

One or More External Representations: What Is Better for Learning?

Author: Anna V. Ermakova

Persistent link: <http://hdl.handle.net/2345/bc-ir:107097>

This work is posted on [eScholarship@BC](#),
Boston College University Libraries.

Boston College Electronic Thesis or Dissertation, 2016

Copyright is held by the author, with all rights reserved, unless otherwise noted.

BOSTON COLLEGE

Lynch School of Education

Department of Counseling, Developmental, and Educational Psychology

Applied Developmental and Educational Psychology Program

ONE OR MORE EXTERNAL REPRESENTATIONS: WHAT IS BETTER FOR LEARNING?

Dissertation

by

ANNA V. ERMAKOVA

submitted in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

June 2016

© Copyright by Anna V. Ermakova

2016

Abstract

One or More External Representations: What Is Better For Learning?

Anna Ermakova

Dissertation Chair: Elida V. Laski

Use of base-10 decomposition strategy for addition in first grade is related to mathematics advantage in the later years (Geary et al., 2013), yet we know little about the strategy's prevalence among first graders or factors contributing to its use. The present study sought to bridge this gap by testing 87 first graders in the greater Boston area. The results confirmed previous findings that showed that in the last 10 years first graders in the US have increased in frequency of base-10 decomposition. Children who had better knowledge of basic number facts used it more frequently, particularly on problems with smaller addends.

Further, the study tested whether an instructional intervention would be effective in increasing reliance on base-10 decomposition. 61 of the original participants were selected to take part in an experimental intervention that taught them to execute the strategy while relying on external representations – sometimes known as manipulatives. Informed by two lines of research, the present study tested the hypothesis that the efficacy of the intervention may depend on whether one or multiple external representations are used for instruction.

The results showed a dramatic increase in first graders' mental base-10 decomposition use as a result of the intervention. Children grew in their use of the strategy at the same rates across genders, levels of basic arithmetic fluency, and working memory. Overall, the results showed that relying on multiple representations during instruction appears more beneficial to

strategy use on mental arithmetic, but this benefit may be conditional on how well the children have mastered and abstracted the strategy. Implications to classroom interventions aimed to increase the use of advanced arithmetic strategies are discussed.

Acknowledgements

I would like to begin by extending my deepest gratitude to my committee members who have supported me through every stage of the dissertation process: from brainstorming ideas, to supporting data collection, to providing feedback on writing and statistical analyses, to final comments and reviews. Dr. Albert – thank you for contributing your expertise in and passion for curriculum and instruction to my dissertation. My research idea was born out of a desire to see practical application of cognitive theory to instructional practice, and I am grateful to have your classroom-based perspective represented in the committee. Dr. Vasilyeva – thank you for being my first research mentor in cognitive psychology who has not only passed on her expertise, but also invited and trusted me to co-lead cross-national research projects that have left a deep mark on my development. The research idea behind this dissertation was born in a class you were teaching in my first year of the PhD program, and I am glad I have held on to and developed it through the years. Finally, to my dissertation chair, Dr. Laski – words of gratitude are not sufficient to communicate how much your support and guidance have meant to me through this process, from theoretical framework to experimental design to presentation of findings in written form. You have sown countless hours into this process, and allowed me to reap growth I could not imagine when I was first beginning. I have grown as a researcher, thinker, writer and storyteller under your mentorship, and I plan to take these skills with me throughout the personal and professional steps I take from here on.

Next, I would like to thank the Boston College Lynch School of Education for supporting my research with Dissertation Development Grants that have enabled me to focus on preliminary data collection and analyses for two summers. Not only am I honored to have been chosen as a

recipient of this competitive award – I also know that this dissertation would not have been possible to complete in due time if it were not for support from the Lynch School.

I am also immensely grateful to Karina Halloran, Katie Benjamin, and Sarah Neece for their help with data collection for this study. You put in hours upon hours of work into this project, and I could not have asked for more diligent, trustworthy, and enthusiastic assistants, who also became friends. A special thanks to Karina for weathering the snowiest Boston winter on record as the primary driver for the research project – I literally could not have done this without you.

Thanks are due to the principals and learning specialists that have opened up their schools' doors to make this research possible, and were so welcoming and supportive throughout the process. I do not take for granted the patience and positive attitude of the teachers who have allowed me to interrupt class in order to meet with their students over the duration of the project. And of course, this study would not have taken place if not for the first graders who participated (and brought a lot of fun and energy to the project) and their parents who have graciously allowed it.

I am grateful to the ladies in my PhD cohort at BC who have supported me through the dissertation process and beyond: Melissa Collins, Jacky Simms, and Miriam Heyman. Thank you for sharing notes and tips, being sounding boards when I have needed it, and in general being such a supportive group! We have gone through many milestones together, and I could not have wished for a more gracious, intelligent, and all-around lovely group of girls to have spent the last five years with. Also thank you to my supervisors and colleagues at Bridge International Academies for being understanding and allowing me the flexibility to work on my dissertation through the months of my employment both in Boston and Nairobi. The chance you have given

me to impact children's lives in Kenya has helped me keep my eyes on the deeper meaning of my research.

Where would I be if not for my family? I thank my parents, Vadim and Svetlana Ermakov, for blessing me with a solid foundation that has inspired me to pursue supporting children's education through my research and field work. I thank my second set of "parents", Ted and Cathy Paul, for being some of my biggest supporters, and for bearing with me working on my dissertation through countless family vacations and holidays. Most importantly, I would like to thank my husband, Tyson, for walking with me through this journey as only a small representation of the journey of life. You have stretched me, given me advice, encouraged me to work more when I needed it and rest more when I needed it. You have more wisdom and kindness than anyone I know, and I am so honored to be your wife. Thank you for helping me reach for the stars!

Above all, I would like to use this opportunity to praise to my God for His never-failing grace and mercy. May He always use my work for His glory... "You make me lie down in green pastures, You lead me beside still waters. My cup overflows..." (Psalm 23)

Table of Contents

	Page
LIST OF TABLES.....	vi
LIST OF FIGURES	vii
CHAPTER 1: INTRODUCTION.....	1
Significance	1
Purpose.....	2
CHAPTER 2: LITERATURE REVIEW.....	3
Addition Strategies in First Grade.....	3
Trends in use of decomposition.....	4
Factors related to use of decomposition.	6
Basic arithmetic fluency.	6
Problem characteristics.....	6
Gender.....	7
Instructional emphases.....	7
Using External Representations for Instruction: Two Perspectives.....	9
Argument in favor of using one external representation for learning.	9
Argument in favor of using multiple external representations for learning.	11
The Present Study.....	13
CHAPTER 3: METHOD.....	21
Participants.....	21
Design.....	21
Pretest/Posttest Measures	22
Addition Tasks.....	22
Addition strategy coding.....	23
Working memory tasks.....	25
Number sets task.....	26
Training Sessions	26
CHAPTER 4: RESULTS.....	29
Overview of Analyses	29
Initial Use of Base-10 Decomposition	29

Effect of Instruction on Use of Base-10 Decomposition	31
Effect of Experimental Condition	36
Ease of abstraction	36
Depth of abstraction.....	37
Summary of Analyses and Key Findings	39
CHAPTER 5: DISCUSSION, CONCLUSION AND IMPLICATIONS	42
Use of Base-10 Decomposition in First Grade.....	43
Effect of Instruction on Use of Base-10 Decomposition	45
Single Representation or Multiple Representations: What Is Better for Learning?	48
Ease of abstraction	48
Depth of abstraction.....	49
Effective Use of External Representations: Factors Other Than Number	51
Limitations and Future Research.....	52
Implications and Conclusions	54
REFERENCES	57
APPENDIX A.....	67
APPENDIX B	68

List of Tables

	Page	
Table 1	Percentage of problems solved using decomposition in research studies in the U.S. from 1987 to 2015.....	5
Table 2	Research predictions based on ease and depth of abstraction hypotheses.....	19
Table 3	Intervention structure: Number and type of practice problems by session.....	28
Table 4	Addition strategies used at pretest: Percentage of all problems.....	30
Table 5	Base-10 frequency and accuracy scores for children following different learning paths.....	38

List of Figures

	Page
Figure 1 Manipulatives used for training.....	14
Figure 2 Pretest to posttest change in percent of base-10 decomposition of all problems and by problem type.....	32
Figure 3 Decomposition frequency by strategy type and gender.....	33
Figure 4 Posttest base-10 decomposition users by pretest user type.....	34
Figure 5 Pretest to posttest change in percent of base-10 decomposition of all problems by learner type.....	35

Chapter 1: Introduction

Mathematics mastery is important to both individuals' professional success and a nation's progress and well-being (National Mathematics Advisory Panel, 2008). Early mathematics performance, in particular, is crucial for later achievement because it lays the foundation for future math learning (e.g., Geary, 2011). An especially important emphasis of early mathematics instruction is mastery of the base-10 decomposition strategy (National Governors Association, 2010).

The base-10 strategy involves decomposing one of the addends to reach the nearest ten, and then adding the remainder (e.g., $15 + 7 = 15 + 5 + 2 = 20 + 2 = 22$). It is a central goal of early mathematics instruction that features prominently in the Common Core Standards for Mathematics; students are expected to use base-10 properties to add single- and double-digit numbers with sums up to 100 by the end of first grade (Geary, 2006; Miura, 1987; National Council of Teachers of Mathematics, 2000; National Research Council, 2001).

Significance

There is good reason to focus on this strategy; the frequency with which children use decomposition strategies in first grade is related to the use of more efficient arithmetic strategies up to fifth grade, as well as greater accuracy on word problems and arithmetic involving fractions up to seventh grade (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Geary, Hoard, Nugent, & Bailey, 2013). Yet, despite the evidence that connects this strategy to greater math fluency through middle school, we know little about its prevalence among first graders or factors contributing to its use.

Further, it is imperative that research examine the efficacy of early instructional interventions aimed at increasing children's use of base-10 decomposition. A common approach to classroom instruction of this strategy specifically and early mathematics in general is using

external representations (ERs) (e.g., Correa, Perry, Sims, Miller, & Fang, 2008; Puchner, Taylor, O'Donnell, & Fick, 2008). Manipulatives, such as Dienes blocks and unifix cubes, are often used as external representations in the instruction of this strategy. Research about the value of ERs for student learning, however, is inconsistent: some studies find that they promote learning while others find they hinder it (e.g., Ball, 1992; Boulton-Lewis & Tait, 1994; Kaminski, Sloutsky, & Heckler, 2006; McNeil & Jarvin, 2007; Resnick & Omanson, 1987; Sowell, 1989).

Purpose

Though base-10 decomposition is predictive of long-term mathematics achievement and is emphasized in curricular standards, the effect of child and problem characteristics on students' choice of this strategy is underresearched. Further, there is little experimental research examining the influence of targeted educational interventions, particularly those using manipulatives, on growth in students' use of base-10 decomposition. In light of these gaps in research, the present study had three primary goals. The first goal was to explore the current rates of prevalence of base-10 decomposition in first grade, as well as factors related to its use. The second goal was to test the effectiveness of a brief instructional intervention in promoting the use of base-10 decomposition among first graders. The third goal was to test the hypothesis that the efficacy of base-10 decomposition instruction using external representations may depend on whether one or multiple ERs are used for instruction.

Chapter 2: Literature Review

The literature review section is divided in three parts. First, it examines research on early addition strategies, particularly base-10 decomposition and factors related to its use. Then, it reviews research in support of two perspectives for effectively using external representations in base-10 decomposition instruction: single versus multiple. Finally, the Present Study section describes the particular goals and predictions of this study.

Addition Strategies in First Grade

Addition is a key goal in early mathematics instruction. During the first years of elementary school, children acquire new addition strategies and become increasingly accurate at solving simple and more complex problems. Generally, by first grade children use three main kinds of strategies to solve addition problems: retrieval, counting, and decomposition (Geary, Fan, & Bow-Thomas, 1992; Shrager & Siegler, 1998). *Retrieval* involves recalling a number fact from memory. This strategy is typically used only on single-digit problems (Geary et al., 2004). *Counting* involves counting from 1 the value of both addends or up from one addend the value of the second (e.g., $4 + 3 = "5, 6, 7"$). *Decomposition* involves transforming the original problem into two or more simpler problems, often using either a previously memorized number fact (e.g., $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$) or the base-10 properties of the number system. Base-10 decomposition, which is the focus of the present study, involves decomposing one of the addends into a part required to reach the nearest decade and the remainder (e.g., $25 + 7 = 25 + 5 + 2 = 30 + 2 = 32$).

Decomposition is considered to be a more efficient and advanced strategy than counting (e.g., Geary, 2006; Shrager & Siegler, 1998). Consider the problem $6+9$. Solving it using base-10 decomposition only involves three steps – decomposing the 9 to $4 + 5$, adding $6 + 4 = 10$, then

$10 + 5 = 15$. In contrast, solving it using counting-on involves 9 steps – enumerating by one 9 times, “7, 8, 9, 10, 11, 12, 13, 14, 15”. Indeed, Siegler (1987) found that first graders solved single-digit addition faster when using decomposition than counting. Further, children who use decomposition more frequently to solve addition problems also tend to be more accurate than their peers who rely more frequently on counting strategies (Fennema et al., 1998; Geary et al., 2004). In fact, the frequency with which children use decomposition mediates gender (Carr, Steiner, Kyser, & Biddlecomb, 2008; Shen, Vasilyeva, & Laski, 2016), income-group (Laski, Schiffman, Vasilyeva, & Ermakova, 2016), and cross-national differences in arithmetic accuracy (Geary, Bow-Thomas, Liu, & Siegler, 1996; Vasilyeva, Laski, & Shen, 2015).

Trends in use of decomposition. The use of decomposition in first grade is positively predictive of arithmetic performance through middle school (e.g., Geary et al., 2004; Geary et al., 2013); yet, our understanding of its current prevalence among first graders and factors that promote its use is limited. Prior research has focused primarily on first graders’ counting strategies. When researchers have examined decomposition, it generally has been combined with other “backup” or “invented” strategies (Fennema et al., 1998; Geary et al., 1996). Only a few studies have included decomposition as a separate category, and those have mostly focused on single-digit addition.

