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Abstract

Do individuals use all information at their disposal when forming expectations about future events? In this paper we present an econometric framework to answer this question. We show how individual information sets can be characterized by simple nonparametric exclusion restrictions and provide a quantile based test for constrained information processing. In particular, our methodology does not require individuals' expectations to be rational, and we explicitly allow for individuals to have access to sources of information which the econometrician cannot observe. As an application, we use microdata on individual income expectations to study which information agents employ when forecasting future earnings. Consistent with models where information processing is limited, we find that individuals' information sets are coarse in that valuable information is discarded. We then quantify the utility costs of coarse information within a standard consumption life-cycle model. Consumers would be willing to pay 0.04% of their permanent income to incorporate the econometrician's information set in their forecasts.

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1 Introduction

Individuals' expectations about uncertain events are a key aspect of modern economics. Knowing what expectations individuals hold is therefore crucial to understand and predict behavior (Manski, 2004). A key ingredient in the process of expectation formation is the information set agents employ. In this paper we estimate the content of information sets using microdata on income expectations. In particular, we show that by focusing directly on the *observed* expectations individuals hold, we can characterize agents' information set despite allowing for two sources of *unobserved* heterogeneity: (i) we do not have to restrict agents' model of belief formation, e.g. by assuming that expectations are rational and (ii) we can allow for individuals' information sets to contain additional information, which is unknown to the researcher. As long as one is not interested in how information enters in the individuals' process of expectation formation but only whether some information is contained in a particular information set, we can express agents' beliefs as a nonseparable model and learn about the content of agents' information sets through simple nonparametric exclusion restrictions. In our application we find that individuals use rather coarse information to predict their future income. In particular, we are not able to reject that agents only use their current income, age, occupational status and local labor market conditions to predict future income growth. In contrast, neither information about their educational status nor their sector of employment are contained in their predictions. After establishing which information individuals use when forming expectations, we test if these information sets are consistent with unconstrained information processing, i.e. whether agents are able to productively use information as long as it is available to them. We do not need to take a stand on why information processing might be limited. Agents might be constrained in the amount of information they can be attentive to as claimed in the literature on rational inattention (see e.g. Sims (2003); Mackowiak and Wiederholt (2000)) and costly information processing more general (see e.g. Reis (2004); Mankiw and Reis (2002)) or they might discard pieces of information they consider of second-order importance by basing their decision on simplified models of their economic surrounding (Gabaix et al. (2006); Gabaix (2014)). Whatever the precise microfoundation, we will say that information processing is limited or costly, whenever the information set agents use is smaller than the one they have access to and such additional information would be useful to predict the variable of interest, in our case future income. We first show that information processing costs cannot be identified without further restrictions. Intuitively, if individuals were to think that some information is not useful to predict the outcome of interest, they will not use it even if information processing is not costly. We then show that under a weak restriction on the agents' model, which in its essence assumes a minimum degree of consistency between the agents' model and the objective data generating process, we can test for costly information processing. In our application, we can comfortably reject that information processing is costless.

How costly are such limits to information processing in utility terms? While our methodology is applicable in a wide range of situations, we quantify the utility costs of costly information processing within a particular model, namely the canonical life-cycle model of consumption. We view this as a natural benchmark, as we analyze individual's income expectations, which are important for optimal consumption behavior. Through the lens of the model, consumers' information sets affect the agents' perceived environment in that they determine how much of the income process is predictable and how much has to be attributed to permanent and transitory shocks. Using the information sets as estimated from the microdata, we find that households overestimate the variance of transitory shocks and slightly underestimate the predictable rate of income growth (always compared to the econometrician). This misconception of the income process they face will change individual behavior. At the estimated parameters, the utility loss of excluding information from their information sets is small in that the average willingness to pay for the econometricians' information set amounts to roughly 0.04% of agents' permanent income. Hence, the information processing costs can be quite low for individuals to rationally choose to not incorporate different sources of information in their income predictions. The reason is that - in the model - occupational characteristics and age do a good job to decompose the observed income process into predictable components and transitory and permanent shocks. With the individuals' model being close to the income process, the utility consequences of omitting expectations are relatively small, especially because individuals are quite well insured against income shocks anyway.

Related literature: Nonparametric tests of rational behavior have a long tradition in applied Microeconomics, in particular consumer demand, see, e.g., Hoderlein (2011) , Lewbel (1995), Haag et al. (2009), Dette et al. (Forthcoming). More specifically, there are various empirical studies of individuals' expectations in general and their information sets in particular. First of all, a large empirical literature exists that tests the rational expectations hypothesis, see e.g. Lovell (1986), Keane and Runkle (1990) and Brown and Matial (1981). This literature has often tested for "informational efficiency", which is similar to our concept of costless information processing and hence closely related to our specification test. Secondly, there are numerous contributions that explicitly study subjective expectation data (Dominitz, 1998; Dominitz and Manski, 1997; Hurd and McGarry, 1995). While data on subjective expectations has often been met with skepticism, Manski (2004) provides evidence that such data is helpful to predict choices and argues that it should be used more often given its wide availability. Finally there is an extensive literature on forecasting that models agents' forecasts as the solution to a well-defined maximization problem for given preferences and information sets (Pesaran and Weale, 2006; Machina and Granger, 2006).

Recently, expectations data have also been explicitly used for particular applications. Guiso et al. (1996) use agents' self-reported income uncertainty in a study of portfolio choice, Carroll (2003) exploits expectations on future inflation and unemployment rates to estimate a structural model of expectation formation, Jappelli and Pistaferri (2000) provide tests for consumption excess sensitivity when explicitly controlling for individuals' income expectations and Coibion and Gorodnichenko (2015) use data on household inflation expectations to explain the missing disinflation during the Great Recession. Finally, Cunha et al. (2005) show how individual information sets can be recovered from a structural model of college choice in a life-cycle framework. While not focusing on the precise content of individual information sets, Coibion and Gorodnichenko (2012) also use expectations data to provide evidence in favor of informational rigidities.

Regarding our application, the life-cycle model of consumption is the workhorse model to analyze consumption behavior and has been tested extensively (see e.g. Hall and Mishkin (1982), Hall (1978), Attanasio and Weber (1995) and Browning and Lusardi (1996) for a review). We focus on a standard setup, where risk-sharing is limited by agents only having access to a non-state-contingent riskless asset. In this environment, consumers engage in precautionary savings.[1] As the amount of information used when forecasting future income determines consumers' income uncertainty, the size of consumers' information sets will affect consumption behavior and we can use the model to quantify the utility costs of coarse information sets.

The structure of the paper is as follows. In the next section we will present our methodology to characterize information sets and give conditions for identification. In section three we apply our econometric technique to microdata on income expectations and measure what information individuals use when forecasting future income. In section four we quantify the economic importance of agents' information on consumption behavior in the context of a standard life-cycle model. Section five concludes. All tables are relegated to Appendix I. Proofs of our theoretical results (Theorem 1 and 2) can be found in Appendix II. Additional empirical results can be found in the supplementary material.

2 Characterizing Information Sets

We consider the following economy: There is a continuum of agents who form expectations about their future income Y. This future income is related to individual level variables like education or experience, but also to aggregate characteristics like relative skill supplies in the individuals' local labor market. To forecast their income, individuals use information, which we model as a set of random variables Q.[2]

Given this information, we denote the objective conditional distribution of Y given Q, i.e.

[1] In the context of the life-cycle model, many recent contributions use both consumption and income data simultaneously to learn about the structure of individual income (Gourinchas and Parker, 2002; Blundell et al., 2008; Krueger and Perri, 2011; Guvenen, 2007).

[2] More precisely, the information is $\sigma(Q)$, where σ denotes the sigma-algebra spanned by Q.

the one of an observer who would have access to both Y and Q, by $F_{Y|Q}$. The conditional distribution of Y given Q as perceived by the agents, however, may be different, and we denote it by $F^I_{Y|Q}$, where the superscript I denotes individuals. Hence, individuals are not required to hold rational expectations, so that $F^I_{Y|Q}$ and $F_{Y|Q}$ do not have to be equal. Individuals are thus characterized by two unobserved objects: their information set Q and their model of belief formation encapsulated in I. Assuming that income is continuously distributed, we denote the respective densities by $f^I_{Y|Q}$ and $f_{Y|Q}$. We henceforth call the subjective distribution agents hold the agents' *forecasting model*, or *model.*

Definition 1. *A (forecasting) model is a conditional distribution function of Y given information Q and type I, i.e.*

$$F^I_{Y|Q}(y, q) = P^I[Y \leq y|Q = q] = \int_{-\infty}^{y} f^I_{Y|Q}(\eta; q)d\eta. \tag{2.1}$$

Similarly to (2.1), the econometrician's beliefs about future income given the information Q (the econometrician's model) are given by

$$F_{Y|Q}(y, q) = P[Y \leq y|Q = q] = \int_{-\infty}^{y} f_{Y|Q}(\eta; q)d\eta. \tag{2.2}$$

Equation (2.1) and (2.2) illustrate that one may think of a forecasting model as a production function. It generates outputs (beliefs about future events) upon usage of inputs (information). How a given information set Q maps into beliefs depends on how agents form their expectations, encapsulated in I. The population of individuals and their accompanying income expectations $F^I_{Y|Q}$ are therefore induced by the underlying unobserved random variables (Q, I).
As mentioned above, we assume to have the distribution of forecasts at the disposal of the econometrician. Hence, from the point of view of the econometrician, we observe realizations of the random variable $F^I_{Y|Q}(Q; y)$, for all y, meaning that we observe many different functions of agents' information Q, and unobserved heterogeneity (in beliefs) embodied in I. Since we assume to observe some characteristics of the individuals which are relevant to their future income, e.g., their education, we observe some elements of Q, but the individuals may well use additional elements, which are not observed by the econometrician.
The two questions we ask are: (1) Which information we as applied researchers have access to in the data, do individuals actually use when forecasting their future income? (2) Suppose we conclude that individuals do not use all information available to the individual, can we assess whether they should have used it, provided information processing was unconstrained? We will address both of these questions in turn.

2.1 What information do individuals use?

In our setup, the question of what information individuals use is formalized by asking which Q individuals *use*. We emphasize the *usage* of information, precisely because individuals might not use all the information they in principle have access to, if information processing is limited. In our application, we employ information from self-reported survey data, i.e. individuals have in principle access to all the information we have. In particular, they provide us with observables $Z = (Z_1', Z_2')'$. The questions is now whether they condition their forecasts on the information contained in Z_2.

To answer this question, we explicitly want to allow for the fact that individuals might use information in addition to Z_2 which is unobservable to the econometrician. Hence, let agents' information set Q be given by $Q = [Q_1', Z_2']'$, where Q_1 simply denotes variables in the individuals' information sets other than Z_2. Note that Q_1 is unobserved. The formal definition of what it means for the information contained in Z_2 to be used is contained in the following definition.

Definition 2. *Consider the setup above. We say that the information in Z_2 is used, conditional on Q_1, whenever for (y, q_1, z_2, z_2', i) with positive probability*[3]*, and $z_2 \neq z_2'$,*

$$F^i_{Y|Q}(y; (q_1, z_2)) \neq F^i_{Y|Q}(y; (q_1, z_2')). \tag{2.3}$$

In words, information is actively used whenever it affects the beliefs of a non-negligible part of the population for a non-negligible part of the income distribution. Put differently, this means that on a non-negligible set, $F^i_{Y|Q}(y; (q_1, z_2))$ is a non-trivial (non-constant) function of z_2. Note in particular that informational usage is defined *conditional* on Q_1. Suppose for example that income was *only* a function of individuals' ability, which is unobservable to the econometrician but used by individuals to forecast future earnings. Now suppose we were to ask whether individuals use the information on educational attainment. If ability and education are correlated, we would find parts of the information in education reflected in individuals' forecasts and we would conclude that individuals use *some* information in their educational attainment to forecast future earnings. We as econometricians cannot say whether there is a Mincerian skill premium or if such skill premium is purely spurious and driven by the correlation between education and ability. But to measure informational content we are not interested in the underlying structural model - whether or not the information contained in education is reflected in individuals' forecasts is the only thing that matters for us. If we were to directly observe ability and individual forecasts, we would correctly conclude that educational information is not used once ability is controlled for.

