Essays on Macroeconomics and Asset Pricing:

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Abstract

A significant theoretical literature suggests that the effects of open market operations and large scale asset purchases are limited when short-term interest rates are constrained by the zero-lower-bound (ZLB). This view is supported by a growing body of empirical evidence that points to the tepid response of the U.S. economy to extraordinary policy measures implemented by the Federal Reserve (Fed) during the past several years. In the first essay, **Effective Monetary Policy at the Zero-Lower-Bound**, I show that permanent open market operations (POMOs), defined as financial market interventions that permanently increase the supply of money, remain relevant at the ZLB and can increase output and inflation. Consequently, I argue that the limited success of Fed policy in recent years may be due in part to the fact that it failed to generate sufficient money creation to support economic recovery following the Great Recession. I then demonstrate that conducting POMOs at the ZLB may improve welfare when compared to a broad range of policy regimes, and conclude by conducting a robustness exercise to illustrate that money creation remains relevant at the ZLB when it is not necessarily permanent.

With these results in hand, I explore the consequences of Fed QE more directly in a framework asset purchases are an independent instrument of monetary policy. In the second essay, **Effective Quantitative Easing at the Zero-Lower-Bound**, I show that the observed lack of transmission between U.S. monetary policy and output economic activity a consequence of the fact the Fed engaged in what I define as sterilized QE: temporary asset purchases that have a limited effect on the money supply. Conversely, I show that asset purchase programs geared towards generating sustained increases in the money supply may significantly attenuate output and inflation losses associated with adverse economic shocks and the ZLB constraint. Furthermore, these equilibrium outcomes may be achieved with a smaller volume of asset purchases. My results imply that Fed asset purchase programs designed to offset the observed declines in the U.S. money supply could have been a more effective and efficient means of providing economic stimulus during the recovery from the Great Recession.

The third essay—which is joint work with Apollon Fragkiskos, Harold Spilker, and Russ Wermers—titled **Buyout Gold: MIDAS Estimators and Private Equity**, we develop a new approach to study private equity returns using a data set first introduced in Fragkiskos et al. (2017). Our innovation is that we adopt a mixed data sampling (MIDAS) framework and model quarterly private equity returns as a function of high frequency factor prices. This approach allows us to endogenize time aggregation and use within-period information that may be relevant to pricing private equity returns in a single, parsimonious framework. We find that our MIDAS framework offers superior performance in terms of generating economically meaningful factor loadings and in-sample and out-of-sample fit using index and vintage-level returns when compared with other methods from the literature. Results using fund-level data are mixed, but MIDAS does display a slight edge. Concerning appropriate time-aggregation, we show that there is significant heterogeneity at the vintage level. This implies highly aggregated private equity data may not properly reflect underlying performance in the cross section.

To Mom and Dad

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Chapter 1

Effective Monetary Policy at the Zero-Lower-Bound

1.1 Introduction

Appropriate monetary policy when nominal interest rates are constrained by the zero-lower-bound (ZLB) became a serious subject of study following Japan's Lost Decade. After bringing short-term interest rates to the ZLB in early 1999, in 2001 the Bank of Japan (BOJ) began conducting large scale asset purchases to target current account balances of domestic banks—a policy dubbed Quantitative Easing (QE)—in an attempt to boost economic growth and fight deflation. Interest in the subject has been bolstered by the policy responses of the U.S. Federal Reserve and other central banks to the Great Recession. Like the BOJ, the Fed lowered its own policy rate to the effective ZLB following the financial crisis that was precipitated by the collapse of Lehman Brothers in October of 2008. Then the Fed began its own asset purchase programs with the intended goals of supporting distressed financial markets and increasing overall economic activity. In both cases, policymakers were confronted with the challenge of implementing expansionary monetary policy to shore up weak economies experiencing a liquidity trap.[1] Whether or not monetary policy should be effective when economies experience a liquidity trap, and if so through which channel it works, is still a matter of debate.

I investigate the ability of open market operations—injections of base money via purchases of government debt—to support economic activity in a liquidity trap. I start by proving a series of relevance results for money creation at the ZLB in a standard New Keynesian setting. These results hold whether money demand is motivated by transaction constraints or the presence of money in the utility function. I then demonstrate how permanent open market operations (POMOs), defined as policies that permanently change the supply of money, bolster output and inflation when nominal interest rates are constrained by the ZLB. I then argue that these results, combined with observations of large and persistent deviations broad set of money aggregates from trend (reported in Figures A.2 through A.7) offer an alternative explanation to the tepid response of the U.S. economy to QE. Fundamentally, large scale asset purchases failed to generate enough money creation to accommodate economic recovery following the Great Recession.

I establish that POMOs improve welfare across a broad spectrum of policy regimes when the ZLB binds. This result holds whether welfare is measured via direct utility losses from business cycle fluctuations, or the consumption equivalent. Furthermore, POMOs attain similar welfare gains to implementing first-best policy. To the extent that Fed policy deviated from the first-best response during the Financial Crisis, this suggests that the Fed could have improved welfare during the recovery from the Great Recession by conducting POMOs.

Permanence is a strong word, and so I relax this assumption regarding the money supply by considering a hybrid policy regime where the central bank engineers an increase in the money supply at the ZLB, but eventually returns to operating via an interest rate feedback rule. In this setting, committing to delayed

[1]A liquidity trap is broadly defined as a scenario where interest rates have fallen to the ZLB so that money and short-term bonds become (nearly) perfect substitutes, rendering any expansionary monetary policy that attempts to lower interest rates further by increasing the money supply impotent.

reform—a return to an interest rate rule some fixed time after the nominal interest rate rebounds—implicitly commits the central bank to only partially unwind any open market operations conducted during the ZLB episode. As a consequence, this class of policies mimics POMOs in that they generate permanent money creation, and are therefore able to increase output and prices at the ZLB.

Numerical simulations suggest POMOs may leave the time-path of the nominal interest rate virtually unchanged while increasing output and inflation. In addition, I show that open market purchases conducted in tandem with a commitment to delayed policy reform—the hybrid regime discussed above—decrease the duration of ZLB episodes and raise the time-path of the nominal interest rate. In other words, this class of policies hastens credit market normalization. This suggests the existence of a channel through which open market operations may influence the macroeconomy that is distinct from interest rate policy.

Discussion of the potency of monetary policy at the ZLB dates back to Keynes's writings on liquidity traps. Keynes concluded that monetary policy would be ineffective when policy rates were constrained by the ZLB which led him to view fiscal policy as the only remedy for economic malaise in such a situation. This assessment was later challenged by the work of Patinkin, Meltzer, Tobin, Friedman, Schwartz, and others who showed that the central bank could influence macroeconomic outcomes through increasing the monetary base via Pigou or portfolio re-balancing effects, even when short-term rates were zero. As a result, many became convinced that an actual liquidity trap could never materialize, or at least that the ZLB would not in and of itself constrain a willing central bank from engaging in expansionary monetary policy. However, in light of the the Japanese economy's tepid response to the BOJ's aggressive QE program and the U.S.'s slow recovery from the Great Recession despite several rounds of large scale asset purchases, interest in Keynes's insights has been revived.

Perhaps the first modern treatment of liquidity traps is Krugman (1998). Krugman employs a simple, static model to discuss policy options at the ZLB and shows that pure money creation is ineffective at affecting nominal quantities if short-term rates fall to zero. When prices are sticky, this result has real consequences as resources will be underemployed. Krugman does note that the central bank can still influence prices and output at the ZLB by committing to a permanently higher money stock and a higher long-run price level by changing private agents' expectations of future policy and economic activity. He remains doubtful that modern central banks could credibly commit to such a policy after cultivating a culture of price stability in the late 20th century. Nevertheless, Krugman recommends the BOJ attempt to "credibly promise to be irresponsible" as a cure for sluggish economic performance by conducting policy to promote expected inflation.

Krugman's seminal analysis prompted two important questions. If expected policy matters, how can central banks demonstrate commitment to future expansionary policy? How much future expansionary policy should be promised and how can it be implemented? To answer this first question, Jung et al. (2005) extend the work of Rotemberg and Woodford (1998) to analyze optimal central bank interest rate policy in a liquidity trap. They subject a simple New Keynesian model to large, adverse shocks and ask: Once the policy rate is taken to the ZLB, when should a central bank raise the nominal interest rate? They find that it is optimal for a welfare maximizing central bank to commit to keep its policy rate below the Wicksellian natural rate for an extended period of time. Eggertsson and Woodford (2003) then generalize this result to the case where the future path of the natural rate is uncertain and obtain the same result. In both cases, commitment to sustaining zero interest rate policy generates expected inflation, which increases current aggregate demand as a consequence of price stickiness and the forward looking nature of the New Keynesian model. To address the second question, Eggertsson and Woodford illustrate that a time-dependent price level target that is increasing in the duration of time the economy is experiencing a liquidity trap implements the optimal interest rate path. Aside from generalizing the work of Krugman (1998), these studies show that it is incumbent on policymakers to promote expected inflation and promise a higher price level to increase welfare, which was precisely Krugman's policy recommendation for Japan.

Along these lines, Armenter (2012) demonstrates that it may be welfare improving for a central bank to maintain zero-interest policy beyond what is called for by optimal commitment when output is significantly more volatile than inflation. In this case welfare losses from inefficiencies due to price dispersion are minimal, so the upside inflation risk of accommodating output is relatively small. Guerron-Quintana et al. (2012) study the impact of forward guidance conducted in tandem with productivity-augmenting policies. While contemporaneous increases in productivity demonstrably decrease output and inflation at the ZLB, they show money creation and governmental action aimed at boosting future productivity through structural reform both improve macroeconomic conditions by generating higher expected inflation and raising aggregate demand. Del Negro et al. (2012) demonstrate that these theoretical results may overstate the empirical effects of forward guidance by looking at the macroeconomic response to FOMC announcements during the recent U.S. ZLB episode. This general conclusion is supported by Keen et al. (2015). In contrast, Bundick and Smith (2016) find there is no disconnect between the theoretical predictions of a simple, estimated New Keynesian model and the observed impact of Fed forward guidance.

Where does this all leave conventional monetary policy? Eggertsson and Woodford (2003) and Curdia and Woodford (2011) derive a series of irrelevance results that suggest the quantity of money, and more generally, the size of the central bank balance sheet and its composition, should have no discernible impact on macroeconomic outcomes unless they are connected somehow with the central bank's efforts to implement accommodative interest rate policy. These authors note that the Pigou and portfolio rebalancing effects earlier authors used to refute the possibility of liquidity traps do not exist in many of the current generation monetary models which assume complete financial markets. Curdia and Woodford (2011) drop this assumption to generate the minimal requirements needed for monetary policy to be able to affect equilibrium determination. The key requirement is that households have different spending opportunities and hence different marginal utilities of income in the same state of the world. This enables monetary policy, which they call balance sheet policy, to change household's pricing kernels and thus relative asset prices and aggregate demand at the ZLB. However, they find that monetary expansion is only effective in so much that it finances the central bank's ability to lend directly to firms when private financial intermediaries can not or will not in tandem with a commitment to sustaining lower interest rates. In other words, money creation is only effective when financial markets are sufficiently disrupted *and* the central bank commits to a lower future path of its policy rate.

A vast literature has sprouted analyzing the effect of the central bank expanding its balance sheet to intervene directly in private lending markets. Two seminal studies in this literature are Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), who utilize models with a banking sector subject to financial frictions as in Bernanke et al. (1999), but focus on crises that originate with financial intermediaries. They find that credit policy, actions aimed at mitigating the disruption of activity between borrowers and lenders via direct intervention in debt markets, can alleviate the impact of financial crises on real activity. Araújo et al. (2013) consider central bank purchases of assets commonly used as collateral in financial markets as a means to support borrowers during times of financial distress. They conclude that these forms of targeted asset purchases may work if financial markets are extremely disrupted and only so long as the central bank can engage in them without drastically reducing the supply of pledgeable collateral in private hands. These frameworks are aptly suited for analyzing the impact of the emergency liquidity facilities established by the Fed in the period directly following the collapse of Lehman Brothers. However, they are not firmly suited for studying the implications of the later, broader, and larger asset purchase programs employed by the Fed nor in general, the role of monetary expansion during prolonged ZLB episodes.

Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2010), and other authors find that QE flattened the yield curve and changed other asset prices, which implies that open market operations at the ZLB may effect the macroeconomy due to some combination of market segmentation or preferred habitat found in Vayanos and Vila (2009). Harrison (2012b) investigates the implications of incorporating balance sheet policy as an independent means of implementing optimal policy at the ZLB in a model with imperfect asset substitutability and market segmentation as in Andrés et al. (2004). This assumption ensures that households treat long-term debt as less liquid then short-term bonds, generating a portfolio re-balancing effect when the central bank distorts the relative supply of assets through open market operations. Harrison finds that central bank purchases of long-term government debt are quite effective and can improve welfare when executed along with interest rate policy at the ZLB. However, this result is contingent on the central bank's ability to purchase an overwhelming majority of the outstanding stock of long-term government securities, which many would view as politically infeasible and/or operationally impossible.

The empirical literature suggests the macroeconomic effects of QE were much more muted. Chen et al. (2012) incorporate portfolio adjustment costs in an estimated, medium-scale model reminiscent of Smets and Wouters (2007) and simulate the macroeconomic effects of QE2, the second round of QE conducted by the Fed around November of 2010. They find that QE2 increased GDP growth by about 0.13% and inflation by only three basis points. These results represent the lower end of the VAR evidence. For example, Chung et al. (2012) find QE2 increased the level of GDP by 0.6% and inflation by 0.1%. Splitting the difference between these two approaches, Wu and Xia (2014) estimate a FAVAR built upon a linear affine term structure model and find that QE lowered unemployment by 0.13% in December, 2013.

Given the over fourfold increase in the Fed's balance sheet as illustrated in Figure A.1, these empirical findings lend credence to the irrelevance results introduced in Eggertsson and Woodford (2003). However, despite their focus on interest rate policy, these authors note that commitment to generating a permanently higher money stock is an effective means to increase aggregate demand in a liquidity trap.

Auerbach and Obstfeld (2005) consider permanent cash transfers as a response to adverse economic shocks in a dynamic model with a strong New Keynesian 'flavor.' They find that a one-time permanent increase in the money supply strengthens an economy suffering from a liquidity trap and improves welfare by lowering the real burden of future taxes. More recently, Robatto (2014) takes a more general approach and concludes that money creation can influence macroeconomic equilibria so long as the policy is not unwound before the economy exits the ZLB, or leads to a permanent change in the stance of policy.[2]

I build upon these last studies and show that their conflicting results are the consequence of different assumptions about the nature of monetary policy upon exiting a liquidity trap. Simply put, Eggertsson and Woodford preclude the central bank from engineering the sort of permanent money creation that Auerbach, Obstfeld, Robatto, and I demonstrate is effective. While theoretically well founded, one still must reconcile these results with the observation that several rounds of Fed QE only marginally impacted real activity. The path of the U.S. money supply suggests an intuitive explanation. Figures A.2 through A.7 illustrate that a broad set of money aggregates experienced significant, negative deviations from trend and have yet to recover. As a consequence, as implemented Fed QE not belong to class of policies—POMOs—that I establish supports real activity at the ZLB. Channeling Friedman and Schwartz (2008), in light of the monetary data my results suggest a better policy response would have focused on generating sufficient money creation to support recovery following the Great Recession.

The remainder of this paper is organized as follows: Section 2 outlines the macroeconomic model. In Section 3 I prove a series relevance results concerning the impact of monetary expansion at the ZLB and perform a series of quantitative exercises to illustrate the macroeconomic effects of POMOs at the ZLB. I then discuss welfare, and conduct a robustness exercise by examining the interaction between open market operations, interest rules, and policy reform and their implications for effective policy. Section 5

[2] e.g. a permanent increase in the monetary base, a permanent change in the relative supply of money relative to short-term nominal debt, etc.

concludes.

1.2 Model Environment

A representative household maximizes expected lifetime utility from consumption of a single, final good and supplies labor to a continuum of monopolistically competitive intermediate-good producers. As a way to motivate transaction demand for money, consumption purchases are subject to a cash-in-advance (CIA) constraint. Aside from money, there is only one other asset in the economy in which the household can store wealth: one-period consumption bonds, B, that pay interest i, and are in zero net supply in equilibrium.

1.2.1 Final Goods Firms

Final goods producers operate in a perfectly competitive environment. They take output from intermediate goods firms and form final goods according to the constant elasticity of substitution bundling technology

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \tag{1.1}$$

where $y(j)$ is output from intermediate firm $j \in [0,1]$. Each final goods producer solves

$$\max_{y_t(j)} P_t \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(j) y_t(j) dj,$$

where P is the price of final output and $P(j)$ is the price of input, j. The first-order conditions yield the optimal price index for final output

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \tag{1.2}$$

and the demand function for intermediate good firm j

$$y_t(j) = y_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta}. \tag{1.3}$$

1.2.2 Intermediate Goods Firms

Intermediate goods producers operate in a monopolistically competitive environment with the production technology

$$y_t(j) = A_t h_t(j), \tag{1.4}$$

where A is an aggregate productivity disturbance. Firms hire labor services from a competitive labor market and take wages as given. Furthermore, pricing decisions are subject to Calvo (1983) frictions: only a fraction $1-\alpha$ of firms may update prices in any given period. The remaining measure α of firms must maintain the same price from their last adjustment. Given the production technology and demand for firm j, all updating firms maximize (real) future discounted profits, $d(j)$:

$$\begin{aligned} &\max_{P_t(j)} \ E_t \sum_{s=0}^{\infty} \alpha^s \Omega_{t,t+s} d_{t+s}(j) \\ &s.t. \ \ d_t(j) = y_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta} \left(\frac{P_t(j)}{P_t} - \gamma_t \right), \end{aligned}$$

where γ is real marginal cost and Ω is a stochastic discount factor determined by households. Given that all firms face the same wage rate, all adjusting firms will pick the same optimal price, P^*. The first order condition for this problem is then

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1}\frac{E_t \sum_{s=0}^{\infty} \alpha^s \Omega_{s,t+s} y_{t+s} \gamma_{t+s} (P_{t+s}/P_t)^{\theta}}{E_t \sum_{s=0}^{\infty} \alpha^s \Omega_{s,t+s} y_{t+s} (P_{t+s}/P_t)^{\theta-1}} \tag{1.5}$$

and price index for final output becomes

$$P_t^{1-\theta} = (1-\alpha)(P_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}. \tag{1.6}$$

1.2.3 Households

Concerning the cash in advance (CIA) constraint, I adopt the timing convention of Lucas (1982):

1. The household enters each period with nominal wealth W_t. Uncertainty over the vector of aggregate disturbances, $\overline{z}_t$, and monetary policy is resolved.

2. Financial markets open. Interest on outstanding debt is paid, new debt is issued, the central bank conducts direct cash transfers, X_t, and the household parts its wealth between money and bonds. The household's financial market budget constraint must satisfy

$$B_t + M_t = W_t + X_t, \tag{1.7}$$

3. Production of both intermediate and final goods occurs.

4. Nominal wages, w_t, are paid and the goods market opens. Consumption satisfies the CIA constraint, $P_t c_t \leq M_t$.

5. Next period's wealth is then determined by the household's excess cash balances, wages, current holdings of nominal bonds, and profits from firm ownership.

$$W_{t+1} = M_t - P_t c_t + w_t h_t + (1+i_t)B_t + \int_0^1 d_t(j)dj \tag{1.8}$$

Note that interest payments on consumer debt, $(1+i_t)B_t$ count toward next period's wealth given the assumption that financial markets open first.

The representative household solves

$$\begin{aligned}
\max_{c_t, h_t(j), M_t, B_t} \quad & E_t \sum_{s=0}^{\infty} \beta^s \left[u(c_{t+s}; z_{t+s}) - v(h_{t+s}; z_{t+s}) \right] \\
s.t. \quad & P_t c_t \leq M_t && (\mu) \\
& M_t + B_t - X_t = \\
& \quad M_{t-1} - P_{t-1}c_{t-1} + w_{t-1}h_{t-1} + (1+i_{t-1})B_{t-1} + \int_0^1 d_{t-1}(j)dj && (\lambda),
\end{aligned}$$

where z_t is the vector containing any exogenous disturbances to household utility, and $v_h(\cdot) > 0$ and $v_{hh}(\cdot) > 0$. The first order conditions are

$$0 = u_c(c_t; z_t) - P_t \mu_t - \beta P_t E_t \lambda_{t+1} \tag{1.9}$$

$$0 = -v_h(h_t; z_t) + w_t E_t \lambda_{t+1} \tag{1.10}$$

$$0 = -\lambda_t + \mu_t + \beta E_t \lambda_{t+1} \tag{1.11}$$

$$0 = -\lambda_t + \beta(1+i_t) E_t(\lambda_{t+1}) \tag{1.12}$$

along with the budget constraints and the transversality condition,

$$\lim_{t\to\infty} \beta^t \lambda_t (B_t/P_t + M_t/P_t) = 0. \tag{1.13}$$

Equations 2.11 and 2.13 yield

$$\frac{u_c(c_t; z_t)}{P_t} = \lambda_t.$$

Therefore 2.14 and 2.12 become

$$u_c(c_t; z_t) = \beta(1 + i_t) E_t u_c(c_{t+1}; z_{t+1}) \frac{P_t}{P_{t+1}} \tag{1.14}$$

$$\frac{v_h(h_t; z_t)}{u_c(c_t; z_t)} = \frac{\omega_t}{1 + i_t}, \tag{1.15}$$

where ω is the real wage.

1.2.4 Monetary Policy

I first abstract from the financial system and open market operations and assume that the central bank conducts policy through manipulating the money supply via direct cash transfers in order to maintain a sequence of targets for gross money growth: $\{\varrho_t^T\}_{t=s}^{\infty}$. The money supply then follows

$$M_t = \varrho_t^T M_{t-1}, \tag{1.16}$$

and the nominal interest rate is left to adjust accordingly. As will be discussed later, if the policy rate runs up against the ZLB, the central bank may engage in monetary expansion by implementing extra, one-time monetary injections.

1.2.5 Equilibrium

An equilibrium in this economy is a collection

$$\{M_0, \{\overline{z}_{t+s}\}_{s=0}^{\infty}, \{y_{t+s}, h_{t+s}, P_{t+s}, \gamma_{t+s}, M_{t+s}, i_{t+s}\}_{s=0}^{\infty}, \{\varrho_{t+s}^{T}\}_{s=0}^{\infty}\}.$$

All markets must clear, so $y_t = c_t$ and $B_t = 0$ for all time. Given that labor is the only factor input, $\gamma_t = \omega_t$ as well. Additionally,

$$\Omega_{t,t+s} = \beta^s \frac{u_c(c_{t+s}; z_{t+s})}{u_c(c_t; z_t)}, \tag{1.17}$$

and the full set of equilibrium conditions are

$$\begin{aligned}
\frac{P_t^*}{P_t} &= \frac{\theta}{\theta-1}\frac{E_t\sum_{s=0}^{\infty}(\alpha\beta)^s u_c(y_{t+s}; z_{t+s}) y_{t+s}\gamma_{t+s}(P_{t+s}/P_t)^{\theta}}{E_t\sum_{s=0}^{\infty}(\alpha\beta)^s u_c(y_{t+s}; z_{t+s}) y_{t+s}(P_{t+s}/P_t)^{\theta-1}} \\
P_t^{1-\theta} &= (1-\alpha)(P_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta} \\
u_c(y_t; z_t) &= \beta(1+i_t)E_t u_c(y_{t+1}; z_{t+1})\frac{P_t}{P_{t+1}} \\
\frac{v_h(y_t/A_t; z_t)}{u_c(y_t; z_t)} &= \frac{\gamma_t}{1+i_t} \\
y_t &= \frac{M_t}{P_t},\ i_t > 0 \\
M_t &= \varrho_t^{T} M_{t-1}
\end{aligned} \tag{1.18}$$

along with some process $\{\overline{z}_{t+s}\}_{s=0}^{\infty}$ for the exogenous disturbances and a sequence $\{\varrho_t^{T}\}_{t=0}^{\infty}$ chosen by the central bank.

1.3 Effective Monetary Policy at the ZLB

In this section I demonstrate that permanent open market operations, defined as any policy that permanently increases the stock of money relative to other assets, are relevant at the ZLB in in the model outlined in the previous section, extending the work of Auerbach and Obstfeld (2005). This relevance result holds whether money demand is motivated by a CIA constraint or due to the presence of money in the utility function. With these results in hand, I then use a simple quantitative model to illustrate the impact of POMOs. I show POMOs are *effective*: they increase output and inflation despite the ZLB. To account for the ZLB constraint, I solve the model using global methods. Using this approach, I then discuss the impact of POMOs in different contexts. Finally, I use these results to discuss the macroeconomic impact of large scale asset purchases and recent Fed policy.

1.3.1 Relevance Results

Cash in Advance

In this section, I demonstrate that money is relevant at the ZLB.[3] As in Auerbach and Obstfeld (2005), assume that at time $t = 0$ the model economy is hit with an adverse economic shock that takes the policy rate to zero. Furthermore, suppose that agents correctly anticipate all future economic disturbances and that at some future date T the economy will exit the ZLB. Using the household's Euler equation 1.14 along with its intratemporal optimality condition 1.15 and the definition $w_t = P_t\gamma_t$, I obtain

$$\frac{w_{t+1}}{v_h(y_{t+1}/A_{t+1};z_{t+1})} = \beta\frac{w_t}{v_h(y_t/A_t;z_t)}(1+i_{t+1}) \tag{1.19}$$

[3] I define relevance to mean that changes in the money supply influence inflation and the real economy, or change the macroeconomic equilibrium.

Under the assumption that the policy rate is constrained by the ZLB in period $t = 0$ through period $t = T - 1$, for $t < T - 1$

$$\frac{w_t}{v_h(y_t/A_t; z_t)} = \beta^{-1} \frac{w_{t+1}}{v_h(y_{t+1}/A_{t+1}; z_{t+1})}. \tag{1.20}$$

Looking ahead, when the economy rises from the ZLB for $t > T - 1$ I have

$$(1 + i_t) = \beta^{-1} \frac{u_c(M_t/P_t; z_t)}{u_c(M_{t+1}/P_{t+1}; z_{t+1})} \frac{P_{t+1}}{P_t} \tag{1.21}$$

given the CIA constraint will bind. Substituting into 1.19 and taking the resulting equation back one period gives

$$\frac{w_t}{v_h(y_t/A_t; z_t)} = \frac{w_{t-1}}{v_h(y_{t-1}/A_{t-1}; z_{t-1})} \frac{u_c(M_t/P_t; z_t)}{u_c(M_{t+1}/P_{t+1}; z_{t+1})} \frac{P_{t+1}}{P_t}. \tag{1.22}$$

Equations 1.20 and 1.22 give a near complete description of nominal marginal cost. All that is left is to determine w_{T-1}. In period $T - 1$,

$$u_c(y_{T-1}; z_{T-1}) = \beta u_c(M_T/P_T; z_T) P_{T-1}/P_T \tag{1.23}$$

$$v_h(y_{T-1}/A_{T-1}; z_{T-1}) = u_c(y_{T-1}; z_{T-1})(w_{T-1}/P_{T-1}), \tag{1.24}$$

so that

$$\frac{w_{T-1}}{v_h(y_{T-1}/A_{t-1}; z_{T-1})} = \beta^{-1} u_c(M_T/P_T; z_T)^{-1} P_T. \tag{1.25}$$

Therefore, when $t > T-1$

$$\frac{w_t}{v_h(y_t/A_t;z_t)} = \beta^{-1}\frac{P_{t+1}}{u_c(M_{t+1}/P_{t+1};z_{t+1})}, \tag{1.26}$$

and when $t \leq T-1$

$$\frac{w_t}{v_h(y_t/A_t;z_t)} = \left(\prod_{s=t+1}^{T-1}\beta^{-s}\right)\frac{P_T}{u_c(M_T/P_T;z_T)}, \tag{1.27}$$

via solving 1.20 forward.

Now consider two sequences for the money supply, $\{M_t\}_{t=0}^{\infty}$ and $\{M_t'\}_{t=0}^{\infty}$, with exactly the same money growth rates but with different initial starting values M_0 and $M_0' = M_0 + \Delta M$, respectively. Then for all t

$$\frac{M_t'}{M_t} = \frac{\prod_{s=1}^{t}\varrho_s(M_0+\Delta M)}{\prod_{s=1}^{t}\varrho_s M_0} = 1+\Delta M/M. \tag{1.28}$$

Suppose that monetary policy is irrelevant at the ZLB and that the path of real variables are identical under both regimes. When the ZLB is binding

$$\left(\frac{w_t'}{v_h(y_t'/A_t;z_t)}\right)\Big/\left(\frac{w_t}{v_h(y_t/A_t;z_t)}\right) = \frac{P_T'u_c(y_T;z_T)}{P_T u_c(y_T';z_T)}, \tag{1.29}$$

which reduces to

$$\frac{w_t'}{w_t} = \frac{P_T'}{P_T} \tag{1.30}$$

under the assumption of irrelevance. During time T, the CIA constraint binds so $M_T = P_T Y_T$, and continuing with the assumption of irrelevance, I obtain

$$P'_T/P_T = (M'_T/y'_T)/(M_T/y_t) = M'_T/M_T \tag{1.31}$$

so

$$P'_T/P_T = 1 + \Delta M/M. \tag{1.32}$$

and thus $w'_t/w_t = 1 + \Delta M/M$ in periods $t \leq T-1$. Inspection of 1.26 provides the same result obtains for $t > T-1$.

A first order, log-linear approximation to 1.5 and 1.6 provides

$$\tilde{P}^*_T = \tilde{\mu} + (1-\alpha\beta)\sum_{s=T}^{\infty}(\alpha\beta)^{s-T}\tilde{w}_s \tag{1.33}$$

$$\tilde{P}_T = (1-\alpha)\tilde{P}^*_T + \alpha\tilde{P}_{T-1} \tag{1.34}$$

where I adopt the notation $\tilde{x} = \log(x)$. Subtracting the log price level when M follows $\{M_t\}_{t=0}^{\infty}$ gives

$$(\tilde{P}^*_T)' - \tilde{P}^*_T = (1-\alpha\beta)\sum_{s=T}^{\infty}(\alpha\beta)^{s-T}(\tilde{w}'_s - \tilde{w}_s). \tag{1.35}$$

If the initial log price level is $\tilde{P}_{-1}$, solving 1.34 forward gives

$$\tilde{P}_T = (1-\alpha)\sum_{s=0}^{t}\alpha^s\tilde{P}^*_{T-s} + \alpha^{T+1}\tilde{P}_{-1}, \tag{1.36}$$

so

$$(\tilde{P}_T)' - \tilde{P}_T = (1-\alpha)\sum_{t=0}^{t}\alpha^s((\tilde{P}^*_{T-s})' - \tilde{P}^*_{T-s}). \tag{1.37}$$

Define $\log(1+\Delta M/M) = \delta_m$. Then, assuming irrelevance 1.35 implies

$$(\tilde{P}^*_T)' - \tilde{P}^*_T = \delta_m. \tag{1.38}$$

and 1.37 gives

$$\tilde{P}'_T - \tilde{P}_T = (1-\alpha)\sum_{t=0}^{t}\alpha^s\delta_m < \delta_m. \tag{1.39}$$

However, if monetary policy is irrelevant, household labor supply during the ZLB period implies

$$\frac{w'_t}{w_t} = \frac{P'_T}{P_T} = \frac{M'_T}{M_T} \tag{1.40}$$

or

$$\tilde{P}'_T - \tilde{P}_T = \tilde{w}'_t - \tilde{w}_t = \delta_m, \tag{1.41}$$

generating a contradiction.

The above result resides in stark contrast to the irrelevance result provided in Eggertsson and Woodford (2003). They show 'balance sheet policy,' which includes increases in the monetary base at the ZLB, fails to impact equilibrium when the central bank commits to an interest rate feedback rule that obeys the Taylor principle (see Taylor (1993)). In their framework, money demand is motivated by incorporating money in the utility function (MIU) and includes the structural relation

$$\frac{u_m(c_t, M_t/P_t, h_t; z_t)}{u_c(c_t, M_t/P_t, h_t; z_t)} = \frac{i_t}{1+i_t} \tag{1.42}$$

which can be solved implicitly for (real) equilibrium money demand

$$\frac{M_t}{P_t} = L(y_t, i_t; z_t),\ i_t > 0. \tag{1.43}$$

They assume that the central bank commits to the money supply rule

$$M_t = P_t L(y_t, \Phi(P_t/P_{t+1}, y_t; \bar{z}_t); z_t)\Psi(P_t/P_{t+1}, y_t; \bar{z}_t) \tag{1.44}$$

where

$$\Psi(P_t/P_{t+1}, y_t; \bar{z}_t) = 1 \text{ if } \Phi(P_t/P_{t+1}, y_t; \bar{z}_t) > 0, \text{ otherwise}$$
$$\Psi(P_t/P_{t+1}, y_t; \bar{z}_t) \geq 1.$$

The assumptions regarding $\Psi(\cdot)$ explicitly call for the central bank to wind down balance sheet policy when the economy exits the ZLB. This directly contradicts results from Robatto (2014), which suggest monetary policy is relevant when it is either permanent, or at least wound down some time after the economy escapes the ZLB. Monetary injections, as described earlier in this section, imply a different, time-varying form for $\Psi(\cdot)$. Specifically, policy is nonstationary and monetary injections are never reversed, which circumvents Eggertsson and Woodford's result.