An analysis of studies of arithmetic strategies conducted over the past 30 years suggests there has been an increase in the use of decomposition. As shown in Table 1, the prevalence of U.S. first graders’ use of decomposition on single-digit problems remained relatively constant between the late 1980’s to mid-2000’s, but recently Vasilyeva and colleagues (2015) reported a sharp increase in children’s use of decomposition, with children using it nearly two to three times as often. Similarly, although there are fewer studies to compare, there seems to have been an

increase in the use of decomposition on mixed-digit addition problems over the last eight to ten years (Geary et al., 2004; Vasilyeva et al., 2015). Geary and colleagues found that first graders used decomposition on eight percent of mixed-digit problems in 2004, and six percent in three years later (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), while Vasilyeva and colleagues found they used decomposition on 42% of problems in 2015. One potential explanation for this trend is the recent emphasis on decomposition in curriculum standards, particularly the Common Core (National Governors Association, 2010) according to which students are expected to use base-10 properties to add single- and double-digit numbers with sums up to 100 by the end of first grade, and subsequent changes to mathematics instruction (e.g., Rampey, Dion, & Donahue, 2009).

Table 1

Percentage of problems solved using decomposition in research studies in the U.S. from 1987 to 2015

Reference	Single-digit addition	Mixed-digit addition
Siegler (1987)	9%	
Geary et al. (1992)	7%	
Geary et al. (1996)	4%	
Geary et al. (2004)	7%	8%
Canobi (2004)	11%	
Geary et al. (2007)	6%	6%
Vasilyeva et al. (2015)	20%	42%

Note: Geary et al. (1996) statistic reflects the highest decomposition frequency in the course of first grade. Geary et al. (2004) statistics were inferred from graphs in the article. The numbers reported for Geary et al. (2004) and Geary et al. (2007) are averaged across all children in the studies.

Factors related to use of decomposition. Various factors might influence the use of decomposition, and base-10 decomposition, in particular. An analysis of the literature points to four likely factors: fluency with number facts, problem characteristics, gender, and instructional emphases.

Basic arithmetic fluency. Fluency – the ability to quickly and accurately retrieve the sums to basic arithmetic problems (e.g., those involving two single-digit addends) – generally contributes to efficient arithmetic strategy choice and accuracy of advanced strategy execution (e.g., Ashcraft, Donley, Halas, & Vakali, 1992; Campbell & Xue, 2001; Geary & Burlingham-Dubree, 1989; Imbo, Vandierendonck, & Rosseel, 2007; LeFevre, Sadesky, & Bisanz, 1996). It may be particularly important in the use of base-10 decomposition because easy retrieval of number facts would allow greater working memory resources to be dedicated to the execution and mental tracking of the steps involved (Ashcraft, 1995; Ashcraft et al., 1992). For instance, to execute the base-10 decomposition strategy for the problem $25 + 7$, one needs to retrieve $5 + 5 = 10$ (to add 5 to the double-digit number to reach the nearest decade) and $5 + 2 = 7$ (to know 2 left from 7 remains to be added to 30). Thus, without knowledge of the basic number facts, the base-10 strategy is likely to be cumbersome and inefficient to execute. Indeed, Cheng (2012) showed that children who were familiar with basic number facts were more likely to use base-10 decomposition on single-digit addition with sums over 10.

Problem characteristics. Problem characteristics are widely accepted to be involved in strategy choice (Siegler, 1987). The magnitude of the sum and the type of problem (e.g., single-digit vs. double-digit) has been found to be related to use of decomposition. For example, Geary and colleagues (1996) found that Chinese kindergartners used decomposition more frequently on problems with sums greater than 10 than on problems with sums up to 10. Other studies found

that kindergartners and first graders used decomposition more frequently on double-digit problems than either mixed- or single-digit problems (Laski, Ermakova, & Vasilyeva, 2014; Vasilyeva et al., 2015).

The unique demands of the base-10 decomposition strategy suggest another characteristic of problems might influence its use on mixed-digit problems: the magnitude of the single-digit addend. As stated above, fluency with number facts up to 10 likely contributes to the ease of executing the base-10 strategy accurately. At the same time, children acquire fluency with smaller numbers earlier than larger ones (Sielger & Robinson, 1982). Thus, it seems likely that children may choose to use the base-10 decomposition strategy more frequently on problems with smaller single-digit addends than on those with larger single-digit addends (e.g., $28 + 3$ vs. $28 + 9$) because of increased likelihood of success.

Gender. Another factor likely related to the use of base-10 decomposition is gender. Fennema and colleagues (1998) found that boys used counting as frequently as girls in first grade, but by second grade preferred invented strategies including variations of decomposition, while girls persisted in relying on counting. Similarly, Carr and colleagues found that first grade boys relied more frequently on mental strategies to solve mixed-digit addition problems, while girls tended to rely on overt strategies, such as counting using manipulatives or fingers (Carr & Davis, 2001; Carr & Jessup, 1997). The reason for this gender difference is not apparent, but the consistent findings suggest it would generalize to the use of base-10 decomposition.

Instructional emphases. As would be expected, individual differences in the use of decomposition seem to be related to differences in instruction. Studies conducted before the Common Core placed greater emphasis on decomposition in US instruction indicated that East Asian students used decomposition more frequently than their American peers (e.g., Geary et al.,

1996), possibly due to curricular differences between the countries (e.g., Fuson & Kwon, 1992); whereas, a recent study found no cross-national difference in students' performance (Vasilyeva et al., 2015). Further, differences in instructional emphases seem to attenuate gender differences in strategy choice. In Taiwan, where instruction highly emphasizes decomposition, there are no gender differences in the use of decomposition, while in the US and Russia where the strategy is less emphasized gender differences are present (Shen et al., 2016).

My literature review revealed two studies that tested instructional interventions specifically designed to promote the use of decomposition. Cheng (2012) found that children who practiced composing single-digit numbers on a frame with ten slots before receiving instruction about base-10 decomposition for single-digit problems with sums greater than 10 used the strategy on the majority of posttest trials. In contrast, children in the control group who had instead practiced classification of objects based on their physical properties continued using counting on most problems even after receiving the same instruction about base-10 decomposition.

Instruction of base-10 decomposition often includes manipulatives. Hiebert and Wearne (1992) compared traditional instruction to an approach designed specifically to teach place value, and double-digit addition and subtraction without regrouping using decomposition. The key difference between the approaches was the use of manipulatives; the experimental approach used fewer external representations, spent more time using each one, and allowed children hands-on practice, rather than only demonstration, in contrast to the traditional approach. At the end of the three-month training period, first graders who had been taught with the experimental approach were more accurate on arithmetic problems than those who had received traditional instruction. This study's findings suggest that the number of manipulatives used influenced learning of

decomposition, but the lack of experimental controls prohibit strong conclusions.

Using External Representations for Instruction: Two Perspectives

External representations (ERs) are commonly used as an aide to student learning in mathematics classrooms, particularly in early elementary grades (e.g., Correa et al., 2008; Puchner et al., 2008). Research about the value of ERs for student learning, however, is inconsistent: some studies find that they promote learning while others find they hinder it (e.g., Ball, 1992; Boulton-Lewis & Tait, 1994; Kaminski et al., 2006; McNeil & Jarvin, 2007; Resnick & Omanson, 1987; Sowell, 1989). One explanation for the inconsistent findings may be differences in the number of external representations used to teach a given concept.

Argument in favor of using one external representation for learning: Ease of abstraction. Some evidence suggests that consistent use of a single representation over time might lead to easier abstraction of conceptual information than reliance on multiple representations. Support for this position is rooted in the principles of cognitive load theory (Sweller, 1988), which suggests that one's cognitive processing capacity is limited and should be minimally taxed during educational activities for effective learning to occur. Boulton-Lewis (1998) suggested that the process of extracting abstract mathematical concepts from ERs poses a high processing load on children's limited cognitive resources. It follows from Sweller's theory that having to attend to a range of features when processing multiple representations may present students with an even higher cognitive load than attending to the features of a single representation, and may prove to be too challenging (Mayer & Moreno, 2003). In support of this view, Boulton-Lewis (1998) found adverse effects of teaching place value concepts using two ERs: bundles of ten sticks and single-unit counters. When elementary school students were encouraged to use both ERs to represent double-digit numbers, they demonstrated a decrease in

understanding of place value, and many of them preferred to represent numbers using the less efficient unit counters, even if they had preferred more efficient bundles of ten sticks before instruction with multiple ERs. Researchers have inferred from this and similar findings that limiting the number of ERs used during instruction may lead to a decrease in cognitive load and promote students' ability to abstract the mathematical concepts instantiated in external representations with greater ease (Boulton-Lewis, 1998; Uttal, 2003).

Further evidence for the potential benefit of consistent use of one ER can be inferred from research on symbolic reasoning – specifically, the dual representation hypothesis (e.g., DeLoache, de Mendoza, & Anderson, 1999). Researchers have suggested that simultaneously treating ERs both as entities in themselves and as demonstrations of mathematical concepts (“dual representation”) may be a difficult task for children, who often lose sight of what the objects are intended to represent and focus instead on their physical features (DeLoache et al., 1999; Uttal, Scudder, & DeLoache, 1997). For example, in studies involving spatial models (representations) of actual rooms, young children have difficulty finding an object based on the location of its symbol in a model room. This difficulty mapping from a representation to an actual location has been interpreted by researchers as being due to children's challenge with treating a symbol (model room) as both a concrete entity and an abstract representation of another entity. While older (5-7 year old) children are able to perform this task successfully, the challenges associated with dual representation likely still apply in more sophisticated instructional contexts.

Indeed, there is evidence that the cognitive demands of having to mentally represent ERs as both symbols for math concepts and objects in and of themselves may impede children's mathematics learning. For example, using highly realistic (perceptually rich) dollar bills to

represent money during mathematics instruction in fourth through sixth grades was related to lower accuracy on word problems than using (perceptually bland) pieces of paper or using no ER at all (McNeil, Uttal, Jarvin, & Sternberg, 2009). If the translation demands associated with ERs interfere with learning, it stands to reason that the effect may be further exacerbated when the demands are increased through use of multiple representations. Instead, consistent practice with a single ER might provide the time for the symbol to lose its superficial appeal and, thus, for children to focus on the mapping between the concept and its symbol (Sowell, 1989; Uttal, 2003; Uttal et al., 1997; Uttal, Marzolf, Pierroutsakos, Smith, Troseth, Scudder et al., 1998).

Finally, this view is supported by empirical evidence that indicates that using multiple ERs is detrimental to learning. Studies of story comprehension have shown that both information recall and processing speed were negatively affected when children who were already using one ER (text) were required to form a new mental representation of the same concepts through being presented with an additional ER (a second text) that did not map well onto the existing mental structure (Fernandez, Yoshida, & Stigler, 1992). An experimental intervention with kindergartners also showed greater benefits of learning numerical operations from one structured type of ER (ten frames) than multiple ERs that varied in structure (various spatial arrangements of objects, such as apples, animals, etc.; Chao, Stigler, & Woodward, 2000). In summary, there is evidence that children find abstraction of conceptual information from external representations challenging, and consistently using one ER to represent a concept has been suggested to increase the ease of abstraction.

Argument in favor of using multiple external representations for learning: Depth of abstraction. On the other hand, there is evidence suggesting that using multiple external representations during instruction may facilitate deeper abstraction of the concepts represented

by them. Used to support this view, structure mapping theory (Gentner, 1983) posits that in order to map knowledge gained from external representations onto abstract mental concepts a learner must form an analogy between the concrete and the abstract. In this theoretical framework, simultaneously interacting with multiple representations allows for a more effective formation of the analogy by drawing children's attention to the abstract concept captured by the ERs, instead of their superficial perceptual attributes (Ainsworth, 1999; Ainsworth, 2006).

This is important because children naturally tend to notice perceptual – rather than substantive – features of objects. For example, one study found four-year olds were more likely to categorize an apple as round than as a fruit (Gentner & Namy, 1999). Providing children with multiple examples of the conceptual category (fruit), however, increased the extent to which they focused on the substantive feature (as evidenced by them matching apples with bananas). On the other hand, when the children were presented with only one example of the conceptual category (apples are fruit), they were equally likely to focus on substantive and perceptual features (matching apples with bananas and with balloons). Thus, learning from multiple representations allowed the young participants to highlight the relevant conceptual features and lose their more natural focus on perceptual ones.

This line of research posits that it is through the process of comparison of multiple representations that children (and adults) benefit from their use. In Kotovsky and Gentner's (1996) study with four-year olds – who did not spontaneously focus on abstract features of representations – it was the comparison of surface similarities of multiple ERs that allowed children to most deeply abstract the concepts behind representations. Children in the study who learned to compare size relations of three squares with similar size relations of three circles were better able to later translate size relations of squares to a novel context: color relations of circles.

Even more strikingly, Loewenstein and Gentner (2001) adapted the task used in prior research to support the dual representation hypothesis (e.g., DeLoache et al., 1999), discussed above, and found that when preschoolers were allowed to compare two representations of the same room, they found it easier to locate the hidden object in the actual room. Further support for the benefit of comparison of multiple representations comes from an experimental study in which college students who were prompted to conduct an intensive comparison of two ERs (e.g., pictures of different examples of heat flow) abstracted the science concept behind the representations more deeply than those who were not guided in their comparison or whose prompt led to less intensive comparison (Kurtz, Miao, & Gentner, 2001).

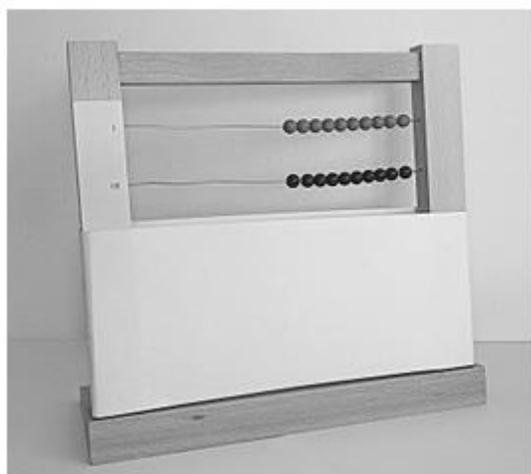
Though the process of comparing multiple representations may take a longer time than learning from one representation, there is empirical evidence that the former is associated with deeper abstraction, as reflected in students' greater ability to generalize the learned material to novel contexts (Bransford & Schwartz, 1999; Gilmore & Green, 1984; Hakel & Helpert, 2005; Schnotz & Kurschner, 2008). One study found that sixth graders who learned fractions concepts from multiple representations outperformed their peers who learned from a single representation both on direct assessments of acquired knowledge and on new types of problems with fractions when prompted to verbalize their reasoning during the solution process (Rau, Aleven, & Rummel, 2015).

