Essays on Information and Financial Frictions in Macroeconomics

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Abstract

This dissertation consists of three independent chapters analyzing the role that information and credit frictions play in goods and financial markets. Within these chapters, I develop dynamic stochastic general equilibrium (DSGE) models to study the implications of these frictions on the macroeconomy, both at the national and international level. In the first chapter, I provide a novel explanation for the observed large and persistent fluctuations in real exchange rates using a model with noisy, dispersed information among price-setting firms. Chapter two studies how entrepreneurs' attitudes towards risk affect business cycles in a framework with agency frictions between borrowers and lenders. Finally, chapter three introduces a liquidity channel in a business cycle model with agency frictions to rationalize the highly volatile behavior of default recovery rates observed in the data.

Real exchange rates have been extremely volatile and persistent since the end of the Bretton Woods system. For many developed economies, real exchange rates are as volatile as nominal exchange rates, and their fluctuations exhibit a half-life in the range of three to five years. Traditional sticky-price models struggle to jointly account for these features under plausible nominal rigidities (Chari, Kehoe, and McGrattan, 2002). Is it possible to reconcile, in a single framework, the enormous short-term volatility of the real exchange rate with its extremely long half-life? The first chapter of this dissertation addresses this question within a framework in which information is noisy and heterogeneous among price-setting firms. In this context, the continuing uncertainty that firms face about the state of the economy and about the beliefs of their competitors, slows down the price adjustment in response to nominal shocks, generating large and long-lived real exchange rate movements. I estimate the model using real output and output deflator data from the US and the Euro Area and show, as an out-of-sample test, that the model successfully explains the observed volatility and persistence of the Euro/Dollar real exchange rate. In a Bayesian model comparison, I show that the data strongly favor the dispersed information model relative to a sticky-price model à la Calvo. The model also accounts for the persistent effects of monetary shocks on the real exchange rate that I document using a structural vector autoregression.

The second chapter, joint with Mikhail Dmitriev, studies how entrepreneurs' attitudes towards risk affect business cycles in a model with agency frictions. Entrepreneurs are inevitably exposed to non-diversified risk, which likely affects their willingness to borrow and to invest in risky projects. Nevertheless, the financial friction literature has paid little attention to how entrepreneurs' desire to take on this risk affects their investment choices in a general-equilibrium setting. Indeed, business cycle models with credit market frictions that feature idiosyncratic risk assume, for tractability, that entrepreneurs are risk neutral (Bernanke, Gertler, and Gilchrist, 1999, BGG). In this chapter, we generalize the BGG framework to the case of entrepreneurs with constant-relative-risk-aversion preferences. In doing so, we overcome the aggregation challenges of this setup and maintain an analytically tractable, log-linear framework. Our main result is that higher risk aversion stabilizes business cycle fluctuations in response to financial shocks, such as wealth redistribution or risk shocks, without significantly affecting the dynamic responses to technology and monetary shocks. Our findings suggest that, within this class of models, the ability of financial shocks to account for a large portion of short-run output fluctuations found in previous work (e.g., Christiano, Motto, and Rostagno (2014)) crucially hinges on borrowers' risk neutrality.

The third chapter, joint with Mikhail Dmitriev, examines the implications of the cyclical properties of default recovery rates for aggregate fluctuations. We document that recovery rates after default in the United States are highly volatile and strongly pro-cyclical. These facts are hard to reconcile with the existing financial friction literature. Indeed, models with limited enforceability à la Kiyotaki and Moore (1997) do not feature defaults and recovery rates in equilibrium, while agency costs models following Bernanke, Gertler, and Gilchrist (1999) underestimate the volatility of recovery rates by one order of magnitude. In this chapter, we extend the standard agency costs model allowing liquidation costs for creditors to depend on the tightness of the market for physical capital. Creditors do not have expertise in selling entrepreneurial assets, but when buyers are plentiful, this disadvantage is minimal. Instead when sellers are abundant, the disadvantage of being an outsider is higher. Following a negative shock, entrepreneurs sell capital and liquidation costs for creditors increase, driving down recovery rates. With higher liquidation costs, creditors cut lending and cause entrepreneurs to sell even more capital. This liquidity channel works independently from standard balance sheet effects, and amplifies the impact of financial shocks on output by up to 50 percent.

Ai miei genitori

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Chapter 1

Information Frictions and Real Exchange Rate Dynamics

1.1 Introduction

Real exchange rates have been extremely volatile and persistent since the end of the Bretton Woods system (Mussa, 1986). For many developed economies, real exchange rates are roughly four times as volatile as output, and their fluctuations exhibit a half-life in the range of three to five years. Moreover, real and nominal exchange rates are highly correlated.[1] In principle, sticky-price models can explain this correlation and the high volatility: if price levels fail to adjust, changes in nominal exchange rates following monetary shocks will readily translate in real exchange rate movements. However, such models cannot produce

[1]Empirical evidence for these facts is presented in Chari, Kehoe, and McGrattan (2002) and Steinsson (2008).

the observed persistence under plausible nominal rigidities, as demonstrated by Bergin and Feenstra (2001) and Chari, Kehoe, and McGrattan (2002).[2] Is it possible to reconcile, in a single framework, the enormous short-term volatility of the real exchange rate with its extremely long half-life?

I study this classic open economy question in a two-country, flexible-price model in which firms have noisy, dispersed information about the economic environment. I show analytically that when firms face strategic complementarities in price-setting, uncertainty about other firms' beliefs results in sluggish price adjustments that can generate large and long-lived real exchange rate fluctuations. The model is estimated on output and output deflator data for the US and the Euro Area using Bayesian methods. I evaluate the quantitative success of the framework by asking whether it reproduces the dynamics of the Euro/Dollar real exchange rate, which were not targeted in the estimation. I find that the estimated model successfully explains these dynamics, as captured by the unconditional volatility and half-life of the real exchange rate, as well as its correlation with the nominal exchange rate. In addition, the model accounts for the persistent effects of monetary shocks on the real exchange rate that I document using a structural VAR. Finally, I conduct a Bayesian model comparison and find that the data strongly favor the dispersed-information framework relative to a sticky-price model à la Calvo.

The main contribution of the paper is to provide a novel explanation for the observed

[2] Subsequent research addresses the persistence anomaly by introducing strategic complementarities (Bouakez, 2005), inertial Taylor rules (Benigno, 2004) and real shocks (Steinsson, 2008; Iversen and Söderström, 2014). While these features increase the persistence of the exchange rate, they are not sufficient to jointly explain the observed half life of the real exchange rate as well as its relative volatility to consumption and output.

real exchange rate dynamics, by showing that the estimated dispersed-information model captures remarkably well the volatility and persistence of the real exchange rate. Closed-economy models in which agents are imperfectly informed are known to be quantitatively successful for explaining the highly persistent effects of monetary disturbances on output and inflation (Melosi, 2014) documented by VAR studies (e.g., Christiano, Eichenbaum, and Evans, 2005). However, little is known about these models' ability to explain the behavior of international relative prices. This paper fills this gap. First, it shows analytically that the model with dispersed information can deliver highly volatile and persistent real exchange rates. Second, it demonstrates quantitatively that the estimated model successfully accounts for the observed real exchange rate behavior. The model's success stems from its ability to generate endogenous persistence in the real exchange rate fluctuations that follow monetary shocks.

The second contribution lies in the quantitative comparison between the dispersed-information model and an alternative benchmark sticky-price model, a comparison currently missing in the open economy literature. To this end, I also estimate a two-country sticky price model à la Calvo. The Bayesian model comparison suggests that the output and output deflator data strongly favor the dispersed-information model relative to the sticky-price model. In sample, the dispersed-information model is already clearly preferred to the Calvo model, but the model with information frictions fares even better in the out-of-sample test, generating far more realistic real exchange rate dynamics.

The information friction that I model is motivated by the mounting evidence about heterogeneity in beliefs among decision makers. To illustrate, Figure 1.1 depicts the times series of the interquartile range of two types of forecasts: the professional analysts' one-year ahead forecasts of CPI inflation and real GDP growth, as taken from the Federal Reserve's Survey of Professional Forecasts. The time series show that there is considerable dispersion in these forecasts. Dispersion in beliefs is pervasive in the economy. Indeed, recent survey data show that there is also widespread dispersion in firms' beliefs about both past and future macroeconomic conditions (Coibion, Gorodnichenko, and Kumar, 2015). This evidence suggests that firms have their own "window on the world" (Amato and Shin, 2006). In this environment, defending a firm's market share will entail some degree of second-guessing competitors' pricing strategies. This second-guessing game might prove particularly challenging in an open economy, in which firms face competition not only from domestic firms but also from foreign exporters.

I follow Woodford (2002) and Melosi (2014) in modeling this heterogeneity in beliefs. Specifically, firms in the model observe private, idiosyncratic signals about nominal aggregate demand and aggregate productivity in the two countries. They also face strategic complementarity in price-setting, which implies that a firm's optimal price depends positively on the prices set by competitors. With private information, strategic complementarity requires firms to respond to higher-order beliefs i.e., beliefs about other firms' beliefs about underlying economic conditions. Beliefs update slowly, as the private signals a firm receives provide relatively little information about other firms' signals. Notwithstanding the absence of nominal rigidities, slow movements in beliefs translate into *endogenously* slow-moving prices. Therefore, while nominal shocks generate swings in the nominal exchange rate, the slow price dynamics can trigger large and persistent real exchange rate fluctuations.

Despite prices' dependence on an infinite hierarchy of beliefs, I can show analytically that the volatility and persistence of the real exchange rate are higher (i) the lower the precision of firms' signals about aggregate demand and (ii) the higher the degree of strategic complementarity. Intuitively, when signals are not very precise, firms learn slowly about changes in nominal aggregate demand and sluggishly update their prices. When strategic complementarities are strong, firms fail to adjust prices quickly in an effort of keeping their own prices in line with those of their rivals. Both of these channels slow down the price adjustment, delivering volatile and persistent real exchange rates following nominal shocks. Notably, strategic complementarity depends on the degree of the economies' openness and on the substitutability between domestic- and foreign-produced goods. Thus, foreign competition provides a channel through which the adjustment of prices might be delayed, one that is naturally absent in closed-economy models.

In the empirical part of the paper, I assess whether the model can quantitatively explain the dynamics of the Euro-Dollar real exchange rate in the period 1971-2011. In the model, low enough signal precisions would be able to generate a highly volatile and persistent real exchange rate, but it is unclear which values should be considered empirically relevant, given the scarce existing evidence on these parameters. To address this shortcoming, I estimate the model parameters via Bayesian methods using real GDP and GDP deflator data for the US and the Euro Area. These data do not directly contain information on the real exchange rate, which is instead defined as the nominal exchange rate adjusted by consumption price indices.

The exclusion of the real exchange rate from the estimation allows me to conduct an out-of-sample test for my model. Specifically, I simulate the model at the estimated parameter values and ask whether it reproduces the dynamics of the Euro/Dollar real exchange rate, which were not targeted in the estimation. I show that the model successfully explains these dynamics, as measured by the volatility, persistence, and half-life of the real exchange rate. The model also delivers the hump-shaped dynamics that are a salient feature of the Euro-Dollar real exchange rate and are central to the observed half life of about 4.5 years. Additionally, the estimated signal-to-noise ratios suggest that firms' signals about nominal aggregate conditions are less precise than signals about productivity, which generates persistence of the real exchange rate from nominal shocks. Using a structural VAR approach, I show that these persistent effects of monetary shocks on real exchange rates are indeed a feature of the data.

I compare these predictions with those of a standard sticky-price model, which I estimate using the same data on real GDP and GDP deflators. The sticky-price model deviates from the dispersed-information model in only two respects: (i) all agents are perfectly informed, and (ii) firms can optimally adjust their prices only in random periods, as in Calvo (1983). Two sets of results emerge. First, the dispersed-information model fits the data significantly better than the sticky-price model, as suggested by the Bayesian model comparison. Second, the model with information frictions is more successful at explaining the out-of-sample dynamics of the real exchange rate. The estimated Calvo model generates low real exchange rate persistence following monetary shocks, confirming the intuition behind the results of Chari, Kehoe, and McGrattan (2002). When technology shocks are added to the picture, the model produces a half-life of the real exchange rate that is twice as large as in the data. Intuitively, this happens because the estimated Calvo model requires large technology shocks to account for the volatility and persistence of output and domestic price indices. However, the size of these technology shocks and their internal propagation in the sticky-price model generates counterfactual predictions for the real exchange rate.

Finally, I investigate the robustness of the predictions of the dispersed-information model to changes in the firms' information set. Specifically, I allow firms to observe noisy signals about equilibrium prices, which are relevant for their price-setting decisions. A re-estimation of the model suggests that these additional signals are relatively noisy, and therefore they carry a low weight in the firms' signal-extraction problem. The implications are that the presence of these additional signals does not substantively ameliorate the fit of the model to the data and, leaves the real exchange rate dynamics unchanged.

This paper contributes to the growing literature that focuses on the aggregate implications of dispersed information among price setters, such as Woodford (2002), Maćkowiak and Wiederholt (2009), Nimark (2008), and Melosi (2014), which builds on the seminal contributions of Phelps (1970) and Lucas (1972). In contrast to most of this literature, which is developed in closed economies and focuses on inflation dynamics, this paper studies the implications for international prices, where uncertainty about foreign demand as well as foreign competitors' actions, plays an important role. My analysis lends further empirical support to the dispersed-information theory, by testing its natural predictions in an open-economy environment. The paper is also naturally related to the literature that studies real exchange rate dynamics in the context of monetary models, such as Johri and Lahiri (2008) and Carvalho and Nechio (2011), in addition to the works already mentioned. Relative to this literature, this paper highlights the importance of a source of endogenous persistence in real exchange rates—dispersed information in environments with strategic complementarities—that has so far been ignored in this context.

Finally, the present study adds to the small literature that focuses on information frictions in open economies. Bacchetta and van Wincoop (2006, 2010) combine information frictions with a finance approach to study other puzzles in international macroeconomics, such as the exchange-rate disconnect and the the forward-discount puzzle. Crucini, Shintani, and Tsuruga (2010) introduce sticky information à la Mankiw and Reis (2002) in a sticky-price model to explain the volatility and persistence of deviations from the law of one price. They find that such a model can explain the empirical half life of eighteen months if information updates occur every 12 months. In contrast, this paper seeks to explain the substantially longer half-life of aggregate real exchange rates by relying only on dispersed information and Bayesian updating, which is consistent with the recent evidence on firms' behavior, as documented by Coibion, Gorodnichenko, and Kumar (2015).

The paper proceeds as follows. Section 2 develops the dispersed-information model. Section 3 provides some analytical results. Section 4 discusses the solution method. Section

5 analyzes the model's impulse responses. Section 6 contains the empirical analysis. Section 7 draws a comparison with the sticky-price model. Section 8 studies the sensitivity of the results to the information structure. Section 9 offers some concluding remarks.

1.2 The Model

The framework is a two-country open-economy monetary model that follows the international macroeconomic tradition initiated by Obstfeld and Rogoff (1995). The setup is similar to Corsetti, Dedola, and Leduc (2010). The world economy consists of two countries of unit mass, denominated H (Home) and F (Foreign), each populated by households, a continuum of monopolistically competitive producers, and a monetary authority. Each country specializes in the production of one type of tradable goods, produced in a number of varieties or brands, with measure equal to the population size. All goods produced are traded and consumed in both countries. Prices are set in the currency of the producer; therefore, the law of one price holds. Deviations of the real exchange rate from purchasing-power parity arise because households exhibit home bias in consumption preferences.

All information is, in principle, available to every agent; however, firms can only pay limited attention to the information available, owing to finite information-processing capacity (Sims, 2003). Following Woodford (2002) and Melosi (2014), this idea is modeled by assuming that firms do not perfectly observe current and past realization of the variables in the model, but rather only observe private noisy signals about the state of nominal aggregate demand and technology.[3] Firms use these signals to draw inferences about other model variables. Households and the monetary authorities are assumed, for tractability, to observe the current and past realization of all the model variables. Below I present the structure of the Home economy in more detail. The Foreign economy is symmetric, and foreign variables will be denoted with an asterisk.

1.2.1 Preferences and Households

The utility function of the representative household in country H is

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \int_0^1 \frac{L_{ht}^{1+1/\psi}}{1+1/\psi} di \right] \right\} \tag{1.1}$$

The representative household has full information, $\mathbb{E}(.)$ denotes the statistical expectations operator, and $\beta < 1$ is the discount factor. Households receive utility from consumption C_t and disutility from working, where L_{ht} indicates hours of labor input in the production of domestic variety $h \in [0, 1]$. Risk is pooled internally, to the extent that all domestic agents receive the same consumption level. The parameter $\psi > 0$ represents the Frisch elasticity of labor supply. Following Woodford (2003, Ch. 3), each of the home varieties (indexed by h over the unit interval) uses a specialized labor input in its production. As noted by Woodford, this type of differentiated labor markets generates more strategic complementarities in price-setting.[4]

[3] The implications of relaxing this assumption are explored in Section 1.8.

[4] Pricing decisions are strategic complements if, when other firms raise their prices, a given firm i wishes to raise its price as well. It is closely related to the concept of "real rigidity", in that it depends solely

Households consume both types of traded goods. The consumption of these goods is denoted by C_{Ht} and C_{Ft}. For each type of goods, one brand or variety is an imperfect substitute for all the other brands, and γ is the elasticity of substitution between brands. Mathematically, consumption baskets of Home and Foreign goods by Home agents are a CES aggregate of Home and Foreign brands, respectively:

$$C_{Ht} \equiv \left(\int_0^1 C_t(h)^{\frac{\gamma-1}{\gamma}} dh \right)^{\frac{\gamma}{\gamma-1}} \qquad C_{Ft} \equiv \left(\int_0^1 C_t(f)^{\frac{\gamma-1}{\gamma}} df \right)^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1$$

The overall consumption basket, C_t, is defined as

$$C_t \equiv \left[\alpha^{\frac{1}{\omega}} (C_{Ht})^{\frac{\omega-1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} (C_{Ft})^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \qquad \omega > 0$$

where α is the weight of the home consumption good and ω is the elasticity of substitution between home and foreign goods, which I alternatively refer to as the trade elasticity. The utility-based consumption price index (CPI) is

$$P_t = \left[\alpha P_{Ht}^{1-\omega} + (1-\alpha) P_{Ft}^{1-\omega} \right]^{\frac{1}{1-\omega}}$$

where P_{Ht} and P_{Ft} are the price sub-indices for the home- and foreign-produced goods,

upon real factors: the structure of production costs and of demand. Strategic complementarities arise also in the presence of decreasing returns or input-output structures in production (Basu, 1995). For a discussion, see Ball and Romer (1990) and Woodford (2003, Ch. 3).

expressed in domestic currency

$$P_{Ht} = \left(\int_0^1 p_t(h)^{1-\gamma} dh \right)^{\frac{1}{1-\gamma}} \qquad P_{Ft} = \left(\int_0^1 p_t(f)^{1-\gamma} df \right)^{\frac{1}{1-\gamma}}$$

Foreign prices are similarly defined. The Foreign CPI is

$$P_t^* = \left[(1-\alpha)(P_{Ht}^*)^{1-\omega} + \alpha (P_{Ft}^*)^{1-\omega} \right]^{\frac{1}{1-\omega}}$$

Let $\mathcal{Q}_t$ denote the real exchange rate, that is, the relative price of consumption: $\mathcal{Q}_t \equiv \frac{\epsilon_t P_t^*}{P_t}$, where ϵ_t is the nominal exchange rate expressed in domestic currency per foreign currency. Even if the law of one price holds at the individual good level (i.e., $P_t(h) = \epsilon_t P_t(h)^*$, which implies $P_{Ht} = \epsilon_t P_{Ht}^*$), the presence of home bias in consumption—that is $\alpha > 1/2$—implies that the price of consumption may not be equalized across countries. Put differently, purchasing-power parity ($\mathcal{Q}_t = 1$) will generally not hold. The terms of trade are defined as the price of imports in terms of exports: $\mathcal{T}_t = \frac{P_{Ft}}{\epsilon_t P_{Ht}^*}$. If the law of one price holds, the real exchange rate will be proportional to the terms of trade

$$q_t = (2\alpha - 1)\, t_t \tag{1.2}$$

where, throughout the paper, lower-case letters denote percentage deviations from steady state.[5] Equation (1.2) implies that an improvement in the terms of trade always appreciates

[5]This result assumes symmetric initial conditions.

the real exchange rate. This is consistent with the empirical evidence (Obstfeld and Rogoff, 2000). Minimizing expenditure over brands and over goods, one can derive the domestic household demand for a generic good h, produced in country H, and the demand for a good f, produced in country F:

$$C_t(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\alpha C_t \qquad C_t(f) = \left(\frac{P_t(f)}{P_{Ft}}\right)^{-\gamma}\left(\frac{P_{Ft}}{P_t}\right)^{-\omega}(1-\alpha)C_t$$

Assuming that the law of one price holds, total demand for a generic home variety h or foreign variety f may be written as

$$Y_t^d(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\left[\alpha C_t + (1-\alpha)\mathcal{Q}_t^{\omega} C_t^*\right] \tag{1.3}$$

$$Y_t^d(f) = \left(\frac{P_t(f)^*}{P_{Ft}^*}\right)^{-\gamma}\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\omega}\left[(1-\alpha)\mathcal{Q}_t^{-\omega} C_t + \alpha C_t^*\right] \tag{1.4}$$

1.2.2 Budget Constraint

The generic home household's budget constraint can be written as

$$P_t C_t + \int q_{H,t}(s_{t+1})\mathcal{B}_{H,t}(s_{t+1})ds_{t+1} \leq \int_0^1 W_{ht}L_{ht}dh + \mathcal{B}_{H,t} + P_t\int_0^1 \Pi_{ht}dh \tag{1.5}$$

$\mathcal{B}_{H,t}(s_{t+1})$ is the holding of state-contingent claims that pay off one unit of domestic currency if the realized state of the world at time $t+1$ is s_{t+1} and $q_{H,t}(s_{t+1})$ is the time-t price of such an asset. W_{ht} is the wage for the h-th type of labor input and Π_{ht} are the real profits of domestic firm h. Maximizing (3.6) subject to (1.5) gives the static first-order condition:

$$C_t^{\sigma} L_{ht}^{1/\psi} = W_{ht}/P_t \tag{1.6}$$

and the following Euler equation

$$1 = \beta(1 + R_{t+1})\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{1.7}$$

where R_{t+1} is the net risk-free rate of return between t and $t+1$.

1.2.3 Monetary Policy

Following Woodford (2002) and Carvalho and Nechio (2011), I leave the specification of monetary policy implicit, and assume that the growth rate of nominal aggregate demands $M_t = P_t C_t$ and $M_t^* = P_t^* C_t^*$ follows exogenous autoregressive processes

$$\Delta m_t = \rho_m \Delta m_{t-1} + u_t^m \tag{1.8}$$

$$\Delta m_t^* = \rho_{m^*} \Delta m_{t-1}^* + u_t^{m^*} \tag{1.9}$$

where $\Delta m_t \equiv \ln M_t - \ln M_{t-1}$ and the monetary shocks u_t^m and $u_t^{m^*}$ are *i.i.d.*, distributed as $\mathcal{N}(0, \sigma_m^2)$ and $\mathcal{N}(0, \sigma_{m^*}^2)$ and uncorrelated across countries.[6] I refer to these shocks as nominal demand shocks or monetary shocks, with the understanding that they capture

[6] This formulation of aggregate demand can also be justified by the presence of a cash-in-advance constraint.

structural shocks that move nominal aggregate demand. The variable M_t can be interpreted as a measure of money supply, such as M1 or M2, or more broadly as a measure of aggregate demand, such as nominal GDP. This specification is widely used in the monetary literature and has been shown to be a good approximation of the process that implements estimated Taylor rules of the types studied in Christiano, Eichenbaum, and Evans (1998).

1.2.4 Exchange Rate Determination

Asset markets are assumed to be internationally complete. Complete markets implies the following risk-sharing condition

$$\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{P_t}{P_{t+1}} = \left(\frac{C_t^*}{C_{t+1}^*}\right)^{\sigma}\frac{\epsilon_t P_t^*}{\epsilon_{t+1}P_{t+1}^*}$$

This equation relates the cross-country differential in the growth rate of consumption to the depreciation of the exchange rate. Assuming symmetric initial conditions, this can be rewritten as

$$\frac{\epsilon_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*}\right)^{\sigma} \tag{1.10}$$

Equation (1.10) is an efficiency condition that equates the marginal rate of substitution between home and foreign consumption to their marginal rate of transformation, expressed as equilibrium prices, i.e., the real exchange rate. A key consequence is that home consumption can rise relative to foreign consumption only if the real exchange rate depreciates.[7]

[7] This implication is known to be at odds with the the data, where real exchange rates and consumption differentials exhibit low or negative correlation (Backus and Smith, 1993). This counterfactual implication

Equation (1.10), combined with the processes for nominal aggregate demand and optimal prices, determines real and nominal exchange rates under complete markets.

1.2.5 Price-setting Decisions

Firms do not perfectly observe the state of aggregate demand and their marginal cost, but at each date they receive private signals about economic conditions. Prices are set in the producer's currency and there are no barriers to trade, so the law of one price always holds. Firm h's expected real profits in period t, conditional on the history of signals observed by that firm at time t, are given by

$$\Pi_{ht} = \mathbb{E}_{ht}\left[\frac{P_t(h)}{P_t}Y_t^d(h) - \frac{W_{ht}}{P_t}L_{ht}\right] \tag{1.11}$$

where $\mathbb{E}_{ht}$ is the expectation operator conditional on firm h's information set, $\mathcal{I}_h^t$. The production function is given by

$$Y_t(h) = A_t L_{ht} \tag{1.12}$$

Total factor productivity, A_t, in the two countries follows the processes

$$\ln A_t = \rho_a \ln A_{t-1} + u_t^a \tag{1.13}$$

could be fixed by assuming incomplete asset markets or by making preference assumptions that break the tight link between marginal utilities and current consumption (e.g., introducing habit formation or non-separable utility). Previous work on this topic suggests that, if anything, these modifications would increase the volatility and persistence of the real exchange rate, thus strengthening my results. For tractability, I proceed with the assumption of complete markets.

$$\ln A_t^* = \rho_{a^*} \ln A_{t-1}^* + u_t^{a^*} \tag{1.14}$$

The shocks u are mean zero and have variances σ_a^2 and $\sigma_{a^*}^2$, respectively.

Each firm in the home country receives the following signals:

$$Z_{h,t} = \begin{bmatrix} z_{h,t}^m \\ z_{h,t}^{m^*} \\ z_{h,t}^a \\ z_{h,t}^{a^*} \end{bmatrix} = \begin{bmatrix} m_t \\ m_t^* \\ a_t \\ a_t^* \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_m & 0 & 0 & 0 \\ 0 & \tilde{\sigma}_{m^*} & 0 & 0 \\ 0 & 0 & \tilde{\sigma}_a & 0 \\ 0 & 0 & 0 & \tilde{\sigma}_{a^*} \end{bmatrix} \begin{bmatrix} v_{h,t}^m \\ v_{h,t}^{m^*} \\ v_{h,t}^a \\ v_{h,t}^{a^*} \end{bmatrix} \tag{1.15}$$

where $v_{h,t}^m, v_{h,t}^{m^*}, v_{h,t}^a, v_{h,t}^{a^*} \sim \mathcal{N}(0,1)$, $a_t = \ln A_{h,t}$ and $a_t^* = \ln A_t^*$. $m_t = \ln M_t$ and $m_t^* = \ln M_t^*$ represent the nominal aggregate demands (or money supplies), and the signal noises are assumed to be independently and identically distributed across firms and over time. Foreign firms receive similar signals drawn from the same distributions. In every period t, firms observe the history of their signals Z_h^t (that is, their information set is $\mathcal{I}_{ht} = \{Z_{h,\tau}\}_{\tau=-\infty}^t$) and maximize (1.11) subject to (1.12) and (1.4). The first-order condition yields

$$P_t(h) = \frac{\gamma}{\gamma - 1} \frac{\mathbb{E}_{ht}\left[\left(\frac{1}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\frac{C_t^W}{P_t}\frac{W_{ht}}{P_t A_t}\right]}{\mathbb{E}_{ht}\left[\left(\frac{1}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\frac{C_t^W}{P_t}\right]} \tag{1.16}$$

where $C_t^W \equiv \alpha C_t + (1-\alpha)\mathcal{Q}_t^\omega C_t^*$. Equation (1.16) states that a firm optimally sets its price to a markup, $\frac{\gamma}{\gamma-1}$, over its *perceived* marginal cost. Following the tradition in this literature, I log-linearize the price-setting equation around the deterministic steady state so that the transition equations of average prices are linear. I assume that firms use the log-linearized model, rather than the original nonlinear model when addressing their signal-extraction problem. This assumption greatly simplifies the analysis, because it allows for the use of the Kalman filter to characterize the dynamics of firms' beliefs. Finally, I assume that at the beginning of time, firms are endowed with an infinite history of signals. This implies that the Kalman gain matrix is time-invariant and identical across firms.

1.2.6 Real Exchange Rate Dynamics

In this section I characterize the solution for the real exchange rate. To simplify the algebra and convey intuition, I henceforth assume log utility for consumption ($\sigma = 1$). Appendix A.1 shows how the model can be solved also for a generic value of σ. As also shown in Appendix A.1, under the producer currency pricing (PCP) assumption, the log-linearized first-order condition for a generic h and f firm, combined with equation (1.2), reads

$$p_t(h) = E_{ht}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t)\right] \tag{1.17}$$

$$p_t^*(f) = E_{ft}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t^* - a_t^*)\right] \tag{1.18}$$

where $\xi = \frac{1+\psi}{\gamma+\psi}$. These equations show the interdependence of the optimal price with their foreign counterpart through the terms of trade. In particular, if home and foreign goods are substitutes ($\omega > 1$), other things equal, a rise in the price of foreign goods (that is, a rise in t_t) causes expenditure switching away from foreign goods toward home goods. The increased demand for home goods increases firm's h marginal cost and makes it optimal to raise $p_t(h)$. If goods are instead complements ($\omega < 1$), a rise in t_t decreases demand both for foreign and home output, hence the optimal price for a home good $p_t(h)$ falls.

The parameter $1-\xi$ is related to the degree of strategic complementarities in price-setting, i.e., it tells by how much the optimal price of an individual firm changes when all the other domestic competitors are changing their prices. Because $\gamma > 1$, then $0 < \xi < 1$. Integrating (1.17) over domestic agents and (1.18) over foreign agents and noting that the log-linear price indices read as $p_{Ht} = \int_0^1 p_t(h)dh$ and $p^*_{Ft} = \int_0^1 p^*_t(f)df$, I obtain

$$p_{Ht} = \bar{\mathbb{E}}_t^{(1)} \left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m_t - a_t) \right] \tag{1.19}$$

$$p^*_{Ft} = \bar{\mathbb{E}}_t^{(1)} \left[(1-\xi)p^*_{Ft} - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m^*_t - a^*_t) \right] \tag{1.20}$$

where $\bar{\mathbb{E}}_t^{(1)}(\cdot) = \int_0^1 \bar{\mathbb{E}}_{it}(\cdot)di$ for $i = h, f$ denotes a first-order average expectation. Note that $\int_0^1 \bar{\mathbb{E}}_{ht}(\cdot)dh = \int_0^1 \bar{\mathbb{E}}_{ft}(\cdot)df$ follows from the symmetry of the information structure. Equations (1.19) and (1.20) can be disentangled following the tradition of the "sum" versus "differences" approach in general equilibrium open-economy models (Aoki, 1981). Specifically, I can take the sum of (1.19) and (1.20) to obtain

$$p_{Ht} + p^*_{Ft} = \bar{\mathbb{E}}_t^{(1)} \left[(1-\xi)(p_{Ht} + p^*_{Ft}) + \xi(m_t + m^*_t) - \xi(a_t + a^*_t) \right] \tag{1.21}$$

which yields the solution

$$p_{Ht} + p^*_{Ft} = \xi \sum_{k=1}^{\infty} (1-\xi)^{k-1} \bar{\mathbb{E}}_t^{(k)} (m_t^W - a_t^W) \tag{1.22}$$

where for any variable x_t, I define $x_t^W \equiv x_t + x_t^*$ and $x_t^D \equiv x_t - x_t^*$. Additionally $\bar{\mathbb{E}}_t^{(k)}(\cdot) = \int_0^1 \bar{\mathbb{E}}_{it}^{(k-1)}(\cdot)di$ denotes the *k-th*-order average expectation. By taking the difference between (1.19) and (1.20) and substituting the solution for the terms of trade, I obtain

$$p_{Ht} - p^*_{Ft} = \bar{\mathbb{E}}_t^{(1)} \left[(1-\varphi)(p_{Ht} - p^*_{Ft}) + \varphi(m_t - m_t^*) - \xi(a_t - a_t^*) \right] \tag{1.23}$$

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$. The solution to the above equation yields

$$p_{Ht} - p^*_{Ft} = \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D - \xi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} a_t^D \tag{1.24}$$

The solution for p_{Ht} and p^*_{Ft} can be found by taking sums and differences of equations (1.22) and (1.24). Proposition 1 follows.[8]

Proposition 1 *Under the assumption of log-utility and complete asset markets, the real exchange rate is given by*

$$q_t = (2\alpha - 1) \left(m_t^D - \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D - \xi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} a_t^D \right) \tag{1.25}$$

[8] Detailed derivations are in Appendix A.1.

where $1-\varphi \equiv 1-\frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$ *governs the degree of strategic complementarity.*

The intuition behind this equation is straightforward. Focus for a moment on the first two terms on the right-hand side of (1.25) and consider a relative shock to nominal demands, m_t^D. Under full-information rational expectations, we expect the shock to have no effect on the real exchange rate, because nominal prices should adjust one for one with the nominal demands. Indeed, with full information $\bar{\mathbb{E}}_t^{(k)} m_t = m_t$ and $\bar{\mathbb{E}}_t^{(k)} m_t^* = m_t^*$ for every k so that $m_t^D = \varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D$, and the real exchange rate responds only to real shocks.

Under imperfect information instead, as long as we have home bias, the real exchange rate also responds to nominal shocks to the extent that higher-order expectations deviate from full-information rational expectations. Equation (1.25) shows also that the persistence of the response of the real exchange rate to relative monetary shocks depends on how quickly the weighted average of higher-order expectations $\varphi \sum_{k=1}^{\infty} (1-\varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D$ adjusts. As shown in section 1.3, the speed of adjustment depends on the degree of strategic complementarities (φ for relative variables) and on the signal-to-noise ratios $\sigma_m/\tilde{\sigma}_m$ and $\sigma_{m^*}/\tilde{\sigma}_{m^*}$. Specifically, the signal-to-noise ratios determine how quickly the different order of expectations in the summation will adjust to shocks. The strategic-complementarity parameter determines the *weights* attached to the different orders. For instance, the average first-order expectation about m_t^D receives a weight φ, the second order receives a weight $\varphi(1-\varphi)$, the third $\varphi(1-\varphi)^2$, and so on.

The last term on the right-hand side of (1.25) indicates that the real exchange rate always responds to relative technology shocks or, in the presence of dispersed information, to the weighted-average of higher-order beliefs about the shock.

1.2.7 Strategic Complementarities in the Open Economy

As discussed above, the strategic-complementarity parameter $(1-\varphi)$ is an important determinant of the dynamics of the real exchange rate, as it affects the weights attached to different orders of expectations. In this section I explain how this parameter crucially depends on the elasticity of substitution between home and foreign goods. In the case of log utility we have

$$(1-\varphi) = 1 - \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi} = 1-\xi\left[1+2\zeta\right] \tag{1.26}$$

where I define $\zeta = \frac{2\alpha(1-\alpha)(\omega-1)}{1+\psi}$. To build intuition let us focus on the case of a closed economy first, obtainable by setting the home-bias parameter α to one (which implies $\zeta = 0$). In this case, the optimal pricing equations (1.17) and (1.18) would read

$$p_t(h) = E_{ht}\left[(1-\xi)p_{Ht} + \xi(m_t - a_t)\right] \tag{1.27}$$

$$p_t^*(f) = E_{ft}\left[(1-\xi)p_{Ft}^* + \xi(m_t^* - a_t^*)\right] \tag{1.28}$$

Here the degree of strategic complementarity is governed by $1-\xi = 1-\frac{1+\psi}{\gamma+\psi} \in (0,1)$. Consider the experiment of increasing p_{Ht}, keeping everything else constant. A domestic firm h responds to a increase in the average price p_{Ht} by increasing its own price. This happens because the increase in p_{Ht} shifts demand away from competitors toward firm's h output, and with specialized labor markets firm's h marginal cost is increasing in its own output. The strength of the increase in $p_t(h)$, measured by $(1-\xi)$, depends on the size of the change in firm's h demand, as captured by the elasticity of substitution between domestic goods γ, and on the slope of the labor supply curve, governed by the Frisch elasticity ψ.

Now consider the same experiment as above in the case in which the economies are open. Rewriting the pricing equations (1.17) and (1.18) using the solution for the terms of trade yields

$$p_t(h) = E_{ht}\left\{[1-\xi(1+\zeta)]p_{Ht} + \xi\zeta(p^*_{Ft} + m^*_t - m_t) + \xi(m_t - a_t)\right\} \tag{1.29}$$

$$p^*_t(f) = E_{ft}\left\{[1-\xi(1+\zeta)]p^*_{Ft} - \xi\zeta(m^*_t - m_t - p_{Ht}) + \xi(m^*_t - a^*_t)\right\} \tag{1.30}$$

Now the response of $p_t(h)$ to an increase in the average domestic price, p_{Ht}, is determined by the strategic-complementarity parameter $[1-\xi(1+\zeta)]$, which will have the same sign as $(1-\varphi)$ in (1.26). Note that this response might be smaller or larger than in the closed-economy case, depending on whether the value of ω is above or below unity. The intuition goes as follows. Under our maintained assumption of log utility in this Section, when $\omega > 1$, home and foreign goods are net substitutes. This diminishes strategic complementarity relative to the closed economy, because an increase in p_{Ht} now shifts demand *away* from all the other domestic goods, partly *toward* firm h's good and partly toward foreign goods. Thus, firm h experiences a milder increase in marginal cost and changes its price by a smaller amount than if it were to operate in a closed economy. Conversely, when $\omega < 1$, home and foreign goods are net complements. An increase in p_{Ht} induces a larger increase in firm's h marginal cost relative to the closed-economy case, and firm h raises its price by a larger amount. These additional effects are captured in the strategic-complementarity parameter $1-\varphi$ via ζ. Thus the substitutability between home and foreign goods has important implications for the degree of strategic complementarity, which in turns affects the dynamics of the real exchange rate through the channels described in Section 1.3.