[3] We say "(y, q, z, i) with positive probability", when all (y, q, z, i) form a set $\mathcal{Y}' \times \mathcal{Q}' \times \mathcal{Z}' \times \mathcal{I}' \subseteq \mathcal{Y} \times \mathcal{Q} \times \mathcal{Z} \times \mathcal{I}$, with $P[\mathcal{Y}' \times \mathcal{Q}' \times \mathcal{Z}' \times \mathcal{I}'] > 0$, and analogously throughout this paper.

Definition 2 is a nonparametric exclusion restriction. However, (2.3) cannot be directly taken to the data as it depends on the unobservables (Q_1, I). To make progress, note first that we can express individuals' information sets Q_1 as a (vector-valued) function $\widetilde{\pi}$ of observables Z_1 and unobservable factors V, which are independent of Z (denoted henceforth as $Z \perp V$). Hence, we can write Q as

$$Q = [Q_1', Z_2']' = [\widetilde{\pi}(Z_1, V)', Z_2']' \equiv \pi(Z_1, Z_2, V), \tag{2.4}$$

where $Z = (Z_1', Z_2')'$ are observed and V is unobserved. At this point, we do not restrict the dimensionality of V. Note again that equation (2.4) is simply a decomposition of Q into observable and unobservable factors, which will be useful to derive a testable restriction to test for informational usage in the sense of Definition 2. Moreover, we assume that I depends solely on V and not on Z. Hence, without loss of generality we can then directly subsume it under V - if it was not a part of V from the outset, we simply denote the original V by $\tilde{V}$, and let $V = (\tilde{V}, I)$.[4]

To derive testable implications, note that (2.4) allows us to write individuals' beliefs as a nonseparable model, i.e. for a fixed value y we have:

$$F^I_{Y|Q}(y; Q) = \varphi(Z, V; y) = \varphi(Z_1, Z_2, V; y). \tag{2.5}$$

We emphasize here that y is a fixed index that describes the function and that Z and V are the actual arguments of the function. Hence, $\varphi(Z, V; y_1)$ relates the dependent variable "conditional probability that $Y < y_1$, which is induced by individual information sets Q and belief formation process I" to observable covariates Z, and unobservables V.

Having all elements of our framework in place, we can now tackle the question of whether individuals actively use the information contained in a variable Z_2 or not. To this end, we form the set of *observable* τ-quantiles (where $\tau \in (0, 1)$) of $F^I_{Y|Q}(y; Q)$ given $Z = (Z_1', Z_2')'$. We introduce the shorthand F^I_y for $F^I_{Y|Q}(y; Q)$, and denote the quantiles just mentioned by

$$k^\tau_{F^I_y|Z_1,Z_2}(z_1, z_2).$$

To understand the usefulness of this quantile function, note that $k^\tau_{F^I_y|Z_1,Z_2}(z_1, z_2)$ is implicitly defined by

$$\tau = P[\varphi(z_1, z_2, V; y) \leq k^\tau_{F^I_y|Z_1,Z_2}(z_1, z_2)|Z_1 = z_1, Z_2 = z_2].$$

Conditional on Z the variation in agents' income expectations is driven by V. This variation reflects both additional unobserved information and differences in beliefs I. By focusing on

[4] Note that this excludes that I is a function of Z. We could allow for this dependence, in which case we could treat I exactly as we treat Q above, i.e., simply define π to be the vector valued mapping from Z, V into Q, I. This is always possible, as we do not restrict the dimension of V or the functional form of π. However, we desist from this greater generality for ease of exposition.

the quantiles, we can essentially "map" these unobservables into the respective quantiles τ. To finally relate this quantity to the unobserved information Q_1, we invoke the following "no averaging out assumption":

Assumption 1. *If, in order to forecast Y, a positive proportion of the population uses Z_2 conditional on information Q_1, it is also the case that when averaging over the unobservables V, the resulting function is a function of Z_2, conditional on Z_1, for a positive probability set. Formally, let*

$$\bar{\varphi}(Z_1, Z_2; y) = \int_V \varphi(Z_1, Z_2, v; y) f_V(v) dv.$$

Then, if

$$\varphi(z_1, z_2, v; y) \neq \varphi(z_1, z_2', v; y),$$

with positive probability, it is also the case that

$$\bar{\varphi}(z_1, z_2; y) \neq \bar{\varphi}(z_1, z_2'; y),$$

with positive probability.

To understand Assumption 1, it is most instructive to understand what it rules out. To start out with, recall that V is independent of Z, and suppose that there are only two types, i.e. $V = v_{high}$ or $V = v_{low}$. Then, our assumption rules out the following: Suppose that for both the high and the low types, the forecast is a function of z_2 conditional on z_1 for at least some y for both types. However, one function, if reweighted by the relative weights of types in the population *exactly* offsets the other. Hence, if the population was comprised to equal proportion of both types, we rule out that one derivative, $\partial_{z_2}\varphi(z_1, z_2, v_{high}, y)$, is positive for all z_2, and the other one, $\partial_{z_2}\varphi(z_1, z_2, v_{low}, y)$, behaves like the *exact* negative of this function for almost *every* z_1, z_2 and y, so that the influence of z_2 exactly cancels out when averaging.[5]
We think of this as a weak assumption, ruling out rather pathological cases. The main implication is that Assumption 1 allows us to use the observed quantiles of the forecasts to assess whether or not individuals use Z_2. This is the content of the following theorem:

Theorem 1. *Consider the model above and suppose Assumption 1 holds true. Then, if*

$$k^{\tau}_{F^I_y|Z_1,Z_2}(z_1, z_2) = k^{\tau}_{F^I_y|Z_1}(z_1) \tag{2.6}$$

for (almost) all (z, τ, y), individuals do not use Z_2 conditional on their information set Q_1, in the sense of Definition 2.

[5]More generally, if I is allowed to depend on Z_2 as well, it rules out that the difference in the derivatives $\partial_{z_2}\varphi(z_1, z_2, v_{high}, y) - \partial_{z_2}\varphi(z_1, z_2, v_{low}, y)$ and the weighted change in the distribution, $\partial_{z_2} f_{I|Z}(z_1, z_2)$ again offset each other exactly everywhere.

Proof. See Appendix II. □

Remark 1.1: In a nutshell, Theorem 1 states the following: If we look at the family of (y, v)-indexed quantile regressions, and if none of these quantile regressions is a function of Z_2 conditional on Z_1, we can conclude that the information contained in Z_2 is discarded by all individuals, at least conditional on the individuals using the additional, partially unobserved information Q_1. In econometric terms, our formalization leads to an omission of variables test for a family of nonparametric regression quantiles. This is a well understood object whose implementation, while not completely trivial, is quite feasible in practice, as we show below.

Remark 1.2: While the quantile function $k^{\tau}_{F^I_y|Z}$ is the appropriate statistic to test for informational content, we can also look at the conditional mean function instead of agents' entire distribution. Doing so delivers an intuitive, but slightly weaker, test for informational usage. In particular, we may consider quantiles of

$$E^I[Y|Q] = m(Z_1, Z_2, V), \tag{2.7}$$

where again $Z = (Z_1', Z_2')'$. Note that (2.7) is directly observed in the data. If Z_2 is not used conditional on Z_1, then

$$m(z_1, z_2, v) = m(z_1, v) \tag{2.8}$$

and we can base our exclusion test on observable quantiles of the conditional mean function

$$k^{\tau}_{E^I[Y|Q]|Z_1,Z_2}(z_1, z_2). \tag{2.9}$$

While intuitive, (2.9) is weaker than (2.6), because we have to strengthen the averaging condition (i.e. Assumption 1) as we are now also averaging over y. In particular, one can think of the family of subjective probabilities as providing at least an univariate reduced form measure of the distribution of forecasts, while the mean averages across this distribution. Looking at the exclusion restriction contained in (2.9) is still useful, however, in that is has less data requirements and is easier to implement.[6]

2.2 Do individuals face costs of information processing?

While Proposition 1 delivers a simple nonparametric test for whether a variable is part of individuals' information sets, it is not helpful for interpreting *why* individuals might exclude some

[6]In our empirical part, we are going to focus on the restriction embedded in (2.9) as our data is not rich enough to estimate a nonparametric exclusion restriction on different quantiles of the subjective expectation data. We are going to come back to this in our empirical part below.

information from their forecast. Hence, we now ask in what sense the finding that some variable is not part of individual information sets is evidence that information processing is limited. In our setup, this can be rephrased as saying: Would someone endowed with the model $F^I_{Y|Q}$ but no constraints on information processing have chosen to use the information that individuals discard? If this was case, we will say that information processing is limited or costly. Hence, the essence of constrained information processing is that there is a demand for information, however, the marginal value may fall short of the marginal processing costs. Again we want to stress that we do not need to take a stand where such limitations to processing stem from, i.e. whether agents' attention is limited in the sense of rational inattention (Sims (2003); Mackowiak and Wiederholt (2000)), because of other considerations of complexity (Gabaix (2014)), or whether individuals actually face utility costs or other constraints to include particular pieces of information in their forecasts (Reis (2004); Mankiw and Reis (2002)).
To test for limits to information processing, we therefore have to define the value of information.

Definition 3. *Consider the setup described above. We say that the information contained in Z_2 is valuable given the model F^I and the information Q_1, whenever*

$$F^I_{Y|Q_1,Z_2}(y,(q_1,z_2)) \neq F^I_{Y|Q_1,Z_2}(y,(q_1,z_2')) \tag{2.10}$$

with positive probability. For notational simplicity we will say that Z_2 is $(F^I,[Q_1,Z_2])-$valuable if (2.10) holds true.

Hence, according to Definition 3, additional information is valuable whenever it changes the individuals' forecasts in some states of the world. Given this definition of information being valuable and our definition of information usage, we can also give a precise definition of what we are looking for in order to identify limits to information processing.

Definition 4. *Consider the setup described above. We say that information processing is limited with respect to Z_2, whenever Z_2 is not used conditional on Q_1, despite Z_2 being $(F^I,[Q_1,Z_2])$-valuable.*

Hence, whenever some information Z_2 would have changed individuals' forecast (given their unobserved model F^I and their unobserved information Q_1) but individuals decide to not use Z_2, we will conclude that their expectation formation process is subject to constrained information processing. The important aspect of Definition 4 is precisely the dependence of the value of information on F^I *and* on Q_1 - both of which are unobserved by the econometrician. Therefore the question is: Can we detect occurrences of costly information processing given data on income expectations without further restrictions on F^I and Q_1? The answer is no. The reason is simply that we can always find an agent's model such that the excluded information is not $\big(F^I,[Q_1,Z_2]\big)$-valuable. Intuitively, if the model agents are using is such that Z_2 is considered noise, Z_2 would not have been used even without processing costs. Hence, in order to give the hypothesis of costly information processing empirical content, we impose the following restriction on the relationship between the agents' and the data generating process.