The Present Study

The present study examined first graders' addition strategies, and tested the benefits of an intervention using a single ER or multiple ERs for teaching base-10 decomposition. Participants were randomly assigned to one of three conditions: (1) base-10 frames, (2) base-10 tiles, or (3) base-10 frames and tiles. As shown in Figure 1, each representation instantiated the base-10

numeric structure and required children to represent double-digit numbers as tens and ones: the frames involved ten green beads on the top row representing ones and ten blue beads on the second row representing tens, while the base-10 tiles involved ten green tiles in a section on the left representing ones and ten blue tiles in a section on the right representing tens. The materials were selected because of their similarities (e.g., identical color coding) and absence of mathematically irrelevant features, which should theoretically increase children's ability to abstract the mathematical concepts within and across them. A pre-to-posttest design was used to measure the effectiveness of the training in each condition. In addition, a microgenetic component allowed for examination of the rate of learning.

Base-10 Frame



Base-10 Tiles



Figure 1. Manipulatives used for training.

The study had three primary goals. The first goal was to describe the extent to which first graders used base-10 decomposition to solve mixed-digit problems. Based on the trends in the literature, I expected first graders' use of decomposition to be comparable to the results of more recent studies, rather than those of older studies. In addition, because I hypothesized the trend of increased use of decomposition to be related to greater instructional emphasis on base-10

knowledge, I expected base-10 decomposition to be the most prevalent kind of decomposition. Further, based on previous findings of gender differences in relation to the use of counting (e.g., Carr & Davis, 2001; Fennema et al., 1998), I expected to also find gender differences with boys using base-10 decomposition more frequently than girls.

To get a more nuanced depiction of first graders' strategy choice, the present study also examined whether problem type and children's arithmetic fluency were related to the frequency with which they used base-10 decomposition. Because using base-10 decomposition on mixed-digit problems involves knowing how to decompose the single-digit addend to arrive at the value needed to change the double-digit number to the next decade, I predicted children with greater fluency with basic addition facts (e.g., easily retrieving the combinations of 5 as $4+1$ and $3+2$) would be more likely to use the strategy. Thus, children with better knowledge of addition facts in the present study should execute the strategy with greater ease, and thus be more likely to select it over more cumbersome counting strategies. Further, children are typically more fluent on the arithmetic facts for smaller numbers before larger ones (Siegler & Robinson, 1982), in part because of more experience with these numbers and in part because they involve fewer combinations. Thus, I also predicted that base-10 decomposition would be used more frequently on mixed-digit problems with a smaller single-digit addend than those with a larger single-digit addend, and that this aspect of the problem would be more predictive of strategy choice than magnitude of the sum.

The second goal of the study was to explore whether strategy choice was malleable, such that a brief instruction could substantially increase the use of base-10 decomposition. On the one hand, existing cross-national studies and intervention studies suggest that instruction can influence strategy choice (Cheng, 2012; Fuson & Kwon, 1992; Geary et al, 1996; Hiebert &

Wearne, 1992; Siegler & Jenkins, 1989; Vasilyeva et al., 2015). Thus, it was possible that with even brief targeted instruction children might opt to use the strategy more frequently when given a choice, and that the change would occur on a broad range of problems. Further, there may also be spillover to the use of other kinds of decomposition.

On the other hand, it is possible that prolonged instructional experience is necessary to induce change. Change in strategy choice is typically depicted as a gradual process of strategy discovery and strengthening of associations between problems and a strategy's accuracy (Siegler & Shipley, 1995). In fact, most cross-national studies and existing intervention studies are characterized by prolonged instructional intervention. For example, Hiebert and Wearne's (1992) intervention extended over the course of three months. If this is the case, then if any improvement occurred, the breadth of change would be expected to be limited. A brief intervention might promote accurate execution of the strategy when children are directed to use it on no-choice tasks, but not extend to how often the strategy is used on a choice task. The change also might be limited to problems on which the strategy is more easily executed (e.g., those with single-digit addends less than 5). Another possibility is that the change might occur specifically in children who are already primed to adopt the strategy (e.g., Cheng, 2012), such that frequency and accuracy of base-ten execution at posttest may depend on children's basic arithmetic fluency or prior use of the base-10 decomposition strategy.

Of course, the extent to which children actually improve on executing the strategy during training should influence the extent to which they generalize it. Previous studies of strategy choice indicate that children are more likely to choose a strategy they can accurately execute. Thus, children who are better able to execute the strategy with or without the instructional materials – the manipulatives – should be more likely to choose to use the strategy when asked to

solve problems mentally following training than those who never “master” the strategy.

The third goal of the study was to test the hypothesis that the efficacy of using external representations for instruction depends on the number of representations used. My approach to testing this hypothesis involved examining the relative strengths and weaknesses of using either a single or multiple external representations in instruction. While there might be an overall advantage of one approach over the other, it seemed more likely, given the reasonable theoretical arguments in favor of each, that the differences for learning between the two would be more subtle. Thus, I conducted specific analyses to explore the two key aspects debated: ease and depth of abstraction of the mathematics concepts instantiated in the representations.

On the one hand, the literature seems to suggest that abstracting conceptual information from manipulatives is an arduous process for young children (e.g., Ball, 1992; McNeil & Jarvin, 2007; McNeil et al., 2009), and that the demands of the task should be minimized as much as possible in order to afford *ease of abstraction*. For this reason, using a single external representation consistently over time is argued to be better than using multiple external representations. One argument in support of this view is that using only one manipulative allows for a greater number of instances of practicing the strategy with that representation, allowing the child to become proficient in using it and to lose interest in the manipulative as an object in and of itself (e.g., Uttal, 2003). If the sheer number of instances using the manipulative influences learning, then the single representation conditions should promote greater and more rapid learning of how to accurately execute the hands-on base-10 strategy than the multiple representation condition. This increased ability to execute the strategy should, in turn, lead children whose instruction includes only one manipulative to choose to use the strategy more frequently than those who were taught with multiple manipulatives. Another argument is that

using one manipulative consistently over time taxes working memory less than using multiple because there are fewer external (and distracting) features for the child to process (Boulton-Lewis, 1998; McNeil & Jarvin, 2007; Uttal, 2003). If this is the case, then individual differences in working memory should influence the extent to which children benefit from using a single versus multiple representations. One would expect no relation between working memory capacity and children's learning in the single representation condition, but a positive relation in the multiple representation condition.

On the other hand, the literature argues that an important result of learning from external representations is *depth of abstraction*, often measured as the ability to extend the knowledge to novel contexts (e.g., Bransford & Schwartz, 1999; Hakel & Helpem, 2005). According to existing research, using multiple representations may be better than using a single external representation to ensure depth of abstraction. In the current context, ability to execute the strategy using the manipulative does not necessarily require the child to abstract the underlying mathematical concepts; rather, acquiring the procedure as it relates to the manipulative is sufficient. In contrast, choosing to and being able to accurately execute the strategy in the absence of the manipulative, once it is acquired with the manipulatives, suggests that the concept has been abstracted. Additionally, it has been suggested that reliance on a single ER encourages formation of a mental representation that reflects the features of the ER – essentially, visualization of the ER itself, rather than abstraction of the concept – and thus hinders application of the concept to new tasks that require that these features be adjusted (Schnotz & Kurschner, 2007). Learning from multiple ERs, on the other hand, has been shown to lead to more flexible mental representations that do not reflect the physical features of the ERs.

If using multiple representations promotes deeper abstraction of the mathematics

concepts, then children who learn to execute it successfully using the external representation (“master”) in the multiple ER condition should demonstrate higher frequency and accuracy of base-10 decomposition when asked to extend that learning to mental arithmetic, than those who mastered the strategy in the single ER condition. Moreover, executing the strategy on mental tasks without making references to ERs (visualizing the manipulatives on which they were trained) would be further indicative of ability to extend knowledge of the hands-on strategy to mental arithmetic. If, indeed, children in the multiple representation condition are better able to transfer the hands-on strategy to mental arithmetic, I predicted that they would use visualization less than children in the single representation condition. A summary of research predictions is reflected in Table 2.

Table 2

Research Predictions Based on Ease and Depth of Abstraction Hypotheses

Single Representation	Multiple Representations
<i>Ease of abstraction hypothesis:</i>	
<ul style="list-style-type: none"> • More practice with one manipulative will promote accuracy of strategy and do so faster 	<ul style="list-style-type: none"> • Less practice with each manipulative will inhibit accuracy of strategy or take more time
<ul style="list-style-type: none"> • Cognitive load is low: Working memory will not be related to frequency of base-10 choice 	<ul style="list-style-type: none"> • Cognitive load is high: Working memory will be positively related to base-10 choice
<i>Depth of abstraction hypothesis:</i>	

- Less frequent choice of base-10 decomposition on novel (mental) task
 - Less accurate use of base-10 decomposition on novel (mental) task
 - Mental model will be visualization-based
- More frequent choice of base-10 decomposition on novel (mental) task
 - More accurate use of base-10 decomposition on novel (mental) task
 - Mental model will be numerically-based
-

Chapter 3: Method

Participants

The present study included 88 first graders (50 girls and 38 boys; mean age = 6.99 years, $SD = .38$) recruited from five urban parochial schools. First grade was chosen because it is a time when children are acquiring the decomposition strategy and when hands-on materials are frequently used in mathematics instruction (Sherman & Richardson, 1995). The majority of participants were White (62%); the remaining participants were 14% Latino, 10% Black, 8% multiracial, and 6% Asian. Most children came from middle- to high-income educated families (80% of the parents reported an annual income of \$100,000 or more; 92% of participants had at least one parent with a Bachelor's degree or higher).

Twenty children were eliminated from participating in the training sessions based on their strategy use at pretest. These children either (1) accurately used base-10 decomposition on the majority of the trials, or (2) were inaccurate on the majority of the trials (usually, this was due to an invalid strategy, such as guessing). The former group was considered to already be at or close to ceiling on their performance and thus not able to detectably grow in base-10 strategy choice based on the intervention. The latter group was considered to not yet possess the basic number sense required to benefit from the intervention. Seven children who qualified did not complete the intervention due to absences. The selected and eliminated samples did not differ from one another on demographic characteristics: age, $t(37.85) = .16, p = .872$, sex, $\chi^2(1) = 2.43, p = .119$, race, $\chi^2(4) = 4.82, p = .306$, household size, $t(63) = .48, p = .634$, household income, $t(66) = 1.07, p = .291$, and parents' highest level of education, $\chi^2(5) = 6.02, p = .304$.

Design

This study used a pretest–posttest design across seven sessions. In sessions 1 and 7, children met individually with an experimenter to complete pretest and posttest measures. In sessions 2-6, children met with an experimenter in small groups to learn and practice solving addition problems with manipulatives. At the end of sessions 4-6, children were asked to solve 3 problems without any assistance or feedback, which served as a microgenetic measure of learning during training.

Pretest/Posttest Measures

At pretest and posttest, children were presented tasks to measure their addition strategy and accuracy. Further, at pretest only, children also completed measures of working memory, and arithmetic fluency.

Addition Tasks. At pretest, children completed a *Choice Task*, on which they were asked to choose any strategy to mentally solve 15 mixed-digit addition problems. All problems included one double-digit addend and one single-digit addend (e.g. $16 + 7$), and were printed on separate sheets of paper (see list of problems in Appendix A). The experimenter read each problem aloud and gave the children as much time as needed to solve it. Children were not provided with any manipulatives or paper and pencil, but were permitted to use their fingers or count aloud.

At posttest, children completed three addition tasks in a set order: No Choice Manipulatives Task, No Choice Mental Task and Choice Mental Task. In the No Choice tasks, children were instructed to solve the problems using the base-10 strategy: “Remember how we’ve been practicing adding numbers in the ‘base-10 way’? Can you use the ‘base-10 way’ to add these numbers [in your mind] *or* [using the ten-frame/ the ten-tiles]?” The experimenter showed and read each problem aloud one at a time and gave the children as much time as needed

to solve the problem. In the *No Choice Manipulatives Task*, children solved six mixed-digit addition problems using the manipulative(s) used during training. Children who were exposed to more than one manipulative during training solved three of the problems with one manipulative, and three with the other, with the order of manipulatives counterbalanced across participants. In the *No Choice Mental Task*, children were instructed to solve six mixed-digit addition problems using the base-10 decomposition strategy. The experimenter did not provide any manipulatives or paper and pencil. The experimenter recorded participants' ability to use the base-10 strategy correctly on the no-choice tasks. The *Choice Task* was identical to the one used at pretest, except that it only included ten trials.

During the training period, children independently solved two (Session 3) to four (Sessions 4 and 5) problems using manipulatives. In the multiple representation condition, they solved the first half of the randomly ordered problems with a base-10 frame and the second half with base-10 tiles. Experimenters made note of accuracy of children's strategy execution on every problem during independent solution portions of the training sessions. The rate of learning measure recorded the number of training sessions it took for children to reach and maintain high accuracy of base-10 execution, considering posttest *No Choice Manipulatives Task* as "session 6". "High accuracy" was defined as all problems solved correctly during the intervention sessions and at least 5 of the 6 problems using manipulatives solved correctly at posttest.

Addition strategy coding. During the Choice Task experimenters observed children's behaviors on each problem to determine strategy use. Any overt signs of strategy use (e.g., if the child counted out loud, the experimenter recorded a counting strategy, if the child verbalized the intermediate addition steps for decomposition, the experimenter noted decomposition) were recorded. When there were no overt behaviors, the experimenter asked the participant how he or

she “figured it out” after an answer was provided. This type of combination of experimenter observations and child self-report has been validated and used for strategy classification in prior research (Rittle-Johnson & Siegler, 1999; Siegler, 1987).