1.3 Analytical Results

To gain intuition about the cyclical properties of the real exchange rate in response to monetary shocks, let us abstract from technological shocks and study the simple case in which money supplies follow a random walk. Precisely, for this section I assume that $a_t = a_t^* = a$ and

$$m_t = m_{t-1} + u_t^m \tag{1.31}$$

$$m_t^* = m_{t-1} + u_t^{m^*} \tag{1.32}$$

which is obtained as a special case from equation (1.8) setting $\rho_m = \rho_{m^*} = 0$. With random walks in nominal spending and linear updating implied by the signal-extraction problem, I can establish Proposition 2.

Proposition 2 *Assuming random-walk processes for nominal spending and complete asset markets (CM), the real exchange rate follows an AR(1) process*

$$q_t = \nu q_{t-1} + (2\alpha - 1)\nu(u_t - u_t^*)$$

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$, $1 - \nu = \varphi \times \kappa_1 + (1 - \varphi) \times \kappa_2 \in (0, 1)$, *and* κ_1, κ_2 *are the non-zero elements of the Kalman gains matrix. The autocorrelation and variance of the real exchange rate are*

$$\rho_{\hat{Q}} = \nu \qquad \sigma^2_{\hat{Q}} = (2\alpha - 1)^2 \left(\frac{\nu}{1-\nu}\right)^2 (\sigma^2_u + \sigma^2_{u^*})$$

Proof. *In Appendix A.2.*

Proposition 2 shows that, under the above assumptions, the real exchange rate follows an AR(1) process. The Proposition highlights how its persistence, ν, depends on the relevant degree of strategic complementarity, φ, and the precision of the signals that determine the weights κ_1 and κ_2 in the Kalman gain matrix. Larger noise and more strategic complementarity increase the persistence of the exchange rate. This is illustrated in Figure 1.2, which depicts the iso-persistence of the real exchange rate as a function of φ and the inverse signal-to-noise ratios $\tilde{\sigma}^2_m/\sigma^2_m$, assumed to be identical for m_t and m^*_t.

A lower φ indicates a higher degree of strategic complementarities, which means that agents put a larger weight on their beliefs about others' actions (and beliefs about others' beliefs about others' actions) relative to their own belief about the current state of nominal demand. This implies that higher-order beliefs receive a higher weight than lower-order beliefs. With high-order beliefs moving more sluggishly than low-order beliefs[9], prices adjust more sluggishly, which in turn implies slower movements in the real exchange rate following a money shock. Additionally, when the relative precision of the signal falls ($\tilde{\sigma}_m^2/\sigma_m^2 \downarrow$), agents will weight their prior more than their signals, failing to change prices and only slowly updating their beliefs when monetary shocks hit the economy. While the shock immediately affects the nominal exchange rate, the slow movement in prices again triggers slow reversion of the real exchange rate to purchasing-power parity.

Finally, notice from Propositions 2 that the a higher ν not only affects the persistence of the exchange rate, but also its volatility. To understand this, consider the response of prices when a monetary shock hits the Home economy. The higher the value of ν, the smaller the adjustment of home prices at the impact of the shock, for the same reasons discussed above. The small impact response of prices drives the amplification of monetary shocks onto the real exchange rate.

1.4 Model Solution

Models with dispersed information and strategic interactions are hard to solve because they feature the "infinite regress" problem in which agents are required to forecast the forecast of others, which results in an infinite dimensional state space (Townsend, 1983). A number of approaches have been developed to solve this class of models. A numerical approach

[9] See Woodford (2002) or Melosi (2014) for further explanation and graphical examples.

consists of guessing and verifying the laws of motion for the vector of higher-order beliefs. Since this vector is infinite-dimensional, in practice it is truncated at a sufficiently high order.[10] Another approach—used, for instance, in Lorenzoni (2009) and which will be used in some extensions below—uses a truncation in the time dimension.

In some cases, one can exploit the fact that only a particular *weighted average* of higher-order expectations matters for the solution of the model (Woodford, 2002; Melosi, 2014). The advantage of this approach is that the state vector has a finite dimension and there is no need to truncate it. The model developed here meets the conditions for the applicability of this method. By looking at equations (1.22) and (1.24), it is clear that determining the dynamics of $\varphi \sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ and $\xi \sum_{k=1}^{\infty}(1-\xi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ for $x = a, m$ is sufficient to determine the endogenous prices p_{Ht} and p^*_{Ft}. In turn, one can use these two variables together with the nominal exchange rate to solve for the rest of the model. Hence, to solve the model I guess that the state of the system includes the exogenous state variables plus the two specific weighted averages of high-order expectations implied by equations (1.22) and (1.24). In particular, I define $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$where $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$ is the vector of exogenous state variables. $X_t^{(k)}$ is shorthand notations for $\bar{\mathbb{E}}_t^{(k)} X_t$. The transition equation for the model can be shown to be

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{1.33}$$

[10] For an example, see Nimark (2011).

where

$$\bar{X}_t = \left[\begin{array}{c} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{array}\right] \quad \bar{B} = \begin{bmatrix} B_{6\times6} & 0 & 0 \\ \Gamma^{\xi,x}_{6\times6} & \Gamma^{\xi,\xi}_{6\times6} & 0 \\ \Gamma^{\varphi,x}_{6\times6} & 0 & \Gamma^{\varphi,\varphi}_{6\times6} \end{bmatrix} \quad \bar{b} = \left[\begin{array}{c} b_{6\times4} \\ \hline \Gamma^{\xi,u}_{6\times4} \\ \hline \Gamma^{\varphi,u}_{6\times4} \end{array}\right] \quad u_t = \begin{bmatrix} u^m_t \\ u^{m^*}_t \\ u^a_t \\ u^{a^*}_t \end{bmatrix}$$

Equation (1.33) is the state transition equation of the system. Firms in the model use the observation equation (1.15) and its foreign counterpart to form expectations about the state vector. The zeros in the $\bar{B}$ matrix reflect the fact that in this model, "sums" variables evolve independently of "differences" variables. The matrices B and b are given by the exogenous processes for monetary policy, whereas matrices Γ are to be determined by solving the signal-extraction problem of the firms using the Kalman filter. One can show that these matrices are functions of the parameters of the model and the Kalman gain matrix associated with the firms' signal-extraction problem. The algorithm used in Woodford (2002) can be easily extended to solve this model.

1.5 Impulse Responses

In this section I study the properties of the model in the more general case in which monetary processes can be autocorrelated ($\rho_m \neq 0$) and the economies are also hit by technology shocks.[11]

1.5.1 Monetary Shocks

Figure 1.3 shows the impulse responses of key variables to a positive monetary shock in the home country for a value of $\rho_m = 0.5$ and different signal-to-noise ratios. Prices for goods produced in the home country increase, but—because price adjustment is incomplete with imperfect information—domestic output also rises. Foreign output falls because, according to the parameterization used, home and foreign goods are net substitute. Consumption rises in both countries, more so in Home given the presence of home bias. The nominal exchange rate (not shown) depreciates as a result of the monetary expansion. The difference between home and foreign goods' prices rises by less than the nominal exchange rate, resulting in a worsening (upward movement) of the terms of trade. The real exchange rate depreciates, as it is proportional to the terms of trade. Finally, domestic inflation rises as the prices of both home goods and foreign goods rise in domestic currency. Conversely, foreign inflation falls, as foreign goods' prices are unchanged, and home goods' prices fall in domestic currency.

The introduction of persistent monetary shocks results in hump-shaped responses for most key macro variables, including the real exchange rate. The hump in the response of the real exchange rate is consistent with the empirical literature (Steinsson, 2008). Interestingly, domestic producer-price inflation displays a hump for persistent monetary shocks. Hence, this model is consistent with the inertial behavior of inflation observed in the data(Christiano,

[11] In this section, the model is parameterized using the calibration and prior means described in Section 1.6.2, unless otherwise noted.

Eichenbaum, and Evans, 2005).

An increase in private signal noise delivers more volatility and persistence in the exchange rate. The intuition for this result is the same as that highlighted in the previous section, whereby with noisy signals, firms put little weight on new information and adjust prices very slowly. Figure 1.4 shows that higher strategic complementarity also contributes to increased volatility and persistence in the exchange rate, as it increases the weight put on other firms' beliefs in price-setting decisions.

1.5.2 Technology Shocks

Figure 1.5 shows the impulse responses to a home technology shock with persistence $\rho_a = 0.95$ for different signal-to-noise ratios. A home technology shock raises domestic output and lowers the prices of home-produced goods. The shock is transmitted internationally via a depreciation of the real exchange rate. Consumption rises in both countries but more markedly in the home country. Varying the signal-to-noise ratio, we observe that more noise tends to dampen the effect of technology shocks, although it contributes to somewhat higher persistence. The intuition behind these result relies on the fact that in this model, output can rise in response to technology shocks only if prices fall, because nominal expenditure is fixed by the levels of money supplies.[12] When signals are more precise firms change prices quickly and output can rise substantially. When signals are noisier, firms fail to lower prices enough, and therefore output increases by a smaller amount.

[12] This corresponds to the case in which monetary authorities do not accommodate technology shocks.

An interesting feature of this model compared to a sticky-price model à la Calvo is that it potentially allows for slow responses to nominal shocks and quicker responses to supply shocks if technology shocks are observed with relatively high precision. A sticky-price model would instead imply a more similar speed of adjustments to different shocks, governed by a single parameter: the exogenous probability of resetting prices.

1.6 Empirical Analysis

This section contains the econometric analysis that evaluates whether the dispersed-information model can account for the empirical properties of the Euro Area/Dollar real exchange rate. The analysis will proceed as follows. First, I will estimate the model parameters using Bayesian techniques. The estimation will help me pin down values for the parameters of the model, in particular the signal-to-noise ratios, for which empirical micro evidence is scarce. I will then use the estimated model to test how well it captures the dynamics of the real exchange rate.

1.6.1 Data and Empirical Strategy

I estimate the parameters of the dispersed-information model using data on the US and Euro Area. The US data comes the FRED database, while the European data comes from the Area Wide Model database.[13] I use the time series of the growth rate of GDP and GDP deflators that I map to the variables $[\Pi_t^H, \Pi_t^F, \Delta Y_t^H, \Delta Y_t^F]$ in the model, where

[13] I am grateful to my advisor, Susanto Basu, for granting me access to the Euro Area data.

$\Pi_t^H = \frac{P_{Ht}}{P_{H,t-1}}$ and $\Pi_t^F = \frac{P^*_{Ft}}{P^*_{F,t-1}}$. For the US, I construct GDP growth by taking the log-difference of real GDP (*GDPC96*) divided by the civilian non institutional population over 16 (*CNP16OV*). The growth rate of the GDP deflator is the log-difference of *GDPDEF*. For the Euro Area, I take the log difference of the real GDP (*YER*) divided by population. Population data for the 17 countries in the Euro Area, consistent with the GDP series, is taken from the OECD database.[14] The sample period goes from 1971:I to 2011:IV. All series are demeaned to be consistent with the model. The US is considered to be the home country. Before estimation the model is stationarized (details are in Appendix A.3).

My empirical strategy is as follows. I estimate the parameters of the dispersed-information model using data on real GDP and GDP deflators for the US and the Euro Area. Using these four observables allows me to pin down the key parameters of the model, including the signal-to-noise ratios related to monetary and technology shocks, for which there is scarce micro evidence. Given the estimated parameters, I subsequently test whether the dispersed information model is quantitatively able to generate the volatility and persistence observed in the Euro/Dollar real exchange rate. This empirical strategy is analogous in spirit to the common practice of calibrating a model to fit certain moments (in this case, the moments of real GDP growth and GDP-deflator inflation rates included in the likelihood function) and testing how well the model reproduces other moments in the data (here, the moments of the real exchange rate). This setup effectively allows me to conduct an out-of-sample test on the real exchange rate, as none of its moments were directly used in the estimation

[14] Population data are available only at annual frequency. I use linear interpolation to obtain the quarterly frequency.

of the model parameters.

It is important to notice that the four series used in the estimation contain very little information about the real exchange rate. First, the real exchange rate in the data is constructed using CPIs rather than GDP deflators. The regressions reported in Table 1.2 show that the four series used in the estimation explain at most 17% of the variation in the real exchange rate. Second, most of the variation in the real exchange rate in the data comes from movements in the nominal exchange rate, which has not been used in the estimation.[15]

1.6.2 Fixing Parameters and Priors

I fix the values of the parameters that are not well identified in the estimation process. Specifically, I set home bias $\alpha = 0.9$ to match the average import-to-GDP ratio for the US over the sample. The parameter γ is set to 7 following Mankiw and Reis (2010), which implies a steady-state markup of 16.7%. Finally, I set σ to 4, a slightly lower value than Steinsson (2008). The calibration is summarized in Table 3.1. I estimate the rest of the parameters.[16]

The prior distributions for the estimated parameters are summarized in Table 1.3. The priors for the standard deviation of shocks and noise terms follow Melosi (2014). There is no clear evidence on the value of the trade elasticity ω, although macro studies usually point toward low values.[17] The prior mean is set to one. The parameter $\xi = \frac{1+\psi}{\gamma+\psi}$ is related to strategic complementarities, and its prior mean is set to 0.4, implying a prior mean for

[15] Regressing the real exchange rate on the nominal exchange rate alone produces an R^2 of 91%.
[16] The discount factor β does not appear in any linearized model equation.
[17] See Corsetti, Dedola, and Leduc (2008) for a discussion.

ψ of 3. The persistence parameters for technology shocks are centered at 0.86 whereas the persistence for monetary shocks is set at 0.5—although for these parameters, I let the data guide the estimation by leaving priors fairly loose.

The model is estimated using Bayesian techniques, as explained in Herbst and Schorfheide (2016). Specifically, I draw from the posterior distribution $p(\Theta|Y)$, where Θ is the parameter vector and Y the data, using a standard random walk Metropolis-Hasting algorithm. The variance-covariance matrix of the proposal distribution, Σ, is set to the variance-covariance matrix of the estimated parameters at the mode of the posterior distribution. I then draw 1,000,000 parameter vectors from the posterior distribution. With this procedure, I get an acceptance rate of about 25%.

1.6.3 Posterior Distribution

In Table 1.4, I present the estimates for the benchmark economy. The table shows the posterior median, together with a 90% posterior confidence band. The posterior of ξ is relatively tight around 0.21, lower than the prior mean, suggesting that the data are informative about this parameter. The posterior median for the persistence of technology shocks in the two countries is 0.98, in line with many other studies. The persistence of money growth processes is 0.45 for the US and 0.76 for the Euro Area. These values are linked to the growth rate of nominal GDP over the sample period for the two countries.[18]

The median estimate for the trade elasticity ω is 0.49. While relatively low, this number

[18]Note that the observables Π_t^H and ΔY_t^H sum to the log of nominal GDP growth in the data and to Δm_t in the model.

is comparable to the estimates of other studies that use a likelihood approach on US and Euro Area data. Lubik and Schorfheide (2006) estimate a trade elasticity of 0.43, while Rabanal and Tuesta (2010)'s estimates are in the range of 0.16-0.94, depending on the model specification. In the present context, low trade elasticity contributes to generating strategic complementarity as discussed in Section 1.2.7.

The parameters that are most important for the present analysis are the signal-to-noise ratios. For monetary or nominal demand shocks, the median estimates are $\sigma_m/\tilde{\sigma}_m = 0.08$ and $\sigma^*_m/\tilde{\sigma}^*_m = 0.07$. For the technology shocks, $\sigma_a/\tilde{\sigma}_a = 0.57$ and $\sigma_a/\tilde{\sigma}_a = 0.78$. These results indicate that firms are more informed about technology shocks than they are about nominal demand shocks by a factor of seven. Melosi (2014), who estimates a closed-economy model similar to the one used here, also finds that firms pay more attention to technology than to nominal demand shocks, and shows that this is consistent with the predictions of a rational inattention model (Sims, 2003), in which firms have to optimally choose how much attention to allocate to the two types of shocks. In a posterior predictive check, Melosi shows that the estimated signal-to-noise ratios are consistent with micro evidence on the absolute sizes of price changes (Nakamura and Steinsson, 2008).

The presence of information frictions implies that firms do not generally set their price equal to the profit-maximizing price, which is defined as the price a particular firm would set if it had complete information. I further validate the estimates of the signal-to-noise ratios by asking how much firms lose, in terms of profits, from being imperfectly informed. Arguably, it would not be plausible to remain poorly informed about the state of the economy if that implied incurring large profit losses. In Appendix A.5, I show that the estimated signal-to-noise ratios imply profit losses well below 1% of steady state revenues—0.5% of steady state revenues for a US firm and 0.8% for a European firm. These profit losses are small, and comparable in size to empirical estimates of the information cost of price adjustment, which is 1.22% of a firm's revenues according to the findings of Zbaracki et al. (2004). The Appendix also shows that the losses in the dispersed-information model are one order of magnitude smaller than the losses that would arise in a sticky-price model à la Calvo that generates similar real effects from monetary shocks.

1.6.4 How Well Does the Model Explain the Real Exchange Rate?

Here I test how well the estimated model captures the dynamics of the real exchange rate observed in the data. The real exchange rate consists of the nominal exchange rate in U.S. dollar per Euros, converted to the real exchange rate index by multiplying it by the Euro area CPI (*HICP*) and dividing it by the U.S. CPI (*CPIAUCSL*). The "synthetic" US/Euro nominal exchange rate prior to the launch of the Euro also comes from the the Area Wide Model Database. As for the Bayesian estimation, the sample period runs from 1971:I to 2011:IV.

Following the empirical approach of Steinsson (2008) and Carvalho and Nechio (2011), I calculate measures of persistence of the real exchange rate based on the estimates of an

AR(p) process of the form.

$$q_t = \mu + \alpha q_{t-1} + \sum_{j=1}^{p} \psi_j \Delta q_{t-j} + \epsilon_t \tag{1.34}$$

where I calculate median unbiased estimates of μ, α, and ψ's using the grid-bootstrap method described by Hansen (1999). I set $p = 5$.

The first three columns of Table 1.5 report several measures of persistence and volatility of the real exchange rate. In the top part of the table, I compute the half-life (HL), up-life (UP), and quarter-life (QL) following a unitary impulse response. The half-life is defined as the largest T such that the impulse response $IR(T-1) > 0.5$ and $IR(T) < 0.5$. The up-life and quarter-life are defined similarly, but with thresholds 1 and 0.25, respectively. All these measures are useful in capturing the non monotonically decaying shape of the exchange rate impulse response. I also consider the more traditional measures of persistence, such as the sum of autoregressive coefficients (captured by α) and the autocorrelation of the HP-filtered exchange rate. All the statistics are reported in years. The second part of Table 1.5 reports measures of volatility and cross-correlation of the real exchange rate.

We can analyze the persistence of the real exchange rate by looking at its response to a unitary impulse, depicted with a black line in Figure 1.6. The response displays a typical hump-shaped behavior, peaking in the second quarter at about 1.3 and not falling below the initial impulse—the up-life—for 9 quarters. The half-life of the exchange rate, the most commonly used measure of persistence, is about 4.4 years, which is well in line with previous evidence. Finally, the quarter-life of the exchange rate is 6.7 years, which implies that the time the exchange rate spends below one half of the initial response but above one quarter of it is 2.3 years, suggesting a moderate acceleration in the rate of decay when short-run dynamics start to die out. These findings are well in line with empirical evidence from other countries (Steinsson, 2008), and point to the presence of a hump shape in the impulse response of the exchange rate also for the US and Euro Area as well. Moreover, Table 1.5 highlights that the real exchange rate is extremely volatile: 5.8 times as volatile as consumption and 4.8 times as volatile as output. Finally, the correlation between the real and nominal exchange rate is 0.99.

To assess the empirical success of the dispersed-information model, I use the following algorithm to compute statistics that are comparable with the data:

- Step 1: Draw a parameter vector from $p(\Theta|Y)$.
- Step 2: Simulate the dispersed-information model for n periods and discard the initial $n/2$ observations. n is chosen such that $n/2$ is equal to the length of the actual data.
- Step 3: Estimate equation (1.34) on the simulated data and compute the relevant statistics.

I repeat the procedure for 10,000 iterations and consider the 90% posterior band. The last three columns of Table 1.5 report the results from the model.

The table shows that the model is notably successful in matching the moments from the data, even though its parameters were not estimated by targeting these moments. The model predicts a median half-life of 5.17 years, which is close to the 4.38 years observed in the data. The model implies a ratio of up-life to half-life that is somewhat larger than in the data. Similar to what is found in the data, the rate of decay of the exchange rate moderately accelerates in later periods, as can be seen in the difference between the half-life and the quarter-life. Finally, the autocorrelation of the HP filtered exchange rate is also in line with the empirical results. Figure 1.6 displays these results visually by superimposing the impulse responses from the model and the data. While the model implies a slightly more pronounced hump in the early quarters after the impulse, the dynamics farther out from the initial impulse tend to be quite similar to the data. The simulated real exchange rates also exhibit the high volatility and the strong correlation with the nominal exchange rate observed in the data. Overall, the estimated dispersed-information model successfully replicates the observed real exchange rate dynamics. These results are noteworthy, considering that the model parameters were not pinned down to match the empirical moments of the real exchange rate and that they imply reasonably small profit losses from limited attention.

1.6.5 Monetary Shocks and Persistent Real Exchange Rates

The previous section showed how the dispersed-information model, driven by monetary shocks and technology shocks, is able to capture the large and persistent fluctuations in real exchange rates. These are statements about the unconditional moments of the real exchange rate. Two interesting questions that remain open are the following: does the estimated model deliver persistent responses of the real exchange rate following monetary shocks alone? And are these dynamics consistent with the observed behavior of the real exchange rate, conditional on monetary shocks?

To address the first question, I simulate the model at the median estimates under the assumption that all fluctuations are due to monetary shocks. The corresponding properties of the real exchange rate are reported in the last column of Table 1.5. The table shows that the implied volatility of the real exchange rate, relative to consumption and output, is similar to the the data. Additionally, the model still generates highly persistent real exchange rate dynamics. The half-life of the exchange rate falls only to 4.13 from 5.17 years when both monetary shocks are productivity shocks were present. A half-life of 4.13 years is very well in line with the observed half life of 4.38 years. These results suggest that monetary shocks in the dispersed information model are able to generate empirically relevant volatility and persistence in real exchange rate dynamics.

I proceed to examine how the real exchange rate responds to monetary shocks in the data. A vast literature attempts to identify the effects of monetary shocks on real exchange rates (e.g., Clarida and Gali, 1994; Rogers, 1999; Eichenbaum and Evans, 1995). The literature highlights the difficulty of the task. Following the empirical macro literature, I identity a monetary shock in the data by means of a structural VAR.

I estimate a two-variable VAR using the real exchange rate and the CPI differential between the US and the Euro Area. Specifically, the variables are collected in the vector $X_t = [\Delta \ln RER_t, \Delta(\ln CPI_t^{US} - \ln CPI_t^{EU})]$.To identify the monetary shock in the VAR, I use the restriction that monetary shocks have no long-run effects on the real exchange rate (e.g., Blanchard and Quah, 1989). This identification scheme is consistent with the dispersed-information model. In keeping with the Bayesian spirit of the paper, I follow Sims and Zha (1998) in specifying the prior distribution for the VAR parameters. I obtain 10,000 posterior draws using the Gibbs sampler.

Figure 1.7 reports the impulse response of the level of the real exchange rate to a monetary shock from the estimated VAR, along with the median impulse response to a home monetary shock in the dispersed-information model.

The impulse response from the VAR highlights the fact that monetary shocks have persistent effects on the real exchange rate. The real exchange rate peaks two quarters after the impulse, displaying hump shaped dynamics, like in the unconditional dynamics. The dispersed information model does really well in capturing these dynamics, as well as the size of the response. The response from the model peaks three quarters after the impulse and then decays at a slightly slower rate compared to the data, but well within the 70% posterior credible set. Again, it should be noted that the parameters in the model were estimated without any reference to real exchange rates.

1.6.6 Business-cycle Moments

To understand how the model performs along other dimensions of the international business cycle, this subsection presents results for several business-cycle statistics commonly analyzed in the literature. Table 1.6 reports the business-cycle moments obtained from the dispersed information model and compares them with the analogous statistics obtained from the data.

For the data, the statistics are based on logged and HP-filtered quarterly data for the period 1971:I to 2011:IV. For the model economy, I simulate time series of 158 quarters from the model and HP-filter the simulated data. In the Table, I report the average statistics across 200 replications.

The table shows that the model produces reasonable results along most of the business-cycle dimensions considered. In terms of volatilities, as in the data, the model predicts that consumption is less volatile than GDP. The model also accurately predicts that nominal exchange rates are more volatile than real exchange rates, which are in turn more volatile than the foreign versus domestic price ratio. This is a considerable improvement relative to the sticky-price model of Chari, Kehoe, and McGrattan (2002), in which the price ratio is much more volatile than in the data. The model also predicts quite volatile net exports.

As to the autocorrelations, the model generates considerable persistence in most of the variables considered, delivering long-lasting dynamics not only in prices but also in quantities such as real GDP, consumption, and net exports, although not so much for employment. In this respect, the dispersed-information model is more successful than the sticky-price model developed by Chari, Kehoe, and McGrattan (2002), which does not generate quite as much persistence in output and consumption.

The model reproduces the positive correlation between home and foreign consumption, output, and employment observed in the data. In terms of the constitutive "pieces" of the real exchange rate, the model also predicts reasonably well the negative correlations between real exchange rate and price ratio, and between nominal exchange rate and price ratio, in addition to the already noted strong positive correlation between nominal and real exchange rates.

There are a few limitations to the model's predictions, which relate to some of the assumptions made in order to keep the model tractable enough to be estimated. The model predicts a strong positive correlation between real exchange rates and relative consumption, which is at odds with the data. This discrepancy is expected, given our assumption of complete asset markets and the results of Backus and Smith (1993). Given the low estimate for the trade elasticity, the findings of Corsetti, Dedola, and Leduc (2008) suggest that assuming incomplete international asset markets is likely to significantly reduce the positive correlation in the model. As commonly found in international real business-cycle models and in contrast to the data, Home and Foreign GDPs exhibit lower cross-correlations than consumptions.[19] Chari, Kehoe, and McGrattan (2002) show that this issue can be addressed by assuming that monetary shocks are correlated across countries.

Finally, the model predicts a pro-cyclical trade balance, while in the data it is counter-cyclical. This is also expected because of the absence of investment in the model. Indeed, by simple national accounting, one can show that if consumption is less volatile than output, the trade balance must be pro-cyclical.[20] Introducing capital accumulation can ameliorate the predictions of the model along this dimension, provided that consumption and investment move in the right direction.[21]

[19] See, for instance Backus, Kehoe, and Kydland (1994) and Heathcote and Perri (2002).

[20] By national accounting in this model, $C = Y - NX$. Hence $Var(C) = Var(Y) + Var(NX) - 2Cov(Y, NX)$, which implies that if $Var(C) < Var(Y)$ then $Cov(Y, NX) > 0$.

[21] For a discussion of the matter, see Raffo (2010).

1.7 Comparison with Sticky-price Model

A natural question that arises in evaluating the empirical success of the imperfect-information model is how well it performs relative to a more traditional sticky-price model à la Calvo (1983). In this section I address the question in two ways. First, I estimate a model with sticky prices à la Calvo and compare its fit to the data relative to the dispersed-information model. Second, I compare the sticky-price model's ability to reproduce the observed real exchange rate dynamics relative to the model with information frictions.

1.7.1 The Calvo Model

Households and monetary authorities are modeled in the same way as in the benchmark economy. Firms can perfectly observe the current and past realization of shocks, but can only reset their prices with a random probability $1 - \theta$. The derivations of the model are standard and can be found, for instance, in Corsetti, Dedola, and Leduc (2010). The dynamics of inflation can be described by the New Keynesian Phillips Curves:

$$\pi_t^H = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{H,t} - \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} \tau_t - \frac{1+\psi}{\gamma+\psi} a_t \right] + \beta \mathbb{E}_t \pi_{t+1}^H \tag{1.35}$$

$$\pi_t^F = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{F,t} + \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} \tau_t - \frac{1+\psi}{\gamma+\psi} a_t^* \right] + \beta \mathbb{E}_t \pi_{t+1}^F \tag{1.36}$$

where $\pi_t^H = p_{H,t} - p_{H,t-1}$, $\pi_t^F = p_{F,t}^* - p_{F,t-1}^*$ and $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. These two equations replace equations (1.22) and (1.24) of the dispersed-information model.

1.7.2 Bayesian Model Comparison

In this section I take a Bayesian approach to compare the dispersed-information model and the sticky-price model. I start by parameterizing the Calvo model. The parameters α,γ, and σ are calibrated to the same values used in the benchmark model. The discount factor β and the Calvo parameter θ cannot be identified separately in the estimation, as they both enter the slope of the Phillips Curve in a non linear fashion. I calibrate the discount factor β to 0.99. I estimate the parameter $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. I set the prior of κ such that the median implies a value of the Calvo parameter $\theta = 0.69$, and the 5th and 95th percentile imply values for θ of approximately 0.5 and 0.90. This range broadly covers the micro and macro estimates for the frequency of price adjustment. The prior for the remaining parameters shared across models is the same as in Section 6. I report my estimates in Table 1.7, along with the median estimates from the dispersed-information model.

The most remarkable difference in posterior estimates across the two models is the estimated standard deviation of productivity shocks. The Calvo model estimates are 3 times as large those of the benchmark model. Note that the latter estimates are consistent with a standard real business cycle calibration of these shocks, while the Calvo estimates are much larger (Kydland and Prescott, 1982). The median estimate for κ implies a value for the Calvo parameter θ of 0.67, and that prices change every three quarters.

The Bayesian approach used in this paper allows me to compare how the dispersed-information and the sticky-price frameworks fit the data overall by computing the posterior probability of each model. I refer to the dispersed-information and the Calvo model with

$\mathcal{M}_{DI}$ and $\mathcal{M}_C$, respectively. I denote the parameter vector associated with each model as Θ_{DI} and Θ_C, respectively. The posterior probability of model $\mathcal{M}_i$ with $i \in \{DI, C\}$ is given by

$$\pi_{T,\mathcal{M}_i} = \frac{\pi_{0,\mathcal{M}_i} p(Y|\mathcal{M}_i)}{\sum_{s\in\{DI,C\}} \pi_{0,\mathcal{M}_s} p(Y|\mathcal{M}_s)}$$

where $\pi_{0,\mathcal{M}_s}$ is the prior probability of model $\mathcal{M}_s$ and Y denotes the dataset used in the estimation. $p(Y|\mathcal{M}_s) = \int \mathcal{L}(\Theta_s|Y, \mathcal{M}_s)p(\Theta_s|\mathcal{M}_s)d\Theta_s$ is the marginal data density (MDD) or marginal likelihood of model $\mathcal{M}_s$, where $\mathcal{L}(\cdot)$ is the likelihood function and $p(\Theta_s|\mathcal{M}_s)$ denotes the prior distribution for the parameter vector Θ_s. As is standard, the prior probabilities, $\pi_{0,\mathcal{M}_s}$, are assumed to be the same across models, that is, $\pi_{0,\mathcal{M}_s} = 1/2$ for all $s \in \{DI, C\}$. Therefore, the model that attains the largest posterior probability is the one with the highest MDD.

The last row of Table 1.7 reports the log of the MDD for the two models. The comparison reveals that the dispersed-information model has a larger posterior probability than the sticky-price model by 43.4 log points. This difference is sizable. It implies that the prior probability ratio in favor of the Calvo model would need to be larger than $7.05e^{18}$ in order for the Calvo model to attain a higher posterior probability than the dispersed-information model. The fact that the Calvo model has fewer parameters than the dispersed-information model is not worrisome, because the MDD penalizes models for the number of their parameters. These findings suggest that the model with information frictions is considerably better suited for explaining the joint dynamics of US and Euro Area key macro variables than the sticky-price model.

1.7.3 Real Exchange Rate Statistics

Now I compare the models' ability to reproduce the real exchange rate dynamics. I focus on the ability of the two models to generate the volatility and the persistence of the exchange rate. Table 1.9 presents statistics for the real exchange rate in the two models unconditionally or conditionally on monetary shocks. Results reported are the median estimates from 200 simulations, using the same methodology described in Section 1.6.4. All statistics are reported in years.

Comparing columns 2 and 4 reveals that the estimated Calvo model delivers both low volatility and low persistence conditional on monetary shocks. The half-life of the real exchange rate in the model is just above 2 years. These results provide additional support for Chari, Kehoe, and McGrattan (2002)'s claim that monetary shocks in sticky price model cannot explain the persistence of the real exchange rate found in the data. While the models considered differ in some assumptions, the result obtained by those authors in a calibration exercise are qualitatively similar to the results obtained here, where the model is instead estimated. Additionally, while the model is able to explain fairly well the volatility of the exchange rate relative to output or consumption, it explains only about half of the absolute volatility of the real exchange rate. Column 4 shows that the dispersed information model is more successful in all these dimensions as already discussed in Section 1.6.4.

In column 3, we observe that when technology shocks are added to the picture, the sticky price model still delivers too little volatility in the real exchange rate, but now it generates counterfactually high persistence. The half life of the exchange rate increases from 2.07 years with only monetary shocks to more than 10 years with both shocks. Hence, the model predicts a half-life that is twice as large as that observed in the data. Additionally, the median quarter-life is around 15 years, about 8 years longer than the quarter life observed in the data. In contrast, column 5 shows that the dispersed-information model delivers a half-life and quarter-life that are only marginally higher than with monetary shocks alone, keeping the real exchange rate dynamics in line with the data. The dispersed-information model also explains more than three quarters of the volatility observed in the data, compared to the 56% explained by the Calvo model.

The difference in the performance of the two models comes from (i) the different size of the estimated technology shocks in the two models and (ii) the different response to technology shocks across model. To the first point, we have seen above that the Calvo model estimates for technology shocks is 3 to 4 times larger than the estimates from the dispersed-information model. Intuitively, this happens because the estimated Calvo models requires large technology shocks to account for the volatility and persistence of output and domestic price indices while the dispersed-information model relies more on monetary shocks to explain that feature of the data.

The second point can be understood by examining the impulse-response function of the real exchange rate to monetary and technology shocks in the two models. The left panels of Figure 1.8 compare the response of the real exchange rate to a home monetary shock across the two models. For ease of comparison, the sizes of the shocks are set to the estimated standard deviations for the dispersed-information model reported in Table 1.7. The panel shows how the dispersed-information model delivers substantially more persistence from these shocks and a much more pronounced hump shape. This different response explains the difference between columns 2 and 4 of Table 1.9.

The right panels show the response of the exchange rate to productivity shocks. A few results emerge. First, the impact response of the real exchange rate is larger in the model with information friction than in the Calvo model. This happens because the presence of sticky prices substantially dampens the effect of productivity in the Calvo model relative to the efficient response. On the other hand, productivity shocks in the dispersed-information model are observed relatively precisely by firms, making the response to these shocks look more like an efficient response. Second, for similar reasons productivity shocks in the Calvo model damp out more slowly relative to the dispersed-information model. One can intuitively see from the picture that the half-life of these shocks is significantly greater in the sticky-price model.

This last fact and the different sizes of estimated technology shocks in the two models are responsible for the considerable difference in unconditional persistence found in the two models in columns 3 and 5 of Table 1.9.

1.7.4 Discussion

The results of this section highlight a number of differences between traditional sticky-price models and models in which slow price adjustment is the endogenous response to information frictions. First, estimation of the two models suggests that the dispersed-information model fits the data on output and output deflators better than the sticky-price model. The former also delivers estimates for the size of productivity shocks that are consistent with real business-cycle calibrations of these shocks. The sticky-price model instead requires substantially larger productivity shocks to explain the data.

Second, comparing the two models' ability to reproduce the empirical properties of the real exchange rates demonstrates the strength of the information-friction model relative to frameworks that model the nominal rigidity exogenously. In this particular case, assuming that prices can be reset with an exogenous probability irrespective of the shocks hitting the economy limits the model's ability to match the data. In the class of model with exogenous nominal rigidities considered here, there is a trade-off between obtaining amplification from nominal shocks and obtaining large effects from real shocks. This trade-off does not necessarily arise in models with dispersed information of the kind considered here, in which prices can respond differently to different kinds of shocks. Indeed the estimation results from the model with information frictions pushes exactly in this direction when seeking to fit the output and prices data, trying to obtain a slow response to monetary shock and a quick response to real shocks. It turns out that that this feature is important when it comes to predicting the real exchange rate.

In contrast, the Calvo model delivers too little persistence with only monetary shocks and too much persistence when both shocks are present. The result that monetary shocks cannot explain the persistence of the real exchange rate is qualitatively reminiscent of the results of Bergin and Feenstra (2001) and Chari, Kehoe, and McGrattan (2002). Differently from those, these findings are obtained in the context of an estimation exercise. The fact that real shocks cannot explain jointly the volatility and persistence of the exchange rate is also discussed in Iversen and Söderström (2014) in the context of a calibrated two-country model. There are modeling differences between the model considered here and theirs. Nonetheless, both models produce the result that with low elasticity of substitution between home and foreign goods, the presence of real shocks, while helping explain the volatility of the exchange rate relative to output and consumption, exaggerates the up-life, half-life, and quarter-life of the real exchange rate.

Taken together, these results suggest that the dispersed-information model outperforms the sticky-price model not only in explaining domestic variables, such as domestic output and prices, but it also better explains international price movements. I further validate this point by including the real exchange rate series among the observables and re-estimating both models. To accommodate the additional observable variable and avoid stochastic singularity, I add a measurement error to the real exchange rate equation. With these modifications, the dispersed-information model and the Calvo model deliver MDDs of 2518.45 and 2479.02, respectively. The difference of about 40 log points is sizable and points to the stronger ability of the model with information frictions to fit the data, consistently with other results in this section.

