
σ	0.0577***	0.5455***	0.343***
	(−57.58)	(−12.32)	(6.36)
λ	0.1325***	0.0966***	
	(−7.37)	(−26.15)	
σ_x	0.0364***	0.4921***	
	(−24.70)	(−48.64)	
ρ	−0.440***	0.0137**	−0.262***
	(−18.40)	(2.20)	(−6.66)
After September 15, 2008 $N = 1990$	SVJL	SVL	SVJL
κ	0.188***	0.0434	0.0137
	(13.22)	(1.41)	(0.75)
θ	0.00807***	0.0286***	0.0181***
	(15.57)	(13.70)	(3.49)
σ	0.2170***	0.0914***	0.0665***
	(−44.44)	(2.62)	(4.51)
λ	0.4098**		0.0298***
	(−1.96)		(2.93)
σ_x	0.0290***		0.140***
	(−14.80)		(8.05)
ρ	−0.351***	0.333***	−0.304**
	(−36.98)	(2.96)	(−2.48)

t statistics in parentheses
$^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$

6 Out-of-sample performance

In this section we compare all the models in terms of forecasting ability both from a risk hedging point of view in a VaR and CVaR sense and also considering the statistical properties of the out of sample simulations. The period we are aiming to forecast is July to December 2016.

6.1 Forecasting ability

In this section we are interested in testing how closely the simulated series, both futures returns and their volatility, resemble the actual data. We compute Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) criteria for the futures return series and for their variance. As an additional measure, we also test the two models on the basis of the predictive accuracy proposed by Diebold and Mariano (1995).

Tables 7 and 8 show the out-of-sample performance of the SV and SVL models for the period July–December 2016 for the futures returns and stochastic variance respectively. The SVL model performs better than the SV model for all the markets considering both the RMSE and MAE criteria for both series. The SVJ model outperforms both the SV and SVL only when forecasting the variance of the returns for the WTI market. For the S&P500 and NG markets, the SVJL is the best choice. Table 9 presents the results of Engle's LM ARCH test on the forecast errors for the SV, the SVL and the SVJL models for all the markets. Considering the series of forecast errors, there is evidence of ARCH effects for the SV, SVJ, SVL and the SVJL models for all the series considered. These results give an opportunity to increase efficiency by modeling ARCH but do not violate any assumptions made when estimating the underlying model. From Table 10, we can see that the Kolmogorov–Smirnov test for normality does not reject its null for the S&P500 market considering all the models. The same conclusion applies for the NG market considering the SVJL model and for the WTI crude oil market considering the SV and SVL models. The Shapiro–Francia test (1972) for normality concurs with those judgments for the forecast errors coming from all the models. The Box–Pierce portmanteau (or Q) test for white noise rejects its null for all the series of forecast errors with the exception of the residuals for the S&P500 market.

Table 7: Out-of-sample performance of SV, SVL and SVJL models: July-December 2016 RMSE and MAE for the returns process.

	SV	SVJ	SVL	SVJL
RMSE				
S&P500	0.08014	0.19278	$\boxed{0.07757}$	0.1104
WTI	0.26398	0.28523	$\boxed{0.26231}$	
NG	0.34547	0.3652	$\boxed{0.33651}$	0.378
MAE				
S&P500	0.06134	0.12978	$\boxed{0.05884}$	0.07281
WTI	0.20573	0.22121	$\boxed{0.20444}$	
NG	0.26411	0.28076	$\boxed{0.25676}$	0.27043

Table 8: Out-of-sample performance of SV, SVL and SVJL models: July-December 2016 RMSE and MAE for the variance process.

	SV	SVJ	SVL	SVJL
RMSE				
S&P500	0.00447	0.013848	0.004074	$\boxed{0.003308}$
WTI	0.021026	$\boxed{0.019163}$	0.020501	
NG	0.066412	0.072497	0.057729	$\boxed{0.03341}$
MAE				
S&P500	0.003215	0.005822	0.002886	$\boxed{0.002195}$
WTI	0.015708	$\boxed{0.014377}$	0.015325	
NG	0.041425	0.046088	0.037161	$\boxed{0.023585}$

6.2 Diebold–Mariano test

This test calculates a measure of predictive accuracy proposed by Diebold and Mariano (1995). We ran the test for each of 350 simulations per model and present summary

Table 9: S&P500, natural gas and WTI futures: Engle's Lagrange multiplier test for autoregressive conditional heteroskedasticity for standardized residuals and squared standardized residuals for SV, SVJ, SVL and SVJL models.

	1 lag	p-val	5 lags	p-val	10 lags	p-val	30 lags	p-val
SV SP res	4.80	0.03	5.20	0.39	6.07	0.81	17.64	0.96
SV SP res squ	0.66	0.42	0.97	0.97	1.31	1.00	12.81	1.00
SV SP J res	0.09	0.76	21.50	0.00	31.89	0.00	5.59	1.00
SV SP J res squ	0.05	0.83	27.44	0.00	36.50	0.00	0.67	1.00
SV SP L res	0.38	0.54	6.07	0.30	7.38	0.69	24.71	0.74
SV SP L res squ	0.00	0.96	1.65	0.90	2.21	0.99	9.59	1.00
SV SP JL res	0.17	0.68	5.11	0.40	7.67	0.66	23.01	0.81
SV SP JL res squ	0.27	0.61	4.30	0.51	5.96	0.82	22.44	0.84
SV NG res	0.01	0.93	0.04	1.00	0.10	1.00	20.19	0.91
SV NG res squ	0.01	0.93	0.04	1.00	0.10	1.00	18.55	0.95
SV NG J res	0.01	0.94	0.04	1.00	0.10	1.00	22.97	0.82
SV NG J res squ	0.01	0.93	0.04	1.00	0.10	1.00	19.40	0.93
SV NG L res	0.05	0.83	0.06	1.00	0.25	1.00	19.37	0.93
SV NG L res squ	0.01	0.92	0.05	1.00	0.11	1.00	13.84	0.99
SV NG JL res	6.68	0.01	7.15	0.21	11.43	0.32	22.10	0.85
SV NG JL res squ	4.05	0.04	4.37	0.50	4.29	0.93	4.76	1.00
SV CL res	0.00	1.00	1.91	0.86	4.71	0.91	17.86	0.96
SV CL res squ	0.15	0.69	0.76	0.98	1.52	1.00	15.42	0.99
SV CL J res	0.00	0.98	2.05	0.84	4.67	0.91	17.12	0.97
SV CL J res squ	0.10	0.75	0.65	0.99	1.30	1.00	10.38	1.00
SV CL L res	0.00	1.00	2.06	0.84	4.75	0.91	17.22	0.97
SV CL L res squ	0.11	0.74	0.75	0.98	1.60	1.00	10.23	1.00

Table 10: S&P500, natural gas and WTI futures: Test statistics and p-values for standardized residuals and squared standardized residuals for SV, SVJ, SVL and SVJL models

	KSmirnov	p-val	SFrancia	p-val	Qtest	p-val
SV SP res	0.094	0.231	3.253	0.001	65.850	0.006
SV SP res squ	0.303	0.000	7.853	0.000	37.832	0.568
SV SP J res	0.296	0.000	8.172	0.000	40.898	0.431
SV SP J res squ	0.452	0.000	9.018	0.000	34.394	0.720
SV SP L res	0.068	0.626	0.928	0.177	85.452	0.000
SV SP L res squ	0.259	0.000	7.420	0.000	43.723	0.316
SV SP JL res	0.072	0.554	0.799	0.212	87.346	0.000
SV SP JL res squ	0.249	0.000	7.090	0.000	39.267	0.503
SV NG res	0.311	0.000	8.799	0.000	6.095	1.000
SV NG res squ	0.483	0.000	9.252	0.000	0.421	1.000
SV NG J res	0.314	0.000	8.809	0.000	5.425	1.000
SV NG J res squ	0.494	0.000	9.254	0.000	0.397	1.000
SV NG L res	0.151	0.007	7.048	0.000	20.472	0.996
SV NG L res squ	0.431	0.000	9.126	0.000	1.917	1.000
SV NG JL res	0.052	0.888	2.377	0.009	34.724	0.706
SV NG JL res squ	0.303	0.000	8.145	0.000	21.069	0.994
SV CL res	0.046	0.958	1.521	0.064	42.030	0.383
SV CL res squ	0.276	0.000	7.644	0.000	34.788	0.704
SV CL J res	0.046	0.957	1.547	0.061	42.792	0.352
SV CL J res squ	0.271	0.000	7.588	0.000	33.762	0.746
SV CL L res	0.046	0.954	1.470	0.071	42.557	0.362
SV CL L res squ	0.263	0.000	7.438	0.000	33.441	0.759

statistics from that set of test results. Given an actual series and two competing predictions, one can apply a loss criterion (such as mean squared error or mean absolute error) and then calculate a number of measures of predictive accuracy that allow the null hypothesis of equal accuracy to be tested. Table 11 reports the results for the futures returns and corresponding variance for all the markets. The test rejects the null that the two models are equally capable in terms of their MSEs at the 95% level of confidence. For the simulations in which the test rejects equal forecast accuracy, we can compare the mean MSE for the two models.

While the results are not conclusive for the futures returns series (see Table 11), in the case of the corresponding variance, we can observe an high number of rejections and for the S&P500 and WTI realized variance of the futures returns the SVL model compared to the SV model has the smaller MSE for all the markets. In summary, for the S&P500 and WTI realized variance of the futures returns, the SVL model has the smaller mean MSE for those simulations in which the Diebold–Mariano test rejects its null hypothesis of equal forecast accuracy. According to Table 12, the SVL model, compared to the SVJ model, shows an higher forecasting accuracy for the S&P500 futures returns and for the natural gas futures variance. Considering Table 13, for the S&P500 and NG realized variance of the futures returns, the SVJL model has the smaller mean MSE compared to the SVL model for those simulations in which the Diebold–Mariano test rejects its null hypothesis of equal forecast accuracy.

Table 11: Diebold–Mariano test for futures returns and their variance SV vs SVL: comparison of forecast accuracy over 350 out-of-sample predictions

	futures returns			variance of futures returns		
	SP500	WTI	NG	SP500	WTI	NG
SV beats SVL	0	0	0	0	0	0
SVL beats SV	39	31	70	$\boxed{196}$	$\boxed{223}$	166
Test inconclusive	311	319	280	154	127	184
Total	350	350	350	350	350	350

Table 12: Diebold–Mariano test for futures returns and their variance SVJ vs SVL: comparison of forecast accuracy over 350 out-of-sample predictions

	futures returns			variance of futures returns		
	SP500	WTI	NG	SP500	WTI	NG
SVJ beats SVL	0	0	0	0	0	0
SVL beats SVJ	$\boxed{336}$	45	60	165	69	$\boxed{304}$
Test inconclusive	14	305	290	185	281	46
Total	350	350	350	350	350	350

Table 13: Diebold–Mariano test for futures returns and their variance SVL vs SVJL: comparison of forecast accuracy over 350 out-of-sample predictions

	futures returns		variance of futures returns	
	SP500	NG	SP500	NG
SVL beats SVJL	0	0	0	0
SVJL beats SVL	71	8	$\boxed{258}$	$\boxed{199}$
Test inconclusive	279	342	92	151
Total	350	350	350	350

7 Forecasting VaR and CVaR

In this section we want to explore whether the forecasts provided by the two models are able to provide a financial investor with a valid tool for hedging risk. Therefore, we derive VaR and CVaR using the simulated volatility series when fixing the parameter values at the GMM estimates and we then backtest them against the actual market futures returns. We perform this analysis for the SV, SVJ, SVL and SVJL models only are they are the best contenders overall.