Children's strategies on each problem were coded as one of four categories: counting, base-10 decomposition, another type of decomposition, and an “other” code. The *counting* strategy was used when a child counted up starting from 1 (count all strategy) or from one of the addends (count from larger addend or count from smaller addend). For example, for $18 + 4$, “count from larger addend” strategy could involve the child saying out loud, “19, 20, 21, 22.” The *base-10 decomposition* strategy was used when a child added to the decade following the larger addend, and then added the remainder (e.g., for $18 + 4$, adding $18 + 2$ to get to 20, and then adding 2 more). The *other type of decomposition* code included trials on which a child separated out tens and ones, and then added the ones in a manner other than base-10 (e.g., for $18 + 4$, decomposing 18 into 10 and 8, counting up 4 from 8, and then adding the resulting 12 to the separated 10). Because it is generally accepted that retrieval applies to stored number facts that typically include single-digit numbers (e.g., Geary et al., 2004) and all problems in this study involved double-digit numbers, a retrieval code was not used. If a child reported that he/she “just knew” and no overt behavioral cues were present, guessed, or described a strategy that did not fit one of the other codes the trial was coded as “*other*.” The experimenter also indicated whether execution of the strategy involved fingers, verbal (guiding oneself or counting out loud, including whisper), a combination of fingers and verbal, or mental (no observable behavior). All trials on which a strategy was not readily apparent were discussed by both experimenters until agreement on the strategy code was achieved.

During the No Choice Mental Task, when children were instructed to only use the base-10 decomposition strategy, the child's explanation of the strategy was coded as either purely numerical or reflective of visualizing the ER/s. For example, to solve $25 + 7$, a numerical explanation only referenced the addends (e.g., "I added 5 to get to 30, and then added 2 more."), while a visualization-based explanation referenced the manipulative as well (e.g., "I imagined adding 5 green tiles, then trading all the green ones for a blue ten, and then adding 2 more green ones.").

Working memory tasks. At pretest, working memory was measured using two tasks from the Working Memory Test Battery for Children (Pickering & Gathercole, 2001): backward digit recall, and backward block recall. Both tasks have been used as predictors of mathematics performance in elementary-school aged children (e.g., Meyer, Salimpoor, Wu, Geary, & Menon, 2010).

The *Digit Recall task* measures verbal working memory (Wechsler, 2003). Child repeat a sequence of orally-presented numbers in backward order. Sequences increase in length from 2 digits up to 9 until the child is no longer able to correctly repeat a sequence of a particular length on two consecutive trials. Test-retest reliability for children ages 6–16 is high ($r = 0.83$; Williams, Weiss, & Rolfhus, 2003).

The *Corsi Blocks task* measures visuospatial working memory. Child tapped a visually-presented series of blocks, arranged in a scattered array in backward order. Sequence lengths increase from 2 blocks to a maximum of 9 blocks until the child is unable to correctly tap a sequence of a particular length on two consecutive trials. Test-retest reliability for older children, ages 11–16, is moderately high ($r = 0.70$ – 0.79 ; Orsini, 1994), and the task has been used successfully with preschool children in prior research (Bull, Espy, & Wiebe, 2008; Orsini et al.,

1987). On both tasks, children's scores were calculated as the total number of sequences correctly recalled and standardized.

Number sets task. Participants' fluency with basic arithmetic facts was assessed using a modified version of the Number Sets Task (Geary, Bailey, & Hoard, 2009). In this task, children are asked to identify sets of objects and numerals that total a target number (either 5 or 9). In this study, children completed the symbolic items involving sets of two quantities represented with Arabic numerals, and the mixed (symbolic/ non-symbolic) items, involving sets with one quantity represented with small shapes, such as dots or stars, and the other represented with Arabic numerals.

Children were given 60 seconds per page for target number 5 and 90 seconds per page for number 9. At the start of each page, the experimenter instructed children to work quickly to circle as many sets as possible that equaled the target number. Participants' scores were standardized using a sensitivity score, d' -prime (Geary et al., 2009; MacMillan, 2002): $d' = z$ scores hits – z scores false alarms.

Training Sessions

Participants received training in one of three experimental conditions: (1) single representation--base-10 frame, (2) single representation--base-10 tiles, or (3) multiple representations--base-10 frames and base-10 tiles. To be certain that any difference found between the single and multiple representation condition was not an artifact of the particular single representation used, I included two single representation conditions. Children were assigned to these conditions using stratified randomization by working memory score, to ensure that cognitive capacity levels of the participants were comparable across conditions. The structure of the training sessions across the three conditions was identical. The only difference

was that in the multiple representation condition, the participants practiced with one manipulative on half the problems, and with the other on the remaining half.

During training, groups of three children (or, when not possible, two) met with an experimenter for five 20-minute sessions over the course of two to three weeks. Child-to-child interactions were limited as much as possible to ensure consistency of intervention across conditions and groups. During the sessions, children listened to the experimenter's explanations then individually practiced using the manipulative(s) with assistance from the experimenter, as necessary.

The sessions were designed to provide scaffolded learning, with decreasing amount of experimenter guidance and increasing amount of independent practice with time. In the first session, the experimenter demonstrated how to count by ones and by tens using the manipulatives. The experimenter then illustrated equivalence of ten ones and one ten, provided two examples of how to represent double-digit numbers as tens and ones, then asked the children to represent two double-digit numbers with the manipulatives and provided feedback, if necessary. Finally, the experimenter modeled execution of base-10 decomposition using the manipulative(s) on two problems, after which the children solved two additional problems following experimenter directions step by step for executing the strategy. Appendix B presents a sample script for Session 1. By session three, experimenter demonstrations were phased out, and independent practice without feedback was introduced.

The problems practiced in each session included single-digit addends ranging from 2 to 9 and double-digit addends ranging from 14 to 88. The independent practice problems at the end of sessions 3-5 served as the microgenetic measure of improvement in accurately executing the base-10 decomposition strategy during training. Thus, the four independent practice problems

were the same across sessions 4 and 5, and included the two problems from the independent portion of session 3. See Table 3 for an overview of the structure of the intervention sessions, including the type and number of practice problems used at each session as part of the scaffolding process.

Table 3

Intervention Structure: Number and Type of Practice Problems by Session

	Session 1	Session 2	Session 3	Session 4	Session 5
Demonstration	17 + 5	9 + 24			
	6 + 38	45 + 7			
Guided practice	9 + 25	7 + 16	18 + 7	8 + 36*	35 + 7*
	47 + 8	5 + 39	4 + 47		
Practice with feedback		34 + 8	9 + 26	9 + 14	47 + 6
		6 + 46	35 + 8	47 + 4	9 + 34
		4 + 27	7 + 49	7 + 39	16 + 8
		18 + 8	17 + 6	23 + 8	5 + 28
Independent practice			29 + 4	6 + 38	6 + 38
			5 + 36	27 + 5	27 + 5
				29 + 4	29 + 4
				5 + 36	5 + 36

Note: Problems were presented in a random order within each portion of the sessions. *In the multiple representation condition, one of the manipulatives was used during guided practice in Session 4 and the other in Session 5, counterbalanced across intervention groups. Both manipulatives were used during practice with feedback and independent practice.

Chapter 4: Results

Overview of Analyses

The results are organized in three main sections, paralleling the goals of the study. The first section focuses on addition performance before training – strategy frequencies overall and by gender, relation between base-10 decomposition and problem characteristics, and between base-10 decomposition and accuracy. The second section explores effect of addition strategy training on mental base-10 decomposition preference overall, as well as by gender, basic arithmetic fluency, and considering problem characteristics. Relation between children’s ability to execute base-10 decomposition accurately when asked and their preference for the strategy is also examined. The third section discusses the effect of experimental condition on participants’ ease and depth of strategy abstraction. Specifically, analyses examining condition effects on children’s rate of learning and differential role of working memory on posttest mental strategy preference are used to test the ease of abstraction hypothesis. In order to test the depth of abstraction hypothesis, I compare those children in the two conditions who demonstrated mastery of base-10 decomposition using manipulatives at posttest on their preference for and accuracy of mental strategy execution. I also explore condition effects on frequency with which participants reported visualizing the manipulatives during mental base-10 decomposition execution.

Initial Use of Base-10 Decomposition

To describe the distribution of strategies and prevalence of decomposition among current first graders, I examined the strategy choice of all children tested at pretest, including those who were excluded from the intervention. As shown in Table 4, while counting was the most prevalent strategy, decomposition was used on 19% of trials, twice as often as reported by Geary

and colleagues in 2004. As expected, base-10 decomposition was substantially more prevalent than other types of decomposition, used on 13% vs. 6% of problems respectively, $t(87) = 2.34$, $p = .022$, $d = 0.31$. Also as expected, girls used base-10 decomposition less frequently than boys, $t(86) = 2.13$, $p = .050$, $d = 0.45$, and counting more frequently than boys, $t(70.36) = 2.39$, $p = .019$, $d = 0.52$.

Table 4

Addition strategies used at pretest: Percentage of all problems (standard deviations presented in parentheses)

	All children	Boys	Girls
Counting	68 (39)	57 (42)	77 (35)*
Decomposition	19 (34)	27 (40)	12 (28)
Base-10	13 (29)	20 (35)	7 (21)*
Other	06 (13)	6 (12)	5 (13)
Other strategies	13 (26)	16 (29)	11 (23)

* $p \leq .05$, gender-based difference

Next, I tested the hypothesis that strategy choice may depend in part on problem characteristics. The size of the problem, defined as the magnitude of sum, was correlated with overall accuracy, $r = .60$, $p = .009$, but not with the frequency with which base-10 decomposition was used. In contrast, the magnitude of the single-digit addend – up to or greater than 5 – was not related to overall accuracy, but was related to the frequency with which base-10 decomposition was used. A paired-samples t -test indicated first graders chose base-10 decomposition more frequently on problems with a single-digit addend up to 5 ($M = 15\%$ of

problems, $SD = 33$) than on problems with a single-digit addend over 5 ($M = 12\%$ of problems, $SD = 27$), $t(87) = 2.85$, $p = .005$, $d = 0.10$.

Further, I used linear regression to examine the hypothesis that basic arithmetic fluency is related to rate of base-10 strategy use. My prediction was confirmed: d' score on the Number Sets Task was a significant predictor of base-10 strategy frequency, $F(1, 86) = 20.92$, $p < .001$, $R^2 = .20$. Children who were more familiar with basic number facts indeed chose to use base-10 decomposition more frequently, regardless of the size of the single-digit addend.

Effect of Instruction on Use of Base-10 Decomposition

Next, I examined whether children's use of base-10 decomposition changed after the brief instruction. These analyses were conducted with only the sample selected for participation, excluding the 20 children who were either at ceiling levels of accurate base-10 decomposition use or floor levels of accuracy at pretest.

A repeated-measures MANOVA indicated a substantial increase in the percentage of problems on which participants used base-10 decomposition for mental calculation on the Choice task, from 4% ($SD = 12$) of problems on pretest to 28% ($SD = 38$) on posttest, $F(1, 60) = 29.05$, $p < .001$, $\eta^2 = .33$. As shown in Figure 2, the same pattern was apparent when for the percentage of problems on which participants used base-10 for each problem type: 7% ($SD = 19$) to 28% ($SD = 42$) on problems with single-digit addends less than 5, $F(1,60) = 21.58$, $p < .001$, $\eta^2 = .26$, and 3% ($SD = 9$) to 27% ($SD = 37$) on problems with single-digit addends greater than 5, $F(1,60) = 29.76$, $p < .001$, $\eta^2 = .33$. A follow-up paired-samples t -test showed that while the selected sample used base-10 decomposition more on problems with single-digit addends up to 5 at pretest, at posttest first graders used the strategy at an equal rate on problems with single-digit addends less than 5 and single-digit addends greater than 5. Further, when first graders opted to

use the strategy on the Choice task at posttest, they executed it as accurately as when they were required to execute it on the No Choice task ($M = .83$, $SD = .22$, and $M = .82$, $SD = .30$, for No Choice and Choice task base-10 decomposition accuracy rates, respectively, $t(24) = .24$, $p = .815$, $d = .04$).

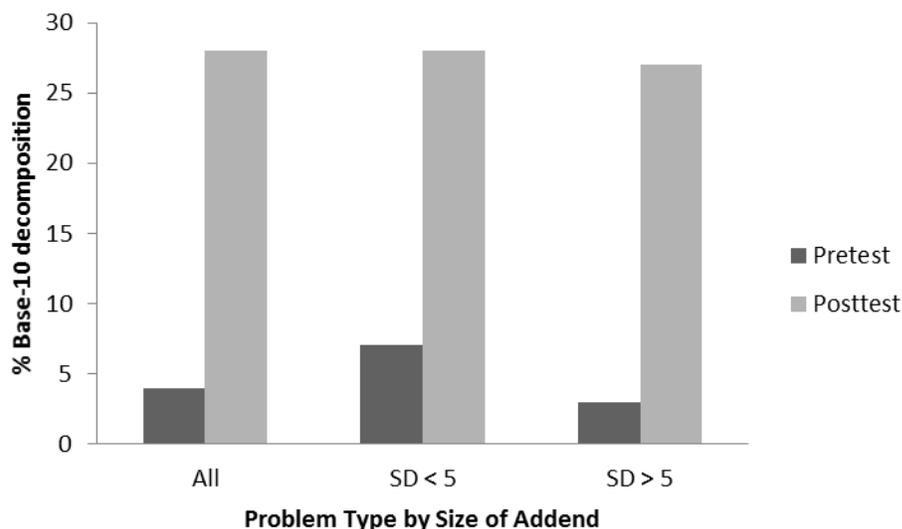


Figure 2. Pretest to posttest change in percent of base-10 decomposition of all problems, and by problem type.

This effect was limited to improvement on the base-10 strategy: a paired-samples t -test indicated no pretest to posttest change in the percentage of problems solved using other types of decomposition. Further, as shown in Figure 3, improvement was also comparable for both genders: a repeated measures ANOVA on the percentage of trials solved with base-10 decomposition found a main effect of time, $F(1,59) = 29.06$, $p < .001$, $\eta^2 = .33$, but no time by gender interaction.

