The Roles of Visuospatial and Verbal Working Memory in Children's Mathematical Performance

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BOSTON COLLEGE Lynch School of Education

Department of Counseling, Developmental, and Educational Psychology

Applied Developmental and Educational Psychology Program

THE ROLES OF VISUOSPATIAL AND VERBAL WORKING MEMORY IN CHILDREN'S MATHEMATICAL PERFORMANCE

Dissertation

by

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Abstract

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The ability to mentally store and manipulate information, termed working memory (WM), is essential to mathematical performance. Yet, little research has investigated the mechanisms through which WM capacity is related to mathematical performance in children. Furthermore, the extent to which children utilize specific WM resources when executing mathematical tasks is poorly understood. Addressing these gaps, this research investigated the nature of relations between WM and children's mathematical performance.

Participants were 56 second and 32 fourth grade students from public and private elementary schools. Students completed tasks measuring their visuospatial-WM and verbal-WM capacities, the strength of their spatial and symbolic representations of numerical magnitude, and arithmetic performance. Arithmetic strategies were also assessed. In a dual-task condition, children's WM resources were experimentally manipulated: children completed mathematical tasks while retaining visuospatial and verbal stimuli in memory.

Results showed that the relation between visuospatial-WM capacity and arithmetic accuracy was mediated by children's spatial numeric representations and the frequency of using a decomposition strategy in solving arithmetic problems. Conversely, the relation between verbal-WM and arithmetic accuracy was mediated by the frequency of using retrieval strategies in solving arithmetic problems. Additionally, the extent to which specific WM resources were involved in children's math performance varied by math task and children's skill level. Verbal-WM resources appeared to be minimally involved in children's spatial numerical representations, but highly involved in symbolic numerical representations and arithmetic calculations. On all math tasks, visuospatial-WM resources were involved to a greater extent among highly skilled children than lowskill children.

The results suggest that WM capacity might improve spatial numerical representations and lead children to use memory-based arithmetic strategies more frequently, resulting in better arithmetic performance. Regardless of WM capacity, children who heavily use visuospatial-WM resources are more successful in executing mathematical tasks than children who rely on these resources minimally. These findings contribute to our understanding of how WM can facilitate children's mathematical performance. Implications for identifying specific challenges in children's mathematical learning are discussed.

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Chapter 1: Introduction

Among a number of socio-emotional and academic predictors of educational achievement, meta-analytic results point to early math achievement as one of the strongest predictors of long-term achievement in both math and reading (Duncan et al., 2007). Yet, national and international assessments of American students' math performance have raised concerns among educators and policy makers, with some fearing that the nation risks losing its status as a financial and economic leader (National Mathematics Advisory Panel, 2008). Recent national test results show that the math performance of elementary school students needs improvement: in all but four states, less than half of fourth-graders scored at or above proficiency, with at least one quarter of students scoring below the basic level in seven states (National Center for Education Statistics, 2011). Moreover, international comparisons of student mathematical performance show that even the highest performing American fourth-graders continue to trail behind their counterparts in nine other countries (Mullis, Martin, Foy, & Arora, 2012). One approach to addressing this problem is to uncover the cognitive skills that play a key role in children's math learning and performance. Doing so would provide a focal point for educators seeking to identify and overcome barriers to students' success in math.

A growing body of literature has taken such an approach, identifying two cognitive abilities as integral to mathematical learning and achievement. One of these cognitive abilities, working memory, is a domain-general limited-capacity system that involves simultaneously encoding, manipulating, and recalling several units of interrelated task-specific information (Baddeley & Hitch, 1974; Daneman & Carpenter, 1980). Associational studies show that working memory capacity is strongly related to mathematical ability in elementary school, even after accounting for IQ (Alloway, 2009; Alloway & Alloway, 2010), verbal ability (Alloway & Passolunghi, 2011), and other executive functions (Bull & Scerif, 2001). Moreover, findings from experimental studies conducted with adults suggest that working memory resources are heavily involved in performing mental arithmetic calculations (DeStefano & LeFevre, 2004; Imbo & LeFevre, 2010; Imbo, & Vandierendonck, 2007a. 2007b; Imbo, Vandierendonck, & Vergauwe, 2007; Trbovich, & LeFevre, 2003).

A second cognitive ability—number sense—is thought to provide a foundation for the development of mathematical skills and knowledge. Although there is no single recognized definition for the term "number sense", it has been described as something that "constitutes an awareness, intuition, ...conceptual structure, or mental number line" (Berch, p.1, 2005). One aspect of number sense that has received much attention in the literature on children's early mathematical development involves associating spatial and symbolic representations of numbers with numerical magnitudes or with quantities of events and objects (Dehaene, 1992; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler and Booth, 2004). Measurement of this aspect of children's number sense has been useful in identifying children's risk for poor math achievement (e.g., Gersten, Jordan, & Flojo, 2005) and implementing early interventions (e.g., Griffin, Case, & Siegler, 2004). For example, this aspect of number sense has been identified as a predictor of school children's development in arithmetic calculations and word problem solving (Fuchs et al., 2010a; Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010b; Griffin et al., 1994), learning to estimate solutions to arithmetic problems (Booth & Siegler, 2008), and growth in a standardized measure of math achievement (Geary, Hoard, Nugent, & Bailey, 2012).

While the literature suggests that number sense and working memory may be essential to children's mathematical performance, the prevalence of deficits in both of these cognitive abilities is cause for concern. In a sample of over 3000 five- to ten-year old children from British mainstream classrooms, 10% demonstrated working memory impairments on a standardized measure called the Automated Working Memory Assessment (AWMA; Alloway, 2007); among these children, 70% demonstrated mathematical difficulties (Alloway, Gathercole, Kirkwood, & Elliot, 2009). Thus, strong evidence that working memory and number sense play a role in children's math performance, combined with a troubling percentage of children who demonstrate difficulties with these aspects of cognition, beg a more thorough understanding of how these variables are related.

To date, despite the fact that a number of associational studies have examined working memory and number sense in relation to arithmetic performance (e.g., Fuchs et al., 2010a, 2010b; Geary, 2011; Geary et al., 2007, 2009b, 2012), there has been limited investigation of potential mechanisms underlying these relations. Additionally, only a few studies have taken an experimental approach to investigating the involvement of working memory resources in children's arithmetic performance (Imbo, & Vandierendonck, 2006; McKenzie, Bull, & Gray, 2003), and none of the existing experimental investigations have examined working memory involvement in children's number sense. The present research addressed these limitations in two parts, by (a) examining possible mechanisms underlying the relation between working memory capacity and arithmetic performance and (b) examining the extent to which working memory resources are involved in children's number sense and arithmetic performance. With regard to the first part, children's number sense and arithmetic problem solving strategies were examined as mediators of the relation between working memory capacity and arithmetic performance. Regarding the second part, working memory resources were manipulated while children completed tasks measuring their number sense and arithmetic calculations, in order to compare working memory involvement for each type of math task and as function of children's grade and skill level.

Chapter 2: Literature Review

Working memory is considered to be foundational to children's mathematical learning and performance (Geary et al., 2012; LeFevre, DeStefano, Coleman, & Shanahan, 2005; Raghubar, Barnes, & Hecht, 2010). However, the mechanisms underlying the relation between working memory capacity and mathematical performance are not well understood. Furthermore, despite a large body of research examining the involvement of working memory resources in adults' math performance, a parallel body of investigations with children has yet to be developed. To address these limitations in our current understanding of this issue, several bodies of literature will be reviewed. First, literature providing a theoretical framework for studying working memory in relation to math performance and literature on the development of working memory in children will be reviewed. Second, literature on two key elements of children's mathematical development—number sense and arithmetic calculations—is reviewed. Third, the existing literature documenting relations between working memory, number sense, and arithmetic performance will be reviewed. Last, gaps in the literature will be summarized and the questions addressed by this research will be presented.

The Structure and Development of Working Memory in Children

Children's performance in various academic domains is increasingly being interpreted from an information processing perspective, with a particular focus on working memory. However, working memory is a complex cognitive system that has been studied from a variety of theoretical angles. While disagreements regarding the exact definition and structure of working memory have persisted over time, most scholars agree that working memory involves short-term storage of information, executive resources such as allocation of attention, and activation of information stored in long-term memory (Miyake & Shah, 1999).