Money in the Utility Function

Money continues to remain relevant when demand is motivated by assuming cash balances enter the utility function. In this case, household period utility is given by

$$u(c_t, M_t/P_t; z_t) - v(h_t; z_t). \tag{1.45}$$

The household's intratemporal optimality conditions in equilibrium change as well. Now

$$\frac{v_h(y_t/A_t; z_t)}{u_c(y_t, M_t/P_t; z_t)} = w_t/P_t \tag{1.46}$$

and

$$\frac{u_m(y_t, M_t/P_t; z_t)}{u_c(y_t, M_t/P_t; z_t)} = \frac{i_t}{1+i_t}. \tag{1.47}$$

Assume that real money demand is satiated at some level $\overline{m}$, so that if $i_t = 0$ central bank supplies $\overline{m}$ and money markets clear. As before, assume that an adverse economic shock takes the economy to the ZLB at time $t = 0$ and that it is correctly anticipated to rebound at some future date T. From the previous section, 1.27 still characterizes the time-path of the ratio of nominal wages to the marginal disutlity from work from $t = 1$ to $T - 1$. Away from the ZLB ($t \geq T$),

$$\frac{w_t}{v_h(y_t/A_t; z_t)} = \frac{P_t}{u_c(y_t, M_t/P_t; z_t)}. \tag{1.48}$$

Now consider two paths for the money supply $\{M_t'\}_{t=T}^{\infty}$ and $\{M_t\}_{t=T}^{\infty}$ *after* the economy rebounds where

$M'_t/M_t = 1 + \Delta M$ for all time. Irrelevance implies

$$\frac{M'_t}{P'_t} = \frac{M_t}{P_t}, \; t \geq T \tag{1.49}$$

or $P'_t/P_t = 1 + \Delta$, otherwise the marginal utility of holding real cash balances would be altered, which would in turn alter the household's stochastic discount factor and the macroeconomic equilibrium. From the household's MRS

$$\frac{w'_t}{w_t} = \frac{P'_t}{P_t} = 1 + \Delta M. \tag{1.50}$$

The supply side of the economy is unchanged given a different motivation of money demand, so imperfect price adjustment suggests $P'_t/P_t < 1 + \Delta M$, generating a contradiction. Monetary policy must influence the equilibrium.

1.3.2 Impact of POMOs at the ZLB

POMOs as Cash Transfers

In this section I evaluate the effects of POMOs. I demonstrate that they are effective at increasing output and inflation at the ZLB. I assume preferences are given by

$$z_t \left(\frac{c_t^{1-1/\sigma}}{1-1/\sigma} - \chi \frac{h_t^{1+\eta}}{1+\eta} \right),$$

yielding the specific FOCs

$$1 = \beta(1+i_t)E_t\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{z_{t+1}}{z_t}\right)\Pi_{t+1}^{-1} \tag{1.51}$$

$$\chi\frac{h_t^{\eta}}{c_t^{-\sigma}} = \frac{\omega_t}{1+i_t}, \tag{1.52}$$

where Π_t is the gross inflation rate. To facilitate the use of global methods so that I can explore the consequences of the ZLB, I assume firms face quadratic adjustment costs of price adjustment as in Rotemberg (1982). Both Calvo staggered pricing and quadratic adjustment costs yield the same implications for aggregate price level determination and inflation, so this assumption is innocuous. The adjustment cost for firm j is given by

$$C(P_t(j), P_{t-1}(j)) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\Pi}\right)^2 h_t,$$

where $\overline{\Pi}$ is target steady-state inflation. In a symmetric equilibrium, where production technology is now $y_t(i) = h_t(i)$,

$$\phi(\Pi_t - \overline{\Pi})\Pi_t = 1 - \theta + \theta\omega_t + \beta E_t\left[\frac{z_{t+1}}{z_t}\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{h_{t+1}}{h_t}\right)(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1}\right]. \tag{1.53}$$

The aggregate resource constraint becomes

$$y_t = \left(1 - \frac{\phi}{2}(\Pi_t - \overline{\Pi})^2\right)h_t. \tag{1.54}$$

I close the model by assuming the preference shock follows

$$z_t = (1-\rho) + \rho z_{t-1} + \sigma_e e_t,\ e_t \text{ iid } N(0,1) \tag{1.55}$$

and that WLOG, the central bank sets $\varrho_t^T = 1$ for all time. This implies the real money supply obeys $m_t = m_{t-1}/\Pi_t$.

I solve the model by policy function iteration as described in Richter et al. (2014). The model's two state variables are z and m_{-1}. For both variables I construct a grid of 30 equally spaced points on the intervals $z \in [0.8, 1.1]$ and $m_{-1} \in [0.95\overline{m}, 1.08\overline{m}]$, where $\overline{m}$ is the steady-state level of the real money supply. I then form a two-dimensional state-space containing all permutations of the state variables along their associated grids. At all 30^2 points I solve for the policies $\omega(m_{-1}, a)$ and $m(m_{-1}, a)$ that are consistent with the model's structural relations and the ZLB constraint, approximating expectations using Guassian quadrature on a grid composed of 30 equally spaced points for e in the interval $[-4, 4]$. To ensure convergence I use the solution to a linear version of the model as initial guesses for $\omega(\cdot)$ and $m(\cdot)$. All computations were performed in Matlab using the parallel computing toolbox.

I then subject the model economy to a large, negative preference shock that takes the nominal interest rate to the ZLB for a number of periods. I assume that the central bank makes some initial and unannounced monetary injection on impact of the shock and commits to keep future money growth rates constant. I vary the size of initial transfers—ΔM in the language of the previous sections—between zero and one percent and explore the implications for the time-paths of output, inflation, real wages, the nominal interest rate, nominal money balances, and the price level.

Parameter values for this experiment are provided in Table B.1. The selected values are fairly standard. The discount factor, β, is based on a quarterly rate that gives an annual risk free rate of approximately four percent. I set η to three, yielding a Frisch elasticity of labor equal to one-third. In line with the macro literature, I set the intertemporal elasticity of substitution at one-half which implies σ is two. I assume firms target a steady-state markup of twenty percent, so θ equals six. With respect to the preference shock, I choose $\rho = 0.85$ and $\sigma_e = 0.005$. Finally, I normalize steady-state output, hours worked, and real money balances to one, which pins down χ.

Figure A.8 provides impulse responses to -10% preference shock. The blue, red, and black lines represent 0%, 0.05%, and 1% increases in the nominal money supply, respectively. Increasing the size of the initial monetary injection decreases the severity of the fall in output and inflation compared to the benchmark of doing nothing (0%). It is evident that monetary policy is relevant even though the nominal interest rate is constrained by the ZLB for about a year after the initial shock. Intuitively, the commitment to a higher trajectory for the money supply increases the price level when the economy rebounds from the ZLB and money regains transaction value. Furthermore, since the money stock is permanently higher, as the economy converges towards the steady-state the price level remains elevated for all time via the Quantity Theory.[4] In a rational expectations equilibrium, agents use this information when forming expectations and anticipate a higher, future price level which must translate into higher inflation given sluggish price adjustment. Higher inflation expectations then boost aggregate demand and contemporaneous output and inflation.

In line with Auerbach and Obstfeld (2005), the trajectory of the nominal interest rate is virtually unchanged regardless of what path the central bank chooses for the nominal money supply. If the monetary authority operates according to an interest rate rule,Eggertsson and Woodford (2003) and Jung et al. (2005) demonstrate the macroeconomic effects achieved by POMOs at the ZLB must be implemented by keeping the nominal interest rate below the Wicksellian natural rate for an extended period of time. The subsequent literature offers many variations on this general principle, but the underlying theme is that the central bank must *extend* the ZLB period and maintain *lower* interest rates in order to be accommodative. In models with money demand, these policies also tend to call for increases in the nominal money supply commensurate with lower interest rates. Thus, there appears to be a disconnect between POMOs and interest rate policy. This points to the potential existence of an independent channel through which money creation can influence the macroeconomy when conventional policy is constrained by the ZLB.

[4] The CIA constraint yields $M_t = P_t \overline{y}$, which is the equation of exchange with velocity fixed at unity.

Implementation of POMOs via Asset Purchases

So far I have assumed that the central bank makes permanent monetary injections via direct transfers to households. While the BOJ has recently mulled such policies, no modern central bank has ever used direct transfers as a means to manipulate the money supply. In fact, it is illegal for many central banks, including the Fed, to engage in such transfers as they are directly the purview of the fiscal authority. However, there are several ways in which the central bank can engineer such policy when confronted with a liquidity trap. Open market purchases of government debt where the central bank explicitly commits to holding newly purchased securities to maturity should yield the desired result.

Consider the simple model from the previous section but now with government spending, financed by issuing bonds or levying lump sum taxes. The aggregate resource constraint becomes

$$y_t = c_t + g_t. \tag{1.56}$$

The government's real budget constraint is given by

$$\tau_t + b_t = g_t + (1 + i_{t-1})b_{t-1}/\Pi_t. \tag{1.57}$$

I assume the central bank commits to purchasing a constant fraction of the outstanding government debt, Q_t, in each period, financed by money creation in accordance with its money growth target. In addition, the central bank returns all interest earned on its debt holdings to the fiscal authority. This implies

$$m_t - m_{t-1}/\Pi_t = Q_t b_t. \tag{1.58}$$

and

$$\tau_t + b_t = (1 - Q_{t-1})(1 + i_{t-1})b_{t-1}/\Pi_t + g_t. \tag{1.59}$$

The fiscal authority follows a tax rule designed to stabilize the national debt

$$\tau_t = \overline{\tau}(b_{t-1}/\overline{b})^{\psi}. \tag{1.60}$$

Finally, to abstract from fiscal policy and simplify the analysis, I assume $g_t = \overline{g}$ for all time.

Lump sum taxes ensure that tax and bond issuance policy does not change households' optimality conditions. As a result, taxes and bond issuance are irrelevant for equilibrium determination. Therefore, the model's only state variables are once again m_{-1} and z. As before, for each variable I construct a grid of 30 equally spaced points along the intervals $m_{-1} \in [0.95\overline{m}, 1.08\overline{m}]$ and $z \in [0.8, 1.1]$. I then form a two-dimensional state-space containing all permutations of the state variables along their respective grids, and solve the model using policy function iteration as outlined in the previous section. With $\omega(\cdot)$ and $m(\cdot)$ in hand, I then back out the implied time-paths for taxes, government debt, and open market purchases of government debt.

Impulse responses to a -10% preference shock are given in Figure A.9. For this experiment, I set $\overline{g}$ to 20% of steady-state output and find $\psi = 0.25$ was sufficient to guarantee a unique, bounded solution. A commitment to a one-time increase in the money supply implemented via an open market purchase of government debt is effective at boosting output and inflation even when the nominal interest rate is constrained by the ZLB. These POMOs require a one-time, significant increase in the size of the central bank's balance sheet (up to 100%), followed by smaller, but still nontrivial purchases that keep the size of the balance sheet elevated for years after the end of the ZLB episode. While the impact on inflation and output is somewhat attenuated, this is primarily due to setting steady-state inflation to 1.005% so as to ensure a positive level of the central bank asset holdings in the steady state. This translated to a $\overline{Q}$ of approximately 5% of outstanding steady state government debt. In addition, these open market operations have no impact on the duration of the ZLB period and minimal effect on rates once the economy rebounds. If anything, POMOs slightly *increase* the time-path of the nominal interest rate. Ultimately, the qualitative results are exactly identical to those of the previous section, demonstrating POMOs are accommodative at the ZLB.

Inflation Inertia

A large literature argues that the benchmark New Keynesian model overstates the effectiveness of accommodative policy at the ZLB. Many studies point to the presence of inflation inertia, which greatly reduces the efficacy of forward guidance, optimal interest rate policy, etc. To address this concern, I now include inflation inertia by amending the cost of price adjustment as follows

$$C(\cdot) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\Pi}\right)^2 h_t + \frac{\xi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \frac{P_{t-1}(j)}{P_{t-2}(j)}\right)^2 h_t. \tag{1.61}$$

Firm profit maximization in a symmetric equilibrium then yields

$$\begin{aligned} \phi(\Pi_t - \overline{\Pi}) = 1 - \theta + \theta\omega_t - \xi(\Pi_t - \Pi_{t-1})\Pi_t + \phi E_t[\Omega_{t,t+1}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1}h_{t+1}/h_t] \\ + \xi E_t[\Omega_{t,t+1}(\Pi_{t+1}^2 - \Pi_t^2)h_{t+1}/h_t] \\ - \xi E_t[\Omega_{t,t+2}(\Pi_{t+2} - \Pi_{t+1})\Pi_{t+1}h_{t+1}/h_t], \end{aligned} \tag{1.62}$$

where again

$$\Omega_{t,t+s} = \beta^s \left(\frac{z_{t+s}}{z_t}\right)\left(\frac{c_{t+s}}{c_t}\right)^{-\sigma}. \tag{1.63}$$

I return to the case of cash transfers and abstract from fiscal policy and asset purchases.

Now the state-space must be expanded to include lagged inflation. The inclusion of an additional state variable and the need to compute expectations two periods ahead greatly adds to the computational complexity of this problem. I construct grids of ten equally spaced points for inflation, the money supply, and the preference shock on the intervals $\Pi_{-1} \in [0.95\overline{\Pi}, 1.05\overline{\Pi}]$, $m_{-1} \in [0.95\overline{m}, 1.05\overline{m}]$, and $z \in [-0.85, 1.1]$. When approximating expectations I use a grid of 20 equally spaced points for $e \in [-4, 4]$. Reducing the density of the state space in this manner allows me to solve the model in a reasonable amount of time while employing Matlab's parallel processing toolbox.

Figure A.10 provides impulse responses to a -10% preference shock. For this experiment I maintain $\phi = 40$ and set $\xi = 20$. Steady state inflation is normalized to unity. The inclusion of inflation inertia only modestly dampens the inflation response to POMOs, while the effect on the output response is virtually nonexistent. This is due in part to the fact that inflation inertia of the type assumed here determines contemporary inflation not only as a function of lagged inflation, but expectations of inflation and economic activity further into the future relative to the benchmark case in Section 3.2.1. As a consequence, policies such as POMOs and forward guidance potentially have even more traction at the ZLB. A more reasonable test would be to motivate the inclusion of a lagged inflation term—and only a lagged inflation term—in firms' optimality conditions. This is left as an avenue for future research.

1.3.3 Policy Discussion

These results demonstrate that permanent open market operations are effective at increasing real activity when the ZLB binds. Following the collapse of Lehman Brothers in October 2008 and the ensuing financial crisis, the Federal Reserve engaged in a series large scale asset purchases (LSAPs), i.e. QE, to accommodate economic activity. In November 2008 the Fed initiated its LSAP program by committing to purchase several hundred billion dollars worth of agency mortgage backed securities (MBS). Early in the following year, these LSAPs were augmented by the initial roll out of QE1. By the time this program concluded in early 2010, the Fed had purchased over 1.25 trillion USD in agency MBS, agency debt, and Treasury securities. Later that year, the Fed initiated QE2 by committing to purchase 600 billion USD in longer-term Treasury securities, ending this round of asset purchases in June 2011. The Fed began another round of QE starting in September 2013. QE3 was more open ended; the Fed announced it would purchase 40 billion USD in agency MBS each month until economic conditions improved. This program continued for over a year, with the FOMC occasionally tweaking the size of monthly asset purchases.

QE was implemented by what the Fed defines as permanent open market operations: *"purchases or sales of securities on an outright basis that add or drain reserves and change the size or composition of the System Open Market Account."* Figure A.11 provides net end-of-month permanent open market transactions as reported by the Federal Reserve Bank of New York between January 2007 and December 2013. A positive value indicates a net purchase, while negative values are open market sales. In addition, the red vertical lines indicate QE announcement dates. The figure clearly depicts an acceleration of open market purchases immediately following FOMC decisions; the data from the end of the sample completely captures the first few months of QE3. Net purchases only briefly fall below zero in late 2012 and as a consequence, significantly increased the size of the Fed's balance sheet as illustrated in Figure A.1. By definition, these financial market interventions come with no commitment to permanence; they may be reversed at any time in the future in accordance with Fed policy.[5] However, the data indicates that the Fed was actively increasing reserves in the banking system via its QE programs. During this time period, the Federal Open Market Committee (FOMC) held its policy rate—the Federal Funds Rate (FFR)—at the effective ZLB.[6] Combined, it appears that the Fed was engaged in conducting POMOs at the ZLB as described in previous sections.

[5] As a consequence, the Fed's definition of permanent open market operations and the definition used previously are significantly different. POMOs throughout this paper are defined as open market transactions that *permanently* change the stock of money.

[6] The FOMC lowered the FFR to a range between zero and twenty-five basis points at its December 2008 meeting. The Committee did not raise its target for the FFR until December 2015.

A look at the reported monetary aggregates suggests otherwise. Figures A.2, A.3, and A.4 plot average, monthly, seasonally adjusted currency in circulation, M2, and MZM[7] versus simple trends from January 1990 to the present. M2 is a broad measure of money that includes liquid assets that can be easily used to conduct transactions: currency, demand deposits, savings deposits, money market securities, money market mutual funds, and small time deposits (CDs). MZM is M2 less small time deposits plus institutional money funds, and so comprises a more liquid set of assets than M2. The data is provided by the Federal Reserve Board and the Federal Reserve Bank of St. Louis. For currency and MZM, linear trends were estimated from the start of the depicted sample through the end of 2006 using the natural logarithm of currency and MZM as dependent variables. The trend for M2 was estimated over the same time period using log M2, but a quadratic trend was used to better match the data. More detail is provided in Tables A.2 and A.3. The vertical red lines still give QE announcement dates. Currency and M2 both share similar features; starting in late 2006 they experience a sharp decline. Money growth recovers but both aggregates never return to the projected trend. MZM tends to be more volatile, but starting in early 2010 it also experiences a discrete drop from trend which persists. These declines are significant. On average, since January 2007, cash has remained approximately 15% below trend while starting in January 2010 MZM has held steady at nearly 7% below trend. QE announcements or actual asset purchases seem to have no impact on these figures.

Figures A.5, A.6, A.7 provide monthly indices for Divisia M3, M4, and M4-, respectively.[8] M3 is defined as M2 plus large time deposits, eurodollar deposits, and repurchase agreements. M4- is M3 plus commercial paper, while M4 is M4- including T-bills. These aggregates include assets that are commonly used for large value payments in financial markets, and represent extremely broad measures of money. Data was obtained from the Center for Financial Stability, and linear trends were estimated using natural logarithms with a window starting in January 1990 and ending in December 2006. The red vertical

[7] Money zero maturity

[8] See Barnett et al. (1984) for details on construction of the Divisia aggregates.

lines once more give QE announcement dates. All three Divisia aggregates maintain trend up until QE1. Following implementation of this asset purchase program, all Divisia indices experience sharp declines. While cash, M2, and MZM, growth recovers following their initial drops, it is clear that Divisa money growth remains depressed even through the announcement and implementation of QE2 and QE3. All three aggregates continue to fall further from trend through the end of the sample (September, 2016).

The behavior of the monetary aggregates suggest that QE as implemented by the Fed did not directly translate into significant increases in the money stock. Therefore, while on the surface these large scale asset purchases mimic effective POMOs, in practice they do not. In fact, it appears that overall Fed policy led to a discrete drop in the money supply. The theory detailed so far suggest that such action should decrease nominal expenditures away from the ZLB, placing downwards pressure on inflation expectations and economic activity. Policy failure in this regard parallels the experience of the U.S. economy at the onset of the Great Depression and the Fed's response as outlined in Friedman and Schwartz (2008). The authors posit that severe monetary contraction was a significant, contributing factor to declines in real economic activity between 1929 and 1933. This view was supported by none other than former Fed Chairman, Ben Bernanke, who stated *"Let me end my talk by abusing slightly my status as an official representative of the Federal Reserve. I would like to say to Milton and Anna: Regarding the Great Depression. You're right, we did it. We're very sorry. But thanks to you, we won't do it again"*(see Bernanke (2002)). In light of the monetary data, it appears that Fed policy failed to increase the money supply in order to support economic recovery following the Great Recession. When coupled with the theoretical results from the previous sections, the failure of Fed policy to generate sufficient money creation provides an account for the limited response of output, inflation, and unemployment to asset purchases found in Chen et al. (2012), Chung et al. (2012), and Wu and Xia (2014), and an alternative explanation for the overall sentiment that U.S. QE and monetary policy in general has been ineffective as outlined in Sims (2016).

One could argue that as excess reserves fall the money supply will recover. Considering the over four-fold increase the Fed's balance sheet, the money supply may also come to be permanently higher in the future. However, the Fed's communication strategy and the private sector's response to Fed policy announcements over the past several years convey a different future scenario. The evolving language of FOMC statements during the ZLB period and the committee's overall emphasis on conducting asset purchases in "the context of price stability" indicate an overall unwillingness on the part of policymakers to accept the necessary up-tick in inflation associated with effective POMOs and other forms of accommodative policy at the ZLB. This view is supported by the behavior of the stock market in response to macroeconomic news announcements (MNAs). Law et al. (2016) find that during the ZLB period, positive MNAs boosted stock prices. However, this result is contingent on markets forecasting future lower interest rates. They find that when markets anticipate future Fed rate hikes in response to good news, these gains are almost completely attenuated. A look at headlines from late 2015, such as *"Dow plunges as job gains raise rate hike fears"* (USA Today, 3/6/2015), *"Ugly day for stocks: Everything down as Fed rate hike fears return"* (CNN Money, 5/26/15), and *'Stocks flop amid fears of Federal Reserve interest rate hike"* (NY Daily News, 11/10/15) reinforce the impression that the Fed would not entertain higher levels of inflation commensurate with POMOs. In addition, the size of the System Open Market Account and the Fed's balance sheet experienced gradual declines pursuant to the termination of QE3 in October 2014 as illustrated in Figure A.1.[9] Combined with observed declines in various measures of money, these stylized facts suggest that the money stock will remain depressed and no new money creation is anticipated—by both the Fed and markets—in the near future. Therefore, POMOs as defined by the Fed do not belong in the class of effective policies analyzed in the previous sections.

These results offer guidance for the conduct of future policy. For POMOs to support economic activity at the ZLB they must be effectively communicated for what they are: one-time, permanent increases in

[9]While the Fed is rolling over investments in longer-term Treasuries, as its MBS portfolio matures the Fed is no longer reinvesting its the proceeds by purchasing additional agency debt.

the stock of money. This is not to say policymakers should necessary target money aggregates or re-adopt money growth rate targets, but rather that they bring attention to the impact of asset purchases on the central bank's balance sheet, how they hope these transactions will affect broad measures of money, and how they forecast changes in the money supply will influence the macroeconomy in the coming future. Along these lines, policymakers should also convey a willingness to tolerate a somewhat elevated level of inflation in the short and medium term. In other words, these financial market interventions must not be sterilized, otherwise the resulting impact on macroeconomy will mimic the lukewarm response of the U.S. economy to Fed QE. Provided policymakers are clear in their expectations and private agents view their actions as credible, the results from previous sections suggest that POMOs can be conducted without unlodging inflation expectations and *within a context of long-run price stability*.

1.4 Relevance and Other Policy Regimes

In this section I explore the welfare consequences of POMOs versus other policy regimes. As a robustness exercise, I relax the previously strong assumptions regarding POMOs and money growth and discuss the interaction between Taylor rules and open market operations, and the implications for policy normalization—returning to Taylor rule and raising interest rates—away from the ZLB.

1.4.1 Welfare and Effective Monetary Policy

I now analyze the welfare impact of effective monetary policy when the economy is potentially constrained by the ZLB and money demand is motivated by a CIA constraint. Woodford (2003) demonstrates that the utility-based welfare criterion for a model economy with transaction frictions takes the form

$$W_0 = -\Gamma E \left\{ \sum_{s=0}^{\infty} \beta^s \left[\hat{\pi}_t^2 + \lambda_x (\hat{x}_t - x^*)^2 + \lambda_i (\hat{i}_t - \hat{i}_t^m)^2 \right] \right\}, \tag{1.64}$$

where a "hat" denotes variables expressed in log-deviations from their respective steady-states. In this specification, x_t is the period-t output gap which is given by the difference between actual and the natural rate of output, while x^* is the steady-state level of the output gap due to the presence of market power in intermediate goods markets. The term i_t^m represents the interest paid on money balances. For this section, I assume the federal government provides production subsidies to intermediate goods producers to set excess capacity to zero in the steady-state ($x^* = 0$), financed by lump-sum taxes. In addition, as implicitly outlined above, $i_t^m = 0$ for all time.[10] Finally, while the response of real variables to the preference shock are nonzero even when prices are fully flexible, the transaction friction due to the CIA constraint is small enough that $\hat{y}_t \approx \hat{x}_t$. As a consequence, the welfare criterion becomes

$$W_0' = -\Gamma E\left\{\sum_{s=0}^{\infty} \beta^s \left[\hat{\pi}_t^2 + \lambda_x \hat{y}_t^2 + \lambda_i \hat{i}_t^2\right]\right\}. \tag{1.65}$$

Woodford (2003) assumes money demand is motivated by the presence of money in the utility function, so that in equilibrium

$$\hat{m}_t = \psi_y \hat{y}_t - \psi_i \hat{i}_t \tag{1.66}$$

In a model with a CIA constraint, $\psi_y = 1$ and $\psi_i = 0$. This is due to the fact financial markets open first, allowing households to distribute their financial assets so that they only carry enough cash to satisfy transaction demand. This implies $\lambda_i = 0$, so the welfare criterion reduces to

$$W_0' = -\Gamma E\left\{\sum_{s=0}^{\infty} \beta^s \left[\hat{\pi}_t^2 + \lambda_x \hat{y}_t^2\right]\right\} \tag{1.67}$$

[10] i_t^m is interest payed on excess money balances. While the Federal Reserve recently started paying interest on excess reserves, in this section I abstract from this policy and assume $i_t^m = 0$.

The unconditional expectation comes out to

$$W_0' = -\frac{\Gamma}{1-\beta}\left[var(\hat{\pi}_t) + \lambda_x var(\hat{y}_t)\right]. \tag{1.68}$$

Equation 1.68 only provides welfare in terms of utility losses. A more useful measure is consumption equivalent welfare losses described in Schmitt-Grohé and Uribe (2007). Consider welfare under two policy regimes, W_0^a and W_0^r. Let λ^c represent the fraction of a representative household's consumption process it would be willing to forgo under regime r in order to be as well off in regime a. In this case, λ^c is implicitly defined by

$$W_0^a = E\sum_{t=0}^{\infty}\beta^t u\big((1-\lambda^c)c_t^r, h_t^r\big). \tag{1.69}$$

To facilitate the calculation of λ^c, I set households' intertemporal elasticity of substitution to unity, so

$$u(c_, h_t; z_t) = z_t\left(\ln c_t - \chi\frac{h_t^{1+\eta}}{1+\eta}\right).$$

This implies

$$\lambda^c = 1 - \exp\big\{(1-\beta)(W_0^a - W_0^r)\big\}. \tag{1.70}$$

To calculate welfare losses using 1.68 and 1.70, I first take a log-linear approximation to the model in

Section 2. This yields the structural relations

$$\begin{aligned}
\hat{y}_t &= E_t \hat{y}_t - 1/\sigma(\hat{i}_t - E_t \hat{\pi}_{t+1}) - 1/\sigma E_t(\hat{z}_{t+1} - \hat{z}_t) \\
\hat{\pi}_t &= \beta \hat{\pi}_{t+1} + \kappa(\eta+\sigma)\hat{y}_t + \kappa \hat{i}_t \\
\hat{m}_t &= \hat{y}_t,\ \hat{i}_t > \underline{i} \\
\hat{m}_t &= \hat{\varrho}_t + \hat{m}_{t-1} - \hat{\pi}_t \\
\hat{z}_t &= \rho \hat{z}_{t-1} + \nu e_t.
\end{aligned} \tag{1.71}$$

Under Calvo pricing, $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$. I solve the log-linear model in Dyare, then use these policy functions to account for the ZLB using the solution method proposed in Guerrieri and Iacoviello (2015).

The value for λ_x is given in Table B.1 and calibrated based on calculations in Woodford (2003):

$$\lambda_x = \frac{(\eta+\sigma)\kappa}{\theta(1+\eta\theta)}. \tag{1.72}$$

Second moments were calculated via simulations. For each of the listed policy regimes I simulate 10,000 draws of $e_t \sim N(0,\sigma^2)$ and calculate the model economy's response to these innovations subject to the ZLB following Holden and Paetz (2012). For each simulation I set $\alpha = 0.7$ and the standard deviation of the preference shock to 0.02, which causes the economy to hit the ZLB approximately 10% of the time under commitment.

Results are listed in Table A.4. I compare POMOs to a suite of standard policy regimes analyzed in the literature on monetary policy. The first column gives utility losses relative to a Taylor Rule regime where $\hat{i}_t = \delta\hat{\pi}_t$, $\delta = 5$. Policy regimes indicated by ΔM represent commitments by the central bank to permanently increase the monetary base by the given amount every time the nominal interest rate is constrained by the lower bound, holding money growth constant at zero at and away from the ZLB. I then measure welfare when the central bank commits to a zero inflation target. The next two policy regimes listed represent optimal policy under both discretion and commitment from the timeless perspective, as discussed in Woodford (2003) and Walsh (2010). The final policy listed is an output gap adjusted price level target suggested in Eggertsson and Woodford (2003).[11] The central bank chooses $\hat{i}_t$ to achieve its price level target, $\hat{p}_t^T$ according to the rule

$$\hat{p}_t = \hat{p}_t^T + \frac{\lambda_x}{\kappa}\hat{y}_t \tag{1.73}$$

if the ZLB is not binding, otherwise it sets

$$\hat{p}_{t+1}^T = \hat{p}_t^T + \beta^{-1}(1+\kappa/\sigma)\Xi_t - \beta^{-1}\Xi_{t-1} \tag{1.74}$$

where $\Xi_t = \hat{p}_t^T - \hat{p}_t$. This operationalizes a commitment to increase its price level target every period the central bank is prevented from hitting its target due to the ZLB. The second column reports consumption costs relative to this regime (regime r) in terms of $100 \times \lambda^c$. A positive value indicates that households are better off when the central bank employs a flexible price level target.[12]

The results suggest that POMOs are superior to pure commitments to a Taylor rule, inflation targeting, and optimal discretion. All monetary injections significantly decreased utility losses and achieved welfare gains associated with optimal commitment and the flexible price level target. Turning towards consumption costs, I obtain similar results. All interventions reduce the consumption cost of economic fluctuations relative to a Taylor rule by nearly 100%, and smaller transfers almost completely eliminate the consumption cost of fluctuations relative to the optimal price level target. A quick glance suggests that under the current parameterization, welfare gains are maximized when the central bank commits to

[11] I refer to this policy as a 'flexible price level target'.

[12] The flexible price target was used as the reference regime in this case because it minimized welfare losses. The time-path of variables under what I define as optimal commitment was calculated at each point in time at the ZLB as the solution to the Ramsey problem conditional on there being no future economic disturbances. Eggertsson and Woodford (2003) explain the flexible price level target implements the Ramsey solution when the future path of the economy is unknown. As such, it outperforms optimal commitment during simulation when the economy fluctuates randomly.

increasing the money supply somewhere between 0.1% and 0.5% each time the ZLB binds. While effective POMOs attenuate declines in output and inflation associated with the ZLB, this comes at the expense of higher future output and inflation. The net impact on volatility and welfare is therefore ambiguous. Table A.5 reports the standard deviations of output and inflation for the different sets of POMOs from Table A.4. While permanent monetary interventions always increase output volatility, moderate increases reduce inflation volatility and the welfare costs of price dispersion. Given that welfare losses increase above a certain threshold, there appears to be a moderate upside risk from higher expected inflation associated with large POMOs. Nonetheless, larger injections still perform remarkably well relative to a host of other policy regimes. To the extent that Fed policy deviated from the first-best solution—in this case, the flexible price level target—or optimal commitment, conducting POMOs could have increased welfare during the recovery from the Great Recession.