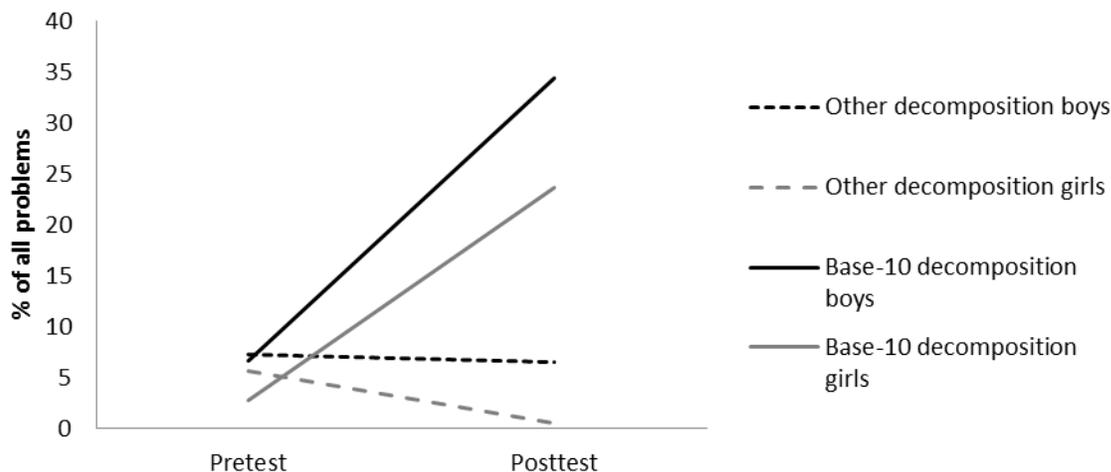


Figure 3. Decomposition frequency by strategy type and gender.

This improvement was not limited to participants who possessed prerequisite knowledge. Individual differences in arithmetic fluency were not related to learning: first graders' d' score on the Number Sets Task was not correlated to the percentage of problems on which they used base-10 decomposition on the Choice Task, or their accuracy on the problems on which base-10 decomposition was used on posttest, controlling for pretest base-10 decomposition frequency and accuracy, respectively. However, children who already chose to use the strategy on some problems prior to instruction were more likely to choose it after instruction. A regression analysis indicated that pretest frequency of base-10 decomposition accounted for 20% of the variance in the percentage of trials on which the strategy was used at posttest, $F(1, 60) = 14.68$, $p < .001$, $R^2 = .20$. Nonetheless, improvement occurred for all children – even those who had not used base-10 decomposition at pretest. As shown in Figure 4, 37% of the participants who had not used the strategy at pretest used it at posttest on at least one trial.

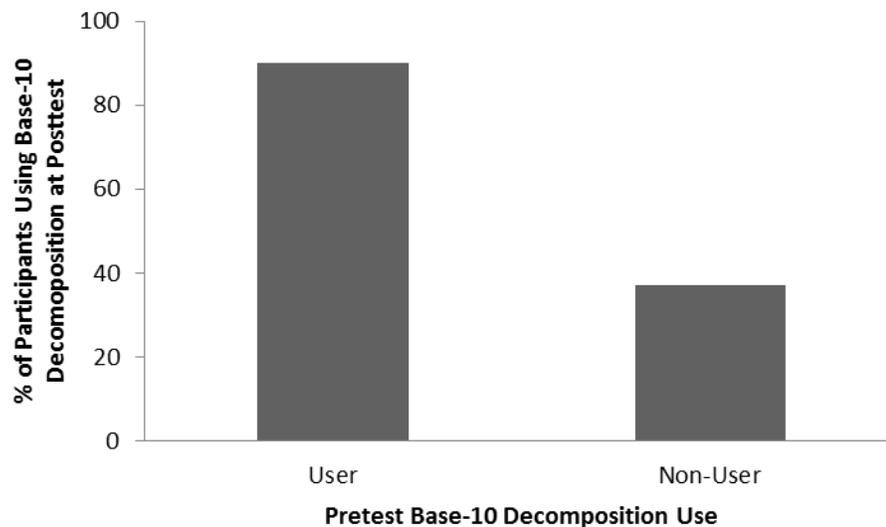


Figure 4. Posttest base-10 decomposition users by pretest user type.

Further, I investigated whether growth in base-10 decomposition frequency was limited to the most successful learners of the hands-on strategy. In order to do this, I compared children who had “mastered” how to execute the strategy using manipulatives and those who had not. Children who used base-10 decomposition accurately on at least 5 out of 6 trials on the Manipulatives task at posttest were coded as “masters” (57% of the sample), while all others were labeled “non-masters.” A repeated-measures ANOVA with percentage of trials solved using base-10 decomposition on pretest and posttest mental Choice tasks as within-subjects variables and learner type (master vs. non-master) as the between subjects variable showed that in addition to a main effect of time – described above – there was a time by learner type interaction, $F(1,59) = 7.41, p = .009, \eta^2 = .11$. A follow-up regression analysis showed that, controlling for frequency of base-10 decomposition at pretest, learner type was predictive of frequency of the strategy on the mental Choice task at posttest, $t(58) = 2.51, p = .015, \beta = .29$. Children who mastered base-10 decomposition using manipulatives as a result of the intervention grew more than children who did not master it. As shown in Figure 5, the gap between masters and non-masters widened from pretest to posttest, with the percent of base-10 decomposition use

growing from 11% ($SD = 25$) to 40% ($SD = 41$) for masters, and 1% ($SD = 3$) to 11% ($SD = 25$) for non-masters. Nevertheless, even children who did not reach the specified level of “mastery” on the Manipulatives task grew in frequency of base-10 decomposition on the mental Choice task as a result of the intervention, $t(25) = 2.20$, $p = .037$, $d = .59$.

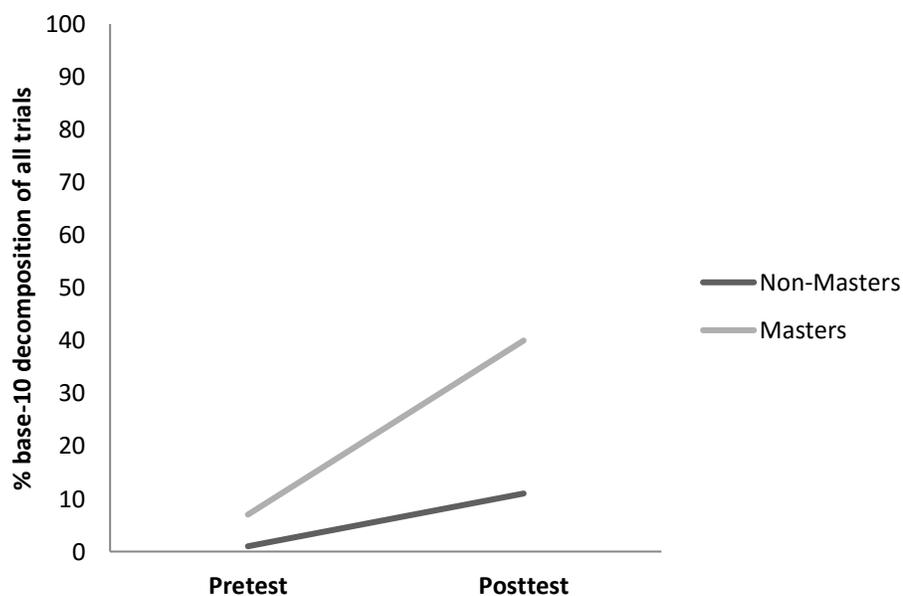


Figure 5. Pretest to posttest change in percent of base-10 decomposition of all problems by learner type.

It is interesting to note that the participants who spontaneously chose base-10 decomposition at pretest carried it out almost exclusively mentally, while those that chose it at posttest seemed to require additional supports, such as fingers and verbal self-guidance – whether in combination or separately. The tool was coded as purely mental on 96% ($SD = 8$) of the base-10 trials at pretest and only 54% ($SD = 41$) of the base-10 trials at posttest. At the same time, a paired-samples t -test showed that using external supports at posttest did not put first graders at a disadvantage on accuracy: participants who chose the strategy at pretest and posttest were able to reach the same level of accuracy of base-10 decomposition.

Finally, I examined the relation between participants' accuracy of base-10 decomposition execution on the two No Choice tasks and their preference for it on the mental Choice task. A linear regression controlling for frequency of base-10 decomposition at pretest, showed that a composite variable of percentage of base-10 decomposition trials solved accurately on the Manipulatives and mental No Choice tasks predicted frequency of base-10 decomposition on the posttest Choice task, $t(2,60) = 2.52, p = .015, \Delta R^2 = .08$. Thus, preference for the strategy was positively related to accuracy of its hands-on and mental execution.

Effect of Experimental Condition

Preliminary analyses indicated no effect of the kind of single-representation used on any of the outcome measures. Thus, these conditions were collapsed in all subsequent analyses.

Ease of abstraction. No difference was found between using either a single representation or multiple representations for ease of abstraction. At posttest, children in both conditions were equally accurate at executing base-10 decomposition when prompted to do so, despite having twice as many instances of practice using the manipulative in the single representation conditions. The mean percent correct of problems solved using base-10 decomposition on the Manipulatives and mental No Choice task was 68% ($SD = 31$) in the single representation condition and 69% ($SD = 33$) in the multiple representation condition, $t(59) = .16, p = .870, d = .03$. There also was no effect of condition on how rapidly participants mastered the strategy. Rate of learning – the number of training sessions, including posttest as the “sixth” session, before individuals reached and maintained high accuracy of base-10 execution (all problems during the intervention sessions and at least 5 out of 6 problems at posttest) – was the same across conditions, 5.19 and 5.43 in the single and multiple representations, respectively, $t(33) = .59, p = .557, d = .26$. Further, a repeated measures ANCOVA with frequency of base-10

decomposition at pretest and posttest as the within-subjects variables, condition as the fixed factor, and working memory score as the covariate showed only an effect of time, but no working memory by time interaction, $F(1,57) = .45, p = .453, \eta^2 = .01$, or working memory by condition interaction, $F(2,57) = .50, p = .609, \eta^2 = .02$. First graders in the multiple ER condition were as likely to choose mental base-10 decomposition as their peers in the single ER condition, regardless of their working memory scores.

Depth of abstraction. Results did suggest a difference between using either a single representation or multiple representations for depth of abstraction. To examine whether acquiring the strategy through instruction with two manipulatives, rather than one, enabled generalization to mental execution of the strategy, I ran a chi-square analysis comparing the number of children who mastered the hands-on strategy in the two conditions. As a reminder, “mastery” was defined as successful execution of base-10 decomposition on 5 out of 6 posttest trials involving manipulatives. The analysis showed no condition differences in the number of masters and non-masters: 57% master (single representation), 43% non-master (single representation), 58% master (multiple representation), and 42% non-master (multiple representation), $\chi^2(1,61) = .02, p = .903, V = .02$.

Next, I conducted learning path analysis (Klahr & Nigam, 2004), in which I compared masters and non-masters in the two conditions (four learning paths in total) on (a) percentage of all Choice task problems they solved using base-10 decomposition or (b) percentage of all base-10 decomposition trials they executed accurately on the Choice task as the dependent variable. A repeated measures ANOVA on the percentage of Choice task problems solved using base-10 decomposition at pretest and posttest, found a significant main effect of time, $F(1, 57) = 31.81, p < .001, \eta^2 = .36$, and a time by learning path interaction, $F(3, 57) = 5.78, p = .002, \eta^2 = .23$. As

reflected in Table 5, non-masters in the two conditions were not significantly different in their frequency of use of mental base-10 decomposition (14% ($SD = 28$) and 6% ($SD = 19$) for single and multiple representation conditions, respectively, $p = .601$), but masters in the multiple representations condition used the strategy more frequently than masters in the SR condition (58% ($SD = 38$) vs. 28% ($SD = 39$), respectively, $p = .044$). Further, masters in the SR condition were not significantly different from non-masters in the MR condition in their frequency of base-10 decomposition choice at posttest ($p = .069$). Additionally, while masters in the multiple representation condition used mental base-10 strategy more frequently than non-masters ($p = .001$), masters and non-masters in the single representation condition used base-10 at the same rate on the mental Choice task at posttest ($p = .137$).

Similarly, an ANOVA of the percentage of all base-10 decomposition trials executed accurately on the Choice task indicated a main effect of learning path, $F(3, 27) = 3.05$, $p = .048$, $\eta^2 = .28$. Pairwise comparisons revealed that masters in both conditions were equally accurate at executing the strategy, as were the non-masters. Mirroring the pattern of findings of frequency, however, non-masters and masters in the single ER condition executed the mental strategy at the same level of accuracy, while masters in the multiple ER condition were more accurate than respective non-masters ($p = .025$; Table 5).

Table 5

Base-10 Frequency and Accuracy Scores for Children Following Different Learning Paths

Learning Path	% of Trials Solved	% of Base-10 Trials solved Accurately	% within condition
	Mean (<i>SE</i>)	Mean (<i>SE</i>)	
Master (SR)	28 (39)	80 (31)	56.8

Master (MR)	58 (38)	91 (11)	58.3
Nonmaster (SR)	14 (28)	58 (49)	43.2
Nonmaster (MR)	06 (19)	17*	41.7

**Note:* Only one nonmaster in the multiple ER condition attempted using base-10 decomposition at posttest; therefore, standard error is not reported.

Congruent findings were found when examining children's reported visualization of the manipulative during mental calculation. An independent-samples *t*-test found children assigned to the multiple representation condition reported visualizing the hands-on procedure less during mental base-10 addition than children assigned to the single representation condition (7% ($SD = 24$) and 30% ($SD = 39$) of all base-10 decomposition trials, respectively), $t(42.09) = 2.34$, $p = .024$, $d = .71$.