Research that has investigated the role of working memory in mathematical performance has typically adopted the classic Baddeley and Hitch (1974, 1994) multicomponential model (see Figure 1). According to this model, there are three primary components of working memory, each with limited capacities for storing and processing information. The multi-componential model postulates that there are two short-term storage components: one encodes verbal information and the other encodes visuospatial information (referred to as the phonological loop and the visuospatial sketchpad, respectively). The third component, termed the central executive, is said to be responsible for manipulation of information on complex tasks, through controlled attention and inhibition of irrelevant information (Baddeley, 1996). A recent addition to the model is the episodic buffer, which is thought to tap information stored in long-term memory in order to facilitate storage and executive processes (Baddeley, 2000, 2012; Baddeley, Allen, & Hitch, 2011).



Figure 1. A current version of the Baddeley and Hitch multi-componential model of working memory. Adapted from "Working memory: Theories, models, and controversies," by A. Baddeley, 2012, *Annual Review of Psychology, 63*, p.16. Copyright 2012 by Annual Reviews.

Working memory, as defined by the multi-componential model, develops substantially across childhood (Gathercole, 1999; Gathercole, Pickering, Ambridge, & Wearing, 2004; Palmer, 2004; Pickering, 2001), with adult levels of capacity reaching two to three times that of young children (Cowan & Alloway, 2009). Specifically, Gathercole and her colleagues (2004) sampled over 700 children, ages 4 to 15, from rural and urban schools in the United Kingdom. Children completed the Working Memory Test Battery for Children (Pickering & Gathercole, 2001) and the Visual Patterns Test (Della Sala, Gray, Baddeley, & Wilson, 1997). Factor analysis suggested that the basic structure of working memory—comprised of the phonological loop, visuospatial sketchpad, and central executive (Baddeley & Hitch, 1974, 1994)—is present and stable from age 6. That is, the relation between distinct components of the working memory system was not found to change with age. However, the functioning of each component of working memory was found to improve linearly with age, beginning at age 4 and leveling off between the ages of 14 and 15. Therefore, while the structure of working memory is in place early on, working memory capacity continues to expand across childhood, making it easier to store and manipulate information with development.

It has also been argued that observable increases in working memory capacity may be due, in part, to the acquisition of knowledge that is relevant to information or stimuli measured on tests of working memory capacity. Support for this general argument comes from a classic study conducted by Chi (1978). Since knowledge and age are often confounded, Chi measured child chess experts' and adult novices' retrieval of spatial arrangements of chess pieces, allowing age and knowledge to be disentangled. As expected, child chess experts recalled more arrangements than adult novices, and, in a separate condition, required fewer training trials to memorize these arrangements. Conversely, adults performed better than children on a digit recall task, presumably because they had more numerical experience than children. These findings, which were later replicated with a larger sample (Schneider, Gruber, Gold, & Opwis, 1993), suggest that knowledge or familiarity with stimuli might facilitate one's ability to recall those stimuli. In summary, the structure and development of working memory in children has been widely studied, both theoretically and empirically. While working memory theorists are divided in their views on the mechanisms underlying individual variations in working memory capacity (Cowan & Alloway, 2009), the Baddeley and Hitch (1974, 1994) multicomponential model has been applied somewhat uniformly across investigations of working memory in relation to children's mathematical performance. Accordingly, the present research examines working memory through the lens of the Baddeley and Hitch model.

Mathematical Development in Children

One of the foundational mathematical competencies children acquire is that of arithmetic calculations. However, this development depends on children's ability to represent numerical information in a manner that is conducive to performing mathematical operations. This understanding of the formal systems used to represent numbers—often referred to as number sense—is rooted in children's innate capacity for making sense of quantitative information. Thus, children's early mathematical development is characterized by a transition from their native concepts of quantity to learned representations of numerical magnitudes, and finally, to the acquisition of arithmetic skills and knowledge.

The measurement and development of number sense in children. Unlike working memory, which is required for executing a range of cognitive processes, the utility of number sense is specific to the domain of mathematics. This important skill, which develops during early mathematical learning, involves mapping quantities onto symbolic and spatial representations of those quantities. Whereas the ability to represent approximate quantities has been documented even in infants (Xu & Spelke, 2000), this early capacity for representing quantity must develop into a more precise and structured system in order for it to facilitate math problem solving (Butterworth, 2005). This transition is difficult for children, as Geary (2006) suggests, because they must adapt their innate system for representing general amounts to a system in which they map number words onto specific quantities (Gallistel & Gelman, 1992).

Number sense is thought to have a range of applications in formal mathematics. As Berch (2005) suggests, "possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information" (p.1). Thus, the term "number sense" has encompassed, often inconsistently, a variety of basic numerical competencies and has been measured using a range of instruments. When measured with certain instruments, number sense has been found to begin developing early in childhood, even prior to children's memorization of specific number facts, and these competencies continue to develop through elementary school (e.g., Geary et al., 2012).

Two instruments that have emerged as particularly useful in capturing children's number sense in relation to their math learning and achievement are the number sets task and number line estimation task. The number sets task measures a child's ability to rapidly apprehend small quantities of objects and symbolic representations of quantity (Geary et al., 2007; Geary, Bailey, & Hoard, 2009a). Two types of stimuli are used in the task: squares containing small quantities of objects and squares containing an Arabic numeral. Several pairs of these stimuli are displayed, and the sum of each pair must be discerned as either a match or a non-match to a target number (see part E of the Appendix). Children's performance on this task has been found to improve with age. In a prospective longitudinal study, Geary and colleagues (2012) measured children's performance on the number sets task annually beginning when the children were in first grade, and ending when they were in fifth grade. Consistent linear growth was observed across the five measurements, indicating that children's ability to rapidly process symbolic representations of quantity improves across the elementary school years.

The second measure, the number line estimation task, is recognized as instrumental in measuring children's spatial representations of numerical magnitude (Dehaene, 1992; Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2008). The task involves estimating the location of a target number on a horizontally displayed number line, typically with marked endpoints. One empirically supported perspective is that, while young children can use their ability to represent approximate magnitudes to estimate the locations of target numbers on the number line, the distances between smaller magnitudes are overestimated, while the distances between larger magnitudes are underestimated (Siegler & Booth, 2004). At this stage, children's estimates are best represented by a logarithmic function. However, with numerical experience, children's estimates of small and large values become more accurate, whereby distances between smaller values begin to approximate distances between larger values, and thus become consistent with a linear function.

The transition of children's estimates from a logarithmic to a linear function, according to one theoretical perspective, represents a shift in children's underlying mental representation of number magnitudes (Siegler & Opfer, 2003; Siegler & Booth, 2004; Booth & Siegler, 2006, 2008). Others have argued, however, that this transition is better explained by improvements in children's proportional judgment, as children estimate magnitudes in relation to reference points on the number line, such as the start, halfway, and end points (Barth & Palladino, 2011). Regardless of the underlying developmental mechanism, the number line task has become a widely used tool to capture children's spatial mapping of numbers (Dehaene, 1992, 1997).

In summary, despite some inconsistency in the definition and measurement of number sense, the number sets and number line estimation tasks have often been used to capture numerical skills and knowledge central to the construct. Furthermore, these two tasks have become commonplace in investigations of the development of children's number sense in relation to their math performance during elementary school. Thus, number sense, in the present study, is the term used to encompass the skills and abilities measured by these two tasks.

The development of arithmetic performance in elementary school. Arithmetic calculations provide the basis for children's mathematical learning in elementary school. Though it is considered one of the most basic mathematical skills one can develop, the importance of mastering these skills is critical. As Geary (2006) discusses, performing

arithmetic calculations involves both conceptual and procedural knowledge. Though there is disagreement about which of these two types of knowledge precedes or informs the other (Gelman & Williams, 1998; Siegler & Crowly, 1994; Sophian, 1997), there is some evidence to suggest that acquiring both types of knowledge is an iterative process (Rittle-Johnson, Siegler, & Alibali, 2001), and it is generally agreed that both play a role in the development of arithmetic performance (Geary, 2006).