1.4.2 Robustness and Policy Normalization

So far I have assumed that away from the ZLB, the central bank conducts policy in order to achieve specific money growth rate targets. Modern central banks largely abandoned money growth targeting in the 1980s and now favor targeting short-term nominal interest rates. Implicitly, many central banks set policy according to a rule that obeys the Taylor principle.[13] By its very definition, a Taylor rule is a commitment to increase nominal interest rates when inflation—and depending on the functional form, output or the output gap—increases. Away from the ZLB, this sort of policy is equivalent to changing money growth so that the supply of money is consistent with the central bank's interest rate target. Infusing money into the economy when the nominal interest rate is at the ZLB will tend to increase expected inflation so long as the assumptions for relevance hold. However, this higher expected inflation will induce the central bank to act in a contradictory manner and increase interest rates, thereby decreasing money growth when

[13] See Taylor (1993).

the ZLB no longer constrains monetary policy. In a rational expectations equilibrium, private agents will correctly anticipate these policy actions and inflation expectations will not change, rendering any increases in the monetary base ineffective.

This result was correctly anticipated in Eggertsson and Woodford (2004). The commitment to a Taylor rule forces the central bank to completely wind down any changes made in the money supply during the ZLB period, making them irrelevant. A similar result holds when the central bank conducts optimal policy. Under commitment, policymakers promise to honor private agents' past expectations of current economic conditions. This limits the range of policy options both away from and at the ZLB. Specifically, the central bank is precluded from making the type of permanent changes that exactly make POMOs relevant. As a consequence, like under a Taylor rule, the monetary authority will unwind all changes in the money supply during the ZLB period when it is optimal to raise the nominal interest rate above zero. This result still holds if the central bank operates with discretion.

As a robustness exercise, I will show that open market operations are still relevant under a Taylor rule provided the central bank does not immediately unwind its actions. Consider a hybrid rule of the form

$$\begin{aligned}
0 &= \varphi_t(\varrho_t - \overline{\varrho}) + (1-\varphi_t)[i_t - \Phi(\pi_t, y_t)] \\
\varphi_t &= \begin{cases} 0 & , \; i_t, i_{t-1}, \ldots, i_{t-k} > 0 \\ 1 & , \; \text{otherwise} \end{cases} \\
M_t^s &= \begin{cases} M_{t-1} + \Delta M & , \; i_t = 0 \text{ and } i_{t-1} > 0 \\ (1+\varrho_t)M_{t-1} & , \; \text{otherwise} \end{cases} \\
\Phi(\pi_t, y_t) &= (\pi_t/\overline{\pi})^{\zeta_\pi}(y_t/\overline{y})^{\zeta_y}
\end{aligned} \tag{1.75}$$

When the economy hits the ZLB the central bank conducts a POMO and sets $\varphi_t = 1$, maintaining constant money growth for k periods following the initial shock. Only then does the central bank normalize policy by setting $\varphi_t = 0$ and returning to interest rate policy.

Figure A.12 gives impulse responses to a -8% preference shock and were computed using a shooting algorithm in Dynare. For the experiment described below, I set $\zeta_\pi = 5$ and $\zeta_y = 2$. On impact, I assume the central bank increases the money supply by ΔM and maintains constant money growth for an additional 25 periods, then returns to following the Taylor rule forever. POMOs continue to be effective so long as the central bank waits some time after the nominal interest rate rebounds to return to the interest rate rule. This lessens the declines in inflation and output even when private agents anticipate with perfect foresight that the monetary authority will normalize policy in the future. This is because under the hybrid specification, returning to a Taylor rule upon exit from the ZLB is a commitment to only partially unwind the central bank's previous actions. As in previous sections, this leads to a permanent increase in the money supply and price level which raises inflation expectations and aggregate demand.

Conventional optimal policy analysis implies that it is incumbent upon a welfare maximizing central bank to commit to holding interest rates at the ZLB for an extended period of time to prevent significant declines in output and inflation following severe economic shocks. Furthermore, policymakers should keep their policy rate below the Wicksellian natural rate upon rebound for some time. Given the natural rate is unobserved, Armenter (2012) demonstrates that it may be welfare improving for a central bank to maintain lower rates for an even longer period of time due to uncertainty over the level of the natural rate. However, some argue that policy uncertainty associated with protracted ZLB periods may dampen the effects of accommodative policy.

Postponing monetary policy normalization generates similar aggregate outcomes as in Armenter (2012), but the implications for policy instruments are markedly different. The inflation response to money creation induces the central bank to engage in a more aggressive manner when the central bank returns to the Taylor rule. As a consequence, permanent open market operations along with delayed policy reform decreases the duration of the ZLB episode while also increasing the time-path of the nominal interest rate relative to the benchmark of simply maintaining a Taylor rule for all time. This is distinctly different from optimal interest rate policy, suggesting that there is an independent role for money creation and open market operations at the ZLB.

These results yield an interesting role for forward guidance, broadly defined as approaches taken by central bankers to convey the future time-path of policy instruments and their role in supporting economic activity. The general approach taken by the FOMC has been what Campbell et al. (2012) call Delphic forward guidance; communicating forecasts of economic activity and likely policy responses without explicit commitments. As the authors note, in the December 2008 FOMC announcement, the committee stated that it expected economic conditions to "warrant exceptionally low levels of the [FFR] for some time." Later this language was amended to replace "some time" with "an extended period." Starting in August 2011, the FOMC began giving time frames for likely interest rate hikes, changing its expected path of policy as economic conditions changed over time. These actions reflect the policy prescriptions of Eggertsson and Woodford (2003) where money is irrelevant. By adopting a hybrid rule, the above results suggest that an alternative approach where the central bank instead engages in open market operations while communicating the time-path of future asset purchases, their likely effects on money, and expectations about the interaction between the money supply and output and inflation. These actions should be enacted together with a commitment to normalize policy—in this context, return to a Taylor rule—only some time in the future[14] will achieve similar qualitative effects. Unlike in Eggertsson and Woodford (2003), monetary policy will not only support economic activity but also decrease the duration of the ZLB episode and reduce future policy uncertainty.

[14]Or conversely, communicate a willingness to not unwind open market operations and maintain steady money growth "for an extended period of time."

1.5 Conclusion

I investigate the ability of permanent open market operations (POMOs) to support economic activity in the wake of adverse shocks when conventional interest rate policy is constrained. Contrary to the well-known Eggertsson and Woodford (2003) irrelevance result, I first show that money creation can influence the real economy at the ZLB in a standard New Keynesian model, and then analyze the impact of permanent cash transfers as in Auerbach and Obstfeld (2005) and Robatto (2014).

I then perform numerical simulations that show POMOs both attenuate declines in inflation and output associated with ZLB using a standard quantitative framework using global solution methods. The intuition is fairly straightforward. Provided the nominal interest rate is expected to rebound at some point in the future, increased money balances will put upwards pressure on the price level. In a rational expectations equilibrium, households will internalize this information and anticipate higher, future prices which must translate into higher expected inflation given sluggish price adjustment, which will then boost contemporaneous output and inflation. I initially model POMOs as cash transfers, but then show they can be implemented with open market operations, and that the inclusion of lagged inflation in the New Keynesian Phillips Curve does not greatly attenuate the effects of these policies.

With these results in hand I discuss Fed policy during the Great Recession. Data on a wide variety of money aggregates—currency, M2, MZM, Divisia M3, Divisia M4-, Divisia M4—all experience significant drops from projected trends between 2006 and 2010. While cash, M2, and MZM growth recovers (but does not revert to trend), growth in the Divisia aggregates remains depressed. Despite significantly increasing the size of its balance sheet via large scale asset purchases, Fed policy failed to make up for this shortfall in the money supply. Work by Law, Song, and Yaron (2016) combined with a significant concentration of headlines from 2014 and 2015 suggest that the private sector did/does not anticipate future money creation either. Using these stylized facts, my theory suggests the tepid response of the inflation and output growth in the U.S. to Fed QE, as outlined in Sims (2016) and various empirical studies, was due to the fact policy failed to generate sufficient money creation to support recovery from the Great Recession.

I then discuss welfare and policy normalization—when to raise interest rates after prolonged ZLB periods. I find that conducting POMOs at the ZLB improves welfare relative to a host of different regimes analyzed in the literature on monetary policy. I find moderate cash injections (between 0.1% and 0.5%) maximize welfare relative to smaller and more significant financial market interventions. These open market transactions attain welfare gains similar to first-best policy, and so I argue that conducting POMOs following the Financial Crisis could have improved welfare during the recovery from the Great Recession. As a robustness exercise, I then illustrate that conducting open market purchases, combined with a commitment to return to a Taylor rule only after the nominal interest rate rebounds, remains a useful tool for increasing output and inflation at the ZLB. I also find that these sorts of hybrid policies shorten the duration of ZLB periods following adverse shocks. This implies that this class of policies can reduce policy uncertainty and support economic activity during ZLB episodes.

Chapter 2

Effective Quantitative Easing at the Zero-Lower-Bound

2.1 Introduction

The U.S. Federal Reserve (the Fed) engaged in a series of extraordinary measures to support economic activity in response to the Financial Crisis. Following the collapse of Lehman brothers in October, 2008, the Fed lowered its policy rate to the effective zero-lower-bound (ZLB) and engaged in a series of large scale asset purchase programs, dubbed quantitative easing (QE). Since November 2008, the Fed has accumulated over 3.6 trillion USD in agency mortgage backed securities, treasury securities, and other financial assets over three separate rounds of QE.[1] A growing body of empirical evidence suggests that these financial market interventions had a limited impact on U.S. production and prices. Chung et al. (2012) find that QE2 increased the level of U.S. GDP by 0.6% and inflation by 0.1%. Chen et al. (2012) only find that QE2 bolstered GDP growth by 0.13% and increased inflation by merely three basis points

[1]As measured by total securities held outright the Federal Reserve System's H.4.1 release, Factors Affecting Reserve Balances.

at an annualized rate. Analyzing labor market conditions, Wu and Xia (2014) calculate that QE3 reduced U.S. unemployment by an additional 0.13% in December 2013.

In 1, I argue that the observed lack of transmission between monetary policy and the U.S. economy during this time period was due in part to the fact the Fed effectively sterilized its QE program. Via its communication policy and actions, the Fed prevented its asset purchases from generating sufficient money creation to support economic activity following the Financial Crisis. Conversely, I show that policies aimed at increasing the level of the money supply are able to attenuate declines in output and inflation associated with adverse economic shocks and the ZLB in a standard New Keynesian setting. I extend this analysis using a framework where households have acess to a broader set of assets and display liquidity preference a lá Andrés et al. (2004) and Harrison (2012b). This framework is advantageous in that it permits a separate channel through which QE may affect the real economy.[2] I show that asset purchases which are reversed when the nominal interest rate rebounds from the ZLB have little impact on output and prices. This is inline with the observed response of the U.S. economy to Fed QE. I then illustrate that asset purchase policies aimed at increasing the level of the nominal money supply at the ZLB bolster production and prices, requiring fewer asset purchases and leaving the long-term inflation rate unchanged. These results imply that the Fed could have stimulated inflation and output growth with smaller, targeted asset market interventions

The model employed throughout the paper has its roots in Tobin's theory of imperfect asset substitutability.[3] Households in the economy view different assets—stocks, short-term bonds, long-term bonds, etc.—as having distinctly different innate characteristics that are separate from their fundamentals-driven returns.[4] For example, consumers may view long-term debt as being inherently less liquid than short-term bonds. Whatever the cause or justification, these factors drive a wedge between risk-adjusted returns. In

[2]Specifically, asset purchase policy is an independent instrument of monetary policy, and may be conducted in tandem with a money growth rule or feedback rule for the nominal interest rate.
[3]See Tobin (1961), Tobin (1969), Tobin (1974), and Tobin (1982).
[4]Tobin dubbed these factors "exogenous interest rate differentials."

addition, imperfect asset substitutability implies the relative supply of assets may influence asset prices, which is important for the conduct of asset purchase policy.[5] Andrés et al. (2004) introduce imperfect asset substitutability by assuming households face both transaction costs and portfolio adjustment costs when trading in long-term debt. This second friction is a convenient way of capturing their assumption that households view long-term debt as less liquid than cash balances. Harrison (2012b) instead assumes households face a portfolio adjustment cost between holdings of short and long-term, risk-free government debt. In Andrés et al. (2004), these costs are paid in units of real goods and services while the portfolio adjustment cost in Harrison (2012b) enters directly into the utility function. While this assumption may seem *ad hoc*, Harrison (2012a) illustrates how this setup yields virtually the same equilibrium conditions as a model where banks back household transaction accounts using a mix of short and long-term debt.

An alternative approach to producing imperfect asset substitutability is introduced in Vayanos and Vila (2009). In this setting, households have innate preferences for investing in certain types of assets, called 'preferred habitat.' As in model with portfolio adjustment costs, the presence of preferred habitat introduces a wedge between fundamental-driven returns and implies monetary policy may affect the real economy by changing the relative supply of assets available to households. Curdia and Woodford (2011) show that both assumptions—explicit portfolio adjustment costs and preferred habitat—generate similar equilibrium conditions to a first order approximation. The authors then demonstrate that asset purchase policy—which they dub balance sheet policy—has a limited impact on economic activity at the ZLB. Specifically, they find that monetary expansion is only effective in so much that it finances the central bank's ability to lend directly to firms when private financial intermediaries cannot or will not, in tandem with a commitment to sustaining lower interest rates. In other words, QE is only effective when financial markets are sufficiently disrupted and the central bank commits to lower the future path of its

[5] Curdia and Woodford (2011) provides a extensive discussion of the literature surrounding imperfect asset substitutability, heterogeneous household spending opportunities, and central bank asset purchase policy.

policy rate.

Curdia and Woodford's results are the consequence of their assumptions about policy away from the ZLB. As in Eggertsson and Woodford (2003) and Eggertsson and Woodford (2004), they assume the central bank returns to following a time-invariant, Taylor-like rule for the nominal rate when it rebounds. This is tantamount to a commitment to unwind any money creation and asset purchases as soon interest rates become positive. In 1 and in this study, I relax this assumption and show that breaks from pre-committed policy rules that enable the balance sheet or the money supply to remain elevated upon exiting the ZLB are effective at increasing output and inflation at the ZLB, contrary to Curdia and Woodford's results. These policies are also advantageous in that they do not require the central bank to adjust their long-run targets for money growth or inflation.

Other approaches at analyzing QE at the ZLB look focus on asset purchases as a way for the central bank to intervene directly in lending markets. Two seminal studies in this literature are Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), who utilize models with a banking sector subject to financial frictions similar to Bernanke et al. (1999), but focus on crises that originate with banks. They find that credit policy—actions aimed at mitigating the disruption of activity between borrowers and lenders via direct intervention in debt markets—can alleviate the impact of financial crises on real activity. Araújo et al. (2013) consider central bank purchases of assets commonly used as collateral in debt markets as a means to support borrowers during times of financial distress. They conclude that these forms of targeted asset purchases may work if credit markets are extremely disrupted and only so long as the central bank can engage in them without drastically reducing the supply of pledgeable collateral in private hands. Approaches along these lines are attractive in that they can generate the contraction in lending observed during the Financial Crisis and Great Recession. They also provide a means through which money creation can generate excess reserves, which increased dramatically following the collapse of Lehman Brothers.[6] Andrés et al. (2004) and Harrison (2012b) do not permit the existence of excess reserves, as they do not explicitly model financial intermediation. I modify their approaches to allow households to hold excess cash[7] if the nominal rate hits the ZLB. While this still does explicitly incorporate the problem of excess reserves, it allows for an analogous equilibrium outcome if nominal rates fall to zero.

The remainder of this paper is organized as follows: Section 2 outlines the macroeconomic model. In Section 3 I discuss the effects of QE at the ZLB, and Section 4 concludes.

2.2 Model

A representative household maximizes lifetime utility from consumption of a single, final good, disutility from providing labor services to intermediate-good producers, and disutility from adjusting its portfolio of long and short-term bonds. The only other financial asset available to the household is money, which is required for consumption purchases due to a cash-in-advance (CIA) constraint, and may be stored at a zero nominal rate. Short-term bonds are standard one-period riskless debt instruments issued by a government entity, the fiscal authority. Andrés et al. (2004) model long-term debt as finite maturity bonds with longer-horizon redemption dates. They assume there is no secondary market for long-term debt to maintain analytical tractability. I follow Harrison (2012b) and model long-term bonds as perpetuities or consols: risk-free debt securities issued by the fiscal authority which have no redemption date. These assets are assumed to have been issued sometime in the infinite past and households may trade them in a secondary market.

[6] Excess reserves still remain extremely elevated today, and are approximately 95% of reserves held at Federal Reserve Banks. See release H.4.1, Factors Affecting Reserve Balances.

[7] Both frameworks assume money demand is driven by the presence of real money balances in the utility function. Their assumptions regarding preferences imply money demand explodes towards infinity at the ZLB. As such, Harrison (2012b) assumes the effective ZLB is 25 basis points, placing a theoretical, if ad hoc, upper bound on money demand.

2.2.1 Government

To focus on the effects of monetary policy, I assume that there is no government spending in the economy and any taxes or transfers to the representative household are lump-sum. To finance these transfers, the government issues short-term debt and consols. Short-term debt refers to one-period bonds, B, which are traded at par and pay a gross interest rate R upon redemption in all states of the world. Consols, B_c trade in markets at a price V, and pay one unit of currency in every period. The nominal consolidated government budget constraint is

$$V_t B_{c,t} + B_t - (1 + V_t) B_{c,t-1} - R_{t-1} B_{t-1} + \Delta_t = T_t, \tag{2.1}$$

where T is lump-sum transfers to or from the representative household, and Δ is the net change in the central bank's balance sheet. Following Harrison (2012b), it is convenient to rewrite the government's budget constraint in terms of the single-period return to perpetuities. Define $B^g_{L,t} = V_t B_{c,t}$ and $R_{L,t} = (1 + V_t)/V_{t-1}$. Then

$$B^g_{L,t} + B_t - R_{L,t} B^g_{L,t-1} - R_{t-1} B_{t-1} + \Delta_t = T_t. \tag{2.2}$$

The central bank may adjust its balance sheet by engineering changes in the money supply or purchasing long-term debt. When conducting asset purchases, the central bank is assumed to commit to purchase a fraction, q, of the outstanding market value of consols. This implies the central bank's balance sheet evolves according to

$$\Delta_t = M_t - M_{t-1} - (q_t B^g_{L,t} - q_{t-1} R_{t-1} B^g_{L,t-1}). \tag{2.3}$$

Letting π denote the gross inflation rate, combining 2.3 and 2.2 provides

$$b_t + m_t + (1-q_t)b^g_{L,t} = \left(m_{t-1} + R_{t-1}b_{t-1} + (1-q_{t-1})R_{L,t}b^g_{L,t-1}\right)/\pi_t + \tau_t, \textbf{??} \tag{2.4}$$

in real terms.

The central bank and fiscal authority work in tandem, so Δ may deviate from zero in equilibrium so long as the fiscal authority offsets theses changes with debt or transfer policy. Following Harrison (2012b), I assume the stock of consols is fixed at $\overline{b}_c$, so that the market value of government perpetuities is $V_t\overline{b}_c$.[8] To stabilize the national debt, the fiscal authority employs a transfer rule

$$\tau_t = \tau(R_{t-1}, b_{t-1}),\ \tau_R, \tau_b < 0. \tag{2.5}$$

This assumption implies that transfers respond slowly to changes in the stock of short-term debt. Thus, variation in the money supply or central bank asset purchases must be balanced with changes in short-term debt issuance.

Regarding monetary policy, I assume the central bank adjusts the nominal rate on short-term debt according to a Taylor rule

$$R_t = \Phi(\pi_t, y_t, R_{t-1}),\ \Phi_\pi, \Phi_y > 0, \tag{2.6}$$

where y is real output. In later sections I allow the central bank to drop this rule for some time and engage in alternative monetary policies. As Harrison (2012b) describes, the presence of portfolio adjustment costs implies there is an independent channel for asset purchases, or QE. The central bank may employ

[8]Note that this quantity may vary over time, even though the real stock of consols is fixed.

a QE rule

$$q_t = \Psi(\pi_t, y_t), \tag{2.7}$$

as a separate instrument of policy. I assume that in 'normal' times, away from the ZLB, the central bank sets $q = 0$, but may set $q > 0$ to support economic activity following adverse shocks.[9]

2.2.2 Households

The representative Household is similar to its counterpart in Harrison (2012b) save for the fact that its consumption purchases are bound by a CIA constraint. I adopt the timing convention of Lucas (1982), so that the household begins each period with full knowledge of the state of the economy—for that period—and enters financial markets where it may part its wealth, A, between the economy's three assets. During this time period, the government also makes adjustment to the money supply, conducts asset purchases, and extracts or distributes lump-sum transfers. As a consequence, the household is bound by

$$M_t + B_t + B_{L,t} = A_t + T_t. \tag{2.8}$$

when trading in financial markets. Production then takes place, and the household provides labor hours, h, to intermediate-goods firms. Nominal wages, W, are then paid and profits, $D(j)$, are distributed to the household as a lump-sum transfer. Subsequently, the goods market opens, where consumption purchases are subject to

$$P_t c_t \leq M_t, \tag{2.9}$$

[9]Unlike in Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Araújo et al. (2013), there is no particular force or friction preventing the central bank from using asset purchases as tool to implement monetary policy in its normal operations. Nonetheless, I set $q = 0$ outside of ZLB episodes to be in line with historical experience.

where P is the price of final goods and services. Finally, end of period wealth is defined as

$$A_{t+1} = M_t - P_t c_t + W_t h_t + R_t B_t + R_{L,t+1} B_{L,t} + \int_0^1 D_t(j) dj, \tag{2.10}$$

and may be used next period to purchase additional securities or cash.

In addition to the transaction friction, the household views long-term government debt as inherently less liquid than short-term bonds. As in Harrison (2012b), this financial market friction is captured by including a convex portfolio adjustment cost in the representative household's utility function. In addition, household preferences may vary endogenously due to the inclusion of a preference shock, a. Consequently, the household problem is

$$\begin{aligned}
\max_{c_t, h_t, m_t, b_t, b_{L,t}} \quad & E_t \sum_{s=t}^{\infty} a_s \beta^{t-s} \left(\ln c_s - \chi \frac{h_s^{1+\eta}}{1+\eta} - \frac{\tilde{\nu}}{2} \left(\delta \frac{b_s}{b_{L,s}} - 1 \right)^2 \right) \\
s.t. \quad & c_t \leq m_t \\
& m_t + b_t + b_{L,t} \leq \big(m_{t-1} - c_{t-1} + w_{t-1} h_{t-1} + R_{t-1} b_{t-1} \\
& \qquad + R_{L,t} b_{L,t-1} + \int_0^1 d_{t-1} dj \big) / \pi_t + \tau_t .
\end{aligned}$$

Setting μ as the Lagrange multiplier associated with the CIA constraint, and λ as the multiplier associated with the budget constraint, the household's first order optimality conditions are

$$0 = a_t/c_t - \mu_t - \beta E_t \lambda_{t+1}/\pi_{t+1} \tag{2.11}$$

$$0 = -a_t \chi h_t^{\eta} + w_t \beta E_t \lambda_{t+1}/\pi_{t+1} \tag{2.12}$$

$$0 = \mu_t - \lambda_t + \beta E_t \lambda_{t+1}/\pi_{t+1} \tag{2.13}$$

$$0 = -a_t \tilde{\nu} \left(\delta \frac{b_t}{b_{L,t}} - 1 \right) \frac{\delta}{b_{L,t}} - \lambda_t + \beta R_t E_t \lambda_{t+1}/\pi_{t+1} \tag{2.14}$$

$$0 = a_t \tilde{\nu} \left(\delta \frac{b_t}{b_{L,t}} - 1 \right) \delta \frac{b_t}{b_{L,t}^2} - \lambda_t + \beta E_t R_{L,t+1} \lambda_{t+1}/\pi_{t+1} \tag{2.15}$$

When combined, 2.11 and 2.13 imply that the real marginal utility of wealth equals the marginal utility of consumption: $\lambda = a/c$.

The inclusion of portfolio adjustment costs drives a wedge between the expected real return of shot-term and long-term debt. To see this, combine 2.14 and 2.15, which results in

$$E_t[\Lambda_{t,t+1}(r_{t+1} - r_{L,t+1})] = \tilde{\nu}\delta \left(\delta \frac{b_t}{b_{L,t}} - 1 \right) \left(\frac{1}{b_t} + \frac{1}{b_{L,t}} \right) c_t \frac{b_t}{b_{L,t}}. \tag{2.16}$$

In this context, $\Lambda_{t,t+1} = \beta(a_{t+1}/a_t)(c_t/c_{t+1})$ is the household's stochastic discount factor, $r_{t+1} = R_t/\pi_{t+1}$, and $r_{L,t+1} = R_{L,t+1}/\pi_{t+1}$. If households did not view perpetuities as less liquid than short-term bonds, then $\tilde{\nu} = 0$, and the standard result that the expected return on these assets are equal would be obtained. Now, with $\tilde{\nu} > 0$, the spread between returns on long and short term debt is driven by the households' portfolio mix. Due to the CIA constraint, the optimality condition governing labor hours suggests that the household's labor supply will also be governed by the distortion of assets in its portfolio, and the relative supply of short-term and long-term debt in the economy.

2.2.3 Final-Goods Firms

Final-goods producers operate in a perfectly competitive environment. These firms purchase differentiated output, $y(j)$, from monopolistically competitive intermediate goods producers to construct final output using the constant elasticity of substitution bundling technology

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \; j \in [0,1]. \tag{2.17}$$

Each final goods producer maximizes profits:

$$\max_{y_t(j)} = P_t \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(j) y_t(j) dj.$$

As stated earlier, P_t is the price level of final output. $P_t(j)$ is the price of input, j. Final goods' firms optimality conditions yield the optimal price index for final output,

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{2.18}$$

and the demand function for intermediate producer j,

$$y_t(j) = y_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta}. \tag{2.19}$$

2.2.4 Intermediate-Goods Firms

Intermediate-goods firms produce differentiated output in a monopolistically competitive environment with the production technology

$$y_t(j) = h_t(j). \tag{2.20}$$

Firms hire labor hours from a competitive labor market and take wages as given. Furthermore, pricing decisions are subject to Calvo (1983) frictions: only a fraction $1-\alpha$ of firms may update prices in any given period. The remaining measure α of firms must maintain the same price from their previous update. Given the production technology and labor market conditions, to minimize costs all firms set real marginal cost according to

$$mc_t = w_t. \tag{2.21}$$

Then all updating firms solve

$$\max_{P_t(j)} E_t \sum_{s=t}^{\infty} \alpha^{s-t} \Lambda_{t,s} \left(\frac{P_t(j)}{P_s} \right)^{-\theta} y_s \left(\frac{P_t(j)}{P_s} - mc_s \right). \tag{2.22}$$

Accordingly, the optimal adjustment price, $P_t(j)^*$, is

$$\frac{P_t(j)}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=t}^{\infty} \alpha^{s-t} \Lambda_{s,t} y_s mc_s (P_s/P_t)^{\theta}}{E_t \sum_{s=t}^{\infty} \alpha^{s-t} y_s (P_s/P_t)^{\theta-1}}. \tag{2.23}$$

Note that the optimal adjustment price for all firms is a function of aggregate variables, so all firms that update prices will set $P(j) = P^*$. Returning to the price index, this implies

$$P_t^{1-\theta} = (1-\alpha)(P_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}. \tag{2.24}$$

2.2.5 Equilibrium

In equilibrium, consumption demand must equal the supply of final goods and services

$$c_t = \mathbb{D}_t^{-1} y_t, \tag{2.25}$$

where $\mathbb{D}$ is a term that represents the distortion due to Calvo pricing frictions and price dispersion. My assumptions regarding monetary policy and asset purchases imply that the market for consols clears according to

$$b_{L,t} = (1 - q_t) b^g_{L,t} = V_t \overline{b}_c. \tag{2.26}$$

Together then, an equilibrium in this economy is a collection

$$\{b_0, b_{L,0}, m_0, \{a_t\}_{t=1}^{\infty}, \{y_t, c_t, \mathbb{D}_t, h_t, mc_t, R_t, R_{L,t}, V_t, w_t, b_t, b_{L,t}, \lambda_t, \mu_t, m_t, \tau_t\}_{t=1}^{\infty}\}$$

when markets clear according to 2.25 and 2.26, and subject to government policy encapsulated by **??** through 2.7, household optimality conditions given by 2.8 through 2.15, and such that final and intermediate goods firms maximize profits via 2.20, 2.23, and 2.24.

In the deterministic steady-state $\mathbb{D} = 1$, so $y = c$. The term δ is calibrated to be the long-run ratio of long to short-term debt, so in the steady-state the adjustment cost is zero. This makes intuitive sense, as in the long-run this ratio is constant. This implies that in the steady-state, the return on short-term debt is equal to the one-period return on consols: $R = R_L = \pi/\beta$. Steady-state net inflation is zero, so this last term reduces to $1/\beta$. Marginal cost is just the real wage rate, which is given by

$$mc = w = \frac{\theta - 1}{\theta} \beta^{-1} \chi y^{1+\eta}. \tag{2.27}$$

This expression implies that the economy faces two long-run frictions: excess capacity due to monopolistic competition, and a labor/leisure distortion due to the fact current wage income cannot be used to purchase contemporaneous consumption goods as a consequence of transaction frictions and the CIA constraint. In the literature, the first friction is usually obviated by assuming the fiscal authority provides steady-state production subsidies to exactly offset the distortion due to monopolistic competition. The second friction may be eliminated by adopting the Friedman rule, but such a policy 1) is not consistent with a zero inflation steady-state, and 2) is suboptimal when prices are sticky in the short-run. Steady-state output is normalized at unity, so $m = y = 1$. I set the level of short-term debt at $b = 1/6$, which is consistent with long-run ratio of the market value of T-bills to monetary aggregates such as M2 and MZM.[10] Once again, δ is the steady-state ratio of long to short-term debt. This implies $overline b_c = \delta b$. Finally, the price of consols is given by $V = (\beta^{-1} - 1)^{-1}$

Now define $\hat{x}_t = \ln(x_t/x)$, the log deviation of a variable from its value in the deterministic steady-state. I now approximate the model's equilibrium conditions around the deterministic steady-state and obtain following sets of equations:

Aggregate Demand (AD) Bloc These relations are derived from the household's intertemporal optimality conditions for savings, bond holdings, and cash balances. To start, to a first order $\hat{c}_t = \hat{y}_t$ as the price distortion term $\mathbb{D}$ is of second order.[11] This implies

$$\hat{y}_t = E_t \hat{y}_{t+1} - \left(\frac{1}{1+\delta} \hat{R}_t + \frac{\delta}{1+\delta} \hat{R}^e_{L,t} - E_t \hat{\pi}_{t+1} - \hat{r}^*_t \right) \tag{2.28}$$

$$\hat{R}_t - \hat{R}^e_{L,t} = \nu(\hat{b}_t - \hat{b}_{L,t}) \tag{2.29}$$

$$\hat{m}_t = \hat{y}_t, \; \hat{R}_t > \ln(\beta), \tag{2.30}$$

where $\nu = (1+\delta)\tilde{v}c/b_L$, $\hat{R}^e_{L,t} = \beta E_t \hat{V}_{t+1} - \hat{V}_t$, and $\hat{r}^*_t = -E_t(\hat{a}_{t+1} - \hat{a}_t)$.[12] Curdia and Woodford (2011) demonstrate that in a general model with household heterogeneity, the dynamic IS or AD curve will relate output to a weighted average of short and long nominal rates. This result manifests itself in equation 2.28, where the weight on the short term rate and the one-period return on consols is determined by

[10] The window for this calibration is 1996 to 2006.

[11] See Benigno and Woodford (2005).

[12] The term r^*_t should not be confused with the Wicksellian natural rate of interest. The CIA constraint implies that even if prices were perfectly flexible, $\hat{y}_t$ would not bet constant at zero in response to purely nominal shocks.

the steady-state ratio of these assets in the representative household's portfolio. Equation 2.29 is just equation 2.16 to a first order approximation, and shows that the expected spread between short and long rates is a function of the relative supply of short-term government debt and perpetuities. The last equation is the CIA constraint, which will hold so long as there is an opportunity cost to holding excess cash balances, i.e. the nominal interest rate lies above the ZLB.

Aggregate Supply (AS) Bloc The next set of relations define the aggregate behavior of firms in production markets and the representative household in labor markets.