As a measure of market risk, VaR has been widely developed since its introduction in RiskMetrics by JP Morgan (1994). It is defined as the maximum potential loss of an underlying asset at a specific probability level over a certain horizon. Despite its popularity, an obvious and distinctive limitation of the VaR approach is that it only specifies the maximum one can lose at a given risk level, but provides no indication for how much more than VaR one can lose if extreme tail events happen. This may lead to an equivalent VaR estimate for two different positions, though they have completely different risk exposures. Artzner et al. (1999) proposed the concept of coherent risk measure, which has become the paradigm of risk measurement. A good alternative is conditional Value-at-Risk (CVaR), which is a coherent risk measure and retains the benefits of VaR in terms of the capability to define quantiles of the loss distribution.

Although the CVaR approach has been widely used for risk analysis, the implementation of backtesting for CVaR models is much harder than for VaR models. Nevertheless, formal backtesting methods can be found in literature, such as the most commonly used approach zero-mean residual test by McNeil and Frey (2000) which relies on bootstrapping, the censored Gaussian method by Berkowitz (2001) and the functional delta approach by Kerkhol and Melenverg (2004).[4] However, applying these methods tend to be difficult and overly complex. The application of these methods is based upon the realization of specific conditions, hence it is possible to backtest CVaR only under specific circumstances. Kerkhol and Melenverg (2004) suggest a viable and simpler alternative to backtesting CVaR on the basis of equal quantiles, after finding a nominal risk level $\widehat{\alpha}$ for CVaR.

We now focus on the models for which we have the most evidence of a substantial impact of the introduction of leverage and jumps on the prediction accuracy of the model. In order to classify the competing models, we follow a two-stage model evaluation procedure where in the first stage models are selected in terms of their statistical accuracy (the backtesting stage), while in the second stage the surviving models are evaluated in terms of their "efficiency" (the efficiency stage).[5]

Stage 1: *Backtesting the VaR and CVaR models*

In order to backtest the accuracy of the estimated VaRs, it is necessary to calculate the empirical failure rates for the estimates. The *Failure Rate* (FR) or *violation rate*, computes the ratio of the number of times returns exceed the estimated VaRs over the total number of observations. The model is said to be correctly specified if the calculated ratio is equal to the pre-specified VaR level (i.e. $\alpha = 5\%$ and $\alpha = 1\%$). If the Failure Rate is higher than α , we can conclude that the model underestimates the risk, and vice versa.

The failure rate $FRVaR_s$ for the downside risk of a long trading position, is calculated as the percentage of negative returns that are smaller than the left quantile VaRs, while the failure rate $FRVaR_d$ for the upside risk of a short trading position is the ratio of positive

[4] A comprehensive discussion of various $CVaR$ backtesting methodologies as well as their implementations at different circumstances is provided by Wimmerstedt (2015).

[5] For details see Sarma et al. (2003).

returns larger than the right quantile VaRs. We define $FRVaR_s$ and $FRVaR_d$ as follows:

$$FRVaR_d = \frac{1}{T}\sum_{t=1}^{T} I_t\left(y_t < -VaR_{d,t}\right)$$
$$FRVaR_u = \frac{1}{T}\sum_{t=1}^{T} I_t\left(y_t > VaR_{u,t}\right)$$

where $VaR_{d,t}$ and $VaR_{u,t}$ are the estimated $VaRs$ for downside and upside risk at time t for a given confidence interval, T is the number of observations and $I_t(\cdot)$ is the indicator function which is defined as:

$$Downside : I_t = \begin{cases} 1 & if \quad y_t < -VaR_{d,t} \\ 0 & if \quad y_t \geq VaR_{u,t} \end{cases}$$
$$Upside : I_t = \begin{cases} 1 & if \quad y_t > VaR_{u,t} \\ 0 & if \quad y_t \leq VaR_{d,t} \end{cases}$$

There are three formal tests based on the above criteria to backtest the VaR estimates. The unconditional coverage test (LR_{uc}), proposed by Kupiec (1995), examines whether the null hypothesis $H_0 : FR = \alpha$ can be satisfied. A good performance of the VaR model should be accompanied by accurate unconditional coverage, that is, the failure rate is statistically expected to be equal to the prescribed VaR level α.

The method proposed by Kupiec (1995) is capable to test the overestimates or underestimates of a VaR model. It does not, however, consider whether the exceptions are scattered or if they appear in clusters.[6] In order to examine whether the VaR violations are serially uncorrelated over time, Christoffersen (1998) proposes the independent likelihood ratio test (LR_{ind}).

In addition, a more selective conditional coverage test (LR_{cc}) which jointly examines the unconditional coverage and independence of violations has been developed by Christoffersen (1998). This test investigates if the failure rate is equal to the expected prescribed risk level and if the exceptions are independently distributed over time. The null hypothesis

[6]Kupiec's (1995) approach is an unconditional test. On the other hand, we need to conditionally examine the VaR performance under the time-varying volatility framework. A good VaR model should be able to reflect this dynamic behavior, which implies that losses exceeding VaR should be independent and unpredictable.

for this test is that the exceptions are independent and that the expected failure rate is equal to the prescribed risk level.

Stage 2: *Efficiency measures*

Lopez (1998, 1999) was the first to propose the comparison between VaR models on the basis of their ability to minimize some specific loss function which reflected a specific objective of the risk manager. Adhering to the Basel Committee's guidelines, supervisors are not only concerned with the number of violations in a VaR model but also with the magnitude of these violations (Basel Committee on Banking Supervision, 1996a,b). In order to address this aspect, following Sarma et al. (2003), we compare the relevant models in terms of the Regulatory Loss Function (RLF) which focuses on the magnitude of the failure and in terms of the Firm's Loss Function (FLF) which, while giving relevance to the magnitude of failures, imposes an additional penalty related to the opportunity cost of capital.[7] We use a non-parametric sign test to check the ability the relevant VaR models to minimize these loss functions.[8]

7.1 Out-of-sample VaR and CVaR estimations

Stage 1: *Backtesting the VaR and CVaR models*

As in Fan et al. (2008) and Youssef at al. (2015), we test the forecasting ability of the SV, SVL, SVJ and SVJL models by computing the out-of-sample VaR and CVaR and comparing them with the actual returns. The out-of-sample VaR predictions are generated from simulated volatilities based on the GMM parameter estimates. A criterion for evaluating our results comes from the consideration that a conservative investor (see for example, Zhao et al. (2015) and Hung et al. (2008)) would choose a greater confidence level and estimate a relatively greater risk (corresponding to $\alpha = 1\%$ in the VaR definition), while a more speculative investor would estimate a smaller risk and face a relatively smaller confidence level, corresponding to $\alpha = 5\%$ in the VaR definition. In order to backtest the

[7] This criterion penalizes large failures more than small failures (see Sarma et al., 2003).

[8] For the sign test see Lehmann (1974), Diebold and Mariano (1995), Hollander and Wolfe (1999) and Sarma et al. (2003).

accuracy of the estimated VaRs and CVaRs, the three formal tests described in the previous section are applied to the model forecasts using empirical failure rate criteria.

Because the LR_{cc} test is the most rigorous among the three tests considered, we will focus on the outcomes of this test. When all the models pass the LR_{cc} test, they are also compared on the basis of the Failure rate (FR): the model whose FR is the closest to α for VaR or $\widehat{\alpha}$ for CVaR, is considered to be the most precise in forecasting risk. If the ratio is greater than α for VaR or $\widehat{\alpha}$ for CVaR, we conclude that the model underestimates risk and vice versa.

Table 14 reports the out-of-sample VaR backtesting results. For the S&P500 futures market, the SV, SVL and SVJL models adequately forecast risk in a VaR sense for risk averse and speculative investors who are long futures but the SV model shows the highest precision. For the WTI futures market, the SV, the SVL and the SVJ models adequately forecast risk in a VaR sense for risk-averse and speculative investors who are long futures. For speculators, the SV and SVL models are more accurate than the SVJ model. These models are also appropriate for risk-averse and speculative investors who are short futures. For speculative investors, the SV model shows the highest precision. For the natural gas futures market, the SVJL model is superior in forecasting risk in a VaR sense for investors holding any position in futures regardless of their degree of risk aversion.

Table 15 reports the out-of-sample CVaR backtesting results.[9] For the S&P500 futures market, both the SV and SVL models adequately forecast risk in a CVaR sense for risk-averse investors with any position in futures, with the SVL model being the most precise. The SV model is more precise for speculative investors who are long futures. None of the four models is able to forecast risk in a CVaR sense for speculative investors who are short futures. For the WTI futures market, both the SV and SVL models adequately forecast risk in a CVaR sense with similar precision, implying that the impact of leverage on tail risk management is not relevant. For the natural gas futures market, neither the SV model

[9]NA denotes uncomputable statistics for the corresponding LR test due to zero failure rate: it means that the model considered provides very accurate CVaR forecasts.

nor the SVL model adequately forecast risk in a CVaR sense.

Table 16 presents a summary of the main conclusions that we can draw from the out-of-sample VaR/CVaR backtesting results.

We classify the results of a test inconclusive when multiple models show the same results in terms of the statistical significance and the failure rate. We declare that none of the models is adequate in the cases where none of the models passes the LR_{cc} test. Overall, the introduction of both leverage and jumps (SVJL model) provides the best forecast for risk in both a VaR and a CVaR sense for investors who have any position in natural gas futures regardless of their degree of risk aversion. In the S&P500 market, the SVJL model provides the most precise forecast of risk in a CVaR sense for risk-averse investors with any position in futures, regardless of their degree of risk aversion.

Table 14: **Out of sample VaR backtesting results using Simulated Volatilities at different risk levels**: α equal to 5% and 1% represent prescribed VaR level corresponding to 95% and 99% CI respectively, LR_{uc} columns show p-values of Kupiec's (1995) unconditional coverage test, LR_{ind} columns are $p - values$ of Christoffersen's (1998) independent test and LR_{cc} columns are p-values of Christoffersen's (1998) conditional coverage test, * denotes significance at its corresponding risk level.