Conceptual knowledge of addition, for example, involves the properties of commutativity and associativity, both of which state, broadly, that the order in which two or more numbers are added does not change their sum. Knowledge of these properties is central to solving addition problems, as it encompasses "an understanding that numbers are sets composed of smaller-valued numbers that can be manipulated in principled ways," (Geary, 2006, p. 789). An understanding of commutativity is seen in children as early as the age of four, through their manipulation of objects. At this age, children have shown that they understand that sets of objects, such as toys, can be broken down into smaller groups of objects and recombined; regardless of how this decomposition and regrouping is performed, they know that the total number of objects stays the same (Canobi, Reeve, & Pattison, 1998, 2002). Thus, children may begin acquiring an understanding of arithmetic principles before they are formally educated on arithmetic computations.

Procedural knowledge, on the other hand, consists of an understanding of the steps involved in solving arithmetic problems; that is, it includes skills in executing strategies in order to arrive at the solution for a given problem. Early procedural knowledge of arithmetic is manifested through children's ability to add small groups of objects by counting them (Fuson, 1982), and by kindergarten, children are typically capable of adding two numbers by counting both addends in their entirety, either on their fingers or verbally (Siegler & Shrager, 1984). Children begin to execute more efficient counting strategies around this stage, such as starting at one of the two addends and counting the other addend to reach the sum (counting on), as opposed to counting both addends in their entirety (counting all). Two types of strategies can be used to count on: the max strategy involves counting on from the smaller of the two addends (less efficient) and the min strategy involves counting on from the larger of the two addends (more efficient).

As children become well versed in counting, they begin to commit addition sums to memory, which facilitates their use of memory-based strategies such as decomposition and retrieval (Siegler & Shrager, 1984). Decomposition involves breaking a problem down into a series of smaller problems that are easier to solve or for which the solutions can be directly retrieved. For example, to solve 12 + 14 = 26, one might break the problem down into 2 + 4 = 6, 10 + 10 = 20, and 6 + 20 = 26. Executing a more sophisticated strategy such as decomposition generally reflects stronger conceptual knowledge of arithmetic, such as the property of commutativity (Canobi et al., 1998).

It is important to note that children's transition from counting strategies to memory-based strategies is not strictly linear. Instead, as Siegler's overlapping waves theory (1996) suggests, children may rely on a variety of strategies to solve a given problem early on, but their learning is marked by finding more effective ways of solving problems, making progressively adaptive choices among the various strategies available to them, and becoming more efficient in executing alternative problem solving strategies. With practice, the development of conceptual knowledge, and exposure to formal instruction, children become increasingly adept at executing memory-based strategies across the elementary school period (Geary et al., 2012), showing substantial growth in arithmetic performance during this time.

Thus, it is clear that children's problem solving strategies are an important aspect of their arithmetic development. In fact, children's selection of certain strategies is predictive of their overall arithmetic performance (Carr & Alexeev, 2011). For example, children who frequently use memory-based strategies to solve arithmetic problems typically show higher levels of math achievement than children who use counting strategies (Carr & Alexeev, 2011; Geary, Hoard, Byrd- Craven, & DeSoto, 2004). Given the significance of children's selection of strategies, the present study assessed children's arithmetic performance not only via the accuracy of their calculations, but also through the frequency with which they executed counting, decomposition, and retrieval strategies.

Relations Between Working Memory, Number Sense, and Arithmetic Performance

As is evident from the literature reviewed above, working memory, number sense, and arithmetic performance have been studied extensively in children. Presented below are some of the key investigations in a large body of literature that has revealed the important contributions of working memory capacity and number sense to children's arithmetic performance. Specifically, existing literature that has investigated links between pairs of all three constructs is reviewed. Some of these investigations have examined relations of working memory capacity and number sense to arithmetic performance in the same analysis, estimating the unique contributions of each of these factors when holding the other constant. Few studies, however, have examined working memory capacity and number sense in relation to each other, resulting in an incomplete understanding of the roles that both factors play in the development of mathematical skills. Thus, the question of how working memory capacity and number sense are used during mathematical problem solving remains largely unanswered.

Working memory capacity and number sense. Evidence that working memory capacity is related to the development of number sense has come, primarily, from studies of children with severe mathematical difficulties. For example, Geary and colleagues (2007) sampled nearly 300 children of average intelligence, some of whom had a mathematical learning disability and some of whom showed typical levels of math achievement. The authors measured children's number sense using the number line and number sets tasks, and measured their working memory capacities. They found that visuospatial and central executive working memory capacities were strongly related to children's performance on both the number line and number sets tasks. A few years later, these findings were replicated by Fuchs and her colleagues (2010b). Thus, there is some evidence to suggest that working memory capacity is related to children's foundational numerical representations, although the possible mechanisms underlying this relation have not yet been identified. Nonetheless, an important conclusion that can be drawn from the existing findings is that both factors should be considered contemporaneously in relation to children's math performance.

Number sense and arithmetic performance. While there has been little research linking working memory capacity to number sense, many investigations of the relation between number sense and various mathematical outcomes in children have been conducted, some of which statistically controlled for working memory capacity. Specifically, the number sets and number line tasks have been found to predict initial differences and growth in children's math achievement in several short- and long-term longitudinal studies, in which a host of domain general abilities, including working memory capacity, were statistically controlled (e.g., Fuchs et al., 2010b; Geary et al., 2007, 2012; Geary et al., 2009b).

In a longitudinal study of over 200 children, Geary and colleagues (2009a) argued that the number sets task could be useful in identifying children at risk for a mathematical learning disability. After controlling for kindergarten working memory capacity, IQ, and math achievement, children's first grade performance on the number sets task predicted their odds of showing severe mathematical difficulty by grade three. Also important, however, is their finding that typically achieving students' performance on the number sets task was correlated with their math achievement from kindergarten through grade three. Thus, early performance on the number sets task is useful not only in predicting severe mathematical difficulties down the road, but also in predicting future levels of performance in typically-achieving children.

Strong links have also been made between children's number line estimation ability and their mathematical learning (Holloway & Ansari, 2009; Schneider, Grabner, & Paetsch, 2009; Siegler & Booth, 2004; Booth & Siegler, 2008). In fact, there is

compelling evidence that number line estimation ability is causally related to arithmetic learning. In one of four experimental conditions, Booth and Siegler (2008) trained over 100 first graders to learn the answers to a small set of addition problems. Children were asked to say the number that they thought was the answer to each problem within 6 seconds, which required them to retrieve the answer from memory or estimate the solution to the problem. Conditions varied according to the use of a number line to display the magnitudes of problem addends. In one condition, these representations were generated by a computer. In another, the child was asked to generate these representations. In a third condition, the child generated the representations, followed by the computer. In the control condition, children were only shown the addends and asked to provide the sum, without any visuospatial representation. Results indicated that children who received computer-generated visuospatial representations showed the greatest improvement in the accuracy of their responses from pretest to posttest, when compared to children in the other three conditions. In other words, training children to spatially represent problem addends facilitated their addition fact learning.

Based on this literature, the contribution of number sense to children's math performance is substantial, beyond significant contributions from working memory capacity. What remains to be clarified, however, are the mechanisms underlying this relation. For example, if number sense predicts children's arithmetic problem solving performance, then it is possible that number sense might facilitate children's ability to successfully implement efficient problem solving strategies. These nuances have yet to be investigated in a single analysis.

Working memory capacity and arithmetic performance. Like number sense, strong working memory capacity may also be crucial to mastering arithmetic skills (LeFevre et al., 2005). A number of associational studies have investigated relations between working memory capacity and math performance in typically achieving children, although few have focused exclusively on arithmetic performance. Many of these studies have shown that, when controlling for a number of other cognitive factors, math performance in elementary school is related to both verbal (Alloway, 2009; Alloway & Alloway, 2010; Alloway & Passolunghi, 2011; Bull & Scerif, 2001) and visuospatial (Alloway & Passolunghi, 2011; Berg, 2008; Swanson & Beebe-Frankenberger, 2008) working memory capacities. For example, Bull and Scerif (2001) measured children's verbal working memory capacity, in addition to other cognitive abilities, and found that in children ages 6 to 8, verbal working memory capacity was significantly predictive of children's mathematical ability, including arithmetic calculations, above and beyond the variance in mathematical ability that was explained by IQ, reading ability, and inhibitory control. In another study, Berg (2008) found that after controlling for processing speed and short-term memory, visuospatial and verbal working memory capacities both uniquely explained significant variance in the accuracy of arithmetic calculations among students in grades three through six.