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{mc}_t, \ \kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{2.31}$$

$$\hat{mc}_t = (1+\eta)\hat{y}_t + \left(\frac{1}{1+\delta}\hat{R}_t + \frac{\delta}{1+\delta}\hat{R}^e_{L,t}\right). \tag{2.32}$$

The first relation is the log-linear approximation of 2.23, and is the standard New Keynesian Phillips Curve (NKPC). The second relation equates firm marginal cost to the representative household's intratemporal optimality condition in equilibrium. The interest rate term is included due to the presence of the CIA constraint, which implies firm marginal cost is driven not only by fluctuations in output, but also interest rate differentials and the relative supply of assets in the economy via the representative household's labor supply decision.

Government and Market Clearing Bloc The final set of log-linear equilibrium conditions specify the shock process, policy rules, the approximate government budget constraint, and market clearing conditions. To start, to a first order log-linear approximation, the tax rule is

$$\frac{\tau}{b}\hat{\tau}_t = -\beta^{-1}\hat{R}_{t-1} - \xi \hat{b}_{t-1}. \tag{2.33}$$

The parameter ξ determines how responsive transfers are to changes in the stock of nominal short-term debt. The approximate interest rate rule is

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \mu \hat{R}_{t-1}, \; 0 < \mu < 1. \tag{2.34}$$

In the long-run, $q = 0$, so I take a linear rather than a log-linear approximation when turning towards the market clearing condition for consols

$$\hat{b}_{L,t} = \hat{V}_t - \hat{q}_t. \tag{2.35}$$

The consolidated government budget constraint is approximated by

$$\begin{aligned} &\hat{b}_t + \frac{m}{b}\hat{m}_t - \delta \hat{q}_t = \\ &\quad \frac{m}{b}\hat{m}_{t-1} - \left(\frac{m}{b} + \beta^{-1}(1+\delta)\right)\pi_t + (\beta^{-1} - \xi)\hat{b}_{t-1} - \beta^{-1}\delta \hat{q}_{t-1}. \end{aligned} \tag{2.36}$$

To close the model, I specify that the preference shock evolves via

$$\hat{a}_t = \rho \hat{a}_{t-1} + \sigma_a e_t, \; e_t \text{ iid } N(0,1). \tag{2.37}$$

2.3 Quantitative Easing at the ZLB

In this section I perform a series of quantitative exercises concerning QE at the ZLB. In all exercises, the economy is hit with a severe preference shock that takes the nominal rate to the ZLB under the Taylor rule, 2.34. Upon hitting the ZLB, the central bank announces a complete set of asset purchase and interest rate (or money growth) policies for the duration of, and following, the ZLB episode.

The ZLB places a hard nonlinear constraint on the model economy. Nominal rates may not fall below zero because households will then place all of their wealth in cash. As a consequence, in equilibrium market forces will prevent both the short and long-rate from falling below zero.[13] The literature has proposed various methods for dealing with the ZLB. Guerrieri and Iacoviello (2015) develop a linear-piecewise solution method first used in Jung et al. (2005) and Eggertsson and Woodford (2003) for general ZLB problems for and use with the Matlab and Dynare. Alternatively, one can employ nonlinear or global solution methods, such as policy function iteration as in Richter et al. (2014).

Kulish and Pagan (2016) develop an algorithm which extends standard methods for solving linear rational expectations models for general structural change. Their method generalizes the Guerrieri and Iacoviello (2015) solution procedure. Transitioning from the ZLB to positive nominal interest rates—even if the central bank does not change its policy rule—is effectively a form of regime change. A commitment by the central bank to change policy at some point during the ZLB episode or after just implies a sequence of structural changes, which the Pagan and Kulish method is aptly suited for analyzing. Given its flexibility, I employ this method, rather than alternative techniques, to compute equilibrium dynamics at the ZLB under alternative policy regimes.[14]

2.3.1 Calibration

Parameter values for all experiments are reported in Table B.1. I set η to three, which implies a Frisch elasticity of one-third, which is in the range of the Congressional Budget Offices's estimates.[15] I follow Harrison (2012b) and set $\xi = 0.025$, which implies transfers are insensitive to changes in debt and minimizes the role of fiscal policy. Between 1996 and 2006, the ratio of Treasury notes and bonds to T-bills was approximately three, which pins down δ.[16] This figure does not change significantly when

[13]This implies the gross rate—used throughout this paper—may not fall below unity.

[14]In practice, I only apply the ZLB constraint to the short-term rate and verify that in equilibrium, the ZLB does not bind for the one-period return on consols.

[15]See Reichling and Whalen (2012)

[16]Yearly data comes from the Securities Industry and Financial Markets Association (SIFMA). Ratios of long to short-term U.S. Treasury securities were calculated using the reported outstanding market value.

TIPs are included in the calculation. Empirical evidence from Andrés et al. (2004) and Bernanke et al. (2004) suggest ν lies between approximately 0.04 and 0.25. I follow Andrés et al. and set $\nu = 0.045$. I choose $\alpha = 0.75$, so that approximately one quarter of all firms may adjust prices every period. This implies κ is approximately 0.086. I set $\phi_\pi = \phi_y = 1$, and that $\mu = 0.3$, so the central bank engages in a slight amount of interest rate smoothing. Finally, I assume the preference shock is relatively persistent and so choose $\rho = 0.8$, and set $\sigma_a = 0.005$

2.3.2 Sterilized QE

The Federal Reserve engaged in several rounds of QE following the collapse of Lehman Brothers and the Financial Crisis. In November of 2008, the Fed committed to purchasing several hundred billion dollars worth of agency mortgage backed securities (MBS) to support distressed financial markets. In early 2009 it augmented these purchases with its first round of QE. The Fed ultimately purchased over 1.25 trillion USD in agency MBS, debt, and Treasury securities over the course of these initial large scale asset purchase programs, colloquially called QE1. The Fed engaged in a second round of QE, QE2, between early 2010 and June 2011, buying an additional 600 billion USD in longer-term Treasury securities. The Fed initiated QE3 in September 2013, with an open-ended commitment to purchase 40 billion USD in agency MBS every month. The Fed adjusted the size of these monthly purchases at times, and the program lasted over a year.

Via QE, the Fed significantly increased the size of its balance sheet, as securities held outright increased from just under one trillion to over four trillion USD.[17] These asset market interventions also resulted in the Fed accumulating a non-trivial portion of the outstanding market value of Treasury and MBS securities. Between 2009 and 2011, the Fed increased its holdings of long-term U.S. Treasury debt from approximately 15% to 20% of marketable Treasury notes and bonds. Between 2011 and 2014 the Fed's

[17] As measured by total securities held outright, from release H.4.1, Factors Affecting Reserve Balances, provided by the Board of Governors of the Federal Reserve System.

share increased to almost 25%.[18] Between 2008 and 2009, the Fed accumulated approximately 15% of outstanding agency MBS debt. It increased its relative holdings to 30% of the outstanding market value between 2012 and 2014.[19] Overall Fed asset holdings—in terms of market value and market share—began declining in late 2014 and 2015. This was primarily driven by the Fed's decision to begin unwinding its MBS portfolio during this time period.[20]

As I discuss in Eiermann 1, QE had little impact on the U.S. money supply. Figures B.1 through B.3 depict the time path of various measures of narrow money plotted against estimated trends.[21] The red lines provide QE announcement dates. All measures experience a discrete drop from trend starting between 2006 and 2009 and have yet to recover. Figures B.4 and B.5 plot broader measure of money against estimated trends. Once again, the red lines provide QE announcement dates.[22] These aggregates also display a significant, negative deviation from trend. However, growth in M4- and M4 also declines so that the gap between these aggregates and their estimated trends has grown over time. All measures suggest that Fed QE failed to generate significant money creation following the Financial Crisis.

In the following exercises, I call policies that mimic these stylized facts—significant asset purchases with limited transmission to the money supply—sterilized QE. To simulate the impact of sterilized QE, I hit the model economy with a large negative demand shock that takes the nominal interest rate to the ZLB under the Taylor rule. Upon hitting the ZLB, the central bank announces it will purchase a specific, constant fraction of the outstanding market value of long-term debt to augment its zero-interest-rate policy. Furthermore, it commits to unwind these asset purchases as soon as the Taylor rule calls for positive

[18] These figures were calculating by taking the ratio of Fed holding of U.S. Treasury notes and bonds—reported by the Board of Governors in release H.4.1, Factors Affecting Reserve Balances—and the sum of marketable notes and bonds outstanding (less TIPs), provided by SIFMA.

[19] These figures are the reported ratio of MBS of all maturities held by the Fed—reported by the Board of Governors in release H.4.1, Factors Affecting Reserve Balances—and the total market value of outstanding agency MBS, reported by SIFMA.

[20] The Fed is not longer reinvesting the proceeds of its MBS portfolio by purchasing additional agency MBS.

[21] Cash in circulation and MZM were obtained from the Board of Governors and the Federal Reserve Bank of St. Louis, respectively. I obtained data on M3 from the Center for Financial Stability. The data are at the monthly frequency and I used a sample starting in January, 1990, and ending in December, 2006. Linear trends were estimated using the natural logarithm of each respective series, which were then transformed to levels.

[22] Trends were estimated in the same manner for M4- and M4, which come from the Center for Financial Stability.

nominal rates. I assume that QE in the model is financed with money creation. That is, the central bank's balance sheet remains neutral so that $\Delta_t = 0$ for the duration of the ZLB episode and asset purchase policy. This choice was designed to limit the impact of asset purchase policy on short-term debt issuance by the fiscal authority, and because this commitment is roughly in line with Fed communication policy before its December, 2015 interest rate hike. During this time period–roughly 2013 through the end of 2015—the Fed communicated readiness to unwind asset purchases so as to keep inflation in check. As stated previously, the Fed explicitly began to draw down its balance sheet before raising interest rates from the ZLB. Furthermore, Law et al. (2016) show that the U.S. stock market reacted negatively to positive macroeconomic news before the December, 2015 rate hike. The authors attribute this negative relationship to private sector expectations; that financial markets viewed good economic news as a prelude to a Fed rate hike which was then considered contractionary. Ultimately, the Fed implicitly communicated temporary QE and the private sector internalized these potential commitments.

The demand shock was calibrated to cause both a significant economic contraction and a ZLB period of nonzero duration. In all exercises I thus hit the economy with a -15%, or three standard deviation, preference shock. Between QE1 and QE2, the Fed increased its holdings of long-term Treasury debt from 15% to 20% of the total, outstanding market share. That figure increased from 20% to nearly 25% in later years. Given the relevant asset in the model is long-term government debt, upon hitting the ZLB I assume the central bank mimics the Fed, and increases $\hat{q}$ from 0% to 5%, then sets asset purchases back to zero when it raises rates according to the Taylor rule. Figures B.6 and B.7 provide impulse responses to the preference shock. The red-dotted line gives economic responses to sterilized QE, while the black line represents responses to the shock in a regime where central bank does not conduct asset purchases.[23] The impact of QE on the policy and 5-year spot rates are marginal.[24]To support asset purchases, the central

[23]The central bank sets the short-term interest rate according to the Taylor rule, and sets $\hat{q}_t = 0$ for all time.

[24]The 5-year spot rate was calculated by taking a log-linear approximation to the expected, cumulative 5-year return on consols: $E_t\left[\prod_{s=0}^{20} \frac{1+V_{t+1+s}}{V_{t+s}}\right]$.

bank increases the nominal money supply by approximately 2.5%. This causes the ratio of money in circulation to nominal spending to increase by 9.5%, which I call excess cash.[25] As in practice, the overall impact on the nominal money supply is marginal, as it converges to 60 basis points below its initial value as the economy approaches the steady-state.[26]

QE attenuates the decline in inflation by approximately eight basis points at a quarterly rate, and approximately 0.32% at an annualized rate. This is relatively high when compared with estimates from Chen et al. (2012). The decline in output is reduced by approximately 26 basis points, which is close to estimates from Chung et al. (2012) for the impact of QE2 on U.S. real GDP. The response of the quantity of short bonds in circulation is similar for both regimes. As stated previously, the fiscal transfer rule is unresponsive to changes in economic conditions, so the fiscal authority offsets money creation by increasing the nominal stock of short-term debt. Declines in inflation greatly increase the real stock of debt and the money supply. Coupled with increases in the (real) money supply and asset purchases, the fiscal authority must greatly increase the stock of real short-term debt to balance the consolidated budget. Accordingly, the market value of consols held by the public increases as households attempt to achieve their optimal portfolio mix. When the central bank conducts QE, the large decline in long-term bonds to the public is due to the fact the central bank withdraws 5% of the market value from circulation, lowering the supply available to households. In sum, sterilized QE has little impact on the macroeconomy. The gains with respect to output and inflation are modest, which is supported by the empirical literature.

Extending the duration of asset purchase policy beyond the ZLB period has only a modest impact on the economy. Figures B.8 and B.9 provide impulse responses where the central bank commits to holding consols on its balance sheet for several years after the nominal interest rate recovers. Sustaining QE for two and four years beyond the ZLB period decreases output losses by a further 40 basis points, while boosting inflation by 0.4% at a quarterly rate when compared against sterilized QE. While these policies

[25] At the ZLB, the CIA constraint becomes slack. Any money not used for the purchase of final goods and services will be held as excess money balances by the representative household.

[26] The nominal money supply, unlike the real money supply, is nonstationary in this framework.

do not effect the time period when the nominal rate rebounds, the higher levels of output and inflation associated with sustained QE cause the central bank to respond with higher nominal rates via the Taylor rule. In equilibrium, the response of debt-issuance policy is relatively the same as under sterilized QE. However, maintaining asset purchase policy implies the stock of consols available to the public remains suppressed for some time. The portfolio rebalance effect causes households to bid up the price of these assets as a consequence, causing the 5-year spot rate to fall by about an additional 1.2-1.3% in response.

2.3.3 Effective QE

I now simulate economic responses under a regime where the central bank engages in asset purchases to engineer sustained increases in the money supply. While the central bank has the freedom to set asset purchase policy independently of conventional monetary policy, it does not have the ability to determine the money supply and the short-term interest rate simultaneously. As a consequence, a commitment to a sustained increase in the money supply must be matched by a commitment to abandon the Taylor rule for some definite and specified time following recovery from the ZLB.

Figures B.10 and B.11 provide impulse responses where the I vary the the duration of the central bank's QE policy, but leave the central bank's targeted increase in the money supply constant at 1%. The black line gives responses to a regime where the central bank increases the money supply but immediately returns to the Taylor rule when it requires positive rates. The red-dotted line is a commitment to maintain a 1% increase in the money supply for two years following the increase in nominal rates, while the blue-dotted line represents a commitment to sustain the increase in the money supply for four years following rebound from the ZLB. The first regime is virtually identical to a regime where the central bank engages in moderate QE—increasing $\hat{q}_t$ by about 2%—but unwinds all asset purchases when the central bank raises rates as soon as the Taylor rule requires. Output and inflation losses decline by a few basis points, and the time-path of nominal assets is relatively the same with respect to sterilized QE.

Alternatively, committing to sustain increases the money supply after the nominal rate rebounds significantly attenuates declines in output and inflation associated with the demand shock and the ZLB. It does not appear that varying the duration of money creation influences the impact of the demand shock. For both policy regimes, output losses are reduced by approximately 1.3-1.4%, and inflation losses are reduced by 1.3% at a quarterly rate. As a consequence, I call this class of policies effective QE. The intuition behind this result is fairly straightforward. As I describe in 1, once the economy exits the ZLB households will face an opportunity cost of holding excess cash and will draw down their reserves. This process will generate an increase in prices. In a rational expectations equilibrium, households will anticipate this future scenario and spend their excess cash at the ZLB to avoid future, higher prices, and inflation. So long as prices are sticky, this will generate increases in both contemporaneous prices and real economic activity.

In order to sustain the money supply and achieve these results, the central bank must engage in larger and larger asset purchases and increase its balance sheet, which mimics the Fed's actions during QE3. However, the central banks holdings of long-term bonds only reach about 2% to 3% of the total outstanding market value, which is two percentage points less than under sterilized QE. The path for the nominal rate is radically different under effective QE. A commitment to maintain a higher money stock is tantamount to maintaining zero interest rates for an extended period of time and at a lower level than what would be called for by a Taylor rule upon exiting the ZLB. There is also a sharp discontinuity around the time the central bank returns to the Taylor rule. This is due to the fact the central bank unwinds its asset purchases as part of its commitment to lower the money supply and return to interest rate policy. This decrease in money growth reduces output growth given sluggish price adjustment. Households anticipate this in a rational expectations equilibrium, and wish to 'save' to avoid a drop in consumption. This drives down interest rates to keep households satisfied with their level of consumption giving savings and investment is zero. This process reduces economic activity, so the central bank must respond aggressively due to the Taylor rule, and in part, as a consequence of interest rate smoothing. This explains why the path of the nominal rate falls after returning to interest rate policy. Regarding the fiscal authority, the treasury does not need to increase the stock of short-term debt to offset money creation and central bank asset purchases to the extent required with sterilized QE. Given the size of QE is reduced, the magnitude of the increase in the market value of consols in the hands of the public is smaller than under sterilized QE, which decreases the decline in the 5-year spot rate due to portfolio rebalancing effects.

I now vary the size of the the central bank's monetary intervention to a demand shock, maintaining the same duration. Figures B.12 and B.13 depict impulse responses where upon hitting the ZLB, the monetary authority increases the nominal money stock by 0%, 1%, 1.5%, and 2%—given by the black, red-dotted, green-dotted, and blue-dotted lines, respectively—and returns to a Taylor rule four years after the nominal rate recovers from the ZLB. As before, sustained increases in the money supply significantly attenuate declines in output and inflation. Furthermore, these gains increase with expansion of the nominal money supply, though the gains are modest. As intuition would suggest, increasing the money supply is associated with larger asset purchases. This implies, to some extent, that a central bank may use the size of asset purchases and their corresponding influence on the money supply to fine tune policy.

While there is significant evidence supporting the view that Fed QE influenced asset prices,[27] a growing empirical literature demonstrates that Fed policy did not influence broader indicators of economic activity, such as output, inflation, and unemployment. The stance of Fed policy, private market expectations, and the path of the U.S. money supply suggest that Fed QE was designed to be temporary and was sterilized. The model and quantitative exercises captures the spirit of these stylized facts and can replicate the observed lack of transmission between QE and U.S. GDP and inflation. Fundamentally, this is due to the fact QE has a limited impact on interest rates and the money supply following the ZLB period. Conversely, I show that by targeting the money supply and delaying adherence to an interest rate rule, the central

[27] See examples are Krishnamurthy and Vissing-Jorgensen (2011) and Gagnon et al. (2010)

bank is able to generate significant increases in economic activity at the ZLB via asset purchases when compared to sterilized QE. Furthermore, the central bank is able to accommodate output growth and inflation with a lower volume of long-term bond purchases. By failing to conduct policy in a manner more supportive of money creation, my results suggest that Fed policy was misdirected following the Financial Crisis. Asset purchase programs aimed at offsetting the observed declines in the money supply could have bolstered real GDP and prices with smaller and more effective financial market interventions.

2.4 Conclusion

A burgeoning empirical literature suggests that Fed QE, coupled with zero-interest-rate policy, had a limited impact on U.S. GDP and inflation. I investigate the observed lack of transmission between U.S. monetary policy and economic activity using a framework where households face portfolio adjustment costs, as in Andrés et al. (2004) and Harrison (2012b). This framework is advantageous in that central bank purchases of long-term debt form an independent channel of monetary policy. As a consequence, one can analyze the extent to which QE may augment monetary policy at the ZLB, and in other contexts.

I perform numerical simulations to analyze the impact of QE at the ZLB. I begin by simulating the effects of sterilized QE—temporary purchases of long-term debt which leave the money supply relatively unchanged. This particular thought experiment was chosen to reflect U.S. monetary policy following the Financial Crisis and during the recovery from the Great Recession. During this time period, the Fed increased the size of its balance sheet by more than fourfold, but narrow and broad measures of the U.S. money supply were unaffected. I find that sterilized QE has a marginal effect on output and inflation: A commitment to increase the central bank's holdings of long-term debt from 0% to 5% of the total market value generates an eight basis point rise in quarterly inflation, and a 40 basis point increase in output. This is roughly in line with historical experience using figures reported by Chen et al. (2012) and Chung et al. (2012).

Subsequently, I analyze a class of policies where the central bank uses asset purchases as a means to generate sustained increases the nominal money supply. To do so, the central bank must also maintain a commitment to delay returning to an interest rate rule. Using the estimated model, I show that policies aimed at increasing the money supply by 1% can reduce output and inflation losses associated with the ZLB by over a full percentage point. The impact effect is not sensitive to the duration of the policy commitment—i.e. the length of time before returning to the policy rule—so long as the central bank ensures the nominal money supply remains elevated immediately following the ZLB episode. In addition, to some extent the central bank can fine tune the size of its asset purchases and their associated impact on the money supply to engineer a desired reduction in output and inflation losses associated with adverse shocks and the ZLB. Furthermore, I find that central bank can manufacture these equilibrium outcomes with a smaller volume of asset purchases when compared with sterilized QE. Given these desirable outcomes, I call policies along these lines effective QE.

My results suggest that Fed policy during the Financial Crisis may have been misdirected. A QE program targeted towards offsetting the observed declines in the U.S. money supply could have bolstered U.S. real GDP and prices with smaller and more effective financial market interventions. This provides an important lesson for the conduct of future monetary policy in the wake of adverse economic shocks. While a broad literature doubts the efficacy of money creation at the ZLB[28], my results imply quite the opposite: money matters.

[28] See Curdia and Woodford (2011)

Chapter 3

Buyout Gold: MIDAS Estimators and Private Equity

3.1 Introduction

Traditional asset pricing techniques are inadequate for analyzing a broad set of alternative asset classes, such as private equity. This problem arises from difficulties in measuring true fund returns without observing the performance of underlying portfolio investments. As Fragkiskos et al. (2017) note, reported portfolio values exhibit returns smoothing[1] as a consequence of stale pricing among venture capital funds, and appraisal based lagged pricing among leverage buyout funds (LBOs). In short, between reporting or pricing events when observations are not available to update portfolio value, fund managers tend to anchor portfolio valuations to prior reported values, generating autocorrelation in the observed returns process. Even although a public market equivalent (PME) may be present,[2] the mismatch in timing between the pricing of the underlying portfolio investments or a PME and their underlying risk exposures

[1] Jenkinson et al. (2013)

[2] For example, see Kaplan and Schoar (2005) among others

induces a horizon-pricing problem via non-synchronous trading.[3] As a consequence—due in part to smoothed returns and horizon-pricing—inference based on observed fund performance underestimates the volatility of the 'true' private equity returns process.

The literature on alternative asset classes offers several approaches to de-smooth observed returns. Geltner (1993) constructs a model of real-estate pricing where smoothed returns are generated by updating and appraisal lags in markets. Geltner (1993) models unobserved, true real-estate returns as an autoregressive process. Geltner then scales the reported, smoothed series by the estimated autoregressive parameters, then generates a de-smoothed series of real-estate index returns that displays a significant increase in reported return volatility and autocorrelation. Getmansky et al. (2004) introduce a de-smoothing technique for analyzing hedge fund returns where smoothed returns are a consequence of performance smoothing and illiquidity exposure. Unlike Geltner (1993), Getmansky et al. assume that true returns are generated by a static factor model while observed returns are a moving average of a fund's true, underlying returns. Given hedge funds and private equity share similar features as an asset class, Pedersen et al. (2014) adapt this approach for analyzing private equity data. They calculate a de-smoothed fund index and find that private equity returns display higher volatility and increased market exposure relative to previous estimates.

Valkanov (2003) uses overlapping data to improve inference. As noted in Kamara et al. (2015), this method of time-aggregation converts both observed returns and factor prices to long-horizon series, increasing the signal to noise ratio in the data. A large number of studies use overlapping data in the finance literature.[4] However, this method increases autocorrelation in the observed series. Harri and Brorsen (2009) list several techniques to correct for this, including using heteroskedasticity and autocovariance consistent standard errors (HAC estimators)[5] or following Britten-Jones et al. (2011).

[3] Kamara et al. (2015)

[4] See Britten-Jones et al. (2011); Daniel and Marshall (1997); Jagannathan and Wang (2005); Jagannathan and Wang (2007); Malloy et al. (2009)

[5] See Hansen and Hodrick (1980); White (1980); Newey and West (1987)

These latter studies highlight reasons to prefer overlapping to non-overlapping data. Given returns smoothing in private equity data, this approach is well suited for analyzing this asset class. Fragkiskos et al. (2017) (henceforth, FSW), use the overlapping data approach of Valkanov (2003) and other de-smoothing procedures to estimate risk exposures for private equity returns. FSW construct returns from a novel data set of quarterly, fund-level cash flow data. Armed with this (unbalanced) panel of private equity returns, they construct vintage-level portfolios and conduct vintage and fund-level analysis. FSW then compare and contrast the merits of different de-smoothing approaches, and find that the overlapping data approach outperforms other methods in terms of both explaining variation in the data and offering superior predictive power.

FSW assume that true returns are priced at the same frequency as observed returns. True returns are driven by variation in the valuation of a fund's underlying portfolio investments, which—as noted in the non-synchronous trading literature—most likely varies at a much higher frequency due to movements in markets, changes in aggregate economic activity, etc. Motivated by this discontinuity, we extend FSW's analysis by using high frequency data to model private equity performance. Following Ghysels et al. (2004) and using a novel data set, we employ a structurally motivated mixed data sampling (MIDAS) estimator using monthly factor returns to price imputed quarterly private equity returns. Compared to the Valkanov (2003) overlapping data and Getmansky et al. (2004) de-smoothing approaches, we find that the MIDAS estimator yields more economically meaningful factor loadings without sacrificing explanatory and predictive power.

By and large, MIDAS estimators are used to forecast low frequency data using high frequency correlates. Foroni and Marcellino (2013) provide an overview of the use of mixed frequency methods—including MIDAS regressions—in the literature. This method was first used to analyze financial data in Ghysels et al. (2004). The authors find that the MIDAS yields more accurate predictions of stock market variance than that of rolling window and GARCH techniques. Clements and Galvão (2008) are the first to use MIDAS to study macroeconomic phenomenon. They find that using MIDAS to predict future U.S. output growth significantly lowers RMSE when compared with quarterly autoregressive or distributed lag models. These results are supported by Andreou et al. (2010) find improved forecasting power from the MIDAS approach to that standard autoregressive techniques when predicting quarterly U.S. inflation and GDP.

To our knowledge, ours is the first study to leverage high frequency data to study private equity performance. We assume that a fund's underlying portfolio investments are priced at a higher frequency than are observed cash flows, and so a fund's true performance is modeled using a high frequency linear factor model. We extend the analysis of Getmansky et al. (2004) and FSW, and assume that observed returns are a weighted average of past, high frequency returns. In the next section, we expound upon this approach and introduce our MIDAS estimator. Section three provides a description of our data, while section four provides our empirical results. We conclude in section five.

3.2 Methodology

In this section we introduce our MIDAS estimator. We then discuss several competing approaches for handling smoothed returns.

3.2.1 MIDAS

Due to returns smoothing and non-synchronous trading, FSW posit that observed private equity returns, r_t^o, are related to contemporaneous and K past values of unobserved true returns, r_t. Similar to Getmansky et al. (2004), they begin with

$$r_t^o = \sum_{j=0}^{K} \theta_j r_{t-j}, \ \sum_{j=0}^{K} \theta_j = 1. \tag{3.1}$$

They assume true returns are generated by a linear factor model,

$$r_t = \alpha + \beta' x_t + \varepsilon_t,\ \varepsilon \text{ iid } N(0, \sigma^2), \tag{3.2}$$

where x_t is a $N \times 1$ vector of factors. Combining 3.1 and 3.2 yields

$$r_t^o = \alpha + \beta' \sum_{j=0}^{K} \theta_j x_{t-j} + \sum_{j=0}^{K} \theta_j \varepsilon_{t-j}, \tag{3.3}$$

which can be estimated with a variety of techniques.

FSW assume that unobserved returns are priced at the same frequency as observed returns. In reality, fund performance is driven by variation in the value of underlying portfolio investments, which while unobserved, likely change at a higher frequency due to market forces. Maintaining the assumption that true returns are generated by a linear factor model, assume

$$r_\tau = \alpha + \beta' x_\tau + \epsilon_\tau,$$

where τ occurs at a higher frequency than t. Assume within every period t, τ frequency variables are determined—but not necessarily observed—M times.[6] Given returns smoothing and non-synchronous trading, we posit that observed returns are a weighted average of past, high frequency and unobserved true returns:

$$r_t^o = \sum_{j=1}^{L} \Phi(j, \Theta) r_t^j, \tag{3.4}$$

where $L = K \times M$. The term r_t^j is a high frequency occurrence of the unobserved true return measured

[6]For example, if t represents a quarterly frequency and τ is a monthly frequency, $M = 3$.

j high frequency sub-periods into the past,[7] while $\Phi(\cdot)$ is a weighting function parameterized by Θ with $\sum_j \Phi(\cdot) = 1$.

Substituting the factor model into this last expression yields

$$r_t^o = \alpha + \beta' \sum_{j=1}^{L} \Phi(j, \Theta) x_t^j + e_t, \textbf{??} \tag{3.5}$$

where $E(e_t) = 0$, $E(e_t^2) = \sigma_e^2$, and $E(e_t e_{t-|k|}) = 0$ for $|k| > K$.[8] Rather than relying on **??**, we extend this specification to allow for factor-specific weights

$$r_t^o = \alpha + \sum_{n=1}^{N} \sum_{j=1}^{L} \beta_n \Phi(j, \Theta_n) x_{n,t}^j + e_t. \tag{3.6}$$

While the literature has not settled on a specific functional form for the weighting function, given its

[7] Returning to the example where t is quarters and τ is months, suppose $K = 2$, so that observed returns are related to unobserved true returns from the current and previous quarters. Then r_t^1 is an occurrence of the true return from the most recent month of the current quarter, while r_t^4 is the first month from the previous quarter, and so on.

[8] To understand why, start with our assumptions that $E(\epsilon_\tau) = 0$, $E(\epsilon_\tau^2) = \sigma^2$, and $E(\epsilon_\tau \epsilon_{\tau-|k|}) = 0$ for $k > 0$. Then

$$E(e_t) = \sum_{j=1}^{L} \Phi(j, \Theta) E(\epsilon_t^j) = 0$$

and

$$\begin{aligned} E(e_t^2) &= \left\{ \sum_{j=1}^{L} \Phi(j, \Theta) E(\epsilon_t^j) \right\}^2 \\ &= \sigma^2 \sum_{j=1}^{L} \Phi(j, \Theta)^2 \\ &= \sigma_e^2. \end{aligned}$$

Finally,

$$E(e_t e_{t-p}) = \begin{cases} \sigma^2 \sum_{j=n \times M+1}^{(K-p) \times M} \Phi(j, \Theta) \Phi(j - M \times p, \Theta) & , 1 < p \leq K \\ 0 & , \text{otherwise} \end{cases}$$

The fact that observed returns depend on lagged, high frequency unobserved returns introduces an MA(K)process in the error term.

parsimony and following the recommendation of Ghysels et al. (2004), we use the beta-polynomial

$$\Phi(j,\Theta_n) = \frac{f(j/L;\theta_{n,1},\theta_{n,2})}{\sum_{j=1}^{L} f(j/L;\theta_{n,1},\theta_{n,2})}$$
$$f(x;\theta_{n,1},\theta_{n,2}) = \frac{x^{\theta_{n,1}-1}(1-x)^{\theta_{n,2}-1}\Gamma(\theta_{n,1}+\theta_{n,2})}{\Gamma(\theta_{n,1})\Gamma(\theta_{n,2})}, \tag{3.7}$$

where $\Gamma(\cdot)$ is the gamma function. This ensures $\Phi(\cdot) > 0$ and $\sum_j \Phi(\cdot) = 1$.