1.8 Sensitivity to Information Structure

In this section I investigate the role of the information structure in generating the persistence of the real exchange rate. The assumptions that firms observe signals about aggregate nominal demand and technology with finite precision is a simple way of capturing the idea that there is a cost in acquiring and processing information. In this context, the lower the cost of acquiring information, the higher the precision of the signals.

Nevertheless, it is worth noting that different signals may carry different information about the variables that matter for firms' decisions. In particular, the literature started by Grossman (1976) and Hellwig (1980) stresses the idea that, under certain conditions, prices may aggregate disparate information that different economic agents have. When making optimal pricing decisions, an important variable for the firms in the model are the aggregate price levels in the two countries. This can be seen from the first-order conditions (1.19) and (1.20), which I repeat here for convenience:

$$p_t(h) = \mathbb{E}_{ht}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t)\right]$$

$$p_t^*(f) = \mathbb{E}_{ft}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t^* - a_t^*)\right]$$

These two equations make clear that a firm's optimal price depends on its expectation about the aggregate price level for domestically produced goods and a measure of relative prices.

Here I entertain the hypothesis that firms observe signals about these prices. Specifically, in addition to the four signals observed in the benchmark model, firms in both the Home and Foreign country now have access to the following two signals:

$$z_{i,t}^{p^H} = p_{H,t} + x_t + v_{i,t}^{p^H}$$

$$z_{i,t}^{p^F} = p_{F,t}^{*} + x_t + v_{i,t}^{p^F}$$

for $i = h, f$. The signals about aggregate prices contain an aggregate noise component, x_t, and an idiosyncratic noise component $v_{i,t}$. All the noise terms are *iid*, normally distributed with mean zero and variances σ_x^2, $\tilde{\sigma}_{p^H}^2$ and $\tilde{\sigma}_{p^F}^2$, respectively. To assess the robustness of my results, I re-estimate the model allowing for the presence of these two additional signals and compare the real exchange rate between my benchmark model and this augmented model. I use flat priors for the additional parameters so as to let the data entirely guide the estimation.

It is worth noting that these new signals are endogenous, and they depend on equilibrium prices. This feature breaks the finite state-space representation of the model solution described in Section 1.4. Hence, I adapt the solution method used in Lorenzoni (2009), allowing it to handle two countries. The details of the algorithm are provided in the Appendix A.6. To solve the model, I write the state-space as the history of the time-t state variables used in the firm's inference problem: $[m_t, m_t^*, a_t, a_t^*, p_{H,t}, p_{F,t}^*]$ and I truncate it at

$t - T$. I choose T sufficiently high, so that by increasing T the impulse response functions of the model do not change.

Table 1.8 presents the posterior mode of the new estimates *vis-à-vis* the benchmark estimates. Most of the parameters that are common across the two model have similar estimates in the two cases. The aggregate noise and the idiosyncratic noises have estimated standard deviations of 25, 46, and 24, respectively. These values are larger then the standard deviations of the signals about monetary shocks, indicating that the data favor the idea that signals about aggregate prices are fairly noisy. The value of the posterior at the mode changes only marginally, suggesting that this extended model is not significantly superior in fitting the data relative to the benchmark model.

How does this affect the main results? Standard signal-extraction theory suggests that an agent should optimally put little weight on more imprecise signals. Hence we can already expect agents to put little weight on these new signals. Figure 1.9 shows the impulse response of the real exchange rate in the benchmark model and in the model with endogenous signals. There are only minor differences between the two, indicating that the dynamics of the model are not quantitatively affected by the presence of endogenous signals on prices once the model is re-estimated. Finally, Table 1.10 confirms these results by comparing the up-life, half-life, and quarter-life of the exchange rate in the two versions of the model and find no significant differences.

1.9 Conclusion

Existing New-Keynesian models with sticky prices struggle to deliver the persistence in the real exchange rate observed in the data under plausible nominal rigidities. In this paper, I argue that the persistence of the real exchange rate, together with its other empirical features, can be explained by a model with strategic complementarity and dispersed information among price-setting firms. In this environment, firms' beliefs about economic conditions and about other firms' expectations become endogenous state variables that result in increased persistence in real exchange rates. Once taken to the data, the model is shown to successfully explain the volatility and persistence of the real exchange rate. The model also generates persistent real exchange rate dynamics following monetary shocks, which is consistent with the empirical evidence documented by a structural VAR. Taken together, my findings suggest that dispersed information is a quantitatively important channel for real exchange rate dynamics that should be taken into account in future research on this topic.

Table 1.1: Calibrated Parameters

Name	Description	Value
γ	Elasticity of substitution between domestic goods	7
α	Home bias	0.9
σ^{-1}	Intertemporal elasticity of substitution	1/4

Table 1.2: Real Exchange Rate Predictability

	ΔRER_t	ΔRER_t	RER_t	RER_t
$\Delta RGDP_t^{US}$	-0.471	-0.456	-2.616	-2.988*
	(-1.00)	(-1.00)	(-1.73)	(-2.00)
$\Delta RGDP_t^{EU}$	0.832	0.656	-1.579	-2.198
	(1.39)	(1.14)	(-0.82)	(-1.16)
$\Delta DEFL_t^{US}$	-1.455	-3.428*	15.28***	6.523
	(-1.40)	(-2.53)	(4.60)	(1.47)
$\Delta DEFL_t^{EU}$	0.533	2.251**	-10.32***	-9.872***
	(0.78)	(2.68)	(-4.69)	(-3.59)
ΔCPI_t^{US}		2.737***		7.591**
		(3.41)		(2.88)
ΔCPI_t^{EU}		-2.735**		-0.216
		(-3.13)		(-0.08)
N	163	163	163	163
R^2	0.025	0.121	0.173	0.216
Adj. R^2	0.000	0.087	0.152	0.187

Notes : t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.3: Priors

Name	Description	Shape	Median	0.05	0.95
ξ	Strategic complementarity	$\mathcal{B}$	0.40	0.24	0.57
ω	Trade elasticity	$\mathcal{N}$	1.00	0.5	1.5
ρ_a	Persistence of technology shock (H)	$\mathcal{B}$	0.86	0.70	0.96
ρ_a^*	Persistence of technology shock (F)	$\mathcal{B}$	0.86	0.70	0.96
ρ_m	Persistence of monetary shock (H)	$\mathcal{B}$	0.50	0.25	0.75
ρ_m^*	Persistence of monetary shock (F)	$\mathcal{B}$	0.50	0.25	0.75
$100\sigma_a$	Std of technology shock (H)	$\mathcal{IG}$	0.68	0.53	0.92
$100\sigma_a^*$	Std of technology shock (F)	$\mathcal{IG}$	0.68	0.53	0.92
$100\sigma_m$	Std of monetary shock (H)	$\mathcal{IG}$	1.80	0.60	6.03
$100\sigma_m^*$	Std of monetary shock (F)	$\mathcal{IG}$	1.80	0.60	6.03
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	$\mathcal{NA}$	0.73	0.31	2.33
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	$\mathcal{NA}$	0.73	0.31	2.33
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	$\mathcal{NA}$	0.11	0.07	0.15
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	$\mathcal{NA}$	0.11	0.07	0.15

Notes : The letters $\mathcal{B}$,$\mathcal{N}$,$\mathcal{IG}$ denote the beta, normal, and inverse gamma distributions. $\mathcal{NA}$ is used for implied priors, which do not belong to any family of theoretical distributions.

Table 1.4: Posterior Estimates

Name	Description	Median	0.05	0.95
ξ	Strategic complementarity	0.21	0.16	0.29
ω	Trade elasticity	0.50	0.39	0.62
ρ_a	Persistence of technology shock (H)	0.98	0.97	0.99
ρ_a^*	Persistence of technology shock (F)	0.98	0.97	0.99
ρ_m	Persistence of monetary shock (H)	0.41	0.29	0.52
ρ_m^*	Persistence of monetary shock (F)	0.74	0.67	0.83
$100\sigma_a$	Std of technology shock (H)	0.97	0.76	1.19
$100\sigma_a^*$	Std of technology shock (F)	0.71	0.58	0.85
$100\sigma_m$	Std of monetary shock (H)	0.89	0.80	0.98
$100\sigma_m^*$	Std of monetary shock (F)	0.77	0.70	0.85
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	0.51	0.42	0.75
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	0.97	0.61	1.01
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	0.11	0.05	0.12
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	0.10	0.04	0.11

Notes : The table reports the median, the 5^{th}, and 95^{th} percentile of the estimates for the parameters of the dispersed-information model.

Table 1.5: Estimation Results

	Data			Model			Model
	Median	0.05	0.95	Median	0.05	0.95	Only M shocks
α	0.96	0.903	0.995	0.97	0.95	0.98	0.96
Half-life (HL)	4.38	2.05	38.54	5.17	3.68	7.43	4.13
Quarter-life (QL)	6.73	2.95	68.10	7.18	5.06	10.63	5.62
UL/UH	0.45	0.15	0.65	0.61	0.52	0.69	0.45
QL-HL	2.35	0.62	13.17	1.98	1.19	3.26	1.49
$\rho(q_{hp})$	0.84	0.75	0.87	0.89	0.87	0.92	0.89
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	5.83	-	-	4.24	3.71	4.76	4.13
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	4.83	-	-	3.65	3.11	4.20	3.64
$\rho(q_{hp}, \varepsilon_{hp})$	0.99	-	-	0.95	0.93	0.97	0.96

Notes : Half-life (HL): the largest T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$. Quarter-life (QL): the largest T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Up-life (UL): the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. ρ and σ correspond to first-order autocorrelation/cross-correlation and standard deviation, respectively. Only M shocks refer to the model driven only by monetary shocks.

Table 1.6: Business Cycle Statistics

Statistic	Data	Model
Standard deviations relative to GDP		
Consumption	0.82	0.86
Employment	0.89	1.17
Nominal Exchange Rate	4.94	4.54
Real Exchange Rate	4.73	3.55
Price Ratio	0.74	1.58
Net Exports	0.38	0.67
Autocorrelations		
GDP	0.87	0.83
Consumption	0.88	0.84
Employment	0.94	0.75
Nominal Exchange Rate	0.84	0.87
Real Exchange Rate	0.83	0.87
Price Ratio	0.89	0.92
Net Exports	0.86	0.85
Cross-correlations		
Home and Foreign GDP	0.52	0.10
Home and Foreign Consumption	0.36	0.57
Home and Foreign Employment	0.46	0.09
Net Exports and GDP	-0.53	0.52
RER and GDP	0.09	0.57
RER and Net Exports	0.18	0.92
RER and Relative Consumption	-0.14	1.00
Real and Nominal Exchange Rate	0.99	0.95
Nominal Exchange Rate and Price Ratio	-0.36	-0.73
RER and Price Ratio	-0.22	-0.50

Notes : With the exception of net exports, standard deviations and correlations in the table are based on logged and HP-filtered US and Euro Area data for the period 1971:I-2011:IV. Net exports are measured as the HP-filtered ratio of real net exports to real GDP. Thus, the standard deviation of net exports is simply the standard deviation of this ratio.

Table 1.7: Posterior Estimates

Name	Description	DI Model	Calvo Model
κ	$(1-\beta\theta)(1-\theta)/\theta$	—	0.16
ξ	Strategic complementarity	0.21	0.15
ω	Trade elasticity	0.49	0.58
ρ_a	Persistence of technology shock (H)	0.98	0.97
ρ_a^*	Persistence of technology shock (F)	0.98	0.96
ρ_m	Persistence of monetary shock (H)	0.45	0.28
ρ_m^*	Persistence of monetary shock (F)	0.76	0.66
$100\sigma_a$	Std of technology shock (H)	0.86	2.14
$100\sigma_a^*$	Std of technology shock (F)	0.81	2.62
$100\sigma_m$	Std of monetary shock (H)	0.90	0.87
$100\sigma_m^*$	Std of monetary shock (F)	0.77	0.75
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	0.57	—
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	0.78	—
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	0.08	—
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	0.07	—
MDD	Log Marginal Data Density	2461.9	2418.5

Notes : The table reports the median estimates for the parameters of the dispersed-information (DI) model and the Calvo model.

Table 1.8: Posterior Estimates Comparison

Name	Description	Benchmark	Endo Signals
ω	Trade elasticity	0.50	0.45
ρ_a	Persistence of technology shock (H)	0.99	0.99
ρ_a^*	Persistence of technology shock (F)	0.99	0.99
ρ_m	Persistence of monetary shock (H)	0.42	0.41
ρ_m^*	Persistence of monetary shock (F)	0.74	0.72
$100\sigma_a$	Std of technology shock (H)	1.47	1.38
$100\sigma_a^*$	Std of technology shock (F)	1.06	1.11
$100\sigma_m$	Std of monetary shock (H)	0.87	0.90
$100\sigma_m^*$	Std of monetary shock (F)	0.75	0.77
$100\tilde{\sigma}_a$	Std noise — technology shock (H)	2.45	2.38
$100\tilde{\sigma}_a^*$	Std noise — technology shock (F)	0.88	1.84
$100\tilde{\sigma}_m$	Std noise — monetary shock (H)	9.55	11.93
$100\tilde{\sigma}_m^*$	Std noise — monetary shock (F)	6.87	7.71
$100\sigma_x$	Std of aggregate noise	-	25.96
$100\sigma_{v_H}$	Std of idiosyncratic noise (H)	-	46.49
$100\sigma_{v_F}$	Std of idiosyncratic noise (F)	-	23.73
$p(\theta\|Y)$	Log Posterior at the Mode	2505.2	2510.7

Notes : The table reports the mode the parameters of the dispersed-information model (Benchmark) and the model with endogenous signals (Endo Signals).

Table 1.9: Real Exchange Rate Statistics Comparison

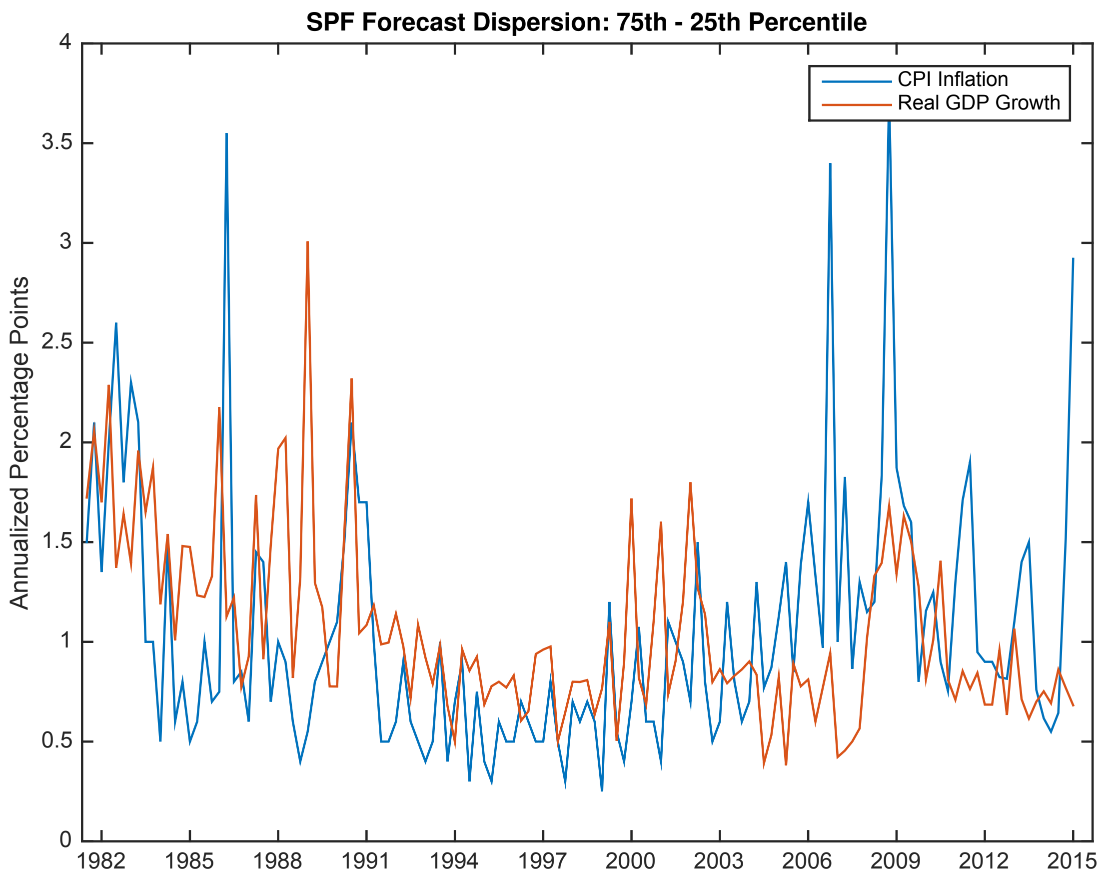
	Data	Calvo	Calvo Only M	DI	DI Only M
Half-life (HL)	4.38	10.25	2.07	5.17	4.13
Up-life (UL)	1.99	5.15	0.97	3.05	2.56
Quarter-life (QL)	6.73	14.96	3.06	7.18	5.62
$\rho(q_{hp})$	0.84	0.86	0.81	0.89	0.89
$100\sigma(q_{hp})$	72.8	40.5	32.6	57.2	55.3
$100\sigma(y_{hp})$	15.7	12.8	10.0	15.5	15.4
$100\sigma(c_{hp})$	13.0	10.5	8.2	13.5	13.4

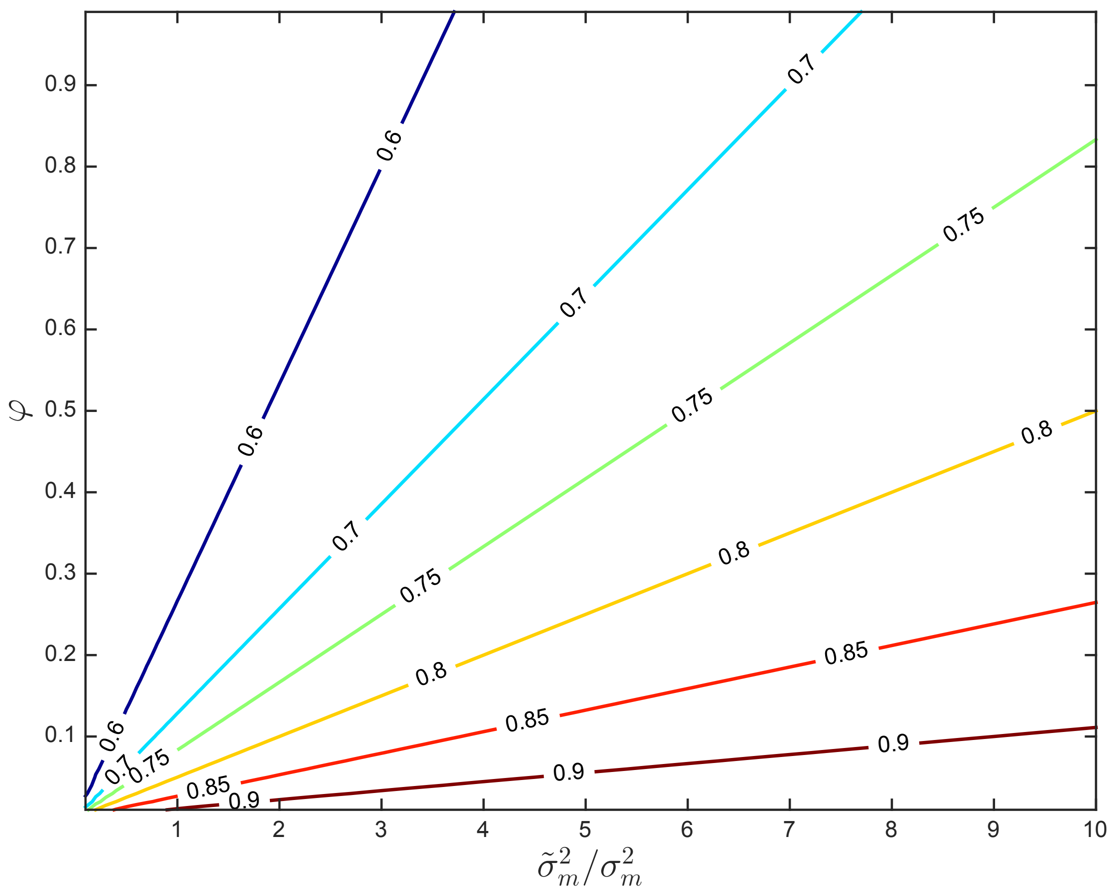
Notes : DI refers the dispersed-information model. Only M refers to the model driven only by monetary shocks.

Table 1.10: Exchange Rate Persistence

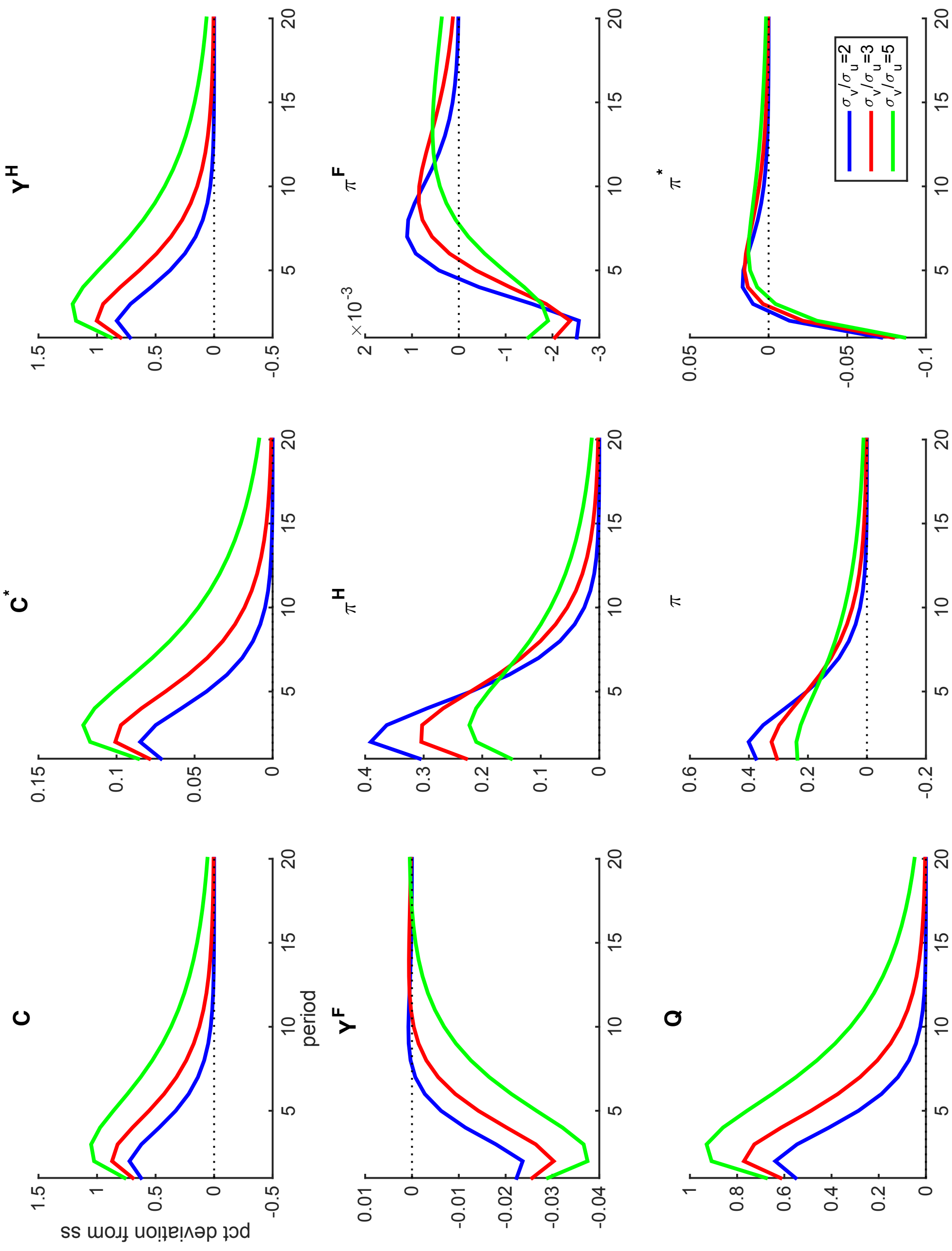
	Data	Baseline	Endogenous Signals
α	0.96	0.97	0.97
Half-life (HL)	4.38	5.17	5.82
Up-life	1.99	3.06	3.26
Quarter-life (QL)	6.73	7.18	8.34
$\rho(q_{hp})$	0.84	0.89	0.88
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	5.83	4.24	4.02
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	4.83	3.65	3.54
Log Posterior	-	2505.2	2510.7

Notes : Half-life (HL): the largest T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$. Quarter-life (QL): the largest T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Up-life (UL): the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. ρ and σ correspond to first-oder autocorrelation and standard deviation, respectively.

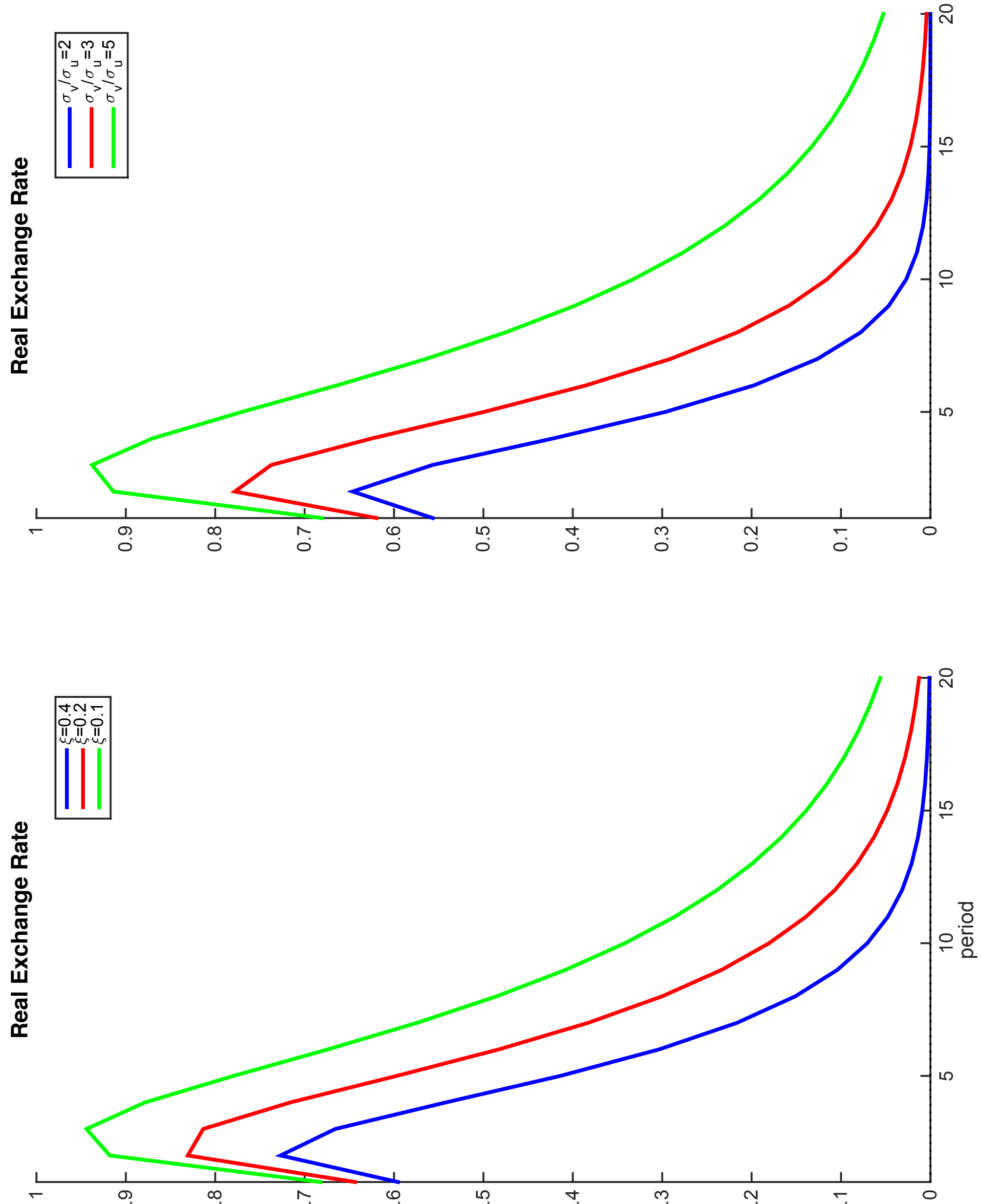

SPF Forecast Dispersion: 75th - 25th Percentile
Annualized Percentage Points
CPI Inflation
Real GDP Growth
0
0.5
1
1.5
2
2.5
3
3.5
4
1982
1985
1988
1991
1994
1997
2000
2003
2006
2009
2012
2015



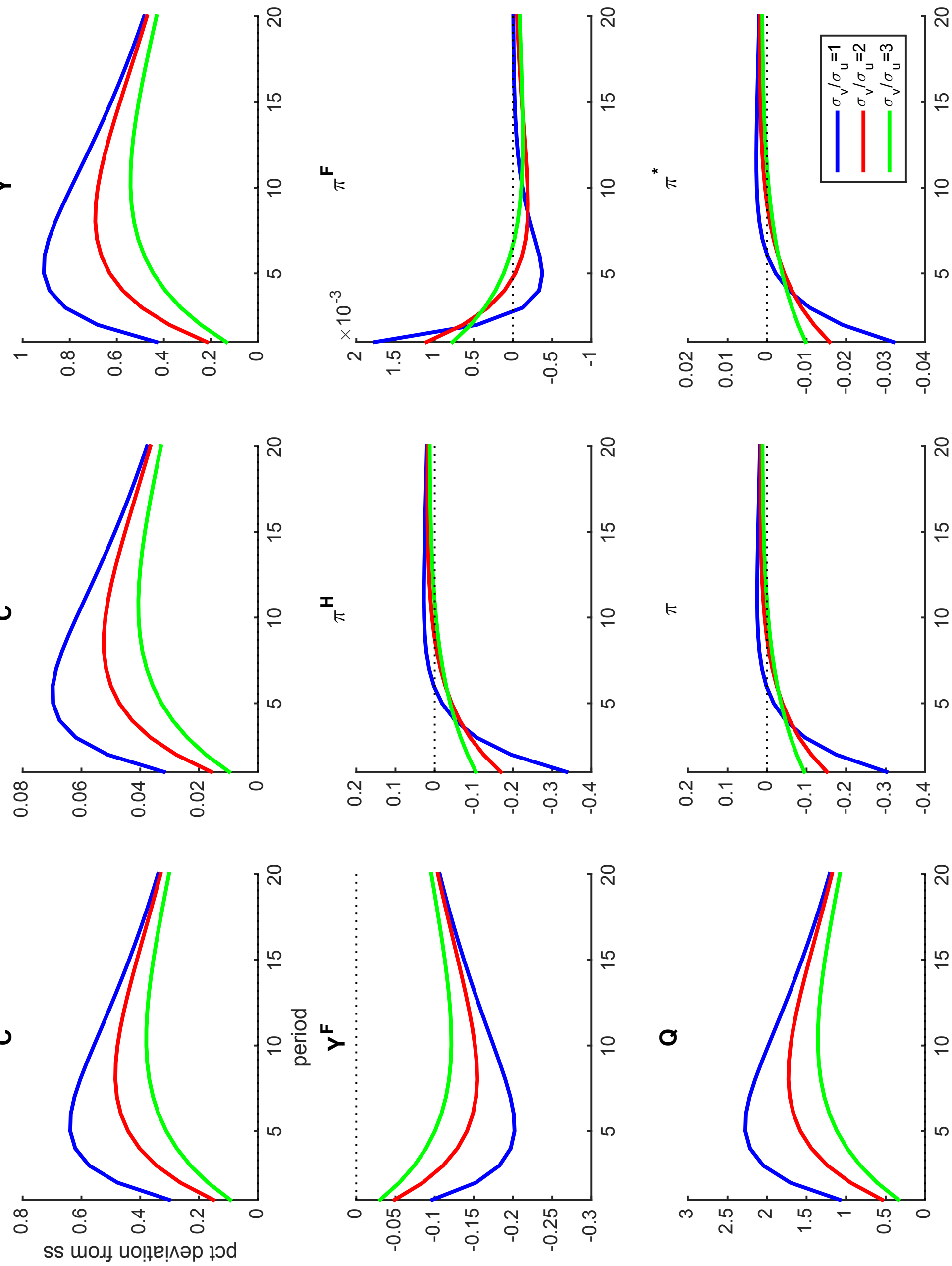
Notes: The figure depicts the iso-persistence curves of the real exchange rate for different values of the inverse signal-to-noise ratio $\tilde{\sigma}_m^2/\sigma_m^2$ and of the strategic-complementarity parameter φ when money supplies follow a random walk. Lower φ indicates more strategic complementarity.



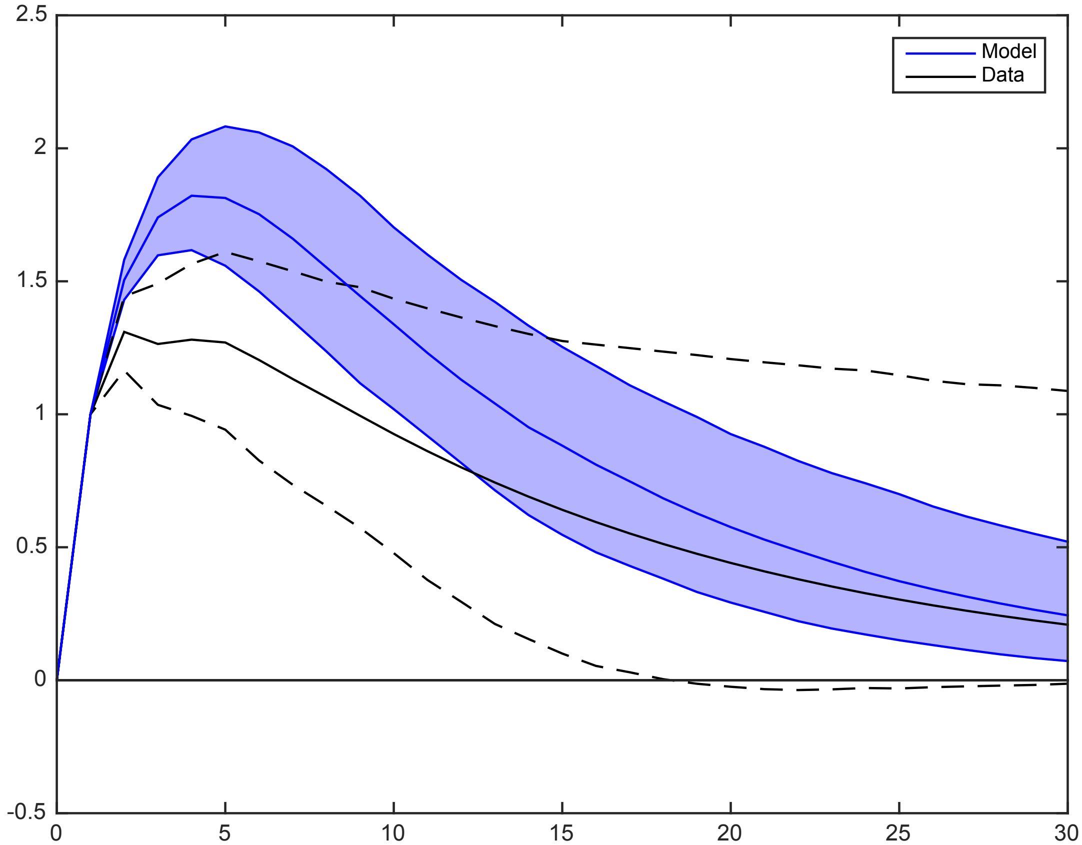
Notes: The figure depicts the impulse responses of key variables following a Home monetary shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u).



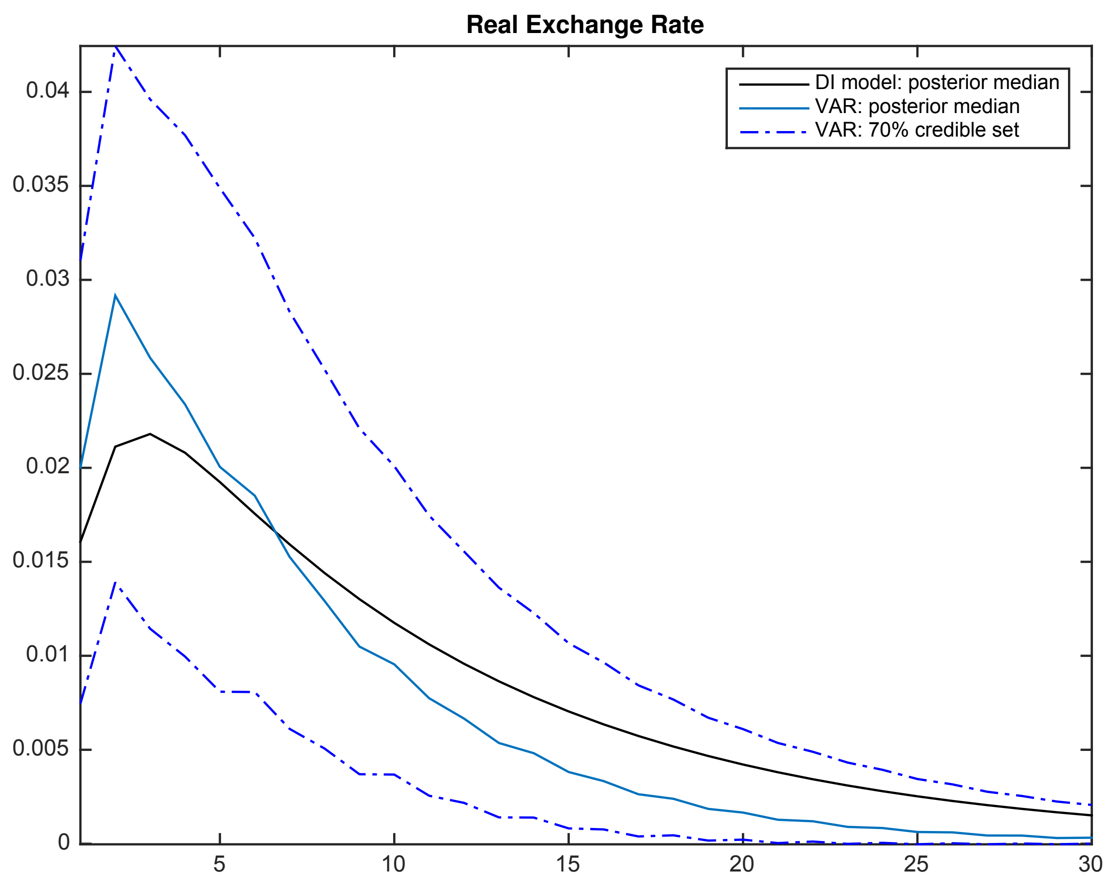
Notes: The figure depicts the impulse responses of the real exchange rate to a Home monetary shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u) and for different values of the strategic-complementarity parameter $(1-\xi)$.