Additional evidence of the working memory capacity and math relation comes from associational studies of children with a mathematical learning disability who show lower working memory capacities than their typically achieving peers (Hitch & McAuley, 1991; McLean & Hitch, 1999; Pickering & Gathercole, 2004; Swanson, 1993;

Swanson & Jerman, 2006)—about 1 standard deviation lower, on average (Geary et al., 2004). While these studies have typically measured general math achievement, as opposed to solely focusing on arithmetic calculations, a substantial portion of the achievement measures that they used involved arithmetic calculations. In a meta-analysis of this body of literature, Swanson and Jerman (2006) analyzed 28 studies comparing children with mathematical difficulties to typically achieving children. They found that, across these studies, verbal working memory capacity emerged as the most important predictor of differences between typically achieving children and children with mathematical difficulties, although it has been argued that this may have been due to the way in which working memory capacity was measured in many these studies (Raghubar et al., 2009). Deficits in visuospatial working memory capacity are also shown to be characteristic of children with mathematical difficulties (Geary et al., 2009b; Passolunghi & Cornoldi, 2008). In particular, children who show deficits in visuospatial processing tend to have difficulty with the basic procedural and conceptual aspects of arithmetic calculations (Geary, 1993; McLean & Hitch, 1999; White, Moffitt, & Silva, 1992).

Taken together, studies of typically achieving children and children with severe mathematical difficulties have identified working memory capacity as a key factor in predicting children's math performance. However, despite this persistent finding, the cognitive process through which this relation is manifested is unclear. Thus, an important gap in this literature parallels that which is found in the literature on number sense and math performance in children. It is clear that these factors are useful in identifying children at risk for mathematical difficulties, but without an understanding of the mechanisms underlying these relations, little can be done to improve these children's chances of success. To address this issue, literature pointing to possible mechanisms is reviewed next.

Possible underlying mechanisms. It is not clear from the extant literature how children's working memory capacity is related to arithmetic performance. However, a number of findings point to number sense and arithmetic problem solving strategies as possible factors mediating this relation. While children's number sense and arithmetic strategies have never explicitly been examined as mediators of the relation between working memory capacity and arithmetic performance, bi-variate relations between these factors raise this possibility.

In one of the longitudinal studies of mathematical learning disability conducted by Geary and colleagues (2007), a kindergarten assessment of children's working memory capacity was linked to their addition problem solving strategies in first grade. Specifically, central executive working memory capacity of children with mathematical learning disabilities predicted the frequency of errors they made when using the retrieval strategy to solve simple and complex addition problems. Visuospatial working memory capacity also emerged as a positive predictor of the frequency with which these children used the min strategy to solve addition problems. The min strategy, as opposed to the max strategy, is the most sophisticated of the counting strategies, in which children count on from the larger of the two addends to reach the sum. It is not surprising that children's use of the min strategy was related to their working memory capacity, as children must first compare the relative magnitudes of the two addends and then decide which of the two is the larger number before counting on.

Arithmetic problem solving strategies were also implicated as a potential mechanism when Geary and colleagues (2009b) investigated characteristics of children in different classes of achievement groups, including mathematical learning disability, low achieving, typically achieving, and high achieving. Certain measurements of children's working memory capacity, number sense, and arithmetic problem solving strategies predicted their odds of group membership. Of particular interest is their examination of memory-based problem solving strategies (retrieval and decomposition). Specifically, larger central executive and visuospatial working memory capacities, higher performance on the number sets task, and more frequent use of memory-based problem solving strategies all predicted children's membership in the high-achieving group, as opposed to the typically-achieving group. Similarly, larger capacities of all working memory components, better number sets task performance, frequent use of memory-based strategies, and frequent use of backup strategies (e.g., reverting to a counting strategy when a memory-based strategy failed) all predicted the odds of membership in the low achieving group as opposed to the mathematical learning disability group.

While the findings of Geary et al. (2007, 2009b) do not reflect a direct test of the mechanisms through which working memory capacity is related to math performance, they do implicate number sense and arithmetic strategies as key contenders. Thus, this work provides a foundation on which to explore these processes in children. In addition to this body of literature, there is another strand of investigation that reflects progress

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toward understanding the role of working memory in math performance. Specifically, a large and growing number of experimental studies have investigated the involvement of specific working memory resources in mathematical problem solving. While this work, which is reviewed below, has not yet provided insight into the role of working memory in number sense, it does provide a window into some of the nuances in the relation between working memory and arithmetic performance.

Involvement of Working Memory Resources in Arithmetic Performance

The wealth of associational investigations of working memory capacity and math performance have implications for identifying children who may be at risk for poor math achievement. However, once these children are identified, it is important to understand effective tools for intervention. Experimental investigations of the involvement of resources from specific components of working memory in math performance provide a foundation for developing such tools, as they can reveal some of the cognitive processes utilized during problem solving, possibly illuminating areas of difficulty for children.

It is important to note that examining the capacity of working memory in relation to math performance, versus examining the extent of involvement of working memory resources in mathematical performance, are two different types of investigations. While one's working memory capacity is measured using span tasks, in which scores reflect the total amount of information that one can remember, the involvement of working memory resources in a given task is measured through an experimental manipulation, as described below.

Investigations of working memory involvement in mathematical performance adhere to a dual-task paradigm, whereby demands are imposed on participants' visuospatial, phonological, or central executive working memory resources during their completion of mathematical tasks. Participants are usually given a recall task, such as a set of letters to remember; before recalling the letters, they are given a math task, such as finding the solution to an addition problem. Thus, they have to manipulate information required for calculating the solution to the problem while trying to retain the verbal information presented earlier. Performance during this dual-task condition is compared to conditions in which there is only a single task: either a math problem solving or a recall task. Generally, it has been found that the dual task causes a decrement in both memory and math performance, compared to the single tasks. The magnitude of this decrement reflects the extent to which working memory resources are involved in carrying out the task. This research has illuminated specific areas of cognitive difficulty underlying arithmetic problem solving, but such work remains largely unparalleled in child populations.

While there is an abundance of this literature, it predominantly includes investigations conducted with college-aged adults, providing a limited understanding of the cognition involved in children's arithmetic calculations, especially given the substantial developments in working memory (Gathercole et al., 2004) and mathematics (e.g., Geary et al., 2012) that are seen across childhood. Furthermore, since number sense is understood to be foundational to children's math learning and performance, investigation of working memory involvement in number sense is still needed to clarify
areas of cognitive difficulty in children's foundational numerical understanding. Below is a review of experimental investigations of working memory in adults' and children's math performance.

Working memory involvement in adults' arithmetic performance.

Experimental studies investigating the involvement of adults' working memory resources in their arithmetic performance have revealed differential involvement of specific components of working memory (Baddeley & Hitch, 1974) in various aspects of arithmetic problem solving (reviewed by DeStefano & LeFevre, 2004). In particular, involvement of specific working memory resources seems to differ based on the arithmetic strategies adults use to generate problem solutions. For example, while verbal working memory has a role in retaining problem information during problem solving (e.g., remembering the values to be added or subtracted; Furst & Hitch, 2000), it also seems to play a role in solving arithmetic problems using counting strategies (Ashcraft, 1995). The central executive and visuospatial components, on the other hand, both seem to play a greater role in solving complex arithmetic problems involving carrying and borrowing (Hubber, Gilmore, & Cragg, 2013; Imbo & LeFevre, 2010; Imbo et al., 2007). The central executive has also been implicated in arithmetic fact retrieval (Seitz & Schumann-Hengsteler, 2002).