Following FSW we select $K = 3$. Our high frequency data is composed of monthly observations of factor prices, so $M = 3$. Consequently, we specify the current observed returns are a weighted average of the twelve most recent months of high frequency factors. We then construct the sum of squared residuals,

$$f(\Theta_n,\beta,\alpha) = \left(r_t^o - \alpha - \sum_{n=1}^{N}\sum_{j=1}^{L} \beta_n \Phi(j,\Theta_n) x_{n,t}^j \right)^2 \tag{3.8}$$

and estimate $\Upsilon = [\Theta_n \ \beta' \ \alpha]$ using nonlinear least squares. Asymptotically, $\hat{\Upsilon} \sim N(\Upsilon, V(\Upsilon))$. A natural estimate of $V(\Upsilon)$ is $V(\hat{\Upsilon}) = s^2(\nabla F(\hat{\Upsilon})'\nabla F(\hat{\Upsilon}))^{-1}$, where $\nabla F(\hat{\Upsilon})$ is the Jacobian of the sum of squared residual function evaluated at the optimum. As noted previously, we expect that the error term exhibits autocorrelation up to K lags. To account for this, we adjust our standard errors by using a Newey and West (1987) HAC estimator of $V(\Upsilon)$ using the Jacobian provided by our numerical solver.[9]

3.2.2 Alternative Approaches

We estimate factor loadings using alternative methods proposed in the literature to facilitate comparison with our MIDAS estimator. Getmansky et al. (2004) directly estimate equation 3.3. Using a concentrated maximum likelihood approach, they jointly estimate α, β, and θ using $K = 2$ for analyzing hedge fund returns. Pedersen et al. (2014) extend this method for use with private equity data. However, their

[9] Computations were performed in Matlab using `lsqnonlin`. We use the numerical Jacobian provided by the solver as $\nabla F(\hat{\Upsilon})$ when calculating standard errors.

estimation approach is subtly different. They note that conditional on the vector of θ_j's, the maximum likelihood estimates of $\gamma = [\alpha\ \beta']'$ and $s^2 = E(\sum_j \theta_j \epsilon_{t-j})^2$ are

$$\begin{aligned} \hat{\gamma} &= (Z'Z)^{-1}Z'r^o \\ \hat{s}^2 &= \frac{1}{T}(r^o - Z\gamma)'(r^o - Z\gamma), \end{aligned} \tag{3.9}$$

where r^o is a vector of observed returns and Z is a matrix of data with $z_t = [1\ \sum_{j=0}^{K} \theta_j x'_{t-j}]$ in each row. Accordingly, they estimate θ_j conditional on the OLS solutions for γ and s^2, then construct Z to estimate these last two parameters. We follow Pedersen's procedure, and calculate standard errors using the Newey and West (1987) HAC estimator of the covariance matrix of $\hat{\gamma}$. Throughout the remainder of this paper we refer to these approaches as the Getmansky-Pedersen or MA(K) methods.

Following Valkanov (2003), FSW estimate private equity returns using a series of different overlapping data estimators. They estimate

$$r_t^o = b_0 + b_1' \sum_{j=0}^{K} x_{t-j} + u_t. \tag{3.10}$$

which they dub YX-overlap. FSW set $\theta_j = 1/(K+1)$, so under this specification $b_0 = (K+1)\alpha$ and $b_1 = K^{-1}\beta$. Given the mismatch between observed returns and the pricing of funds' underlying investments, FSW correct for this non-synchronous trading problem by estimating

$$\sum_{j=0}^{K} r_t^o = b_0 + b_1' \sum_{j=0}^{K} z_{t-j} + v_t, \tag{3.11}$$

where $z_t = \sum_j x_{t-j}$, which they call YX^2-overlap.[10] Both specifications induce autocorrelation in the

[10] Summing over factors causes significant weight to placed on factor prices observed in the middle of the overlapping data window. To compensate for this, we follow FSW and instead calculate $\tilde{z}_t = \sum_{j=0}^{K} \omega_j x_{t-j}$ so b_1 retains its interpretation from 3.10.

error term, which they account for by adjusting test statistics according to Kamara et al. (2015).[11] Given the problems with using overlapping data noted in Harri and Brorsen (2009), we estimate 3.11 using FGLS to generate efficient estimates of b.

3.3 Data

3.3.1 Returns Data

We use the same, novel data set from FSW. Our data comes from commercial private equity data provider PitchBook. PitchBook is a data vendor that obtains quarterly fund-level information on private equity vehicles from institutional investors. In addition to buyout and venture data, PitchBook also provides information on more recent vintages of infrastructure funds, and to a lesser extent, real estate funds. Similar to a competing vendor, Preqin, these data are obtained from U.S. institutional investors subject to Freedom of Information Act request compliance (FOIA), as well as responses from fund general partners and news wires in cases of missing data. As with other data collected via FOIA requests, the sample is limited to large U.S. pension funds, which likely sensors our data from the performance of other groups of investors such as endowments.[12]

The quality of data obtained via FOIA requests is easily cross-checked among other investors invested in the same fund and thus reported cash flows are likely highly accurate. However, this may only be valid when the number of investors reporting in a given quarter remains relatively the same over time.

[11] Consider specification 3.9. According to the assumed data generating process,

$$u_t = \sum_{j=0}^{K} \theta_j \varepsilon_{t-j}.$$

Thus u_t is generating by an MA(K) process with autocovariance generating function

$$\gamma(q) = \begin{cases} \sigma_\varepsilon^2 \sum_{j=0}^{K-|q|} \theta_j \theta_{j+|q|} & \text{, if } |q| \leq K \\ 0 & \text{, otherwise} \end{cases}$$

A similar result holds for specifications 3.10 and 3.11.

[12] Lerner et al. (2007) find private equity returns are higher for endowments and other investment groups.

Data providers may institute a filtering process to reduce the variation in reported cash flows taken from the average of medians reported by invested investors to obviate this problem. However, this could inject additional noise into the measurement process. PitchBook states that they do not filter the data. Thus, additional variation in cash flows between quarters can result from the addition or subtraction of investors reporting differing valuations to the data vendor. This tends to be more prevalent in earlier vintages when the numbers of investors in any given fund are small. Since we are not privy Preqin's data, we are uncertain of their reporting methodology. However, given the nature of the asset class, we suspect their data is subject to the same potential biases.

We construct our return series from time and fund-specific data on cash flows—distributions (D) and contributions (C)—and "remaining values." This latter category represents fund managers' valuations of unrealized gains in their respective underlying portfolio investments, which we called net asset valuations (NAV). With these data we construct holding period returns, r_t^o, via

$$r_t^o = \frac{NAV_t - NAV_{t-1} + D_t - C_t}{NAV_{t-1}}. \tag{3.12}$$

Simply put, the return to holding a stake in a private equity investment is the change in the value of a particular fund's portfolio—plus realized gains in terms of net distributions—over last period's valuation.[13]

To reduce noise in the data, we ensure that reported cumulative cash flows maintain their high water marks (HWM) through the time-series. For example, if in the first quarter of 2002 the reported cumulative contribution of a given fund was $100 million, then in following quarter it can be no less than this amount. In cases where cumulative cash flows fall below the HWM, then we carry forward the prior HWM to the new quarter. Again, falling below the HWM can result from variation in the median cash flow reported

[13] This specification implicitly assumes that contributions and distributions occur at exactly the same time. If this were not the case, one could employ the Modified Dietz method to calendar-weight cash flows to discount contributions and distributions appropriately.

by PitchBook when the number of investors reporting to PitchBook changes from period to period, and where those reported cash flows across investors varies.

Due to the inherent differences in valuation processes underlying buyout and venture funds, our study focuses on the larger buyout industry for U.S. domiciled funds. Because our approach focuses on the time-series properties of aggregate, vintage-level, and fund-level cash flows, the number of funds found in the cross-section is not particularly important. Rather, the number of continuous cash flow observations is, so we follow Ewens et al. (2013) and restrict our base tests to those funds with at least 20 quarters of cash flow data. However, to remain consistent with other studies,[14] and Barber and Yasuda (2014) and to reduce measurement error in the earlier vintages, we begin our study with the 2000 vintage of buyout funds and we end with the 2009 vintage, which satisfies our criteria of 20 quarterly observations as our data sample ends in 2013. We follow Brown et al. (2015) and filter later observations when NAV falls below 2% of fund size assuming the fund is no longer active when this occurs. Lastly, given the noise generated from the measurement and reporting process, we truncate funds from the data set whose standard deviation of returns lie in the 10% tail of the distribution, the majority of which suffer from a small-denominator issue when calculating quarterly returns. For a complete report on descriptive statistics in the data, see FSW.

3.3.2 Factors

The literature has identified several factors relevant to risk-adjusting private equity returns. As our study attempts to price returns to private equity in relation to other asset classes, we narrow our factor selection to a parsimonious model of the most significant explanatory variables used in the literature. Following Franzoni et al. (2012) we use the four-factor model of Pástor and Stambaugh (2003), which augments the three-factor model of Fama and French (1988) with a factor for innovations in market liquidity. Owing

[14] See for example, Ang et al. (2014)

to the leverage employed by the private equity industry, we follow Pedersen et al. (2014) and include a factor for changes in the high-yield corporate spread.

Fama and French Three-Factor Model

The Fama-French three-factor model attempts to explain certain risk premia along with an overall public market exposure of private equity. It's important to note that these factors are free of transaction costs. Therefore, an investment in private equity that loads on these factors could be seen as an alternative to capture the risk premia associated with those factors without incurring transaction costs.

Excess Return to the Market (MKT) The market factor is the excess market return of all NYSE, AMEX and NASDAQ stocks over the one month Treasury bill rate. The beta of private equity against the market factor has been found to be close to one. Franzoni et al. (2012) estimate a beta of 1.3 for buyout funds found in the CEPRES database,[15] while Jegadeesh et al. (2015) find market factor loadings for listed private equity funds and fund-of-funds not significantly different from one.

Small-Minus-Big (SMB) The SMB factor measures the spread return between a portfolio of small capitalization stocks and large capitalization stocks. As such, a positive coefficient indicates a focus on small cap investing. The majority of private equity deals involve the purchase of small-cap firms. However, exposure to this factor can vary as large-cap firms may behave like small-cap firms and vice versa. Indeed, the existing literature on private equity does not have a conclusive answer for LBOs. For example, Jegadeesh et al. (2015) find a positive beta associated with the the small-minus-big (SMB) factor, whereas Pedersen et al. (2014) and Franzoni et al. (2012) estimate a negative (albeit insignificant) coefficient. Almost by definition, Harris et al. (2014) find a positive beta between venture capital and small-cap returns.

[15] CESPRES is the Center for Private Equity Research that was originally formed as a joint venture in 2001 between a subsidiary of Deutsche Bank and the University of Frankfurt.

High-Minus-Low (HML) This factor captures return differences between a portfolio of high and low book-to-market firms. Hence, a positive coefficient reflects an investment bias towards value stocks versus growth stocks. Among buyout funds, the literature has generally found a positive value bias, with Franzoni et al. (2012) and Jegadeesh et al. (2015) citing a beta of 1.0 and 0.3, respectively. However, Pedersen et al. (2014) find a significant growth bias for buyout funds (beta of -1.4), which is typically consistent with venture capital funds

Credit Spreads (HIY)

In a leveraged buyout, a private equity firm acquires a company using equity raised from investors in the fund coupled with significant levels of debt, which can be as high as 90% of the total acquisition value. Funding is usually obtained from the syndicated loan market, wherein multiple banks and non-bank lenders issue debt, or leveraged loans, to the fund's underlying corporate entity. The leveraged loan is often comprised of multiple tranches involving loans of different maturities, currencies, schedules, and seniorities. The loans typically pay Libor plus a spread, and are secured by the private company's assets. The debt often includes a junior, unsecured portion in the form of high-yield bonds or "mezzanine debt," the performance of which are impacted by the cost or availability of debt and co-vary with credit spreads. The level and timing of debt a private equity firm raises varies through time, depending on how many and when acquisition of portfolio companies are made. In accordance with the timing of draw-downs, we expect that most debt is taken towards the early stages of a funds life and to some extent, during its middle years.

Interest rates determine how much debt private equity firms target; low (high) interest rates allow for more (less) debt to be incurred. Taxes affect variances in debt issuance as well. Since interest paid on debt is tax deductible, the debt is expected to be higher when the corporate tax rate is high, or a firm has high and steady taxable cash flows. Another factor to consider is that fund managers have an incentive to take on as much leverage as possible, given their incentive carry of 20% of excess returns acts like a call option on the firm's excess returns.

The relationship between leverage and private equity performance begins at the portfolio company level. When debt conditions are favorable, private equity firms should be willing to pay a higher price and take on more leverage to acquire a company. While this reduces the company's realized return, the additional leverage amplifies returns at the fund level. Also, a company is able to sustain higher leverage precisely when it is expected to have strong cash flows relative to EBITDA. As a consequence, we posit that increase an increase in credit spreads is associated with deteriorating credit market conditions.[16] This causes lenders to become more risk averse, lowering the pool of potential lenders and decreasing available funding. In this environment, private equity funds will be less able to employ leverage as a tool fund investments, which would reduce its measured return.

Liquidity (LIQ)

Due to their high leverage, private equity investments are sensitive to the capital constraints faced by the providers of debt to private equity.[17] When credit markets are stressed, private equity managers face difficulties refinancing their investments and making new ones. As a result, funds may be forced to liquidate their investments at unfavorable prices or accept higher interest rates on debt. In both cases, measured returns should decline.

Dissecting the exposure to the Pastor and Stambaugh liquidity measure, Franzoni et al. (2012) find empirical support for the theoretical linkage of Brunnermeier and Pedersen (2009) between funding liquidity and market liquidity.Brunnermeier and Pedersen (2009) find a negative relationship between changes in lending standards, as measured by the Federal Reserve's Senior Loan Officer Survey and market liquidity. The availability of capital, or funding liquidity, can thus be proxied by the market

[16]See Brunnermeier and Pedersen (2009)
[17]Axelson et al. (2013)

liquidity factor as Franzoni et al. (2012) estimate a significant relationship between private equity returns and the liquidity risk factor. This relationship holds given that when funding liquidity is tight; traders become reluctant to take on capital-intensive positions in high margin securities, hence lowering market liquidity. On the other hand, market illiquidity increases the risk of refinancing a trade, which increases margins and lowers funding liquidity. This result obtains when margin setting financiers are unsure whether price changes are due to fundamental news or to liquidity shocks.

As long as capital is sufficient enough so that funds to do fun up their borrowing constraints, market liquidity is insensitive to changes in capital and margins. But once funding constraints bind, market liquidity declines and prices are affected more by funding liquidity than fundamentals, as was the case in the Financial Crisis of 2007. Since speculator funding constraints affect all securities, then market liquidity is correlated across assets. To the extent that public and private equity markets are integrated, market liquidity affects private equity returns via a funding liquidity channel. In other words, private equity returns depend on the availability of capital to debt providers, funding liquidity, and as a consequence on market liquidity as well. This implies a positive relationship between market liquidity and private equity returns.

3.4 Results

3.4.1 Factor Loadings

We begin by employing our MIDAS estimator to analyze private equity risk exposures using the Cambridge Associates Buyout Index. Cambridge Associates is an investment consulting firm that provides services to broad set of private and public agents. They provide a set of publicly available private equity benchmarks from cash flow and position data using a proprietary database which includes over 6,000 funds. For our first set of results we use quarterly data on U.S. buyout funds from 1985 through the end of 2015. We also provide results using the Getmansky-Pedersen and the FWS overlapping approaches.

A typical private equity fund reports performance measures and distributes funds to investors near the middle or end of a given quarter. This implies that any information within the quarter that precedes our observations on NAVs and net cash flows may be reflected in this data, and potentially useful for inference. MIDAS is uniquely suited to leverage this aspect of the data generating process. When estimating (6), we also include the *first month of the next quarter's* data on factor prices.[18] This procedure is called 'nowcasting' in the MIDAS literature. We denote the Getmansky-Pedersen approach as MA(K) and FWS' XY-overlap approach as simply XY-overlap. For these last two methods, quarterly factor returns were calculated as within-quarter, cumulative, monthly factor returns.

Table C.1 gives estimated factor loadings and alphas. In general, coefficients obtained using the MIDAS approach are higher than when using the competing MA(K) and YX-overlapping methods. All approaches yield positive and statistically significant market exposure. Yet, only the MIDAS estimator gives a value that is consistent with the literature.[19] In addition, all approaches yield positive estimated abnormal returns, though only the MIDAS estimate of approximately 3-4% is statistically significant. The MIDAS estimator gives a statistically significant value of 0.93 for HML, which is on the high end of the range found by Franzoni et al. (2012) and Jegadeesh et al. (2015), but ultimately consistent with their results. No technique yields a significant loading on SMB, and the sign differs based on the method used. This is not surprising, given the ambiguous results coming from Jegadeesh et al. (2015), Pedersen et al. (2014), and Franzoni et al. (2012). All three approaches yield positive betas on liquidity exposure. The MIDAS and YX-overlap procedures values are statistically significant, which confirms their intuition and results. Finally, the MIDAS and MA(K) estimators yield negative loadings on the high yield spread, though the MIDAS estimator alone generates a statistically significant value. This confirms the intuition that adverse credit market conditions negatively impact fund returns through their ability to leverage

[18]In terms of notation, this implies using x_{t+1}^{12} as another regressor in **?**.

[19]See Franzoni et al. (2012), Pedersen et al. (2014), and Jegadeesh et al. (2015)

equity from investors.

Also provided in Table C.1 are measures of goodness of fit. We report traditional R^2 and in-sample RMSE. In addition, we report the RMSE of one-step-ahead forecasts using an expanding or rolling window. To do so, we cut the sample in half and estimate alpha and factor loadings using only the first $T/2$ observations. We then use the model to forecast next period's observed return, $r_t^{o,f}$, and compare it to the data. We then iterate on this procedure by expanding the estimation window by an additional observations every round, ultimately calculating

$$RMSE_{RW} = \sqrt{\frac{2}{T}\sum_{t=T/2+1}^{T/2}(r_t^o - r_t^{o,f})^2}. \tag{3.13}$$

The MIDAS estimator provides improvements over the MA(K) and YX-overlap methods when considering all three benchmarks. While gains in in-sample and out-of-sample $RMSE$ are modest, the MIDAS estimator explains over 25% more of the variation in the data when compared with the YX-overlap approach and lowers R^2 by nearly 7% compared to the Getmansky-Pederson method. Given its ability to provide economically meaningful risk exposures, explanatory power, and out-of-sample predicting power, from a holistic perspective we find that the MIDAS estimator yields superior overall results in this context.

To account for possible non-synchronous or long-horizon pricing problems in the data, we follow Kamara et al. (2015) and FSW by estimating risk exposures using overlapping observations. Specifically, we estimate the MIDAS, MA(K), and YX-overlap models using $z_t = \sum_j x_{t-j}$ as regressors. Results are reported in Table C.2. All three approaches now yield an abnormal return of approximately 5-6%, though now the MIDAS and the YX-overlap[20] yield significant values. All three methods produce statistically significant coefficients for MKT, and the magnitude of these values does not change significantly when using overlapping data. Once again, only the MIDAS technique produces a value consistent with the lit-

[20]This is technically now the YX^2-overlap method described previously and in FSW.

erature. Using overlapping data causes all methods to produce statistically significant loadings on HML, and all now lend credence to results in the literature that buyout funds exhibit a positive value bias. The results for SMB are positive but insignificant, confirming the ambiguity result from the literature. The point estimates for LIQ are also all positive, but only significant now for the MIDAS estimator. Using overlapping data flips the sign of HIY when using YX-overlap, and now—as intuition suggests—all techniques yield a negative loading on HIY. Again, only the MIDAS approach yields a significant value. Using overlapping data causes the point estimate to increase in magnitude and statistical significance.

Turning to goodness of fit, using overlapping observations increases R^2 for all estimators. The MIDAS estimator provides modest improvements over the MA(K) and YX-overlap approaches. Using MIDAS increases R^2 by 6% over the Getmansky-Pedersen approach, and by nearly 15% over the YX-overlap method. Gains in in-sample and out-of-sample are more modest than before, with the exception that MIDAS lowers rolling-window RMSE by approximately 9% over the MA(K) estimator. Considering the evidence, we conclude that the MIDAS estimator yields improved performance when correcting for potential non-synchronous trading and horizon-pricing issues with overlapping data.

Figures C.1 and C.2 plot monthly estimated vs. quarterly observed returns using the Cambridge Associates Index. Month returns were computed using

$$\hat{r}_\tau = \hat{\alpha} + \hat{\beta}' x_\tau, \tag{3.14}$$

where τ represents a period of one month, and $\hat{\alpha}$ and $\hat{\beta}$ are estimates of these parameters obtained via the MIDAS procedure according to 3.8. Figure C.1 gives results using non-overlapped data, while figure C.2 gives estimates using overlapped data. Pricing private equity at the month frequency using non-overlapping observations increases volatility nearly threefold. Removing the larger, abnormal negative return—approximately −62%—reduces this figure, but monthly returns remain approximately twice as volatile as observed quarterly returns. Using overlapping observations—and once again removing the calculated abnormal return eaerly in the sample—yields estimated monthly returns that are more than four times more volatile than observed quarterly returns.[21] Given the MIDAS estimator's superior performance at the index level, these results imply that underlying returns in private equity may fluctuate substantially more than their reported values.

Tables C.3 through C.5 give estimates for value-weighted returns from vintage-level portfolios. These portfolios were constructed using fund-level returns using cash flow and valuation data from PitchBook. For each inception date, we constructed a value-weighted portfolio composed of funds that met our selection criteria. We then calculated quarterly returns using this series. Loadings on MKT are not consistent across vintages using any method, though MIDAS is the only approach that yields coefficients that are positive and a magnitude that is consistent with the literature when statistically significant. Contrary to the aggregate level index and evidence from the literature, HML is virtually negative across all vintages and methods. When significant, SMB changes sign depending on the approach and vintage. Results for this factor are inconclusive, but as stated before, this is consistent with the empirical evidence: different studies offer different estimates for this parameter. Coefficients associated with LIQ are predominantly positive, especially when we find they are statistically significant across all techniques. This is in line with our previous results. The results on HIY are mixed across methods and vintages— and for many funds the coefficient is positive and statistically significant—which runs counter to our intuition that increases in risk premia should reduce fund leverage and hence decrease their measured returns. Despite inconclusive results in the realm of risk exposures, the MIDAS approach offers significant improvements in terms explanatory power at the vintage level over time. The in-sample and out-of-sample RMSE for all three techniques is comparable for most vintages, but MIDAS is superior for the 2005 and 2007 vintages.

Tables C.6 through C.8 provide estimates for value-weighted vintage portfolios using overlapping ob-

[21]The variance of the excess CA index return over the one-month Treasury is approximatley 0.24%. The excluding the early outlier, the variance of excess monthly returns using non-overlapping data is approximately 0.59%, and the variance of excess monthly returns using overlapping observations is about 0.96%.

servations. As expected, when significant we find that loadings on MKT are predominantly positive across vintages and estimators. However, the loadings using the Getmansky-Pedersen approach and YX-overlap are much lower than the values reported in the literature. Point estimates for HML are entirely negative for all vintages using the Getmansky-Pedersen and YX-overlap techniques. These results are in line with Pedersen et al. (2014) which demonstrates private equity has a significant growth bias. The results using MIDAS are mixed. We find support for Pedersen et al. (2014) using early vintages, but evidence that bolsters Franzoni et al. (2012)'s conclusion that private equity exhibits value bias in later vintages.[22] Our results for SMB are mixed as well, but all methods predominantly yield positive and significant point estimates for LIQ, confirming our earlier results. Finally, while our results are again mixed for HIY using MIDAS and Getmansky-Pedersen, we estimate positive and significant exposure for most vintages using YX-overlap. Given the expected relationship between risk premia, leverage, and private equity returns, we find this last result a potential weakness of the YX-overlap method. The MIDAS approach yet again provides comparable or superior results using our goodness of fit tests. MIDAS yields higher R^2 for most vintage portfolios, and the difference is especially stark for earlier vintages. MIDAS and YX-overlap in-sample and out-of-sample statistics are comparable across vintages, but MIDAS offers a clear improvement of the Getmansky-Pedersen estimator in terms of rolling window RMSE. While our results are largely mixed across vintages and methods, nonetheless we find that on several margins the MIDAS estimator yields preferable results in terms of both fit and model interpretation. Therefore, we conclude that using high frequency data may improve upon earlier methods used to analyze private equity returns at the vintage level.

We report estimates obtained using the MIDAS and YX-overlap estimators using fund-level data in Tables C.9 through C.12. We do not provide estimates for the Getmansky-Pedersen method as this estimator failed to converge for a significant number of funds, a source of weakness of this approach. Results

[22] This runs somewhat counter to intuition. Later vintage funds are at earlier stages in their life cycle and should exhibit characterstics that closer resemble venture capital; the value on SMB should be negative.

at the fund level are mixed. Using non-overlapped and overlapped data abnormally large MKT loadings using the MIDAS approach, and counter to previous results in this study and obtained in the literature, the coefficient for many funds is negative and significant. Regardless of whether we use overlapping or non-overlapping observations, both approaches yield ambiguous results concerning the values associated with HML and SMB. The MIDAS estimator yields a mixed bag of positive and negative loadings on LIQ, while the YX-overlap approach yields mostly positive and many statistically significant values, as our insights suggest. However, a majority of point estimates for HIY using the YX-overlap technique are positive, which runs counters to our intuition. The MIDAS estimator provides negative—and many statistically significant—point estimates for fund-level loadings on the high-yield spread, so this method dominates on this margin. The MIDAS estimator generates higher R^2 for most funds, and comparable if not better in-sample and out-of-sample RMSE.

We suspect that fund-level noise that is averaged out in the aggregate—either via the construction of vintage portfolios or higher level performance indexes—plays a significant role in driving returns. This lowers the signal to noise ratio, which in turn reduces the explanatory power of our estimates. That being said, while both approaches display some inadequacies at the fund-level, MIDAS does appear to have a slight edge. This suggests that there is promise in using high frequency data in the study of private equity.

3.4.2 MIDAS Weights

Figures C.3 and C.4 plot the weighting schemes implied by our estimates of Θ_n using the Cambridge Associates returns index. There is significant weight placed on earlier, high frequency observations—including the leading month—of MKT, SMB, LIQ, and HIY, while the data places more weight on observations of HML four to six months in the past. Interestingly, a majority of weight is placed on the first lead for the market and small-minus-big factors. All together, this suggests that using within quarter information is likely very important for private equity analysis. Using overlapping observations leaves the weight on MKT and the high yield spread virtually unchanged, and practically inverts the weights associated with different lags and leads of monthly HML, SMB, and LIQ. This suggests that the appropriate time aggregation scheme for these variables may be dependent on treatment of the data. Figures C.5 through C.9 report estimated weights for our constructed vintage-level portfolios. There appears to be significant heterogeneity in the optimal weighting scheme across vintages for all factors, and this result is echoed in Figures C.10 through C.14, which provide estimated weights using overlapping observations.

There appears to be a significant discrepancy between weights estimated using index and vintage level data. Our results suggests that using highly aggregated private equity performance data may fail to capture the dependency of observed returns on past factor prices in the cross section. This presents two challenges to current methods used in the study of private equity. First, failing to properly aggregate across time may lead to improper estimates of private equity risk exposures. Second, even if this first obstacle is overcome, using weights estimated with aggregate data may generate misleading forecasts of fund level returns. In both cases, our results suggest that researchers and practitioners alike must exercise caution when making fund level inference using aggregate data.

3.5 Conclusion

We develop a new approach to study private equity returns using a new data set first introduced in Fragkiskos et al. (2017). Our innovation is that we adopt a mixed data sampling (MIDAS) framework and model quarterly private equity returns as a function of monthly—and potentially higher frequency—factor prices. This allows us to let the data generate the optimal time-aggregation scheme—up to certain parameters—and use within quarter factor returns to estimate risk exposures using a single, straightforward framework. This method is motivated in part by the existence of smoothed returns in the data. As such, we compare results using our MIDAS framework to results obtained using other approaches to de-smooth returns in the literature: what we call the Getmansky-Pedersen or MA(K) technique (Getmansky et al. (2004) and Pedersen et al. (2014)), and the overlapping observation method—YX-overlap— employed in Fragkiskos et al. (2017), which takes into consideration varying horizons over which risk-factors are priced (Kamara et al. (2015)) combined with the overlapping observation findings in Britten-Jones et al. (2011).

Our MIDAS estimator offers superior performance in terms of generating economically meaningful factor loadings and in-sample and out-of-sample fit using index-level and constructed vintage-portfolio returns when compared with the other approaches. At the index level, MIDAS provides noticeable increases in R^2 and in-sample and out-of-sample RMSE. Simultaneously, it yields estimates of factor loadings which are consistent with the past literature and intuition. Estimates using constructed vintage portfolios offer less noticeable differences, but MIDAS nonetheless yields more desirable results on these margins. Results using fund-level data are mixed, but MIDAS does display a slight edge.

Our estimated monthly weights suggest that within period information is important in determining and pricing private equity returns. Our estimates are also starkly different across factors when using index and vintage level data. This suggests appropriate time aggregation is highly dependent on the observed series and the level of aggregation in the cross section. Our results imply highly aggregated private equity data may not properly reflect underlying performance in the cross section.