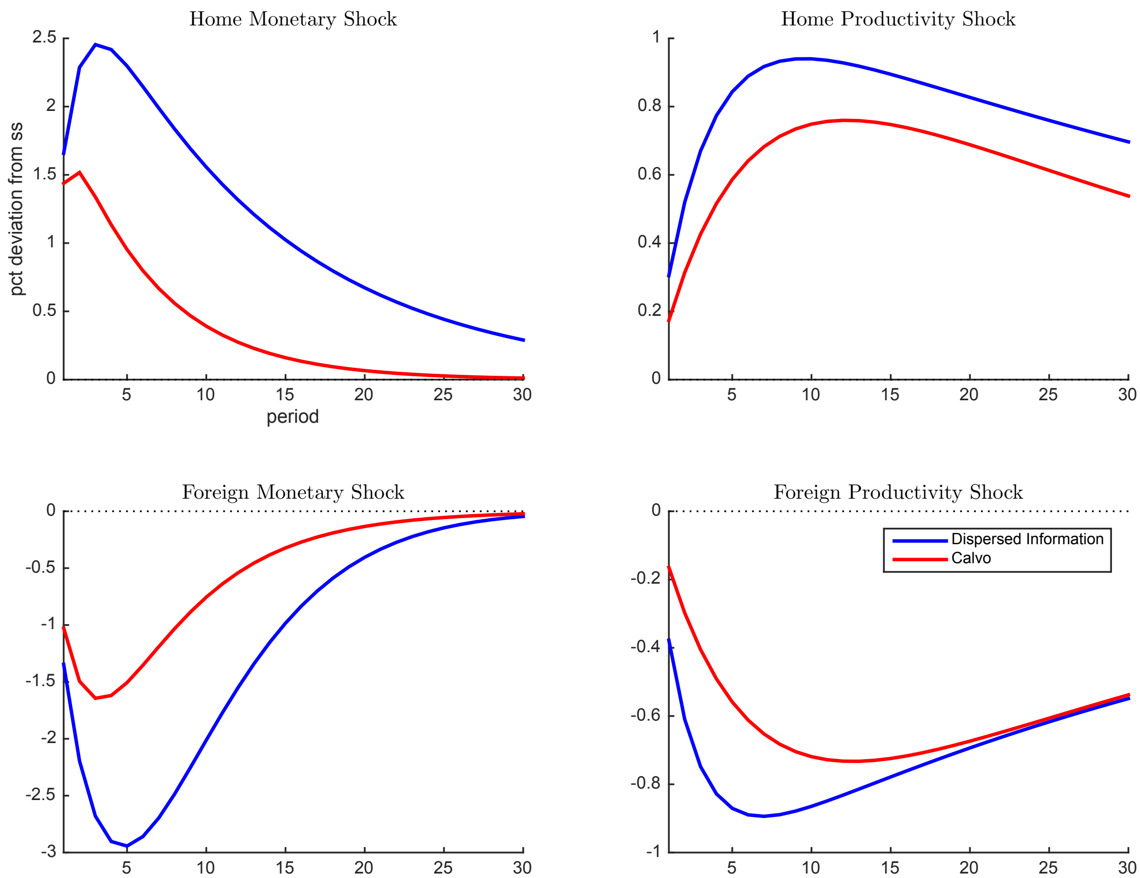
Notes: The figure depicts the impulse responses of key variables following a Home technology shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u).



Notes: The black lines depict the median response and the associated 90% confidence band of the exchange rate from the data. The blue lines represent analogous objects from the simulated dispersed-information model, as explained in Section 1.6.4.

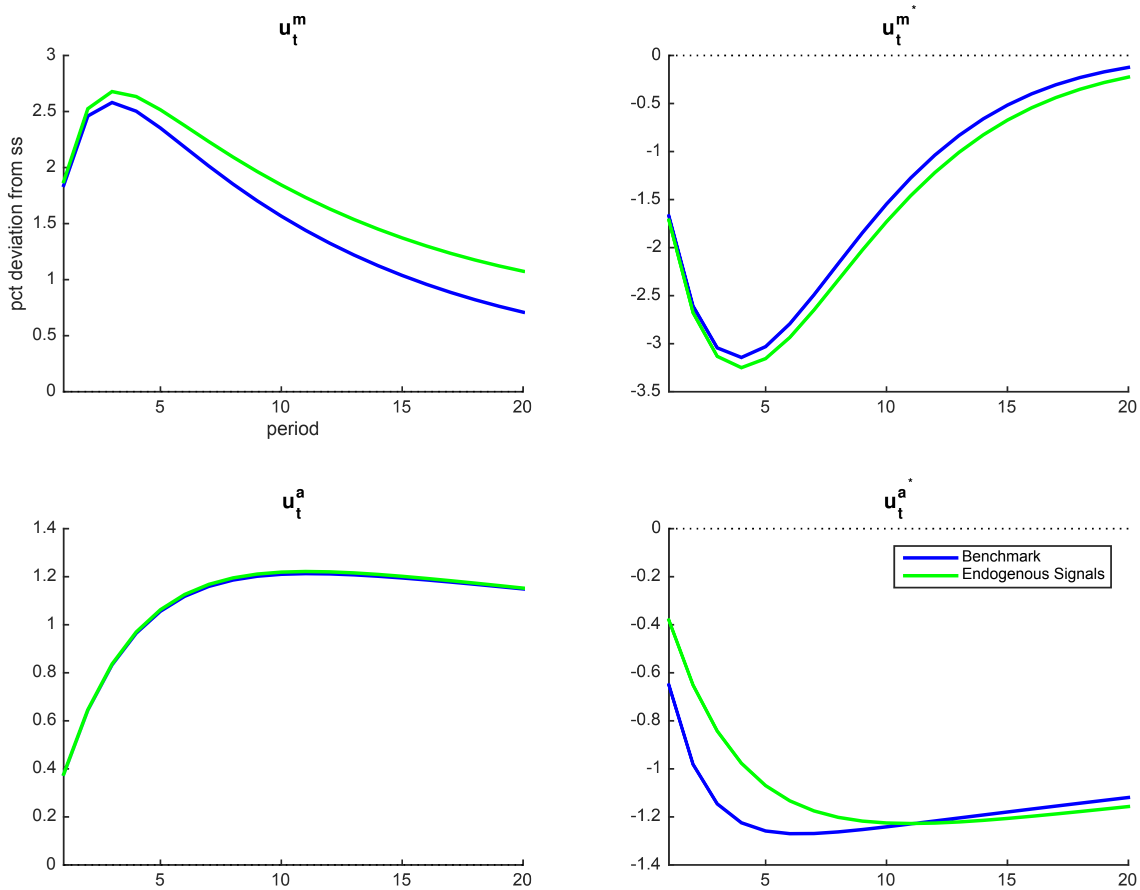


Notes: the black lines depict the median response of the exchange rate to a one standard deviation Home monetary shocks in the model. The blue lines represent the median response and the 70% credible set of the real exchange rate to a one standard deviation monetary shock in the VAR.



Notes: The figure depicts the response of the real exchange rate to the four structural shocks. For all the panels, the shock sizes have been normalized to the median estimate of the standard deviation from the dispersed-information model.

Figure 1.9: Impulse Response Comparison with Endogenous Signals



Notes: The figure depicts the response of the real exchange rate to the four structural shocks for the benchmark dispersed-information model and for the model with endogenous signals. The impulse responses are simulated using the posterior mode of the parameter estimates.

Chapter 2

Risk Aversion and the Financial Accelerator

2.1 Introduction

According to Knight (1921), bearing risk is one of the defining features of entrepreneurship. Entrepreneurs are inevitably exposed to non-diversified risk, which affects their willingness to borrow and invest in risky projects. Nevertheless, the financial frictions literature has paid little attention to how entrepreneurs' desire to take on this risk affects their choices in a general equilibrium setting. Indeed, business cycle models with credit market frictions assume either no idiosyncratic risk (Kiyotaki and Moore, 1997), risk-neutral entrepreneurs (Bernanke, Gertler, and Gilchrist, 1999, BGG), or full diversification (Forlati and Lambertini, 2011; Liu and Wang, 2014).

The objective of this paper is to study how entrepreneurs' attitudes towards undiversifiable risk affect business cycles in a model with financial frictions. To this end, we generalize the BGG framework to the case of entrepreneurs with constant-relative-risk-aversion (CRRA) preferences, yet maintaining an analytically tractable, log-linear framework. Notably, our linearization does not result in certainty equivalence because our steady state, while being deterministic in the aggregate sense, still features non-zero volatility of idiosyncratic productivity. In the steady state of our model, every entrepreneur is still exposed to significant idiosyncratic risk, which has a first-order effect.

Our main results are as follows. First, risk-averse borrowers choose a lower leverage in steady state than their risk-neutral counterparts, *ceteris paribus*. Intuitively, risk-averse agents try to reduce the volatility of their returns, which in the model is achieved by cutting leverage. Second, in partial equilibrium, when entrepreneurs are risk averse, leverage becomes more sensitive to fluctuations in excess returns to capital and to shocks to the variance of idiosyncratic productivity — so called "risk shocks" following Christiano, Motto, and Rostagno (2014). This finding is consistent with the results of Chen, Miao, and Wang (2010), who study investment and financing decisions for entrepreneurial firms in a dynamic capital structure model with incomplete markets. The higher sensitivity of leverage to excess returns has important general equilibrium implications and tends to stabilize business cycle fluctuations. Indeed, we find that the response of output to financial shocks such as risk and wealth shocks is 60 to 70 percent smaller when entrepreneurs are risk-averse than when they are risk-neutral. Finally, the responses of key macro variables to technology and monetary shocks are more similar for risk-averse and risk-neutral borrowers, although about 20 percent smaller in the former case.[1]

In our framework, as well as in BGG, a risk shock increases defaults and the cost of borrowing, inducing entrepreneurs to borrow less and to reduce their purchases of capital goods. In general equilibrium, lower demand for capital depresses the price of capital, triggering two additional effects. First, it generates the BGG financial accelerator: a lower capital price reduces net worth, which lowers investment demand leading to further decreases in the price of capital, net worth and demand for capital. Second, a low price of capital increases the expected returns to capital because the price is expected to revert back up to steady state. Higher expected returns tend to increase borrowing and investment demand.

Since the leverage chosen by risk-averse entrepreneurs is more sensitive to expected returns to capital, this second effect tends to increase investment more when entrepreneurs are risk averse relative to when they are risk neutral. Thus, even though the risk shock causes borrowing to decrease in partial equilibrium, higher future returns to capital almost entirely offset the fall in general equilibrium, when entrepreneurs are risk averse. With almost no change in credit, demand for capital does not fall as much. As a result, investment and output decline much more moderately when entrepreneurs are risk averse compared to when they are risk neutral. The effect of expected returns to capital on borrowing also explains the muted effects of wealth shocks for risk-averse entrepreneurs, as we discuss below.

[1] We find that the response of key macro variables to government spending shocks is very similar for risk-averse and risk-neutral borrowers, although about 15% smaller in the risk-averse case. We do not report these results in the simulations.

Instead, the response of endogenous variables to technology and monetary shocks differs less between the risk-averse and the risk-neutral case, since the credit channel described above is not the primary channel of transmission for these shocks. However, as before, the credit channel still delivers more amplification for risk-neutral *vis-à-vis* risk-averse borrowers.

On the methodological side, we are the first to our knowledge to incorporate risk aversion in a model of idiosyncratic, uninsurable risk such as BGG, while keeping the analytical tractability of a log-linear framework. Modeling costly state verification problems with risk-averse borrowers has several difficulties which we need to address. To begin with, the optimal contract is no longer a debt contract, as for the case of risk neutrality (Townsend, 1979). Under a standard debt contract, in case of default the lender confiscates all the net worth of the borrower. Such an arrangement is no longer optimal for the risk-averse borrower because it would imply a zero-consumption scenario. We build on the results by Gale and Hellwig (1985) and Tamayo (2014) in the costly state verification literature who show that, in a static, partial-equilibrium setting, risk-averse entrepreneurs would offer a different optimal contract to the lender. This contract ensures that the borrower retains some of his net worth even in the case of default. We extend Tamayo's financial contract to a general equilibrium framework that features optimal history-independent loans with predetermined returns for lenders.[2]

The second difficulty lies in the aggregation of individual histories in the presence of

[2] Precisely, we derive the optimal one-period contract with deterministic monitoring. For CSV in partial equilibrium, the literature has also focused on dynamic contracts with deterministic monitoring (Wang, 2005), dynamic contracts with stochastic monitoring (Monnet and Quintin, 2005) and self-enforcing stochastic monitoring (Cole, 2013).

uninsurable idiosyncratic risk and non-linear preferences, whose combination implies that every entrepreneur chooses a different leverage. This form of heterogeneity normally requires giving up the traditional frameworks with a limited number of agents in favor of a more computational approach, e.g., Krusell and Smith (1998). We instead allow entrepreneurs to be risk-averse and make two assumptions that lead to identical leverage choices for potentially different entrepreneurs. Specifically, we allow only newborn entrepreneurs to work, so that labor income does not affect the financial decision of entrepreneurs. Moreover, we assume that all net worth is reinvested in every period and entrepreneurs consume only in the case of death, which occurs with an exogenous probability. These two assumptions keep the aggregation of individual histories simple, and ensure, as in BGG, that only aggregate net worth matters for the economy dynamics.

Our results contribute to the literature of costly state verification in DSGE models where frictions arise because of information asymmetries. The CSV framework brings into the business cycle picture the possibility of endogenous defaults, endogenous spreads and cross-sectional variation among borrowers, therefore naturally accommodating questions regarding risk.[3] Recent applications include Chugh (2013), who studies risk shocks in a model with costly state verification and finds that cross-sectional firm-level evidence provides little empirical support for the presence of large risk shocks. On the other hand, Ferreira (2014) identifies risk shocks using sign restrictions in a VAR and finds that these shocks explain

[3]Financial frictions can also be rationalized using other mechanisms. Recent examples with enforcement/collateral constraints include Jermann and Quadrini (2012), Gertler and Karadi (2011), Gertler and Kiyotaki (2010). House (2006) studies financial frictions in a model with adverse selection in the spirit of Stiglitz and Weiss (1981). For an excellent survey of the literature see Brunnermeier, Eisenbach, and Sannikov (2012).

a sizable portion of the fall in economic activity during the Great Recession. Dmitriev and Hoddenbagh (2013) study risk shocks in a BGG model with optimal state-contingent contracts and find that they have little effects. Martinez-Garcia (2014) finds that the BGG model is producing too countercyclical and large spread between Baa corporate bond yield and the 20-year Treasury bill rate since the Great Moderation. As we show below, the presence of risk-averse entrepreneurs decreases the volatility of excess returns to capital, suggesting that our model generates spreads dynamics more in line with the data. Finally, in contrast with all these papers, risk shocks in our framework affect not only the cost of lending by changing bankruptcy costs, but also the entrepreneur's willingness to borrow.

The paper proceeds as follows. Section 2 derives the static optimal contract in partial equilibrium. Section 3 introduces aggregate risk and dynamics. Section 4 incorporates the resulting contract into the general equilibrium framework. Section 5 contains our quantitative analysis and results. Section 6 concludes.

2.2 Static Optimal Contract in Partial Equilibrium

In this section we study the optimal contract between a risk-averse borrower (the entrepreneur) and a risk-neutral lender. In the financial frictions literature popularized by BGG, borrowers are assumed to be risk neutral and hence indifferent to aggregate or idiosyncratic risk. In the present context instead, the borrower is a risk-averse agent who is subject to uninsurable risk. Lenders are risk-neutral with respect to the idiosyncratic (i.e. entrepreneur-specific) risk because, as will be true in the general equilibrium model developed below, they can diversify their lending activity across a large number of projects.

The static contract between the lender and borrower follows the traditional CSV framework and resembles the optimal contract developed by Tamayo (2014).[4] Entrepreneurs invest in a risky asset (capital) in the amount of QK, where K denotes the quantity of capital purchased and Q its relative price. The return on the investment is $QKR^k\omega$, where R^k indicates aggregate returns to capital and $\log(\omega) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ the idiosyncratic return component that is specific to the entrepreneur with pdf $\phi(\omega)$. ω is independently distributed across entrepreneurs. We assume that the lender cannot observe the realization of the idiosyncratic shock to the entrepreneurs unless he pays monitoring costs μ which are in fixed percentage of total assets. In each state of the world $\omega \in \Omega$, the risk-averse entrepreneur chooses to report $s(\omega)$ and the report is verified in the verification set $\Omega^V \subset \Omega$. Following the literature, we assume that reports are always truthful so that $s(\omega) = \omega$ for all $\omega \in \Omega$, which implies that the repayment function depends only on ω.[5]

Definition 1 *A contract under CSV is an amount of borrowed funds B, a repayment function $R(\omega)$ in the state of nature ω and a verification set Ω^V, where the lender chooses to verify the state of the world.*

[4]For earlier treatments of the contracting problem see Townsend (1979) and Gale and Hellwig (1985).
[5]See Tamayo (2014) for details.

The static problem in the presence of only idiosyncratic risk ω can be formulated as

$$\max_{K,R()} \frac{\int_0^\infty [QKR^k(\omega - R(\omega))]^{1-\rho}\phi(\omega)d\omega}{1-\rho} \tag{2.1}$$

$$BR \leq QKR^k \int_0^\infty R(\omega)\phi(\omega)d\omega - \mu QKR^k \int_{\omega\in\Omega^V} \omega\phi(\omega)d\omega \tag{2.2}$$

$$QK = B + N \tag{2.3}$$

$$0 \leq R(\omega) \leq \omega \quad \forall\omega \tag{2.4}$$

The first equation is the expected utility of the entrepreneur from the investment return. The second equation is a participation constraint for the lender; it says that he should be paid on average the gross safe rate of return, R. The third equation just says that the entrepreneur uses the loan (B) and his own net worth (N) for acquiring capital. The final inequality constraint states that repayments should be non-negative and cannot exceed the total value of assets. The following Proposition is a special case of Tamayo's Theorem 1 case iii).

Proposition 3 *Under the optimal contract that solves the problem (2.1) subject to (2.2), (2.3), (2.4), the repayment function $R(\omega)$ can be written as that*

- $\exists\ \bar{\omega}$ *and* $\underline{\omega}$, *such that*

$$R(\omega) = \begin{cases} 0 & \textit{if} \quad \omega < \underline{\omega} \\ \omega - \underline{\omega} & \textit{if} \quad \underline{\omega} \leq \omega \leq \bar{\omega} \\ \bar{R} & \textit{if} \quad \omega > \bar{\omega}, \textit{where } \bar{\omega} \geq \bar{R} \geq \bar{\omega} - \underline{\omega} \end{cases}$$

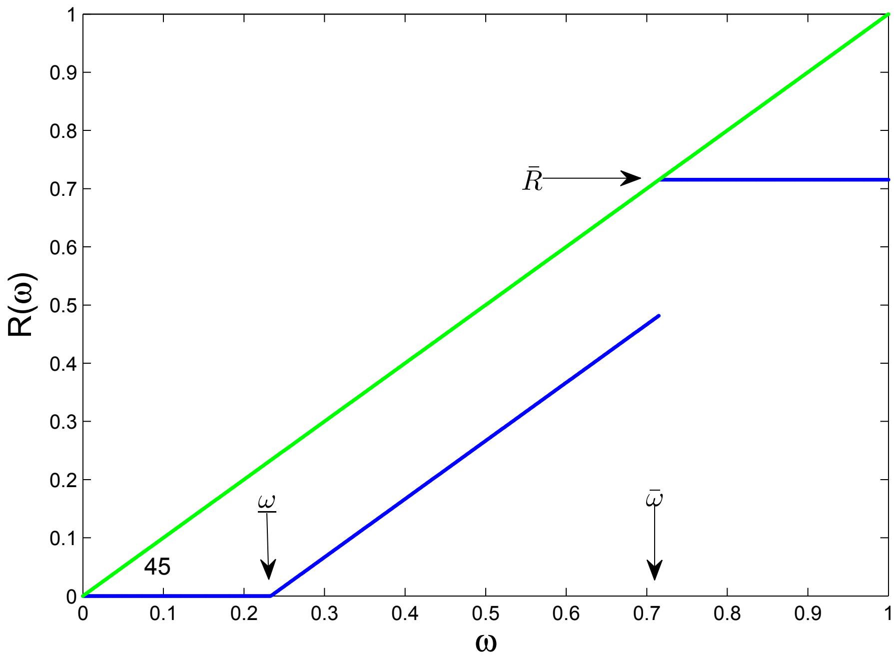
$$\Omega^V = [0, \bar{\omega})$$

Proof *See Appendix B.1.*

The optimal contract is illustrated in Figure 2.1. When the lender monitors the borrower ($\omega \leq \bar{\omega}$), he does not seize all assets. If the borrower's returns are very small ($\omega < \underline{\omega}$), the lender receives no repayment; if the borrower is a little more successful ($\underline{\omega} < \omega < \bar{\omega}$), he keeps a fixed amount $\underline{\omega}$ of resources, while the lender seizes the rest. As in Townsend (1979)'s debt contract, when the borrower is not monitored, the lender receives a flat payoff. The structure of the optimal contract in the defaulting region is the result of the borrower's attempt to smooth his return across different states of the world.[6] Therefore, optimal risk sharing requires that the borrower be initially prioritized in the repayment. At the same time the lender is indifferent to the structure of the repayment function, as long as his net payment covers the opportunity cost of his funds on average.

[6]Effectively, in the region $\omega \in (\underline{\omega}, \bar{\omega})$ the borrower always receives $\underline{\omega}$.

Figure 2.1: Optimal Contract With Risk-averse Entrepreneurs



Corollary 1 *When $\rho \to 0$ then $\underline{\omega} \to 0$, $\bar{R} \to \bar{\omega}$, so that the optimal contract replicates the original BGG contract.*

Corollary 1 states that when the borrower becomes risk-neutral, the optimal contract converges to the debt contract of BGG. In this case the repayment function is completely characterized by $\bar{\omega}$, as $\bar{R}$ becomes equal to $\bar{\omega}$ and $\underline{\omega}$ goes to zero. In other words, the debt contract of BGG is a special case of the richer risk-sharing agreement described in Proposition 1.

An interesting implication of Proposition 1 is that, notwithstanding the complexity of the problem under risk-aversion, the repayment function $R(\omega)$ is completely characterized by the thresholds $(\underline{\omega}, \bar{\omega})$ and by the non-default repayment $\bar{R}$. This allows us to reformulate the contracting problem as follows:

$$\mathcal{L} = \max_{\bar{\omega}, \underline{\omega}, \bar{R}, \kappa, \lambda} \frac{(\kappa R^k)^{1-\rho} g(\bar{\omega}, \underline{\omega}, \bar{R})}{1-\rho} + \lambda \Big(\kappa R^k h(\bar{\omega}, \underline{\omega}, \bar{R}) - (\kappa - 1) R \Big)$$

where $\kappa \equiv \frac{QK}{N}$, $g(\bar{\omega}, \underline{\omega}, \bar{R})$ and $h(\bar{\omega}, \underline{\omega}, \bar{R})$ are correspondingly:

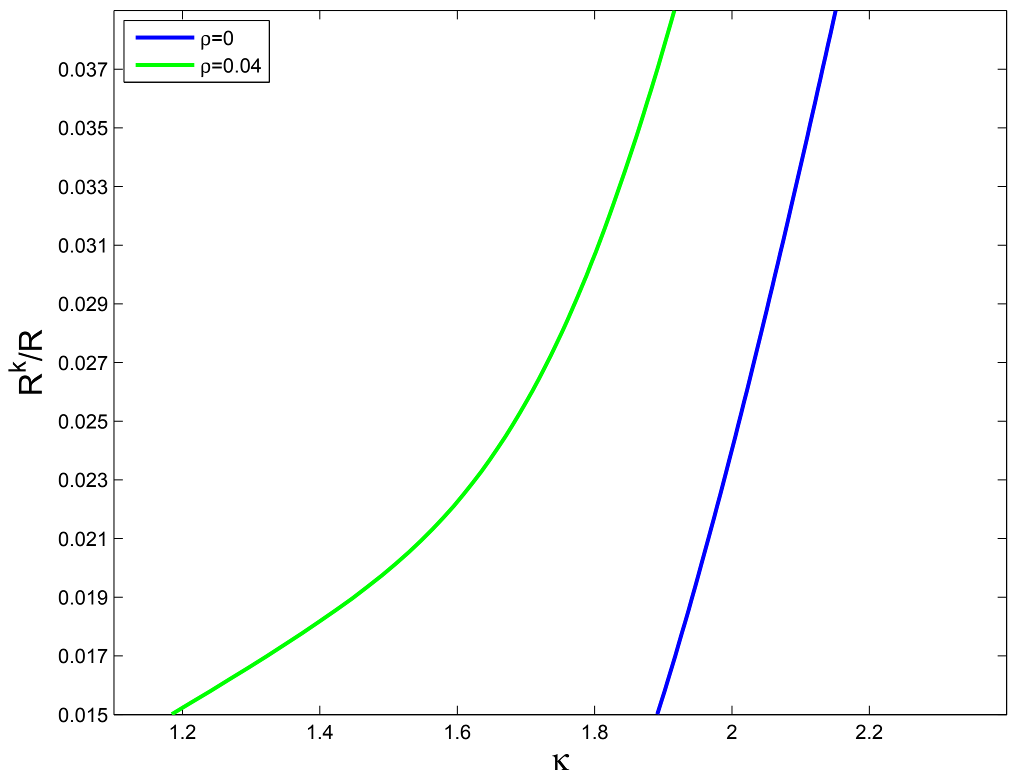
$$g(\bar{\omega}, \underline{\omega}, \bar{R}) = \int_0^{\underline{\omega}} \omega^{1-\rho} \phi(\omega) d\omega + \underline{\omega}^{1-\rho} \int_{\underline{\omega}}^{\bar{\omega}} \phi(\omega) d\omega + \int_{\bar{\omega}}^{\infty} (\omega - \bar{R})^{1-\rho} \phi(\omega) d\omega \tag{2.5}$$

$$h(\bar{\omega}, \underline{\omega}, \bar{R}) = (1-\mu) \int_{\underline{\omega}}^{\bar{\omega}} \omega \phi(\omega) d\omega - \underline{\omega} \int_{\underline{\omega}}^{\bar{\omega}} \phi(\omega) d\omega + \bar{R} \int_{\bar{\omega}}^{\infty} \phi(\omega) d\omega - \mu \int_0^{\underline{\omega}} \omega \phi(\omega) d\omega \tag{2.6}$$

The optimal $\kappa, \bar{\omega}, \underline{\omega}, \bar{R}$ are only functions of exogenous variables R^k, R and parameters σ_ω, μ. The first-order conditions for this problem are reported in Appendix B.1.

Figure 2.2 shows the relationship between the (annualized) discounted returns to capital (R^k/R) and leverage κ. The relationship is positive as higher returns to capital lower expected defaults, thereby reducing agency costs and allowing entrepreneurs to borrow more. From the Figure we also see that for any given excess return to capital, as risk-aversion increases, leverage decreases. This is what we should expect as, when risk aversion rises, entrepreneurs will try to reduce the volatility of their returns by cutting leverage. In other words, a precautionary motive arises that reduces the equilibrium leverage.

Figure 2.2: Optimal Leverage



2.3 Dynamic Optimal Contract in Partial Equilibrium With Aggregate Risk

In this section we extend the contract to a dynamic setting where entrepreneurs maximize their expected consumption path and returns to capital are subject to aggregate risk. For the moment, aggregate returns to capital and the risk-free rate are still exogenous. We largely use notation from Dmitriev and Hoddenbagh (2013).

At time t, the entrepreneur j purchases capital $K_t(j)$ at a unit price of Q_t, which he will rent to wholesale goods producers in the next period. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_t K_t(j) = N_t(j) + B_t(j). \tag{2.7}$$

In period $t+1$, entrepreneur j is hit with an idiosyncratic shock $\omega_{t+1}(j)$ and an aggregate shock R^k_{t+1}, so that he is able to deliver $Q_t K_t(j) R^k_{t+1} \omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega_{t+1}(j)$ is a log-normal random variable with distribution $\log(\omega_{t+1}(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_{\omega,t}, \sigma^2_{\omega,t})$ so that the mean of ω is equal to 1.[7] The realizations of ω are independent across entrepreneurs and over time. When the realization of $\omega_{t+1}(j)$ exceeds $\bar{\omega}_{t+1}$ the entrepreneur is able to repay the loan at the contractual rate Z_{t+1}. That is,

$$B_t Z_{t+1} = Q_t K_t R^k_{t+1} \bar{R}_{t+1} \tag{2.8}$$

Following BGG, we assume that entrepreneurs die with constant probability $1-\gamma$. It is well known, for instance from the work of Krusell and Smith (1998), that if agents are risk-averse and subject to uninsurable idiosyncratic risk, there is no simple way of aggregating individual histories and one would need to keep track of the wealth distribution of all the entrepreneurs. Consider the case where entrepreneurs receive a wage income in every period. In this case, different entrepreneurs would choose different leverages, depending on their net worth. For example, entrepreneurs with a very low net worth would realize

[7] The timing is meant to capture the fact that the variance of ω_{t+1} is known at the time of the financial arrangement, t.

that, even in the case of very low idiosyncratic returns to capital, if they survive to the next period, they would be able to make up for their losses with their wages. Given their low net worth today, the variance of their net worth tomorrow is still pretty low even for a high leverage, therefore it will be optimal to choose a high leverage. Consider instead an entrepreneur with a very high net worth today. In case of a low idiosyncratic realization tomorrow, he would lose almost all his wealth and end up consuming only his wage. This entrepreneur will choose a lower leverage than the low-net-worth entrepreneur. The issue of different leverages does not arise in BGG because entrepreneurs are risk-neutral and thus are indifferent to the variance of their future wealth.

To resolve the aggregation problem, we assume that entrepreneurs work only in the first period of their lives and that they consume all their net worth only upon the event of death. If entrepreneurs survive they do not consume anything and reinvest all their proceeds. In order to keep aggregate dynamics of net worth the same of BGG, we assume that in the first period entrepreneurs provide $\frac{1}{1-\gamma}$ units of labor, so that total labor income is identical in both models. Entrepreneur j's value function is

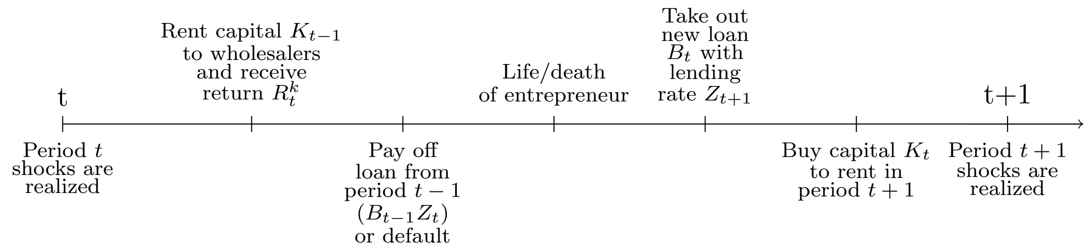
$$V_t^e(j) = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t \frac{(C_{t+s}^e(j))^{1-\rho}}{1-\rho} \tag{2.9}$$

where $C_{t+s}^e(j)$ is the entrepreneur j's consumption in case of his death,

$$C_t^e(j) = N_t(j) \tag{2.10}$$

defined as wealth accumulated from operating firms. The timeline for entrepreneurs is plotted in Figure 2.3.

Figure 2.3: Timeline for Entrepreneurs



The dynamic problem can be formulated recursively as follows:

$$\max_{K_t,\bar{R}_{t+1},\bar{\omega}_{t+1},\underline{\omega}_{t+1}} \mathbb{E}_t\left[\frac{(\kappa_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1},\underline{\omega}_{t+1},\bar{R}_{t+1},\sigma_{\omega,t})\Psi_{t+1}}{1-\rho}\right] \tag{2.11}$$

$$s.t. \Psi_t = 1+\gamma\mathbb{E}_t\left[(\kappa_t R_{k,t+1})^{1-\rho} g(\bar{\omega}_{t+1},\underline{\omega}_{t+1},\bar{R}_{t+1},\sigma_{\omega,t})\Psi_{t+1}\right] \tag{2.12}$$

$$s.t. \beta\kappa_t R_{k,t+1} h(\bar{\omega}_{t+1},\underline{\omega}_{t+1},\bar{R}_{t+1},\sigma_{\omega,t}) = (\kappa_t-1)R_t \tag{2.13}$$

As in BGG, R_t is the safe rate known at time t. Lenders require to be paid R_t on average, which implies that the contract must specify a triplet $\{\underline{\omega}_{t+1},\bar{\omega}_{t+1},\bar{R}_{t+1}\}$ contingent on R_{t+1}^k.[8] This assumption about the repayment to the lenders makes entrepreneurs effectively bear the aggregate risk. The following Proposition summarizes the solution to the dynamic contracting problem.

[8] Later in the general equilibrium model R_t will be equal to the inverse of the household's stochastic factor.

Proposition 4 *Solving problem (2.11)-(2.13) and log-linearizing the solution gives the following relationship between leverage and the expected discounted return to capital*

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - R_t) \tag{2.14}$$

where $\nu_p > 0$*. Moreover, when the standard deviation of idiosyncratic productivity varies over time, the relationship becomes*

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - R_t) + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{2.15}$$

with $\nu_\sigma < 0$*.*

Proof *Equations (2.14) and (2.15) are obtained in Appendix B.2.*

Following our assumptions about entrepreneurial wage and consumption, all entrepreneurs choose the same leverage regardless of their net worth, so that aggregate leverage κ_t will simply be equal to the leverage chosen by each entrepreneur. Moreover, to a first-order approximation, the complex financial agreement between borrowers and lenders boils down to the single equation (2.14) that links leverage to the expected excess return or the capital wedge. Note that equation (2.14) is identical in form to the one in BGG (equation (4.17) in their paper). The presence of risk-aversion only changes the elasticity of leverage to the excess returns ν_p and to the volatility of idiosyncratic productivity ν_σ, if σ_ω is allowed to change over time. In this sense, our framework fully nests the BGG framework, and this is what allows us to compare the two models in a meaningful way.

When borrowers are risk averse ($\rho > 0$) the values of the elasticities ν_p and ν_σ will be different from the risk-neutral case. For all the calibrations that we considered we have that

$$\frac{\partial \nu_p}{\partial \rho} > 0 \qquad\qquad \left|\frac{\partial \nu_\sigma}{\partial \rho}\right| > 0$$

To understand this result it is useful to think about how ρ affects steady-state leverage and marginal monitoring costs. Marginal monitoring costs represent the marginal cost of increasing leverage and, importantly, they are a convex function of leverage itself. Therefore, when leverage is lower, marginal monitoring costs are also lower and less sensitive to leverage. An increase in risk aversion reduces steady-state leverage, as explained in Section 2.2. Lower leverage means that the steady state is in a region where marginal monitoring costs are flatter relative to the risk neutral case. Hence, the response of κ_t to a given change in excess returns to capital (ν_p) will be larger when steady state leverage is lower because in that region marginal monitoring costs are less sensitive to changes in κ_t.

Proposition 2 indicates that, for a given change in prices, leverage is more volatile when entrepreneurs are risk averse. If leverage varies more also in general equilibrium we might expect investment and output to be more volatile, so that risk aversion would constitute an additional channel of amplification of shocks through the financial accelerator. However, in general equilibrium, excess returns to capital adjust endogenously to changes in the economic environment and it might well be that this adjustment acts as a stabilizer rather than as an amplifier of shocks. Hence, we proceed with the analysis by embedding the optimal contract just derived in the BGG general-equilibrium framework. This allows us to study the effect of the financial accelerator with risk-averse entrepreneurs when expected discounted returns to capital are determined endogenously.

2.4 The Model in General Equilibrium

We now embed our partial equilibrium framework in a standard dynamic New Keynesian model, where returns to capital and returns to lenders are determined endogenously. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers.

2.4.1 Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}, \tag{2.16}$$

where C_t is household consumption, and H_t is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_{t-1} D_t - D_{t+1} + R_{t-1}^n \frac{B_t}{P_t} - \frac{B_{t+1}}{P_t} \tag{2.17}$$

where W_t is the real wage, T_t is lump-sum taxes, Π_t is lump-sum profits received from final goods firms owed by the household, D_t are deposits in financial intermediaries (banks) that pay a real non-contingent gross interest rate R_{t-1} and B_t are nominal bonds that pay a gross non-contingent interest rate R^n_{t-1}.

Households maximize their utility (2.16) subject to the budget constraint (2.17) with respect to consumption, labor, bonds, and deposits, yielding the following first order conditions:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} R_t, \tag{2.18}$$

$$C_t^{-\sigma} = \beta R_t^n \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right\} \tag{2.19}$$

$$W_t C_t^{-\sigma} = \chi H_t^{\eta}. \tag{2.20}$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$.

2.4.2 Retailers

The final consumption good consists of a basket of intermediate retail goods, which are aggregated together in a CES fashion by the representative household:

$$C_t = \left(\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.21}$$

The demand for retailer i's unique variety is

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} C_t, \tag{2.22}$$

where p_{it} is the price charged by retail firm i. The aggregate price index is defined as

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{2.23}$$

Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. They have price-setting power and are subject to Calvo (1983) price rigidities. With probability $1-\theta$ each retailer is able to change its price in a particular period t. Retailer i maximizes the following stream of real profits:

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \right\}, \tag{2.24}$$

where P_t^w is the wholesale goods price and $\Lambda_{t,s} \equiv \beta \frac{U_{C,t+s}}{U_{C,t}}$ is the household's (i.e. shareholder's) stochastic discount factor. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \left(\frac{p_{it}^*}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \left[\frac{p_{it}^*}{P_{t+s}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{t+s}^w}{P_{t+s}}\right] \right\} = 0. \tag{2.25}$$

From this condition, it is clear that all retailers that are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \;\; \forall i$. The price level will evolve according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} . \tag{2.26}$$

Dividing the left and right hand side of (2.26) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} , \tag{2.27}$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (2.25) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{\Lambda_{t,s}(1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w\right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{\Lambda_{t,s}(1/p_{t+s})^{1-\varepsilon} Y_{t+s}\right\}}, \tag{2.28}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_{t+s}}$ and $p_{t+s} = P_{t+s}/P_t$.