Summary of Analyses and Key Findings

In summary, children used decomposition on 19% of all pretest addition problems, with base-10 decomposition being the most prevalent type of decomposition used (13% of the pretest problems). Girls used base-10 decomposition less frequently and counting more frequently than boys. Magnitude of the single-digit addend was negatively related to the frequency with which base-10 decomposition was used, though the effect size of the relation was small. Basic arithmetic fluency positively predicted frequency of the strategy.

Children selected for the intervention increased their use of base-10 decomposition on the mental Choice task from 4% of problems at pretest to 28% at posttest. Problem type, gender and number fluency were not related to frequency of base-10 decomposition after the intervention. Participants who used base-10 decomposition for addition at pretest did so almost exclusively mentally (96% of the problems); in comparison, those who used it on the Choice task at posttest

tended to rely on fingers and verbal counting more, solving an average of 54% of the problems without the use of external supports.

Preference for base-10 decomposition on the mental Choice task was accompanied by as high accuracy of execution (82%) as when they were required to execute the strategy on the mental No Choice task (83% accuracy). Further, children's ability to execute base-10 decomposition accurately when required (on the two No Choice tasks) was positively predictive of their preference on the strategy (on the mental Choice task at posttest).

First graders who used base-10 decomposition at pretest were more likely to use the strategy at posttest, though 37% of those who did not use the strategy at pretest relied on it at least once at posttest. Children who executed base-10 decomposition strategy using manipulatives at a high level of accuracy ("masters") were more likely to use it mentally when given a choice at posttest, but even those who did not fully master the hands-on strategy grew in their use of it from pretest to posttest.

Contradicting the ease of abstraction hypothesis for single representation advantage, accuracy of base-10 decomposition execution on the No Choice task was equivalent in the two conditions. The participants also learned the strategy at the same rate regardless of condition. Finally, working memory was not related to frequency of base-10 decomposition on the mental Choice task for participants in the multiple representation condition.

In contrast, the depth of abstraction hypothesis for multiple representation advantage was supported. Though there were no condition differences in the number of masters and non-masters of base-10 decomposition on the Manipulatives task, those children who mastered the strategy using manipulatives in the multiple representation condition were more likely to rely on it on the novel mental Choice task than masters in the single representation condition. Additionally,

participants trained in the multiple representation condition reported visualizing the hands-on procedure less during mental base-10 addition than children assigned to the single representation condition.

Chapter 5: Discussion, Conclusion and Implications

Early mathematics learning is predictive of long-term mathematics achievement (Duncan et al., 2007; Geary, 2011; LeFevre et al., 2010). Effective math interventions in elementary school thus hold the potential of positively influencing one's math performance through the years of formal schooling, and even into one's professional life (National Mathematics Advisory Panel, 2008). A particularly important goal of math instruction in elementary school is students' mastery of base-10 decomposition for addition, which has been predictive of advanced arithmetic strategies up to middle school (Geary, 2006; Geary et al., 2013; National Council of Teachers of Mathematics, 2000; National Research Council, 2001).

A majority of elementary school teachers believe that using external representations during math instruction is beneficial or even necessary for students' learning (Puchner et al., 2008; Sherman & Richardson, 1995). Though this is a popular view among practitioners, empirical evidence of the effectiveness of ERs is inconclusive: their use sometimes promotes and sometimes hinders learning (Boulton-Lewis & Tait, 1994; Kaminski et al., 2006; McNeil & Jarvin, 2007; Uttal, 2003). In the context of base-10 decomposition instruction in first grade, the present study tested the hypothesis that the efficacy of external representations may depend on whether one or multiple ERs are used for instruction.

The study had three main goals. The first goal was to give an account of base-10 decomposition use on mixed-digit problems in first grade, particularly as it relates to arithmetic fluency, gender, and problem characteristics. The second was to examine the effect of a brief intervention on first graders' use of base-10 decomposition. The third goal was to test the hypothesis that the efficacy of using external representations for instruction depends on the number of representations used.

Use of Base-10 Decomposition in First Grade

In the last 10 years, research studies have noted an increase in the use of the decomposition strategy for addition in first grade. In 2004 and then in 2007, Geary and colleagues found that first graders used decomposition on 8% and 6% of the problems with mixed-digit addends, respectively, while Vasilyeva and colleagues observed a rate of 42% in 2015. First graders in the present study spontaneously used decomposition on 19% of mixed-digit problems – a rate two to three times that of the 2004 and 2007 studies, though lower than that of the 2015 study. Decomposition may have been less prevalent in the current sample than in Vasilyeva and colleagues' (2015) sample due to socioeconomic or curricular differences. Indeed, the present sample appears to have come from a slightly lower socioeconomic background. Prior research (Laski et al., 2016) has related income and arithmetic strategy, with children coming from higher socioeconomic backgrounds choosing to use decomposition more than their peers coming from lower socioeconomic backgrounds. Additionally, it is possible that the curricula of the schools in Vasilyeva's study emphasized base-10 number properties and arithmetic strategies more than the curricula of the schools in the present study.

Nevertheless, the present findings provide support for an increase in first graders' use of decomposition over the last decade. This is likely due to a recent focus on base-10 decomposition in US educational standards, instruction, and curricular materials (National Governors Association, 2010; Shen et al., 2016). Indeed, the Common Core Standards for Mathematics, which were released in 2010 – that is, in the time between the 2007 and 2015 studies – expect first graders to add single- and double-digit numbers with sums up to 100 using base-10 properties of number. The observed increase in decomposition strategies among first graders may be attributable to increase in base-10 decomposition specifically. As a matter of

fact, the base-10 strategy was the most prevalent decomposition strategy among first graders in the present study – used twice more frequently than other types of decomposition.

Existing research has not examined factors related to first graders' choice of base-10 decomposition for addition in detail. Prior studies have found that size of the problem predicted decomposition use (Geary et al., 1996; Laski et al., 2014; Vasilyeva et al., 2015). The current findings indicated that base-10 decomposition, in particular, it is the size of the single-digit addend that predicts how frequently first graders use the strategy on mixed-digit problems specifically. This may be because children tend to be more fluent with smaller numbers than larger ones (Siegler & Robinson, 1982), and thus may perceive a higher likelihood of success in carrying out the strategy that requires composing and decomposing the smaller numbers. Indeed, the larger the number, the more number facts a child needs to know to decompose it effectively. For instance, one needs to be familiar with only two number facts to fluently decompose the number 3 ($1 + 2$ and $2 + 1$), while decomposing the number 9 takes familiarity with eight ($1 + 8$, $8 + 1$, $2 + 7$, $7 + 2$, and so forth). The importance of basic arithmetic fluency was indeed confirmed in the study: children were more likely to use base-10 decomposition when they were fluent with basic number facts.

Finally, as in earlier studies, boys relied on base-10 decomposition more than girls, while girls relied on counting more than boys. There was no difference in the rates of other types of decomposition between genders, suggesting that differences in rates of “cognitive strategies” (e.g., Carr & Davis, 2010; Fennema et al., 1998) may have been driven by differences in base-10 decomposition. Additionally, this finding supports Shen and colleagues' (2016) claim that though the overall prevalence of base-10 decomposition has increased over the last decade, differences in its use by gender have remained, and girls are still at a disadvantage.

Effect of Instruction on Use of Base-10 Decomposition

With the exception of Cheng (2012) which provided nine training sessions, most evidence for the importance of instruction for strategy choice has up to this point come from comparisons of expansive curricular programs (e.g., Fuson & Kwon, 1992; Shen et al, 2016) , or from prolonged experimental interventions (e.g., Hiebert & Wearne, 1992; Siegler & Jenkins, 1989). The present study further demonstrated that even brief instruction can be effective in increasing the rate of a desirable strategy. As a result of only five hands-on training sessions (lasting an average of 19 minutes each), first graders in this study increased in their choice of base-10 decomposition from 4% to 28% of the problems. Moreover, they were as accurate executing the strategy spontaneously as they were when specifically instructed to use it, suggesting that the increase in frequency was not due to a “priming effect,” but constituted a meaningful adoption of the strategy. In fact, choice of base-10 decomposition at posttest was predicted by how well children were able to execute it. Mastery of the strategy in the presence and absence of the manipulatives resulted in higher rates of base-10 decomposition choice during mental addition among first graders in the study. Shrager and Siegler’s (1998) model of change in strategy choice suggested that children choose the strategies which they perceive to lead to the highest likelihood of successful execution. The present findings show that this is applicable not only to spontaneous strategy discovery, but also to strategy instruction: encouraging children’s proficiency with an advanced strategy is likely to lead to them choosing it more for themselves.

The change in prevalence of base-10 decomposition occurred on a broad range of problems. As a result of the brief targeted intervention, participants became as likely to choose the strategy on problems with larger single-digit addends as on problems with smaller ones. The practice they received composing and decomposing single-digit numbers of varying magnitudes

likely made them more confident in the likelihood of successfully carrying out base-10 decomposition at posttest, even on problems with larger single-digit addends. This finding underscores the importance of building children's basic arithmetic fluency in order to affect their preference of base-10 decomposition for addition. Additional evidence that this training may have affected strategy choice through increasing participants' arithmetic fluency comes from the finding that pretest fluency levels were no longer predictive of posttest strategy choice. Consistent with this explanation, number composition training has indeed been shown to increase rates of base-10 decomposition use in previous studies (Cheng, 2012).

In addition to increasing first graders' choice of base-10 decomposition by increasing the likelihood of their accuracy on it, the intervention appears to have highlighted the benefits of base-10 decomposition for children with a wide range of skills. First graders who chose to use base-10 decomposition before the intervention did so almost exclusively mentally. Yet, after the intervention, children who chose the strategy carried it out using fingers or verbal self-guidance on almost half of the problems. On the one hand, this may suggest that the brevity of the instruction resulted in fragile abstraction, where children were still unable to fully adopt the mental strategy and instead had to rely on external supports. On the other hand, however, this finding may be revealing that the instruction enabled a wider range of learners to take advantage of the strategy. While only those who had been able to carry it out mentally relied on it before, the intervention may have convinced even children who were only able to carry it out using external supports of the benefits (e.g., efficiency) of using the strategy, promoting its use among them.

Evidence from earlier studies also suggests that first grade girls fail to abandon manipulatives in favor of mental addition strategies longer than boys (e.g., Fennema et al., 1998).

Yet, the present study shows that an effective intervention is able to attenuate for this difference. The brief intervention was as effective in promoting base-10 decomposition for girls as it was for boys, as the two groups increased their use of the strategy at the same rate.

Prior models of strategy choice focus mostly on spontaneous strategy discovery, guided by children's prior knowledge and characteristics of the problem at hand (Shrager & Siegler, 1998). However, it is reasonable to suspect that children's strategies do not change spontaneously most of the time, but are rather affected by formal instruction. Based on the findings of this study, instruction may not only equip children with skills necessary to execute advanced strategies accurately, but also increase their confidence in the efficiency of using a strategy on its own merit, even if its execution is cumbersome to begin with, such as in the case of using fingers.

Moreover, Shrager and Siegler (1998) suggested that after a strategy is spontaneously discovered, its generalization – that is, increase in the frequency of its use – occurs slowly and gradually. The present intervention demonstrates that effective formal instruction can allow generalization to take place at a relatively fast rate. Indeed, evidence from other fields of learning attests to the ability of brief instruction to rapidly influence children's mental representations and approaches. For example, Rosenthal and Zimmerman (1972) altered Piaget's classic conservation experiment by demonstrating volume equivalence to participating children. As a result, children who had failed the conservation task prior to the demonstration instead succeeded on it. Similarly, Saxon and Towse (1998) demonstrated to a sample of kindergartners who had represented numbers as collections of single units regardless of their size that double-digit numbers can also be represented as collections of tens and ones. Due to this demonstration the participants' number representation preference shifted from single-unit to tens-based on an

independent practice task. The findings of the present study likewise show that instruction-guided strategy change may occur more rapidly than spontaneous strategy change previously investigated by researchers (e.g., Shrager & Siegler, 1998; Siegler & Shipley, 1995). Paralleling Vygotsky's (1978) argument that learning leads development – i.e., that interactions with a more knowledgeable other are able to push the child to develop skills he or she would not otherwise possess at a particular point in time – these findings suggest that instruction can meaningfully influence strategy preference, accelerating the process otherwise guided by children's own metacognitive processes (Shrager & Siegler, 1998).

Single Representation or Multiple Representations: What Is Better for Learning?

Theoretical and empirical evidence reviewed in this paper points to two opposing views of how to effectively use external representations: consistent use of a single ER versus reliance on multiple ERs. Claims in favor of using a single representation for learning are based on its benefits for ease of abstraction, while those in support of using multiple representations often cite depth of abstraction as the primary benefit. Both are discussed in light of the present study's findings below.

Ease of abstraction. Most of the literature in favor of using one representation is built on the underlying argument that using manipulatives poses a high cognitive load on young children, preventing them from gaining proficiency with the underlying concepts or strategies (e.g., Boulton-Lewis, 1998; McNeil & Jarvin, 2007; Uttal, 2003). This view suggests that, in contrast to learning from multiple representations at the same time, consistently using one promotes ease of abstraction by reducing the cognitive load generally associated with manipulative use. There are two ways in which a single ER is theorized to accomplish this: by (a) offering more practice with the representation, which allows for more familiarity with it and the content it represents;

and (b) minimizing the amount of superficial features that a child needs to process and that proportionately contribute to working memory demands.

In contrast to these claims, the present study found that the two experimental conditions did not differ on ease of abstraction or factors hypothesized to lead to it. Children in the single and multiple representation conditions were able to execute the strategy in the presence and absence of the ER/s with the same rate of accuracy, despite the fact that the single representation group received twice the amount of practice with their representation. Further, working memory capacity was not related to the frequency with which participants in either condition used base-10 decomposition after the intervention. This suggests that the cognitive load of learning from two ERs simultaneously is no greater than the cognitive load of learning from one, at least for first graders. Future research should explore whether cognitive load is a greater concern for younger children, for whom this finding may have been different.

Depth of abstraction. Supporters of multiple representations claim that the process of comparison of several ERs leads children to disregard their superficial features and abstract the concepts more deeply, as evidenced by their ability to extend their acquired knowledge to novel tasks (e.g., Ainsworth, 2006; Gentner, 1983; Kotovsky & Gentner, 1996). In contrast, learning from a single ER has been suggested to result in children's mental representations reflecting both the substantive and superficial features of the ER, limiting their flexibility in new contexts (e.g., Schnotz & Kurschner, 2008).