Appendix A

Effective Monetary Policy at the Zero-Lower-Bound

A.1 Tables

Table A.1: Calibrated Parameter Values

Parameter	Value
β	0.99
σ	2
η	3
θ	6
ϕ	40
χ	0.825
ρ	0.85
ν	0.005
ψ	0.25
ξ	20
ζ_π	5
ζ_y	2
α	0.7
κ	0.1316
λ_x	0.0058

Table A.2: Gap Between Observed Series and Estimated Trends

Variable	T	Mean	Std. Dev	Min	Max
Curency	117	15.2302	1.6794	8.9012	17.6374
MZM	81	6.9952	2.1637	1.6846	10.9303

Table A.3: Estimated Trends

Variable	Currency	M2	MZM
t	0.0061	-0.0219	0.0064
	(0.0001)	(0.0013)	(0.0001)
t^2		0.0000	
		(0.0000)	
const.	0.2314	16.8691	2.0710
	(0.0441)	(0.6178)	(0.0577)
T	204	204	204
R^2	0.9933	0.9879	0.9752
RMSE	0.0295	0.0282	0.0607

Table A.4: Welfare Comparison

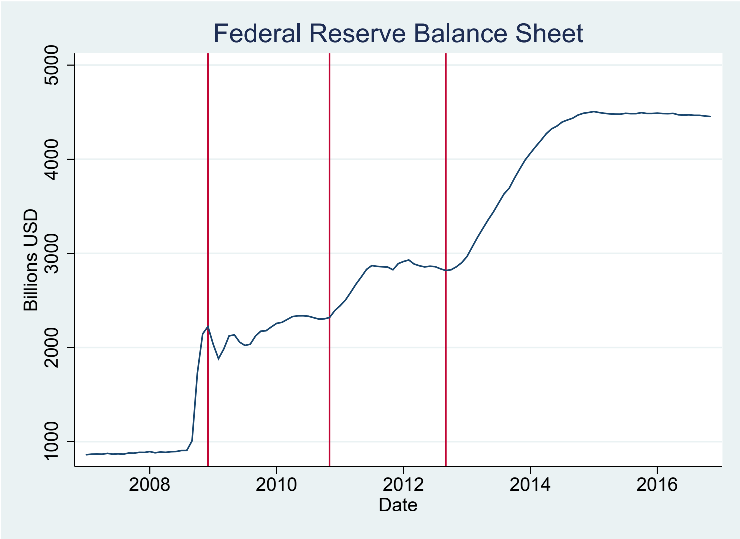
Policy	Utility Loss	Consumption Cost
Taylor rule	100.0000	0.0124
$\Delta M = 0\%$	1.7842	0.0001
$\Delta M = 0.01\%$	1.7712	0.0001
$\Delta M = 0.05\%$	1.7262	0.0001
$\Delta M = 0.1\%$	1.6867	0.0001
$\Delta M = 0.5\%$	2.0072	0.0001
$\Delta M = 1\%$	4.0062	0.0004
Inflation target	17.9580	0.0021
Optimal Discretion	18.3959	0.0022
Optimal Commitment	3.4260	0.0003
Price level target	1.0073	0.0000

Table A.5: Output and Inflation Volatility as a Function of Different Monetary Interventions

Policy	$100 \times \sigma_y$	$100 \times \sigma_\pi$
$\Delta M = 0\%$	0.1377	0.0713
$\Delta M = 0.01\%$	0.1379	0.0710
$\Delta M = 0.05\%$	0.1384	0.0701
$\Delta M = 0.1\%$	0.1393	0.0693
$\Delta M = 0.5\%$	0.1520	0.0756
$\Delta M = 1\%$	0.1790	0.1070

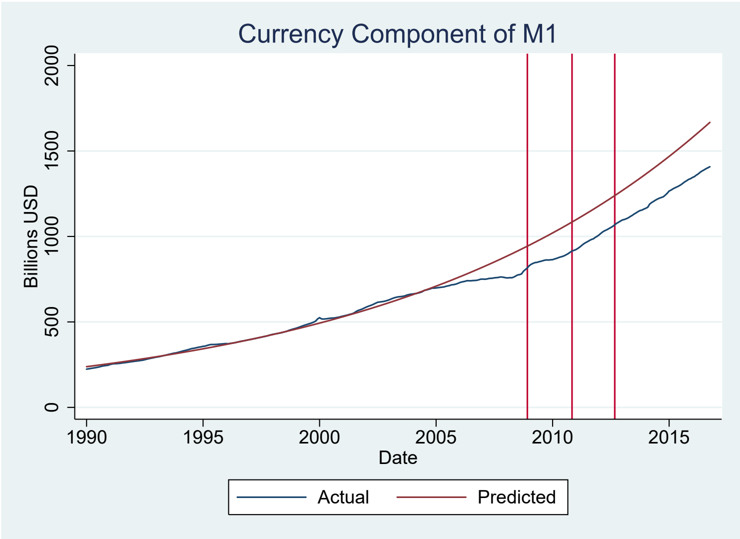
A.2 Figures

Figure A.1: Assets Held Outright with QE Announcement Dates



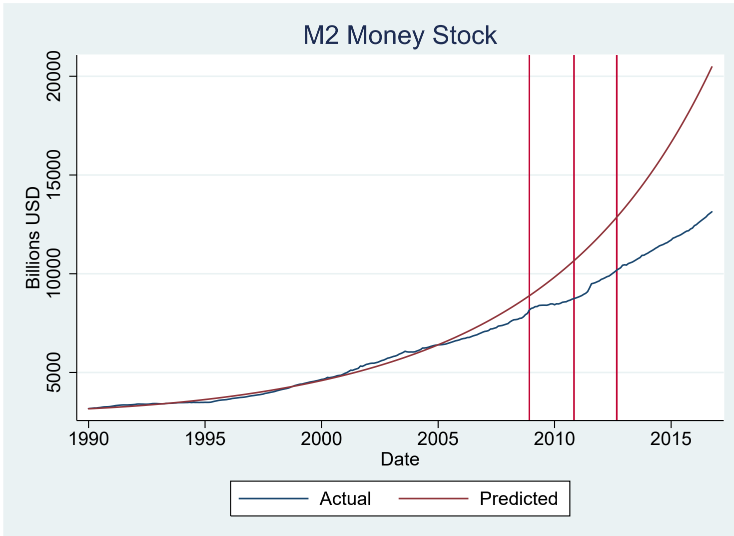
Source: Board of Governors of the Federal Reserve System

Figure A.2: Currency in Circulation vs. Estimated Trend



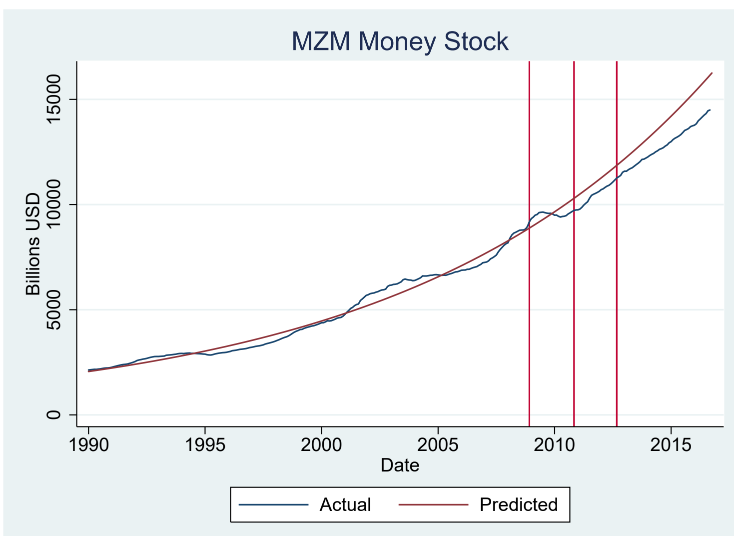
Source: Board of Governors of the Federal Reserve System

Figure A.3: M2 Money Stock vs. Estimated Trend



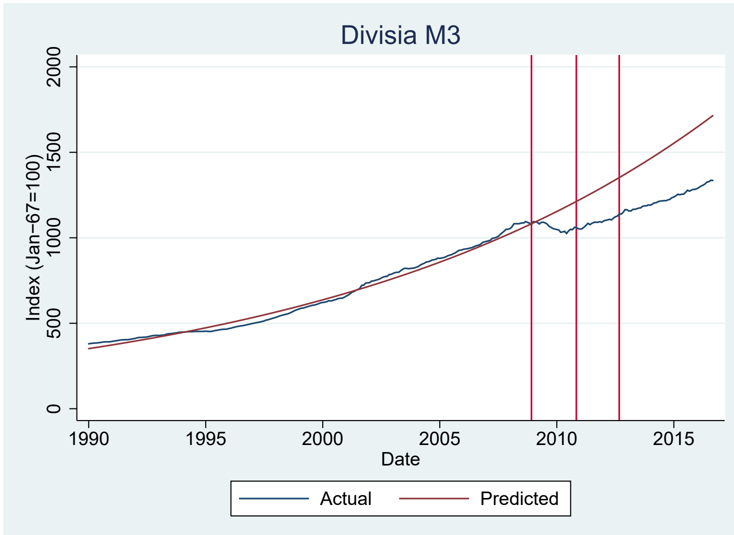
Source: Board of Governors of the Federal Reserve System

Figure A.4: Money Zero Maturity vs. Estimated Trend



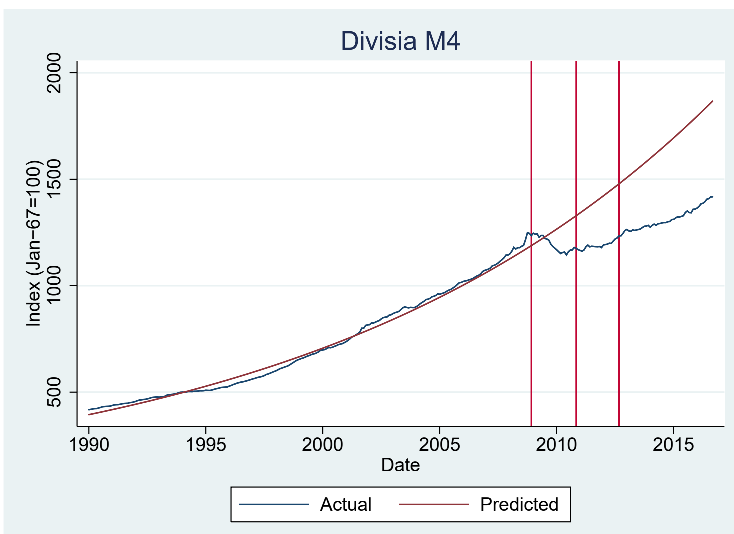
Source: Federal Reserve Bank of St. Louis

Figure A.5: Divisia M3 vs. Estimated Trend



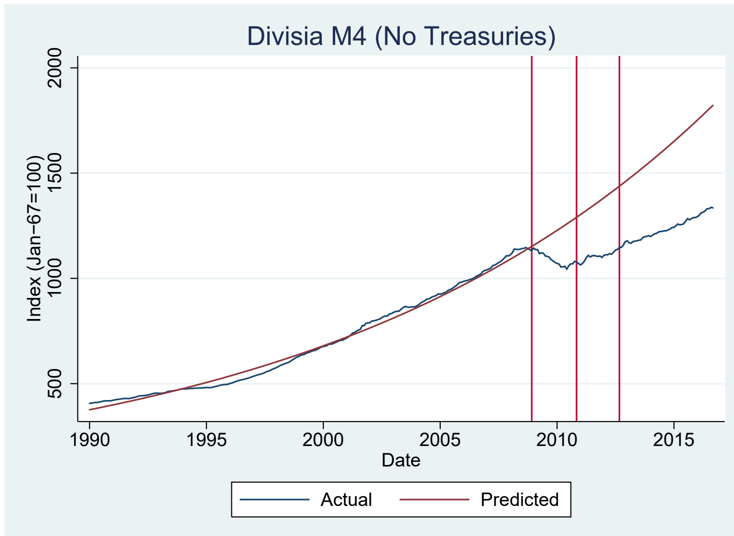
Source: Center for Financial Stability

Figure A.6: Divisia M4 vs. Estimated Trend



Source: Center for Financial Stability

Figure A.7: Divisia M4— vs. Estimated Trend



Source: Center for Financial Stability

Figure A.8: Impulse Responses with Permanent Monetary Injections

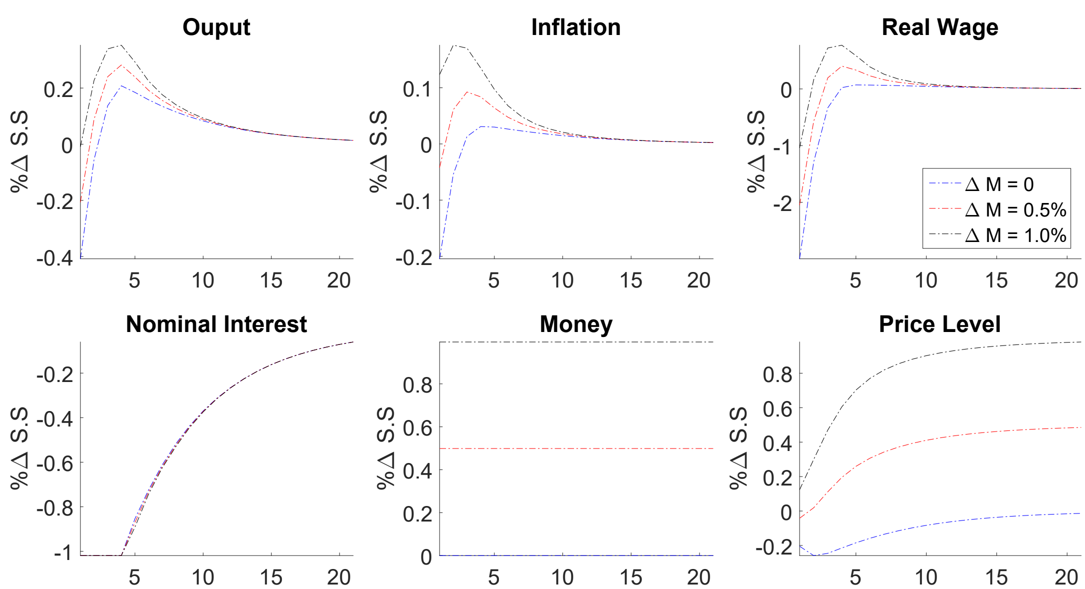


Figure A.9: Impulse Responses with POMOs Engineered with OMOs

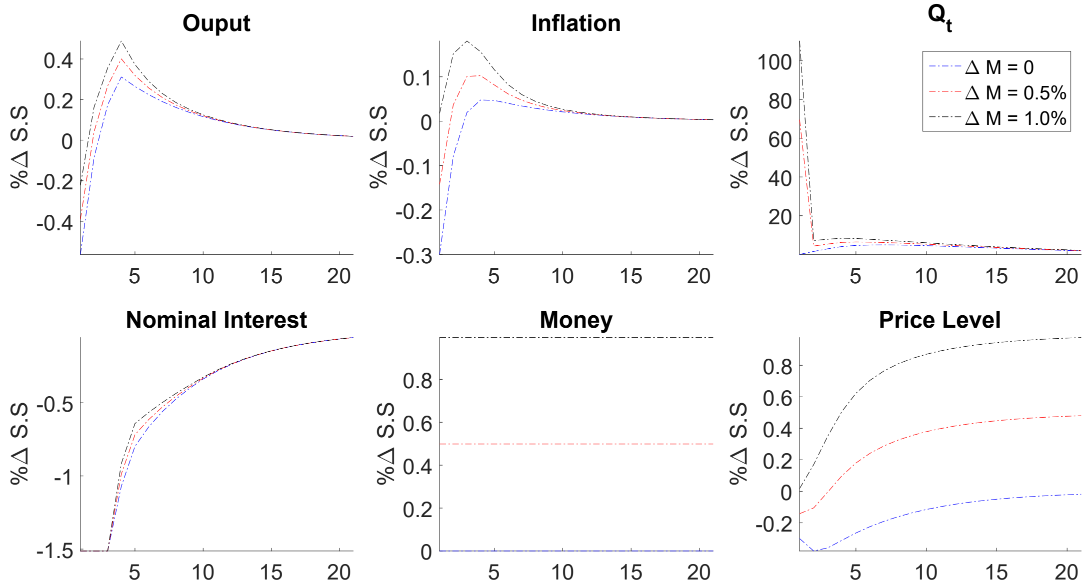


Figure A.10: Impulse Responses in the Presence of Inflation Inertia

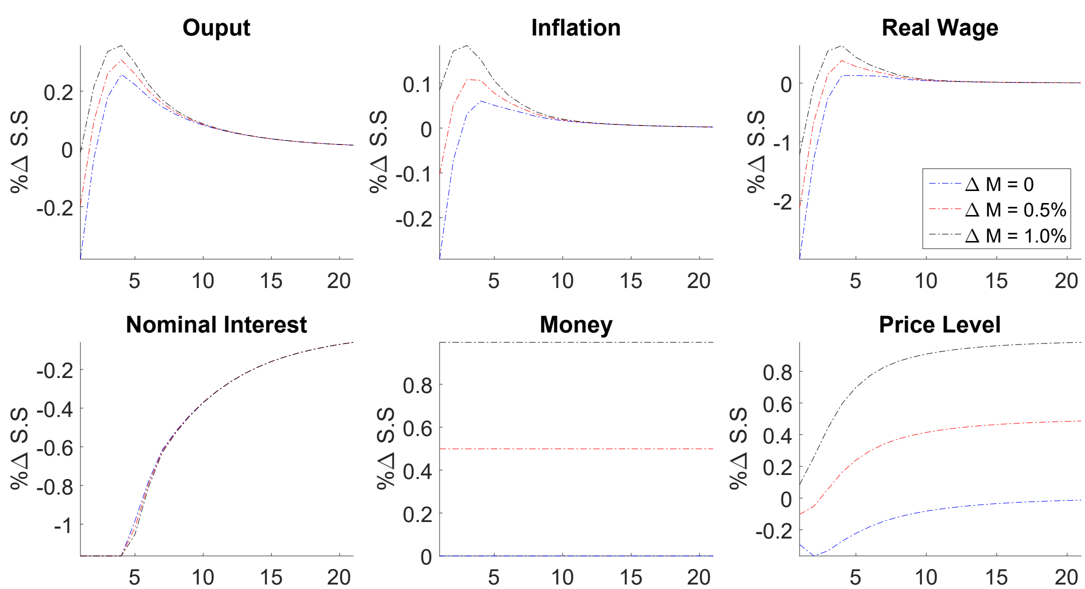
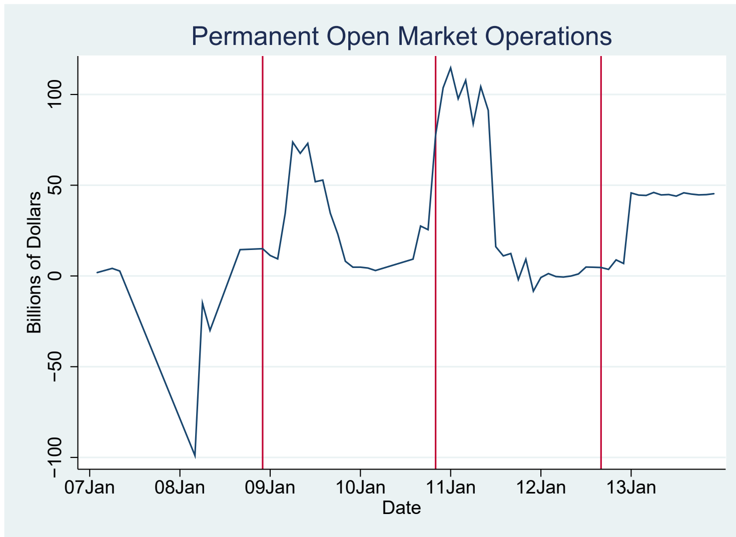
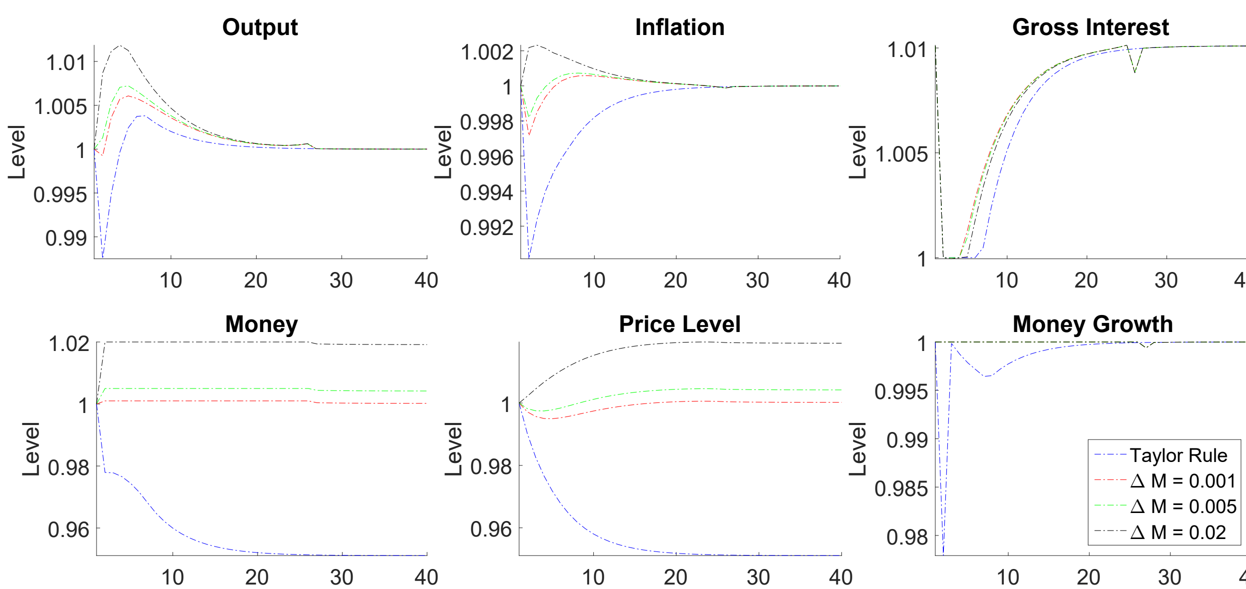


Figure A.11: Changes in the Fed's SOMA with QE Announcement Dates



Source: Federal Reserve Bank of New York

Figure A.12: Impulse Responses with the Hybrid Rule



Appendix B

Effective Quantitative Easing at the Zero-Lower-Bound

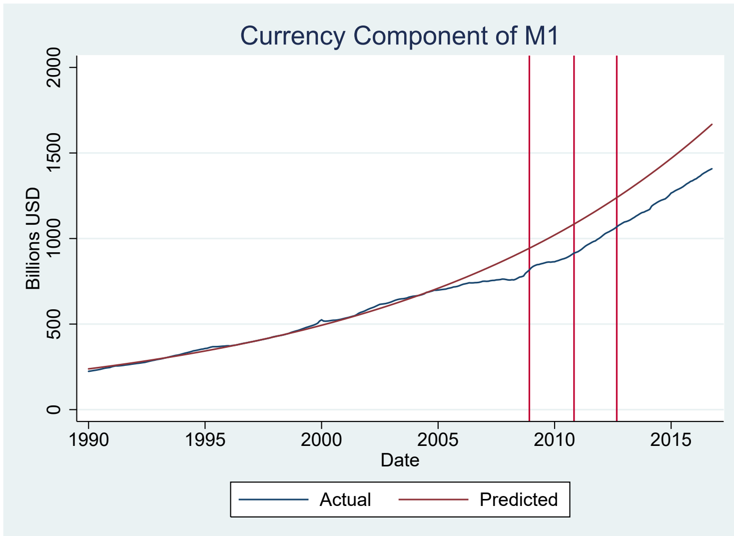
B.1 Tables

Table B.1: Calibrated Parameter Values

Parameter	Value
β	0.99
η	3
ν	0.045
δ	3
α	0.75
ξ	0.025
ϕ_π	1
ϕ_y	1
μ	0.3
ρ	0.8
σ_a	0.005

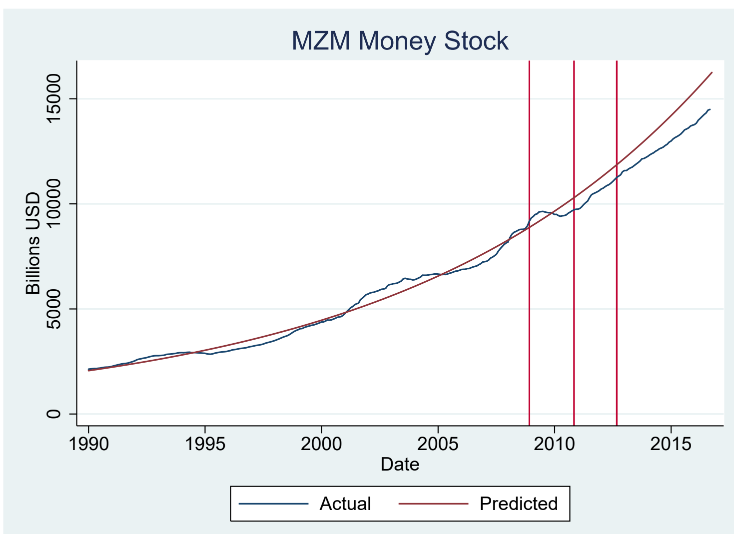
B.2 Figures

Figure B.1: Currency in Circulation vs. Estimated Trend



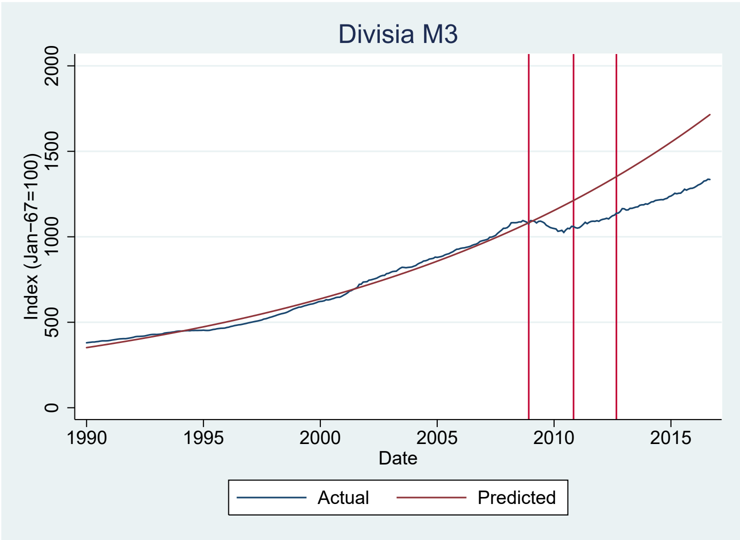
Source: Board of Governors of the Federal Reserve System

Figure B.2: Money Zero Maturity vs. Estimated Trend



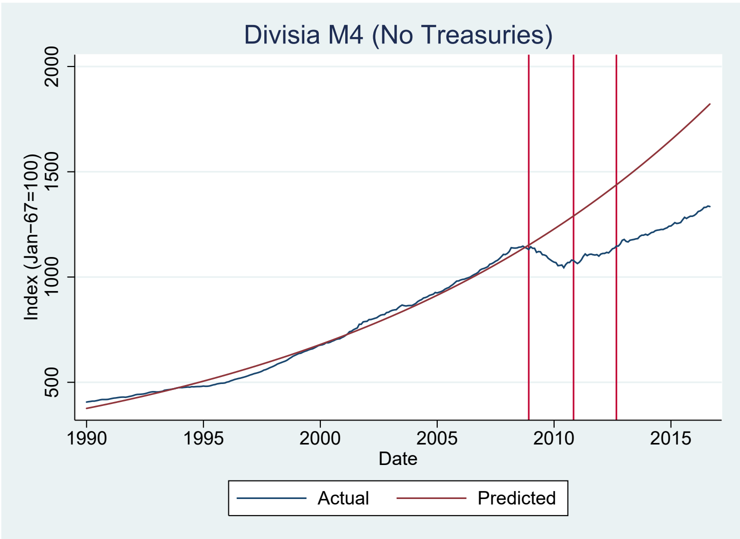
Source: Federal Reserve Bank of St. Louis

Figure B.3: Divisia M3 vs. Estimated Trend



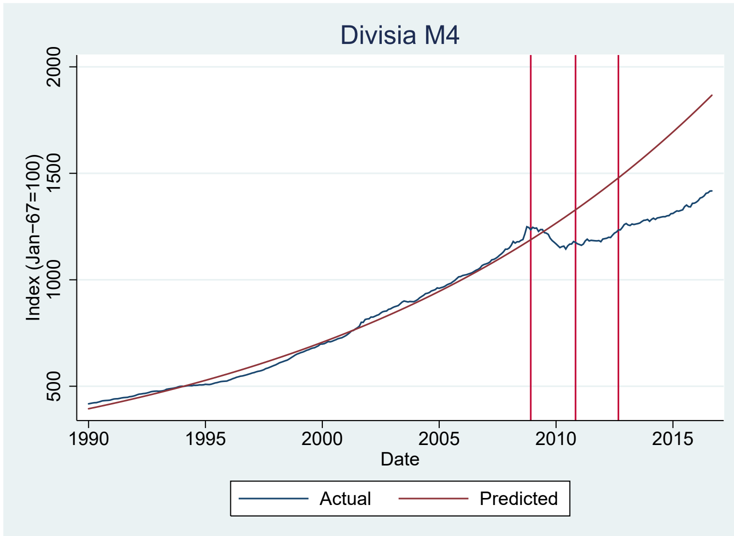
Source: Center for Financial Stability

Figure B.4: Divisia M4— vs. Estimated Trend



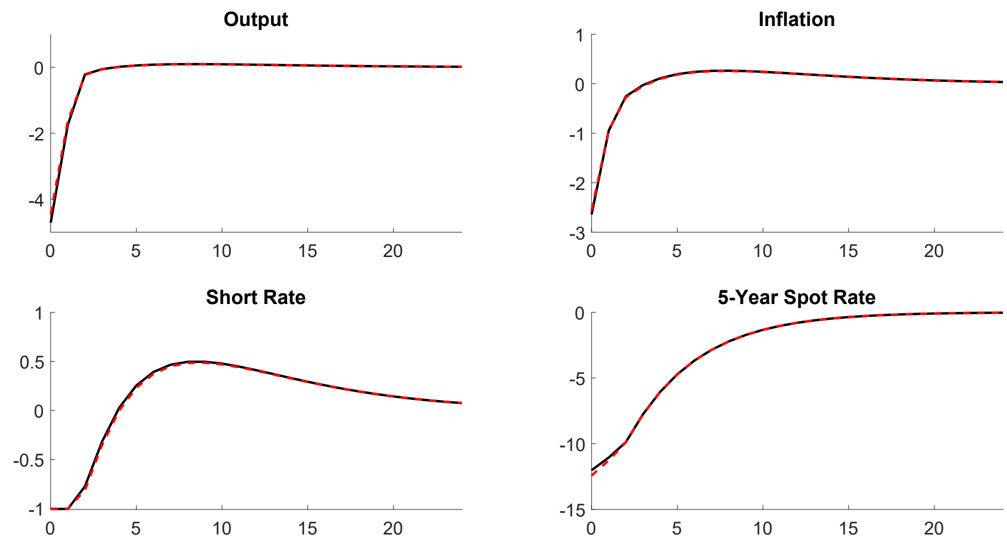
Source: Center for Financial Stability

Figure B.5: Divisia M4 vs. Estimated Trend



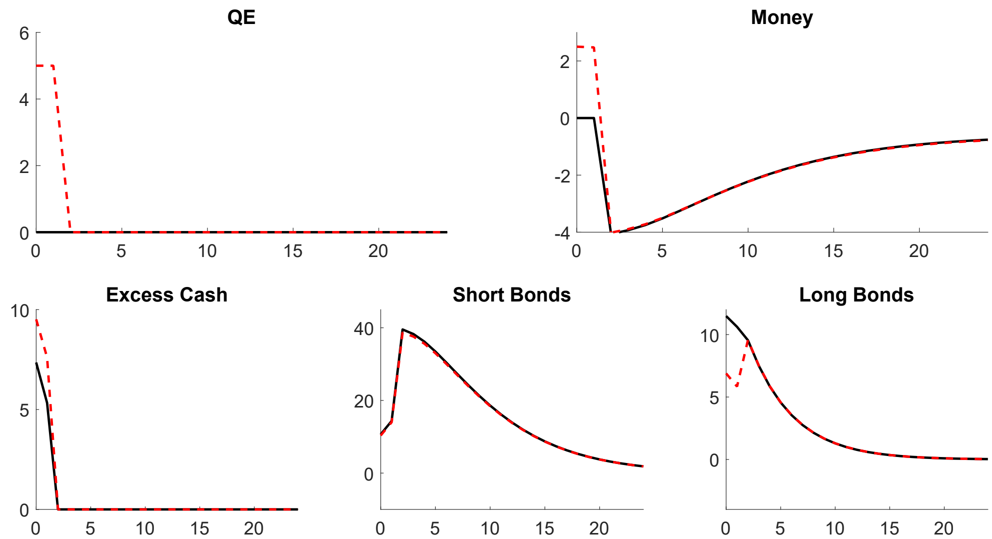
Source: Center for Financial Stability

Figure B.6: Impulse Responses with Sterilized QE (1)



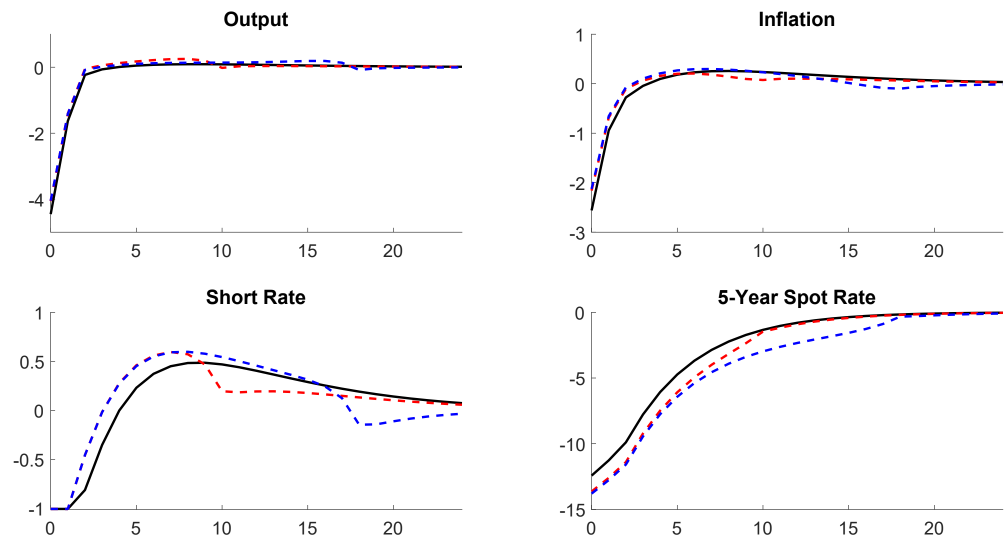
Impulse responses to a -15% preference shock. The dotted red line gives economic responses where the central bank conducts zero-interest-rate policy in tandem with QE. The black line represents responses to a regime where then central solely conducts zero-interest-rate policy. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.7: Impulse Responses with Sterilized QE (2)



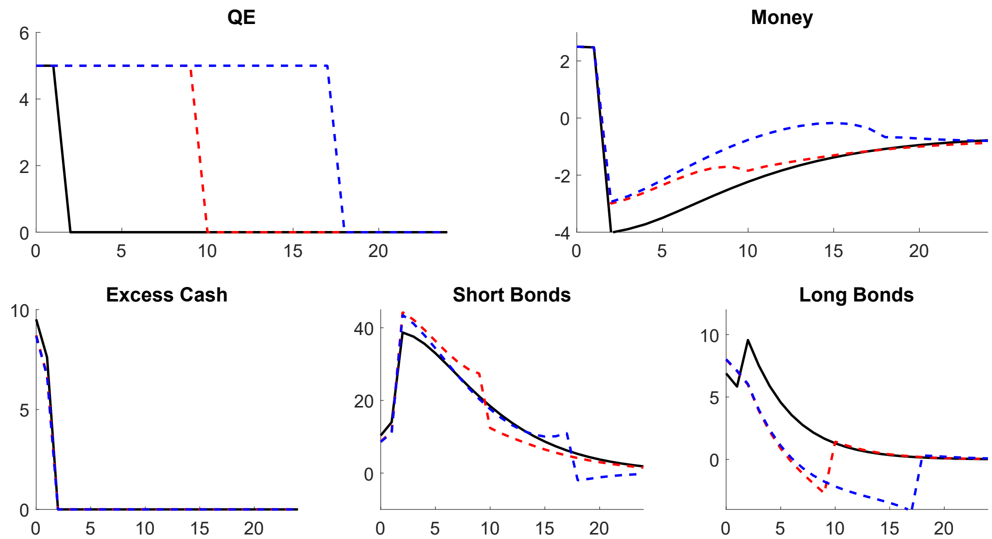
Impulse responses to a -15% preference shock. The dotted red line gives economic responses where the central bank conducts zero-interest-rate policy in tandem with QE. The black line represents responses to a regime where then central solely conducts zero-interest-rate policy. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.8: Impulse Responses with Sustained QE (1)