Assumption 2. *If Z_2 is $(F, [Q_1, Z_2])$-valuable, then Z_2 is $(F^I, [Q_1, Z_2])$-valuable, with positive probability.*

Assumption 2 requires a minimal amount of consistency between the agents' view of the world and the structural model of the economy. Hence, we refer to Assumption 2 as an assumption of *weak rationality.* Intuitively, it requires the following: whenever the econometrician with $Q = [Q_1, Z_2]$ at his disposal would not discard Z_2, we require that a positive proportion of individuals would not do so either. These individuals could disagree with the econometrician about how Z_2 enters and how important it is, they could disagree about the structural model, or they could disagree about the distribution of all the variables. But they have to agree that Z_2 determines the distribution of the income forecast, conditional on Q_1, in some way. Hence, while rational expectations require that $F^I_{Y|Q_1,Z_2}(y, q_1, z_2) = F_{Y|Q_1,Z_2}(y, q_1, z_2)$ for all (y, q_1, z_2), Assumption 2 only requires that $F^I_{Y|Q_1,Z_2}(y, q_1, z_2)$ has to depend on z_2 if $F_{Y|Q_1,Z_2}(y, q_1, z_2)$ does. Note also that Assumption 2 is about *hypothetical* situations: we do *not* assume that we know what Q_1 actually is. We just require that *if* we had the agents' information set at our disposal, there was no disagreement whether or not Z_2 is valuable. Hence, we consider Assumption 2 to be rather weak and it turns to be sufficient to detect costly information processing in the data. Assumption 2 and Definition 4 suggest a strategy to test for limited information processing. Suppose we found out that Z_2 was *not* used by individuals. If we had Q_1 at our disposal, we could simply test if we as econometricians would find Z_2 valuable (conditional on Q_1). Alas, Q_1 is not observed. However, we can try to infer as much as possible about this information from the observable distribution of forecasts. To do so, for the forecasts at different values of y we use again the shorthand $F^I_{Y|Q}(y; Q) = F^I_y$, which is a random variable, indexed by y. For simplicity, assume that y takes J finite values and that we can thus observe the collection of random variables $\{F^I_{y_j}\}_{j=1}^J$. This means that we have J functions of the underlying randomness (Q_1, Z_2, I). However, conditional on Z_2, it is in general not the case that we are able recover Q_1 or I from these functions (i.e., the individuals could have additional information that is not reflected in the current forecast). Hence, we can only proxy (Q_1, I) by $\{F^I_{y_j}\}_{j=1}^J$, conditional on Z_2, and we would have to assume that whatever is leftover - let us call it S - is independent of all other information.

While this insight allows to directly construct tests, we propose to use an alternative, but equivalent, formulation. To this end, we use an insight from Matzkin (2003), who shows that if we are not interested in the structural model, we can represent the heterogeneity equivalently as a nonseparable function with a scalar unobserved variable that enters monotonically and which can be identified with the quantiles of $F^I_{y_1}$, conditional on Z. For instance, if we take the random variable "Individual's probability that future income is below y_1" as dependent variable, we can represent

$$F^I_{y_1} = k^{V_1}_{F^I_{y_1}|Z}(Z), \tag{2.11}$$

where $V_1 = v_1$ represents now both the v_1-quantile of $F^I_{y_1}|Z$, as well as a reduced form representation of the heterogeneity in $F^I_{y_1}$ given the information in Z. Of course, we can do this procedure for any value of y_j, and hence get a collection of random variables $V_1, ..., V_J$ which, conditional on Z, reflect the same information as $F_{y_1}, ..., F_{y_J}$ with $F_{y_j} = F_{Y|Q}(y_j; Q)$, $j = 1, ..., J$. That is, $\sigma(Z, F_{y_1}, ..., F_{y_J}) = \sigma(Z, V_1, .., V_J)$. This implies that we may equally well use the following conditional probability, which will be the crucial object to test whether information processing is costly:

$$P\left[Y < y_j | Z_1, Z_2, V_1, .., V_J\right], \tag{2.12}$$

Note that both of these information sets contain less information than $\sigma(Q_1, Z_2, I) = \sigma(Z_1, Z_2, V)$, where V is the more general, potentially infinite dimensional unobservable introduced in the previous section. This suggests to think of $V_1, .., V_J$ as the observed components of V reflected in the forecasts, and we thus use V_O to denote these (O stands for observed). This notation is chosen to emphasize that (Z_1, Z_2, V_O) is not equivalent to (Q, I). Indeed, we may write $(Q, I)' = \rho(Z, V_O, S)$, with $S \perp Z$. In terms of the notation of the previous section, we have thus $V = (V_O', S')'$, i.e., S reflects information individuals may in principle possess but which is not reconstructable from their forecast. Finally, we can use again the fact that there is a one to one relation between the conditional probability in equation (2.12) for all y_j, and the τ-quantile of Y given Z, V_O, i.e.,

$$k^{\tau}_{Y|Z,V_O}(z, v_O)$$

for all $\tau \in (0, 1)$. A test for costless information processing can now be based on the following Theorem.

Theorem 2. *Consider the setup described above and let Assumption 2 hold. If individuals do not use Z_2 conditional on Q_1, then there are limits to information processing in the sense of Definition 4 whenever*

$$k^{\tau}_{Y|Z_1,Z_2,V_O}(z_1, z_2, v_O) \neq k^{\tau}_{Y|Z_1,V_O}(z_1, v_O) \tag{2.13}$$

with positive probability.

Proof. See Appendix II. □

While a formal proof may be found in the appendix, the reasoning is as follows: First, if the quantile in equation (2.13) is a function of Z_2, then it is also the case that the probability in equation (2.12) is a function of Z_2. Since this probability is an average of $P\left[Y < y_j | Z_1, Z_2, V_1, .., V_J, S\right]$

over S, it also implies that the latter probability is a function of Z_2 for at least a small positive probability set. However, this is equivalent to saying that the objective conditional probability $P[Y < y_j|Q_1, Z_2]$ is a function of Z_2. Since subjective and objective conditional probabilities coincide in terms of the set of variables deemed valuable (due to Assumption 2), it also means that for some positive proportion of the population, Z_2 should have been included in their information set.

Theorem 2 therefore allows to devise a test for limited information processing as follows: Suppose that, in a first step, an applied researcher uses Theorem 1 to conclude that individuals do not use the information contained in Z_2. In a second step, the researcher then tests whether the inequality in equation (2.13) holds. This test takes again the now familiar "omission of variables from nonparametric regression quantiles" form. If we find that the conditional quantile of Y given Z and V_O is indeed a function of Z_2 (for at least a small positive probability set) we can conclude that it would have been beneficial for the individuals to use Z_2, given their information Q_1. Hence, if they decided not to use it, it must have been because the costs of using it outweighed the benefits - implying that there are limits to include Z_2 in their forecasts.

Remark 2.1: Given a perfect data set, the "omission of variables from nonparametric regression quantiles" test stemming from equation (2.13) is what we suggest be performed. In our dataset, however, the observations on the events $F_{y_1}, \ldots, F_{y_J}$ are quite poor (see next section). Therefore, we only use a single function of the underlying information, i.e., the conditional mean $E^I[Y|Q]$, which we denote by E^I. Note however that the above logic is still valid, and hence we are able to use

$$P[Y < y_j|Z_1, Z_2, V_O],$$

for a grid $y_1, ..., y_J$, where V_O is now the scalar $V_O = F_{E^I|Z}(E^I; Z)$, and hence $E^I = k_{E^I}^{V_O}(Z)$. Finally, instead of looking at $P[Y < y_j|Z_1, Z_2, V_O]$ for a grid $y_1, ..., y_J$, we now look at the equivalent set of quantiles

$$k^{\tau_j}(Y|Z_1, Z_2, V_O),$$

for a grid $\tau_1, ..., \tau_J$ of quantiles of Y given Z and V_O.

Remark 2.2: At a conceptual level, our exclusion restriction contained in Theorem 2 shares some resemblance with tests for informational efficiency within the context of models with rational expectations: with rational expectations, variables in agents' information sets should have no explanatory power for the outcome of interest once agents' forecasts are controlled for. A key objection to this test is the existence of aggregate shocks that realize only once in a given period - see Keane and Runkle (1990). The reason is that individual forecasts in period $t-1$ are averages over all possible realizations of the aggregate shock, yet realizations in period t contain a single realization. For instance, in period $t-1$ a boom or a bust may have looked equally likely, yet in period t only the boom realizes. Hence, the outcome of interest

is a function of the realization of the shock, while individuals only include the distribution of shocks in their forecasts.
While our approach is in principle also affected by this issue, it is dramatically less so. The reason is that we do not insist on a forecast that is on average correct - we are merely interested in whether or not variables are in agents' information sets. To see the difference, consider the above example and note what we would have to rule out for our analysis to still be valid in the presence of aggregate shocks: we *cannot* admit that in period $t-1$, where boom and bust looked equally likely to the individual, the average expectation of the event that future income Y is below a threshold y, i.e., $Y < y$, given the individuals' information was *not* a function of, say, education, while after the boom has realized, it is a function of education. For this to be the case, future income in both states of the world (i.e. boom and bust) must again be determined by two *exactly* offsetting functions of education for *every level of* y. In other words, the type of assumption we require for our analysis to be valid in the presence of aggregate shocks is exactly of the form of Assumption 1. As before, we think of cases that are ruled out by this assumption to be theoretically possible, but rather implausible in practice. Hence, we are not particularly concerned about the consequences of the Keane-Runkle critique for our approach.

Remark 2.3 (*Implementation*): As we will detail in the next section, we propose to form standard frequentist nonparametric sample counterpart estimators to the nonparametric identification results in this section. A possible alternative is to use Bayesian analysis. Since the nonparametric aspect of our approach is important, yet Bayesian nonparametric analysis is less well developed, we feel that a frequentist approach is justified in our case. However, we do not believe that there are any aspects that would fundamentally benefit one type of analysis over the other, and thus encourage Bayesian work on what we deem is a very important question.

3 Empirical Analysis

In this section we apply the framework developed in Section 2 to cross-sectional microdata on individuals' income expectations. As in the theory laid out above, we will first measure the content of individual information sets and then ask if the microdata is consistent with models where information processing is limited.

3.1 Data Sources

The data we use is from the 'Survey of Household Income and Wealth' (SHIW), collected by the Bank of Italy.[7] The SHIW provides detailed information on individual characteristics, sources

[7] The data and all the programs used to generate the results of this paper are available upon request.

of income, and financial assets for about 8,000 households (roughly 24,000 individuals). In 1991, the survey included a question on individual income and inflation expectations, which was also used by Jappelli and Pistaferri (2000) and Guiso et al. (1996) to study consumption growth and portfolio choices, respectively. While the focus of our paper lies on income expectations and thus the SHIW is exactly the right data to use, we note that there are many other interesting data sources that contain information on individuals' expectations. Most notably, the Health and Retirement Study (HRS) survey includes regular questions on expectations about future events such as retirement age, inheritance, or life expectancy. While each of these questions gives rise to topics worthwhile studying in their own right, we relegate the application of our methodology to this kind of expectation data to future research.

The SHIW does not only elicit point estimates on respondents' expectations (say about their mean income growth), but asks individuals about their entire subjective distribution about future income growth. More precisely, the question about individual income expectations has the following wording: "Think about your entire working income or pension payments and its evolution over the next year from now. On this card you see several possible categories of growth rates. Which possibilities concerning your income change do you rule out? Assume you could distribute 100 points on the remaining categories: how many points would you give to each category?". Overall, there are 12 categories with the 10 inner intervals spanning a range between 0% and 25% and the boundary intervals being wider than the inner ones.

Table 1 displays the cumulative distribution of $F^I_{Y|Q}(y;Q)$ for selected growth scenarios y. In particular, a given cell for a rate of income growth y and a probability p reports the share of individuals who believe that the probability of their future income growth being at most y is below or equal to p. As expected, the entire distribution of $F^I_{Y|Q}(y;Q)$ shifts towards the upper endpoint at unity exhibiting increasing point mass as y increases. That is, while for instance 51.67% of the respondents ruled out an income growth of less than or equal to 3% within the next year, only 8.57% deem that their income growth will exceed 8% with probability one. By contrast, roughly 34% (85%) of the respondents believe that their income growth will be less than or equal to 3% (8%).[8] As a second observation, notice that, independently of income growth y, only 10-15% of all respondents attach probabilities other than zero or one to different income growth events. In fact, a closer examination of the data reveals that, out of 3,196 individuals with valid (non-missing) expectation data, only 1,126 people distributed points across more than one category, and only 447 individuals allocated the 100 points to more than two categories.