α		Failure rate			LR_{cc}			LR_{uc}			LR_{ind}		
		SP	CL	NG	SP	CL	NG	SP	CL	NG	SP	CL	NG
SV													
5%	$VaR_{l,t}$	$\boxed{4.06\%}$	7.32%	13.0%	0.69	0.24	0*	0.62	0.27	0*	0.51	0.23	0.93
	$VaR_{s,t}$	$\boxed{7.32\%}$	$\boxed{7.32\%}$	14.63%	0.03*	0.24	0*	0.27	0.27	0*	0.6	0.23	0.02*
1%	$VaR_{l,t}$	0.81%	0.81%	6.50%	0.96	0.96	0*	0.83	0.83	0*	0.9	0.9	0.52
	$VaR_{s,t}$	3.25%	4.06%	6.50%	0*	0.03	0*	0.05	0.01	0*	0.06	0.51	0.45
SVJ													
5%	$VaR_{l,t}$	13.82%	8.13%	8.13%	0*	0.14	0.14	0.01*	0.18	0.83	0*	0.13	0.30
	$VaR_{s,t}$	18.67%	9.76%	11.382%	0*	0.03*	0.01*	0.61	0.10	0.73	0*	0.02*	0.02*
1%	$VaR_{l,t}$	8.94%	0.81%	3.252%	0*	0.83	0.05	0.01*	0.9	0.60	0*	0.96	0.12
	$VaR_{s,t}$	13.01%	5.69%	4.878%	0*	0*	0*	0.13	0.35	0.27	0*	0*	0*
SVL													
5%	$VaR_{l,t}$	2.44%	7.32%	8.94%	0.32	0.24	0.17	0.15	0.27	0.07	0.7	0.23	0.99
	$VaR_{s,t}$	4.88%	8.13%	11.38%	0.02*	0.13	0.01*	0.95	0.14	0*	0.21	0.18	0.63
1%	$VaR_{l,t}$	0.81%	0.81%	5.69%	0.96	0.96	0*	0.83	0.83	0*	0.9	0.9	0.39
	$VaR_{s,t}$	4.06%	4.06%	4.06%	0*	0.03	0.03	0.01	0.01	0.01	0.13	0.51	0.51
SVJL													
5%	$VaR_{l,t}$	2.44%		$\boxed{4.88\%}$	0.32		0.7	0.15		0.95	0.6973		0.43
	$VaR_{s,t}$	8.13%		$\boxed{6.50\%}$	0.02*		0.40	0.14		0.4636	0.75		0.29
1%	$VaR_{l,t}$	0.81%		$\boxed{1.63\%}$	0.96		0.77	0.83		0.5223	0.9		0.8
	$VaR_{s,t}$	2.44%		$\boxed{1.63\%}$	0.01*		0.77	0.17		0.5223	0.75		0.8

Table 15: **Out of sample CVaR backtesting results using Simulated Volatilities at different nominal risk levels**. $\widehat{\alpha}$ of 1.96% and 0.38% represent nominal $CVaR$ level corresponding to 95% and 99% CI respectively, LR_{uc} columns show p-values of Kupiec's (1995) unconditional coverage test, LR_{ind} columns are $p-values$ of Christoffersen's (1998) independent test and LR_{cc} columns are $p-values$ of Christoffersen's (1998) conditional coverage test, * denotes significance at its corresponding risk level.

$\widehat{\alpha}$		Failure Rate			LR_{cc}			LR_{uc}			LR_{ind}		
		SP	CL	NG	SP	CL	NG	SP	CL	NG	SP	CL	NG
SV													
1.96%	$CVaR_{l,t}$	1.6%	0.8%	11.4%	0.91	0.57	0*	0.78	0.3	0*	0.8	0.9	0.57
	$CVaR_{s,t}$	3.2%	4.1%	9.8%	0*	0.26	0*	0.34	0.14	0*	0.07	0.5	0.93
0.38%	$CVaR_{l,t}$	0.8%	0.8%	5.7%	0.78	0.78	0*	0.5	0.5	0*	0.9	0.9	0.39
	$CVaR_{s,t}$	2.4%	2.4%	4.9%	0.04	0.042	0*	0.01	0.01	0*	0.7	0.7	0.47
SVJ													
1.96%	$CVaR_{l,t}$	0.11	0.02	0.04	0*	0.78	0.14	0.03	0.80	0.51	0*	0.92	0.26
	$CVaR_{s,t}$	0.14	0.06	0.07	0*	0.02*	0*	0.20	0.36	0.68	0*	0.03	0*
0.38%	$CVaR_{l,t}$	0.07	0.01	0.02	0*	0.50	0.01	0.14	0.90	0.70	0*	0.78	0.04
	$CVaR_{s,t}$	0.12	0.04	0.05	0*	0*	0*	0.10	0.51	0.27	0*	0.00	0*
SVL													
1.96%	$CVaR_{l,t}$	0.8%	0.8%	6.5%	0.6	0.57	0*	0.3	0.3	0*	0.9	0.9	0.5
	$CVaR_{s,t}$	4.1%	4.1%	7.3%	0*	0.26	0*	0.14	0.14	0*	0.13	0.51	0.26
0.38%	$CVaR_{l,t}$	0%	0.8%	3.2%	NA	0.78	0*	0.33	0.5	0*	NA	0.9	0.1
	$CVaR_{s,t}$	1.6%	2.4%	4.1%	0.23	0.04	0*	0.1	0.01	0*	0.8	0.7	0.5
SVJL													
1.96%	$CVaR_{l,t}$	1.6%		$\boxed{4.1\%}$	0.92		0.26	0.78		0.14	0.8		0.51
	$CVaR_{s,t}$	2.4%		$\boxed{4.1\%}$	0.02*		0.26	0.71		0.14	0.75		0.51
0.38%	$CVaR_{l,t}$	0%		$\boxed{0.8\%}$	NA		0.78	0.33		0.5	NA		0.9
	$CVaR_{s,t}$	$\boxed{0\%}$		$\boxed{1.6\%}$	NA		0.24	0.33		0.1	NA		0.8

Table 16: **Out of sample VaR / CVaR backtesting interpretation**: in this table "*best*" means that either only one model is statistically adequate to forecast risk or one of the models is more precise than the other in cases where multiple models are adequate; "test inconclusive" means that all the three models can properly forecast risk with exactly the same precision.

	Risk averse short futures			Risk averse long futures			Speculative short futures			Speculative long futures		
	SP500	WTI	NG	SP500	WTI	NG	SP500	WTI	NG	SP500	WTI	NG
VaR												
SV best		$\boxed{\checkmark}$						$\boxed{\checkmark}$		$\boxed{\checkmark}$		
SVJ best												
SVL best		$\boxed{\checkmark}$										
SVJL best			$\boxed{\checkmark}$			$\boxed{\checkmark}$			$\boxed{\checkmark}$			$\boxed{\checkmark}$
Test inconclusive				$\boxed{\checkmark}$	$\boxed{\checkmark}$						$\boxed{\checkmark}$	
None adequate	$\boxed{\checkmark}$						$\boxed{\checkmark}$					
CVaR												
SV best		$\boxed{\checkmark}$						$\boxed{\checkmark}$		$\boxed{\checkmark}$		
SVJ best											$\boxed{\checkmark}$	$\boxed{\checkmark}$
SVL best		$\boxed{\checkmark}$		$\boxed{\checkmark}$				$\boxed{\checkmark}$				
SVJL best	$\boxed{\checkmark}$		$\boxed{\checkmark}$	$\boxed{\checkmark}$		$\boxed{\checkmark}$			$\boxed{\checkmark}$	$\boxed{\checkmark}$	$\boxed{\checkmark}$	$\boxed{\checkmark}$
Test inconclusive					$\boxed{\checkmark}$							
None adequate							$\boxed{\checkmark}$					

Stage 2: *Efficiency measures.*

Table 17 compares the best performing models within the VaR backtesting process using the Regulatory loss function (RLF) and Firm's loss function (FLF) as ranking criteria. Panel A presents the average loss values for the RLF and the FLF for the competing models at various risk levels in the three markets. The models with the lowest average loss values are underlined. Panel B reports the standardized sign statistics values. S_{AB} denotes the standardized sign statistics with null of *"non-superiority"* of the SVL model over the SVJL model while S_{BA} represents the standardized sign statistics with null of *"non-superiority"* of the SVJL model over the SVL model. S_{CD} denotes the standardized sign statistics with null of *"non-superiority"* of the SVL model over the SVJ model while S_{DC} represents the standardized sign statistics with null of *"non-superiority"* of the SVJ model over the SVL model. "*" denotes significance at the corresponding level.

SVL vs SVJL

The results in Panel A show that the SVL model achieves a lower average loss than the SVJL model under the RLF approach while the SVJL model scores a lower average loss under the FLF approach. To address the statistical significance of the losses, we report the values of the standardized sign test in Panel B. For the RLF criterion, the competing models are not significantly different from each other. Under the FLF criterion, the SVJL model is significantly better than the SVL model for firms who are long S&P500 futures and for firms who are short natural gas futures at a 95% confidence level. The SVL model outperforms the SVJL model only for firms who are short S&P500 futures at a 95% confidence level.

SVL vs SVJ

The results in Panel A indicate that, under the RLF criterion, the SVL model is more likely to achieve lower average losses than the SVJ model for financial regulators who focus on the risk affecting long positions in futures, while the SVJ model has the potential to achieve a smaller average loss compared to the SVL model for financial regulators who focus on the risk affecting short positions in futures. Considering the FLF approach, firms who use the VaR criterion for tail risk management while minimizing the opportunity cost of capital in the S&P500 and WTI crude oil futures markets should prefer the SVJ model while firms operating in the natural gas futures market would be better off adopting the SVL model.

In order to examine the statistical significance of the losses, we report the values of the standardized sign test in Panel B. For the RLF criterion, the competing models are not significantly different from each other. Under FLF criterion, the SVJ model is significantly better than the SVL model for firms who are long futures in the SP&500 and WTI markets at both 95% and 99% confidence level. On the contrary, the SVL model is significantly better than the SVJ model for firms who are short futures in the SP&500 and WTI markets at both the 95% and 99% levels of confidence.

Our results indicate that under the RLF criterion, financial regulators who are interested in minimizing VaR in all the futures markets considered, would be indifferent to the choice of models. Under the FLF criterion, the introduction of both jumps and leverage in the SVJL model would benefit speculative firms who are long S&P500 futures and speculative firms who are short natural gas futures and use VaR for risk management while wanting to minimize the opportunity cost of capital. Under the same logic, firms who are short S&P500 or WTI crude oil futures would be better off considering the SVL model with leverage only, while firms who are long WTI crude oil futures and those who are long S&P500 futures would be better off by adopting the SVJ model for their VaR forecasts.

Table 18 presents the summary results of RLF and FLF loss functions for the models chosen at the CVaR backtesting stage. Its interpretation is similar to the one described for Table 17 but it is in terms of CVaR.

SVL vs SVJL

Under the RLF criterion, both the two models score similarly. Under the FLF criterion, the SVJL model is more likely to perform better than the SVL model for both firms who are short and long futures. Similarly, the standardized sign test values for the RLF criterion in Panel B indicate that there are no significant differences between the two models. The

SVJL model is more likely to perform better than SVL only for firms who are long S&P500 futures at a 1.96% nominal level while SVL scores better for firms who are short S&P500 futures at a 1.96% nominal level.

SVL vs SVJ

There is no absolute advantage of one over the other under the RLF criterion. Under the FLF criterion, the SVJ model performs better than the SVL model for firms who hold any positions in the SP&500 and the WTI futures markets. Similarly, the standardized sign test values for the RLF criterion in Panel B indicate that there are no significant differences between the two models. Under the FLF criterion, the SVJ model is significantly better than the SVL model for firms who are long futures in the SP&500 and WTI markets at both the 95% and 99% confidence levels. On the contrary, the SVL model is significantly better than the SVJ model for firms who are short futures in the SP&500 and WTI markets at the 95% and 99% levels of confidence.