The present study found that among children who mastered the base-10 decomposition strategy using ERs, those in the multiple representation condition were indeed more likely to choose the strategy on the novel mental addition task. This is interesting, considering that there were no differences between strategy "masters" in the two conditions on accuracy with which

they carried out the strategy. Thus, in addition to skills necessary to execute the strategy successfully, children's choice of base-10 decomposition was determined by another factor, referred to in this study as depth of abstraction. Training with a single manipulative may have led to an equal amount of skill with the strategy, but less flexibility when applying it to contexts that do not involve a hands-on manipulative. Indeed, children who learned from a single ER in the present study were more likely to visualize the manipulative even when executing the strategy without it. This indicates that their mental representations may have been limited by the external features of the manipulative on which they were trained, as has been suggested in the literature (Schnotz & Kurschner, 2008). Thus, consistent with the literature (Ainsworth, 1999; Ainsworth, 2006; Schnotz & Kurschner, 2008), multiple representations did lead to greater flexibility of mental representation: comparing two manipulatives appears to have led to a mental representation that embodied the concepts they represented, not their superficial features. Of course, it is possible that learning from a single manipulative leads to this level of abstraction eventually as well, but may require more time.

In general, one would expect that achieving deep abstraction of a strategy would take a longer time than simply becoming proficient with its execution. Based on this, children in the multiple representation condition could be expected to take longer to master base-10 decomposition than children in the single representation condition, but have deeper knowledge of it when they did. However, the results of this study speak to the contrary: children who learned from two manipulatives did not differ from their peers on the rate at which they mastered the strategy, and yet adopted a more generalizable mental model of it than those who learned from one manipulative. In other words, they did not sacrifice efficiency of learning to achieve depth of learning.

Effective Use of External Representations: Factors Other Than Number

Why was the intervention in this study successful in increasing the rate of base-10 decomposition, while many prior interventions using ERs have been unsuccessful (e.g., Boulton-Lewis, 1998)? Why was processing multiple representations beneficial to learning, contrary to what has been shown before (e.g., Chao et al., 2000)? One possible reason is that the intervention followed empirically based recommendations on using ERs to ensure they pose a minimal cognitive load (Ball, 1992; Resnick & Omanson, 1987; Sowell, 1989; Uttal, 2003). For example, the materials were mathematically transparent with different color codes representing tens and ones; were not previously familiar to the children; and were minimally interesting as objects in and of themselves. Most experimental interventions that have used external representations to date have not adequately controlled for the quality of their intervention. Some have used manipulatives whose intrinsic features diminished their effectiveness (e.g., the “unstructured” manipulatives in the multiple representations condition of Chao et al., 2000). Others have used them in a way less than optimal for learning (e.g., presenting a manipulative after a concept was already known; Boulton-Lewis, 1998). Yet others have not controlled for comparison group instruction quality experimentally (e.g., Hiebert & Wearne, 1992). The present study sought to address these shortcomings, and found that that when high quality materials are used, they may in fact promote, rather than hinder learning, providing empirical support for existing theoretical recommendations for effective manipulative use (e.g., Ball, 1992; Uttal, 2003; Uttal et al., 1998) that have been minimally tested experimentally up to this point. Further, when the quality of manipulatives is controlled, using multiple manipulatives at one time is more beneficial to children’s learning than learning from a single manipulative.

Manipulatives in the multiple representations condition in the present study were also chosen using research- and theory-based recommendations. For example, for multiple representations to facilitate abstraction, they need to be similar in the way that they represent the conceptual information (e.g., Gentner, 1983). Thus, while the two manipulatives were different along superficial dimensions, they emphasized the base-10 structure of number in the same way: tens were blue and ones were green, regardless of whether they were beads or tiles. The regrouping procedure was also carried out similarly, by “trading” 10 ones for one 10. Thus, multiple representations may present an advantage for learning specifically when conceptual correspondences between them are salient.

Finally, experimenters in the present study also emphasized connections between the concepts and the manipulatives, and between the two manipulatives in the multiple ER condition. This also likely contributed to the success of the intervention. As children find it challenging to spontaneously find correspondences between the abstract concepts and their concrete representations, researchers have strongly emphasized the importance of explicit instructor connections between the manipulatives and the mathematical concepts they represent in ensuring students’ concept abstraction (e.g., Ball, 1992; Uttal, 2003).

Limitations and Future Research

Like any study, the present investigation had some limitations. One was its small sample size, which created statistical power concerns in some of the analyses, particularly when groups were compared within the sample. Future research should seek to examine the issues raised by this study with larger samples, and samples of different backgrounds. Children in the present study were mostly White, of middle to high socioeconomic background, and born to highly educated parents. The results would need to be replicated with children who do not share these

characteristics with the present sample to be generalizable. Additionally, the benefits of using one versus many ERs may be different for children who are younger or older than the present sample, as their cognitive capacity and knowledge base are different from those of first graders.

This study showed that first graders' strategy choice can be affected as a result of as few as five sessions. Nevertheless, longitudinal follow-up would be required to answer the question of how persistent this effect is. Additionally, it would be interesting to look at the differences between the two experimental conditions in a longer intervention, and whether they would change or intensify with more prolonged practice. Understanding which approach to external representations is more beneficial in cases when interventions have to be brief versus prolonged is particularly important for informing classroom practice.

Additionally, the intervention in the present study was carried out in small groups of students. This approach was chosen to increase the ecological validity of the study by resembling the classroom context, and reduce the amount of time students needed to spend out of class. Nevertheless, it is possible that group members affected each other's performance, whether by providing motivation or distraction. Future interventions should either be conducted individually, or group effects should be measured in larger samples.

One could argue that even the single representation condition contained multiple representations: the manipulative and the printed arithmetic problems. Indeed, written numerals are considered to be an external representation. Presenting children with one abstract and one concrete or one abstract and two concrete representations in the single and multiple representation conditions, respectively, allowed for the two conditions to receive an intervention of comparable quality. A pure single representation condition would have required that manipulatives be presented without the problems written out, likely putting children in that

condition at a disadvantage. Besides, some prior studies comparing single and multiple representations have also defined SR and MR in terms of concrete representations that often accompanied text or number expressions (e.g., Rau, Alevan, & Rummel, 2015). On the other hand, using two types of manipulatives was a conservative way of assessing the effectiveness of multiple representations. Future research should examine the benefits of higher numbers of ERs in order to establish a threshold past which using multiple representations is detrimental to learning.

Finally, the manipulatives chosen for the intervention study were very similar in their mathematically relevant features, making connections between them more salient. However, one advantage of multiple representations cited in prior research is that their unique properties may complement one another in conveying a concept and thus make up for potential limitations in any one representation (Cox & Brna, 1995; Friedlander & Tabach, 2001). One could argue that participants in the present study did not have an opportunity to draw on unique properties of different representations because they were too similar to one another in how they represented base-10 decomposition (in contrast to differences between such representations as a table and a Venn diagram). Future studies should take into account not only the number, but also the qualities of ERs, and test the benefits of similar ERs to those that instead complement each other's uniqueness.

Implications and Conclusions

Despite these limitations, the present study makes a promising case for the effectiveness of instructional interventions in promoting the base-10 decomposition strategy for addition. The findings suggest that emphasizing base-10 decomposition in the national curricular standards, textbook lessons, classroom discourse, and instructional materials is likely to lead to higher rates

of its use. Moreover, even brief targeted interventions may be enough to substantially increase the prevalence of the strategy across genders and ability levels.

The importance of explicitly teaching the strategy and the most effective ways of doing so needs to also be emphasized in teacher training programs. Specifically, practicing teachers and teachers in training should be encouraged to choose mathematically transparent manipulatives, make explicit correspondences between them, and make connections between the manipulatives and the concepts they represent, to maximize their students' learning of the strategy.

When using external representations such as manipulatives as instructional materials, teachers are recommended to encourage only children's ability not only to learn the hands-on strategy, but also extend it to mental arithmetic. While different numbers of external representations appear to lead to similar rates of learning and accuracy of strategy execution with hands-on materials, multiple representations may promote deeper abstraction of hands-on strategies than using a single one. The present study provides evidence that multiple ERs encourage a more flexible mental representation than a single ER, which appears to constrain the mental representation to the context and features of the manipulative used for learning. Thus, while children who learn from multiple representations may not have an advantage in ease of learning the strategy, the evidence suggests they have an upper hand in how deeply they learn it.

Scientific research is important to further investigation of factors that contribute to successful change in children's arithmetic strategy choice as a result of educational interventions, particularly if these factors extend beyond those that have been found to affect children's spontaneous strategy discovery. Additionally, as research and educational assessments investigate children's learning as a result of instruction, how learning is measured needs to be

nuanced. Concepts such as ease and depth of abstraction, for example, need to be more clearly defined and measured, with the ultimate goal of devising interventions that promote both.

References

- Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*(2), 131-152.
- Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183-198.
- Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions, *Mathematical Cognition, 1*, 3-34.
- Ashcraft, M. H., Donley, R. D., Halas, M. A., & Vakali, M. (1992). Working memory, automaticity, and problem difficulty. *Advances in psychology, 91*, 301-329.
- Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator: The Professional Journal of the American Federation of Teachers, 16*(2), 14-18.
- Boulton-Lewis, G. M. (1998). Children's strategy use and interpretations of mathematical representations. *The Journal of Mathematical Behavior, 89*, 133-155.
- Boulton-Lewis, G. M., & Tait, K. (1994). Young children's representations and strategies for addition. *British Journal of Educational Psychology, 64*(2), 231-242.
- Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24*, 61-100.
- Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology, 33*(3), 205-228.
- Campbell, J. I., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology, 130*(2), 299-315.

- Canobi, K. H. (2004). Individual differences in children's addition and subtraction knowledge. *Cognitive Development, 19*(1), 81-93.
- Carr, M., & Davis, H. (2001). Gender differences in arithmetic strategy use: A function of skill and preference. *Contemporary Educational Psychology, 26*(3), 330-347.
- Carr, M., & Jessup, D. L. (1997). Gender differences in first-grade mathematics strategy use: Social and metacognitive influences. *Journal of Educational Psychology, 89*(2), 318-328.
- Carr, M., Steiner, H. H., Kyser, B., & Biddlecomb, B. (2008). A comparison of predictors of early emerging gender differences in mathematics competency. *Learning and Individual Differences, 18*(1), 61-75.
- Chao, S.-J., Stigler, J. W., & Woodward, J. A. (2000). The effects of physical materials on kindergartners' learning of number concepts. *Cognition and Instruction, 18*(3), 285-316.
- Cheng, Z. J. (2012). Teaching young children decomposition strategies to solve addition problems: An experimental study. *The Journal of Mathematical Behavior, 31*(1), 29-47.
- Correa, C. A., Perry, M., Sims, L. M., Miller, K. F., & Fang, G. (2008). Connected and culturally embedded beliefs: Chinese and US teachers talk about how their students best learn mathematics. *Teaching and Teacher Education, 24*(1), 140-153.
- Cox, R., & Brna, P. (1995). Supporting the use of external representations in problem solving: The need for flexible learning environments. *Journal of Artificial Intelligence in Education, 6*(2/3), 239-302.
- DeLoache, J. S., de Mendoza, O. A. P., & Anderson, K. N. (1999). Multiple factors in early symbol use: Instructions, similarity, and age in understanding a symbol-referent relation. *Cognitive Development, 14*(2), 299-312.

- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Sexton, H. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428.
- Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). A longitudinal study of gender differences in young children's mathematical thinking. *Educational Researcher*, 27(5), 6-11.
- Fernandez, C., Yoshida, M., & Stigler, J. W. (1992). Learning mathematics from classroom instruction: On relating lessons to pupils' interpretations. *The Journal of the Learning Sciences*, 2(4), 333-365.
- Friedlander, A., & Tabach, M. (2001). Promoting multiple representations in algebra. In A. Cuoco (Ed.), *The roles of representation in school mathematics. 2001 Yearbook of the National Council of Teachers of Mathematics* (pp. 173-184). Reston, VA: National Council of Teachers of Mathematics.
- Fuson, K. C., & Kwon, Y. (1992). Korean children's single-digit addition and subtraction: Numbers structured by ten. *Journal for Research in Mathematics Education*, 23(2), 148-165.
- Geary, D. C. (2006). Development of mathematical understanding. In W. Damon (Gen. Ed.), *Handbook of child psychology, 6th Ed.* (777-810). New York: John Wiley & Sons.
- Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539-1552.
- Geary, D. C., Bailey, D. H., & Hoard, M. K. (2009). Predicting mathematical achievement and mathematical learning disability with a simple screening tool: The Number Sets Test. *Journal of Psychoeducational Assessment*, 27(3), 265-279.

- Geary, D. C., Bow-Thomas, C. C., Liu, F., & Siegler, R. S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development, 67*(5), 2022-2044.
- Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47*(2), 175-192.
- Geary, D. C., Fan, L., & Bow-Thomas, C. C. (1992). Numerical cognition: Loci of ability differences comparing children from China and the United States. *Psychological Science, 3*(3), 180-185.
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88*(2), 121-151.
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*(4), 1343-1359.
- Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PloS one, 8*(1), e54651.
- Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7*(2), 155-170.
- Gentner, D., & Namy, L. L. (1999). Comparison in the development of categories. *Cognitive Development, 14*(4), 487-513.