While $F^I_{Y|Q}$ has been at the focus of the analysis so far, we could in principle also look at other

[8] To see this, note that 66% (15%) of people think that the probability of experiencing income growth of at most 3% (8%) is below or equal 90%. Hence, 34% (85%) of people think that the probability of experiencing income growth of at most 3% (8%) exceeds 90%. Given the categorical nature of the expectations data, these are responders who expect the respective income growth with probability one.

measures such as $\mathbb{P}^I(Y \in [y_l, y_u)|Q)$, i.e. the probability of income growth Y falling into a specific growth category $[y_l, y_u)$. However, Table 2, which displays the cumulative distribution of $\mathbb{P}^I(Y \in [y_l, y_u)|Q)$ for different categories $[y_l, y_u)$, demonstrates that the phenomenon of excessive point mass at the endpoints 0 and 1 and little variation in between observed in Table 1 persists. Moreover, the distributions (and thus quantiles) are very uneven across growth categories $[y_l, y_u)$ complicating a comparison of quantiles across different growth categories further. For instance, while for the $0-3\%$ category the data exhibits variation only between the 56% and the 71% quantile (which lie at the endpoints 0 and 1, respectively), this variation occurs between the 77% and the 94% quantile for the 5-6% growth category. This feature, together with the aforementioned lack of variation in the data, impede an analysis on the basis of different quantiles of either $F^I_{Y|Q}(y;Q)$ or $\mathbb{P}^I(Y \in [y_l, y_u)|Q)$, and motivate the use of a functional of $F^I_{Y|Q}(y;Q)$, namely the conditional mean $E^I[Y|Q]$ (see Section 3.3 below).
Besides the expectation data, the SHIW survey also contains data on realized income growth Y and on various economic characteristics. It will be those characteristics for whose exclusion we will test for. Note especially that the entire data is self-reported, i.e. our analysis does not suffer from the problem that individuals might not have access to the information the researcher tests for. So if we conclude that some variable Z is not included in the income expectations, we can rule out the case that Z was not known to the individuals. They clearly knew Z but decided to not use it when forming their income expectations. This aspect of the data is important because it allows us to exclusively focus on the aspect of limited processing - in principle individuals have access to a wide range of information but they optimally choose to not include parts of it in their forecasts. From an economic point of view we are interested in the capacity of individuals' to forecast their labor income. Hence, we focus only on working males, which are between 20 and 65 years old.

3.2 Reduced Form Results

Before turning to the nonparametric test, we take a reduced form look at the data to gauge the validity of the reported income expectations. In Table 3 we regress the realized income growth (for both labor and capital income) on individuals' mean expectations $E^I[Y|Q]$ and other characteristics. We see that there is a robust positive correlation between expected and realized income growth for labor income. We view the results in Table 3 as reassuring in that individuals' reported expectations do in fact have predictive power for realized growth rates. Additionally, the results also show that individuals seem to predict their labor income and not their capital income. In the last two columns we regress the growth rate of capital income on individuals' mean expectations and do not find any discernible pattern, because the coefficients are very imprecisely estimated. The last column focuses only on individuals reporting non-zero capital income growth. The standard errors decline substantially and the estimated coefficient

is statistically zero.
Now consider a first pass to measure the informational content of individual information sets. In Table 4 we report the results of a regression of individuals' expected income growth on various characteristics. This provides us with a reduced form sense about which information individuals do pay attention to and which not. While current (log) income, local labor market conditions (which are captured by the area dummies) and occupational characteristics are highly significant and therefore not excluded from individuals' income expectations, age and education are not part of individual information sets. In the following, we will test these hypothesis nonparametrically, as implied by our theory.

3.3 Testing for Informational Content

We now turn to the test of individuals' information sets. As outlined in Section 2, in theory this could be achieved by testing the quantile exclusion restrictions in Theorem 1 since we can conclude that individuals do not use Z_2 conditional on Z_1 if

$$k^{\tau}_{F^I_y|Z_1,Z_2}(z_1, z_2) = k^{\tau}_{F^I_y|Z_1}(z_1) \tag{3.1}$$

for (almost) all (z, τ, y). However, due to the aforementioned limitations of our data, in what follows we will focus on a functional of that distribution, namely the conditional mean of individuals' expectations, $E^I[Y|Q]$, and test for the informational content of the distribution of $E^I[Y|Q]$ as a proxy of the test in Theorem 1.[9]
Let $k^{\tau_j}_{E^I|Z}(Z)$ denote the conditional quantile function of individuals' income expectations for a set of τ_j's with $j = 1, \ldots, J$ where each $\tau_j \in (0, 1)$. The test is then implemented using the following procedure:

1. We first estimate the conditional quantile functions $k^{\tau_j}_{E^I|Z_1}(Z_1 = z_1)$ and $k^{\tau_j}_{E^I|Z_1,Z_2}(Z_1 = z_1, Z_2 = z_2)$ using a (semi-)linear specification for $j = 1, \ldots, J$.
2. Given the estimates $\hat{k}^{\tau_j}_{E^I|Z_1}(z_{1,i})$, we generate the residuals

$$\hat{\varepsilon}_{\tau_j,i} = E^i[Y|Q] - \hat{k}^{\tau_j}_{E^I|Z_1}(z_{1,i}). \tag{3.2}$$

3. With these residuals at hand, we construct B bootstrap samples with

$$E^i[Y|Q]^{\star} = \hat{k}^{\tau_j}_{E^I|Z_1}(z_{1,i}) + \hat{\varepsilon}^{\star}_{\tau_j,i},$$

[9] As outlined in Remark 1.2, this requires us to strengthen Assumption 1 and we assume that changes in the subjective density do not average out when averaging over y in what follows.

where $\hat{\varepsilon}^{\star}_{\tau_j,i}$ are the bootstrap residuals, which have been constructed on the basis of $\hat{\varepsilon}_{\tau_j,i}$ using the wild bootstrap method of Haerdle and Mammen (1993), with a simple adjustment to suit the asymmetric loss function in quantile estimation as suggested by Feng et al. (2011). Crucially, note that $E^i[Y|Q]^{\star}$ is generated under the null, i.e. using the model where the exclusion restriction is imposed.

4. Next, we compute the empirical equivalent of the τ_j-th quantile test statistic:

$$\rho^{\tau_j} = \int [k^{\tau_j}_{E^I|Z_1}(z_1) - k^{\tau_j}_{E^I|Z_1,Z_2}(z_1, z_2)]^2 \omega(z_1, z_2) dz_1 dz_2$$

as:

$$\hat{\rho}^{\tau_j} = \frac{1}{n} \sum_i [\hat{k}^{\tau_j}_{E^I|Z_1}(z_{1,i}) - \hat{k}^{\tau_j}_{E^I|Z_1,Z_2}(z_{1,i}, z_{2,i})]^2 \omega(z_{1,i}, z_{2,i}), \tag{3.3}$$

where $\omega(z_{1,i}, z_{2,i})$ is a suitable weighting function.[10]

5. Using the B bootstrap samples we then estimate the distribution of ρ^{τ_j}, say $\hat{H}_{\rho^{\tau_j}}$, on the basis of equation (3.3).

6. We conclude that Z_2 is excluded from the information sets of the individuals (conditional on Z_1) if $\hat{\rho}^{\tau_j}$ does not exceed the 95% quantile of $\hat{H}_{\rho^{\tau_j}}$.

Before turning to the actual test results, we provide an overview of the variation in the (conditional) mean income expectations $E^I[Y|Q]$ and in the realized income growth data in Table 5. Notice that in particular around the 0.15 and the 0.25 quantiles, $E^I[Y|Q]$ displays very little variation owed to the heaped reporting of individuals, which suggests considerable point mass around the value 0.015.[11] As a consequence, we will focus on quantiles for our test above 0.25, namely the 0.35, the 0.5, and the 0.65 quantiles (Tables for the full set of quantiles can be found in the supplementary material).
For each test, we use three different specifications for the semi-linear conditional quantile function:

$$k^{\tau_j}_{E^I|Z}(z) = g_{\tau_j}(y^*, a) + w'\beta_{0,\tau_j},$$

[10] In practice, we take the weighting function

$$\omega(z_1) = \begin{cases} 1 & \text{if } z_i \leq q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right) \\ 0 & \text{if } z_i > q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right) \end{cases},$$

where $q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right)$ is the 95%-quantile of $(z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)$ with $\bar{z}_1$ and Σ_{Z_1} denoting the sample mean vector and the variance-covariance matrix of z_1, respectively.

[11] In fact, there is a discrete jump between the 0.05 and the 0.06 quantile from -0.0175 to 0.015.

where $g(\cdot)$ is a nonlinear function, y^* denotes the natural logarithm of (current) income, a age in years, w a vector that contains other covariates such as occupation, area, sector or education dummies, and $z = (y^*, a, w')'$. The first specification is of a linear form and taken to be $k^{\tau_j}_{E^I|Z}(y^*, a, w) = \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\alpha_j}$. The second and third specifications are nonlinear, where $g(\cdot, \cdot)$ is either modeled as:

$$g_{\tau_j}(y^*, a) = \sum_{i=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2,$$

or

$$g_{\tau_j}(y^*, a) = \sum_{i=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} ln\,(y)^2\,,$$

and $\sum_{i=1}^{K} \gamma_{i,\tau_j} p_i(\cdot)$ denotes a linear combination of base functions of a fourth order (cubic) B-spline (the inner knots are chosen to be the $\{0.25, 0.5, 0.75\}$ quantiles of the data).

Using this procedure, we can now test for the exclusion of different pieces of information. Our tests will always be based on the reasoning laid out above, i.e. we will test for the exclusion of some information Z_2 via the test "Z_1 vs $[Z_1, Z_2]$". We start out from a natural benchmark, namely the case where individuals only perceive a rudimentary life-cycle profile, i.e. only use their age and their current income to predict future income growth so that $Z_1' = [linc, age]$.[12] Given Z_1, we then test for the exclusion of *all* other individual characteristics we consider, namely the information contained in individuals' regional, occupational, sectoral and educational characteristics, i.e. $Z_2' = [area, occ, sec, educ]$. The results are contained in Table 6. The first column contains the actual specification for the conditional quantile function, the second column the test statistic (calculated as in equation (3.3)), the third column the critical value, i.e. the 95% quantile of the distribution of the test statistic. The last column finally contains the p-value. Table 6 therefore shows that we can confidently reject the hypothesis that agents only use current income and age to predict future income growth across all specifications for all three selected quantiles. In particular, individuals use information contained in their educational attainment, their sector of employment, their occupation or their locality to predict future income growth.

We will now decompose which information individuals pay attention to. We begin by considering $Z_1' = [linc, age, occ, area)]$ and $Z_2' = [sec, educ]$, i.e. we formally test the null hypothesis that individuals do not condition on their educational and sectoral characteristics, once Z_1 is controlled for. The first part of Table 7 shows indeed that we cannot reject this null hypothesis. For all specifications and all quantiles, the test statistic is below the critical value at

[12] In what follows, we use the obvious abbreviations *linc*, *occ*, *sec*, and *educ* to denote *natural logarithm of (current) income*, *occupational affiliation*, *sectoral affiliation*, and *educational status*.

conventional levels of significance.[13] This contrasts with the second part of Table 7, where we test $Z_1' = [linc, age, sec, educ]$ and $Z_2' = [occ, area]$ and clearly reject the null hypothesis at all conventional levels, implying that occupational affiliation and regional characteristics play a role even after conditioning on Z_1.

To confirm that it is actually the information contained in both the regional and occupational characteristics that enter individuals' information sets besides current income and age, we conduct two further robustness tests for the expectation data. First we test $Z_1' = [linc, age, occ]$ against $[Z_1', Z_2']'$, where $Z_2' = [area, sec, educ]$. Then we reverse the role of localities and occupation and test $Z_1' = [linc, age, area]$ against $[Z_1', Z_2']'$, where $Z_2' = [occ, sec, educ]$. Both of these exercises are contained in Table 8 and both of these show that we comfortably reject either of these alternatives. Hence, there is useful information in both variables, which individuals use when forming their expectations about future income.