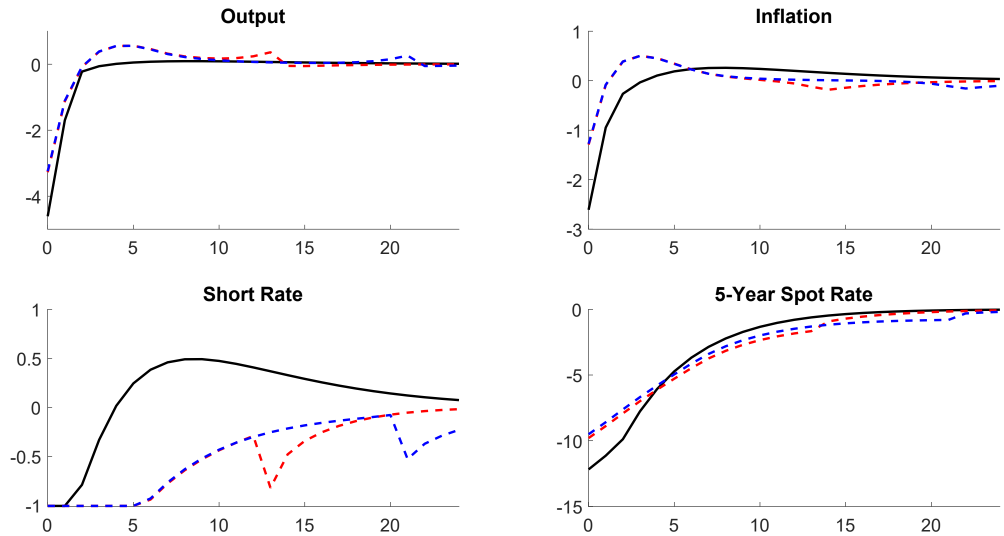
Impulse responses to a -15% preference shock. The dotted red line gives economic responses where the central bank conducts interest rate policy in tandem with QE. The black, red dotted, and blue dotted lines give responses when the central bank reverses asset purchases, zero, two, and four years after the nominal rate rebounds, respectively. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.9: Impulse Responses with Sustained QE (2)



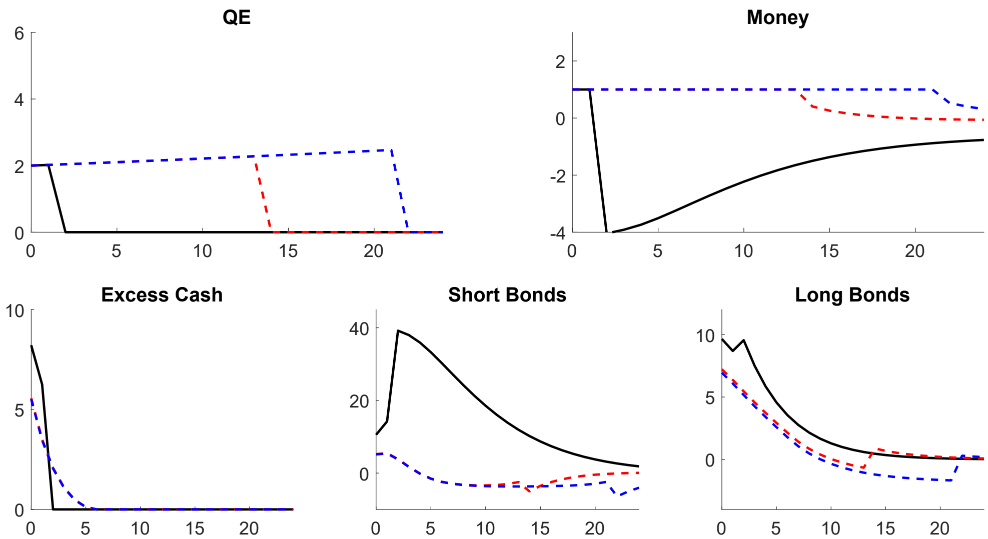
Impulse responses to a -15% preference shock. The dotted red line gives economic responses where the central bank conducts interest rate policy in tandem with QE. The black, red dotted, and blue dotted lines give responses when the central bank reverses asset purchases, zero, two, and four years after the nominal rate rebounds, respectively. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.10: Impulse Responses with Effective QE, Duration Experiment (1)



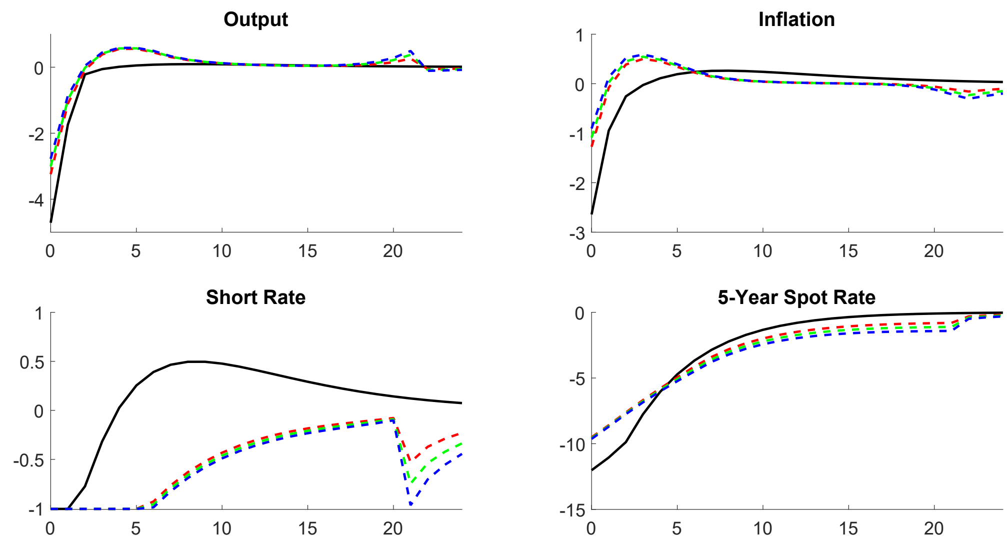
Impulse responses to a -15% preference shock when the central bank implements a 1% increase in the money supply using QE. The black, red-dotted, and blue-dotted lines give responses when the central bank reintroduces the Taylor rule zero, two, and four years after the nominal rate rebounds, respectively. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.11: Impulse Responses with Effective QE, Duration Experiment (2)



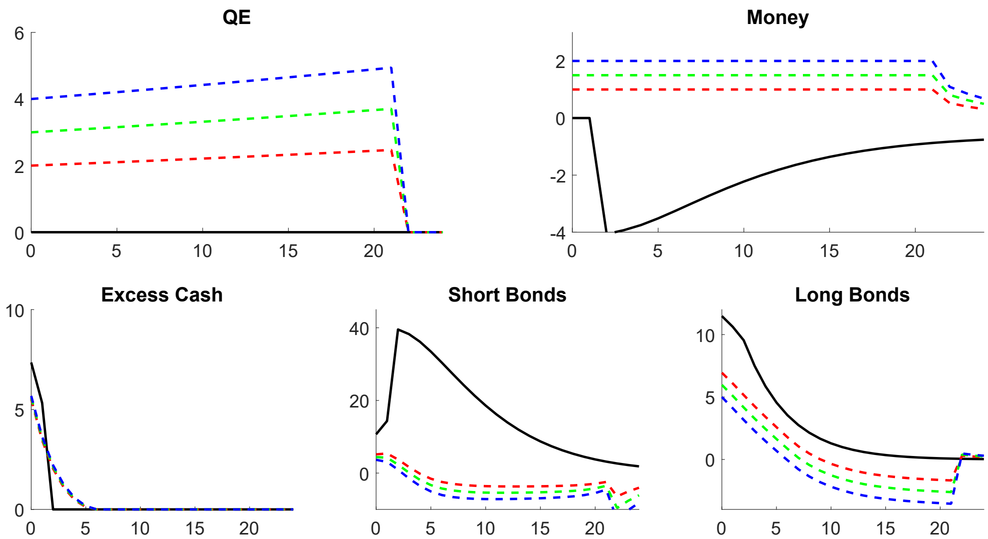
Impulse responses to a -15% preference shock when the central bank implements a 1% increase in the money supply using QE. The black, red-dotted, and blue-dotted lines give responses when the central bank reintroduces the Taylor rule zero, two, and four years after the nominal rate rebounds, respectively. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.12: Impulse Responses with Effective QE, Size Experiment (1)



Impulse responses to a -15% preference shock when the central bank implements an increase in the money supply using QE, and returns to a Taylor rule five years after the nominal rate recovers from the ZLB. The black, red-dotted,-green dotted, and blue-dotted lines represent responses to 0%, 1%, 1.5%, and 2% increases in the money supply, respectively. All values are reported as percentage deviations from the deterministic steady-state.

Figure B.13: Impulse Responses with Effective QE, Size Experiment (2)



Impulse responses to a -14% preference shock when the central bank implements an increase in the money supply using QE, and returns to a Taylor rule five years after the nominal rate recovers from the ZLB. The black, red-dotted, green-dotted, and blue-dotted lines represent responses to 0%, 1%, 1.5%, and 2% increases in the money supply, respectively.

Appendix C

Buyout Gold: MIDAS Estimators and Private Equity

C.1 Tables

Table C.1: Cambridge Associates Index

	MIDAS	MA(K)	YX-overlap
α	0.0368***	0.0111	0.0029
MKT	1.3060***	0.3301***	0.7229***
HML	0.9264**	0.0691	0.2380
SMB	0.2342	0.0015	−0.0622
LIQ	0.4362**	0.0491	0.1067*
HIY	−0.5457*	−0.0250	0.0364
R^2	0.6228	0.5571	0.3615
$RMSE$	0.0298	0.0319	0.0388
$RMSE_{RW}$	0.0391	0.0560	0.0482

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of the Cambridge Associates U.S. buyout index, and MA(K) is the Getmansky-Pedersen de-smoothing approach. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY is Moody's seasoned Baa corporate bond yield over the federal funds rate. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.2: Cambridge Associates Index with Overlapped Data

	MIDAS	MA(K)	YX-overlap
α	0.0612***	0.0519	0.0531**
MKT	1.7166***	0.4222***	0.6834***
HML	1.4747***	0.1812**	0.2453*
SMB	0.3001	0.0382	0.0672
LIQ	0.2551*	0.0394	0.0783
HIY	−0.9713***	−0.0523	−0.0035
R^2	0.7653	0.7026	0.6180
$RMSE$	0.0657	0.0735	0.0845
$RMSE_{RW}$	0.0820	0.1779	0.1019

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of the Cambridge Associates U.S. buyout index, and MA(K) is the Getmansky-Pedersen de-smoothing approach. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY is Moody's seasoned Baa corporate bond yield over the federal funds rate. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.3: Vintage Portfolios, MIDAS

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	0.0239	1.1825**	−1.2868	−1.4092**	0.7083***	0.2813	0.4672	0.0587	0.0866	56
2001	0.0785	0.9303	−2.1849*	−1.2532	−0.9810	−1.0681	0.4737	0.0442	0.0624	54
2002	−0.0125	1.3460*	1.2665	0.6625	0.9759**	0.7497	0.4395	0.0790	0.1345	50
2003	0.0602**	1.1803*	2.3297	2.1056	1.1054*	−0.7158**	0.4646	0.0651	0.1126	45
2004	0.0483	−0.8692	−1.8704**	−1.1739	0.7162	−0.2811	0.4002	0.0395	0.0728	43
2005	−0.0273	0.6648**	−1.7561**	0.7136*	1.2360*	0.6700*	0.5362	0.0285	0.0606	37
2006	−0.0706	0.2856	4.8133	−3.9256	0.3471	0.7118	0.3890	0.0732	0.2657	37
2007	−0.0419**	0.9910***	1.9268*	−0.7615	1.6074***	0.7242**	0.9605	0.0127	0.0264	29
2008	−0.0157	1.4228**	−3.6633	−0.5663	0.9532	0.5301*	0.8263	0.0352	0.0652	27
2009	0.0299	0.8167	1.8241	−2.1357	0.2515	−0.0713	0.7146	0.0179	0.0256	23

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.4: Vintage Portfolios, Getmansky et al. (2004)

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	−0.0142	0.0728**	−0.0630	0.0406	0.0228	0.1107*	0.1488	0.0728	0.0903	56
2001	−0.0247***	0.0461	−0.0315	0.0725**	0.0559***	0.1373***	0.4902	0.0419	0.0552	54
2002	0.1055***	−0.2563**	−0.0745	−0.3235**	0.1239**	−0.2861	0.2975	0.0866	0.1354	50
2003	0.6538***	−0.0063	0.0660	−0.0839	−0.1817**	−1.8279***	0.2841	0.0723	0.1669	45
2004	−0.0032***	0.0015	−0.0062	0.0005	0.0120***	0.0216***	0.4337	0.0378	0.0708	43
2005	0.0041	0.0459*	−0.1145**	−0.0096	0.0000	0.0073	0.3048	0.0341	0.1205	37
2006	0.0015	0.1247*	0.3205	−0.3636	−0.0023	−0.0560	0.3253	0.0769	0.0511	37
2007	0.0869**	−0.1350**	−0.2747	−0.5547***	0.1300**	−0.2640*	0.6450	0.0380	0.1941	29
2008	−0.0104	−0.0166	−0.4390***	−0.0586	0.3257***	0.2125*	0.7870	0.0386	0.0669	27
2009	0.4937***	−0.1607***	−0.0023	0.1174	0.0781**	−0.4451	0.6949	0.0165	0.0537	23

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.5: Vintage Portfolios, YX-overlap

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	−0.0498	0.1797	−0.4375	0.1988	0.2325	0.5274	0.1314	0.0752	0.0825	53
2001	-0.1305^{**}	0.0260	−0.0728	0.3424^{*}	0.4559^{***}	0.7590^{***}	0.4165	0.0469	0.0410	51
2002	−0.1384	0.4554	−0.9179	0.4427	0.4629^{***}	0.9978	0.1297	0.0866	0.0997	47
2003	0.1724	−0.4817	0.0157	−0.3053	0.0959	−0.5804	0.0488	0.0871	0.0834	42
2004	0.0116	−0.2859	−0.1568	−0.3397	0.3297^{***}	0.175 54	0.2236	0.0466	0.0351	40
2005	0.0600	0.2051	-0.8398^{*}	−0.1408	0.0439	−0.0550	0.2602	0.0363	0.0344	34
2006	0.0015	0.1247^{*}	0.3205	−0.3636	−0.0023	−0.0560	0.3253	0.0769	0.0511	37
2007	-0.1225^{**}	1.1519^{**}	-0.9641^{**}	0.3396	−0.1466	0.6401^{***}	0.2199	0.0844	0.0551	34
2008	−0.1537	0.3496	-2.2806^{***}	0.5195	1.2999^{***}	1.3745^{**}	0.7210	0.0467	0.0431	24
2009	−0.0526	0.3900	0.1058	0.5618	−0.1013	0.3587	0.0873	0.0307	0.0258	20

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.6: Vintage Portfolios, MIDAS with Overlapping Data

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	0.0080	2.5161***	−3.2441***	−0.2965	0.7780***	0.4255	0.8621	0.0714	0.1159	53
2001	0.0804***	−0.2793	−1.7253**	−2.2153***	0.8011	−0.9001***	0.8129	0.0760	0.1148	51
2002	0.0718***	−1.8541***	0.5654	−2.0915*	2.0766***	−0.3275*	0.7518	0.0704	0.1126	47
2003	0.0967***	1.6270	−1.0817	0.1531	−0.6022*	−1.1303***	0.8158	0.0733	0.1309	42
2004	0.0069	0.1291	0.6361	0.4884	0.3458	0.2532	0.1550	0.0897	0.1298	40
2005	0.0029	0.3048	1.2110*	−1.0966**	1.0524***	0.2063	0.8842	0.0306	0.0500	34
2006	−0.0889**	4.2352***	6.6534***	−3.5364	1.3662***	0.9078*	0.8282	0.1008	0.2313	34
2007	−0.0417*	1.5030**	1.6098***	0.3258	1.3904***	0.7186**	0.9978	0.0098	0.0232	26
2009	−0.0200	2.8659***	−2.1623	−1.4120	0.4215	0.3920	0.9790	0.0329	0.0632	24
2009	0.0484	0.8914***	3.8685**	−1.5014	0.3803*	−0.4849	0.9724	0.0097	0.0155	20

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.7: Vintage Portfolios, Getmansky et al. (2004) with Overlapping Data

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	−0.1030	0.1083**	−0.3309**	0.0584	0.1199***	0.2542**	0.5421	0.1212	0.3251	53
2001	−0.2121***	0.0404	−0.2410***	0.1362***	0.1930***	0.3311***	0.8219	0.0667	0.1600	51
2002	0.2273***	−0.1899***	−0.0240	−0.2337***	0.1036**	−0.1257*	0.5536	0.0934	0.1994	47
2003	−0.1926***	0.1687*	−0.4474*	0.3043***	0.1958***	0.3355***	0.4573	0.1257	0.3460	42
2005	0.0182	0.0864***	−0.1515*	−0.0771	0.0130*	0.0268	0.7563	0.0441	0.6627	34
2006	−0.2564**	0.5731***	−0.1287	0.0582	−0.1376***	−0.2697*	0.8069	0.1062	1.3114	34
2007	1.7469**	−0.0531	−0.0889	−0.6240***	0.0938***	−1.5531***	0.9876	0.0215	0.2521	26
2008	−0.1804***	0.0873	−0.6344***	0.1234***	0.4045***	0.4051***	0.9532	0.0446	0.8224	24
2009	0.2337	−0.2255*	−0.4387*	0.6300	0.0579	0.7607	0.7014	0.0288	0.1991	20

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation. Estimates for 2004 did not converge and so are excluded from analysis.

Table C.8: Vintage Portfolios, YX-overlap with Overlapping Data

Vintage	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
2000	-0.2251	0.1546	-0.8644^{**}	0.1503	0.3305^{***}	0.6316^{**}	0.5406	0.1245	0.1302	50
2001	-0.4653^{***}	0.0082	-0.5320^{***}	0.2934^{**}	0.5105^{***}	0.7685^{***}	0.8050	0.0751	0.0642	48
2002	-0.2446	0.0540	-0.5075^{*}	0.1134	0.4486^{***}	0.6080^{**}	0.4772	0.1079	0.1182	44
2003	-0.5206^{**}	-0.3907	-1.2129^{*}	0.8343^{***}	0.5685^{***}	0.9273^{**}	0.4950	0.1304	0.1111	39
2004	-0.1433	-0.1655^{**}	-0.1671^{**}	-0.2112^{**}	0.3577^{***}	0.3867^{***}	0.7653	0.0442	0.0357	37
2005	0.1586^{***}	0.1777	-0.4779^{*}	-0.4289^{**}	0.0966^{***}	0.0162	0.7699	0.0433	0.0385	31
2006	-0.4503	1.4424^{***}	-0.7649	0.1279	-0.4008^{**}	0.5298	0.7584	0.1173	0.1056	31
2007	-0.4566^{***}	0.8516	-2.3813^{***}	0.4807	0.5495	1.0110^{***}	0.7243	0.1054	0.0698	23
2008	-0.7546^{***}	0.3808	-2.2589^{***}	0.6432^{***}	1.3940^{***}	1.5461^{**}	0.9419	0.0526	0.0557	21
2009	-0.7546^{***}	0.3808	-2.2589^{***}	0.6432^{***}	1.3940^{***}	1.5461^{***}	0.9419	0.0526	0.0557	17

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a value-weighted portfolio containing funds from a specific vintage. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.9: Funds, MIDAS

Fund	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
Alta Communications VIII	0.0611*	0.6306	2.5788	1.2380	0.8274**	−1.6631**	0.5768	0.0690	0.1124	55
Baker Communications Fund II	−0.0166	4.5266***	−14.0835***	−1.5695	−1.1063**	0.4674	0.4541	0.1296	0.1800	55
Bear Stearns Merchant Banking Partners II	−0.0290	2.0364**	2.7961	2.1417*	0.9145*	0.1400	0.2177	0.1632	0.2224	54
Blackstone Communication Partners I	0.0490	1.9129	2.7486	−2.6326	1.4108	−1.3793	0.2614	0.2679	0.4199	53
Blue Point Capital Partners	0.0676*	1.6124*	−2.4724	2.9990	0.3555	−0.9030	0.3912	0.0975	0.1764	50
Boston Ventures VI	0.0813	3.3740**	−2.6835*	0.8402	−2.0678**	−1.4440	0.4990	0.0629	0.1082	55
Carlyle Partners III	−0.0165	4.8079***	−3.7413	3.5265***	0.4179	0.8284*	0.4698	0.0987	0.1765	46
Cortec Group Fund III	0.0221	−0.5265	−1.4137	−1.9606*	−0.5701	0.5473	0.2762	0.0963	0.1447	43
DLJ Merchant Banking Partners III	0.1381**	−2.3896	2.4247	−3.2450*	2.0238**	−1.1992	0.3585	0.1149	0.1608	54
2003 Riverside Capital Appreciation Fund IV	−0.0335	−3.3144	−2.5047	2.1552	3.5726	1.4436	0.4481	0.0744	0.0823	41
Alta Communications IX	0.1077*	−0.8682	4.1348	−4.2910*	0.3334	−2.0133**	0.3304	0.0752	0.1397	39
Blackstone Capital Partners IV	0.1083*	1.4503	9.9294	4.8842	1.5669	−1.5639	0.2818	0.1673	0.2177	44
Castle Harlan Partners IV	0.1972***	−1.8586*	2.3666*	−4.1572***	−0.6155***	−3.0826***	0.3701	0.0885	0.1485	43
Cerberus Institutional Partners III	0.0534	0.2743	0.6222	0.8188	0.3374	−0.5900	0.3277	0.0639	0.1433	32
First Reserve Fund X	0.3430***	0.9561*	6.8604*	−2.7868	−4.2632*	−5.6522***	0.4893	0.1177	0.2997	37
Glencoe Capital Partners III	−0.2868***	12.4967***	−18.9741***	6.3744*	−4.4245**	4.5774***	0.5148	0.1230	0.4055	37
GTCR Fund VIII	0.0606	−4.7097***	−5.6268***	4.6986**	3.2017***	−0.5661	0.6554	0.0838	0.1584	41
KKR Millennium Fund	−0.0545	−3.3421	5.8672	4.0579*	5.7042	1.0811	0.3487	0.1479	0.3760	45
Levine Leichtman Capital Partners III	−0.0529	2.2152	−8.7048*	5.2769**	1.6533	1.7908	0.4289	0.1560	0.2179	41
Markstone Capital Partners	0.1500	52.2896	73.8081	−64.3471**	−49.6037	−13.3673	0.4488	1.6093	1.7176	40
OCM Principal Opportunities Fund III	0.1716***	−4.3561*	5.4010	−5.2547**	1.3460	−2.4670***	0.3288	0.0735	0.1431	40
Rhone Partners II	0.1694	9.9891	−7.7274	4.9901	−8.2071	−2.4437	0.2627	0.2428	0.3828	40
Silver Lake Partners II	0.0064	2.5239***	−0.6405	−0.3256	0.2042	−0.0333	0.5289	0.0574	0.0886	38
Swander Pace Capital Partners III	0.1270	5.9330	1.7383	−2.6202	−4.0576*	−2.0218	0.3708	0.1276	0.3859	39
Thayer Equity Investors V	−0.0642	2.6879*	−2.6326	2.8477	0.4026	1.3107	0.5097	0.0668	0.2037	37

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a specific fund. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.10: Funds, MIDAS with Overlapped Data

Fund	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
Alta Communications VIII	0.0108	1.5376	1.7970	1.4294	1.3013^{*}	−0.7675	0.7989	0.1440	0.2162	52
Baker Communications Fund II	0.0055	4.7296^{***}	-15.4740^{***}	0.5892	-2.2217^{*}	−0.0043	0.7778	0.1783	0.3351	52
Bear Stearns Merchant Banking Partners II	0.0403	1.1295	2.0081	1.5894	0.9329^{*}	−0.8820	0.3852	0.2039	0.2639	51
Blackstone Communication Partners I	0.1194	5.6101	5.1518	4.4083	0.2969	−2.8577	0.4940	0.5386	0.8893	50
Blue Point Capital Partners	0.1593^{***}	-1.6468^{***}	-2.2914^{**}	1.9943^{***}	-1.8764^{***}	-2.2798^{***}	0.7373	0.1190	0.1783	47
Boston Ventures VI	0.1337^{**}	1.8928	1.4444^{*}	−1.6760	−1.3202	-2.2929^{***}	0.7508	0.1385	0.2408	52
Carlyle Partners III	-0.0829^{**}	9.7053^{***}	-4.8216^{***}	5.6156^{***}	-1.5028^{***}	1.6340^{***}	0.8570	0.1090	0.2154	43
Cortec Group Fund III	0.0106	1.5123	−1.4717	0.9962	-1.8069^{***}	0.6384	0.5988	0.1308	0.2270	40
DLJ Merchant Banking Partners III	0.1394^{***}	-1.6763^{***}	1.6879	-2.6055^{*}	0.4814	-1.5491^{***}	0.3982	0.1539	0.2350	51
2003 Riverside Capital Appreciation Fund IV	0.0229	1.1570	1.1228	−1.9625	0.1430	0.1653	0.4189	0.0849	0.1497	38
Alta Communications IX	0.1147^{***}	−2.5696	4.6598^{***}	-3.6972^{***}	1.6814^{**}	-2.0419^{***}	0.7920	0.0704	0.1142	36
Blackstone Capital Partners IV	0.1214^{*}	−2.2650	7.7617^{**}	3.7174	0.6431	−1.3113	0.6417	0.1694	0.2899	41
Castle Harlan Partners IV	0.2095^{***}	−3.2094	2.7841^{*}	-5.3277^{***}	−0.0490	-2.9558^{***}	0.6183	0.1286	0.2253	40
Cerberus Institutional Partners III	0.1228^{***}	-1.2813^{***}	0.7137	0.7278	−0.3595	-1.6289^{***}	0.8665	0.0487	0.1072	29
First Reserve Fund X	0.2656^{***}	−1.9799	6.7652^{**}	−4.4369	−0.0929	-3.9472^{**}	0.5960	0.1780	0.4159	34
Glencoe Capital Partners III	-0.2445^{***}	3.9728^{***}	-17.5613^{***}	4.4202^{**}	−1.6004	4.4268^{***}	0.8903	0.1321	0.2782	34
GTCR Fund VIII	0.1046	-7.9577^{***}	-5.8778^{*}	1.7136	4.2997^{**}	−0.9385	0.7518	0.1267	0.2927	38
KKR Millennium Fund	−0.0331	-6.6368^{***}	6.2656^{***}	4.7066^{***}	5.8190^{***}	1.1778^{**}	0.7491	0.1402	0.2299	42
Levine Leichtman Capital Partners III	0.0796	−0.8263	3.6073	2.8319	1.9137	−0.7281	0.3025	0.2336	0.3803	38
Markstone Capital Partners	2.2151^{**}	158.2438^{**}	41.3610^{***}	-144.4137^{**}	-63.4265^{**}	-49.0506^{**}	0.6437	2.1066	5.5821	37
OCM Principal Opportunities Fund III	0.2013^{***}	-4.4943^{***}	4.9119^{***}	-5.9607^{***}	0.8438	-2.8960^{***}	0.7886	0.0668	0.2035	37
Rhone Partners II	0.3775^{***}	-8.5998^{***}	15.6668^{**}	10.0288^{***}	7.5127^{***}	-4.9717^{**}	0.7725	0.2872	0.7894	37
Silver Lake Partners II	0.0321	3.5496^{**}	0.7949	1.3164	−1.0205	−0.5631	0.7595	0.0917	0.2431	35
Swander Pace Capital Partners III	0.0681^{**}	7.0570^{***}	-5.1414^{**}	-4.2541^{**}	-4.3530^{***}	-1.1612^{***}	0.8265	0.0998	0.1433	36
Thayer Equity Investors V	−0.0370	1.4115	−2.6716	1.9691	0.3859	1.0252	0.6546	0.0950	0.2114	34

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a specific fund. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.11: Funds, YX-overlap

Fund	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
Alta Communications VIII	0.1527	0.1829	0.0763	0.0958	0.2594	−0.8609	0.3023	0.0914	0.1069	52
Baker Communications Fund II	−0.3895**	0.7716**	0.5808	1.2173**	0.6359***	1.7609***	0.1929	0.1608	0.1737	52
Bear Stearns Merchant Banking Partners II	−0.5097*	0.7595	0.9917	1.3526	0.3769	2.3479*	0.1606	0.1741	0.1870	51
Blackstone Communication Partners I	−0.5976*	0.4678	4.8191	1.8243	0.1420	2.0140*	0.1947	0.2872	0.1697	50
Blue Point Capital Partners	−0.0268	−0.2713	1.2361	0.5629	0.3231	0.0288	0.1628	0.1182	0.0843	47
Boston Ventures VI	−0.2070**	0.3760	0.1470	0.9419**	0.4786***	1.0276***	0.3690	0.0725	0.0649	52
Carlyle Partners III	−0.0177	0.6450*	0.4069	0.6576	0.1144	0.2374	0.0886	0.1295	0.1539	43
Cortec Group Fund III	−0.0824	−0.4264	0.2420	−0.1375	0.0805	0.6200	0.0683	0.1074	0.0991	40
DLJ Merchant Banking Partners III	0.2901	−0.0439	−0.5708	−0.4169	0.0564	−1.0469	0.0782	0.1407	0.1334	51
2003 Riverside Capital Appreciation Fund IV	0.0420	0.3469	−1.0686**	−0.1884	0.1243	0.1696	0.1875	0.0581	0.0612	38
Alta Communications IX	0.0751	−0.5408*	0.9487	−0.6868	0.4114***	−0.4259**	0.2246	0.0848	0.1010	36
Blackstone Capital Partners IV	−0.1883	−0.2900	2.2965	0.3597	0.3533	0.8186	0.1220	0.1552	0.1547	41
Castle Harlan Partners IV	−0.0962	−0.5845	1.6267**	−0.9692**	0.6369**	0.4284	0.2474	0.0988	0.1117	40
Cerberus Institutional Partners III	0.0443	−0.4334	−0.9236	−0.5351	0.7942*	0.2090	0.2259	0.0715	0.0530	29
First Reserve Fund X	−0.1886	0.5042	−0.5500	1.0014	0.5305	1.0648**	0.1357	0.1009	0.1025	34
Glencoe Capital Partners III	−0.1141	1.3396***	−1.6656	−1.1001	−0.0542	0.8571	0.2587	0.1566	0.1265	34
GTCR Fund VIII	−0.1459	−0.4208	−0.1752	−0.1026	0.7142**	0.8999*	0.1471	0.1055	0.0714	38
KKR Millennium Fund	−0.2018	0.0163	−0.1607	0.1613	0.7874**	1.2202**	0.1155	0.1373	0.1575	42
Levine Leichtman Capital Partners III	0.6748	−0.8702	−1.2787	−2.2905	0.0884	−2.6225	0.1805	0.1910	0.1833	38
Markstone Capital Partners	−0.0973	−0.2205	−2.4792	−0.4029	1.0794**	0.9159	0.0883	0.2312	0.0577	37
OCM Principal Opportunities Fund III	0.0442	−0.5274	−0.1815	−1.1094**	0.4639***	0.0173	0.1313	0.0877	0.1001	37
Rhone Partners II	−0.4581**	0.5521	−0.8472	2.3000	1.6692*	2.6476**	0.1951	0.2420	0.0995	37
Silver Lake Partners II	−0.0012	0.0884	−1.5946***	−1.2713**	0.6617***	0.4752**	0.4547	0.0647	0.0605	35
Swander Pace Capital Partners III	−0.3800**	1.8433**	−2.0245	3.4946**	0.1085	1.9403**	0.2708	0.1288	0.0871	36
Thayer Equity Investors V	−0.0185	0.7057*	−1.1790	0.8870	0.0190	0.3415	0.1005	0.0932	0.0853	34

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a specific fund. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

Table C.12: Funds, YX-overlap with Overlapped Data

Fund	α	MKT	HML	SMB	LIQ	HIY	R^2	$RMSE$	$RMSE_{RW}$	T
Alta Communications VIII	0.8186^{*}	0.2708	−0.7886	−0.1978	0.2872^{*}	−0.9510	0.4453	0.2482	0.2823	49
Baker Communications Fund II	-1.9068^{***}	1.0292^{***}	0.5764	1.5715^{***}	0.7098^{***}	2.1508^{***}	0.6193	0.2395	0.2405	49
Bear Stearns Merchant Banking Partners II	-0.8848^{**}	0.3993	−0.4849	0.6830^{***}	0.5244^{**}	1.2518^{***}	0.3390	0.2143	0.2209	48
Blackstone Communication Partners I	-4.5323^{**}	2.0563^{**}	2.3534	4.1959^{***}	0.8842^{***}	4.7192^{***}	0.4536	0.5761	0.3321	47
Blue Point Capital Partners	0.0372	−0.0229	−0.2028	0.6684^{**}	0.3949^{***}	0.0421	0.6061	0.1329	0.0928	44
Boston Ventures VI	-1.1396^{***}	0.4791^{**}	−0.2102	1.2536^{***}	0.6707^{***}	1.4439^{***}	0.7471	0.1389	0.1308	49
Carlyle Partners III	−0.0012	1.1323^{***}	−0.7603	0.5815^{*}	0.3080^{**}	0.4072	0.4590	0.2167	0.2474	40
Cortec Group Fund III	−0.2638	-0.4255^{**}	0.6004	−0.1235	−0.0132	0.5034	0.2963	0.1552	0.1162	37
DLJ Merchant Banking Partners III	0.6667^{**}	0.0659	−0.7107	−0.1214	0.1100	-0.4873^{*}	0.2079	0.1794	0.1994	48
2003 Riverside Capital Appreciation Fund IV	0.0581	0.3215^{*}	-1.4039^{***}	−0.0132	0.1747^{***}	0.3243	0.4777	0.0751	0.0793	35
Alta Communications IX	0.2004^{**}	-0.4286^{***}	0.7482^{***}	-0.4430^{*}	0.3897^{***}	-0.3047^{***}	0.7448	0.0847	0.0953	33
Blackstone Capital Partners IV	-1.2960^{***}	0.4241^{*}	0.0789	1.5254^{***}	0.5801^{***}	1.6696^{***}	0.4546	0.1921	0.1902	38
Castle Harlan Partners IV	0.0278	-1.0451^{***}	1.9262^{**}	-1.2648^{**}	0.5584^{***}	−0.0684	0.5047	0.1606	0.1763	37
Cerberus Institutional Partners III	−0.2106	−0.0220	−0.7443	0.0329	0.5708^{***}	0.5129^{***}	0.5737	0.0721	0.0576	26
First Reserve Fund X	-1.2796^{*}	0.6688^{**}	−1.1006	1.8557	0.8473^{**}	1.7712^{**}	0.3579	0.1944	0.1679	31
Glencoe Capital Partners III	−1.9921	2.0642^{***}	-2.0589^{**}	0.9936	0.3678	2.6349^{*}	0.6390	0.2400	0.1881	31
e GTCR Fund VIII	−0.0256	-0.7404^{**}	−0.5682	−0.8131	0.7661^{***}	0.3503	0.5591	0.1570	0.1002	35
KKR Millennium Fund	-1.4432^{***}	0.3305^{**}	-1.0538^{*}	0.7242^{**}	1.0860^{***}	2.0418^{***}	0.6514	0.1555	0.1727	39
Levine Leichtman Capital Partners III	0.1696	−0.0883	0.3385	0.7794	0.2313	−0.1210	0.3129	0.2237	0.2128	35
Markstone Capital Partners	0.0787	-0.9364^{***}	-1.8952^{***}	−0.4050	1.0993^{***}	0.4107	0.5176	0.2729	0.1036	34
OCM Principal Opportunities Fund III	0.2147	-0.6023^{***}	−0.3632	-1.2723^{***}	0.5591^{***}	0.0189	0.7061	0.0882	0.0887	34
Rhone Partners II	-3.5122^{***}	0.4454	0.5906^{***}	4.0145^{***}	2.1778^{***}	4.3178^{***}	0.7639	0.2968	0.1636	34
Silver Lake Partners II	−0.1362	0.4384^{***}	-2.2618^{***}	−0.5079	0.5764^{***}	0.6528^{**}	0.7286	0.1036	0.0895	32
Swander Pace Capital Partners III	−1.0044	0.7860	0.4849	1.0273	0.2068	1.1993	0.2384	0.2109	0.1175	33
Thayer Equity Investors V	−0.1882	0.8063^{***}	-1.5317^{*}	0.8901^{**}	0.1161	0.5425	0.2576	0.1367	0.1150	31

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log excess return of a specific fund. MKT (market), HML (high-minus-low), and SMB (small-minus-big) are the Fama-French factors, while LIQ is the Pastor-Stambaugh liquidity factor. HIY the Bank of America Merrill Lynch high yield option adjusted spread. $RMSE_{RW}$ is the root mean squared error for one-step ahead forecasts with rolling window estimation.