In one of these studies, Imbo and Vandierendonck (2007) taxed college students' central executive and verbal working memory resources while they solved complex addition and subtraction problems. They also examined participants' selection and implementation of problem solving strategies by implementing a choice/no-choice

manipulation, which involved four conditions. In the "choice" condition, participants were free to solve problems using any strategy of their choosing. In the three "no-choice" conditions, participants were asked to solve the problems using one of three strategies: retrieval (saying the answer that immediately came to mind), decomposition (breaking the problem down into a series of smaller problems), or counting (counting one-by-one to reach the sum or difference). Participants' central executive working memory resources were taxed while they executed these arithmetic strategies by asking participants to determine if an audible tone was low or high, relative to another tone, at random intervals. They engaged their verbal working memory resources by having participants continuously repeat the word "the". Both of these manipulations were implemented while participants completed the math task. They found that central executive working memory was involved in executing all three types of strategies in the "no-choice" conditions, while verbal working memory was involved only in executing decomposition and counting strategies. However, there was no evidence that either component of working memory was involved in selecting strategies in the "choice" condition.

As Imbo and Vandierendonck (2007) discuss, their findings that specific components of working memory are involved in implementing arithmetic problem solving strategies are consistent with other literature. Retrieving the solution to a problem from long-term memory may involve scanning through several candidate answers to select the correct one; the finding that this process involved central executive working memory is consistent with the idea that a key role of the central executive component of working memory is to interact with long-term memory (Baddeley, 1996). However, when individuals are unable to retrieve the solution to an arithmetic problem, they must select and implement heuristics to decompose the problem or to keep track when implementing a counting procedure (Ashcraft, 1995; Logie, Gilhooly, & Winn, 1994). These processes involve allocation of attention and inhibition of irrelevant information, which is another important role played by the central executive component (Baddeley, 1996). With regard to verbal working memory, they argue that their finding is consistent with literature which has shown that verbal working memory, in addition to the central executive, is involved in keeping track of totals when counting up or down to reach the solution to a problem, or keeping track of intermediate results when executing a decomposition strategy (Ashcraft, 1995; Heathcote, 1994; Passolunghi & Siegel, 2004).

In another study, specific components of working memory were found to be differentially involved in adults' performance when solving addition problems presented in vertical versus horizontal formats; these problem formats primed participants to implement different types of problem solving strategies (Trbovich & LeFevre, 2003). Specifically, the visuospatial component of working memory seems to be important for solving problems presented in a vertical format, while the verbal component may play a larger role in solving problems presented in a horizontal format. Therefore, while both visuospatial and verbal working memory play a role in arithmetic problem solving, their roles may vary as a function of problem presentation format, and in turn, the strategy used to solve the problem. As Trbovitch and LeFevre discuss, the presentation of problems in a vertical format may prime individuals to solve the problem using a columnar algorithm, which would require visualization of the problem in this particular

format, hence the use of visuospatial working memory. The presentation of problems in a horizontal format, on the other hand, seems to prime participants to use a decomposition or counting strategy which, as Imbo and Vandierendonck (2007) found, requires resources from verbal working memory.

In addition to variations in working memory involvement in problem solving as a function of problem solving strategies, there is also evidence to suggest that working memory involvement might vary as a function of individual differences, such as how skilled individuals are in arithmetic. This research, which is reviewed next, could have important implications for educators after further investigation. In particular, a parallel body of research would need to be conducted with children.

Individual differences in working memory involvement. Few studies of working memory involvement in adults' math performance have approached this topic from an individual differences perspective. However, an understanding of individual differences in the cognitive underpinnings of mathematical problem solving may provide insights into how children showing difficulty with mathematics can be better served in their mathematics education.

In a dual-task investigation of working memory involvement in addition problemsolving strategies, adult participants were asked to complete a series of single-digit addition problems (Imbo & Vandierendonck, 2010). In the dual-task condition, the central executive component of working memory was taxed using the same procedure the authors used in their 2007 study, described above. Participants' accuracy, response time, and problem solving strategies were recorded. They were also given a separate measure of arithmetic skill, which allowed the investigators to compare working memory involvement in performance between high- and low-skill individuals. They found that high-skill individuals were more adaptive in their strategy selection based on the demands of the task, compared to low-skill individuals. Specifically, when compared to low-skill participants, high-skill participants used retrieval more often in the dual-task than in the single-task condition, especially on problems with sums larger than 10.

Retrieval, of course, is an adaptive strategy choice when there are competing demands on working memory (in this case, demands from the secondary central executive task), as it is less demanding of working memory resources than counting or decomposition strategies. Furthermore, in examining response times, they found smaller effects of central executive load on response times of high-skill individuals than low-skill individuals. Taken in the context of the finding that high-skill individuals chose retrieval more often in the dual-task condition, this finding suggests that high-skill individuals require central executive resources to a lesser extent than low-skill individuals on simple addition problems because they adapted to task demands by selecting the retrieval strategy.

Similarly, Imbo and LeFevre (2009) found that Chinese-educated individuals required central executive working memory resources to a lesser extent than Canadianeducated individuals; importantly, the Chinese-educated participants showed higher levels of arithmetic skill than Canadian-educated participants. However, a year later these authors (2010) obtained contrasting results in another dual-task study, in which they manipulated visuospatial and verbal working memory resources during Chinese- and

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Canadian-educated college students' subtraction and multiplication problem solving. Once again, the arithmetic skill level of Chinese-educated students exceeded that of the Canadian-educated students. However, they found that Chinese-educated college students generally showed the same degree of involvement of visuospatial and verbal working memory as Canadian-educated college students, rather than less involvement.

Taken together, these three studies show some preliminary evidence that while high-skill individuals may require central executive resources to a lesser extent than lowskill individuals, they may require visuospatial and verbal working memory resources to a similar extent. However, the fact that the 2007 and 2009 studies involved addition, while the 2010 study involved subtraction and multiplication, combined with the fact that arithmetic skill level was confounded with cultural background in both studies, makes it difficult to draw any solid conclusions in the differential involvement of these specific components of working memory as a function of skill level.

Summary. It is clear from the findings presented in this section, and a large body of similar investigations (see DeStefano & LeFevre, 2004), that working memory does not operate in a unitary fashion during adults' mathematical problem solving. Furthermore, some studies have suggested that the involvement of working memory in problem solving might vary as a function of individual differences, such as arithmetic skill-level. With respect to children, this issue has not been addressed in depth, and yet it would be important to disentangle how different components of working memory relate to math problem solving in children. This would benefit our theoretical understanding of math reasoning from a developmental perspective and would help identify targets for educational interventions.

Working memory involvement in children's arithmetic performance. To date, only two experimental studies on the involvement of working memory resources in children's math performance have been conducted (Imbo, & Vandierendonck, 2006; McKenzie et al., 2003). These studies have focused on different age groups, and have varied in their manipulation of working memory. Both studies examined individual differences in working memory involvement, based on age (McKenzie et al., 2003) and their level of skill in implementing certain strategies (Imbo & Vandierendonck, 2006).

McKenzie and colleagues (2003) investigated working memory in relation to arithmetic problem solving using a dual-task procedure with Norwegian children ages 6 to 7 and 8 to 9. Children completed two- and three-term single-digit addition problems in three within-subjects conditions: one in which they solved addition problems without interference, another in which they solved addition problems with visuospatial interference, and a third in which they solved addition problems with verbal interference. Visuospatial interference involved a matrix of squares and verbal interference involved a recording of a children's story; children were expected to remember this information while they completed the arithmetic task. In the younger group, visuospatial interference resulted in the greatest decrement to baseline performance, while in the older group, both visuospatial and verbal interference produced drops in performance compared to baseline, although the effects were smaller for older children than for younger children. The authors argued that younger children may rely more on visuospatial strategies to solve arithmetic problems, while older children may employ more verbally-based strategies.

Imbo and Vandierendonck (2006) examined the central executive component of working memory in relation to the addition problem strategies of Belgian children in grades 4 through 6. Addition problems included two single-digit terms with sums greater than 10. In the dual-task condition, children also completed a secondary task that demanded central executive working memory resources. With regard to strategies, half of the children were assigned to a choice condition, in which they were free to solve the problems using a strategy of their choosing. The other half was assigned to a no-choice condition, in which they were required to solve the problem using retrieval, decomposition, or a counting strategy. Consistent with their previous findings (2007), the central executive component of working memory was heavily involved in children's use of all three strategies. However, decomposition and counting strategies were more demanding of central executive working memory resources than the retrieval strategy, perhaps because they require several processes, including manipulation of information, keeping track of counting, and retrieving several arithmetic facts from long-term memory. In examining individual differences, they found that children who were less efficient in executing retrieval and decomposition displayed greater involvement of central executive working memory in problem solving, showing again that the role of the central executive component of working memory might be reduced when participants are more skilled in executing particular strategies.