- Gilmore, D. J., & Green, T. R. G. (1984). Comprehension and recall of miniature programs. *International Journal of Man-Machine Studies*, 21, 31-48.
- Hakel, M., & Halpern, D. (2005). How far can transfer go? Making transfer happen across physical, temporal, and conceptual space. In J. P. Mestre, *Transfer of learning from a modern multidisciplinary perspective* (pp. 357-370). Information Age Publishing Inc.
- Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education*, 23(2), 98-122.
- Imbo, I., Vandierendonck, A., & Rosseel, Y. (2007). The influence of problem features and individual differences on strategic performance in simple arithmetic. *Memory & Cognition*, 35(3), 454-463.
- Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2006). Do children need concrete instantiations to learn an abstract concept? In R. Sun & N. Miyake (Eds.), *Proceedings of the 28th annual conference of the Cognitive Science Society* (pp. 411–416). Mahwah, NJ: Erlbaum.
- Klahr, D., & Nigam, M. (2004). The equivalence of learning paths in early science instruction effects of direct instruction and discovery learning. *Psychological Science*, 15(10), 661-667.
- Kotovskiy, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. *Child Development*, 67(6), 2797-2822.
- Kurtz, K. J., Miao, C. H., & Gentner, D. (2001). Learning by analogical bootstrapping. *The Journal of the Learning Sciences*, 10(4), 417-446.

- Laski, E.V., Ermakova, A., & Vasilyeva, M. (2014). Early use of decomposition for addition and its relation to base-10 knowledge. *Journal of Applied Developmental Psychology*, 35,444-454.
- Laski, E. V., Schiffman, J., Vasilyeva, M., & Ermakova, A. (2016). Arithmetic accuracy in children from high-and low-income schools. *AERA Open*, 2(2), 2332858416644219.
- LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, 81(6), 1753-1767.
- LeFevre, J. A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 22(1), 216-230.
- Loewenstein, J., & Gentner, D. (2001). Spatial mapping in preschoolers: Close comparisons facilitate far mappings. *Journal of Cognition and Development*, 2(2), 189-219.
- MacMillan, N. A. (2002). Signal detection theory. In J. Wixted & H. Pashler (Eds.), *Stevens' handbook of experimental psychology, 3rd edition, vol. 4: Methodology in experimental psychology* (pp. 43–90). New York, NY: John Wiley.
- Mayer, R. E., & Moreno, R. (2003). Nine ways to reduce cognitive load in multimedia learning. *Educational Psychologist*, 38(1), 43-52.
- McNeil, N., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice*, 46(4), 309-316.
- McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction*, 19(2), 171-184.

- Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences, 20*(2), 101-109.
- Miura, I. T. (1987). Mathematics achievement as a function of language. *Journal of Educational Psychology, 79*(1), 79-82.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Governors Association, Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.
- Orsini, A. (1994). Corsi's block-tapping test: Standardization and concurrent validity with WISC—R for children aged 11 to 16. *Perceptual and Motor Skills, 79*, 1547-1554.
- Orsini, A., Grossi, D., Capitani, E., Laiacona, M., Papagno, C., & Vallar, G. (1987). Verbal and spatial immediate memory span: normative data from 1355 adults and 1112 children. *The Italian Journal of Neurological Sciences, 8*(6), 537-548.
- Pickering, S., & Gathercole, S. E. (2001). *Working memory test battery for children (WMTB-C)*. London: Psychological Corporation.
- Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics

- manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.
- Rampey, B. D., Dion, G. S., & Donahue, P. L. (2009). *NAEP 2008: Trends in academic progress*. NCEES 2009-479. Jessup, MD: National Center for Education Statistics.
- Rau, M. A., Alevan, V., & Rummel, N. (2009). Intelligent tutoring systems with multiple representations and self-explanation prompts support learning of fractions. In V. Dimitrova, R. Mizoguchi, B. du Boulay, & A. Graesser (Eds.), *Artificial intelligence in education* (pp. 441-448). Fairfax, VA: IOS Press, Inc.
- Rau, M. A., Alevan, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology*, 107(1), 30-46.
- Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41-96). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Rittle-Johnson, B., & Siegler, R. S. (1999). Learning to spell: Variability, choice, and change in children's strategy use. *Child Development*, 70(2), 332-348.
- Rosenthal, T. L., & Zimmerman, B. J. (1972). Modeling by exemplification and instruction in training conservation. *Developmental Psychology*, 6(3), 392-401.
- Saxton, M., and Towse, J. N. (1998). Linguistic relativity: The case of place value in multi-digit numbers. *Journal of Experimental Child Psychology*, 69, 66-79.

- Schnotz, W., & Kurschner, C. (2008). External and internal representations in the acquisition and use of knowledge: Visualization effects on mental model construction. *Instructional Science, 36*(3), 175-190.
- Shen, C., Vasilyeva, M., & Laski, E. V. (2016). Here, but not there: Cross-national variability of gender effects in arithmetic. *Journal of Experimental Child Psychology, 146*, 50-65.
- Sherman, H., & Richardson, L. (1995). Elementary school teachers' beliefs and practices related to teaching mathematics with manipulatives. *Educational Research Quarterly, 18*(4), 27-36.
- Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science, 9*(5), 405-410.
- Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology, 116*(3), 250-264.
- Siegler, R. S., & Jenkins, E. (1989). *How children discover strategies*. Hillsdale, NY: Erlbaum.
- Siegler, R. S., & Robinson, M. (1982). The development of numerical understandings. *Advances in Child Development and Behavior, 16*, 241-312.
- Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In T. Simon, & G. Halford (eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31-76). Hillsdale, NJ: Erlbaum.
- Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20*(5), 498-505.
- Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*(2), 257-285.

- Uttal, D. H. (2003). On the relation between play and symbolic thought: The case of mathematics manipulatives. In O. Saracho & B. Spodek (Eds.), *Contemporary perspectives in early childhood* (pp. 97-114). Greenwich, CT: Information Age.
- Uttal, D. H., Marzolf, D. P., Pierroutsakos, S. L., Smith, C. M., Troseth, G. L., Scudder, K. V., & DeLoache, J. S. (1998). Seeing through symbols: The development of children's understanding of symbolic relations. In O. N. Saracho and B. Spodek (Eds.), *Multiple perspectives on play in early childhood education*. Albany, NY: State University of New York Press.
- Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*(1), 37-54.
- Vasilyeva, M., Laski, E. V., & Shen, C. (2015). Computational fluency and strategy choice predict individual and cross-national differences in complex arithmetic. *Developmental Psychology, 51*(10), 1489-1500.
- Wechsler, D. (2003). *Wechsler intelligence scale for children-WISC-IV*. New York, NY: Psychological Corporation.
- Williams, P. E., Weiss, L. G., & Rolfhus, E. (2003). *Clinical validity (WISC-IV Technical Report No. 3)*. Retrieved December 8, 2004, from <http://harcourtassessment.com/hai/Images/pdf/wisciv/WISCIVTechReport3.pdf>.

Appendix A

Addition Problems Presented at Pretest

$19+6^*$

$7+44^*$

$38+4^*$

$5+46^*$

$6+25^*$

$7+27$

$33+8^*$

$17+9^*$

$8+28^*$

$26+5$

$8+39$

$35+6$

$4+18$

$29+5^*$

$7+14^*$

*problems presented on the mental Choice task at posttest

Appendix B

Intervention Sessions Script Example:

One Representation, Ten-Frame Condition

Session 1

Introduction

Experimenter:

You have been adding numbers in different ways for some time now. Today we will be learning [or practicing, for later sessions] about adding numbers using a way called ‘*Going through Ten*’. It is called ‘*Going through Ten*’ because when you add numbers using this way, you figure out how to get to the nearest ten first and then add on the extras. We will use base-10 frames to practice adding our numbers.

Manipulative demonstration

Look, here is a ten-frame. Each row has beads on it. We can use these beads to show numbers. We will use the first two rows – green and blue. The green beads on the first row are called ‘ones’, and the blue beads on the second row are called ‘tens’. Each of green bead is worth 1 [points to green beads], and each blue bead is worth 10 [points to the blue beads]. So, 10 green beads on the first row are the same as 1 blue bead on the second row. Watch: 10 of these [pushing to the right 10 green beads, counting, “1, 2, 3, 4, 5, 6, 7, 8, 9, 10”] are the same as 1 of these [pushing to the right 1 blue bead, saying “ten”].

When we add our numbers, we will count the green ones at the top by ones, like I just did. And we will count the blue ones at the bottom by tens like this, [pushing beads to the right] “10, 20, 30, 40, 50, 60, 70, 80, 90, 100.”

Base-10 number composition demonstration

Okay! Are you ready to make numbers now? First, I will show you some numbers on my ten-frame and then I will give you your own base-10 frames to show numbers by yourselves.

Let's start with an easy number. Here is number 9 [experimenter holds up a paper with the number printed on it]. To show this number, we're just gonna push 9 green beads over to this side [counting out ones-beads] "1, 2, 3, 4, 5, 6, 7, 8, 9!" Now this shows 9 [pointing to the number card] and this shows 9 [pointing to the ten-frame].

Bigger numbers are made up of tens and ones. Here is number 14 [experimenter holds up a paper with the number printed on it]. Let's show it on the ten-frame. We could start with the green beads at the top or with the blue beads at the bottom. First, let's start with the green beads at the top. Count with me [experimenter pushes beads to the right], "1, 2, 3, 4, 5, 6, 7, 8, 9, 10" – oh no, I need 14, but I have run out of green beads! Remember how I said that 10 of these are just like 1 of these [pointing to the tens-row]? I can use 1 of the blue ones on the bottom instead of the 10 at the top. So, let's put the ones back and use 1 ten instead! [Experimenter pushes ones-beads back, and counts out a ten-bead, saying "ten!"] Remember, our number is 14, so let's keep counting: "ten..." [pointing to the ten-bead], [counting out ones-beads] "11, 12, 13, 14!" I have shown number 14! Now this shows 14 [pointing to the number card] and this shows 14 [pointing to the ten-frame].

The first digit always shows how many tens we have. So, there is a '1' for one ten here [points to the numeral 1 on the card], and one blue bead for one ten here [points to the blue bead on the ten-frame]. The second digit always shows how many ones we have. So, there is a '4' for four ones here [points to the numeral 4 on the card], and four green beads for four ones here [points to the green bead on the ten-frame]. So, 14 is 1 ten and 4 ones [points to the numeral card].

Now here is number 36 [experimenter holds up a paper with the number printed on it]. Let's show it on the ten-frame. Now we know that we need to start with the tens, don't we? Count with me, [counting out ten-beads] "10, 20, 30". If I keep counting here, I will go to 40, but 40 is more than 36, and I only need 36. So, now I will count my ones [counting out ones-beads], "31, 32, 33, 34, 35, 36!" See, here is number 36.

Remember, the first digit always shows how many tens we have. So, there is a '3' for three tens here [points to the numeral 3 on the card], and three blue beads for three tens here [points to the blue beads on the ten-frame]. The second digit always shows how many ones we have. So, there is a '6' for six ones here [points to the numeral 6 on the card], and six green beads for six ones here [points to the green beads on the ten-frame]. So, 36 is 3 tens and 6 ones [points to the numeral card].

[Experimenter gives out base-10 frames to each child] Now it's your turn to show me a number! How do you show number 23? [Experimenter holds up a paper with the number printed on it] [As children work, experimenter walks around and helps those that are not doing it correctly by reminding them of the directions.]

Practice: 2 NUMBERS.

Base-10 strategy demonstration with mixed-digit numbers

You all tried really hard to show your numbers! Good job! Now we will add numbers using the '*Going through Ten*' way! Remember, when we add two numbers using the '*Going through Ten*' way, our job is to get to the next 10 first then add on the extras.

Let me show you how to add $17+5$ [experimenter shows a paper with $17+5$ on it]. To begin with, I will make the biggest number on the ten-frame – you already know how to do that. My biggest number is 17. Remember, first I show how many tens are in 17. 17 has 1 ten. [points

to the digit on the numeral card, and counts out 1 ten] Then, I show the ones. 17 has 7 ones [points to the digit on the numeral card, and counts out 7 ones without counting out loud] – that’s 17.

The problem is $17 + 5$ so now I need to add 5 more ones. First, I’ll use these 3 ones to get to the next ten [pointing to 3 ones], but I need to add 5, and I don’t have that many ones! So, I will make the next ten [pushes beads over], by trading all the green ones for 1 blue ten [substitutes ones-beads for a ten-bead] – which will make 20 – and then I’ll add the extra ones I still need. Remember, we added 3, so now to get to 5 I need to add 2 more, because 5 is $3+2$. [Counts out two one-beads, “4, 5!”] Now I have put together both of my numbers – 17 and 5. Let’s see what number they added up to: [experimenter counts out the tens, “10, 20”] twenty, [and counts the ones, “21, 22”] 22! $17+5$ equals 22.

I added these numbers by Going through ten! To add 5, I first figured out how many ones I needed to get to the next 10 – it was 3 ones, to get to 20 [pointing to double-digit number on card] – and then added the 2 extra ones that were left over and got 22. Two tens and two extra (pointing to the digits on the numeral card).

Let’s add two more numbers, and then you can do it yourselves!

This time, let’s add $6+38$ [experimenter shows a paper with $6+38$ on it]. To begin with, I will count out my biggest number on the ten-frame – you already know how to do that. My biggest number is 38. Remember, first I show how many tens are in 38. 38 has 3 tens [points to numeral card, and counts out 3 tens]. Then, I show the ones. 38 has 8 ones [points to numeral card, and counts out 6 ones without counting out loud] – that’s 38.

The problem is $6 + 38$, so now I need to add 6 more ones. I will use these 2 ones to get to the next ten, but I need to add 6, and I don't have that many ones! So, I will make the next 10 [pushes ones-beads over], by trading all the green ones for 1 blue ten [substitutes ones-beads for a ten-bead] — which will make 40 -- and then I'll add the extra ones I still need. Remember, we added 2, so now to get to 6 I need to add 4 more ones because 6 is $2+4$. [Counts, "3, 4, 5, 6!"]

Now I have put together both of my numbers – 6 and 38. Let's see what number they add up to: [experimenter counts out the tens, "10, 20, 30, 40"] forty, [counts out the ones, "41, 42, 43, 44"] 44! $6 + 38$ equals 44.

I added these numbers by Going through ten! To add 6, I first figured out how many ones I needed to get to the next 10 – it was 2 ones to get to 40 – and then added the 4 extra ones that were left over and got 44. 4 tens and 4 extra ones (pointing to digits on numeral card).

Test trials

[Experimenter records child's accuracy on each problem]

Now I want you to do your best as you solve some problems on your own!

Here is one: add $15+6$ using the going through ten way.

Now add $28+4$ using the '*Going through Ten*' way.