C.2 Figures

Figure C.1: Estimated Monthly vs. Observed Quarterly Returns

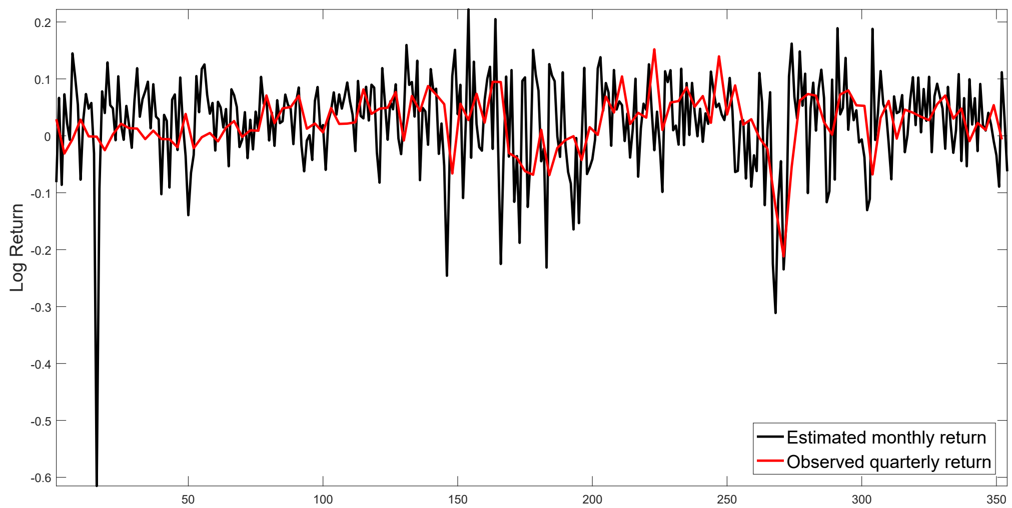


Figure C.2: Estimated Monthly vs. Observed Quarterly Returns

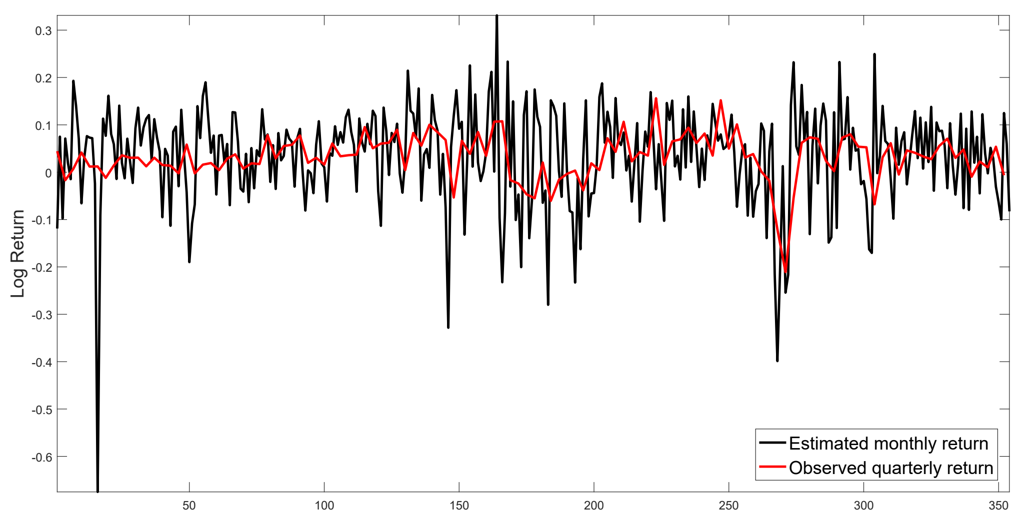


Figure C.3: Factor Weights, Cambridge Associates Index

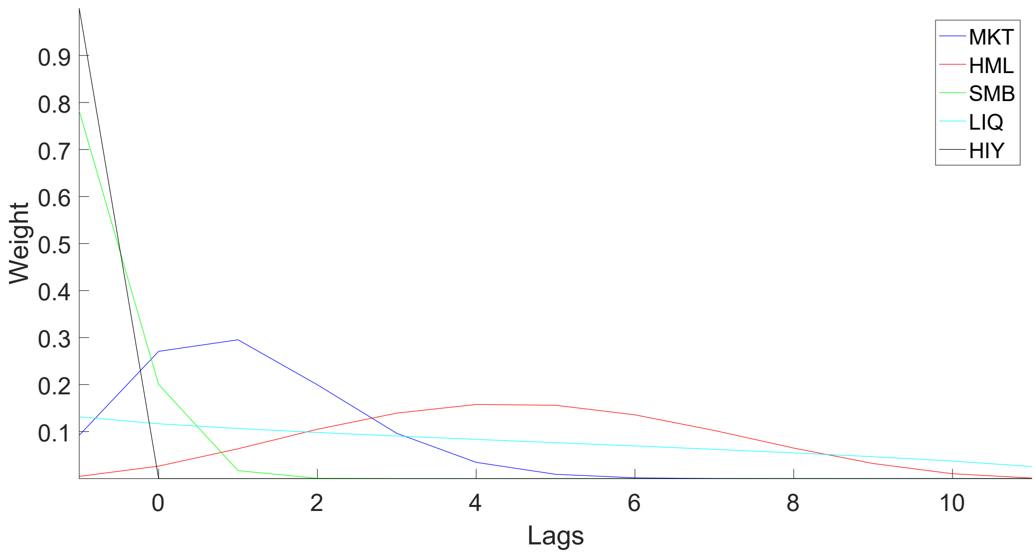


Figure C.4: Factor Weights, Cambridge Associates Index with Overlapped Data

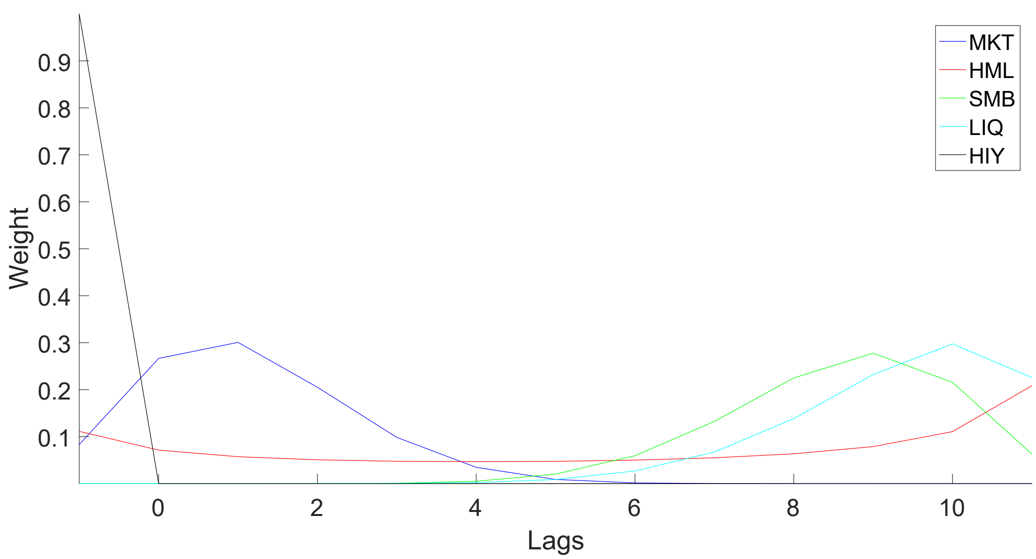


Figure C.5: Factor Weights, Vintage Porftolios

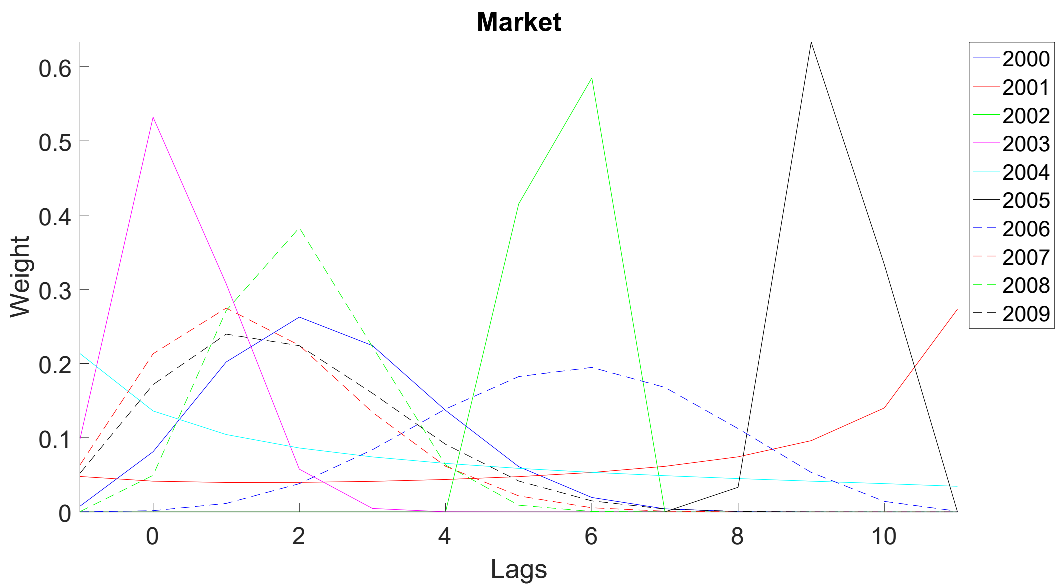


Figure C.6: Factor Weights, Vintage Porftolios

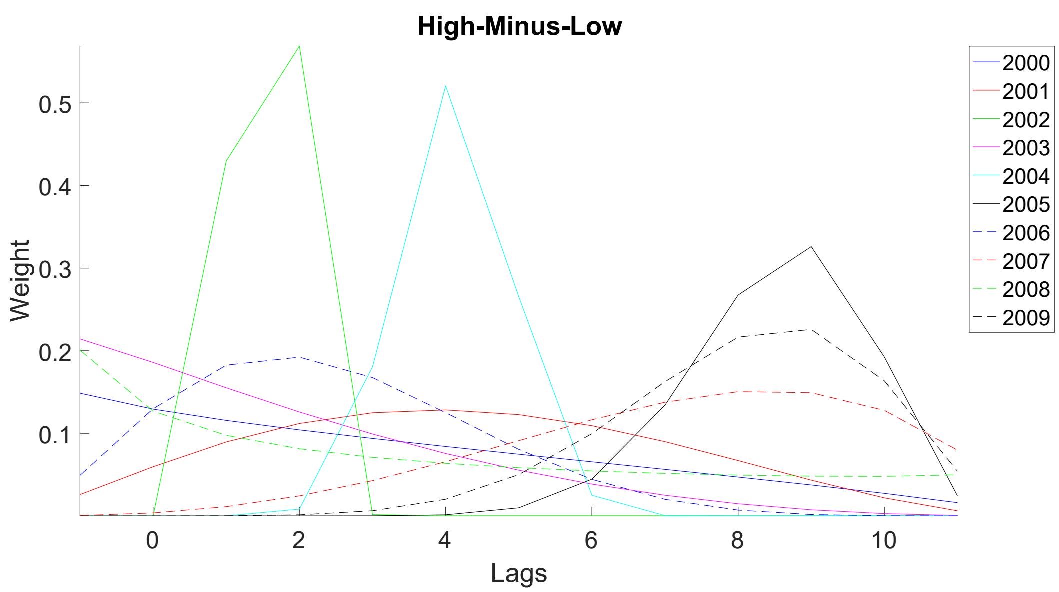


Figure C.7: Factor Weights, Vintage Porftolios

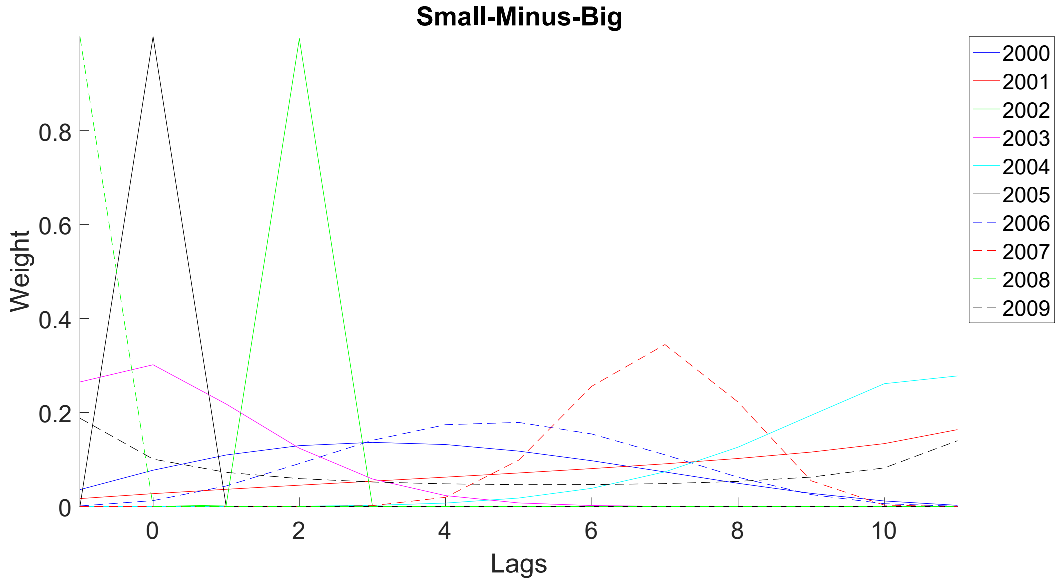


Figure C.8: Factor Weights, Vintage Porftolios

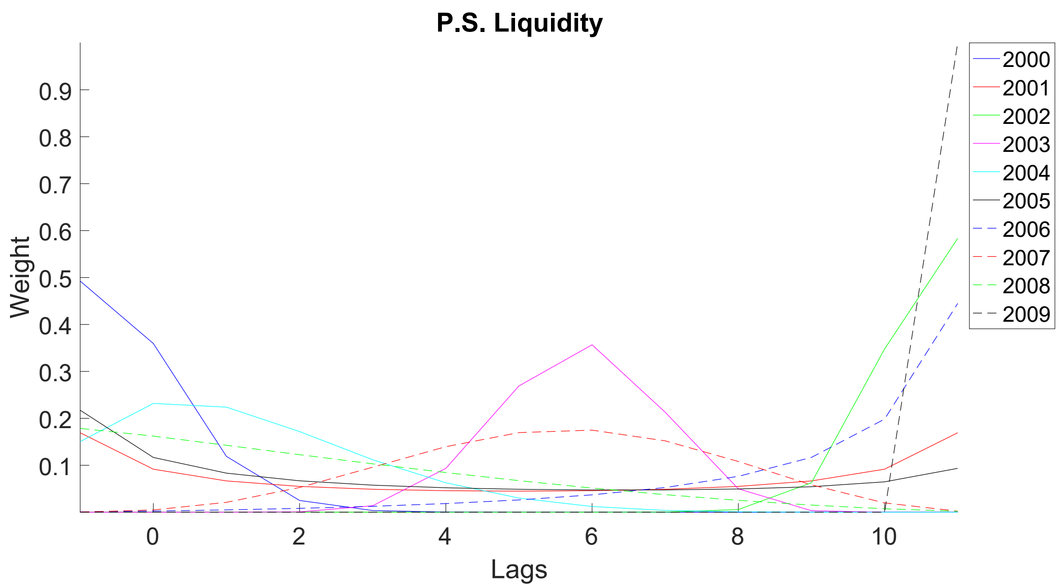


Figure C.9: Factor Weights, Vintage Porftolios

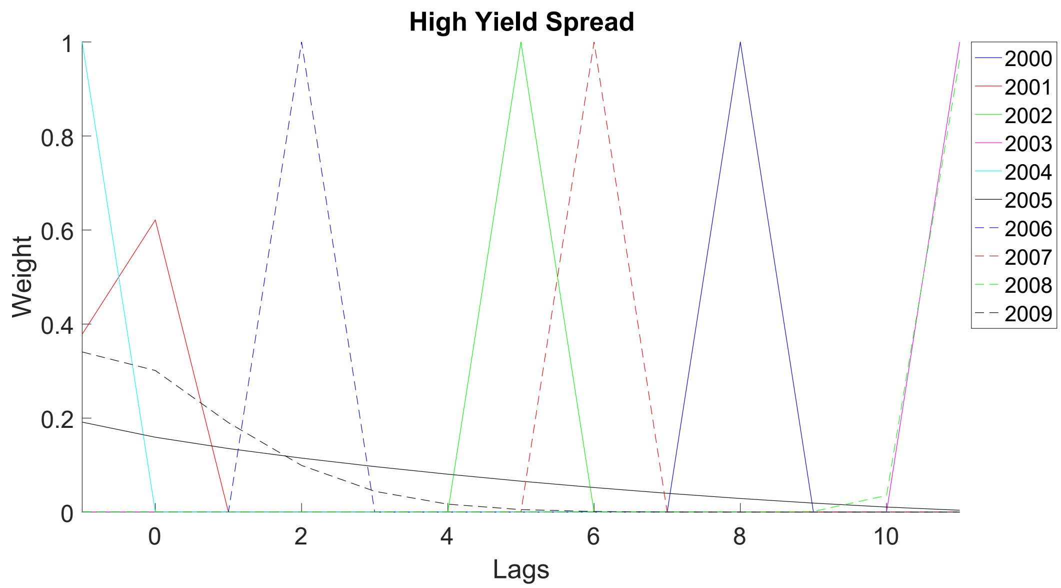


Figure C.10: Factor Weights, Vintage Porftolios with Overlapped Data

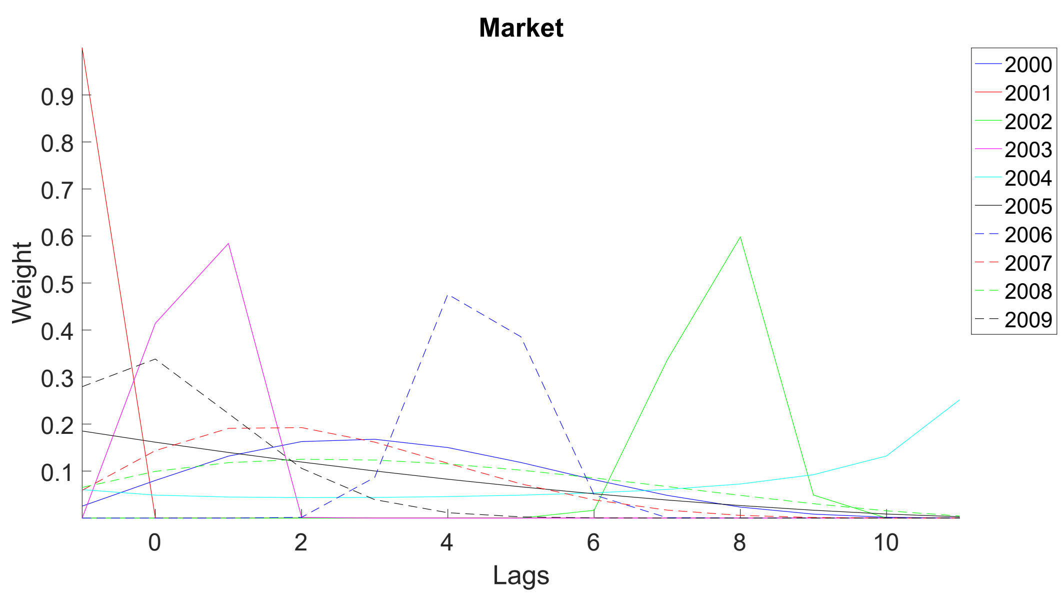


Figure C.11: Factor Weights, Vintage Porftolios with Overlapped Data

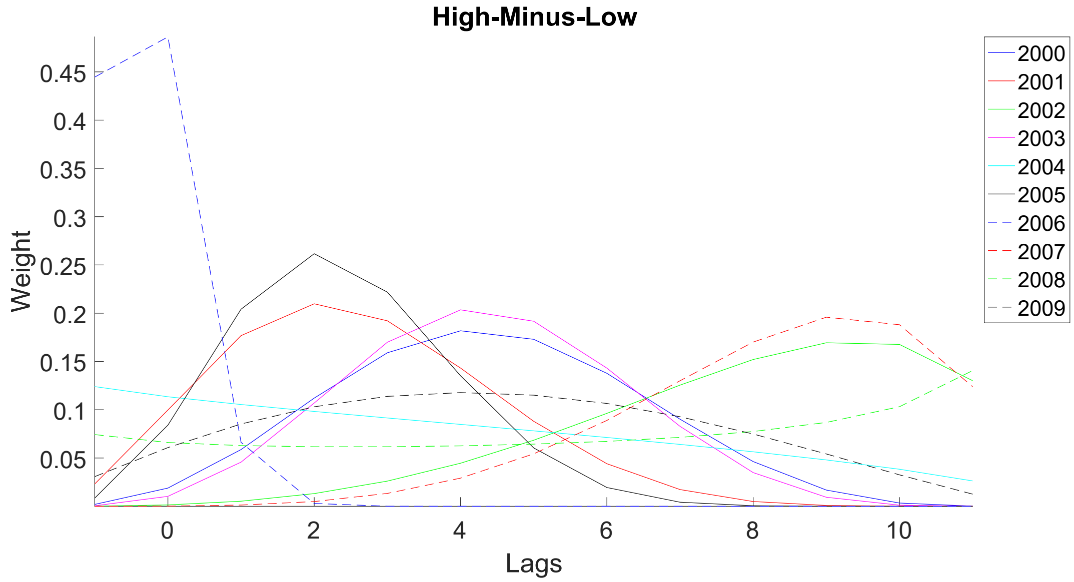


Figure C.12: Factor Weights, Vintage Porftolios with Overlapped Data

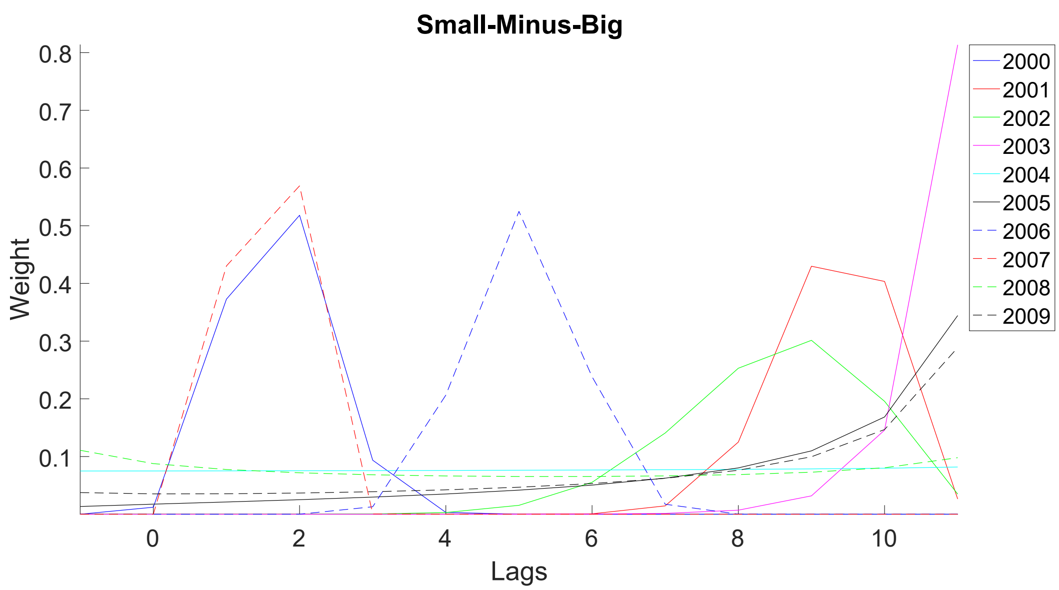


Figure C.13: Factor Weights, Vintage Porftolios with Overlapped Data

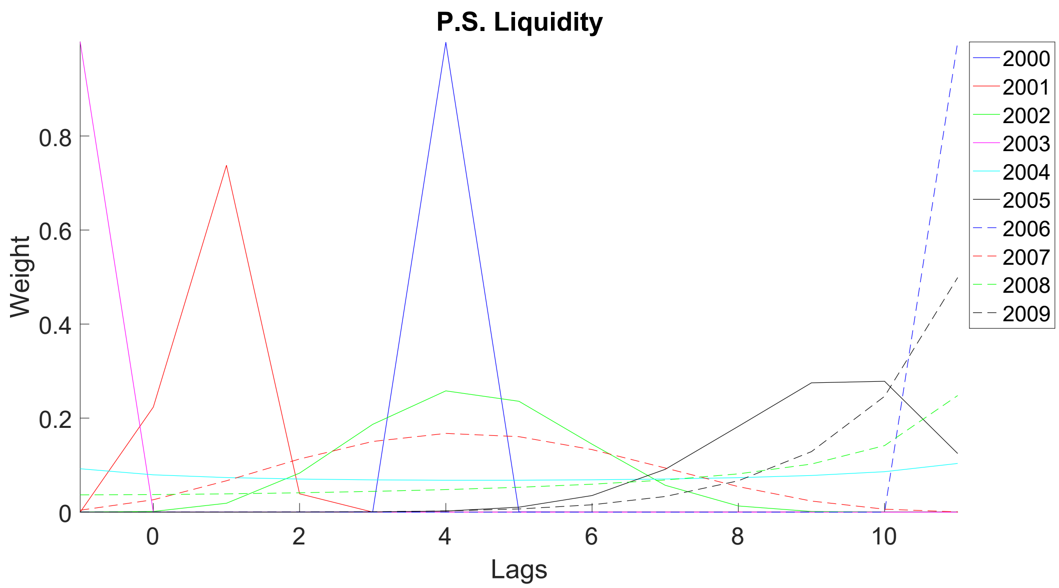
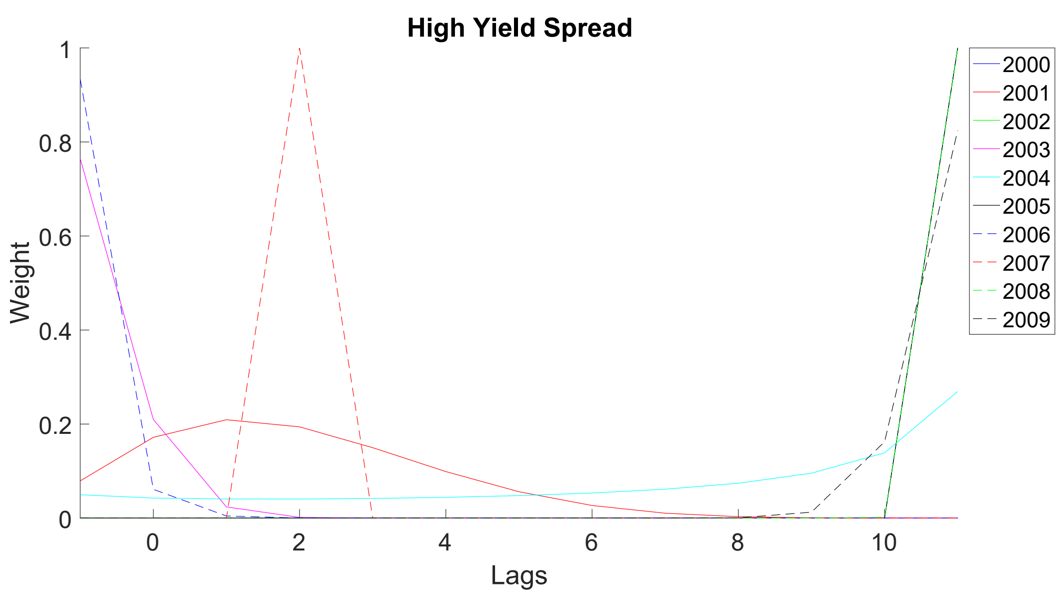


Figure C.14: Factor Weights, Vintage Porftolios with Overlapped Data



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