The Present Research

The purpose of this dissertation is to address gaps in the extant literature by more closely examining the nature of the relation between working memory and arithmetic problem solving in children. While associational studies have shown that the capacities of visuospatial and verbal working memory are related to children's arithmetic performance, the mechanisms underlying this relation remain unclear. However, bi-variate relations from a number of studies have raised the possibility that number sense and arithmetic strategies mediate the relation between working memory and arithmetic performance.

The involvement of working memory resources in children's mathematical performance is also unclear. It is currently unknown how and to what extent working memory resources are involved in children's number sense, despite associational findings that central executive and visuospatial working memory capacities are related to children's number sense. Additionally, the extent of visuospatial and verbal working memory involvement in children's arithmetic performance requires further investigation. Last, individual differences in working memory involvement as a function of children's grade or skill level needs to be examined. Specific limitations of the available literature and the rationale underlying the questions addressed by the present study are discussed below.

Gaps in the literature and underlying rationale. The primary limitation among associational investigations of working memory and math performance in children is that while working memory capacity, number sense, arithmetic strategies, and arithmetic performance have all been linked to each other in this line of work, the nature of interrelations between all three variables is unclear. One of the studies conducted by Geary and colleagues (2007) produced some of the only available evidence that working memory is related to number sense; specifically, that the central executive and visuospatial working memory capacities predict performance on the number line and number sets tasks. Other studies have found children's performance on the number line and number sets tasks to be related to their math achievement (e.g., Geary et al., 2009b; Holloway & Ansari, 2009; Schneider, Grabner, & Paetsch, 2009; Siegler & Booth, 2004; Booth & Siegler, 2008). While these findings, in combination, suggest that number sense might serve as a mediator of the relation between working memory and math performance, this meditational chain has not been explicitly tested.

Additionally, associational investigations of working memory capacity in relation to children's arithmetic problem solving strategies (Geary et al., 2004, 2007, 2009b)—as well as one experimental study of working memory involvement in children's strategies (Imbo & Vandierendonck, 2007)—provide some evidence that arithmetic problem solving strategies might serve as an underlying mechanism. However, this has not been directly examined. Last, the link between number sense and arithmetic strategies remains poorly understood, and yet children with mathematical difficulties often demonstrate poor number sense and execution of arithmetic strategies, raising the possibility that these two factors may not only be related, but also that they may serve as serial mediators of the relation between working memory capacity and math performance.

A few important limitations of the existing experimental investigations of working memory in children should also be addressed. First, of the two available studies

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of working memory involvement in children's arithmetic performance (Imbo & Vandierendonck, 2006; McKenzie et al., 2003), neither study examined the involvement of working memory in children's number sense. However, a link between working memory and number sense has been reported in the associational literature (e.g., Geary et al., 2007). Moreover, given that number sense is understood to be foundational to children's mathematical learning and performance (Berch, 2005), it would be useful to know not only that working memory is involved in math performance, but also whether it is involved in the skills foundational to math performance—namely, number sense.

Second, while the studies conducted by Imbo and Vandierendonck (2006) and by McKenzie and colleagues (2003) showed that central executive, verbal, and visuospatial working memory are all involved in children's single-digit arithmetic problem solving, the involvement of specific working memory components in multi-digit arithmetic problem solving is not yet known. While mastery of simple arithmetic serves as a foundation for developing skills in more complex arithmetic operations, and it is useful to understand how working memory is involved in this basic mathematical skill, understanding the nature of working memory involvement in more difficult problems is important, as these problems may require greater use of procedural strategies, such as decomposition, as opposed to retrieval. In other words, children may memorize the solutions to single-digit addition problems, but they may employ a variety of more cognitively demanding strategies for multi-digit problems.

Third, differences in the involvement of working memory in math problem solving between children with high- versus low-levels of mathematical skill has not been

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directly examined in the extant literature, despite preliminary evidence that highly skilled adults may rely on central executive resources to a lesser extent than low-skill adults (Imbo & Vaniderendonck, 2007; Imbo & Lefevre, 2009). Understanding individual differences in the involvement of working memory would provide a starting point for developing interventions that address cognitive difficulties underlying children's difficulty with mathematics.

Research questions and hypotheses. In response to these limitations in the extant literature, two sets of research questions were addressed.

(1) Do number sense and arithmetic problem solving strategies mediate the relation between working memory capacity and arithmetic performance? Are visuospatial and verbal working memory capacities related to arithmetic accuracy through the same cognitive processes, or are there different mediators for each type of working memory?

To address these questions, assessments of children's visuospatial and verbal working memory capacities, number sense, arithmetic strategies, and arithmetic accuracy were made. These measurements allowed for an explicit test of the prediction that number sense and problem-solving strategies could serve as mediators of the relation between working memory and arithmetic accuracy. This hypothesis was based on a number of bi-variate relations between these variables that have been documented in the literature. First, while the relation between working memory capacity and number sense has rarely been investigated, relations between the visuospatial working memory capacity and both the number line and number sets tasks have been documented (Fuchs et al.,

2010b; Geary et al., 2007). Second, working memory capacity has been linked to children's arithmetic strategies (Geary et al., 2009b). Third, while relations between number sense and arithmetic strategies are poorly understood, the fact that they explain shared variance in children's mathematical performance (e.g., Fuchs et al., 2010b; Geary et al., 2007) suggests that a relation between the two should be directly tested. Fourth, although research has not typically focused exclusively on children's arithmetic performance, number sense is strongly predictive of children's math achievement after controlling for working memory capacity (Fuchs et al., 2010a; 2010b; Geary, 2011; Geary et al., 2009a, 2012). Last, it is well-known that children's arithmetic strategies are an important aspect of their arithmetic performance and development (Carr & Alexeev, 2011; Geary et al., 2004; Geary, 2006). Given this body of evidence, it is possible to hypothesize that number sense and problem solving strategies mediate the relation between working memory and arithmetic performance. However, the literature does not provide enough information to make predictions with regard to the specific measures of number sense and arithmetic strategies that would serve as mediators for visuospatial versus verbal working memory.

(2) To what extents are visuospatial and verbal working memory involved in children's number sense and arithmetic performance? Does the extent of involvement differ by grade level, or for high- and low-performing students?

To address these questions, children's visuospatial and verbal working memory resources were experimentally manipulated in a dual-task condition. This allowed for an examination of the involvement of these resources in children's number sense and multidigit arithmetic performance, as well as a comparison of the extent to which each type of working memory is involved. It was predicted that both visuospatial and verbal working memory would be involved in arithmetic problem solving, based on the findings of McKenzie and colleagues (2003), and on findings from studies conducted with adults (Trbovich & Lefevre, 2003; Imbo & LeFevre, 2010). Although there is some evidence that visuospatial working memory capacity is related to number sense (Geary et al., 2007; Fuchs et al., 2010b), the involvement of working memory resources in number sense has never been examined, and thus this aspect of the question was approached from an exploratory perspective.

To capture individual differences in arithmetic skill, elementary school children in grades 2 and 4 were chosen to participate in this study. Grades 2 and 4 were chosen, specifically, because by grade 2 children have typically received between one and three years of formal education in mathematics, can be expected to have fairly strong number sense (Geary et al., 2012; Laski & Siegler, 2007; Siegler & Booth, 2004), and have had some practice in solving multi-digit arithmetic problems; by grade 4, children have typically developed a very strong number sense (Geary et al., 2012; Siegler & Opfer, 2003) and are fairly skilled at solving multi-digit arithmetic problems. Thus, by selecting children from these two grades, it was expected that a range of mathematical skill and maturity would be captured. This allowed for an examination of variations in working memory involvement as a function of grade or performance level. Since there were inconsistent findings in the literature with regard to variations in working memory involvement by skill level (e.g., Imbo & Vandierendonck, 2007; Imbo & LeFevre, 2009; Imbo & LeFevre, 2010), no specific predictions were made with regard to this question.