Our results indicate that under the RLF criterion, financial regulators who are interested in minimizing CVaR in all the futures markets considered, would be indifferent to the choice of models. Under the FLF criterion, the introduction of both jumps and leverage in the SVJL model would benefit speculative firms who are long S&P500 futures (at 5% significance level), use CVaR for risk management while wanting to minimize the opportunity cost of capital. Under the same logic, for both VaR and CVaR forecasting, firms who are short S&P500 or WTI crude oil futures would be better off considering the SVL model with leverage only while firms who are long WTI crude oil futures and those who are long S&P500 futures would be better off by adopting the SVJ model.

Table 17: **Out of sample RLF and FLF loss function approach applied to the models surviving the VaR Backtesting stage.**

Volatility models and VaR methods		RLF						FLF					
		5%			1%			5%			1%		
Panel A: Average loss values													
		S&P500	CL	NG	S&P500	CL	NG	S&P500	CL	NG	S&P500	CL	NG
SVL	Long	$\underline{0}$		$\underline{0.03}$	$\underline{0}$		$\underline{0.02}$	0.01		0.02	0		0.01
	Short	0		$\underline{0.07}$	0		$\underline{0.11}$	−0.01		$\underline{-0.01}$	0		$\underline{-0.01}$
SVJL	Long	0		0.04	0		0.04	$\underline{0.01}$		$\underline{0.02}$	$\underline{0}$		$\underline{0.01}$
	Short	$\underline{0}$		0.09	$\underline{0}$		0.2	$\underline{0}$		−0.01	$\underline{0}$		0
SVL	Long	$\underline{0.00}$	$\underline{0.01}$	$\underline{0.03}$	$\underline{0.00}$	$\underline{0.01}$	$\underline{0.02}$	0.01	0.01	$\underline{0.02}$	0.00	0.00	$\underline{0.01}$
	Short	0.00	0.05	0.07	$\underline{0.00}$	0.04	0.11	−0.01	−0.01	$\underline{-0.01}$	0.00	0.00	$\underline{0.00}$
SVJ	Long	0.00	0.01	0.04	0.00	0.02	0.05	$\underline{0.00}$	$\underline{0.01}$	0.02	$\underline{0.00}$	$\underline{0.00}$	0.01
	Short	$\underline{0.00}$	$\underline{0.05}$	$\underline{0.06}$	0.00	$\underline{0.04}$	$\underline{0.09}$	$\underline{0.00}$	$\underline{-0.01}$	−0.01	$\underline{0.00}$	$\underline{0.00}$	0.00
Panel B: Sign statistics													
S_{AB}	Long	10.73		10.55	10.91		10.91	2.25		−1.53	2.25		−1.35
	Short	10.19		10.55	11.09		11.09	−2.07*		2.07	−2.07		2.07
S_{BA}	Long	10.73		9.29	10.91		9.82	−2.07*		1.71	−2.07		1.53
	Short	10.19		8.75	10.37		10.19	2.25		−1.89*	2.25		−1.89
S_{CD}	Long	8.21	9.83	9.47	9.11	10.91	10.37	4.42	2.61	−1.35	4.24	2.98	−1.17
	Short	7.12	9.29	9.47	8.39	10.19	10.37	−4.24*	−2.98*	1.17	−4.24*	−2.98*	1.17
S_{DC}	Long	10.73	10.55	9.83	10.91	11.09	10.19	−4.24*	−2.43*	1.53	−4.06*	−2.80*	1.35
	Short	10.55	10.37	8.93	10.73	10.73	10.37	4.42	3.16	−0.99	4.42	3.16	−0.99

Table 18: **Out of sample RLF and FLF loss function approach applied to the models surviving the CVaR Backtesting stage**

Volatility models and CVaR methods		RLF						FLF					
		1.96%			0.37%			1.96%			0.37%		
Panel A: Average loss values													
		S&P500	CL	NG	S&P500	CL	NG	S&P500	CL	NG	S&P500	CL	NG
SVL	Long	$\underline{0}$		$\underline{0.02}$	NA		$\underline{0.02}$	0		0.01	0		0
	Short	0		$\underline{0.07}$	0		$\underline{0.08}$	0		$\underline{0}$	0		0
SVJL	Long	0		0.02	NA		0.04	$\underline{0}$		$\underline{0.01}$	$\underline{0}$		$\underline{0}$
	Short	$\underline{0}$		0.1	NA		0.14	$\underline{0}$		0	$\underline{0}$		$\underline{0}$
SVL	Long	$\underline{0.00}$	0.02	$\underline{0.02}$	NA	$\underline{0.00}$	$\underline{0.02}$	0.00	0.01	$\underline{0.01}$	0.00	0.00	$\underline{0.00}$
	Short	$\underline{0.00}$	0.06	0.07	$\underline{0.00}$	0.04	0.08	0.00	0.00	$\underline{0.00}$	0.00	0.00	$\underline{0.00}$
SVJ	Long	0.00	$\underline{0.02}$	0.05	0.00	0.01	0.04	$\underline{0.00}$	$\underline{0.01}$	0.01	$\underline{0.00}$	$\underline{0.00}$	0.00
	Short	0.00	$\underline{0.05}$	$\underline{0.07}$	0.00	$\underline{0.03}$	$\underline{0.07}$	$\underline{0.00}$	$\underline{0.00}$	0.00	$\underline{0.00}$	$\underline{0.00}$	0.00
Panel B: Sign statistics													
S_{AB}	Long	10.91		10.73	11.09		11.09	2.43		−1.17	2.25		−1.53
	Short	11.09		10.91	11.09		11.09	−2.07*		2.07	−2.07		2.07
S_{BA}	Long	10.91		9.65	11.09		10.37	−2.25*		1.35	−2.07		1.71
	Short	10.37		9.47	10.73		10.19	2.25		−1.89	2.25		−1.89
S_{CD}	Long	8.93	10.73	10.37	9.47	10.91	10.55	4.60	2.98	−1.17	4.24	2.98	−1.17
	Short	8.21	10.19	9.83	8.39	10.37	10.37	−4.24*	−2.98*	1.17	−4.24*	−2.98*	1.17
S_{DC}	Long	10.91	11.09	10.19	11.09	11.09	10.55	−4.42*	−2.80*	1.35	−4.06*	−2.80*	1.35
	Short	10.73	10.73	9.65	10.91	10.91	10.37	4.42	3.16	−0.99	4.42	3.16	−0.99

8 Conclusions

In this paper we have proposed a model for futures returns that provides both individual investors and firms who have positions in financial and energy commodity futures a valid tail risk management tool.

Our paper contributes to the existing literature, which is generally based on lower frequency data, by examining the informational content of high-frequency data using a Generalized Method of Moments (GMM) approach. While existing literature examines models for commodity market futures returns with either leverage or jumps, we find that a stochastic volatility model which embeds both features, the SVJL model, is effective in fitting the volatility of natural gas and stock indexes futures returns. While empirical studies in energy markets embed either leverage or jumps in the futures return dynamics, we show that the introduction of both features improves the ability to forecast volatility as an indicator for risk for both the S&P500 and natural gas futures markets. Unlike most of the existing studies in energy derivative markets based on daily data, our empirical analysis makes use of high-frequency (tick-by-tick) data from the futures markets, aggregated to 10-minute intervals during the trading day. The intraday variation is then utilized to generate daily time series of prices, returns and realized variance. We find significant evidence of a leverage effect for S&P500 and crude oil markets: a negative shock to returns increases volatility in these markets. We also find evidence of an inverse leverage effect for the natural gas market: volatility becomes higher when energy returns increase.

We show that the use of the SVJL model improves the ability to forecast volatility for the S&P500 and for the natural gas futures markets using both the RMSE and MAE criteria. We also show that overall, the introduction of both leverage and jumps in the SVJL model provides the best forecast for risk in both a VaR and a CVaR sense for investors who have any position in natural gas futures regardless of their degree of risk aversion. In the S&P500 market, the SVJL model provides the most precise forecast of risk in a CVaR sense for risk-averse investors with any position in futures, regardless of their degree of risk aversion. Focusing on a firm's internal risk management, the introduction of both jumps and leverage in the SVJL model would benefit speculative firms who are short natural gas futures aiming at minimizing tail risk in a VaR sense, as well as speculative firms who are long S&P500 futures and use either VaR or CVaR as financial risk management criteria while wanting to minimize the opportunity cost of capital.

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Appendix I: Realized Variance and Moment conditions

Following Bollerslev and Zhou (2002), who use continuously observed futures prices, we build a conditional moment estimator for stochastic variance models based on matching the sample moments of *Realized Variance* with population moments of the *Integrated Variance*. In this paper, realized variance is computed as the sum of high-frequency (10-minute interval) intraday squared returns.

8.1 No jumps

The returns on futures at time t over the interval $[t-k, t]$ can be decomposed as

$$r(t,k) = \ln F_t - \ln F_{t-k} = \int_{t-k}^{t} \mu(\tau)\, d\tau + \int_{t-k}^{t} \sigma(\tau)\, dW_\tau$$

When no jumps are considered, the Quadratic Variation coincides with Integrated Variance from the population and it is defined as

$$QV(t,k) = IV(t,k) = \int_{t-k}^{t} \sigma^2(\tau)\, d\tau$$

The *Realized Variance* from the sample is defined as:

$$RV(t,k,n) = \sum_{j=1}^{n \cdot k} r\left(t-k+\frac{j}{n}, \frac{1}{n}\right)^2$$

$$RV(t,k,n) \xrightarrow{p} IV(t,k)$$

as $n \longrightarrow \infty$

where n is the sampling frequency.