2.4.3 Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage W_t and W_t^e, and rent capital from entrepreneurs at rental rate R_t^r. Note that capital purchased in period t is used in period $t+1$. Following BGG, the production function of the representative wholesaler is given by

$$Y_t = A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}, \tag{2.29}$$

where A_t denotes aggregate technology, K_t is capital, H_t is household labor, H_t^e is entrepreneurial labor, and Ω defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}. \tag{2.30}$$

One can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

$$\frac{P_t^w}{P_t} = p_t^w = \frac{1}{\mathcal{X}_t}, \tag{2.31}$$

where $\mathcal{X}_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}. \tag{2.32}$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$\frac{1}{\mathcal{X}_t}\alpha\frac{Y_t}{K_{t-1}} = R_t^r, \tag{2.33}$$

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)\frac{Y_t}{H_t} = W_t, \tag{2.34}$$

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)\frac{Y_t}{H_t^e} = W_t^e. \tag{2.35}$$

Given that equilibrium entreprenerial labor in equilibrium is 1, we have

$$\frac{\Omega}{\mathcal{X}_t}(1-\alpha)Y_t = W_t^e. \tag{2.36}$$

2.4.4 Capital Producers

While entrepreneurs hold capital between periods, perfectly competitive capital producers hold capital within a given period, and use available capital and final goods to produce new capital. Capital production is subject to adjustment costs, according to

$$K_t = I_t + (1-\delta)K_{t-1} - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}, \tag{2.37}$$

where I_t is investment in period t, δ is the rate of depreciation and ϕ_K is a parameter that governs the magnitude of the adjustment cost. The capital producer's objective function is

$$\max_{I_t} K_t Q_t - I_t, \tag{2.38}$$

where Q_t denotes the price of capital. The first order condition of the capital producer's optimization problem is

$$\frac{1}{Q_t} = 1 - \phi_K \left(\frac{I_t}{K_{t-1}} - \delta \right). \tag{2.39}$$

2.4.5 Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household's funds among various entrepreneurs. The bank takes nominal household deposits, D_t, and lends out the nominal amount B_t to entrepreneurs. In equilibrium, deposits will equal loanable funds ($D_t = B_t$). Households receive a predetermined real rate of return R_t on their deposits.

2.4.6 Entrepreneurs

We have already described the entrepreneur's problem and timing in detail in Section 3. At the beginning of each period entrepreneurs rent out the capital they bought at the end of the previous period to perfectly competitive wholesalers. Later, wholesalers return to the entrepreneurs depreciated capital and pay them the rental rate. After that, entrepreneurs sell their capital and settle their position with the banks, either by repaying their loans or by defaulting. Following the arrangements with the banks, nature decides which entrepreneurs are going to survive, and which entrepreneurs are going to die and consume all of their net worth. Subsequently, new entrepreneurs are born with zero net worth and supply inelastically one unit of labor in the aggregate. Then, newborn and surviving old entrepreneurs borrow money from banks and buy capital from capital producers.

Wholesale firms rent capital at rate $R^r_{t+1} = \frac{\alpha Y_t}{\mathcal{X}_t K_{t-1}}$ from entrepreneurs. After production takes place entrepreneurs sell the undepreciated capital back to capital goods producers for the unit price Q_{t+1}. Aggregate returns to capital are then given by

$$R^k_{t+1} = \frac{\frac{1}{\mathcal{X}_t}\frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{2.40}$$

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1-\gamma$, which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma\big(Q_t K_t R^k_{t+1} - (Q_t K_t - N_t)R_t - \mu Q_t K_t R^k_{t+1}\int_0^{\bar{\omega}_{t+1}} \omega\phi(\omega)d\omega\big) + W^e_{t+1}. \tag{2.41}$$

The terms inside the brackets reflect the aggregate returns to capital to entrepreneurs, net of loan repayments and monitoring costs. Aggregate entrepreneurial consumption is given by

$$C^e_t = (1-\gamma)(N^e_t - W^e_t) \tag{2.42}$$

Given that each entrepreneur chooses the same leverage, we can define leverage as the ratio of aggregate capital expenditure to aggregate net worth

$$\kappa_t = Q_t K_t / N_t. \tag{2.43}$$

2.4.7 Goods Market Clearing

The goods market clearing condition is

$$Y_t = C_t + I_t + G_t + C_t^e + \mu Q_{t-1} K_{t-1} R_t^k \int_0^{\bar{\omega}_t} \omega \phi(\omega) d\omega \tag{2.44}$$

where the last term reflects aggregate monitoring costs.

2.4.8 Monetary Policy

As in BGG, we assume that there is a central bank which conducts monetary policy by choosing the nominal interest rate R_t^n according to the following rule

$$\log(R_t^n) - \log(R^n) = \rho^{R^n} \Big(\log(R_{t-1}^n) - \log(R) \Big) + \xi \pi_{t-1} + \epsilon_t^{R^n} \tag{2.45}$$

where ρ^{R^n} and ξ determine the relative importance of the past interest rate and past inflation in the central bank's interest rate rule. Shocks to the nominal interest rate are given by ϵ^{R^n}. It should be noted that the interest rule in BGG differs from the conventional Taylor rule, where current inflation rather than past inflation is targeted.

2.4.9 Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A, \tag{2.46}$$

$$\log(G_t/Y_t) = (1-\rho^G)\log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon_t^G, \tag{2.47}$$

$$\log(\sigma_{\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \epsilon_t^{\sigma_\omega} \tag{2.48}$$

where ϵ^A, ϵ^G and ϵ^{σ_ω} denote exogenous shocks to technology, government spending and idiosyncratic volatility, and (G_{ss}/Y_{ss}) and $\sigma_{\omega,ss}$ denote the steady state values for government spending relative to output and idiosyncratic volatility respectively. Recall that σ_ω^2 is the variance of idiosyncratic productivity, so that σ_ω is the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (2.45).

2.4.10 Equilibrium

The nonlinear model has 26 endogenous variables and 26 equations. The endogenous variables are: R, R^n, H, C, π, p^*, p^w, $\mathcal{X}$, Y, W, W^e, I, Q, K, R^k, N, C^e, k, $\bar{\omega}$, $\underline{\omega}$, $\bar{R}$, Ψ, λ, G, A, σ_ω, where the new variable λ corresponds to the Lagrange multiplier for the optimality conditions used in the Appendix. The equations defining these endogenous variables are: (2.18), (2.19), (2.20), (2.27), (2.28), (2.30), (2.31), (2.34), (2.36), (2.37), (2.39), (2.40), (2.41), (2.42), (2.43), (2.44), and financial contract participation (2.13), discounting condition (2.12) and optimality conditions (B.17), (B.18), (B.19), (B.20). The exogenous processes for technology, government spending and idiosyncratic volatility follow (2.46), (2.47) and (2.48) respectively. Nominal interest rate shocks are defined by the Taylor rule in (2.45).

2.4.11 Log-linear Model

The log-linear model has 19 equations and 19 variables, because algebraic manipulations with the Calvo model allow to replace (2.27), (2.28) and (2.31) with (2.52), and drop p^* and p^w, while simplifying the financial contract allows to replace (2.12), (2.13), (B.17), (B.18), (B.19), (B.20) with (2.62) and drop $\bar{\omega}, \underline{\omega}, \bar{R}, \Psi$. The equations are

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1} - \hat{C}_t\Big) + \hat{R}_t = 0, \tag{2.49}$$

$$\hat{R}^n_t = \hat{R}_t + \mathbb{E}_t\hat{\pi}_{t+1}, \tag{2.50}$$

$$\hat{Y}_t - \hat{H}_t - \hat{\mathcal{X}}_t - \sigma\hat{C}_t = \eta\hat{H}_t, \tag{2.51}$$

$$\hat{\pi}_t = -\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{2.52}$$

$$\hat{Y}_t = \hat{A}_t + \alpha\hat{K}_{t-1} + (1-\alpha)(1-\Omega)\hat{H}_t, \tag{2.53}$$

$$\hat{K}_t = \delta\hat{I}_t + (1-\delta)\hat{K}_{t-1}, \tag{2.54}$$

$$\hat{Q}_t = \delta\phi_K(\hat{I}_t - \hat{K}_{t-1}), \tag{2.55}$$

$$\hat{R}^k_{t+1} = (1-\epsilon)(\hat{Y}_{t+1} - \hat{K}_t - \hat{\mathcal{X}}_{t+1}) + \epsilon\hat{Q}_{t+1} - \hat{Q}_t, \tag{2.56}$$

$$Y\hat{Y}_t = C\hat{C}_t + I\hat{I}_t + G\hat{G}_t + C^e\hat{C}^e_t + \phi N(\hat{\phi}_t + \hat{N}_{t-1}), \tag{2.57}$$

$$\hat{\phi}_t = \hat{Q}_{t-1} + \hat{K}_{t-1} - \hat{N}_{t-1} + \nu^m_\sigma\hat{\sigma}_{\omega,t-1} + \nu^m_p(\mathbb{E}_{t-1}R_{k,t} - \hat{R}_{t-1}), \tag{2.58}$$

$$\hat{\kappa}_t = \hat{K}_t + \hat{Q}_t - \hat{N}_t, \tag{2.59}$$

$$C^e\hat{C}^e_t = (1-\gamma)(N\hat{N}_t - W^e\hat{W}^e_t), \tag{2.60}$$

$$\hat{W}^e_t = \hat{Y}_t - \hat{\mathcal{X}}_t, \tag{2.61}$$

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - \hat{R}_t) + \nu_\sigma\hat{\sigma}_{\omega,t}, \tag{2.62}$$

$$\hat{R}^n_t = \rho^{R^n}\hat{R}^n_{t-1} + \xi\hat{\pi}_t + \rho^Y\hat{Y}_t + \epsilon^{R^n}_t, \tag{2.63}$$

$$\hat{N}_t = \gamma\Big(\kappa R_k(\hat{\kappa}_{t-1} + \hat{R}_{k,t}) - \kappa R\hat{\kappa}_{t-1} - (\kappa-1)R\hat{R}_{t-1} - \phi\hat{\phi}_t\Big) + \frac{W^e}{N}(\hat{W}^e_t) + \frac{N-W^e}{N}\hat{N}_{t-1}, \tag{2.64}$$

$$\hat{A}_t = \rho^A \hat{A}_{t-1} + \epsilon_t^A, \tag{2.65}$$

$$\hat{G}_t = \rho^G \hat{G}_{t-1} + \epsilon_t^G, \tag{2.66}$$

$$\hat{\sigma}_{\omega,t} = \rho^{\sigma_\omega} \hat{\sigma}_{\omega,t-1} + \epsilon_t^{\sigma_\omega} \tag{2.67}$$

2.5 Quantitative Analysis

In section 2.3 we discussed the role of risk aversion in determining the elasticities of leverage with respect to the expected discounted returns to capital and to the standard deviation of idiosyncratic productivity. In particular, we have highlighted the fact that in partial equilibrium leverage becomes more responsive to the latter with higher risk aversion, as marginal monitoring costs build up more slowly. While the partial equilibrium analysis suggests higher sensitivity of leverage and, hence, higher amplification under risk aversion, the general equilibrium effect depends on the endogenous adjustment of prices and returns. In this section we investigate quantitatively the general equilibrium effects of technology, monetary, idiosyncratic volatility, and wealth shocks for different coefficients of risk aversion.

2.5.1 Calibration and Benchmarks

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the risk aversion parameter $\sigma = 1$, so that the utility of households is logarithmic in consumption, and the elasticity of labor supply to 3 ($\eta = 1/3$). The share of capital in the Cobb-Douglas production function is $\alpha = 0.35$. Capital adjustment costs are $\phi_k = 10$, to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly capital depreciation is $\delta = .025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = .0275$, yielding an annualized business failure rate of eleven percent. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price setting, we set the Calvo parameter $\theta = 0.75$, so that 25% of firms can reset their prices in each period, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R^n} = 0.9$ and the parameter on lagged inflation to $\xi = 0.11$. We set the persistence of the shocks to technology at $\rho^A = 0.99$, and keep the standard deviation at 1 percent. Following BGG, for monetary shocks we consider a 25 basis point shock (in annualized terms) to the nominal interest rate with persistence $\rho^{R_n} = 0.9$.

For our purposes, the most important part of the calibration regards the volatility to idiosyncratic productivity and the risk-aversion parameter. We want to compare the impulse responses of the model with risk-averse entrepreneurs to those of the benchmark model with risk-neutral ones. Following Christiano, Motto, and Rostagno (2014, CMR), we set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$. As to the standard deviations of idiosyncratic volatility shocks σ_ω, we choose two different values for each coefficient of risk-aversion. If we set σ_ω to be the same for the different coefficients of risk-aversion, the model with the smaller ρ would imply a higher steady-state leverage. It follows that a shock of a given size would have a stronger effect on impact, since similar movements in prices and returns to capital would induce larger fluctuations in net worth when leverage is higher. Thus, when we increase risk-aversion, we decrease the idiosyncratic volatility to numerically align the steady-state leverage and the excess returns to capital in two models.[9]

Following BGG, when entrepreneurs are risk-neutral we set σ_ω to 0.28, which implies a steady-state leverage 2.1 and a value of R^K/R of 1.0084, corresponding to an annualized excess return of 3.3 percent. In the case of risk-averse entrepreneurs, we set $\rho = 0.5$ and $\sigma_\omega = 0.085$, which generate leverage of 2.1 and R^K/R of 1.0076, corresponding to annualized excess returns of 3 percent. Why this particular coefficient of risk aversion and level of idiosyncratic volatility? If we look at the literature on cross-sectional volatility of sales growth, Castro, Clementi, and Lee (2010) obtain a value for firm-specific volatility of TFP between 0.04 and 0.12. Comin and Mulani (2006), Davis, Haltiwanger, Jarmin, and Miranda (2007) and a more recent study by De Veirman and Levin (2014) report the volatility for the annual growth of sales to be between 0.24 and 0.3, however that volatility corresponds to a much smaller standard deviation of quarterly idiosyncratic productivity. We simulate our model in the steady state, where aggregate shocks are absent, but idiosyncratic shocks still affect firms and find that $\sigma_\omega = 0.08$ and $\sigma_\omega = 0.1$ imply a value of volatility of annual sales of 0.24 and 0.3, which is the range observed in the data. We settle for a value of σ_ω of 0.085 and subsequently choose a value for ρ that delivers a leverage of two. The results

[9] We do not report the results for the two models with different risk-aversion and other identical parameters. In the model with higher risk-aversion and lower leverage the effect on the endogenous variables on impact is smaller for all shocks.

reported in our simulations are robust to the choice of ρ and σ_ω, as long as we select these two parameters to match the leverage and the average excess returns observed in the data.

2.5.2 Leverage, Capital Returns and Amplification

Our calibration implies that the two cases we consider — risk-averse and risk-neutral entrepreneurs — have very similar steady states in terms of leverage and capital returns. The first two columns of Table 1 show that in the risk-neutral calibration, the steady-state leverage and R^k are 2.1 and 1.0186, respectively. The risk-averse calibration delivers similar values — leverage of 2.1 and R^k equal to 1.0176 — using a higher risk aversion and a lower volatility of idiosyncratic productivity. We do not report the other steady-state variables but they are very similar across the two models.[10]

Table 2.1: Steady-state Comparison

	κ	R^k	ν_p	ν_σ
Risk-neutral case (σ_ω=0.28, ρ= 0.0)	2.098	1.0186	18.74	-0.71
Risk-averse case (σ_ω=0.085, ρ= 0.5)	2.084	1.0176	125.36	-1.94

$$\hat{\kappa}_t = \nu_p(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \nu_\sigma\hat{\sigma}_{\omega,t}$$

Despite the fact that steady states are similar, entrepreneurial risk-aversion still affects the way in which the economy reacts to shocks. This different sensitivity is captured by the different values of the two elasticities ν_p and ν_σ in equation (2.15) for the two calibrations.

[10] From the model equations one can see that if leverage, capital returns and defaults are identical, then the two steady states will coincide. Although with higher risk aversion defaults are smaller, they are in both cases very small compared to GDP so that in practice the steady states are almost identical.

Table 1 shows that these elasticities are higher in absolute value for the risk-averse case. As we discussed in section 2.3, an increase in ρ increases both elasticities in absolute value. The decrease in σ_ω further increases ν_p and decreases ν_σ although most of the change in the elasticities between our two preferred scenarios is really driven by the increase in ρ.[11] Notably, in our risk-averse calibration the elasticity ν_p grows by about seven times whereas the elasticity ν_σ grows only by about three times relative to our risk-neutral calibration.

How would higher sensitivity of leverage to excess returns and to the volatility of idiosyncratic productivity affect business cycles? In partial equilibrium, for a given change in prices or idiosyncratic volatility, the larger fluctuations in leverage should strengthen amplification. However, in general equilibrium the impact of ν_p and ν_σ is less obvious because the movement of prices is endogenous and it differs with and without risk-aversion.

To predict the outcome it is helpful to think about the elasticity ν_p in two extreme cases: the frictionless case and the risk-neutral case. In a world without financial frictions $\nu_p \to \infty$. Even the smallest increase in expected capital returns makes entrepreneurs be willing to hold an infinite amount of capital, owing to constant returns, so that in equilibrium returns to capital are equal to the safe rate. At the opposite end of the spectrum, when entrepreneurs are risk-neutral, ν_p is small, reflecting the fact that even if capital returns rise, borrowing cannot increase much because marginal borrowing costs increase very quickly with leverage. In this case large swings in excess returns are required to generate movements in leverage.

[11] Starting from $\sigma_\omega = 0.28$ and $\rho = 0$ and reducing σ_ω to 0.085 only increases ν_p from 18.74 to 43.17 and decreases ν_σ from -0.71 to -0.89. Therefore, most of the change in the elasticities is due to the change in ρ, rather than to the change in σ_ω.

Given that ν_p in the risk-averse case is larger than in the risk-neutral case, we should expect excess returns in the risk-averse case to still react to shocks (because financial frictions are still present), but more mildly than in the risk-neutral case. With smaller movements in the returns to capital and, therefore, the price of capital, we expect smaller fluctuations in net worth and less volatile business cycles. The simulations in the following section confirm our intuition.

2.5.3 Simulations

In this section we simulate our model and study the impulse responses of key macroeconomic variables to different shocks, comparing the case of risk aversion and the case of risk neutrality.

Risk and Wealth Shocks

After a risk shock, the probability of a low realization of ω increases, thus banks increase the interest rates charged on loans to cover the higher costs of default. Entrepreneurs respond by borrowing less and by reducing the quantity demanded of capital goods, given the fewer resources available to them. In general equilibrium, the drop in investment demand reduces the price of capital, which has two additional effects. On the one hand, the lower price of capital reduces the net worth of the entrepreneur, which further decreases the demand for capital goods through the standard financial accelerator mechanism described in BGG. On the other hand, there is an additional general equilibrium effect, which partially offset the fall in credit, as explained in CMR. Precisely, when the price of capital falls, it is expected to revert to steady state in the future. Other things equal, this raises the expected returns to capital, increasing credit received by entrepreneurs and the demand for capital. For this reason, the decline in credit is smaller than the decline in net worth.

Figure 2.4 shows the impulse responses to a risk shock for risk-neutral and risk-averse entrepreneurs. In both cases, these dynamic responses are consistent with the intuition given above. Credit falls, net worth falls even more, resulting in an increase in leverage. Investment declines as a result of lower demand, caused by the lack of entrepreneurial financial resources. The fall in investment is greatly responsible for the drop in output. Even thought the responses are qualitatively similar in the two cases, they are very different quantitatively, with the presence of risk-averse entrepreneurs greatly buffering the fall in investment and output. The difference in the dynamic responses is due to the different response of credit across the two cases. In the risk neutral case, the positive effect on borrowing of higher future returns to capital is weak (given the low value of ν_p) and only mildly offsets the negative impact of the risk shock on borrowing. As a result credit, falls significantly when entrepreneurs are risk neutral. Instead when entrepreneurs are risk averse, their demand for investment goods is much more sensitive to changes in prospective returns to capital (this is captured by the higher value of ν_p). Thus, even though the risk shock causes credit to decrease in partial equilibrium, higher future returns to capital, almost entirely offset the fall in general equilibrium. With almost no change in credit, demand for capital does not fall as much. As a result investment and output decline much more moderately when entrepreneurs are risk averse.

Figure 2.5 shows the impulse responses to a wealth shock that transfers in a lump-sum fashion 1% of the initial net worth of entrepreneurs to households. The intuition for these responses is similar to the intuition for the risk shock. A drop in wealth reduces the net worth of entrepreneurs, and the analysis of the debt contract suggests that this reduces borrowing. With fewer credit, entrepreneurs reduce the demand for capital goods, driving down their price. The change in the price of capital triggers the general equilibrium effects described above. As noted by CMR, after a wealth shock, the positive effect on credit—which works through the increase in expected returns to capital—is stronger than the financial accelerator effect, resulting in an increase in credit in equilibrium.[12] Nevertheless the increase in credit is not sufficient to cover the fall in net worth, hence the resources available to entrepreneurs fall. For this reason the wealth shock causes a decline in investment and output. Similarly to the case of the risk shock, our analysis suggests that the expected-returns-to-capital effect is even stronger when entrepreneurs are risk averse, because these types of entrepreneurs are more sensitive to changes in returns to capital. As a result, credit rises by more, net worth falls by less and, hence, the investment and output drops are considerably smaller.

[12] CMR's shock is an equity shock rather than a wealth shock. In particular they assume a stochastic process for the parameter γ, the fraction of entrepreneurs who survive. An unexpected fall in γ reduces net worth immediately. Their equity shock and our wealth shock are essentially equivalent.

Technology and Monetary Shocks

Figure 2.6 plots the impulse responses of the two models under risk-neutrality and risk-aversion to a technology shock. In both cases the direction of the responses is the same and follows the intuition of BGG. In particular, the productivity shock immediately stimulates the demand for capital, leading to an investment boom. The increase in investment raises asset prices, which raises net worth and reduces the capital wedge. The decline in the wedge further stimulates investment and the financial accelerator mechanism arises: an initial increase in investment increases asset prices and net worth, which further stimulates investment. The financial accelerator model also delivers more persistence than standard New Keynesian models because net worth reverts to steady state very slowly, as can be seen from the Figure. As usual for all models with sticky prices, a one percent increase in total factor productivity leads to less than one percent response of GDP for both models, since marginal costs go down, while prices do not adjust completely on impact, and as a result markups in the economy increase.

The responses of output, investment, consumption and other macroeconomic variables is similar across the two scenarios. The output response is almost identical because consumption and investment behave very similarly in the two cases. As we expected, the response of excess returns to capital is much milder in the risk-averse case, about one fifth of the response of the risk-neutral case. Movements in net worth and leverage are somewhat larger in the risk-averse case but the price of capital increases in a very similar fashion across the two scenarios, which leads to similar responses in investment.

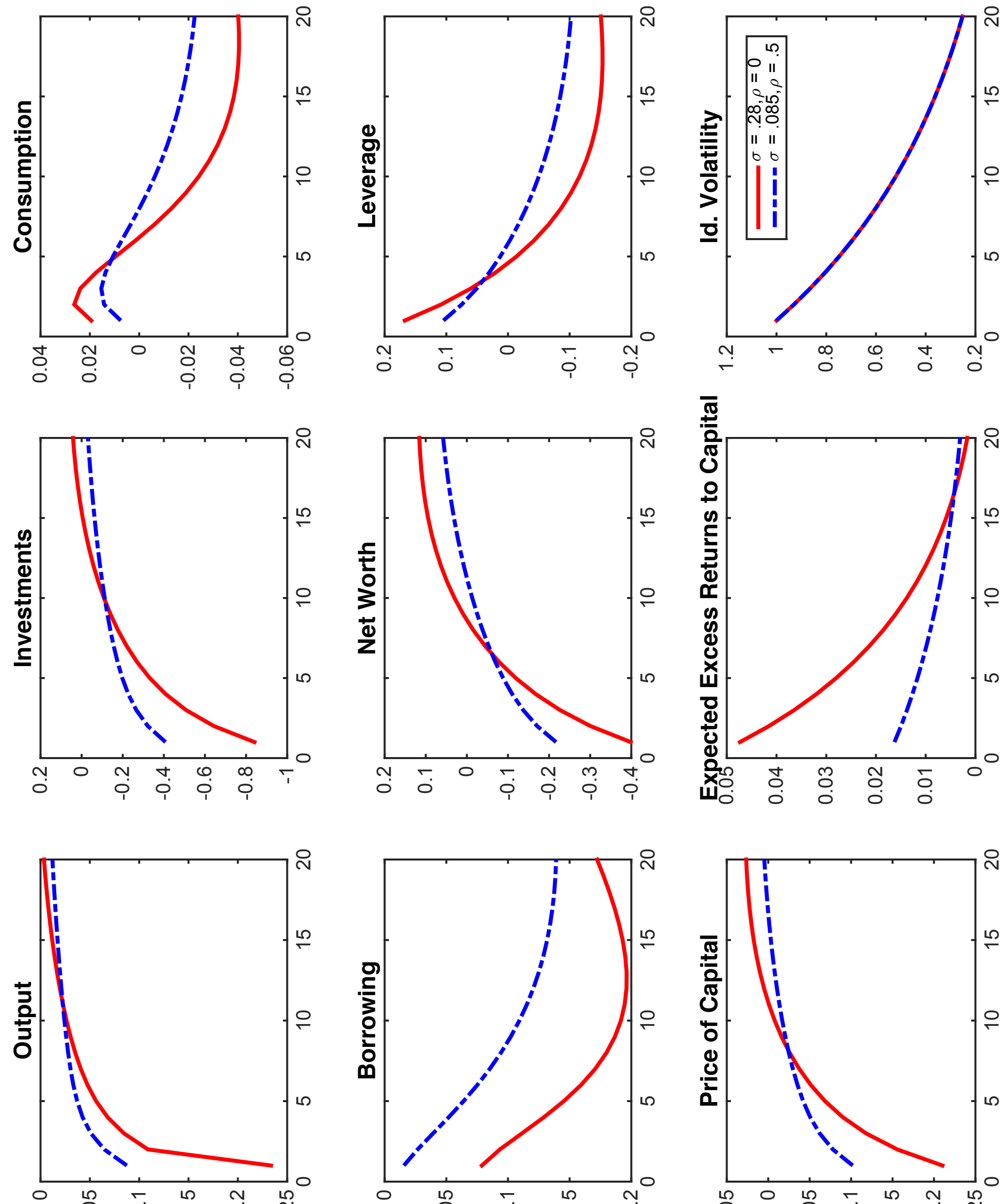
One of the appealing feature of general equilibrium models with costly state verification and risk-neutral borrowers is that they amplify monetary shocks and they make the responses of macro variables more persistent, thanks to the endogenous dynamics in net worth. Figure 2.7 shows the impulse responses of the two models with varying degrees of risk-aversion with respect to 25 basis-point shock to the interest rate. The model with higher risk aversion and more precautionary behavior displays responses to monetary shocks that are about twenty percent smaller on impact vis-a-vis the risk-neutral case. As for the technology shock, excess returns to capital go down much less in the case of risk-aversion. Nevertheless, because of the higher sensitivity of leverage to this wedge, the response of leverage and net worth is quantitatively similar - only about 20% smaller in the risk-averse scenario. The price of capital and investment go up to a smaller extent in the risk-aversion case, therefore, we observe a somewhat smaller reaction of output to the same shock. Nevertheless, the responses are similar in the two cases. The endogenous adjustment of excess returns to capital is such that the financial accelerator mechanism is fundamentally robust to the presence of risk-averse entrepreneurs in response to technology and monetary shocks.

2.6 Conclusion

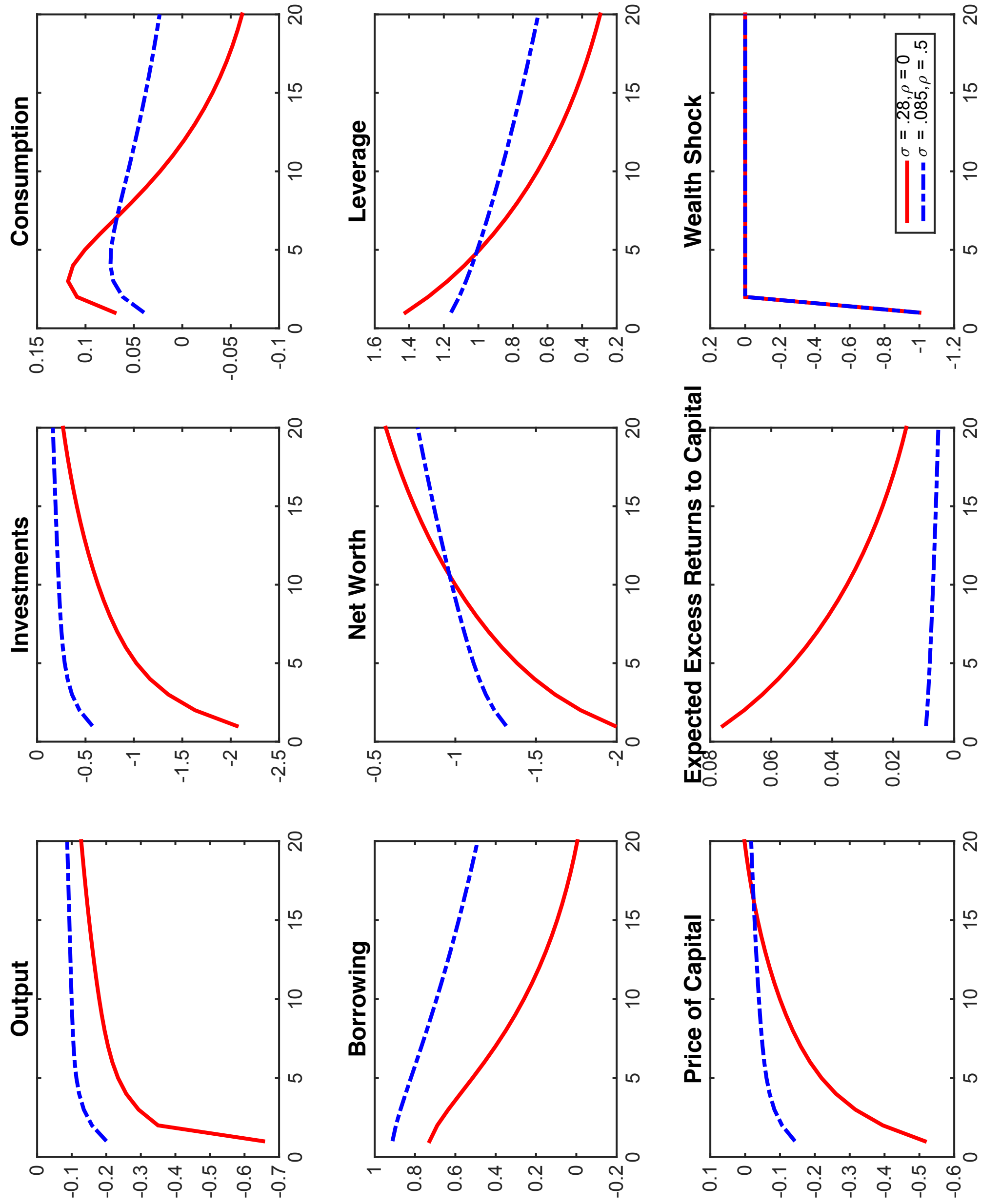
In this paper we extend the BGG framework to allow borrowers to have constant relative risk-aversion preferences instead of being risk neutral. This new framework is tractable as the popular BGG model of financial frictions but allows us to address new questions regarding risk that were not answerable in the original model. We use this new framework to ask the most natural of the questions, that is whether traditional macroeconomic shocks as well as recently popularized shocks have different effects when entrepreneurs are risk averse as opposed to risk neutral. We find that the model with risk-averse borrowers compared to the model with risk-neutral borrowers demonstrates similar responses for technology and monetary shocks, but significantly weaker responses for shocks to the volatility of idiosyncratic productivity or "risk-shocks" and for wealth shocks. These results relate to the literature that stresses the importance of changes in uncertainty or idiosyncratic risk in explaining salient features of business cycles, such as Christiano, Motto, and Rostagno (2014) and Gilchrist, Sim, and Zakrajšek (2014). Our simulations suggest that the quantitative importance of financial shocks such as risk shocks or wealth shocks is sensitive to the risk attitude of the entrepreneurs in the model.

For subsequent research our framework can be extended in several directions. It is possible to have several types of entrepreneurs with different preferences and leverage, while maintaining analytical tractability. Such specification would allow the average level of risk-aversion to be time-varying, since positive shocks would redistribute resources towards agents with higher leverage and lower risk-aversion. In this case, a sequence of good shocks would decrease average risk-aversion and increase leverage, which might make economy more fragile to negative shocks.

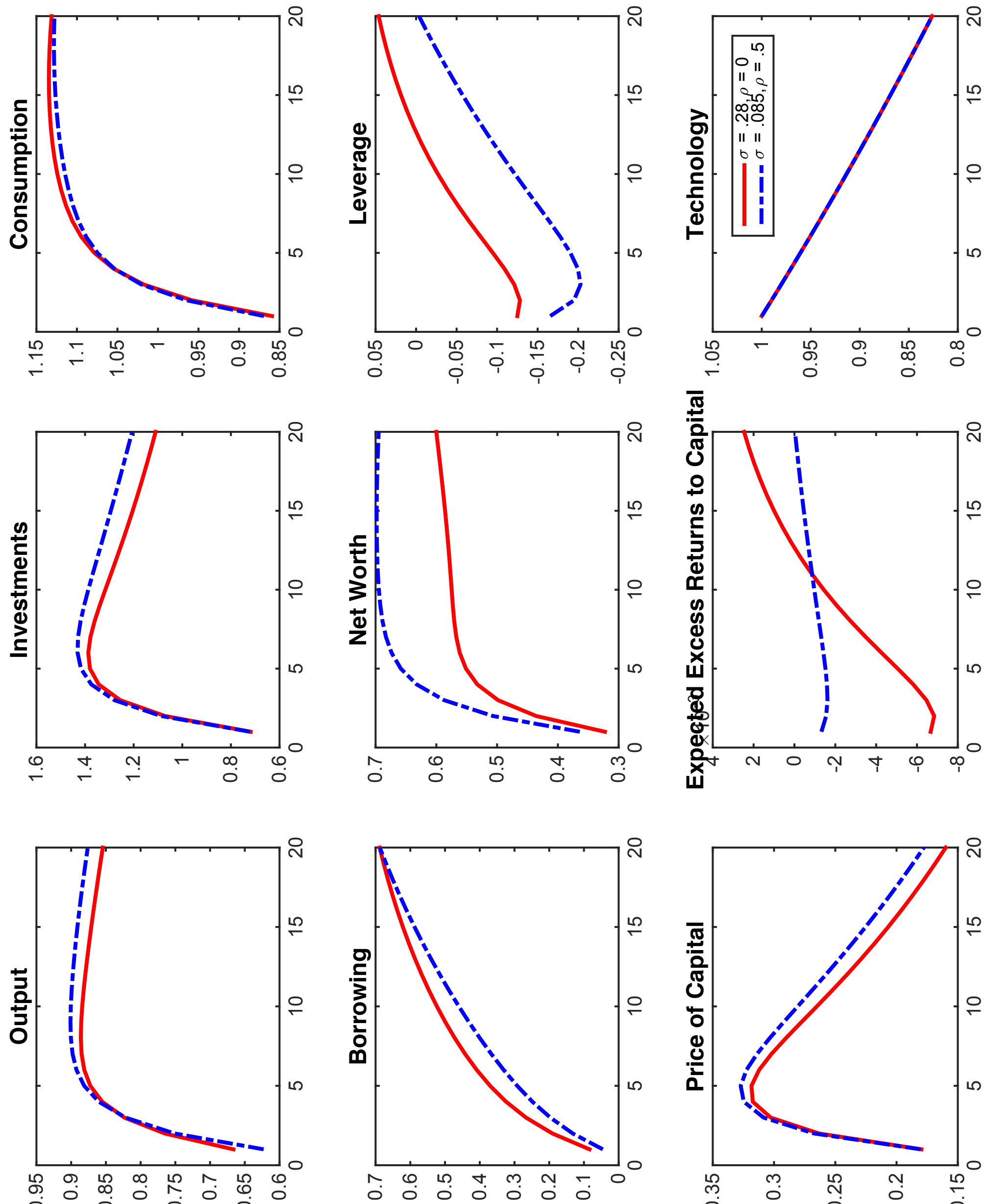
Our framework also allows for contracts with optimal risk-sharing of aggregate risk between lenders and borrowers. In the current framework returns to lenders are predetermined, and entrepreneurs effectively carry all aggregate risk, so it would be interesting to investigate, whether the amplification of monetary and technology shocks is robust to the trade of state-contingent claims on the aggregate state of the world. From Dmitriev and Hoddenbagh (2013), Carlstrom, Fuerst, Ortiz, and Paustian (2014), and Carlstrom, Fuerst, and Paustian (2016) we know that the financial accelerator is not robust to the presence of state-contingent contracts for risk-neutral entrepreneurs. Dmitriev and Hoddenbagh (2014) demonstrate that amplification is not robust to state-contingent contracts in costly enforcement environment, developed by Kiyotaki and Moore (1997) and extended by Iacoviello (2005) to risk-averse agents environment. The robustness of the accelerator to state-contingent contracts in costly state verification framework with risk-averse agents remains an important question for future research.