Chapter 3: Methods

Participants

Participants were 58 second grade children (26 females) and 32 fourth grade children (15 females) from 3 public and 3 private elementary schools in the northeast. Once permission was granted by school principals, who were initially contacted via email, teachers who agreed to participate were asked to send home consent forms with their students. Children whose parents returned consent forms indicating that they permitted their child to participate were included in the sample. Among these children, only those who have their written informed assent to participate were included. On the assessment of arithmetic performance (described below), two children responded incorrectly to all items. Data for these children were excluded from all analyses (the exclusion of these children is reflected in the numbers reported above).

Measures & Procedure

General study procedure. Children completed all tasks individually with the experimenter in one session lasting approximately 25 minutes. All tasks were completed on a laptop computer using E-Prime 2.0 Professional software (Psychology Software Tools, Inc., 2002) in a quiet space in the child's school. The testing session involved three within-subjects conditions: a dual-task condition (memory recall + math problem solving), a single memory task condition, and a single math task condition. The dual-task condition was designed to demand working memory resources, specifically from either the phonological loop or the visuospatial sketchpad (Baddeley & Hitch, 1974, 1994). That is, while children completed tasks measuring their number sense and arithmetic

problem solving in the dual-task condition, an extra cognitive load was placed on their visuospatial and verbal working memory resources. This procedure was adapted from that used by Imbo and LeFevre (2010) with adults, and is consistent with the dual-task paradigm used in experimental studies of adults' working memory and math problem solving (DeStefano & LeFevre, 2004). After completing the three single task conditions, a brief assessment of children's arithmetic problem solving strategies was administered. After the testing session was complete, children were thanked for their participation and given their choice of a pencil or a sticker as a token of appreciation.

Experimental conditions. Each of the three within-subjects conditions included 16 trials. Every condition began with instructions, followed by 2 to 4 practice trials, depending on the number of unique types of items presented in that condition. Children were given verbal feedback at the end of each practice trial and were invited to ask questions, if they had any. Next, the 16 experimental trials were presented in random order. The content and procedures within each condition are discussed next, and details about the stimuli presented in each condition are discussed in the following section. Information about each condition is also listed in Table 1, and a visuospatial representation of the procedures in each condition is depicted in Figure 2. Before beginning the experimental conditions, children were told the following, *"Sometimes, I will ask you to solve math problems. Other times, I will ask you to remember something. Your job is to pay attention and follow instructions."* At the end of each condition, children were told, *"Good job! Let's move on."*

Table 1

Number of Items i	in Each Condition and Block	

	Memory Stimuli			
	Visuospatial	Verbal	None	
Single Math Task			16	
Addition			4	
Subtraction			4	
Number Line			4	
Number Sets			4	
Single Memory Task	8	8		
Math and Memory Dual Task	8	8		
Addition	2	2		
Subtraction	2	2		
Number Line	2	2		
Number Sets	2	2		

Note. Within each condition, items were presented in random order.



Figure 2. Experimental procedure by condition.

Single math task condition. In this condition, children were presented with math tasks. Note that there was no secondary memory recall task in this condition. Within the

16 trials in this condition, there were four math tasks, each involving four trials, but all trials were presented in a random order, rather than being presented in blocks. Two tasks were designed to measure children's number sense, and the other two tasks were designed to measure arithmetic performance. Of the two number sense tasks, one included number line estimation trials and the other included number sets identification trials; of the two arithmetic performance tasks, one consisted of addition trials and the other consisted of subtraction trials. Each trial consisted of a single math item/problem, with one exception: number line trials involved two number line items because they could generally be solved in half the amount of time as the other types of math tasks.

At the beginning of this condition, children were given the following instructions, which were presented on the computer screen and read aloud by the experimenter: "*Now you will solve some math problems. When you know your answer, say it right away. Let's practice.*" They were then given four practice problems, one representing each type of math task. Following feedback and any questions, the 16 experimental trials were presented. Preceding each trial, the experimenter asked, "*Ready?*" to prepare the child. In each trial, the math item/problem was displayed in the center of the computer screen and the child was prompted to solve the problem. For number sets, addition, and subtraction tasks, the items/problems remained on the screen for 15 s, during which the child was expected to say his or her answer aloud. For the number line task, each trial totaled 15 s, but the two items were displayed for 7 s each, with a 1 s interval between the two items. At the end of each trial, the math item/problem disappeared and a white screen appeared before moving on to the next trial. The child's responses were recorded by the

experimental software; on addition and subtraction items, the experimenter typed in the child's responses, while on number line and number sets items, the child's mouse click responses (described in greater detail below) were recorded by the software.

Single memory task condition. In this condition, children were presented with memory tasks; there was no secondary math task in this condition. There were 16 randomized trials involving two types of memory tasks, of eight items each. One task measured children's visuospatial working memory capacity and the other measured children's verbal working memory. Each trial consisted of a single memory item.

The condition began with these instructions, which were presented on the computer screen and real aloud by the experimenter: "*Now, you will use your memory*. *Sometimes you will see circles. Other times you will hear letters. Your job is to remember them. Let's practice.*" Children were then given two practice trials: one visuospatial working memory trial, and one verbal working memory trial. After feedback was given and any questions were answered, the 16 trials began. Preceding each trial, instructions appeared on the computer screen, which the experimenter read aloud. On verbal working memory trials, the instructions read, "*Remember what you hear*"; for visuospatial trials, the instructions read, "*Remember what you see*". Next, the child was presented with the visuospatial or verbal memory stimuli (described in greater detail in the following section). Visuospatial stimuli remained on the computer screen for 4 s, and audio recordings of verbal stimuli were played on the computer speakers for approximately 5 s. After the stimuli were presented, a white screen appeared, during which the child was expected to wait silently for 15 s. After 15 s, the screen changed. On verbal trials, a gray

screen appeared, and the child was asked say aloud all the letters they remembered in the order that they heard them. On visuospatial trials, a response screen for the visuospatial stimulus appeared, and the child was asked to take the computer mouse and click on the screen locations where they had previously seen the stimuli (described in more detail in the subsequent section). After the child provided a response, the next trial began. The experimental software recorded the child's response on both types of trials; on verbal trials, the experimenter typed in the child's mouse and on visuospatial trials, the software recorded the locations of the child's mouse clicks.

Dual-task condition. This condition combined the procedures and content of the single math task and single memory task conditions, resulting in a dual-task procedure; that is, each trial included a math item/problem embedded in a memory trial. The math items/problems and memory stimuli presented in this condition were parallel to those presented in the single task conditions. The specific math and memory tasks were crossed so that there were four trials for each of the following categories: (1) Arithmetic Math (2 addition, 2 subtraction) + Verbal Recall; (2) Arithmetic Math (2 addition, 2 subtraction) + Verbal Recall; (2) Arithmetic Math (2 addition, 2 subtraction) + visuospatial Recall; (3) Number Sense (2 number line, 2 number sets) + Verbal Recall; (4) Number Sense (2 number sets) + Visuospatial Recall, although the order of items were randomized, as was the case with the single-task conditions.

The condition began with the following instructions, which were presented on the computer screen and read aloud by the experimenter: "*Now you need to use your memory and solve math problems. First, you'll see circles or hear letters. Try to remember them! Then, solve the math problem. Say your answer as soon as you know it. Let's practice.*"

They were then asked to complete two practice problems and ask any questions before the 16 experimental trials began. At the start of each visuospatial trial, a screen appeared with the text "*Remember what you see*"; at the start of each verbal trial, a screen appeared with the text "*Remember what you hear*". Next, a visuospatial or verbal memory stimulus was presented for 4 s. Following the memory stimulus, a math item/problem appeared on the screen for 15 s, during which the child was expected to provide an answer. Note that the math problem in this condition replaced the 15 s blank screen presented in the single memory task condition. After 15 s, the screen changed either to gray or to the visuospatial response slide, depending on the modality of the memory stimuli presented prior to the math problem. The child was asked to recall the memory stimuli at this point, just as they had done in the single memory task condition.