Residual 1

From Bollerslev and Zhou (2002), page 56 *Appendix A.1* equation $(A.3)$

$$\begin{aligned} e_1 &= E\left[\mathcal{V}_{t+1,t+2} \middle| \mathcal{G}_t\right] - \mathcal{V}_{t+1,t+2} \qquad (6) \\ &= \alpha E\left[\mathcal{V}_{t,t+1} \middle| \mathcal{G}_t\right] + \beta - \mathcal{V}_{t+1,t+2} \end{aligned}$$

where $\mathcal{V}_{t+1,t+2}$ is the *realized variance* and $\mathcal{G}_t$ is the information set.

$$\begin{aligned}
dt &= T - t = t + 1 - t = 1 \\
&= t + 2 - (t+1) = 1 \\
\alpha &= e^{-\kappa} \\
\beta &= \theta\left(1 - \alpha\right) \\
a &= \frac{1}{\kappa}\left(1 - \alpha\right) = \frac{\beta}{\kappa\theta} \\
b &= \theta\left(1 - a\right) \\
&= \theta - \frac{\beta}{\kappa}
\end{aligned}$$

Residual 2

$$\begin{aligned}
E\left[\left.\mathcal{V}_{t+1,t+2}^{2}\right|\mathcal{G}_t\right] &= H1 \cdot E\left[\left.\mathcal{V}_{t,t+1}^{2}\right|\mathcal{G}_t\right] + I \cdot E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{G}_t\right] + J \\
e_2 &= E\left[\left.\mathcal{V}_{t+1,t+2}^{2}\right|\mathcal{G}_t\right] - \mathcal{V}_{t+1,t+2}^{2} \\
&= H1 \cdot E\left[\left.\mathcal{V}_{t,t+1}^{2}\right|\mathcal{G}_t\right] + I \cdot E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{G}_t\right] + J - \mathcal{V}_{t+1,t+2}^{2}
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
A &= \left(\frac{\sigma}{\kappa}\right)^2 \cdot \left(\frac{1}{\kappa} - 2\alpha - \frac{1}{\kappa}e^{-2\cdot\kappa}\right) \\
B &= \left(\frac{\sigma}{\kappa}\right)^2 \cdot \left(\theta\left(1 - 2\alpha\right) + \frac{\theta}{2\kappa}\left(\alpha + 5\right)\left(\alpha - 1\right)\right) \\
C &= \alpha \cdot \frac{\sigma^2}{\kappa}\left(1 - \alpha\right) = \alpha \cdot a \cdot \sigma^2 \\
D &= \frac{\sigma^2\theta}{2\kappa}\left(1 - \alpha\right)^2 = a^2 \cdot \kappa \cdot \sigma^2 \cdot \theta
\end{aligned}$$

$$\begin{aligned}
H1 &= \alpha^2 \\
G1 &= \beta^2
\end{aligned}$$

$$
\begin{aligned}
I &= \frac{1}{a}\left[a^2\left(C+2\alpha\beta\right)+\left(\alpha-\alpha^2\right)\left(2ab+A\right)\right] \\
N &= \left[a^2\left(D+\beta^2\right)+\beta\left(2ab+A\right)+\left(1-\alpha^2\right)\left(b^2+B\right)\right] \\
J &= -bI+N
\end{aligned}
$$

Residual 3

Residuals 3 is built in order to deal with the leverage aspect as they focus on the relationship between futures returns and their variance.

This moment condition derives from the paper by Garcia et al. (2011) page 32 top equation in Box I:

$$
e_3 = \frac{E\left[\left. p_{t,t+1}\mathcal{V}_{t+1,t+2}\right| \mathcal{G}_t\right]-b}{a} - p_{t,t+1}V_{t,t+1} \tag{8}
$$

Considering the relationship between the population variance V_t and the realized variance $\mathcal{V}_{t,t+1}$

$$
\begin{aligned}
E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{F}_t\right] = aV_t+b \\
V_t = \frac{E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{F}_t\right]-b}{a}
\end{aligned}
$$

$$
V_t^2 = \frac{Ab+ab^2-aB-(A-2ab)E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{F}_t\right]+aE\left[\left.\mathcal{V}_{t,t+1}^2\right|\mathcal{F}_t\right]}{a^3}
$$

$$
\begin{aligned}
E\left[\left. p_{t,t+1}V_{t,t+1}\right|\mathcal{G}_t\right] &= \frac{\sigma\rho e^{-\kappa}}{\kappa}\left(E\left[\left. V_{t,t+1}\right|\mathcal{G}_t\right]\kappa+\theta\left(e^{\kappa}-k-1\right)\right) \\
\frac{E\left[\left. p_{t,t+1}\mathcal{V}_{t+1,t+2}\right|\mathcal{G}_t\right]-b}{a} &= \frac{\sigma\rho e^{-\kappa}}{\kappa}\left(\frac{E\left[\left.\mathcal{V}_{t,t+1}\right|\mathcal{G}_t\right]-b}{a}\kappa+\theta\left(e^{\kappa}-k-1\right)\right)
\end{aligned}
$$

$$
e_3 = \frac{\sigma\rho e^{-\kappa}}{\kappa}\left(\frac{\mathcal{V}_{t,t+1}-b}{a}\kappa+\theta\left(e^{\kappa}-k-1\right)\right)-p_{t,t+1}V_{t,t+1}
$$

where $\mathcal{G}_t \subset \mathcal{F}_t$ is the "*discrete time filtration*".

8.2 Jumps

When we allow for discrete jumps, the returns on futures at time t over the interval $[t-k,t]$ can be decomposed as

$$\begin{aligned} r(t,k) &= \ln F_t - \ln F_{t-k} \\ &= \int_{t-k}^{t} \mu(\tau)\, d\tau + \int_{t-k}^{t} \sigma(\tau)\, dW_\tau + \int_{t-k}^{t} x(\tau)\, dN(\lambda\tau) \end{aligned}$$

In this case, Integrated Variance and Quadratic Variation do not coincide:

$$\begin{aligned} IV_{jumps}(t,k) &= \int_{t-k}^{t} \sigma^2(\tau)\, d\tau + \sum_{t-k\leq s\leq t} (x(s)\, dN(\lambda s))^2 \\ &= QV(t,k) + \sum_{t-k\leq s\leq t} (x(s)\, dN(\lambda s))^2 \end{aligned}$$

Barndorff-Nielsen and Shephard (2004) proposed the Realized Bipower Variation as a consistent estimate of integrated variance component in the presence of jumps:

$BV(t,k;n) = \frac{\pi}{2} \sum_{i=2}^{n\cdot k} \left| r\left(t-k+\frac{ik}{n}, \frac{1}{n}\right)\right| \left| r\left(t-k+\frac{(i-1)k}{n}, \frac{1}{n}\right)\right|$

$$\begin{aligned} RV(t,k,n) - BV(t,k;n) &\longrightarrow QV(t,k) - IV(t,k) \\ QV(t,k) - IV(t,k) &= \sum_{t-k\leq s\leq t} (x(s)\, dN(\lambda s))^2 \\ \text{as } n &\longrightarrow \infty \end{aligned}$$

A similar approach was used in Baum and Zerilli (2016).

Residual 1

From Bollerslev and Zhou (2002) *Appendix B* page 62 ($B.14$) affects all the moment conditions (impact of jumps)

At time $(t,t+1)$

$$e_{1J} = E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] + \lambda\sigma_x^2 dt - RV_{t,t+1}$$

since

$$E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] = E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] + \lambda\sigma_x^2 dt$$

combining with equation $(A.3)$ on page 56, *Appendix A.1*

Residual 2

At time $(t+1, t+2)$

$$\begin{aligned} e_{2J} &= E\left[RV_{t+1,t+2} \middle| \mathcal{G}_t \right] - RV_{t+1,t+2} \\ &= E\left[BP_{t+1,t+2} \middle| \mathcal{G}_t \right] + \lambda\sigma_x^2 dt - RV_{t+1,t+2} \\ &= \alpha E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] + \beta + \lambda\sigma_x^2 dt - RV_{t+1,t+2} \\ &= \alpha \left(E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] - \lambda\sigma_x^2 dt \right) + \beta + \lambda\sigma_x^2 dt - RV_{t+1,t+2} \\ &= \alpha E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] + \lambda\sigma_x^2 (1-\alpha) dt - RV_{t+1,t+2} \\ &= \alpha E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] + \gamma - RV_{t+1,t+2} \end{aligned}$$

where $\gamma = \lambda\sigma_x^2 (1-\alpha) dt = \left(RV_{t,t+1} - E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] \right)(1-\alpha)$

$E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right]$ is the observed realized Variance

$E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right]$ is the observed BiPower variation

$RV_{t+1,t+2}$ is the realized variance in the next period

Residual 3

From Bollerslev and Zhou (2002) *Appendix B* page 62 $(B.14)$ affects all the moment conditions (impact of jumps)

At time $(t, t+1)$

$$\begin{aligned} e_{3J} &= E\left[RV_{t,t+1}^2 \middle| \mathcal{G}_t \right] - RV_{t,t+1}^2 \\ &= E\left[BP_{t,t+1}^2 \middle| \mathcal{G}_t \right] + 2\lambda\sigma_x^2 E\left[RV_{t,t+1}^2 \middle| \mathcal{G}_t \right] dt - \lambda\sigma_x^4 dt - RV_{t,t+1}^2 \end{aligned}$$

combining with equation ($A.9$) on page 58, *Appendix A.1*

Residual 4

At time $(t+1, t+2)$

$$e_{4J} = E\left[RV^2_{t+1,t+2} \middle| \mathcal{G}_t \right] - RV^2_{t+1,t+2}$$

but

$$\begin{aligned} E\left[RV_{t+1,t+2} \middle| \mathcal{G}_t \right] &= E\left[BP_{t+1,t+2} \middle| \mathcal{G}_t \right] + \lambda\sigma_x^2 dt \\ &= \alpha E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] + \gamma \end{aligned}$$

$$E\left[BP^2_{t+1,t+2} \middle| \mathcal{G}_t \right] = H1 \cdot E\left[BP^2_{t,t+1} \middle| \mathcal{G}_t \right] + I \cdot E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] + J$$

$$E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] = E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] - \lambda\sigma_x^2 dt$$

$$E\left[BP^2_{t,t+1} \middle| \mathcal{G}_t \right] = E\left[RV^2_{t,t+1} \middle| \mathcal{G}_t \right] - 2\lambda\sigma_x^2 E\left[RV^2_{t,t+1} \middle| \mathcal{G}_t \right] dt + \lambda\sigma_x^4 dt$$

$$\begin{aligned} e_{4J} &= E\left[BP^2_{t+1,t+2} \middle| \mathcal{G}_t \right] + 2\lambda\sigma_x^2 E\left[RV_{t+1,t+2} \middle| \mathcal{G}_t \right] dt - \lambda\sigma_x^4 dt - RV^2_{t+1,t+2} \\ &= H1 \cdot E\left[BP^2_{t,t+1} \middle| \mathcal{G}_t \right] + I \cdot E\left[BP_{t,t+1} \middle| \mathcal{G}_t \right] + J + \\ &+ 2\lambda\sigma_x^2 E\left[RV_{t+1,t+2} \middle| \mathcal{G}_t \right] dt - \lambda\sigma_x^4 dt - RV^2_{t+1,t+2} \\ &= H1 \cdot \left(E\left[RV^2_{t,t+1} \middle| \mathcal{G}_t \right] - 2\lambda\sigma_x^2 E\left[RV^2_{t,t+1} \middle| \mathcal{G}_t \right] dt + \lambda\sigma_x^4 dt \right) \\ &+ I \cdot \left(E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] - \lambda\sigma_x^2 dt \right) \\ &+ J + 2\lambda\sigma_x^2 \left(\alpha E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] + \gamma \right) - \lambda\sigma_x^4 dt - RV^2_{t+1,t+2} \\ &= H1 \cdot E\left[RV^2_{t,t+1} \middle| \mathcal{G}_t \right] + N3 \cdot E\left[RV_{t,t+1} \middle| \mathcal{G}_t \right] + N4 \end{aligned}$$

where

$$\begin{aligned} N3 &= 2\lambda\sigma_x^2 \left(\alpha - H1 \right) + I \\ N4 &= \lambda\sigma_x^2 \left(\sigma_x^2 \left(H1 - 1 \right) + 2\gamma + 2\beta - I \right) + J \end{aligned}$$

$E\left[\left.V_{t,t+1}^2\right|\mathcal{G}_t\right]$ can be observed from the realized variance