Finally, we test whether age, which was found to be insignificant in the reduced form case (see Table 4), actually plays a role once we employ our more flexible test. Thus, we test $Z_1' = [linc, occ, area]$ against $[Z_1', Z_2']'$, where $Z_2 = [age]$.[14] While for individuals around the 0.35 quantile age does not appear to play an important role when forming their predictions, we observe that it does clearly matter for the 0.5 and the 0.65 quantiles (see Table 9). The latter result holds irrespective of whether we test against a linear or a nonlinear term of age. As the different quantiles absorb individual heterogeneity in unobserved information and hence refer to different types in the population, Table 9 shows that at least a subgroup of individuals in the population actively uses the information in age to predict their future income growth. This underlines the importance of allowing for nonlinear specifications in our setup.

In sum, the four variables log income, age, occupation and area provide a sufficient description of the individuals' information sets in our data and we cannot reject that educational status and sectoral affiliation do not predict individual mean income expectations, once the former characteristics are controlled for. While the absence of education from the information set might seem surprising at first sight, we want to stress again that this exclusion holds *conditional* on the other characteristics. For instance, in the context of labor income, agents might gather information about their future job prospects by observing their professional peers. In such a situation, education would for example only play an indirect role in that it matters to attain a specific occupational status, but not have a direct impact on the forecast itself. In fact, putting

[13] To check whether any of these two variables matters individually, we repeated this test against the alternatives $Z_2 = [educ]$ and $Z_2 = [sec]$ individually. As expected, in both cases the null hypothesis could not be rejected at any reasonable significance level across all quantiles (an exception being the 0.35 quantile for the linear specification) leading us to the conclusion that neither educational attainment nor sectoral affiliation seem to enter the individuals' information sets once occupation, area, age, and income were controlled for.

[14] Notice that when conducting a test against omission of age in the nonlinear case, we test either against a linear and quadratic (second specification) or a fourth order B-spline (third specification) term of age.

this argument to a (crude) test by examining $Z_1' = [linc, age]$ vs. $[Z_1', Z_2']' = [linc, age, educ]$ reveals that the null of exclusion of educational status from the information set (conditional on Z_1) can be comfortably rejected for various quantiles across specifications once occupational and sectoral affiliation have been omitted.[15]

3.4 Testing for limits to information processing

In a second step, we are now going to ask whether there is evidence that the omission of education and sectoral information is due to information processing costs, or whether the information in these variables is indeed redundant in the sense that even a decision maker without limitations to information processing had decided to discard this information. To gauge whether information processing is actually limited, we implement a test on the basis of Theorem 2. More specifically (and as outlined in Remark 2.1), we will do so by examining $k^{\tau_j}(Y|Z_1, Z_2, V_O)$ for our selected quantiles, where the V_O's with $V_O = F_{E^I|Z}(E^I; Z)$ are constructed as quantile ranks of $E^I[Y|Q]$ for each conditioning set. However, while in principle the conditioning set contains $Z' = \{Z_1', Z_2'\}'$, stratifying the data by the entire vector led to a lack of observations in numerous cells.[16] We therefore construct V_O in two different ways, namely as a function of Z_1 only, and, for comparison reasons, of $\{Z_1^{c\prime}, Z_2'\}'$, where Z_1^c denotes the continuous elements. That is, the former vector consists of $[linc, age, occ, area]$, the latter one is given by $[linc, age, sec, educ]$. For each of these conditioning sets, we then construct ten quantile ranks (0-10%, 10-20%, etc.) of $E^I[Y|Q]$.[17] Also, notice that to conduct this test, we restrict our sample to individuals with observations on both expected and realized growth, which reduces the sample size to 1418.[18]
Turning to the first part of the results in Table 10, we observe that we can comfortably reject the null hypothesis of costless information processing across all quantiles and specifications at conventional levels of significance. This test outcome is confirmed when examining the second set of results for the case where V_O is constructed as a function of Z_2 as well. Hence, the conclusions from this table are twofold: first, it appears that the construction of V_O, albeit not fully in line with the theoretical setup, does have very little influence on the actual test outcome leading us to conjecture that the misspecification is rather innocuous in our case. Second and more importantly, we can clearly reject the hypothesis of costless information processing since the fact that we as econometricians consider $Z_2' = \{educ, sec\}$ to be valuable conditional

[15] Results available upon request.

[16] Recall that discrete variables such as occupational status, education etc. are coded in multiple dummy variables.

[17] To construct the conditioning sets for the continuous variables, we used a kernel function $K(u) = \mathbb{I}\{|u| < h\}$ with $u = (z_1^c - Z_{1i}^c)$ and bandwidth $h = std(Z_{1i}^c)$, where $std(\cdot)$ denotes the standard deviation.

[18] Unreported results show that the key results of testing $Z_1' = [linc, age, occ, area)]$ against $[Z_1', Z_2']'$, where $Z_2' = [sec, educ]$, are not affected by this reduction of the sample size.

on knowing individuals' mean income expectations implies that agents should consider that information valuable as well by our assumption of weak rationality (Assumption 2).[19]

As a final remark, notice that all hypotheses above have been tested independently. This decision, albeit theoretically somewhat questionable, appears relatively innocuous in our case as most of the fundamental conclusions in this section have been drawn on the basis of test statistics that lay very far from either side of the corresponding critical value. In particular, note that applying a standard adjustment method such the Holm-Bonferroni procedure with a significance level of $\alpha = 0.05$ to the test sequence above does not alter any of the test conclusions.[20]

4 The Value of Information

The fact that consumers seem to exclude information from their information set which we as econometricians would include is consistent with the presence of limits to information processing. In this section, we are going to quantify the utility consequences of the resulting coarse information sets using a simple version of the standard life-cycle model as an example. This model is not only a natural starting point to analyze the value of information, but it also follows naturally from our econometric application: predicting future income is precisely the crucial forecasting problem individuals have to perform.

Our approach is the following. We consider a life-cycle problem, where individual face income risk and markets are exogenously incomplete in that only a risk-less bond is available. There are no other constraints on borrowing. Parametrizing the income process requires us to distinguish between the predictable component of future income and the perceived innovation. It is at this point, where differences in the agents' information set come in. Given the same microdata on income realizations, variations in the information set used to predict future income growth, will lead to different decompositions of the income process into its predictable and unpredictable components and to different behavior as encapsulated in the policy function. To estimate the willingness to pay for information, we will therefore first solve for the optimal consumption and savings policies under the individuals' information set. We will then simulate life-cycle profiles using these policy functions but having income evolve under the law of motion, which

[19] As a final check for the usefulness of the discarded information, we also replicated the entire analysis in Step 1 using the data on realized income growth as a dependent variable (see supplementary for complete results). For instance, as seen in Table 11, we can comfortably reject that the information in educational attainment or sectoral affiliation is not predictive for income realizations, once the remaining observable information is controlled for. Note that these tests are different from the ones reported in Table 10: in the latter we explicitly control for the estimated unobserved information V individuals employ to predict income growth.

[20] If we also account for the test sequence described in Footnote 13, only some of the conclusions on the 0.35 and the 0.5 quantile for the two nonlinear specifications are altered.

we as econometricians could infer from the data. These simulated life-cycle profiles allow us to estimate the utility loss of making decisions under a smaller information set and the willingness to pay for the econometricians' information set.

4.1 The Environment

We consider a parametrization of the life-cycle model that is standard in the literature.[21] An infinitely lived consumer chooses consumption to maximize expected utility

$$U = E\left[\sum_{t=1}^{\infty} \beta^t u\left(C_t\right)\right], \tag{4.1}$$

subject to the per-period budget constraint

$$A_{t+1} = R\left(A_t + Y_t - C_t\right), \tag{4.2}$$

where Y_t denotes personal income at time t, A_t are individuals' asset holdings at time t and $R = 1 + r$ is the gross interest rate. Given an initial condition A_0 and the No-Ponzi condition, (4.1) and (4.2) fully characterize the agents' optimal consumption plan. We parametrize $\{Y_t\}_t$ in the standard way as

$$Y_t = P_t T_t, \tag{4.3}$$

where P_t denotes permanent income and T_t is a transitory income shock. The stochastic process for permanent income is given by

$$P_t = G_t P_{t-1} N_t, \tag{4.4}$$

where G_t denotes the predictable growth in permanent income and N_t is a shock to permanent income. (4.3) and (4.4) provide a very parsimonious parametrization of the income process, which nevertheless has been shown to capture salient features of individual income data reasonably well (see e.g. Gourinchas and Parker (2002)). Individuals only need to know the distribution of shocks T_t and N_t and the predictable growth process $\{G_t\}_t$ to know the entire joint distribution of their income process. In particular, suppose that T_t and N_t were log-normally distributed with parameters (μ_T, σ_T^2) and (μ_N, σ_N^2). Then, $(\mu_T, \sigma_T^2, \mu_N, \sigma_N^2, \{G_t\}_t)$ fully characterizes the income process. The concept of permanent income implies that $E\left[Y_t|P_t\right] = P_t$, so that (4.3) requires $\mu_T = -\frac{1}{2}\sigma_T^2$. Similarly, we can always normalize $\mu_N = -\frac{1}{2}\sigma_N^2$ and adjust G_t accordingly.[22]

[21] See for example Carroll (1997) or Gourinchas and Parker (2002) for similar approaches and Deaton (1991) for a related model which allows for borrowing constraints.

[22] Suppose the true process has $ln\left(N_t\right) \sim \mathcal{N}\left(\mu, \sigma^2\right)$. Then $ln\left(P_t\right) | \sim \mathcal{N}\left(g_t + \mu + p_{t-1}, \sigma_N^2\right)$. As μ is known to the agent, we can always incorporate in the predictable component g_t and normalize $\mu_N = -\frac{1}{2}\sigma_N^2$.

How would the agents *in this model* predict $(\sigma_T^2, \sigma_N^2, \{G_t\}_t)$? We assume that they follow the rationale of econometricians and hence follow the approach laid out in Carroll and Samwick (1997). Letting $y_t \equiv ln(Y_t)$ (and for the other variables analogously), the growth rate of income is given by

$$y_{t+1} - y_t = p_{t+1} + t_{t+1} - p_t - t_t = g_{t+1} + n_{t+1} + t_{t+1} - t_t. \tag{4.5}$$

Similarly, the h-step difference is

$$r_{h,t} \equiv y_{t+h} - y_t = \sum_{m=1}^{h} g_{t+m} + \sum_{m=1}^{h} n_{t+m} + v_{t+h}^i - t_t^i. \tag{4.6}$$

According to the logic of the model, g_{t+1} is the predictable component of income growth, i.e. given *their* information set Q, the agents would estimate

$$E\left[y_{t+1} - y_t | Q\right] = g_{t+1} - \frac{1}{2}\sigma_N^2. \tag{4.7}$$

From (4.6) and (4.7), individuals could then calculate the residual

$$\omega_{h,t} \equiv r_{h,t} - \sum_{m=1}^{h} E\left[y_{t+m} - y_{t+m-1} | Q\right] = \sum_{m=1}^{h}\left(n_{t+m} + \frac{1}{2}\sigma_N^2\right) + v_{t+h}^i - t_t^i \sim \mathcal{N}\left(0, h\sigma_N^2 + 2\sigma_T^2\right).$$

Hence, given more than 2 observations of income (i.e. a sufficiently long panel), σ_N^2 and σ_T^2 can be estimated from $\left\{\omega_{h,t}^2\right\}_h$.
It is clearly seen from (4.7) how differences in the information set Q will lead to different interpretations of the same data $\{y_{i,t}\}_{i,t}$. Not only will the predictable component of income growth be different, but the backed out residual $\omega_{h,t}$ will also have different statistical properties, which will lead the decision maker to arrive at different estimates for the variance of transitory and permanent shocks.
Table 12 reports the results of this exercise for the two different information sets we estimated in Section 3.3. In the first row we report the parameters of the process individuals perceive by only using regional and occupational information in addition to age and current income to forecast their future income. Given their information set, they conclude that transitory shocks had a variance of 0.0552 and permanent shocks one of 0.0145. In the second row we report the implied model of the econometrician, who also realizes that educational and sectoral information is valuable. By incorporating these sources of information, the perceived variance of both shocks decline. However, Table 12 also suggests that the differences induced by variations in the information set are not very large.