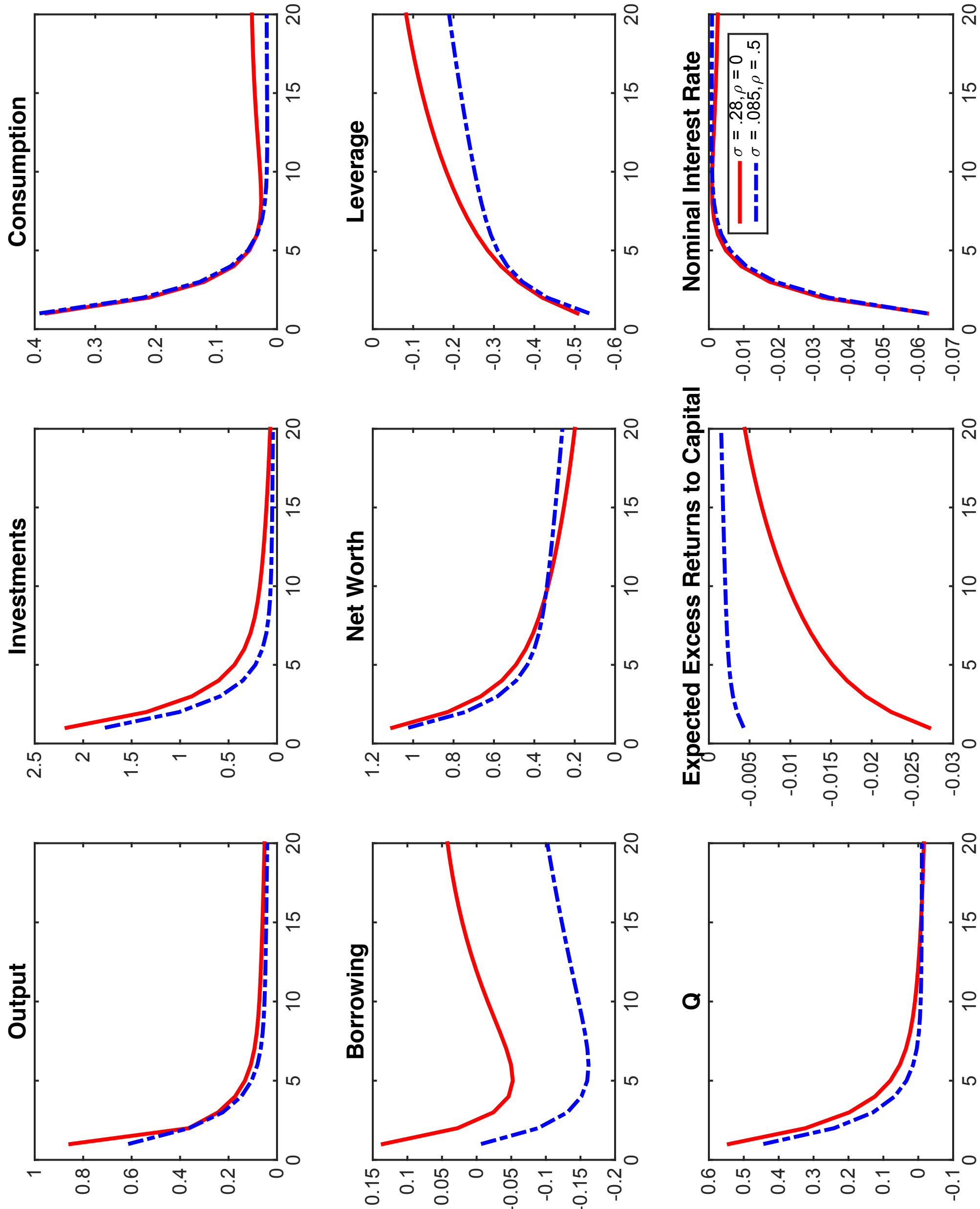
Notes: The figure depicts the impulse responses of key variables following a risk shock for the model with risk neutrality (red lines) and for the model with risk aversion (blue lines). All variables are in percentage deviations from their steady state value.



Notes: The figure depicts the impulse responses of key variables following a wealth redistribution shock for the model with risk neutrality (red lines) and for the model with risk aversion (blue lines). All variables are in percentage deviations from their steady state value.



Notes: The figure depicts the impulse responses of key variables following a technology shock for the model with risk neutrality (red lines) and for the model with risk aversion (blue lines). All variables are in percentage deviations from their steady state value.



Notes: The figure depicts the impulse responses of key variables following a monetary shock for the model with risk neutrality (red lines) and for the model with risk aversion (blue lines). All variables are in percentage deviations from their steady state value.

Chapter 3

Implications of Default Recovery Rates for Aggregate Fluctuations

3.1 Introduction

Default recovery rates for corporate bank loans in the United States are strongly procyclical and highly volatile, ranging from 53 to 88 percent over the last 25 years. This finding does not seem surprising at first. Shleifer and Vishny (1992) pointed out that during a recession it is harder for a bank to sell the assets of a firm in financial distress, since the most productive use of these assets would be exercised by similar firms, which are likely to experience comparable financial difficulties. Furthermore, in times of recession, other financial institutions are trying to sell similar assets due to widespread bankruptcies. All of these factors make markets less liquid and cause recovery rates to deteriorate sharply during economic downturns.

General equilibrium models with financial frictions since Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999, BGG) have so far focused on explaining the dynamics of spreads and defaults, but put little emphasis on the behavior of recovery rates. By itself this is not an issue, since these models might be able to generate realistic patterns of recovery rates without explicitly trying to match them. However, we demonstrate that in the existing models recovery rates are almost flat over the cycle and rarely move by more than two percent from their average value.[1] This suggests that current models tend to underestimate the cost of bankruptcy in a recession and overestimate them in a boom. So long as bankruptcy costs impede the flow of funds from lenders to borrowers, these results imply that current frameworks might be understating the severity of financial frictions and their effects on macroeconomic aggregates.

The natural research question of this paper is how can existing models be modified in order to explain the behavior of the recovery rates and other business cycle variables. One of the simplest and most natural approaches consists in incorporating the Shleifer and Vishny (1992) insight into a dynamic general equilibrium model. Indeed, if liquidating an asset requires a match with a potential buyer, then during a recession when markets are illiquid, finding a corresponding match becomes harder, which would make liquidation costs countercyclical and recovery rates procyclical. When markets are very liquid and buyers are

[1] Here we mean costly state verification approach following Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) , Christiano, Motto, and Rostagno (2014) and others, since models following the costly state enforcement approach after Kiyotaki and Moore (1997) do not have default or recovery rates.

plentiful, liquidation costs should decrease, while illiquidity should make it more difficult to find a match, driving liquidation costs upward. We denote this effect by liquidity channel and we embed it into a state-of-the-art model of financial frictions.

Formally, we extend a standard agency cost model by allowing liquidation costs for creditors to depend on the tightness of the market for physical capital. Building on Blanchard and Galí (2010), we assume that banks pay liquidation costs that depend on the ratio of the capital sold to capital purchased by entrepreneurs. These costs are small when the majority of entrepreneurs are trying to buy capital; in this case banks can sell liquidated assets relatively easy. On the other hand, when most of entrepreneurs sell physical capital, liquidation costs for banks increase. Naturally in the agency cost framework, most entrepreneurs are net buyers of capital in a boom and net sellers in a recession, making liquidation costs countercyclical. It turns out that this additional friction allows us to successfully explain the existing dynamics of defaults, spreads and balance sheets, as well as recovery rates.

In a related paper, Choi and Cook (2012) study the effect of a concave production function for liquidation services in a small-scale financial accelerator model, and show that this concavity can generate higher volatility of recovery rates. We differ from their work in three respects. First, we use a different approach to modeling liquidation, which relies on the tightness of the market for capital goods. Second, we build on the medium-scale DSGE model by Christiano, Motto, and Rostagno (2014), which explains the joint behavior of macroeconomic and financial variables. Third, we use Moody's dataset instead of FDIC's, which focuses on corporate debt and grants a tighter link between spreads, defaults, and recoveries in the data.

We make several contributions to the literature. First, we show that standard nominal rigidities and balance sheet channels in agency costs models are not sufficient to generate the pattern of recovery rates observed in the data. Second, we introduce a liquidity channel to the agency costs model, which allows us to reconcile the model and the data. Third, we demonstrate that the liquidity channel strengthens the effect of financial shocks on output and asset prices. Indeed, when a negative shocks hits the economy, not only do markups go up and balance sheets deteriorate, but markets also become less liquid due to the fact that most entrepreneurs are trying to sell physical capital. As a result, banks become more reluctant to lend to entrepreneurs even if the latter have strong balance sheets, since, even if the probability of default for the entrepreneur is the same, the illiquidity of the markets drives down the potential recovery rate for the bank. In other words, expected bankruptcy costs go up for all borrowers, regardless of their balance sheets. We find that the liquidity channel amplifies the impact of financial shocks by a factor that is between 25 and 50 percent, depending on the nature of the shock. Finally, we find these additional negative effects to be persistent and present up to 20 quarters after the shock has hit the economy.

3.2 Recovery Rates and the Business Cycle

In this section we document the cyclical properties of recovery rates and investigate whether current macroeconomic models with financial frictions are able to explain them. Recovery rates measure the extent to which the creditor recovers the principal and accrued interest due on a defaulted debt. Our data come from Moody's (2016) "Annual Default Study: Corporate Default and Recovery Rates", which contains information about defaulted corporate bonds and loans recoveries, measured by the market value of defaulted debt as a percentage of par one month after default. The aggregate data are available at annual frequency from 1990 until 2014 and reflect the experience of over 20,000 corporate issuers in Moody's proprietary database. Figure 3.1 shows the dynamics of the recovery rate on first-lien loans along with those of real GDP growth for the United States.[2]

As the Figure shows, recovery rates tend to vary systematically over time. Recovery rates on first-lien loans exhibit substantial volatility, ranging from 53.4 percent in 1993 to 87.7 percent in 2004 within our sample. The Figure also makes clear that recovery rates closely track the business cycle. Loan recovery rates rose above 80 percent in the early 2000s while the economy was booming. As the financial crisis unravelled, recoveries started plummeting, reaching about 53 percent in the midst of the Great Recession. These findings are consistent with previous evidence by Frye (2000a,b) and Schuermann (2004), who show that in a recession, recovery is about a third lower than in an expansion. Over our sample period, recoveries and GDP growth exhibit a contemporaneous correlation of 0.41.[3]

Aggregate recovery rates exhibit a systematic relationship with defaults, as documented by Altman et al. (2005). As one may expect, recovery rates are lower when the aggregate

[2]While the recovery rates are available for different types of assets, including secured and unsecured bonds, in this study we focus on bank loans, which have been the traditional focus of the financial accelerator literature. Nevertheless, other types of assets' recovery rates exhibit strong pro-cyclicality and even higher volatility. In this sense, we take a conservative stance on volatility of recovery rates both in terms of frequency and type of assets.

[3]The correlation between recoveries and HP-filtered GDP is 0.30.

default rate increases. In Figure 3.2 we provide some evidence of this relationship by showing the pattern of recoveries and the delinquency rates on business loan for the United States.[4] Between 2006 and 2009, delinquency rates increased from 1.27 to 3.91 percent, while recovery rates fell from 83.6 to 53.6 percent. The two time series are negatively correlated with a correlation coefficient of -0.42, while the correlation of delinquencies with real GDP growth is -0.33. Our evidence on default and recoveries is in line with previous research which highlights a similar macroeconomic dependence of recovery rates (Mora, 2012).

We now examine the behavior of aggregate recovery rates through the lens of a general equilibrium model with financial frictions. A strand of the macroeconomic literature has focused on the ability of these models to explain the behavior of spreads and defaults over the business cycle but so far their implications for recoveries remains unexplored. For our analysis we use the model of Christiano, Motto, and Rostagno (2014, CMR), which builds on the seminal contribution of BGG in the financial accelerator literature. Our choice is guided by two facts. First, this class of models features equilibrium defaults and associated bankruptcy costs. Hence, it is straightforward to construct a measure of the aggregate recovery rate in the model that can be compared with the data. Second, the estimated model of CMR is successful at explaining the time-variation in defaults observed in the data. Indeed, in one of their posterior predictive checks, the authors show that this model successfully accounts for the dynamics of delinquency rates for the United States over the

[4] This data correspond to the series "Delinquency Rate On Business Loans, All Commercial Banks" on the FRED database.

last two decades. Therefore it is natural to ask whether the model is able to explain the dynamics of recovery rates, conditional on its success in accounting for fluctuations in defaults.

To answer this question, we compute the aggregate implied recovery rate from CMR using their publicly available codes and compare it against the data.[5] In their paper, the authors do not try to match recovery rates, but they do a good job at explaining other financial variables such as credit, spreads, net worth and the slope from the term structure, as well as a set of traditional macroeconomic variables. As can be seen from the results presented in Figure 3.3, the implied recovery rate from CMR do not exceed 72 percent or fall below 68 percent even during 2008-2009 Great Recession. On the other hand, the recovery rate from the Moody's dataset features a much higher volatility.

These findings suggest that current models of financial frictions tend to underestimate the cost of bankruptcy in a recession and overestimate them in a boom. So long as bankruptcy costs impede the flow of funds from lenders to borrowers, these results imply that current frameworks might be understating the severity of financial frictions and their effects on macroeconomic aggregates. In the next sections we introduce a new channel in the financial accelerator model that is able to explain the behavior of recoveries and we study its effect on aggregate fluctuations.

[5] We derive a formula for the model-implied recovery rate in the Appendix.

3.3 The Liquidity Channel

In this section we introduce the contracting problem between financial intermediaries on the lending side and entrepreneurs on the borrowing side, and we demonstrate how the contract is affected by liquidity of the markets. While the contracting problem closely follows BGG, bankruptcy costs per unit of asset (or capital) are going to be affected by the liquidity of capital markets. Here the price of capital goods and the expected returns to capital are taken as given by lenders and borrowers. The subsequent section will endogenize these prices and returns in our general equilibrium environment.

3.3.1 Entrepreneurs

There is a continuum of entrepreneurs indexed by j. Entrepreneurs are the only agents accumulating capital in the model. At time t, entrepreneur j purchases raw capital, $\bar{K}_{t+1}(j)$, at a unit price of Q_t. The entrepreneur uses his net worth, $N_t(j)$, and a one-period loan $B_{t+1}(j)$ from a financial intermediary (or bank) to purchase his desired level of capital:

$$Q_t\bar{K}_{t+1}(j) = N_t(j) + B_{t+1}(j). \tag{3.1}$$

After the purchase, the entrepreneur converts the raw capital into effective capital services. At the beginning of period $t+1$, the entrepreneur is hit with an idiosyncratic shock that the lender cannot directly observe. Entrepreneur j earns income by supplying capital services and from capital gains; he then goes to another bank and gets a new loan in order to refinance his previous loan. The new bank perfectly observes the balance sheet of the entrepreneur, as the entrepreneur reveals his private information to the potential creditor. If a new loan is not extended and the entrepreneur is not able to refinance, he defaults, the old banks seizes his assets and tries to sell them in the market for physical capital. On the contrary, if the entrepreneur is able to refinance, he repays the loan to the old bank and uses his residual resources to buy the additional capital. Sometimes the new loan is not sufficient to repay the old loan, and in this case the entrepreneur covers the difference by selling some units of physical capital.

There are two differences relative to the standard agency cost frameworks in macroeconomic models. First, in our model most of the physical capital stays with entrepreneurs, who go to the market to buy or sell only the additional units. In the standard framework, physical capital moves back and forth between households or capital agencies and entrepreneurs. The assumption that entrepreneurs keep and accumulate capital is important for the notion of liquidity. In bad states of the world the majority of entrepreneurs are selling capital and are making markets very illiquid, which puts additional pressure on banks, that try to sell seized assets. In the standard framework, this effect would be much weaker, since the continuous movement of the entire capital stock between households and entrepreneurs would make capital markets always liquid. Our assumption also matches more closely real life phenomena. Indeed, the stock of capital moves very slowly with business cycles, while the flows of capital are very pro-cyclical and volatile, which perfectly serves the concept of liquidity in our framework.

Second, we assume that when the entrepreneur refinances the loan, he reveals his private information to the new lender, but not to the old lender. We consider this assumption highly plausible, since the borrower has a strong motivation to reveal his situation ex-ante in order to get the loan and to respond to all information requests from the lender, otherwise the lender could simply reject the loan application. On the other hand, once the loan is approved, entrepreneur has smaller incentives ex-post to provide the lender with his private information. Under these assumptions, we have a standard costly state verification setup and we follow the contracting problem between lenders and borrowers after BGG.

3.3.2 The Loan Contract

The contracting between the entrepreneur and the financial intermediary is subject to a typical agency problem. In period $t+1$, entrepreneur j is hit with an idiosyncratic shock, $\omega_{t+1}(j)$, and an aggregate shock, R^k_{t+1}, so that he is able to deliver $Q_t K_t(j) R^k_{t+1}\omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega_{t+1}(j)$ is a log-normal random variable with distribution $\log(\omega_{t+1}(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_{\omega,t}, \sigma^2_{\omega,t})$ so that the mean of ω is equal to 1. We denote by $f_t = f(\omega, \sigma_{\omega,t})$ and by $F_t = F(\omega, \sigma_{\omega,t})$ the probability density function and cumulative distribution function of ω_t, respectively.[6] The realizations of ω are independent across entrepreneurs and over time. We assume that the lender cannot observe the realization of the idiosyncratic shock to the entrepreneurs unless he pays monitoring costs μ_{t+1} which are expressed in percentage of total assets. The loan obtained by the entrepreneur takes the

[6]The timing is meant to capture the fact that the variance of ω_{t+1} is known at the time of the financial arrangement, t.

form of a standard debt contract, where Z_{t+1} denotes the promised gross rate of return on the loan. Let, $\bar{\omega}_{t+1}$, be the value of ω below which an entrepreneur is not able to repay the principal and the interest on the loan. This cutoff is defined by

$$B_{t+1}(j)Z_{t+1} = Q_t\bar{K}_{t+1}(j)R^k_{t+1}\bar{\omega}_{t+1} \tag{3.2}$$

Entrepreneurs with $\omega < \bar{\omega}$ are not able to refinance and, hence, declare bankruptcy. In this case, the lender seizes the entrepreneurial assets and tries to find a match on the market in order to sell these assets. The ex-post $t+1$ payoff to the entrepreneur with net worth $N_t(j)$ is given by

$$\int_{\bar{\omega}_{t+1}}^{\infty} [Q_t\bar{K}_{t+1}(j)R^k_{t+1}\omega - B_{t+1}(j)Z_{t+1}]dF_t(\omega) = [1 - \Gamma_t(\bar{\omega}_{t+1})]R^k_{t+1}\kappa_t N_t(j) \tag{3.3}$$

where

$$\kappa_t \equiv \frac{Q_t\bar{K}_{t+1}(j)}{N_t(j)}$$

$$\Gamma_t(\bar{\omega}_{t+1}) \equiv [1 - F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$$

$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$$

and κ_t denotes leverage, from which we have dropped the index j in anticipation of the result that leverage is independent of net worth (see below).

Financial intermediaries collect deposits from the household, to which they promise a competitively determined, non-state contingent, nominal interest rate R_t. The financial intermediary diversifies his lending across a large number of entrepreneurs. Thus, its participation constraint can be written as

$$[1 - F_t(\bar{\omega}_{t+1})]Z_{t+1}B_{t+1}(j) + (1 - \mu_{t+1}) \int_0^{\bar{\omega}_{t+1}} K_{t+1}(j)Q_t R^k_{t+1} \omega dF_t(\omega) \geq R_t B_{t+1}(j) \quad (3.4)$$

where the left hand-side of (3.4) is the expected gross return on the loan and the right hand side is the opportunity cost of lending for the financial intermediary. This equation states that returns to lenders consist of the payoff from firms that did not default, and from the seized assets of entrepreneurs that could not repay their loans net of liquidation costs.

Using the definition of leverage and equation (3.2), the participation constraint can be re-written as

$$R^k_{t+1}[\Gamma_t(\bar{\omega}_{t+1}) - F_t(\bar{\omega}_{t+1})] = \frac{\kappa_t - 1}{\kappa_t} R_t \quad (3.5)$$

Following BGG, we assume that entrepreneurs go out of business with exogenous probability $(1 - \gamma_t)$. In this event, after collecting their earnings from renting their capital, the entrepreneur sells his capital, pays back his loan and consumes his residual net worth. The exiting entrepreneurs are replaced by an inflow of new entrepreneurs that receive an initial start-up transfer from the household, W^e_t. Therefore, the entrepreneurial objective function is described by

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \left[\left(\Pi^s_{i=0} \gamma_{t+i} \right) N_{t+s}(j) \right] \right\} \quad (3.6)$$

The law of motion for aggregate net worth, N_t, is given by

$$N_t = \gamma_t[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^k Q_{t-1}\bar{K}_t + W_t^e \tag{3.7}$$

The debt contract specifies a pair $(B_{t+1}(j), Z_{t+1})$ that maximizes the utility of the entrepreneur given by (3.6) subject to the participation constraint of lenders defined by (3.5). As it is evident, the problem of choosing B_{t+1} is equivalent to choosing κ_t, independently of net worth. Furthermore, using (3.2) we can re-express Z_{t+1} in terms of $\bar{\omega}_{t+1}$, so that our contract is described by the pair $(\kappa_t, \bar{\omega}_{t+1})$. Dmitriev and Hoddenbagh (2013) show that maximization of inter-temporal utility with linear preferences is identical to the maximization of the next period expected payoff in (3.3) to the first order approximation. As in BGG, in this model we can solve for the aggregate variables N_t, κ_t and $\bar{\omega}_{t+1}$ without keeping track of the distribution of net worth.

3.3.3 Financial Intermediaries

Financial intermediaries accept deposits from households and provide one-period loans to entrepreneurs. While able to diversify the idiosyncratic risk by lending to a large number of entrepreneurs, financial intermediaries are still subject to aggregate risk. Financial intermediaries play also an important role in liquidating the assets of entrepreneurs who go bankrupt. The liquidation cost that they face, μ_t, is going to be proportional to the market value of the assets.

Liquidating assets is costly for banks. Banks specialize in financial intermediation, not in selling distressed assets, hence they lack sufficient skills to properly assess the value of capital goods. When markets are very liquid, the lack of skills is not very problematic and banks can easily find buyers who would pay competitive prices. When markets are very illiquid and banks need to liquidate assets they are instead forced to take a discount on the true market value.

The notion of asset market liquidity that we consider, θ_t, is defined as the ratio of aggregate net sales over net purchases of capital by entrepreneurs in the capital goods market

$$\theta_t = \frac{\int_0^1 \max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]dj}{\int_0^1 \max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]dj} \tag{3.8}$$

An analytical expression for θ_t is derived in the Appendix. In equation (3.8) the term $\max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]$ in the numerator defines the net sales of capital units by entrepreneur j. When the entrepreneur is a net purchaser of capital on the market, this term becomes zero. Correspondingly, the term $\max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]$ in the denominator denotes net purchases of physical capital by entrepreneur j. In the steady state entrepreneurs with high idiosyncratic productivity realizations become net buyers of capital, while unproductive entrepreneurs become net sellers. However, during recessions the fall in asset prices reduces aggregate returns, and makes most of entrepreneurs start selling capital. This effect will make markets very illiquid for banks that try to sell seized assets.

We assume that liquidation cost of the banks are a decreasing function of market liquidity

$$\mu_t = \mu\big(\theta_t/\theta_{ss}\big)^{\varphi} \tag{3.9}$$

where $\varphi > 0$ and θ_{ss} is the steady state value of θ_t. This approach parallels Blanchard and Galí (2010), who model frictions in labor markets by assuming that hiring costs are an increasing function of labor market tightness. Our formalization implies that banks can liquidate assets immediately, as long as they are willing to pay the liquidation cost, μ_t, which is a function of the liquidity of the market for capital goods. An alternative formulation of the problem would see banks paying a search cost to find a match with a potential buyer. In this case some of the capital will not be matched and stay on the balance sheets of the banks, which would add a lot of complexity by making the problem of financial intermediaries dynamic in the presence of agency costs. Moreover, the gains of developing such a model are limited by the absence of data on "vacancies" for capital, which make the matching approach less attractive in capital markets relative to labor markets. Nevertheless, both approaches share the idea that the cost of liquidating the capital is a decreasing function of the liquidity in the capital market. Our approach has the advantage of keeping the model much more tractable and compatible with the current generation of agency costs setups.

3.4 General Equilibrium

Our general equilibrium model extends CMR, allowing liquidation cost to vary with the business cycle, as developed in the previous section. This medium-scale DSGE model has been shown to be well suited to explain the joint co-movement of financial and traditional key macroeconomic variables. The New Keynesian backbone of the model follows Christiano, Eichenbaum, and Evans (2005), augmented with technology shocks in the production of installed capital, following the contribution of Justiniano, Primiceri, and Tambalotti (2010). The agency problem between lenders and entrepreneurs comes from BGG. Our framework is isomorphic to CMR's model when we set $\varphi = 0$.

3.4.1 Final Goods Producers

Perfectly competitive firms produce a homogeneous final good, Y_t, from a continuum of intermediate goods, $Y_{j,t}, j \in [0,1]$ using the following Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{f,t}}} dj \right)^{\lambda_{f,t}}, \qquad 1 < \lambda_{f,t} < \infty \tag{3.10}$$

where $\lambda_{f,t}$ is a price markup shock. All the shocks processes will be described below. Maximization of profits, together with the zero-profit condition, implies that the price of the final good, P_t, is the familiar CES aggregate of intermediate goods' prices.

The homogenous final good can be converted into consumption goods, C_t, one for one. A different technology converts $\Upsilon^t \mu_{\Upsilon,t}$ units of final goods into one unit of investment goods, where $\mu_{\Upsilon,t}$ is an investment-specific technology (IST) shock. These two technologies are operated by perfectly competitive firms so that the equilibrium price of investment goods is $P_t/(\Upsilon^t \mu_{\Upsilon,t})$.

3.4.2 Intermediate Goods Producers

Each intermediate good j is produced by a monopolist using the following production function

$$Y_{j,t} = \max[\epsilon_t K_{j,t}^{\alpha}(z_t l_{j,t})^{1-\alpha} - \Phi z_t^*, 0] \tag{3.11}$$

where $K_{j,t}$ and $l_{j,t}$ denote the amount of effective capital and labor employed by firm j. ϵ_t is a stationary technology shock, while the variable z_t follows a process with a stationary growth rate. Φ is a fixed cost in production chosen so that profits are zero in steady state and $z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$ to ensure the existence of a balanced growth path. Supplier j sets his price to maximize his profits subject to Calvo-style frictions (Calvo, 1983). In particular, in every period t a random subset ξ_p of suppliers cannot optimally set its price, but adjusts it according to $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where the indexation follows $\tilde{\pi}_t = (\pi_t^{target})^{\iota}(\pi_{t-1})^{1-\iota}$ and $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$. π_t^{target} represents a target inflation rate for the monetary policy rule, described below. The complementary set of suppliers $1-\xi_p$ re-optimizes prices to maximize the profit function:

$$E_t \sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[P_{j,t} \left(\prod_{k=1}^{s} (\pi_{t+k}^{target})^{\iota} (\pi_{t+k-1})^{1-\iota} \right) Y_{j,t+s} - W_{t+s} l_{j,t+s} - P_{t+s} r_{t+s}^{k} K_{j,t+s} \right] \tag{3.12}$$

where the demand for the intermediate product $Y_{j,t}$ comes from the final goods producers, W_t indicates the nominal wage and Λ_t is the marginal utility of nominal income for the representative household.

3.4.3 Capital Goods Producing Sector

Perfectly competitive firms purchase investment goods and transform them into new capital. The technology used by these firms takes I_t units of investment goods and transforms them into $(1 - S(\zeta_{I,t} I_t / I_{t-1})) I_t$ units of new capital. Thus, the flow profit function for a capital good producer is given by

$$Q_t(1 - S(\zeta_{I,t} I_t / I_{t-1})) I_t - P_t / (\Upsilon^t \mu_{\Upsilon,t}) I_t \tag{3.13}$$

The function $S(x)$ captures the presence of adjustment costs in investment, and is such that $S(x) = S'(x) = 0$ and $S''(x) = S''$, where x denotes the steady-state value of $\zeta_{I,t} I_t / I_{t-1}$ and S'' will be a model parameter. $\zeta_{I,t}$ is a shock to the marginal efficiency of investment (MEI) in producing capital goods.

3.4.4 Labor Market

The structure of the labor market follows Erceg, Henderson, and Levin (2000). The specialized labor types, $h_{i,t}$, are combined by perfectly competitive employment agencies into a homogenous labor input using the following technology:

$$l_t = \left(\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} \, di \right)^{\lambda_w}, \qquad 1 < \lambda_w < \infty \tag{3.14}$$

The homogenous labor input is then sold to the intermediate firms. The wage paid by these firms for the homogenous labor input

$$W_t = \left(\int_0^1 (W_t^i)^{\frac{1}{1-\lambda_w}} \, di \right)^{1-\lambda_w} \tag{3.15}$$

can be obtained by solving the profit maximization problem of the employment agencies.

3.4.5 Households

The representative household maximizes its lifetime utility by choosing the optimal path of consumption and labor input

$$E_t \sum_{s=0}^{\infty} \beta^s \zeta_{c,t+s} \left\{ \log(C_{t+s} - bC_{t+s-1}) - \Psi_L \int_0^1 \frac{h_{i,t+s}^{1+\sigma_L}}{1+\sigma_L} di \right\} \qquad b, \sigma_L > 0 \tag{3.16}$$

where C_t denotes household consumption, b parameterizes the degree of consumption habits and $\zeta_{c,t}$ indicates a preference shock. The household provides a continuum of differentiated labor inputs, $h_{i,t} \in [0, 1]$.

We can write the flow budget constraint for the household as

$$(1+\tau^c)P_tC_t + B_{t+1} \leq (1-\tau^l)\int_0^1 W_t^i h_{i,t} di + R_tB_t + \Pi_t \tag{3.17}$$

The left-hand side of the budget constraint encompasses the sources of expenditure. The household purchases consumption goods, C_t, that are taxed at a rate τ^c,at price P_t, and bonds, B_{t+1}. The household sources of revenues are the earnings from labor and from bonds. Π_t denotes lump-sum payments to the household, including profits from intermediate goods, transfers from entrepreneurs, and lump-sum transfers from the government net of lump-sum taxes. Following Erceg, Henderson, and Levin (2000), we assume that there is a monopoly union for each type of labor input that sets the wage rate, W_t^i, according to a Calvo-style friction. Specifically, in every period a random subset of unions $1-\xi_w$ sets their wage optimally by maximizing

$$E_t \sum_{s=0}^{\infty} \beta^s \xi_w^s \left\{ \Lambda_{t+s} W_t^i h_{i,t+s} - \zeta_{c,t+s} \Psi_L \frac{h_{i,t+s}^{1+\sigma_L}}{1+\sigma_L} \right\} \tag{3.18}$$

subject to the labor demand function coming from the intermediate goods producers. The complementary set of unions adjusts their wage according to $W_t^i = (\mu_{z^*,t})^{\iota_\mu}(\mu_{z^*})^{1-\iota_\mu}\tilde{\pi}_{w,t}W_{t-1}^i$, where μ_{z^*} is the growth rate of z_t^* in the non-stochastic steady state and

$$\tilde{\pi}_{w,t} = (\pi_t^{target})^{\iota_w}(\pi_{t-1})^{1-\iota_w}, \quad 0 < \iota_w < 1. \tag{3.19}$$

3.4.6 Aggregate Returns and Law of Motion for Capital

While the entrepreneur's problem and timing were described in the previous section, here we explicitly determine the aggregate returns, R^k_{t+1}, and the law of motion for capital. At the beginning of period $t+1$, after observing the aggregate rate of returns and prices in period $t+1$, each entrepreneur determines the utilization rate, u_{t+1}, of its capital and supplies effective capital services $u_{t+1}\omega\bar{K}_{t+1}(j)$ for a competitive market rental rate, r^k_{t+1}.[7] At the end of period $t+1$ the entrepreneur is left with $(1-\delta)\omega K_{t+1}(j)$, which is sold in a competitive market for the price Q_{t+1}. Hence, the aggregate component of the entrepreneurs' return, R^k_{t+1}, is given by

$$R^k_{t+1} \equiv \frac{(1-\tau^k)[u_{t+1}r^k_{t+1} - a(u_{t+1})]\Upsilon^{-(t+1)}P_{t+1} + (1-\delta)Q_{t+1} + \tau^k\delta Q_t}{Q_t} \tag{3.20}$$

where a is an increasing and convex function capturing the cost of capital utilization and τ^k indicates the tax rate on capital income. The utilization rate is set to its optimal level, which satisfies

$$\Upsilon^{-(t+1)}a'(u_{t+1}) = r^k_{t+1} \tag{3.21}$$

In steady state, $u = 1$, $a(1) = 0$ and $\sigma_a \equiv a''(1)/a'(1)$.

In equilibrium, aggregate demand for capital goods must be equal to aggregate supply. Aggregate demand is given by the demand for capital goods by all entrepreneurs, $\bar{K}_{t+1}$.

[7] The utilization rate is not indexed by j as it is independent of the entrepreneur's net worth. This can be seen below in equation (3.21).

Aggregate supply is given by the undepreciated capital of all entrepreneurs, $(1-\delta)\bar{K}_t$, plus the new capital goods produced in period t, $(1-S(\zeta_{I,t}I_t/I_{t-1}))I_t$. Hence the law of motion for aggregate capital is

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + (1-S(\zeta_{i,t}I_t/I_{t-1}))I_t \tag{3.22}$$

The utilization rate transforms raw capital into effective capital services according to

$$K_t = u_t\bar{K}_t \tag{3.23}$$

3.4.7 The Government and The Resource Constraint

A monetary authority sets the nominal interest rate, in linearized form, following the feedback rule:

$$R_t - R = \rho_p(R_{t-1}-R) + (1-\rho_p)\left[\alpha_\pi(\pi_{t+1}-\pi_t^{target}) + \alpha_{\Delta y}\frac{1}{4}(g_{y,t}-\mu_z^*)\right] + \frac{1}{400}\epsilon_t^p, \tag{3.24}$$

where ϵ_t^p is a shock to monetary policy in annual percentage points and ρ_p is a smoothing parameter in the policy rule. The monetary authority responds to deviation of expected inflation from target, $\pi_{t+1}-\pi_t^{target}$, and to deviations of quarterly growth in gross domestic product from its steady state, $g_{y,t}-\mu_z^*$.

Fiscal policy is fully Ricardian. Government consumption expenditure, G_t, is given by

$$G_t = z_t^* g_t \tag{3.25}$$

where g_t follows a stationary stochastic process.

Finally, the resource constraint can be written as

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\frac{\bar{K}_t}{\Upsilon^t} \tag{3.26}$$

The last term on the constraint indicates the output cost of adjusting capital utilization. D_t represents the resource cost associated with liquidation by financial intermediaries

$$D_t = \mu_t G(\bar{\omega}_t) R_t^k \frac{Q_{t-1}\bar{K}_t}{P_t} \tag{3.27}$$

where, relative to CMR, μ_t is determined endogenously.

3.4.8 Shocks and Information

The model described above includes 11 aggregate shocks: $\epsilon_t, \mu_{z,t}, \lambda_{f,t}, \pi_t^*, \zeta_{c,t}, \zeta_{I,t}, \mu_{\Upsilon,t}, \gamma_t, \sigma_t, \epsilon_t^p$ and g_t. Relative to CMR we abstract from modeling long-term interest rates and its associated shock . Each shock is modeled as a first-order autoregressive process. The autocorrelation of monetary policy and equity shock is set to zero. Following CMR baseline specification, we allow agents to receive information about the realization of risk shocks before innovation is realized. In particular we consider the following representation of the risk shock:

$$\sigma_t = \rho_\sigma \sigma_{t-1} + \underbrace{\xi_{0,t} + \xi_{1,t-1} + \ldots + \xi_{p,t-p}}_{=u_t} \tag{3.28}$$

The innovation u_t is a sum of i.i.d. random variables with zero mean that are orthogonal to $x_{t-j}, j \geq 1$. In period t, agents observe $\xi_{j,t}, j = 0, 1, \ldots, p$ and we refer to $\xi_{j,t}, j > 0$ as news shocks. The news shocks exhibit the following correlation:

$$\rho_{\sigma,n}^{|i-j|} = \frac{E\xi_{i,t}\xi_{j,t}}{\sqrt{(E\xi_{i,t}^2)(E\xi_{j,t}^2)}}, \qquad i, j = 0, \ldots, p. \tag{3.29}$$

where $-1 \leq \rho_{\sigma,n}^{|i-j|} \leq 1$. Under this specification, the parameters associated with the risk shock are: $\rho_\sigma, \rho_{\sigma,n}, \sigma_\sigma$ and $\sigma_{\sigma,n}$. The other shocks have only two parameters: an autocorrelation and a standard deviation parameter.

3.5 Calibration

Standard Parameters. Our calibration for the standard part of the model follows CMR. The values for the parameters that are related to the long-run properties of our model are summarized in Table 3.1. We set the capital's share, α, to 0.4, the Frisch elasticity of labor supply, σ_L to 1, and the depreciation rate for capital to 0.025. The mean growth rate of the unit root technology, μ_z, is fixed at 0.41 percent and the rate of investment specific technological change at 0.42 percent. These values are chosen to be consistent with the average growth rate of per capita GDP over the sample period and to account for the average rate of decline in the price of investment goods. We set the steady state value of g_t so that government expenditure is 20 percent of GDP in steady state, consistent with the data. Steady-state inflation is fixed at 2.4 percent on an annual basis. The household's discount factor is set to 0.9987. As in Christiano, Eichenbaum, and Evans (2005) we set the steady state markups in the product market $\lambda_{f,t}$ and in the labor market λ_w to 1.2 and 1.05. The tax rates on consumption, capital income and labor follow CMR. We fix the habit formation parameter, b, to 0.74, CMR posterior mode.

For the part of the model that relates to financial frictions, we set the steady-state probability of entrepreneurs exiting business, $1-\gamma$, is set to $1-0.985$. Liquidation costs in steady state, μ, are set to 0.21 and the steady-state volatility of idiosyncratic productivity to 0.26. These values imply a steady-state leverage $\bar{K}/N$ of 2.015, an annualized default probability of 2.24 percent and a value of R^k/R of 1.0073 corresponding to annualized excess return to capital of 4 percent. Furthermore, our calibration implies a share of consumption and investment in GDP of 0.52 and 0.27, in line with the data. Note that our modification to the CMR framework does not affect the steady state of the model but only its dynamics.