$V_{t+1,t+2}^2$ is the realized variance in the next period

Residual 5

$$e_{5J} = \frac{\sigma\rho e^{-\kappa}}{\kappa}\left(\frac{RV_{t,t+1} - b}{a}\kappa + \theta\left(e^{\kappa} - k - 1\right)\right) - p_{t,t+1}V_{t,t+1}$$

where

$$E\left[\left.RV_{t,t+1}\right|\mathcal{G}_t\right] = E\left[\left.BP_{t,t+1}\right|\mathcal{G}_t\right] + \lambda\sigma_x^2 dt$$

Appendix II: Figures

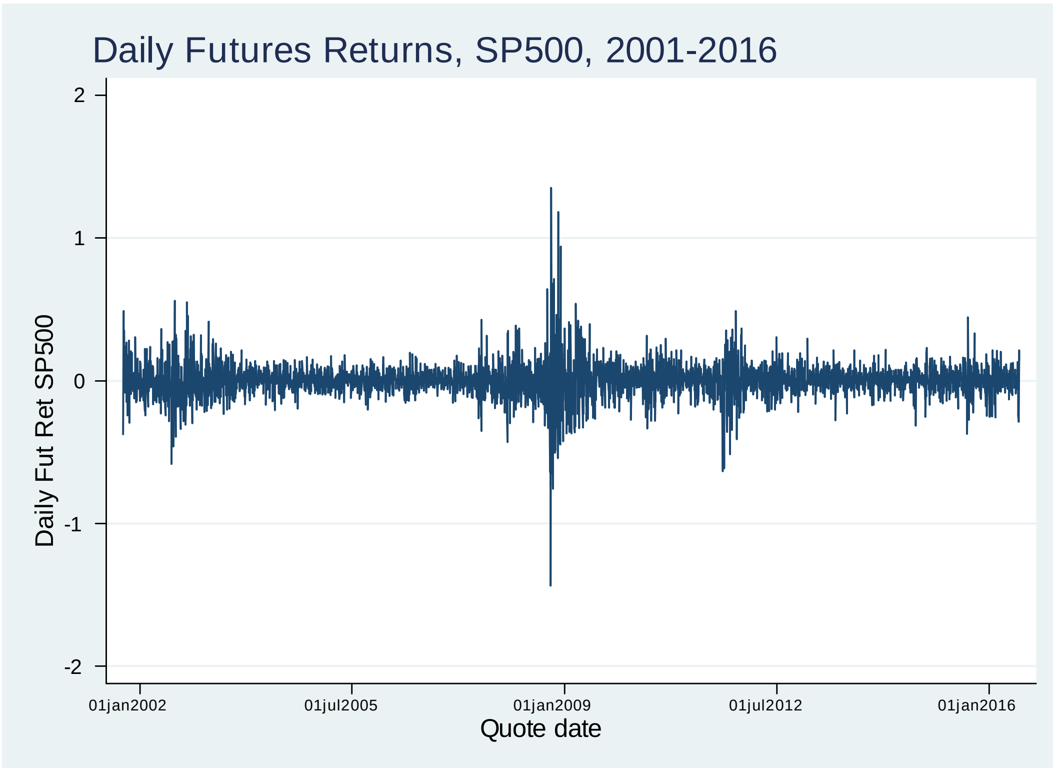

Figure 1

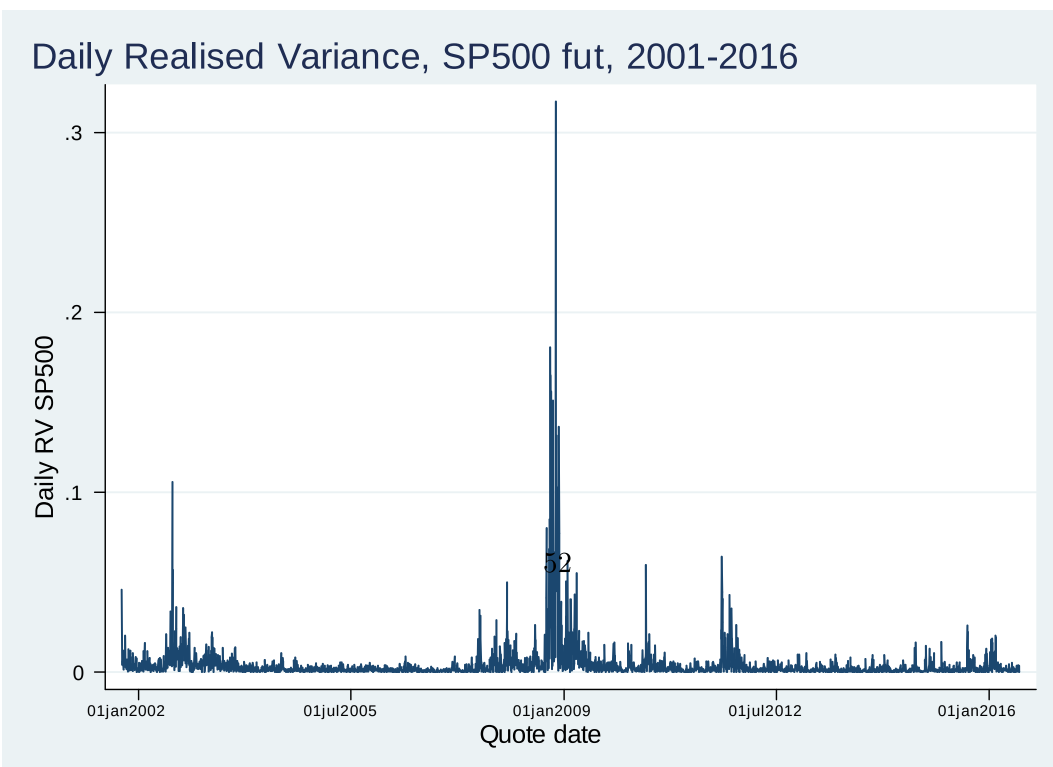

Figure 2

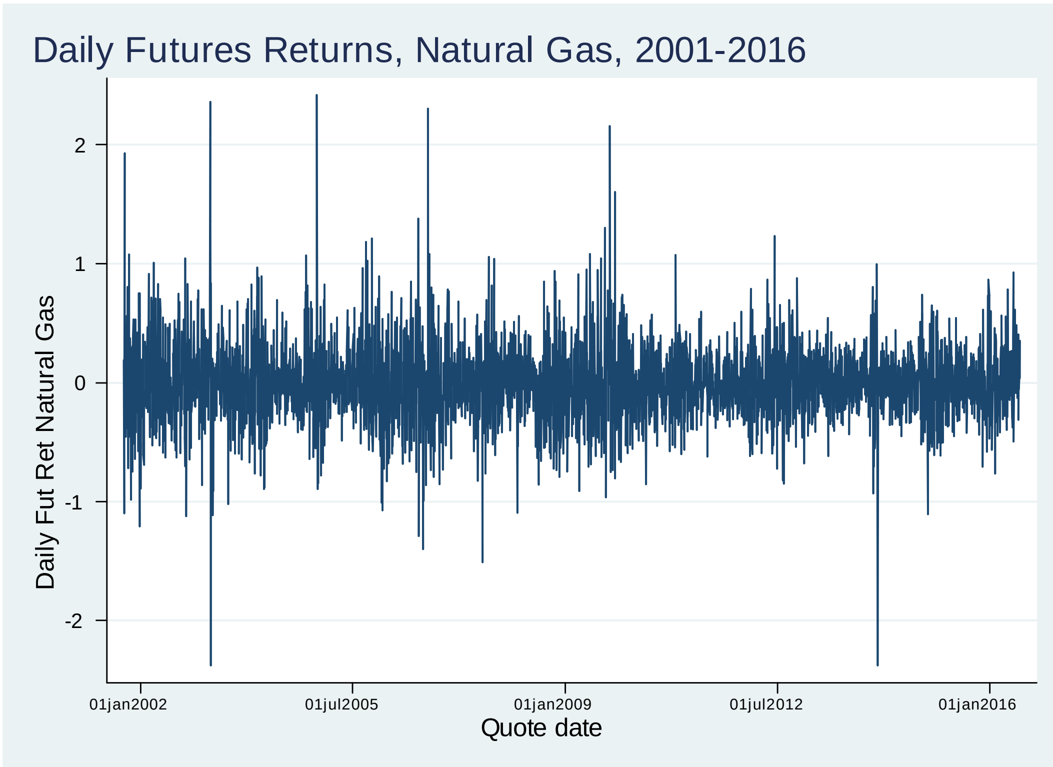

Figure 3

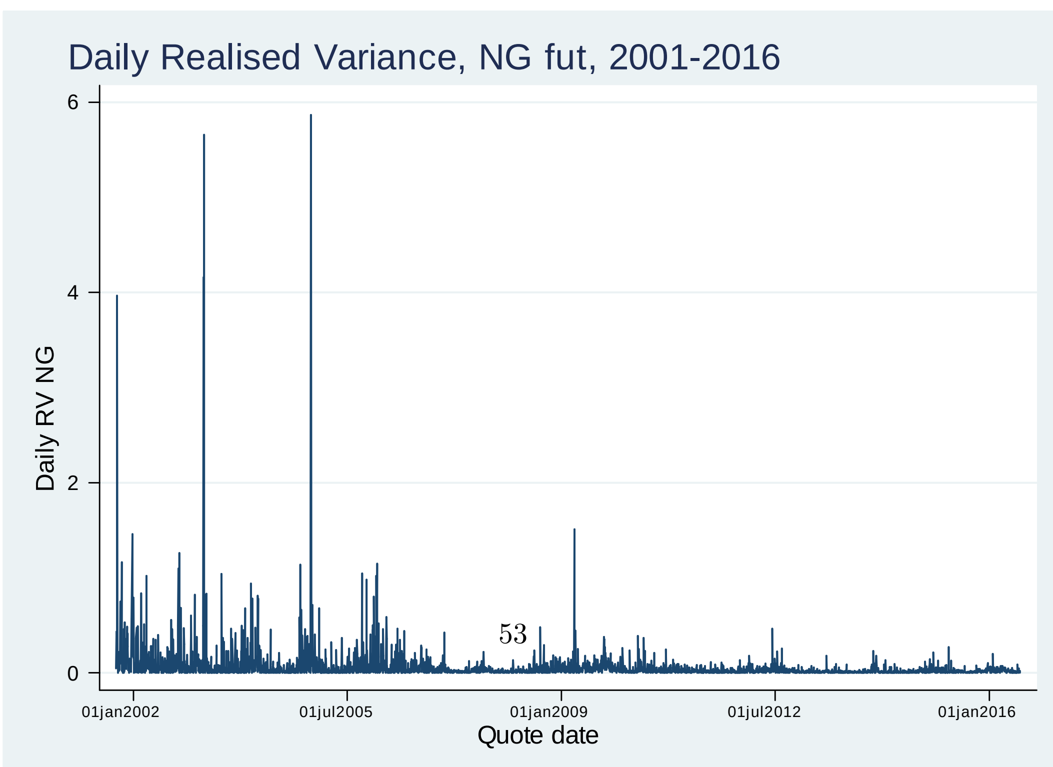

Figure 4

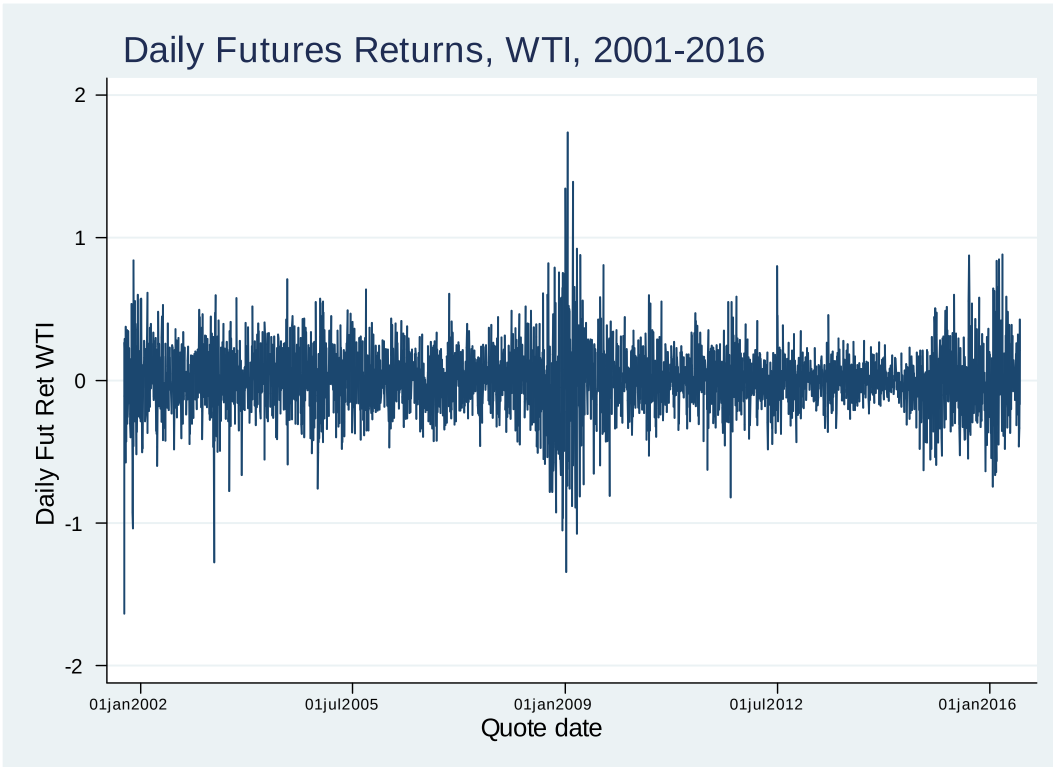

Figure 5

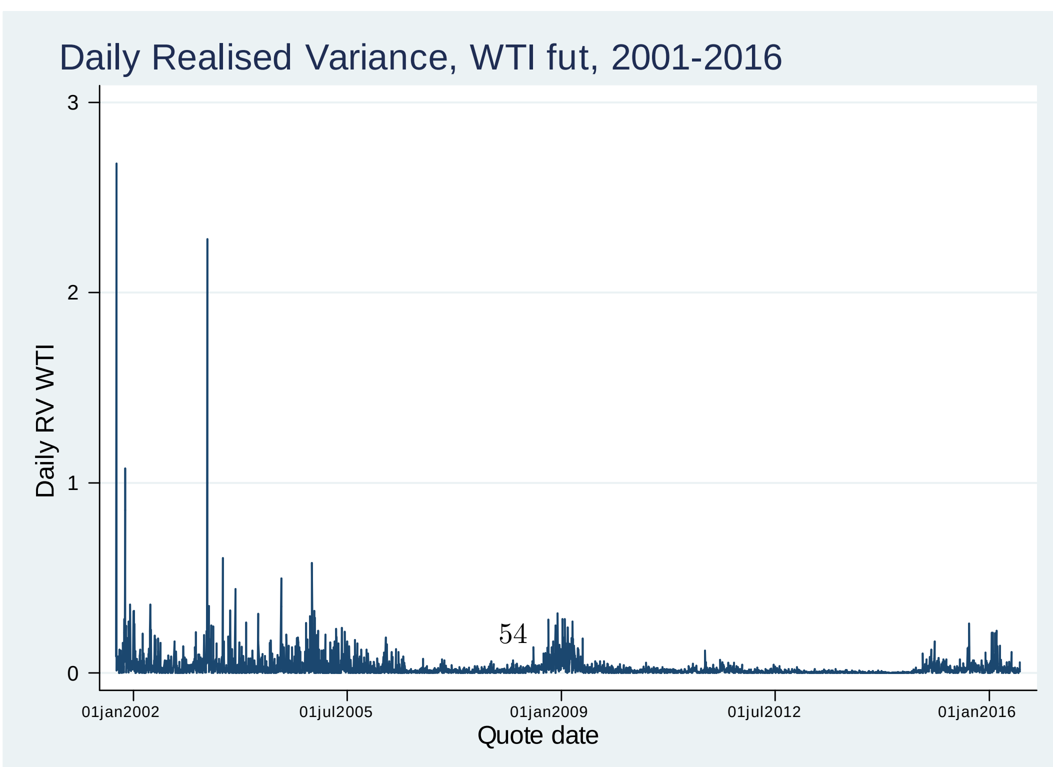

Figure 6

Appendix III: t and J tests on the moment conditions

Table 19: **GMM estimates for SV model for the S&P500 futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2} \middle| G_t\right] - V_{t+1,t+2}$	-0.0052	-0.00271	0.997841
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	-0.0052	-0.00844	0.993267
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	-0.0052	-0.05157	0.958872
$E\left[V^2_{t+1,t+2} \middle| G_t\right] - V^2_{t+1,t+2}$	-0.00851	-0.00735	0.994135
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	-0.00851	-0.01141	0.9909
$E\left[V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17} \middle| G_t\right] - V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17}$	-0.00851	-0.04573	0.963526
Hansen's J	χ^2_3	1.34382	0.7188

t statistics, p-values and J test on the moment conditions

Table 20: **GMM estimates for SVJ model for the S&P500 futures: 09/2001–06/2016**

	MuMoments	pval
$E\left[V_{t+1,t+2} \middle\| G_t\right] - V_{t+1,t+2}$	0.000068	0.999994
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle\| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	0.000068	0.999995
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle\| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	0.000068	0.999950
$E\left[V^2_{t+1,t+2} \middle\| G_t\right] - V^2_{t+1,t+2}$	0.000865	0.999991
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	0.000865	0.999930
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17} \middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	0.000865	0.999938
$E\left[V_{t,t+1} \middle\| G_t\right] - V_{t,t+1}$	0.002765	0.999498
$E\left[V_{t,t+1}BP^2_{t-16,t-15} \middle\| G_t\right] - V_{t,t+1}BP^2_{t-16,t-15}$	0.002765	0.999712
$E\left[V_{t,t+1}BP^2_{t-2,t-1} \middle\| G_t\right] - V_{t,t+1}BP^2_{t-2,t-1}$	0.002765	0.993492
$E\left[V^2_{t,t+1} \middle\| G_t\right] - V^2_{t,t+1}$	-0.000559	0.999998
$E\left[V^2_{t,t+1}V^2_{t-10,t-9} \middle\| G_t\right] - V^2_{t,t+1}V^2_{t-10,t-9}$	-0.000559	0.999986
$E\left[V^2_{t,t+1}BP^2_{t-18,t-17} \middle\| G_t\right] - V^2_{t,t+1}BP^2_{t-18,t-17}$	-0.000559	0.999803
Hansen's J	$\chi^2_7 = 6.96664$	0.4324

p-values and J test on the moment conditions

Table 21: **GMM estimates for SVL model for the S&P500 futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2} \middle| G_t\right] - V_{t+1,t+2}$	0.0001	0	1
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	0.0001	0	1
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	0.0001	0	1
$E\left[V_{t+1,t+2}V_{t-20,t-19} \middle| G_t\right] - V_{t+1,t+2}V_{t-20,t-19}$	0.0001	0.000025	0.99998
$E\left[V^2_{t+1,t+2} \middle| G_t\right] - V^2_{t+1,t+2}$	0.000232	0	1
$E\left[V^2_{t+1,t+2}V^2_{t-20,t-19} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-20,t-19}$	0.000232	0	1