As usual, it is convenient to write the problem recursively. Conditional on permanent income P_t, the only additional state variable is cash-on-hand $X_t = A_t + Y_t$. This yields the recursive formulation

$$\begin{aligned}\mathcal{V}(X,P) &= \max_{A'}\left\{u\left(X-\frac{1}{R}A'\right)+\beta E_{Q^I}\left[\mathcal{V}(X',P')\,|P\right]\right\} \\ \text{s.t.}\quad & X' = A' + Y' \\ & Y' = GPN'T',\end{aligned} \tag{4.8}$$

where E_{Q^I} denotes the expectations taken over the perceived joint distribution of N' and T' given the agent's information set Q^I. We assume that u takes the constant relative risk aversion (CRRA) form $u(c) = \frac{c^{1-\theta}}{1-\theta}$. (4.8) can then be solved numerically in a straightforward manner to yield policy functions π_c^I and π_a^I, where the superscript "I" stresses that these policies are contingent on the individuals' information set. To solve this model, we take standard parameter values, which are displayed in Table 13 below.

4.2 The Utility Costs of Constrained Information Processing

By how much would consumers do better if they were to use a more complete information set? To answer this question, we are going to adopt the following procedure. Let π_c^I and π_a^I be the policy functions of a consumer with too small an information set and let $\{Y_t^F\}_t$ be the income process under the full information set, i.e. when using all the valuable information to estimate the predictable component of income growth g_t. In our application, this refers to the last row of Table 12. Now suppose a consumer were to base his behavior on (π_c^I, π_a^I) when facing the income process $\{Y_t^F\}_t$. How much would he be willing to pay to be able to use the policy functions (π_c^F, π_a^F), which are the solution to the life-cycle problem, when the income process is indeed perceived to be $\{Y_t^F\}_t$?

To calculate these welfare losses numerically, we are simulating M life-cycle profiles using the income process $\{Y_t^F\}_t$, but behavior based on (π_c^I, π_a^I).[23] Hence: consumers face an income process, which has slightly *less* transitory uncertainty than they thought when they made their consumption and savings plans. With N and T being both entirely idiosyncratic shocks, this corresponds exactly to the empirical distribution of future histories, a consumer could experience.

To measure the willingness to pay for superior information, we then redo this analysis for behavior based on (π_c^F, π_a^F), i.e. for the policy functions derived under the correct income process. The difference in ex-ante values of these two scenarios is exactly the utility loss of

[23] In practice we take $M = 50.000$.

using a coarse information set. Formally, let $V_F^I(x)$ and $V_F^F(x)$ be the value of facing the income process $\left\{Y_t^F\right\}_t$ with behavior governed by $\left(\pi_c^I, \pi_a^I\right)$ and $\left(\pi_c^F, \pi_a^F\right)$ at a level of cash-on-hand, relative to permanent income, x. We then define the willingness to pay for information $\Delta^{I,F}(x)$ implicitly by

$$V_F^F\left(x\left(1+\Delta^{I,F}(x)\right)\right) = V_F^I(x). \tag{4.9}$$

Hence, $\Delta^{I,F}(x)$ is the required relative change in cash-on-hand, which would make an informed consumer equally well off as the less informed consumer. By construction we have $V_F^I(x) < V_F^F(x)$ so that $\Delta^{I,F}(x) < 0$. The results of this exercise in our application are contained in Table 14, which reports $\Delta^{I,F}(x)$ for different quantiles of the stationary distribution of cash-on-hand.

The utility loss form coarse information is small - at least within our example of a life-cycle model. On average, consumers would be willing to pay roughly 0.04% of their cash-on-hand (relative to permanent income). The utility loss is small as consumers are sufficiently well self-insured to not be materially affected by their slight overestimate of uncertainty. In fact: precisely because they consider the world as more risky, they will accumulate a bigger buffer stock of savings compared to the well-informed counterpart. Hence, uninformed consumers hold slightly “too much” assets, which however does not have large utility consequences.

5 Conclusion

What information do individuals use when they form expectations about future events? In this paper we present an econometric framework to answer that question. In particular, we show that by focusing directly on agents’ observable subjective expectations, one can characterize the content of their information sets without assuming a particular model of belief formation (e.g. that expectations are rational) and one can allow individuals to use additional information, which the econometrician does not observe. We apply our methodology to the case of individuals’ income expectations. Using microdata on agents’ beliefs about income growth, we show that information sets are relatively coarse: while individuals do incorporate occupational characteristics, their age (or their labor market experience) and local labor market conditions in their income forecasts, we do not find evidence for educational characteristics or sectoral affiliation to matter. As this information is self-reported, i.e. in principle available, we interpret this informational coarseness as being consistent with costly information processing. To gauge the utility consequences of this behavior, we consider a simple example of a standard consumption life-cycle model using consumers’ information sets from the microdata. On average consumers would be willing to pay 0.04% of their permanent income to be able to actively use the information set of the econometrician.

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Appendix I

	Probabilities p										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\leq 3\%$	51.67	53.39	54.9	56.68	57.4	62.25	62.78	63.62	64.84	66.25	100
$\leq 5\%$	29.59	30.56	31.94	33.19	34.66	37.88	39.04	40.23	41.48	42.48	100
$\leq 6\%$	19.49	20.11	20.99	22.18	23.18	25.21	26.02	27.21	28.68	29.9	100
$\leq 7\%$	13.32	13.86	14.33	14.89	15.61	17.52	18.61	19.27	20.39	21.24	100
$\leq 8\%$	8.57	9.01	9.51	9.76	10.29	11.7	12.48	13.29	13.95	14.95	100

Notes: The table displays the cumulative distribution of $F^I_{Y|Q}(y;Q)$ in percent for the sample of individuals with valid expectation data (3,196 individuals) across different income growths $y \in \{3\%, 5\%, 6\%, 7\%, 8\%\}$.

Table 1: Cumulative Distribution of $F^I_{Y|Q}$ for Different Growth Categories

	Probabilities p										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$0-3\%$	56.3	58.09	59.62	61.43	62.28	67.47	68.06	68.88	70.22	71.47	100
$3-5\%$	63.53	66.62	69.6	72.72	74.73	82.3	83.08	83.95	85.17	85.52	100
$5-6\%$	76.85	79.76	82.67	86.39	87.83	92.21	93.02	93.65	94.15	94.28	100
$6-7\%$	83.7	86.36	89.08	91.84	92.62	95.87	96.09	96.34	96.65	96.78	100
$7-8\%$	87.43	89.93	91.71	93.43	94.43	96.87	97.15	97.44	97.72	97.72	100

Notes: The table displays the cumulative distribution of $\mathbb{P}^I\Big(Y \in [y_l, y_u)\Big|Q\Big)$, the probability of income growth Y falling into a specific growth category $[y_l, y_u)$, in percent for the sample of individuals with valid expectation data (3,196 individuals). The specified growth categories for $[y_l, y_u)$ are the intervals $[0,3), [3,5), [5,6), [6,7), [7,8)$.

Table 2: Cumulative Distribution of $\mathbb{P}^I\Big(Y \in [y_l, y_u)\Big|Q\Big)$ for Different Growth Categories

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Realized Growth Labor Inc.					Realized Growth Capital Inc.	
$E^I[Y\|Q]$	0.517**	0.637**	0.504**	0.336**	0.112**	4.080	-0.224
	(0.114)	(0.170)	(0.120)	(0.101)	(0.0491)	(4.197)	(0.353)
ln(wages)		-0.233*	-0.382**	-0.482**	-0.145**	-0.187	-0.0600
		(0.139)	(0.194)	(0.226)	(0.0113)	(0.387)	(0.0572)
age		0.0195*	0.0266**	0.00594	0.0112**	0.181**	0.00974
		(0.0110)	(0.00706)	(0.00832)	(0.00221)	(0.0919)	(0.0123)
age^2		-0.000259**	-0.000329**	-0.0000389	-0.000122**	-0.00230**	-0.000106
		(0.000127)	(0.0000867)	(0.000106)	(0.0000260)	(0.00109)	(0.000137)
N	2075	2075	2075	1827	1665	1841	1571
R^2	0.004	0.066	0.115	0.168	0.132	0.005	0.015

Notes: Robust standard errors in parentheses (* $p < 0.10$, ** $p < 0.05$). Specifications 3 to 7 control for a full set of education, industry, occupation and area fixed effects. Specification 4 is restricted to individuals with positive (future) income growth only, while specification 5 controls for outliers by not considering individuals with extreme income growth. Specification 7 conditions on individuals reporting non-zero capital income growth.

Table 3: Predictive Power of Income Expectations

	(1)	(2)	(3)	(4)	(5)	(6)
	$E^I[Y\|Q]$	$E^I[Y\|Q]$	$E^I[Y\|Q]$	$E^I[Y\|Q]$	$E^I[Y\|Q]$	$E^I[Y\|Q]$
log(wages)	0.00611**	0.00342	0.00112	0.00777**	0.00657**	0.00270
	(0.00243)	(0.00281)	(0.00295)	(0.00253)	(0.00255)	(0.00317)
age	0.000332	0.000420	0.000452	0.000201	0.000348	0.000350
	(0.000604)	(0.000606)	(0.000602)	(0.000604)	(0.000603)	(0.000608)
age^2	-0.00000796	-0.00000860	-0.00000945	-0.00000688	-0.00000811	-0.00000841
	(0.00000706)	(0.00000706)	(0.00000705)	(0.00000705)	(0.00000706)	(0.00000708)
	1	2	3	4	5	6
F-Test: Education		1.66 (0.155)				0.46 (0.76)
F-Test: Occupation			6.34 (0.00)			4.16 (0.01)
F-Test: Region				7.56 (0.00)		7.16 (0.00)
F-Test: Sector					1.74 (0.156)	3.23 (0.0214)
N	3196	3196	3196	3196	3196	3196
R^2	0.008	0.010	0.014	0.017	0.009	0.025

Notes: Robust standard errors in parentheses (* $p < 0.10$, ** $p < 0.05$). Specifications 2 to 5 control for education, occupation, area, and sectoral fixed effects, respectively. Specification 6 controls for all different types of fixed effects. The F-tests are on the coefficients of the corresponding variable dummies (the information of Education, Occupation, Region, and Sector is coded in multiple dummies for each variable).