The remainder of the parameters affect the model's dynamics. We calibrate these parameters using CMR's posterior mode. CMR estimate were pinned down by Bayesian techniques to match the dynamics of eight macroeconomic series and four financial series. The parameter values are summarized in Table 3.2. The persistence parameters tend to be relatively high for all shocks, except for the persistent component of technology growth, implying that

$\log z_t$ follows roughly a random walk. CMR estimates point to sizable nominal rigidities both in prices and wages. Overall these parameter values are reasonable and in line with previous research.

Calibrating φ. The elasticity of liquidation costs, φ, is calibrated using our data on recovery rates. While our model is parameterized at a quarterly frequency, recovery rates are only available at annual frequency. We address the issue by constructing in the model the following moving average that we map directly into the recovery data

$$R_\tau^c = \frac{R_t^c + R_{t-1}^c + R_{t-2}^c + R_{t-3}^c}{4R^c} \tag{3.30}$$

where τ is a yearly time subscript. We choose a value of φ equal to 16, such that the volatility of recovery rates in the model matches the volatility of recovery rates in the data. This value of φ implies that that if the ratio of sellers to buyers of capital increases by 1 percent, liquidation costs will go up by 2 percentage points. Finally, note that when we set φ equal to zero our model becomes isomorphic to CMR's.

3.6 Impulse Response Analysis

Figure 3.4 outlines the dynamic effect of a positive entrepreneurial survival shock on the economy. Following the shock, fewer entrepreneurs exit the economy, which drives aggregate entrepreneurial net worth up and allows entrepreneurs to invest more. Consequently investment increases, driving asset prices, output and hours upward. Higher asset prices boost aggregate returns, which leads to a rise of worth and a decrease in defaults. This is an example of standard financial accelerator mechanism, and it holds for the basic model and the extended version with liquidity channel. Despite these similarities two models generate very different dynamics of recovery rate. While in the basic model recovery rate stays practically flat, in the model with liquidity channel recovery rate skyrockets by 25 percent. This spike in recovery follows from the fact that with a higher survival rate fewer entrepreneurs exit the industry and sell their businesses. In turn, this implies that capital markets are more liquid, making it much easier for banks to sell seized assets. The surge in recovery rates decreases the cost of lending, which in turn drives investment, net worth, output and asset prices further up. As a result, the impact of a survival shock on output is almost twice as high for the model with liquidity channel.

The dynamic effect of risk shocks on the economy is demonstrated on Figure 3.5. Higher risk causes the increase in defaults and bankruptcy costs, and as a result investment contracts and drive asset prices down, causing net worth and output to fall. Decrease in asset prices leads to the decline of recovery rate by 2 percentage point in the basic model and by 8 percentage point in the model with the liquidity channel. This sharper decrease in the recovery rate is caused by markets becoming less liquid due to the surge in defaults and reduction in investment. The fall in recovery rate for the extended model makes cost of lending higher, and this leads to the next contraction of investment, net worth, price of capital and output. Overall, the presence of the liquidity channel amplifies the impact response of output to the risk shock by 25 percent.

The mechanism is similar for a contractionary monetary shock, illustrated in Figure 3.6. In the case of the basic model the negative demand shock causes a contraction in investment and asset prices. This initial effect translates into net worth losses and leads to the next round of financial tightening, decreasing in investment, prices, and net worth and leading to a surge in defaults. All of these processes make markets less liquid and, therefore, the model with the liquidity channel generates a stronger decline in recovery rates relative to the basic model, where it essentially stays at the steady state level. The additional fall in recovery rate makes external financing more costly and causes investment and asset prices to go down, which again leads to the deterioration of net worth and strengthens the recession.

The liquidity channel amplifies shocks for all the cases above. This is not a coincidence, since investment are procyclical, making capital markets less liquid during a recession and more liquid during a boom. The illiquidity causes the recovery rates to fall during the recession, increasing the financial wedge stronger, which, through a deterioration of balance sheets, has an additional negative impact on the economy. Balance sheet effects work in a similar way through a procyclicality of investment that causes asset prices and, hence, net worth to be procyclical as well. Although these two channels are distinct, they tend to reinforce each other. Lower net worth causes a decline in investment and makes market less liquid, but the liquidity channel causes recovery rates to go down, reducing lending and investment, which leads to a drop in asset prices and, consequently, net worth.

3.7 Conclusion

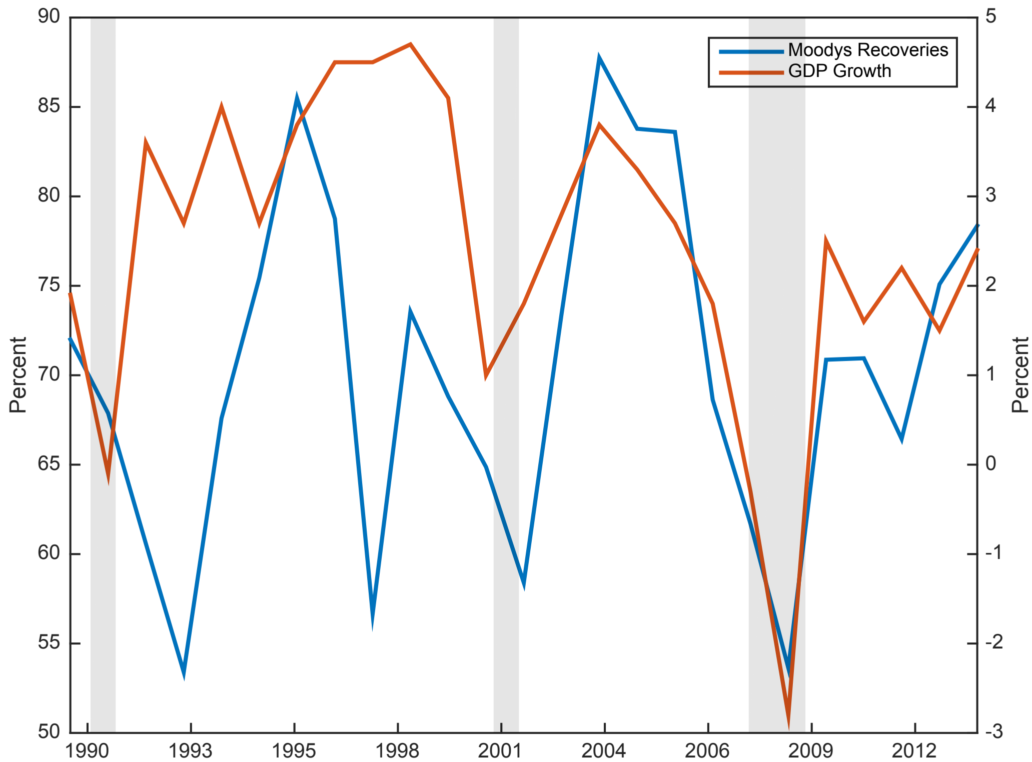
In this paper we document that recovery rates from default in the United States are very volatile and strongly pro-cyclical. We demonstrate that current models of financial frictions significantly understate this volatility by one order of magnitude relative to the data. This finding suggests that current models may underestimate the severity of frictions in financial markets. We therefore extend the financial friction model of Christiano, Motto, and Rostagno (2014), allowing liquidation costs for defaulted assets to depend on the liquidity of the market for these assets. This modification allows us to explain the behavior of standard business cycle variables as well as recovery rates. Our impulse response analysis suggests that the effect of financial shocks on output and asset prices is strongly amplified in the presence of liquidity channel.

Table 3.1: Calibration - Parameters Related to the Steady State

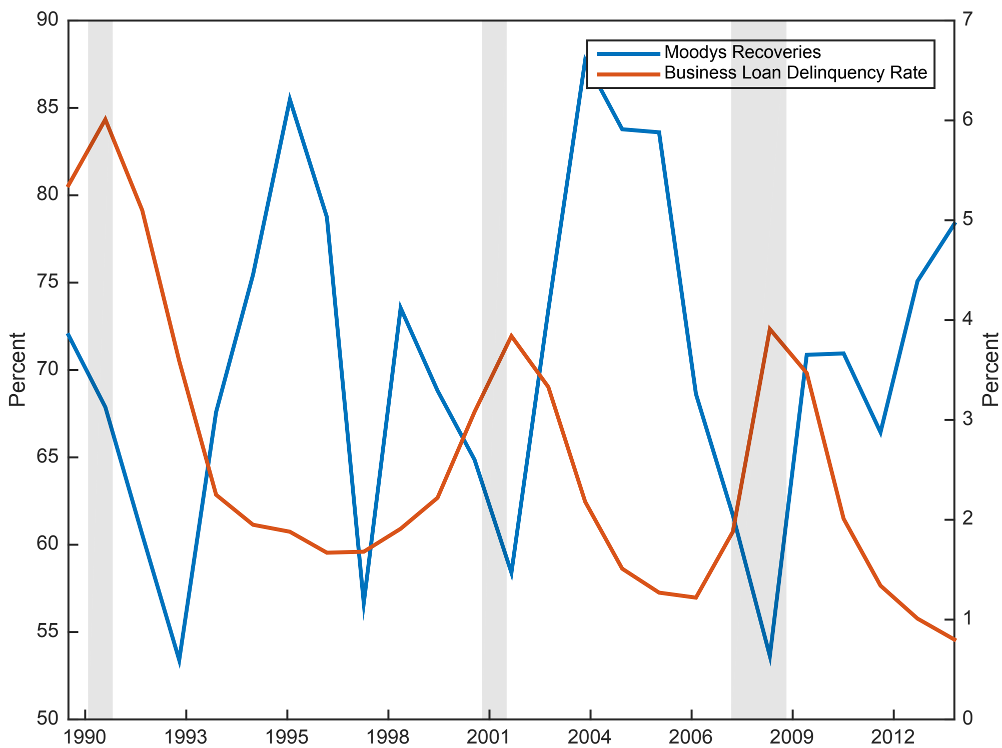
Name	Description	Value
β	Discount rate	0.9987
σ_L	Inverse Frisch elasticity of labor supply	1
Ψ_L	Disutility weight on labor	0.7705
b	Habit formation	0.74
λ_w	Steady-state mark-up for suppliers of labor	1.05
λ_f	Steady-state mark-up for intermediate goods firms	1.2
μ_z	Mean growth rate of unit root technology process	0.41
Υ	Steady-state rate of investment-specific technological change	0.42
δ	Capital depreciation rate	0.025
α	Share of capital in production function	0.4
γ	Fraction of entrepreneurial net worth retained	0.985
μ	Steady-state monitoring costs	0.21
σ	Steady-state standard deviation of idiosyncratic productivity	0.26
W^e	Transfers received by entrepreneurs	0.005
η_g	Share of government spending in GDP in steady state	0.2
π^{target}	Steady-state inflation rate (APR)	2.43
τ^c	Tax rate on consumption	0.05
τ^k	Tax rate on capital income	0.32
τ^l	Tax rate on labor income	0.24

Table 3.2: Calibration - Other Parameters

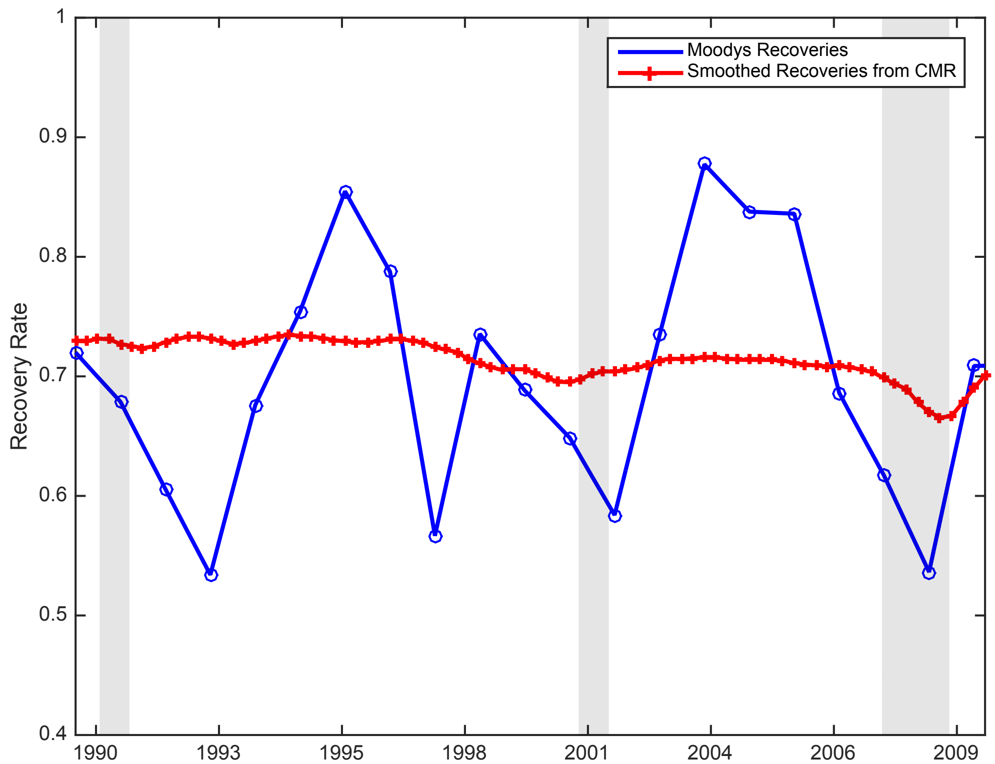
Name	Description	Value
ξ_w	Calvo wage stickiness	0.81
σ_a	Curvature, utilization cost	2.54
S''	Curvature, investment adjustment cost	10.78
ξ_p	Calvo price stickiness	0.74
α_π	Policy weight on inflation	2.4
ρ_p	Policy smoothing parameter	0.85
ι	Price indexing weight on inflation target	0.90
ι_w	Wage indexing weight on inflation target	0.49
Υ	Wage indexing weight on technology shock	0.94
$\alpha_{\Delta y}$	Policy weight on output growth	0.36
φ	Elasticity of liquidation costs	16
$\rho_{\sigma,n}$	Correlation among signals	0.39
ρ_{λ_f}	Autocorrelation, price markup shock	0.91
ρ_{μ_Υ}	Autocorrelation, IST shock	0.99
ρ_g	Autocorrelation, government spending shock	0.94
ρ_{μ_z}	Autocorrelation, persistent technology	0.15
ρ_ϵ	Autocorrelation, transitory technology	0.81
ρ_σ	Autocorrelation, risk shock	0.97
ρ_{ζ_c}	Autocorrelation, preference shock	0.90
ρ_{ζ_I}	Autocorrelation, MEI shock	0.91
Standard deviations		
$\sigma_{\sigma,n}$	Anticipated risk shock	0.028
$\sigma_{\sigma,0}$	Unanticipated risk shock	0.07
σ_{λ_f}	Price markup shock	0.011
σ_{μ_Υ}	IST shock	0.004
σ_g	Government spending shock	0.023
σ_{μ_z}	Persistent technology shock	0.0071
σ_γ	Survival probability shock	0.0081
σ_γ	Temporary technology shock	0.0046
σ_{ϵ^p}	Monetary policy shock	0.49
σ_{ζ_c}	Preference shock	0.023
σ_{ζ_I}	MEI shock	0.055
σ_{ζ_I}	Measurement error, net worth	0.018



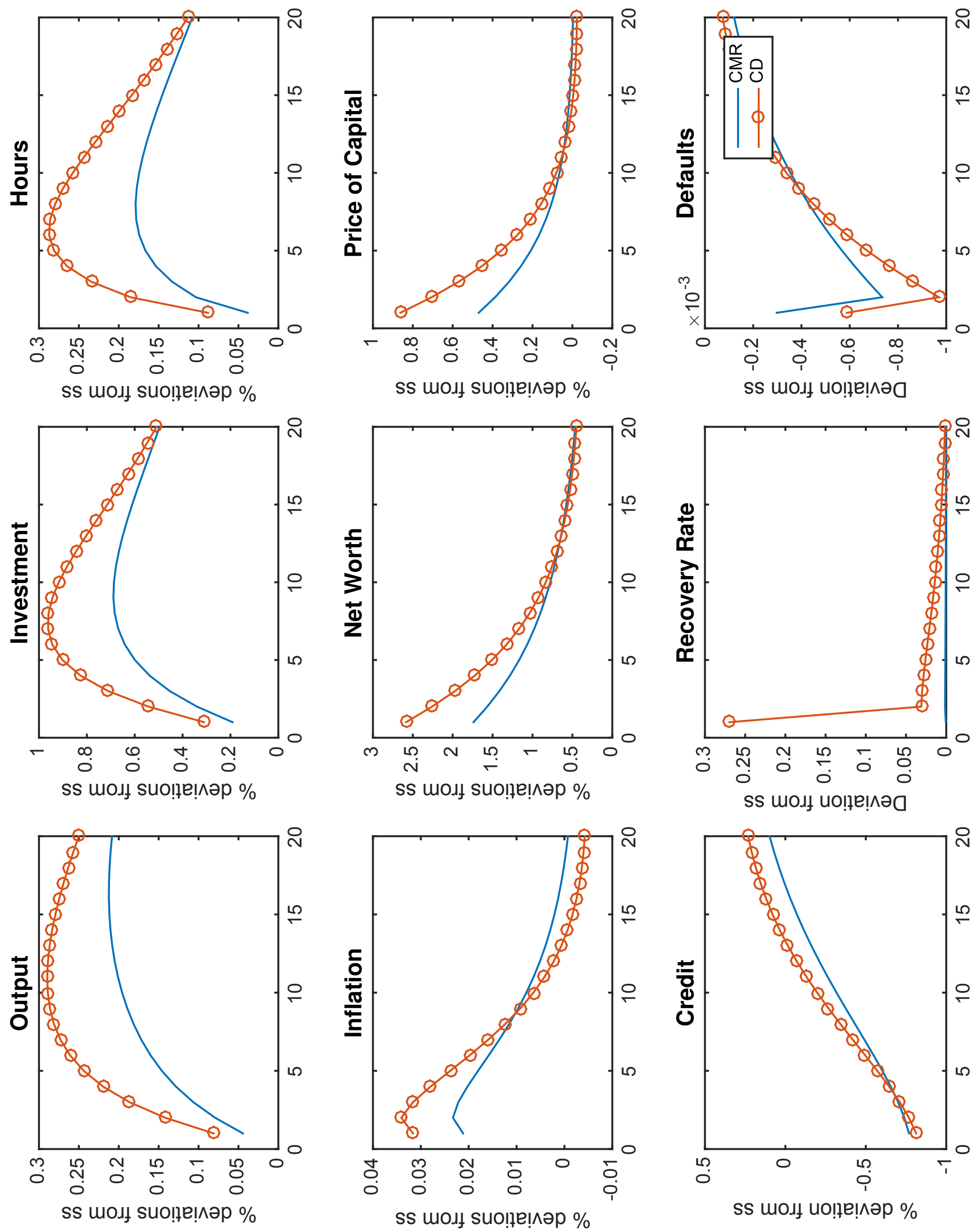
Notes: Recovery rates (left axis) come from Moody's "Annual Default Study: Corporate Default and Recovery Rates". GDP growth (right axis) comes from the FRED database. Shaded bars indicate NBER recessions.



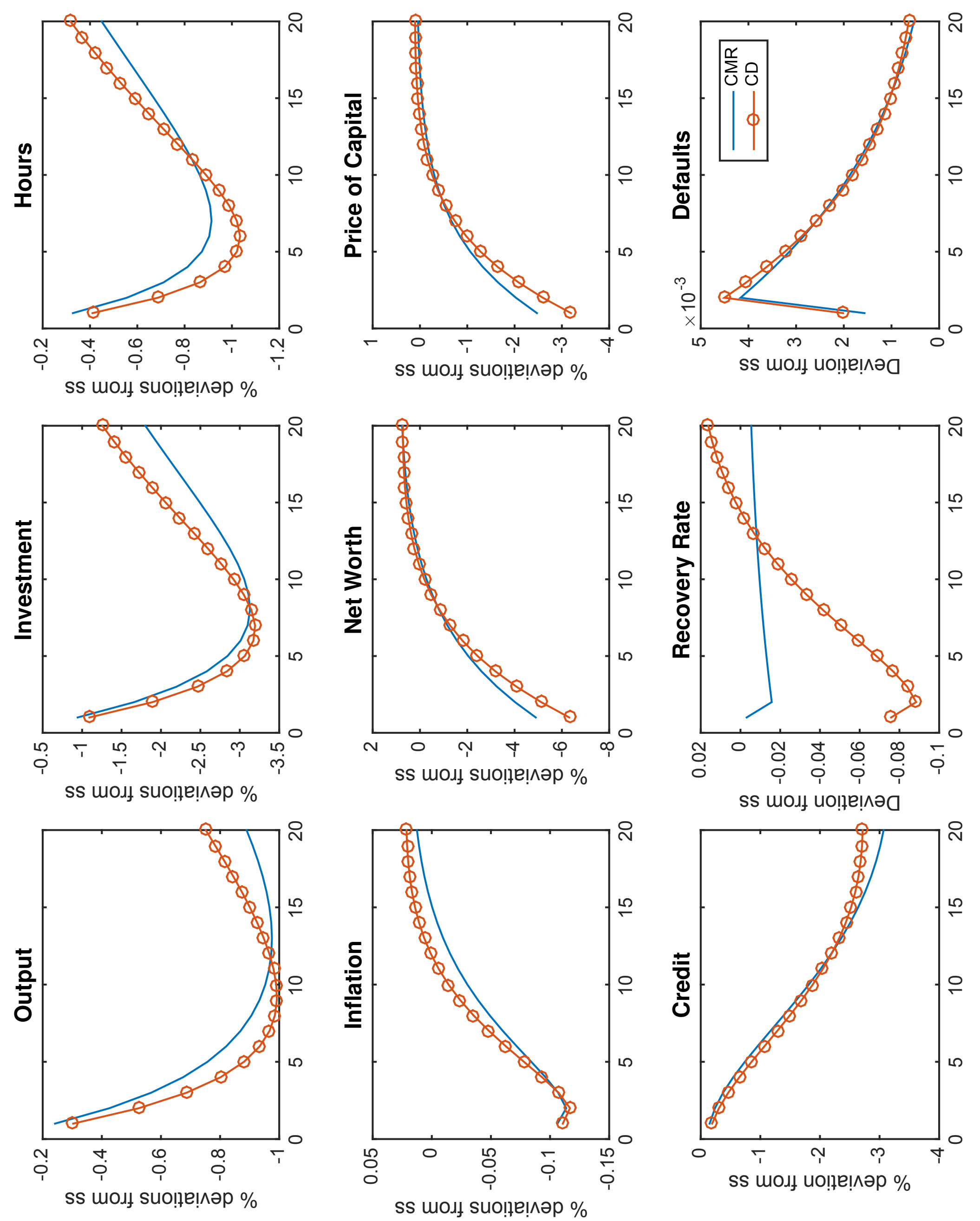
Notes: Recovery rates (left axis) come from Moody's "Annual Default Study: Corporate Default and Recovery Rates". Business Loan Delinquency Rates (right axis) come from the FRED database. Shaded bars indicate NBER recessions.



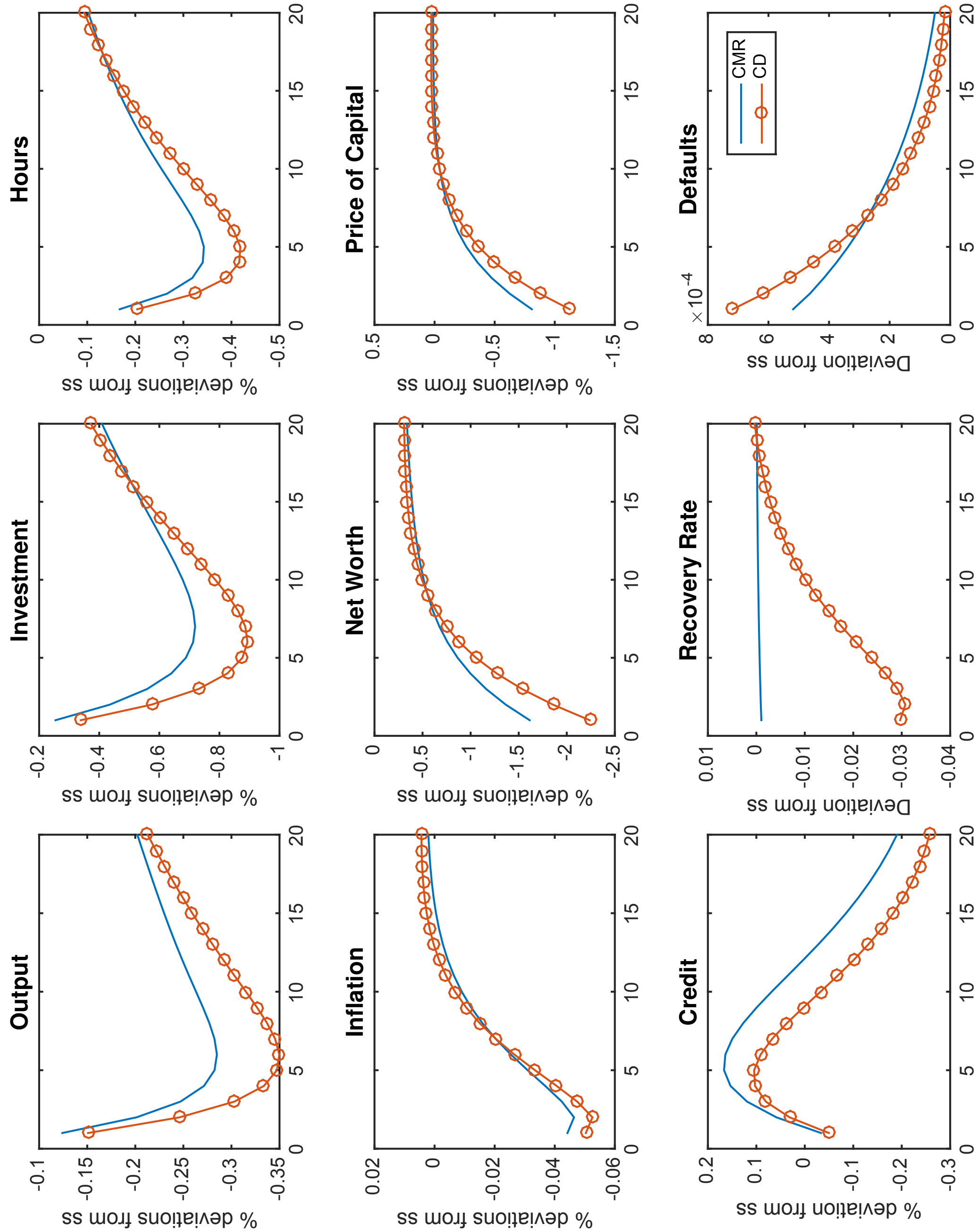
Notes: The data on recovery rates come from Moody's "Annual Default Study: Corporate Default and Recovery Rates". The implied series from the model comes from the estimated Christiano, Motto, Rostagno (2014) model, as explained in the main text. Shaded bars indicate NBER recessions.



Notes: The figure depicts the impulse responses of key variables following an entrepreneurial exit shock for the baseline model (CMR) (blue lines) and for the model with time-varying liquidation costs (orange lines).



Notes: The figure depicts the impulse responses of key variables following a risk shock for the baseline model (CMR) (blue lines) and for the model with time-varying liquidation costs (orange lines).



Notes: The figure depicts the impulse responses of key variables following a monetary shock for the baseline model (CMR) (blue lines) and for the model with time-varying liquidation costs (orange lines).

Appendix A

Information Frictions and Real Exchange Rate Dynamics

A.1 Solution for p_{Ht} and p_{Ft}

Log-linearizing the FOC, one obtains

$$p_t(h) = \mathbb{E}_{ht}(w_{ht} - a_t) \tag{A.1}$$

Add and subtract p_t inside the expectation

$$p_t(h) = \mathbb{E}_{ht}(w_{it} - p_t + p_t - a_t) \tag{A.2}$$

Now substitute $w_{ht} - p_t$ from the log-linear version of (1.6) to obtain

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi} l_{it} + p_t - a_t) \tag{A.3}$$

Substitute the production function for l_{it}

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi}(y_{it} - a_{it}) + p_t - a_t) \tag{A.4}$$

Now substitute the log-linearized demand for y_{it}

$$p_t(h) = \mathbb{E}_{ht}[\sigma c_t + p_t - (1 + \frac{1}{\psi})a_t + \frac{1}{\psi}(-\gamma(p_t(h) - p_{Ht}) - \omega(p_{Ht} - p_t) + \alpha c_t + (1-\alpha)(\omega q_t + c_t^*)]$$

Add and subtract p_{Ht} and rearrange to obtain

$$\begin{aligned} p_t(h) =& \mathbb{E}_{ht}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right)a_t + p_{Ht} - \left(\frac{\psi+\omega}{\gamma+\psi}\right)(p_{Ht} - p_t) + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right)c_t\right] + \\ & \mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c_t^*)\right] \end{aligned}$$

Now recall that $p_{Ht} - p_t = -(1-\alpha)\tau_t$, so

$$\begin{aligned} p_t(h) =& \mathbb{E}_{it}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right)a_t + p_{Ht} + (1-\alpha)\left(\frac{\psi+\omega}{\gamma+\psi}\right)\tau_t + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right)c_t\right] + \\ & \mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c_t^*)\right] \end{aligned}$$

Recall that $q_t = (2\alpha - 1)\tau_t$, hence

$$p_t(h) = \mathbb{E}_{ht}\left[p_{Ht} - (\gamma+\psi)^{-1}[(1+\psi)\,a_t + (1-\alpha)\,(\psi+\omega+\omega(2\alpha-1))\,\tau_t + (\alpha+\psi\sigma)\,c_t + (1-\alpha)c_t^*]\right]$$

$$p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)\,(\psi+2\alpha\omega)\,\tau_t + (\alpha+\psi\sigma)\,c_t + (1-\alpha)c_t^* - (1+\psi)\,a_t]\right\} \tag{A.5}$$

A similar equation can be derived for $p_t(f)$. Rewrite this as

$$p_t(h) = \mathbb{E}_{it}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)\,(\psi+2\alpha\omega)\,\tau_t + (1+\psi\sigma)\,c_t - (1-\alpha)(c_t - c_t^*) - (1+\psi)\,a_{it}]\right\}$$

using the fact that $c_t - c_t^* = (2\alpha-1)\sigma^{-1}\tau_t$, I can write

$$p_t(h) = \mathbb{E}_{it}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)\left(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}\right)\tau_t + (1+\psi\sigma)\,c_t - (1+\psi)\,a_{it}]\right\}$$

use the money process and the link between relative prices to write

$$\begin{aligned} p_t(h) =& \mathbb{E}_{ht}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)\left(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}\right)\tau_t\right\} + \\ & \mathbb{E}_{ht}\left\{(1+\psi\sigma)\,(m_t-(1-\alpha)\tau_t-p_{Ht}) - (1+\psi)\,a_t]\right\} \end{aligned}$$

and finally

$$p_t(h) = \mathbb{E}_{ht}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ht}\right\} +$$
$$\mathbb{E}_{ht}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a_t\right\}$$

Similarly, for the foreign country I have

$$p^*_t(f) = \mathbb{E}_{ft}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p^*_{Ft}\right\} -$$
$$\mathbb{E}_{ft}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m^*_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a^*_t\right\}$$

Notice that the last two equations collapse to (1.17) and (1.18) when $\sigma = 1$. By averaging these two equation over firms one obtains

$$p_{Ht} = \bar{\mathbb{E}}^{(1)}_t\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ht}\right\} +$$
$$\bar{\mathbb{E}}^{(1)}_t\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a_t\right\}$$

and

$$p^*_{Ft} = \bar{\mathbb{E}}^{(1)}_t\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p^*_{Ft}\right\} -$$
$$\bar{\mathbb{E}}^{(1)}_t\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m^*_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a^*_t\right\}$$

When taking the sum of these two equations the terms of trade cancel out

$$p_{Ht} + p^*_{Ft} = \bar{\mathbb{E}}^{(1)}_t \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)(p_{Ht} + p^*_{Ft}) + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m^W_t - \left(\frac{1+\psi}{\gamma+\psi}\right) a^W_t \right\}$$

Recursively substituting $p_{Ht} + p^*_{Ft}$ on the right-hand side yields

$$p_{Ht} + p^*_{Ft} = \tilde{\xi} \sum_{k=1}^{\infty} (1-\tilde{\xi})^{k-1} E^{(k)}_t \left(m^W_t - \frac{1+\psi}{1+\psi\sigma} a^W_t \right) \tag{A.6}$$

where

$$\tilde{\xi} = \frac{1+\psi\sigma}{\gamma+\psi} \tag{A.7}$$

Taking instead the difference of the average prices equations yields

$$p_{Ht} - p^*_{Ft} = \mathbb{E}^{(1)}_t \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)(p_{Ht} - p^*_{Ft}) + \right\}$$
$$\mathbb{E}^{(1)}_t \left\{ 2 \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi} \right] \tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m^D_t - \left(\frac{1+\psi}{\gamma+\psi}\right) a^D_{it} \right\}$$

Now I need to solve for τ_t in terms of $p_{Ht} - p^*_{Ft}$ and m^D_t

$$\varepsilon_t = q_t + p_t - p^*_t = (2\alpha-1)\tau_t + m^D_t - c^D_t \tag{A.8}$$

$$= (2\alpha-1)\tau_t - (2\alpha-1)\sigma^{-1}\tau_t + m^D_t = (2\alpha-1)(1-\sigma^{-1})\tau_t + m^D_t \tag{A.9}$$

So

$$\tau_t = p^*_{Ft} - p_{Ht} + \varepsilon_t = p^*_{Ft} - p_{Ht} + (2\alpha-1)(1-\sigma^{-1})\tau_t + m^D_t \tag{A.10}$$

$$(1-(2\alpha-1)(1-\sigma^{-1}))\tau_t = -(p_{Ht}-p^*_{Ft}) + m^D_t \tag{A.11}$$

or

$$\tau_t = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}(-(p_{Ht}-p^*_{Ft}) + m^D_t) \tag{A.12}$$

Substituting this above

$$p_t(h) - p_t(f)^* = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi} - 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right)(p_{Ht}-p^*_{Ft})\right\} +$$
$$\mathbb{E}_{it}\left\{\left(\frac{1+\psi\sigma}{\gamma+\psi} + 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right)m^D_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a^D_t\right\}$$

Hence the solution for the price difference can be expressed as

$$p_{Ht} - p^*_{Ft} = \tilde{\varphi}\sum_{k=1}^{\infty}(1-\tilde{\varphi})^{k-1}E^{(k)}_t\left(m^D_t - \frac{1+\psi}{(\gamma+\psi)\tilde{\varphi}}a^D_t\right) \tag{A.13}$$

where

$$\tilde{\varphi} = \left(\frac{1+\psi\sigma}{\gamma+\psi} + 2\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{(\gamma+\psi)(1-(2\alpha-1)(1-\sigma^{-1}))}\right]\right) \tag{A.14}$$

It can be easily verified that if $\sigma = 1$, then $\tilde{\xi} = \xi$ and $\tilde{\varphi} = \varphi$ and one goes back to the equations (1.22) and (1.24) in the main text. The solution for the real exchange rate follows by using the relationship $q_t = (2\alpha-1)t_t$.

A.2 Proof of Proposition 2

The random-walk hypothesis implies that $m_t^D = m_{t-1}^D + u_t$, where $u_t \equiv u_t^m - u_t^{m^*}$. The proof follows the guess-and-verify approach used by Woodford (2002). The guess is that:

$$p_{Ht} - p_{Ft}^* = \nu(p_{Ht} - p_{Ft}^*) + (1-\nu)m_t^D \tag{A.15}$$

Denote with i subscript a generic firm in either Home or Foreign. Equation (1.15) shows that firms in each country receive two signals about the money supplies: one for Home ($z_{i,t}^m$) and one for Foreign ($z_{i,t}^{m^*}$). Given the properties of the signals, it is as if firm i received one signal about the difference in money supplies: $s_{i,t} = m_t^D + \eta_{i,t}$ with $\eta_{i,t} = v_{i,t}^m - v_{i,t}^{m^*}$. Writing compactly the process for the money supplies, the guess for the price difference, and this signal in a state-space representation, we have:

$$\begin{bmatrix} m_t^D \\ p_{Ht} - p_{Ft}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1-\nu & \nu \end{bmatrix} \begin{bmatrix} m_{t-1}^D \\ p_{H,t-1} - p_{F,t-1}^* \end{bmatrix} + \begin{bmatrix} 1 \\ 1-\nu \end{bmatrix} u_t \quad \Longrightarrow x_t = Mx_{t-1} + du_t \tag{A.16}$$

$$s_{i,t} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} m_t^D \\ p_{Ht} - p_{Ft}^* \end{bmatrix} + \eta_{i,t} \quad \Longrightarrow s_{i,t} = ex_t + \eta_{i,t} \tag{A.17}$$

Here I have defined the new vector x_t and matrices M, d and e to write the problem as a state-space system. The Kalman filter implies:

$$\mathbb{E}_{it}(x_t) = \mathbb{E}_{i,t-1}(x_{t-1}) + \kappa[s_{i,t} - eM\mathbb{E}_{i,t-1}(x_{t-1})] \tag{A.18}$$

with $\kappa = [\kappa_1, \kappa_2]'$ being a 2×1 vector of Kalman gains. Given the symmetry of signals across countries, integrating the last expression over the continuum of Home or Foreign firms yields:

$$\bar{\mathbb{E}}_t^{(1)}(x_t) = \kappa eMx_{t-1} + (M - \kappa eM)\bar{\mathbb{E}}_{t-1}^{(1)}(x_{t-1}) + \kappa edu_t \tag{A.19}$$

Now note that equation (1.23), absent technology shocks, may be written as:

$$p_{Ht} - p_{Ft}^* = (1-\varphi)\bar{\mathbb{E}}_t^{(1)}(p_{Ht} - p_{Ft}^*) + \varphi\bar{\mathbb{E}}_t^{(1)}(m_t^D) \tag{A.20}$$

On the right-hand side of this expression, the average expectations of m_t^D and $p_{Ht} - p_{Ft}^*$ can be replaced using equation (A.19) after performing the matrix algebra. This yields:

$$p_{Ht} - p_{Ft}^* = \nu(p_{H,t-1} - p_{F,t-1}^*) + [\varphi\kappa_1 + (1-\varphi)\kappa_2]m_t^D + [(1-\nu) - \varphi\kappa_1 - (1-\varphi)\kappa_2]\bar{\mathbb{E}}_{t-1}^{(1)}(m_{t-1}^D)$$

This verifies the original guess in equation (A.15) and shows that $(1-\nu) = \varphi\kappa_1 + (1-\varphi)\kappa_2$. Now recall that with log utility the real exchange rate is given by $q_t = (2\alpha - 1)(p_{Ft}^* + \varepsilon_t -$

$p_{Ht}) = (2\alpha - 1)(m_t^D - p_{Ht} + p_{Ft}^*)$. Using the solution for the price difference yields:

$$
\begin{aligned}
q_t &= (2\alpha - 1)[m_{t-1} + u_t - (1-\nu)m_{t-1} - \nu(p_{H,t-1} - p_{F,t-1}^*) - (1-\nu)u_t] \\
&= \nu q_{t-1} + (2\alpha - 1)\nu u_t
\end{aligned}
$$

The expressions for the autocorrelation and the standard deviation of the real exchange rate immediately follow. ■

A.3 Stationarizing the Model

In rewriting the model in a stationary representation, I can exploit the following facts:

- The level of the money supply is nonstationary, but money growth is stationary.
- Price levels and, more generally, higher-order beliefs about money supplies are nonstationary but deviations of these beliefs from the true levels of the money supplies are stationary.