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	0.000232	0	1
$E\left[V^2_{t+1,t+2}V^2_{t-22,t-21} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-22,t-21}$	0.000232	0.000025	0.99998
$E\left[p_{t,t+1}V_{t+1,t+2} \middle| G_t\right] - p_{t,t+1}V_{t+1,t+2}$	-0.00023	0	1
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V^2_{t-15,t-14} \middle| G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V^2_{t-15,t-14}$	-0.00023	0	1
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V^2_{t-12,t-11} \middle| G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V^2_{t-12,t-11}$	-0.00023	0	1
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V^2_{t-18,t-17} \middle| G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V^2_{t-18,t-17}$	-0.00023	$-2.9E-05$	0.999977
Hansen's J	χ^2_8	11.177	0.1919

.t statistics, p-values and J test on the moment conditions

Table 22: **GMM estimates for SVJL model for the S&P500 futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2} \middle\vert G_t\right] - V_{t+1,t+2}$	$-2.2E-05$	0	1
$E\left[V_{t+1,t+2}V^2_{t-20,t-19} \middle\vert G_t\right] - V_{t+1,t+2}V^2_{t-20,t-19}$	$-2.2E-05$	0	1
$E\left[V_{t+1,t+2}V^3_{t-18,t-17} \middle\vert G_t\right] - V_{t+1,t+2}V^3_{t-18,t-17}$	$-2.2E-05$	$-2E-06$	0.999998
$E\left[V^2_{t+1,t+2} \middle\vert G_t\right] - V^2_{t+1,t+2}$	$-3E-06$	0	1
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	$-3E-06$	0	1
$E\left[V^2_{t+1,t+2}V^4_{t-25,t-24} \middle\vert G_t\right] - V^2_{t+1,t+2}V^4_{t-25,t-24}$	$-3E-06$	0	1
$E\left[p_{t,t+1}V_{t+1,t+2} \middle\vert G_t\right] - p_{t,t+1}V_{t+1,t+2}$	$-3E-06$	0	1
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14} \middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14}$	$-3E-06$	0	1
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V_{t-12,t-11} \middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V_{t-12,t-11}$	$-3E-06$	0	1
$E\left[V_{t,t+1} \middle\vert G_t\right] - V_{t,t+1}$	0.000021	0	1
$E\left[V_{t,t+1}V^{\frac{1}{2}}_{t-16,t-15} \middle\vert G_t\right] - V_{t,t+1}V^{\frac{1}{2}}_{t-16,t-15}$	0.000021	0	1
$E\left[V_{t,t+1}V^4_{t-22,t-21} \middle\vert G_t\right] - V_{t,t+1}V^4_{t-22,t-21}$	0.000021	0	1
$E\left[V^2_{t,t+1} \middle\vert G_t\right] - V^2_{t,t+1}$	$-3E-06$	0	1
$E\left[V^2_{t,t+1}V^2_{t-10,t-9} \middle\vert G_t\right] - V^2_{t,t+1}V^2_{t-10,t-9}$	$-3E-06$	0	1
$E\left[V^2_{t,t+1}V_{t-18,t-17} \middle\vert G_t\right] - V^2_{t,t+1}V_{t-18,t-17}$	$-3E-06$	0	1
Hansen's J		$\chi^2_9 = 1.60417$	0.9963

p-values and J test on the moment conditions

Table 23: **GMM estimates for SV model for the Natural Gas futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2} \middle\| G_t\right] - V_{t+1,t+2}$	-0.0052	-0.00271	0.997841
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle\| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	-0.0052	-0.00844	0.993267
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle\| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	-0.0052	-0.05157	0.958872
$E\left[V^2_{t+1,t+2} \middle\| G_t\right] - V^2_{t+1,t+2}$	-0.00851	-0.00735	0.994135
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	-0.00851	-0.01141	0.9909
$E\left[V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17} \middle\| G_t\right] - V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17}$	-0.00851	-0.04573	0.963526
Hansen's J	χ^2_3	1.34382	0.7188