Table 4: Individual Information Sets: Reduced Form Estimates

τ-Quantile:	$E^I[Y\|Q]$	Y
0.05	-0.0175	-0.3267
0.15	0.015	-0.1582
0.25	0.015	-0.0965
0.35	0.0175	-0.0495
0.45	0.0377	-0.0157
0.55	0.04	0.0149
0.65	0.0475	0.0529
0.75	0.0587	0.1048
0.85	0.075	0.1756
0.95	0.1145	0.3845
# of obs.	3196	1755

Table 5: Quantiles of Individuals' (Conditional) Mean Expectations $E^i[Y|Q]$ and Realizations Y_i

$Z_1' = \{ln(Income), Age\}$ $Z_2' = \{Occupation, Area, Education, Sector\}$				
Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% CV	P value
	0.35	5.744	1.735	0.000
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.5	89.646	5.332	0.000
	0.65	4.086	1.557	0.000
	0.35	8.212	1.968	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.5	14.224	1.878	0.000
	0.65	3.628	1.612	0.000
	0.35	4.893	1.694	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^2 + w'\beta_{0,\tau_j}$	0.5	10.227	2.361	0.000
	0.65	3.376	1.602	0.000

Note: The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 6: Basic Income Profile

$Z_1' = \{ln(Income), Age, Occupation, Area\}$ $Z_2' = \{Sector, Education\}$				
Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% CV	P value
	0.35	1.484	2.184	0.164
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.5	1.197	1.665	0.244
	0.65	1.339	1.591	0.136
	0.35	0.926	1.866	0.460
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.5	1.045	1.620	0.408
	0.65	1.136	1.503	0.312
	0.35	1.465	2.061	0.148
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j}$	0.5	1.014	1.707	0.400
	0.65	1.404	1.493	0.064
$Z_1' = \{ln(Income), Age, Sector, Education\}$ $Z_2' = \{Occupation, Area\}$				
Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% CV	P value
	0.35	9.526	2.442	0.000
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.5	5.333	2.041	0.000
	0.65	2.451	1.601	0.000
	0.35	9.996	2.154	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.5	3.868	1.762	0.000
	0.65	2.349	1.598	0.000
	0.35	7.680	2.116	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j}$	0.5	3.193	1.655	0.000
	0.65	1.958	1.497	0.004

Note: The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 7: Sufficient Information Set

Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% CV	P value
$Z_1' = \{ln(Income), Age, Occupation\}$; $Z_2' = \{Area, Education, Sector\}$				
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.35	14.667	2.365	0.000
	0.5	5.963	1.713	0.000
	0.65	2.786	1.573	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.35	12.411	2.145	0.000
	0.5	4.088	1.775	0.000
	0.65	2.580	1.648	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^2 + w'\beta_{0,\tau_j}$	0.35	12.865	2.132	0.000
	0.5	6.063	2.448	0.000
	0.65	2.211	1.569	0.000
$Z_1' = \{ln(Income), Age, Area\}$; $Z_2' = \{Occupation, Education, Sector\}$				
Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.35	3.791	1.651	0.000
	0.5	4.847	1.912	0.000
	0.65	2.306	1.525	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.35	3.242	1.793	0.004
	0.5	2.265	1.855	0.012
	0.65	1.841	1.526	0.004
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j}$	0.35	2.915	1.619	0.000
	0.5	2.335	1.663	0.012
	0.65	2.287	1.445	0.000

Note: The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 8: Testing Robustness of Final Information Set

$Z_1' = \{ln(Income), Occupation, Area\}$ $Z_2 = \{Age\}$				
	Expected Income Growth			
Specification: $k^{\tau_j}_{E^I\|Z}(z)$	Quantile	Statistic	95% Crit. Value	P value
	0.35	0.452	2.740	0.720
$= \gamma_{0,\nu} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.5	6.758	2.713	0.008
	0.65	2.261	2.011	0.020
	0.35	0.552	2.468	0.684
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.5	3.717	2.543	0.008
	0.65	1.881	1.855	0.044
	0.35	0.680	2.705	0.572
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j}$	0.5	5.984	2.367	0.008
	0.65	2.360	1.692	0.000

Note: In the nonlinear specifications, we test with $Z_2' = [\text{age}, \text{age}^2]$ and $Z_2 = [g(\text{age})]$. The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 9: Nonlinearities in the Age Profile

$\{Z_1', V_O\}' = \{ln(Income), Age, Occupation, Area, V\}$ $Z_2' = \{Education, Sector\}$				
(1) $V_O = F_{E^I\|Z}(E^I; Z)$, $Z = \{Z_1\}$				
Specification: $k_y^{\tau_j}(y^*, a, w, v_o)$	Quantile	Statistic	95% CV	P value
	0.35	3.7248	3.8006	0.056
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	9.4809	3.7235	0.000
	0.65	16.2833	3.3557	0.000
	0.35	9.0759	3.751	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	8.8684	3.283	0.000
	0.65	6.3322	3.9692	0.008
	0.35	5.3653	3.2124	0.016
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	10.1637	4.1438	0.000
	0.65	8.0034	3.8275	0.004
(2) $V_O = F_{E^I\|Z}(E^I; Z)$, $Z' = \{Z_1^{c\prime}, Z_2'\}'$				
Specification: $k_y^{\tau_j}(y^*, a, w, v_o)$	Quantile	Statistic	95% CV	P value
	0.35	5.2276	3.8588	0.012
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	9.6453	4.6645	0.000
	0.65	13.1217	3.0385	0.000
	0.35	9.6582	3.7812	0.004
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	8.2847	3.5995	0.004
	0.65	4.1463	3.4879	0.032
	0.35	5.4381	3.7761	0.012
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j} + v_o\beta_{1,\tau_j}$	0.5	8.7521	3.5148	0.004
	0.65	4.8307	3.7125	0.028

Note: $Z_1' = \{ln(income), age, occupation, area\}$, $Z_2' = \{sector, education\}$, and $Z_1^{c\prime} = \{ln(income), age\}$. The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 10: Testing for Costly Information Processing

$Z_1' = \{ln(Income), Age, Occupation, Area\}$ $Z_2' = \{Sector, Education\}$				
Specification: $k_y^{\tau_j}(z)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\tau_j} + \gamma_{1,\tau_j} y^* + \gamma_{2,\tau_j} a + w'\beta_{0,\tau_j}$	0.35	2.529	1.668	0.000
	0.5	3.948	1.674	0.000
	0.65	3.146	1.589	0.000
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(y^*) + \delta_{1,\tau_j} a + \delta_{2,\tau_j} a^2 + w'\beta_{0,\tau_j}$	0.35	2.926	1.531	0.000
	0.5	2.601	1.572	0.000
	0.65	2.498	1.564	0.004
$= \sum_{j=1}^{K} \gamma_{i,\tau_j} p_i(a) + \delta_{1,\tau_j} y^* + \delta_{2,\tau_j} y^{*2} + w'\beta_{0,\tau_j}$	0.35	2.756	1.570	0.000
	0.5	3.782	1.527	0.000
	0.65	2.718	1.492	0.000

Note: The three panels refer to the respective specifications for the quantile function. Column 2 contains the respective quantile we are testing for. Column 3 contains the test-statistic $\hat{\rho}^{\tau_j}$. Column 4 contains the 95% quantile of the bootstrap distribution of the test-statistic. Column 5 contains the P-value of the test statistics. We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 11: Results Using the Data on Income Realizations

Information set Q	σ_T^2	σ_N^2	$E[g_t]$
Income, *Age*, *Occupation*, *Area*	0.0552	0.0145	0.0346
Income, *Age*, *Occupation*, *Area*, *Education*, *Sector*	0.0547	0.0143	0.0345

Note: This Table contains the parameters for the income process as perceived by someone with the respective information set. See Section 4.1 for details.

Table 12: Perceived Income Processes as a Function of the Information Set

Parameter	Value
β	0.94
R	1.02
θ	1
$(G, \sigma_T^2, \sigma_N^2)$	see Table 12

Note: This Table contains parameters for the life-cycle model. See Section 4.1 for details.

Table 13: Parameter Values for Life-Cycle Problem

Quantile of Cash-on-Hand Distribution						Mean
0.1	0.25	0.35	0.5	0.65	0.75	
-0.0407	-0.0389	-0.0389	-0.0389	-0.0413	-0.0395	-0.0388

Note: This Table contains the utility loss of coarse information for different quantiles of the distribution of cash-on-hand. See Section 4.2 for details.

Table 14: Willingness to Pay for Information

Appendix II

Proof of Theorem 1: For simplicity, assume that $(V, I) \perp Z$. Note first that

$$k^v_{F^I_y|Z_1,Z_2}(z_1, z_2) = k^v_{F^I_y|Z_1}(z_1)$$

for (almost) all (z, v, y), if and only if,

$$F_{F^I_y|Z_1,Z_2}(y^v; z_1, z_2) = F_{F^I_y|Z_1}(y^v; z_1),$$

where y^v is the value associated with the v quantile. Next, observe that

$$\begin{aligned} F_{F^I_y|Z_1,Z_2}(y^v; z_1, z_2) &= \int 1\{\theta < y^v\} F_{F^I_y|Z}(d\theta; z_1, z_2) \\ &= \int 1\{\varphi(z_1, z_2, v; y) < y^v\} F_V(dv). \end{aligned}$$

Finally, by the logical negative of Assumption 1, we conclude that $F_{F^I_y|Z_1,Z_2}(y^v; z_1, z_2) = F_{F^I_y|Z_1}(y^v; z_1)$ everywhere implies that $\varphi(Z_1, Z_2, V; y)$ has to be trivial in Z_2 for the population (that there does not exist a nonzero probability set such that it is non-trivial). *Q.E.D.*

Proof of Theorem 2: In the following, denote $P[Y < y_j|Z_1, Z_2, V_1, .., V_J] = \gamma(y_j, Z_1, Z_2, V_O)$. Start out by observing that

$$k^{\tau}_{Y|Z_1Z_2V_O}(z_1, z_2, v_O) \neq k^{\tau}_{Y|Z_1V_O}(z_1, v_O) \tag{5.1}$$

on a positive probability set, iff

$$\gamma(y; z_1, z_2, v_O) \neq \gamma(y; z_1, v_O), \tag{5.2}$$

on a positive probability set. Then, because

$$\begin{aligned}
\gamma(y; z_1, z_2, v_O) &= P[Y < y|Z_1 = z_1, Z_2 = z_2, V_O = v_O] &(5.3)\\
&= \int P[Y < y|Z_1 = z_1, Z_2 = z_2, V_O = v_O, S = s]\, F_S(ds), &(5.4)
\end{aligned}$$

it also has to be the case that $P[Y < y|Z_1 = z_1, Z_2 = z_2, V_O = v_O, S = s]$ is a nontrivial function of z_2 on a positive probability set. This is equivalent with saying that $P[Y < y|Q_1 = q_1, Z_2 = z_2]$ is a nontrivial function of z_2, i.e., Z_2 is $(F, [Q_1, Z_2])$ valuable. Due to Assumption 2 this implies that Z_2 is also $(F^I, [Q_1, Z_2])$ valuable, a potential contradiction, if we found in the previous section that Z_2 is not being used conditional on Q_1 and that information processing is costless. *Q.E.D.*

Supplementary Material - Not For Publication

Table 15: Full Results of Table 6 - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age	sector, education, area, occupation	0.35	5.745	1.735	0.000
		0.45	3.902	1.531	0.000
		0.55	55.124	5.999	0.000
		0.65	4.086	1.557	0.000
		0.75	3.130	1.556	0.000
		0.85	3.111	1.697	0.000
		0.95	1.514	1.956	0.172
income, age	sector, education, area, occupation	0.35	8.212	1.968	0.000
		0.45	3.080	1.479	0.000
		0.55	20.027	2.655	0.000
		0.65	3.628	1.612	0.000
		0.75	2.796	1.655	0.000
		0.85	2.690	1.564	0.000
		0.95	2.264	2.036	0.016
income, age	sector, education, area, occupation	0.35	4.893	1.694	0.000
		0.45	4.054	1.536	0.000
		0.55	51.252	4.412	0.000
		0.65	3.376	1.602	0.000
		0.75	2.945	1.616	0.000
		0.85	3.190	1.637	0.000
		0.95	1.597	2.180	0.188

Table 16: Full Results of Table 7 (1st part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation, area	sector, education	0.35	1.484	2.184	0.164
		0.45	0.903	1.532	0.624
		0.55	1.536	1.708	0.092
		0.65	1.339	1.591	0.136
		0.75	1.376	1.592	0.108
		0.85	1.412	1.593	0.124
		0.95	1.099	2.058	0.328
income, age, occupation, area	sector, education	0.35	0.926	1.866	0.460
		0.45	0.745	1.520	0.804
		0.55	1.270	1.584	0.168
		0.65	1.136	1.503	0.312
		0.75	0.923	1.638	0.556
		0.85	1.175	1.551	0.252
		0.95	1.644	2.086	0.100
income, age, occupation, area	sector, education	0.35	1.465	2.061	0.148
		0.45	0.804	1.508	0.752
		0.55	1.241	1.651	0.208
		0.65	1.404	1.494	0.064
		0.75	1.072	1.578	0.368
		0.85	1.287	1.543	0.168
		0.95	0.894	2.211	0.468