The exogenous state variables are $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$. The state-transition equation is given by:

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{A.21}$$

where

$$\bar{X}_t = \begin{bmatrix} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_{6\times 6} & 0 & 0 \\ \Gamma^{\xi,x}_{6\times 6} & \Gamma^{\xi,\xi}_{6\times 6} & 0 \\ \Gamma^{\varphi,x}_{6\times 6} & 0 & \Gamma^{\varphi,\varphi}_{6\times 6} \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_{6\times 4} \\ \hline \Gamma^{\xi,u}_{6\times 4} \\ \hline \Gamma^{\varphi,u}_{6\times 4} \end{bmatrix} \quad u_t = \begin{bmatrix} u^m_t \\ u^{m^*}_t \\ u^a_t \\ u^{a^*}_t \end{bmatrix}$$

where $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$ are the weighted averages of higher-order beliefs that matter for the solution of the model. The matrices B and b are given exogenously, and the matrices Γ can be found as the solution to the fixed-point problem. Now define for any exogenous variable, x_t, the deviation of the variable itself from its weighted average of HOEs: $x_t^{-\xi} \equiv x_t - \xi \sum_{k=1}^{\infty}(1-\xi)x_t^{(k)}$ and the similar object for φ, so that in vectors this is $X_t^{-\xi} = X_t - \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$. Because the weighted average of HOEs converges in the long run to the respective variables, the dynamics of $X_t^{-\xi}$ and $X_t^{-\varphi}$ will be stationary. Furthermore, notice that the Kalman filter iteration implies that $\Gamma^{\xi,x} + \Gamma^{\xi,\xi} = \Gamma^{\varphi,x} + \Gamma^{\varphi,\varphi} = B$. Using this fact and equation (A.21), one can show that

$$X_t^{-\xi} = \Gamma^{\xi,\xi} X_{t-1}^{-\xi} + [b - \Gamma^{\xi,u}]u_t \tag{A.22}$$

$$X_t^{-\varphi} = \Gamma^{\varphi,\varphi} X_{t-1}^{-\varphi} + [b - \Gamma^{\varphi,u}]u_t \tag{A.23}$$

Finally, we stationarize the exogenous part of the system by rewriting it in terms of money growth rates

$$\underbrace{\begin{bmatrix} \Delta m_t \\ \Delta m_t^* \\ a_t \\ a_t^* \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \rho_m & 0 & 0 & 0 \\ 0 & \rho_{m^*} & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & \rho_{a^*} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta m_{t-1} \\ \Delta m_{t-1}^* \\ a_{t-1} \\ a_{t-1}^* \end{bmatrix}}_{Y_{t-1}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{a} \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix} \tag{A.24}$$

So finally, our stationary state can be written as

$$\underbrace{\begin{bmatrix} Y_t \\ X_t^{-\xi} \\ X_t^{-\varphi} \end{bmatrix}}_{\bar{Y}_t} = \underbrace{\begin{bmatrix} A & 0 & 0 \\ 0 & \Gamma^{\xi,\xi} & 0 \\ 0 & 0 & \Gamma^{\varphi,\varphi} \end{bmatrix}}_{\Phi} \begin{bmatrix} Y_{t-1} \\ X_{t-1}^{-\xi} \\ X_{t-1}^{-\varphi} \end{bmatrix} + \underbrace{\begin{bmatrix} a \\ b - \Gamma^{\xi,u} \\ b - \Gamma^{\varphi,u} \end{bmatrix}}_{C} u_t \tag{A.25}$$

Define the stationary variables: $mp_t \equiv m_t - p_{Ht}$ and $mp_t^* = m_t^* - p_{Ft}^*$. The solution for these variables can be written as

$$mp_t = e_h \bar{Y}_t \tag{A.26}$$

$$mp_t^* = e_f \bar{Y}_t \tag{A.27}$$

The other model equations can be written as

$$\tau_t = [1 - (2\alpha - 1)(1 - \sigma^{-1})]^{-}1(mp_t - mp_t^*) \tag{A.28}$$

$$c_t = mp_t - (1 - \alpha)\tau_t \tag{A.29}$$

$$c_t^* = mp_t^* + (1 - \alpha)\tau_t \tag{A.30}$$

$$y_{Ht} = \omega(1 - \alpha)\tau_t + \alpha c_t + (1 - \alpha)(\omega q_t + c_t^*) \tag{A.31}$$

$$yf = -\omega(1 - \alpha)\tau_t + (1 - \alpha)(c_t - \omega q_t) + \alpha c_t^* \tag{A.32}$$

$$q_t = \sigma(c_t - c_t^*) \tag{A.33}$$

$$\pi_t^H = \Delta m_t - mp_t + mp_{t-1} \tag{A.34}$$

$$\pi_t^F = \Delta m_t^* - mp_t^* + mp_{t-1}^* \tag{A.35}$$

$$\pi = \alpha\pi_t^H + (1 - \alpha)(\Delta\varepsilon_t + \pi_t^F) \tag{A.36}$$

$$\pi^* = (1 - \alpha)(\pi_t^H - \Delta\varepsilon_t) + \alpha\pi_t^F \tag{A.37}$$

$$\Delta\varepsilon_t = \Delta m_t - \Delta m_t^* + (2\alpha - 1)(1 - \sigma^{-1})(\tau_t - \tau_{t-1}) \tag{A.38}$$

$$\Delta m_t = \rho_m \Delta m_{t-1} + u_t^m \tag{A.39}$$

$$\Delta m_t = \rho_m^* \Delta m_{t-1} + u_t^{m^*} \tag{A.40}$$

$$a_t = \rho_a a_{t-1} + u_t^a \tag{A.41}$$

$$a_t^* = \rho_a^* a_{t-1}^* + u_t^{a^*} \tag{A.42}$$

These equations can be written compactly as

$$Z_t = \Xi \bar{Y}_t \tag{A.43}$$

Equations (A.25) and (A.43) form the stationary state-space representation of the model.

A.4 Simulating Firms' Prices

Consider the nonstationary system as defined above

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{A.44}$$

Recall that the prices solutions are equations (A.6) and (A.13), which can be solved to get the individual prices as a function of the state

$$p_{Ht} = \frac{(m_t^{\xi} + m_t^{*\xi} + m_t^{\varphi} - m_t^{*\varphi}) - (\chi_w(a_t^{\xi} + a_t^{*\xi}) + \chi_d(a_t^{\varphi} - a_t^{*\varphi}))}{2} \tag{A.45}$$

$$p_{Ft}^{*} = \frac{[m_t^{\xi} + m_t^{*\xi} - (m_t^{\varphi} - m_t^{*\varphi})] - [\chi_w(a_t^{\xi} + a_t^{*\xi}) - \chi_d(a_t^{\varphi} - a_t^{*\varphi})]}{2} \tag{A.46}$$

where I defined $\chi_w = \frac{1+\psi}{(\gamma+\psi)\tilde{\xi}}$ and $\chi_d = \frac{1+\psi}{(\gamma+\psi)\tilde{\varphi}}$ and

$$\tau_t = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}[m_t - m_t^{*} - (m_t^{\varphi} - m_t^{*\varphi}) + \chi_d(a_t^{\varphi} - a_t^{*\varphi})] \tag{A.47}$$

Define two vectors v_h and v_f and v_τ such that $p_{Ht} = v_1 \bar{X}_t$ and $\tau_t = v_2 \bar{X}_t$. So

$$v_h = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,1,0,-1,0,-\chi_d,+\chi_d] \tag{A.48}$$

$$v_\tau = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}[1,0,-1,0,0,0,0,0,0,0,0,0,-1,0,1,0,\chi_d,-\chi_d] \tag{A.49}$$

$$v_f = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,-1,0,1,0,\chi_d,-\chi_d] \tag{A.50}$$

Recall that the home price was

$$p_t(h) = \mathbb{E}_{it}\left\{\left(1-\frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ht}\right\}+ \tag{A.51}$$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t+\left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t-\left(\frac{1+\psi}{\gamma+\psi}\right)a_{it}\right\} \tag{A.52}$$

or

$$p_t(h) = \mathbb{E}_{it}\left\{\left(1-\frac{1+\psi\sigma}{\gamma+\psi}\right)v_1\bar{X}_t\right\}+ \tag{A.53}$$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]v_2\bar{X}_t+\left(\frac{1+\psi\sigma}{\gamma+\psi}\right)e_1\bar{X}_t-\left(\frac{1+\psi}{\gamma+\psi}\right)e_5\bar{X}_t\right\} \tag{A.54}$$

For foreign prices

$$p_t^*(f) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) p_{Ft}^*\right\} -$$
$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) m_t^* - \left(\frac{1+\psi}{\gamma+\psi}\right) a_{it}^*\right\}$$

or

$$p_t^*(f) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right) v_f \bar{X}_t\right\} -$$
$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right] v_\tau \bar{X}_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right) e_3 \bar{X}_t - \left(\frac{1+\psi}{\gamma+\psi}\right) e_6 \bar{X}_t\right\}$$

So you can write it as

$$p_t(h) = P_h \bar{X}_{t|t}^{(1)}(h) \qquad\qquad p_t^*(f) = P_f \bar{X}_{t|t}^{(1)}(f) \tag{A.55}$$

where P are the appropriate matrices. The state space for the firm h is given by

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{A.56}$$

$$z_{it} = D\bar{X}_t + v_{it} \tag{A.57}$$

where

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0_{1\times 12} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0_{1\times 12} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0_{1\times 12} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1\times 12} \end{pmatrix} \tag{A.58}$$

Beliefs about the state evolve according to

$$\bar{X}^{(1)}_{t|t}(i) = \bar{B}\bar{X}^{(1)}_{t-1|t-1}(i) + K[z_{it} - D\bar{B}\bar{X}^{(1)}_{t-1|t-1}(i)] \tag{A.59}$$

where $K = \Sigma D'(D\Sigma D' + \Sigma_v)^{-1}$ and

$$\Sigma = \bar{B}\Sigma\bar{B} - \bar{B}\Sigma D'(D\Sigma D' + \Sigma_v)^{-1}D\Sigma\bar{B}' + \bar{b}\Sigma_u\bar{b}' \tag{A.60}$$

These are the same matrices K, Σ that one finds with the model solution.

A.5 Calculating Profit Losses

Profit Losses in the Dispersed-information Model

Modeling imperfect information with noisy signals is a simple way of formalizing the idea that a cost is associated with gathering and processing the information that is relevant for firms' optimal pricing decisions. In the context of the present model, that information consists of aggregate economic conditions and the prices set by domestic and foreign competitors. One way to evaluate the plausibility of the estimated signal-to-noise ratios is to consider the individual profit loss that a firm incurs when they observe signals only with finite precision. Indeed, one may argue that if paying limited attention to macroeconomic conditions leads to high profit losses, a firm should pay more attention to those conditions. On the other hand, if profit losses are small, then a firm's cost of acquiring more information would outweigh the gain in profits that derive from obtaining more information.

I here explore this reasoning in the context of my model estimates. Recall that the price set by firm h in the home country in the model with dispersed information is given by equation (1.19). Instead, the price that a firm would set under full information, expressed in log-deviations from the steady state, is

$$p_t^{\diamond}(h) = (1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}\boldsymbol{t}_t + \xi(m_t - a_t) \tag{A.61}$$

Firm's h expected per-period profit loss due to imperfect information is then given by

$$\mathbb{E}\left[\Pi_{h,t}(P_t^{\diamond}(h), \cdot) - \Pi_{h,t}(P_t(h), \cdot)\right]$$

After taking a log-quadratic approximation to the profit function, this expression simplifies to

$$-\frac{\Pi_{11}}{2}\mathbb{E}\left[(p_t^{\diamond}(h) - p_t(h))^2\right]$$

where Π_{11} is the curvature of the profit function with respect to the firm's own price. As shown in the Appendix, the expression $(p_t^{\diamond}(h) - p_t(h))^2$ can be computed analytically once the model is solved, using the matrices from the firm's signal-extraction problem. Using the posterior mode, I find that the expected profit loss from imperfect tracking of economic conditions is 0.5% of steady-state revenue for a US firm and 0.8% for a European firm.[1] These numbers suggest that the profit losses from imperfect information are small and plausible. Empirical evidence on the cost of price adjustment indicates that the cost of price adjustment in US industrial manufacturing is 1.22% of a firm's steady-state revenues (Zbaracki et al., 2004). The figure implied by the estimated model is well below the empirical value, suggesting that it is rational for firms to settle on an equilibrium with imperfect information, as the cost of being fully informed would outweigh the profit gain.

Profit Losses in the Calvo Model

The presence of nominal rigidities in the Calvo model implies that generally, firms do not set their prices to the level that would maximize their profits. Firms are thus subject to profit losses that can be compared to the losses in the dispersed-information model.

The profit-maximizing price, $P_t^{\diamond}(h)$, or the price that a firm would set under flexible prices, taking as given the level of demand and the level of aggregate prices, is given in equation (A.61), because the structure of the economy is the same as in the dispersed information model. Instead, the price that a firm in the Home country sets if subject to the Calvo friction, $P_t^C(h)$, is the optimal reset price if it has a chance to update its price, or its old price otherwise. That is, $P_t^C(h) = (1-\theta)P_t^R(h) + \theta P_{t-1}^C(h)$. The optimal reset

[1] Virtually all the expected profit loss comes from the imperfect tracking of monetary shocks. Specifically, the expected profit loss due to imperfect tracking of monetary shocks for a US firm is 0.33% of steady-state revenues. The profit loss due to imperfect tracking of technology shocks is only 0.006% of steady-state revenues.

price, in log-linear terms, is given by

$$p_t^R(h) - p_{H,t} = (1-\theta\beta)mc_t(h) + \theta\beta E_t\{(p_{t+1}^R(h) - p_{H,t+1}) + \pi_{t+1}^H\} \tag{A.62}$$

where $mc_t(h) = \left[\frac{\sigma\psi+1}{\gamma+\psi} y_{H,t} - \frac{2(1-\alpha)\alpha\psi(\sigma\omega-1)}{\gamma+\psi}\tau_t - \frac{1+\psi}{\gamma+\psi} a_t\right]$. In this case there is no closed-form solution for the expected profit-loss expression, but the model can be simulated to compute the expectation.

To make the profit losses comparable across models, I use the following calibration. For the parameters that are common across models, I calibrate the Calvo model using the median estimates from the dispersed-information model. This set of parameters includes the volatility and persistence of shocks. This choice keeps the models comparable, as the profit losses, calculated with a quadratic approximation, are affected by the size of the shocks. As evident from the Phillips Curves equations, the sticky-price model additionally requires to calibrate the discount factor, β, and for the probability of non-price adjustment, θ. I set β to 0.99, the standard value in the literature. For θ, I search for the value that allows me to match the impulse response of the real exchange rate from the two models following a home monetary shock. I find that value to be $\theta = 0.86$, which implies a median price duration of 7 quarters. Empirical estimates of the median price duration range between 4 months and 8-10 months (Bils and Klenow, 2004; Nakamura and Steinsson, 2008). In this sense, the sticky-price model requires "too much price stickiness" to explain the real exchange rate persistence, even in the presence of strategic complementarities that flatten the Phillips Curve for a given degree of nominal rigidities. This confirms the intuition of Chari, Kehoe, and McGrattan (2002) mentioned in the introduction.

Using this calibration, I find that the Calvo friction delivers an expected per-quarter profit loss of 5.11% and 7.77% of steady-state revenue for a US firm and European firm, respectively. These losses are one order of magnitude larger than in the dispersed-information model and quite substantial. In particular, they are greater than the estimated cost of price adjustment in Zbaracki et al. (2004).

Why are the differences so large? Recall the expression for the profit-maximizing price in (A.61). This equation makes clear the role of strategic complementarities. The stronger the strategic complementarities, the larger $(1-\xi)$, and the more the optimal price of a particular firm depends on the aggregate price level. In the dispersed-information model, large strategic complementarities imply that firms place large weights on higher-order beliefs relative to lower-order beliefs. Because higher-order beliefs adjust more sluggishly, all prices in equilibrium adjust sluggishly and they tend to be close together. At the same time, high strategic complementarity implies that the profit-maximizing price is also close to the average price in the economy. As a result, the difference between a particular firm's equilibrium price and its profit-maximizing price tends to be small, implying small profit losses.

In the Calvo model, while strategic complementarity still requires the profit-maximizing price to be close to the average price, an individual firm's price may be arbitrarily far away from the average if the firm did not have the chance to reset the price for a long time. Thus, expected profit losses increase rapidly with the probability of non-price adjustment. In this case, with the value of θ required to match the persistence in the real exchange rate, the losses are substantial.

A.6 Solving the Model with Endogenous Signals

This algorithm is an adaptation of Lorenzoni (2009)'s solution method.

Law of Motion For The State

Define the vectors $z_t = [m_t, m_t, a_t, a_t^*, p_{Ht}, p_{Ft}^*]'$ and $\mathbf{z_t} = [z_t, z_{t-1}, z_{t-2}, ...]$. We are looking for a linear equilibrium of the form:

$$\mathbf{z_t} = \mathbf{A}\mathbf{z_{t-1}} + \mathbf{B}\mathbf{u_t} \tag{A.63}$$

where $\mathbf{u_t} = [e_t^m, e_t^{m^*}, e_t^a, e_t^{a^*}]$. The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} A_m \\ A_{m^*} \\ A_a \\ A_{a^*} \\ A_{p_H} \\ A_{p_F^*} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} B_m \\ B_{m^*} \\ B_a \\ B_{a^*} \\ B_{p_H} \\ B_{p_F^*} \end{bmatrix}$$

where $A_m, A_{m^*}, A_a, A_{a^*}, B_m, B_{m^*}, B_a, B_{a^*}$ are known exogenous vectors and $A_{p_H}, A_{p_F^*}, B_{p_H}, B_{p_F^*}$ are to be determined. The pricing equations can be written as

$$p_{Ht} = \Psi_H E_t[\mathbf{z_t}] \tag{A.64}$$

$$p_{Ft} = \Psi_F E_t[\mathbf{z_t}] \tag{A.65}$$

for known selector vectors Ψ_H, Ψ_F.

Individual Inference

We can write the vector of signals for a Home firm as

$$\mathbf{s_t^H} = \mathbf{F}\mathbf{z_t} + \mathbf{G}\mathbf{v_t} \tag{A.66}$$

Bayesian updating requires

$$E_{h,t}[\mathbf{z_t}] = E_{h,t-1}[\mathbf{z_t}] + \mathbf{C}(\mathbf{s_t^H} - E_{h,t-1}[\mathbf{s_t^H}]) \tag{A.67}$$

Define $\mathbf{\Omega} = Var_{h,t-1}[\mathbf{z_t}]$. The Kalman gain $\mathbf{C}$ and the matrix $\mathbf{\Omega}$ must satisfy

$$\mathbf{C} = \mathbf{\Omega F'}(\mathbf{F\Omega F'} + \mathbf{GVG'})^{\mathbf{-1}} \tag{A.68}$$

$$\mathbf{\Omega} = \mathbf{A}(\mathbf{\Omega} - \mathbf{CF\Omega})\mathbf{A'} + \mathbf{B\Sigma B'} \tag{A.69}$$

Fixed Point

The average first-order beliefs can be expressed as a function of the state as

$$\mathbf{z_{t|t}} = \mathbf{\Xi^H z_t} \tag{A.70}$$

Using the updating equation and aggregating across home firms

$$\mathbf{z_{t|t}} = (I - \mathbf{CF})\mathbf{A z_{t-1|t-1}} + \mathbf{CF z_t} \tag{A.71}$$

So the matrix $\mathbf{\Xi^H}$ must satisfy

$$\mathbf{\Xi^H z_t} = (I - \mathbf{CF})\mathbf{A\Xi^H z_{t-1}} + \mathbf{CF z_t} \tag{A.72}$$

A similar matrix $\mathbf{\Xi^F}$ is defined for foreign firms' first-order beliefs. These matrices allow me to rewrite equations (95) and (96) as

$$p_{H,t} = \mathbf{\Xi^H z_t} \tag{A.73}$$

$$p_{F,t} = \mathbf{\Xi^F z_t} \tag{A.74}$$

The equilibrium is characterized by the vectors $A_{p_H}, A_{p_F^*}, B_{p_H}, B_{p_F^*}$ and matrices $\mathbf{\Xi^H}, \mathbf{\Xi^F}$ which are consistent with the law of motion of the state equation and with the signal-extraction problem of the firms. The equilibrium can be computed iterating over the relevant equations until convergence is achieved. The convergence criterion is given by the quadratic distance of the impulse-response functions of p_H and p_F^* to the exogenous shocks in $\mathbf{u_t}$, with weights given by the variances of the shocks.

Appendix B

Risk Aversion and the Financial Accelerator

B.1 Proof of Proposition 1

The proof follows Tamayo (2014). First, note that when the report is not verified ($\omega \notin \Omega^V$) the repayment function must only depend on the report $\tilde{\omega}$, i.e. we have $R(\tilde{\omega})$. Therefore, the entrepreneur will choose $\omega^* = \arg\min_{\tilde{\omega}} R(\tilde{\omega})$ so the contract may as well set $R(\tilde{\omega}) = \bar{R}$. Second, under the optimal contract, in the verification region $R(\omega) \leq \bar{R}$ because otherwise the contract would not be incentive compatible. Specifically, the entrepreneur would prefer to misreport $\omega \notin \Omega^V$ and pay $\bar{R}$. Finally, it can also be shown that Ω^V must be a lower interval (for the proof see Lemma 3 in Tamayo (2014)). These findings can be summarized by saying that the optimal repayment function follows:

$$R(\omega) = \begin{cases} R(\omega) \le \bar{R}, & \text{if } \omega \le \bar{\omega} \\ \bar{R}, & \text{if } \omega > \bar{\omega} \end{cases} \tag{B.1}$$

Now let us rewrite the contracting problem using the above results as

$$\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) \tag{B.2}$$

$$\text{s.t. } \kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \right) \ge (\kappa - 1) \tag{B.3}$$

$$\bar{R} \le \bar{\omega} \tag{B.4}$$

$$R(\omega) \le \omega \ \forall \omega \le \bar{\omega} \tag{B.5}$$

$$R(\omega) \ge 0 \ \forall \omega \le \bar{\omega} \tag{B.6}$$

where we have plugged in the constraint (2.3), used the definition of leverage $\kappa = \frac{QK}{N}$ and rescaled the objective function and constraints by the exogenous parameters N and R. Assign the multipliers $\lambda, \xi, \gamma_1(\omega)$ and $\gamma_2(\omega)$ to the constraints. The Lagrangian reads:

$$\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) + \tag{B.7}$$

$$\lambda \left[\kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega})\right) - (\kappa - 1)\right] + \tag{B.8}$$

$$\xi(\bar{\omega} - \bar{R}) + \int_0^{\bar{\omega}} \gamma_1(\omega)(\omega - R(\omega))\phi(\omega) d\omega + \int_0^{\bar{\omega}} \gamma_2(\omega)(R(\omega))\phi(\omega) d\omega \tag{B.9}$$

The first order necessary conditions with respect to $R(\omega), \bar{R}, \bar{\omega}$ after appropriate rescaling of the multipliers can be written as[1]:

$$-\gamma_1(\omega)\phi(\omega) - \left(\kappa \frac{R^K}{R}\right)^{1-\rho} \{[\omega - R(\omega)]^{-\rho}\phi(\omega) + \lambda \left(\kappa \frac{R^K}{R}\right) \phi(\omega) + \gamma_2(\omega)\phi(\omega) = 0 \text{ for every } \omega \leq \bar{\omega} \tag{B.10}$$

$$-\xi - \left(\kappa \frac{R^K}{R}\right)^{1-\rho} \int_{\bar{\omega}}^{\infty} [\omega - \bar{R}]^{-\rho} d\Phi(\omega) + \lambda \left(\kappa \frac{R^K}{R}\right) [1 - \Phi(\bar{\omega})] = 0 \tag{B.11}$$

$$-\frac{\xi}{\phi(\bar{\omega})} - \left(\kappa \frac{R^K}{R}\right)^{1-\rho} [\bar{\omega} - R(\bar{\omega})]^{1-\rho} + \left(\kappa \frac{R^K}{R}\right)^{1-\rho} [\bar{\omega} - \bar{R}]^{1-\rho} - \lambda \left(\kappa \frac{R^K}{R}\right) [R(\bar{\omega}) - \bar{R} - \mu] = 0 \tag{B.12}$$

[1] We do not need the first-order condition with respect to κ to prove the proposition.

and the complementary slackness conditions:

$$0 = \lambda \left\{ \kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} \bar{R}(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \right) - (\kappa - 1) \right\} \tag{B.13}$$

$$0 = \xi[\bar{\omega} - \bar{R}] \tag{B.14}$$

$$0 = \gamma_1(\omega)[\omega - R(\omega)] \tag{B.15}$$

$$0 = \gamma_2(\omega)R(\omega) \tag{B.16}$$

Suppose that $\gamma_1(\omega) > 0$ for all $\omega < \bar{\omega}$. Then it must be that $\gamma_2(\omega) = 0$, from the complementary slackness conditions. Then equation (75) would imply that $\lambda > \left(\kappa \frac{R^K}{R}\right)^{-\rho} (0)^{-\rho}$ which is not possible. Hence it must be true that $\gamma_1(\omega) = 0$ for all $\omega \leq \bar{\omega}$ and a standard debt contract is not optimal. We know from (75) that $\gamma_1(\omega) = 0 \iff (\omega - R(\omega))^{-\rho} \geq \lambda$. Now there are two possible cases. Suppose $\gamma_2(\omega) = 0$ for all $\omega \leq \bar{\omega}$. Then the contract specifies that $R(\omega) = \omega - \lambda^{-1/\rho} \left(\frac{R}{R^K \kappa}\right)$. By complementary slackness it should be the case that $R(\omega) > 0$ for all ω, which is not possible because if $\omega = 0$, $R(\omega) > 0$ would not be feasible. Then it must be the case that $\gamma_2(\omega) > 0$ for some ω which implies $R(\omega) = 0$ and $\omega \leq \lambda^{-1/\rho} \left(\frac{R}{R^K \kappa}\right)$ for the same ω. Hence there is a lower interval where $R(\omega) = 0$. Call the upper bound of this interval $\underline{\omega} \equiv \lambda^{-1/\rho} \left(\frac{R}{R^K \kappa}\right)$. Therefore $R(\omega) = 0$ if $\omega \leq \underline{\omega}$ and $R(\omega) = \omega - \underline{\omega}$ if $\underline{\omega} \leq \omega \leq \bar{\omega}$.■

B.2 FOCs for the Dynamic Contract and Proof of Proposition 2

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t\left\{\frac{(\kappa_t R^k_{t+1})^{1-\rho} g(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t})\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\left(\kappa_t R^k_{t+1} h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) - (\kappa_t - 1)R_t\right)\right\}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t\left\{(\kappa_t R^k_{t+1})^{1-\rho} g_{t+1}\Psi_{t+1} - \lambda_{t+1}R_t\right\} = 0 \tag{B.17}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R^k_{t+1} h_{\bar{\omega},t+1} = 0 \tag{B.18}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\underline{\omega},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R^k_{t+1} h_{\underline{\omega},t+1} = 0 \tag{B.19}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{R},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R^k_{t+1} h_{\bar{R},t+1} = 0 \tag{B.20}$$

Now we can express λ_{t+1} from $\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = 0$

$$\lambda_{t+1} = -\frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho}\frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \tag{B.21}$$

Now we plug this condition into the three other equations and obtain

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t \left\{ (\kappa_t R^k_{t+1})^{1-\rho} g_{t+1} \Psi_{t+1} + \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \right\} = 0 \quad \text{(B.22)}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\underline{\omega},t+1} \Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \kappa_t R^k_{t+1} h_{\underline{\omega},t+1} = 0 \quad \text{(B.23)}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{R},t+1} \Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R^k_{t+1})^{1-\rho} g_{\bar{\omega},t+1} \Psi_{t+1}}{1-\rho} \frac{1}{\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} \kappa_t R^k_{t+1} h_{\bar{R},t+1} = 0 \quad \text{(B.24)}$$

we can transform this system to

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t \left\{ (R^k_{t+1})^{1-\rho} \Psi_{t+1} \left(g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \right) \right\} = 0 \quad \text{(B.25)}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = g_{\underline{\omega},t+1} - g_{\bar{\omega},t+1} \frac{h_{\underline{\omega},t+1}}{h_{\bar{\omega},t+1}} = 0 \quad \text{(B.26)}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = g_{\bar{R},t+1} - \frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} h_{\bar{R},t+1} = 0 \quad \text{(B.27)}$$

Since in the equation ($B.25$) Ψ_{t+1} and $\hat{R}_{k,t+1}$ enter as multiplicative terms and the term $g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t$ is equal to zero in the steady state, Ψ_{t+1} and $\hat{R}_{k,t+1}$ have no effect in the first order approximation. Therefore, to find the approximate solution it is sufficient to consider the following system:

$$\kappa_t R^k_{t+1} h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (\kappa_t - 1) R_t \tag{B.28}$$

$$\mathbb{E}_t \left\{ g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t \right\} = 0 \tag{B.29}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{B.30}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{B.31}$$

We can substitute k_t and obtain

$$\frac{\mathbb{E}_t \left[\frac{g_{\bar{\omega}} R_{t+1}}{(1-\rho) R_{k,t+1} h_{\omega,t+1}} \right]}{E_t g_{t+1}} = \frac{1}{1 - \frac{R_{k,t+1}}{R_t} h_{t+1}} \tag{B.32}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{B.33}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{B.34}$$

Whenever the gradient of this system has full rank at the steady state, we will be able to find an approximate solution of $\hat{\bar{\omega}}_{t+1}, \hat{\underline{\omega}}_{t+1}, \hat{\bar{R}}_{t+1}$ as functions of $\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t$, $\hat{R}_{k,t+1} - \hat{R}_t$ and $\hat{\sigma}_{\omega,t}$. Using this fact and log-linearizing equation (B.28) will give us

$$\hat{k}_t = \nu_p (\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t) + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{B.35}$$

Appendix C

Implications of Default Recovery Rates for Aggregate Fluctuations

C.1 Expression for Recovery Rates

In keeping with the data, we measure recovery rates in the model as the market value of defaulted debt as a percentage of its face value (or par). In the financial accelerator model of BGG, there is a continuum of borrowers (or entrepreneurs), indexed by (j) who purchase raw capital, $\bar{K}$, at a unit price of Q. The entrepreneur j uses his net worth, $N(j)$, and a one-period loan $B(j)$ from a financial intermediary to purchase his desired level of capital. The entrepreneur is subject to an aggregate return, R^k, and an idiosyncratic return, ω, where $\log(\omega) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ so that the mean of ω is equal to 1. We denote by $f(\omega)$ and by $F(\omega)$ the probability density function and cumulative distribution function of ω, respectively. Thus, the value of the entrepreneur's assets ex-post is $Q\bar{K}(j)R^k\omega$. The loan obtained by the entrepreneur takes the form of a standard debt contract, where Z denotes the promised gross rate of return on the loan. Let, $\bar{\omega}$, be the value of ω below which an entrepreneur is not able to repay the principal and the interest on the loan. This cutoff is defined by

$$B(j)Z = Q\bar{K}(j)R^k\bar{\omega} \tag{C.1}$$

Entrepreneurs with $\omega < \bar{\omega}$ are not able to refinance and, hence, declare bankruptcy. Due to bankruptcy costs, the financial intermediary is only able to recover a fraction $(1-\mu)$ of the entrepreneur's asset. Thus the average recovery rate, conditional on default is given by:

$$R_c = \int_0^1 \int_0^\infty \frac{(1-\mu)\omega Q\bar{K}(j)R^k}{F(\bar{\omega})B(j)Z} dF(\bar{\omega})dj \tag{C.2}$$

Now multiply both the numerator and denominator by $\bar{\omega}$, and substitute out for $B(j)Z$ using (C.1) to obtain

$$R_c = \int_0^1 \int_0^\infty \frac{(1-\mu)\omega}{F(\bar{\omega})\bar{\omega}} dF(\bar{\omega})dj = \frac{(1-\mu)G(\bar{\omega})}{F(\bar{\omega})\bar{\omega}} \tag{C.3}$$

where $G(\bar{\omega}) \equiv \int_0^\infty \omega dF(\omega)$.

C.2 Derivation of θ_t

Here we derive an analytical expression for θ. Consider an entrepreneur with net worth $N_t(j)$ His returns are given by

$$N_t(j) = [\omega_t R_t^k N_{t-1}(j)\kappa_{t-1} - (\kappa_{t-1} - 1)Z_t] \tag{C.4}$$

where κ is the common leverage across entrepreneurs. The new amount of capital chosen by the entrepreneur is $\bar{K}_{t+1}(j) = N_t(j)\kappa_t/Q_t$, so we have

$$\bar{K}_{t+1} = [\omega_t R_t^k N_{t-1}(j)\kappa_{t-1} - N_{t-1}(j)(\kappa_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} \tag{C.5}$$

and $\bar{K}_t(j) = N_{t-1}(j)k_{t-1}/Q_{t-1}$. Thus, net purchases or sales of capital for entrepreneur j are equal to

$$\bar{K}_{t+1}(j) - \bar{K}_t(j) = [\omega_t R_t^k N_{t-1}(j)k_{t-1} - N_{t-1}(j)(k_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} - N_{t-1}(j)\frac{\kappa_{t-1}}{Q_{t-1}} \tag{C.6}$$

Define $\tilde{\omega}$ as the value of ω for which an entrepreneur neither buys nor sells capital. $\tilde{\omega}$ is pinned down by

$$0 = [\tilde{\omega}_t R_t^k N_{t-1}(j)\kappa_{t-1} - N_{t-1}(j)(\kappa_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} - N_{t-1}(j)\frac{\kappa_{t-1}}{q_{t-1}} \tag{C.7}$$

Then

$$\tilde{\omega}_t R_t^k N_{t-1}(j) k_{t-1} = (N_{t-1}(j)(k_{t-1}-1)Z_t)\frac{\kappa_t}{Q_t} + N_{t-1}(j)\frac{\kappa_{t-1}}{Q_{t-1}} \tag{C.8}$$

Using this last expression, we can rewrite (C.6) as

$$\bar{K}_{t+1}(j) - \bar{K}_t(j) = R_t^k N_{t-1}(j)\kappa_{t-1}(\omega_t - \tilde{\omega}_t) \tag{C.9}$$

Summing across all entrepreneurs and taking into account the the new entrants as well as those who leave business we can write

$$\theta_t = \frac{\int_0^1 \max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]dj}{\int_0^1 \max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]dj} \tag{C.10}$$

$$= \frac{-\gamma_t R_t^k N_{t-1}\kappa_{t-1}\frac{\kappa_t}{Q_t}\int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)N_{t-1}\kappa_{t-1}R_t^k\frac{\kappa_t}{Q_t}}{\gamma_t R_t^k N_{t-1}\kappa_{t-1}\frac{\kappa_t}{Q_t}\int_{\tilde{\omega}_t}^{\infty}(\omega-\tilde{\omega}_t)dF(\omega) + W_t^e\frac{\kappa_t}{Q_t}} \tag{C.11}$$

$$= \frac{-\gamma_t R_t^k N_{t-1}\kappa_{t-1}\int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)N_{t-1}\kappa_{t-1}R_t^k}{\gamma_t R_t^k N_{t-1}\kappa_{t-1}\int_{\tilde{\omega}_t}^{\infty}(\omega-\tilde{\omega}_t)dF(\omega) + W_t^e} \tag{C.12}$$

$$= \frac{-\gamma_t \int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)}{\gamma_t \int_{\tilde{\omega}_t}^{\infty}(\omega-\tilde{\omega}_t)dF(\omega) + \frac{W_t^e}{R_t^k N_{t-1}\kappa_{t-1}}} \tag{C.13}$$

$$= \frac{-\gamma_t(\Gamma_t(\tilde{\omega}_t) - \tilde{\omega}) + 1 - \gamma_t}{\gamma_t(1-\Gamma(\tilde{\omega}_t)) + \frac{W_t^e}{R_t^k N_{t-1}\kappa_{t-1}}} \tag{C.14}$$

where

$$\tilde{\omega}_t = \frac{\frac{\kappa_{t-1}}{\kappa_t}\frac{Q_t}{Q_{t-1}} + (\kappa_{t-1}-1)Z_t}{R_t^k \kappa_{t-1}} = \bar{\omega}_t + \frac{1}{\kappa_t}\frac{Q_t}{Q_{t-1}R_t^k} \tag{C.15}$$

and

$$\Gamma(\tilde{\omega}) = \int_0^{\tilde{\omega}} \omega f(\tilde{\omega}) d\omega + \tilde{\omega}(1 - F(\tilde{\omega})) \tag{C.16}$$

$$= \Phi\left(\frac{\log(\tilde{\omega})}{\sigma} - \frac{\sigma}{2}\right) + \tilde{\omega}\left[1 - \Phi\left(\frac{\log(\tilde{\omega})}{\sigma} + \frac{\sigma}{2}\right)\right] \tag{C.17}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

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