.t statistics, p-values and J test on the moment conditions

Table 24: **GMM estimates for SVJ model for the Natural Gas futures: 09/2001–06/2016**

	MuMoments	pval
$E\left[V_{t+1,t+2} \middle| G_t\right] - V_{t+1,t+2}$	-0.005914	0.999994
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	-0.005914	0.999995
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	-0.005914	0.999950
$E\left[V^2_{t+1,t+2} \middle| G_t\right] - V^2_{t+1,t+2}$	-0.009844	0.999991
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	-0.009844	0.999930
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17} \middle| G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	-0.009844	0.999938
$E\left[V_{t,t+1} \middle| G_t\right] - V_{t,t+1}$	0.002155	0.999498
$E\left[V_{t,t+1}BP^2_{t-16,t-15} \middle| G_t\right] - V_{t,t+1}BP^2_{t-16,t-15}$	0.002155	0.999712
$E\left[V_{t,t+1}BP^2_{t-2,t-1} \middle| G_t\right] - V_{t,t+1}BP^2_{t-2,t-1}$	0.002155	0.993492
$E\left[V^2_{t,t+1} \middle| G_t\right] - V^2_{t,t+1}$	-0.005225	0.999998
$E\left[V^2_{t,t+1}V^2_{t-10,t-9} \middle| G_t\right] - V^2_{t,t+1}V^2_{t-10,t-9}$	-0.005225	0.999986
$E\left[V^2_{t,t+1}BP^2_{t-18,t-17} \middle| G_t\right] - V^2_{t,t+1}BP^2_{t-18,t-17}$	-0.005225	0.999803
Hansen's J	$\chi^2_7 = 5.81155$	0.5619

p-values and J test on the moment conditions

Table 25: **GMM estimates for SVL model for the Natural Gas futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2}\middle\| G_t\right] - V_{t+1,t+2}$	-0.0058	-0.00262	0.997913
$E\left[V_{t+1,t+2}V_{t-16,t-15}\middle\| G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	-0.0058	-0.00277	0.997792
$E\left[V_{t+1,t+2}V_{t-17,t-16}\middle\| G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	-0.0058	-0.0678	0.945947
$E\left[V^2_{t+1,t+2}\middle\| G_t\right] - V^2_{t+1,t+2}$	-0.01161	-0.01107	0.991166
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9}\middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	-0.01161	-0.0099	0.992101
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17}\middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	-0.01161	-0.18512	0.853143
$E\left[p_{t,t+1}V_{t+1,t+2}\middle\| G_t\right] - p_{t,t+1}V_{t+1,t+2}$	-0.00281	-0.00116	0.999072
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14}\middle\| G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14}$	-0.00281	-0.00544	0.995658
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V_{t-18,t-17}\middle\| G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V_{t-18,t-17}$	-0.00281	-0.02369	0.9811
Hansen's J	χ^2_5	3.50114	0.6232

.t statistics, p-values and J test on the moment conditions

Table 26: **GMM estimates for SVJL model for the Natural Gas futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2}\middle\vert G_t\right] - V_{t+1,t+2}$	-0.00061	-0.00029	0.999771
$E\left[V_{t+1,t+2}V_{t-10,t-9}\middle\vert G_t\right] - V_{t+1,t+2}V_{t-10,t-9}$	-0.00061	-0.00194	0.998452
$E\left[V_{t+1,t+2}V^3_{t-18,t-17}\middle\vert G_t\right] - V_{t+1,t+2}V^3_{t-18,t-17}$	-0.00061	-0.00024	0.999809
$E\left[V^2_{t+1,t+2}\middle\vert G_t\right] - V^2_{t+1,t+2}$	-0.00602	-0.00574	0.995417
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9}\middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	-0.00602	-0.14815	0.882232
$E\left[V^2_{t+1,t+2}V^4_{t-25,t-24}\middle\vert G_t\right] - V^2_{t+1,t+2}V^4_{t-25,t-24}$	-0.00602	-0.01217	0.99029
$E\left[p_{t,t+1}V_{t+1,t+2}\middle\vert G_t\right] - p_{t,t+1}V_{t+1,t+2}$	0.000105	0.00003	0.999976
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14}\middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-15,t-14}V_{t-15,t-14}$	0.000105	0.000015	0.999988
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V_{t-12,t-11}\middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-12,t-11}V_{t-12,t-11}$	0.000105	0.000264	0.999789
$E\left[V_{t,t+1}\middle\vert G_t\right] - V_{t,t+1}$	0.052523	0.057867	0.953858
$E\left[V_{t,t+1}V^{\frac{1}{2}}_{t-6,t-5}\middle\vert G_t\right] - V_{t,t+1}V^{\frac{1}{2}}_{t-6,t-5}$	0.052523	4.326995	0.000016
$E\left[V_{t,t+1}V^4_{t-2,t-1}\middle\vert G_t\right] - V_{t,t+1}V^4_{t-2,t-1}$	0.052523	0.344424	0.730547
$E\left[V^2_{t,t+1}\middle\vert G_t\right] - V^2_{t,t+1}$	-0.00536	-0.00157	0.998747
$E\left[V^2_{t,t+1}V^2_{t-10,t-9}\middle\vert G_t\right] - V^2_{t,t+1}V^2_{t-10,t-9}$	-0.00536	-0.00151	0.998795
$E\left[V^2_{t,t+1}V_{t-18,t-17}\middle\vert G_t\right] - V^2_{t,t+1}V_{t-18,t-17}$	-0.00536	-0.00746	0.994051
Hansen's J		$\chi^2_9 = 6.00869$	0.7390

p-values and J test on the moment conditions

Table 27: **GMM estimates for SV model for the WTI futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2}\middle\vert G_t\right] - V_{t+1,t+2}$	0.000451	0.000079	0.999937
$E\left[V_{t+1,t+2}V_{t-16,t-15}\middle\vert G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	0.000451	0.000044	0.999965
$E\left[V_{t+1,t+2}V_{t-17,t-16}\middle\vert G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	0.000451	0.000586	0.999532
$E\left[V^2_{t+1,t+2}\middle\vert G_t\right] - V^2_{t+1,t+2}$	0.001443	0.000015	0.999988
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9}\middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	0.001443	0.000355	0.999717
$E\left[V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17}\middle\vert G_t\right] - V^2_{t+1,t+2}V^{\frac{1}{2}}_{t-18,t-17}$	0.001443	0.000482	0.999615
Hansen's J	χ^2_3	1.16069	0.7624

.t statistics, p-values and J test on the moment conditions

Table 28: **GMM estimates for SVJ model for the WTI futures: 09/2001–06/2016**

	MuMoments	pval
$E\left[V_{t+1,t+2} \middle\vert G_t\right] - V_{t+1,t+2}$	0.000068	0.999994
$E\left[V_{t+1,t+2}V_{t-16,t-15} \middle\vert G_t\right] - V_{t+1,t+2}V_{t-16,t-15}$	0.000068	0.999995
$E\left[V_{t+1,t+2}V_{t-17,t-16} \middle\vert G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	0.000068	0.999950
$E\left[V^2_{t+1,t+2} \middle\vert G_t\right] - V^2_{t+1,t+2}$	0.000865	0.999991
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9} \middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	0.000865	0.999930
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17} \middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	0.000865	0.999938
$E\left[V_{t,t+1} \middle\vert G_t\right] - V_{t,t+1}$	0.002765	0.999498
$E\left[V_{t,t+1}BP^2_{t-16,t-15} \middle\vert G_t\right] - V_{t,t+1}BP^2_{t-16,t-15}$	0.002765	0.999712
$E\left[V_{t,t+1}BP^2_{t-2,t-1} \middle\vert G_t\right] - V_{t,t+1}BP^2_{t-2,t-1}$	0.002765	0.993492
$E\left[V^2_{t,t+1} \middle\vert G_t\right] - V^2_{t,t+1}$	-0.000559	0.999998
$E\left[V^2_{t,t+1}V^2_{t-10,t-9} \middle\vert G_t\right] - V^2_{t,t+1}V^2_{t-10,t-9}$	-0.000559	0.999986
$E\left[V^2_{t,t+1}BP^2_{t-18,t-17} \middle\vert G_t\right] - V^2_{t,t+1}BP^2_{t-18,t-17}$	-0.000559	0.999803
Hansen's J	$\chi^2_7 = 4.53499$	0.7165

p-values and J test on the moment conditions

Table 29: **GMM estimates for SVL model for the WTI futures: 09/2001–06/2016**

	MuMoments	tstat	pval
$E\left[V_{t+1,t+2}\middle\vert G_t\right] - V_{t+1,t+2}$	0.000451	0.000017	0.999987
$E\left[V_{t+1,t+2}V_{t-19,t-18}\middle\vert G_t\right] - V_{t+1,t+2}V_{t-19,t-18}$	0.000451	0.00002	0.999984
$E\left[V_{t+1,t+2}V_{t-17,t-16}\middle\vert G_t\right] - V_{t+1,t+2}V_{t-17,t-16}$	0.000451	0.000548	0.999563
$E\left[V^2_{t+1,t+2}\middle\vert G_t\right] - V^2_{t+1,t+2}$	0.001539	0.000007	0.999994
$E\left[V^2_{t+1,t+2}V^2_{t-10,t-9}\middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-10,t-9}$	0.001539	0.000024	0.999981
$E\left[V^2_{t+1,t+2}V^2_{t-18,t-17}\middle\vert G_t\right] - V^2_{t+1,t+2}V^2_{t-18,t-17}$	0.001539	0.001058	0.999156
$E\left[p_{t,t+1}V_{t+1,t+2}\middle\vert G_t\right] - p_{t,t+1}V_{t+1,t+2}$	0.000108	0.000006	0.999995
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V_{t-18,t-17}\middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-18,t-17}V_{t-18,t-17}$	0.000108	0.000001	0.999999
$E\left[p_{t,t+1}V_{t+1,t+2}p_{t-16,t-15}V_{t-16,t-15}\middle\vert G_t\right]$ $-p_{t,t+1}V_{t+1,t+2}p_{t-16,t-15}V_{t-16,t-15}$	0.000108	0.000097	0.999923
Hansen's J	χ^2_5	1.37354	0.9272

.t statistics, p-values and J test on the moment conditions