Table 17: Full Results of Table 7 (2nd part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, sector, education	area, occupation	0.35	9.527	2.442	0.000
		0.45	2.681	1.633	0.000
		0.55	6.160	1.991	0.000
		0.65	2.451	1.601	0.000
		0.75	1.948	1.584	0.008
		0.85	2.197	1.626	0.008
		0.95	1.837	2.224	0.096
income, age, sector, education	area, occupation	0.35	9.996	2.155	0.000
		0.45	2.623	1.505	0.000
		0.55	5.160	1.864	0.000
		0.65	2.349	1.598	0.000
		0.75	1.874	1.538	0.004
		0.85	1.892	1.483	0.008
		0.95	2.378	1.977	0.008
income, age, sector, education	area, occupation	0.35	7.680	2.117	0.000
		0.45	2.852	1.682	0.000
		0.55	4.674	1.775	0.000
		0.65	1.958	1.497	0.004
		0.75	1.826	1.496	0.008
		0.85	2.070	1.707	0.008
		0.95	1.227	2.154	0.288

Table 18: Full Results of Table 8 (1st part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation	area, education, sector	0.35	14.668	2.365	0.000
		0.45	3.402	1.702	0.000
		0.55	12.354	2.761	0.000
		0.65	2.786	1.573	0.000
		0.75	2.816	1.590	0.000
		0.85	2.601	1.565	0.000
		0.95	2.274	1.939	0.024
income, age, occupation	area, education, sector	0.35	12.411	2.145	0.000
		0.45	2.648	1.544	0.000
		0.55	9.387	2.211	0.000
		0.65	2.580	1.648	0.000
		0.75	2.295	1.540	0.000
		0.85	2.677	1.633	0.000
		0.95	2.710	2.057	0.008
income, age, occupation	area, education, sector	0.35	12.865	2.132	0.000
		0.45	2.721	1.597	0.000
		0.55	4.647	1.998	0.000
		0.65	2.211	1.569	0.000
		0.75	2.157	1.585	0.000
		0.85	2.417	1.542	0.000
		0.95	1.344	2.373	0.204

Table 19: Full Results of Table 8 (2nd part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, area	occupation, education, sector	0.35	3.792	1.660	0.000
		0.45	1.791	1.560	0.012
		0.55	2.981	1.664	0.000
		0.65	2.306	1.525	0.000
		0.75	1.805	1.566	0.020
		0.85	2.043	1.571	0.004
		0.95	1.242	2.558	0.232
income, age, area	occupation, education, sector	0.35	3.242	1.793	0.004
		0.45	1.521	1.545	0.064
		0.55	2.729	1.708	0.004
		0.65	1.841	1.526	0.004
		0.75	1.386	1.616	0.096
		0.85	1.623	1.556	0.036
		0.95	1.179	2.147	0.304
income, age, area	occupation, education, sector	0.35	2.915	1.619	0.000
		0.45	1.837	1.511	0.004
		0.55	2.263	1.561	0.004
		0.65	2.287	1.445	0.000
		0.75	1.619	1.597	0.044
		0.85	1.814	1.607	0.012
		0.95	1.009	2.178	0.392

Table 20: Full Results of Table 9 - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, occupation, area	age	0.35	0.452	2.740	0.720
		0.45	2.122	2.277	0.060
		0.55	1.891	2.345	0.124
		0.65	2.261	2.011	0.020
		0.75	3.483	2.453	0.004
		0.85	2.331	2.067	0.016
		0.95	1.208	2.406	0.304
income, occupation, area	age, age^2	0.35	0.552	2.468	0.684
		0.45	2.249	1.891	0.012
		0.55	2.005	2.099	0.064
		0.65	1.881	1.855	0.044
		0.75	1.737	1.648	0.036
		0.85	1.219	1.696	0.240
		0.95	0.910	2.330	0.456
income, occupation, area	g(age)	0.35	0.680	2.705	0.572
		0.45	1.427	1.671	0.116
		0.55	2.784	1.730	0.000
		0.65	2.360	1.692	0.000
		0.75	2.725	1.741	0.000
		0.85	3.552	1.710	0.000
		0.95	4.767	2.612	0.000

Table 21: Full Results of Table 6 - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age	sector, area, education, occupation	0.05	1.627	2.059	0.144
		0.15	4.940	1.770	0.000
		0.25	5.199	1.706	0.000
		0.35	6.352	1.662	0.000
		0.45	8.245	1.551	0.000
		0.55	7.246	1.531	0.000
		0.65	7.132	1.557	0.000
		0.75	7.765	1.707	0.000
		0.85	6.938	1.819	0.000
		0.95	3.534	2.204	0.004
income, age	sector, area, education, occupation	0.05	1.556	2.063	0.156
		0.15	4.419	1.652	0.000
		0.25	4.305	1.571	0.000
		0.35	5.283	1.469	0.000
		0.45	7.577	1.600	0.000
		0.55	6.652	1.510	0.000
		0.65	6.163	1.566	0.000
		0.75	5.528	1.635	0.000
		0.85	4.538	1.724	0.000
		0.95	3.469	2.278	0.004
income, age	sector, area, education, occupation	0.05	1.600	2.158	0.148
		0.15	4.236	1.877	0.000
		0.25	4.716	1.615	0.000
		0.35	6.576	1.506	0.000
		0.45	7.449	1.609	0.000
		0.55	6.315	1.605	0.000
		0.65	5.858	1.503	0.000
		0.75	6.174	1.692	0.000
		0.85	5.533	1.808	0.000
		0.95	3.912	2.354	0.004

Table 22: Full Results of Table 7 (1st part) - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation, area	sector, education	0.05	1.142	2.393	0.364
		0.15	2.278	1.754	0.020
		0.25	2.786	1.851	0.004
		0.35	2.529	1.668	0.000
		0.45	3.202	1.678	0.000
		0.55	2.951	1.573	0.000
		0.65	3.146	1.589	0.000
		0.75	2.953	1.651	0.000
		0.85	3.148	1.806	0.000
		0.95	3.068	2.315	0.004
income, age, occupation, area	sector, education	0.05	1.371	2.276	0.256
		0.15	2.614	1.781	0.008
		0.25	2.925	1.634	0.000
		0.35	2.926	1.531	0.000
		0.45	2.969	1.604	0.000
		0.55	2.254	1.534	0.000
		0.65	2.498	1.564	0.004
		0.75	2.154	1.492	0.000
		0.85	2.560	1.610	0.000
		0.95	2.165	2.273	0.056
income, age, occupation, area	sector, education	0.05	1.962	2.112	0.076
		0.15	2.659	1.746	0.004
		0.25	2.557	1.595	0.000
		0.35	2.756	1.570	0.000
		0.45	2.779	1.550	0.000
		0.55	2.197	1.540	0.004
		0.65	2.718	1.492	0.000
		0.75	2.609	1.618	0.000
		0.85	3.046	1.537	0.000
		0.95	3.227	2.344	0.004

Table 23: Full Results of Table 7 (2nd part) - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, sector, education	occupation, area	0.05	0.876	2.283	0.480
		0.15	1.531	1.650	0.080
		0.25	1.990	1.669	0.008
		0.35	2.259	1.618	0.000
		0.45	2.047	1.587	0.000
		0.55	2.216	1.629	0.004
		0.65	2.605	1.682	0.000
		0.75	1.961	1.598	0.012
		0.85	1.588	1.621	0.064
		0.95	1.930	2.492	0.096
income, age, sector, education	occupation, area	0.05	0.688	2.240	0.636
		0.15	1.389	1.886	0.172
		0.25	1.703	1.617	0.040
		0.35	1.850	1.494	0.000
		0.45	2.160	1.723	0.000
		0.55	1.959	1.565	0.004
		0.65	1.604	1.555	0.032
		0.75	1.370	1.646	0.156
		0.85	0.917	1.602	0.548
		0.95	2.002	1.947	0.040
income, age, sector, education	occupation, area	0.05	0.536	2.014	0.796
		0.15	1.424	1.612	0.116
		0.25	1.673	1.541	0.032
		0.35	1.965	1.491	0.000
		0.45	2.402	1.521	0.000
		0.55	2.416	1.531	0.000
		0.65	1.792	1.496	0.020
		0.75	1.436	1.577	0.108
		0.85	1.362	1.592	0.120
		0.95	1.084	2.377	0.356

Table 24: Full Results of Table 8 (1st part) - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation	sector, area, education	0.05	1.535	2.494	0.160
		0.15	3.085	1.791	0.000
		0.25	4.021	1.638	0.000
		0.35	4.038	1.549	0.000
		0.45	4.633	1.560	0.000
		0.55	4.211	1.653	0.000
		0.65	4.667	1.672	0.000
		0.75	3.871	1.639	0.000
		0.85	3.702	1.601	0.000
		0.95	2.581	2.319	0.032
income, age, occupation	sector, area, education	0.05	1.523	2.107	0.172
		0.15	3.295	1.713	0.000
		0.25	3.687	1.536	0.000
		0.35	4.273	1.617	0.000
		0.45	4.942	1.509	0.000
		0.55	3.125	1.499	0.000
		0.65	3.529	1.569	0.000
		0.75	3.095	1.562	0.000
		0.85	3.025	1.744	0.000
		0.95	2.753	2.017	0.008
income, age, occupation	sector, area, education	0.05	1.559	2.057	0.168
		0.15	2.895	1.888	0.000
		0.25	3.499	1.610	0.000
		0.35	4.271	1.487	0.000
		0.45	4.418	1.497	0.000
		0.55	3.025	1.532	0.000
		0.65	3.931	1.623	0.000
		0.75	3.481	1.543	0.000
		0.85	2.910	1.707	0.000
		0.95	1.980	2.108	0.080

Table 25: Full Results of Table 8 (2nd part) - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, area	sector, occupation, education	0.05	1.825	1.856	0.060
		0.15	4.839	1.656	0.000
		0.25	4.640	1.853	0.000
		0.35	5.938	1.656	0.000
		0.45	7.481	1.610	0.000
		0.55	6.953	1.601	0.000
		0.65	7.280	1.611	0.000
		0.75	8.044	1.794	0.000
		0.85	6.167	1.901	0.000
		0.95	4.557	2.649	0.004
income, age, area	sector, occupation, education	0.05	2.561	2.069	0.020
		0.15	4.233	1.801	0.000
		0.25	4.207	1.735	0.000
		0.35	5.043	1.635	0.000
		0.45	6.302	1.600	0.000
		0.55	6.031	1.590	0.000
		0.65	5.941	1.571	0.000
		0.75	6.356	1.623	0.000
		0.85	4.507	1.674	0.000
		0.95	3.374	2.075	0.012
income, age, area	sector, occupation, education	0.05	1.555	1.959	0.136
		0.15	4.378	1.780	0.000
		0.25	4.115	1.608	0.000
		0.35	5.679	1.566	0.000
		0.45	6.562	1.707	0.000
		0.55	6.172	1.521	0.000
		0.65	5.919	1.594	0.000
		0.75	6.775	1.583	0.000
		0.85	4.838	1.897	0.000
		0.95	1.538	5.625	0.164

Table 26: Full Results of Table 9 - Rational Model

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, occupation, area	age	0.05	0.060	2.869	1.000
		0.15	0.291	1.907	0.968
		0.25	0.777	1.848	0.644
		0.35	0.599	1.885	0.796
		0.45	1.294	1.826	0.244
		0.55	1.436	1.877	0.164
		0.65	1.035	1.894	0.416
		0.75	0.906	1.752	0.516
		0.85	0.747	1.896	0.680
		0.95	2.514	2.744	0.056
income, occupation, area	age, age^2	0.05	0.137	2.449	1.000
		0.15	1.493	1.812	0.128
		0.25	2.796	1.693	0.000
		0.35	3.355	1.771	0.000
		0.45	3.411	1.679	0.000
		0.55	3.072	1.637	0.000
		0.65	2.432	1.763	0.000
		0.75	3.851	1.670	0.000
		0.85	3.642	1.679	0.000
		0.95	8.015	2.500	0.000
income, occupation, area	g(age)	0.05	1.599	2.427	0.144
		0.15	3.958	1.778	0.000
		0.25	4.488	1.765	0.000
		0.35	5.620	1.713	0.000
		0.45	7.405	1.708	0.000
		0.55	9.782	1.606	0.000
		0.65	8.217	1.771	0.000
		0.75	8.477	1.613	0.000
		0.85	9.335	1.798	0.000
		0.95	4.282	2.945	0.000