The order of the two single task conditions was counterbalanced. However, the dual-task condition was always presented last: the single task conditions familiarized children with the math and memory procedures, before these procedures were combined into a dual-task procedure. This decision was based on the conclusion that the dual-task would already be quite challenging for children, and should not impose the added burden of trying to understand the memory and math procedures all at once, at the risk of impaired performance.

Arithmetic strategy assessment. After completing the three conditions, the child's arithmetic strategies were assessed. They were given a selection of 8 arithmetic problems that were presented during the experimental procedure, and were given 15 s to solve each problem. On each item, after the child provided a numeric response, the experimenter

asked, "*How did you solve that problem*?" and wrote down the child's reported strategy, as well as any observable behaviors the child displayed during problem solving.

Mathematical tasks, memory stimuli, and scoring. Complete sets of all mathematical tasks and memory stimuli are presented in the Appendix and described below.

Verbal working memory. Verbal working memory items were audio voice recordings of strings of 5 letters (e.g., "FCPDX"), which was adopted from Imbo and LeFevre (2010). Vowels were excluded to preclude children from encoding letter strings as non-words, and there were no repeated letters, making each letter an independent stimulus. Additionally, every consonant in the alphabet was presented at least once. There were two sets of 5-letter strings, one used in the single memory task condition and one used in the dual-task condition.

The goal on each trial was for the child to recall each of the letters in the correct order. Performance was measured in terms of accuracy, which was defined as the proportion of letters (out of five on each item) that the child recalled in the correct order. For example, if the stimuli were "XDHWV" and the child said, "XDHWB", the child's score would be 0.80 (i.e., 4 correct out of 5) for that item; if the child responded, "XHDW", their score would be 0.40 (i.e., 2 correct out of 5) for that item. Children's scores were averaged across 8 items in the single memory task condition, 4 arithmetic items in the dual-task, 2 number line items in the dual-task, and 2 number sets items in the dual-task condition.

Visuospatial working memory. Visuospatial working memory items were

displayed on the computer screen. Stimuli were three black dots (diameter approximately .5 inch) in varying locations within a 6 x 6 matrix. The 36 cells within the matrix had invisible borders, but the outer borders of the matrix were black. Thus, to anyone who viewed them, the visuospatial stimuli appeared as three dots within a white square with a black border. Each visuospatial pattern was constructed so that no dot was in the same column or row as another dot, and so that the three dots never formed a straight line. Therefore, each pattern formed some sort of triangle, but care was taken to ensure that no two triangles were identical. These stimuli were adapted from the 4×4 matrix used by Trbovich and LeFevre (2003). A 6 x 6 matrix was used to provide a wider variety of patterns formed by the three dots. Furthermore, the larger matrix made it possible to limit the number of times dots appeared near the edges or corners of the matrix, because it might have been possible for children to encode the locations of dots near the boarders verbally (e.g., "The dot was near the top left corner"), rather than spatially. This approach allowed for a more pure assessment of visuospatial working memory capacity, rather than an assessment that confounded visuospatial and verbal working memory.

The goal on each trial was for the child to remember the location of each dot. The child responded directly by clicking on the response screen with the mouse. The response screen was identical to the stimulus screen, except that no dots were displayed within the borders of the matrix. For each mouse click the child made on this screen, a black dot centered on the mouse click coordinates appeared and the *x* and *y* coordinates were recorded by the computer. Performance was measured in terms of accuracy, which was defined as the number of dots, out of three on each item, for which the child correctly

recalled the location in which it had been presented. Since it would be virtually impossible to click the exact location of any of the dots, a margin of error was identified for each response. Specifically, this margin of error was defined by the cells that contained the stimulus dots within the $6 \ge 6$ matrix. In other words, the *x* and *y* coordinates were categorically coded as either falling within the area of the correct cell (correct response), or outside of that area (incorrect response). To code the child's responses, the distance between the *x* and *y* coordinates of the child's response and the coordinates of the borders of the correct cell was calculated. If this distance exceeded the dot's radius, in pixels, then the response was coded as incorrect; if the distance was less than the dot's radius, or if the entire dot fell within the area of the correct cell, then the child's response was coded as correct. In other words, the child's click was considered to be incorrect only if no portion of the dot fell within the boundaries of the correct cell.

Number sense. Two types of items were used to measure children's number sense: the number sets task (Geary et al., 2009a) and number line estimation task (Siegler & Booth, 2004). These tasks were chosen for two reasons. First, these are commonly used measures of children's basic number sense. Second, these were the measures used in previous investigations of the relation between working memory and number sense (Fuchs et al., 2010b; Geary et al., 2007).

For number line items, children were asked to estimate where a given value between 0 and 1000 falls on a number line with marked endpoints of 0 and 1000. Each number line item is presented as a white screen displayed on the computer, with a black horizontal line that is placed in the vertical and horizontal center of the screen and that stretches nearly the complete horizontal length of the screen. At the top-center of the screen, the target number to be estimated by the child was displayed in black font. As mentioned previously, two number line items were presented on each 15s trial, for two reasons. First, pilot data suggested that estimation of one value on the number line takes fewer than 15 seconds (approximately 7 seconds). Second, the task was adapted from a number of studies using the number line estimation task (e.g., Booth & Siegler, 2008; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003), which sample approximately 26 values within the scale (e.g., 0 to 1000, 0 to 100, etc.). Since the present study measures other types of math ability and working memory performance, it would not be feasible to ask children to estimate the magnitudes of 26 values within each condition. Instead, 8 were sampled in the single math task condition, and 8 values were sampled in the dual-task condition, with 2 values on each of the four 15 s number line trials in each condition.

Children used the computer mouse to click on the point along the number line where they estimated the target value to be. The x and y coordinates of the pixel location of the mouse click were recorded by the experimental software. Since the number line did not span the entire screen, the width of the number line was subtracted from the width of the screen and this difference was divided by 2 to obtain the width of the space from the left edge of the screen to the left edge of the number line. This value was then subtracted from the x coordinates of the child's responses for each number line item to get the true positions of the child's estimates. Accuracy on the number line task was represented as proportion absolute error (PAE), which was calculated by taking the absolute value of the difference between the child's estimate and the actual target magnitude, then dividing this difference by 1000 (the scale of the number line). Thus, lower PAE represents greater accuracy of estimates.

The number sets task involved a child's ability to identify number sets as symbolized by Arabic numerals and groups of objects (adapted from Geary et al., 2009a). Children were given a target number of 5 or 9 and a set of rectangles containing two or three adjacent squares that shared borders, similar to dominos. Each half (or third) of a domino contained an Arabic numeral or a quantity of small shapes (e.g., circles, stars, diamonds, squares, etc.). The child was asked to determine whether the sum of the quantities represented in each domino matched the target number. Each item contained dominos that matched the target value and dominos that did not match the target value. Some dominos contained only quantities of shapes, some contained only Arabic numerals, and some contained a mix of these two representations. The total quantity displayed on each domino was less than 10. Items with 5 as the target number displayed 9 dominos, 4 of which matched the target; items with 9 as the target number displayed 6 dominos, 3 of which matched the target. The x and y coordinates of the child's mouse clicks were recorded by the computer. Each click was matched to a particular domino, based on the x and y coordinates of the outside borders of the domino. A small margin of error was allowed when coding children's responses as correct or incorrect, in case the child was imprecise in responding. The margin of error was defined as 50% of the vertical or horizontal distance between the domino in question and the adjacent dominos, so that whichever domino the click was nearest to was the one that was used to represent

the child's selection. Accuracy was defined as the proportion of hits on the item, out of 4 possibilities on items with a target of 5, and out of 3 on possibilities on items with a target of 9.

Arithmetic problem solving. The single math task and dual-task conditions each contained 4 addition and 4 subtraction problems. Each problem included one single-digit value and one double-digit value under 100. Doubles (e.g., 44, 88, etc.) were excluded. Accuracy, defined as a correct response to the problem, was coded for each item. For children's responses to addition problems, the proportion of correct responses was calculated across 4 items in the single-task condition, 2 visuospatial items in the dual-task condition, and 2 verbal items in the dual-task condition; parallel calculations were made for subtraction problems.

To determine whether children's scores on addition and subtraction items should be combined or treated as separate scales, an analysis of internal consistency was conducted on accuracy scores. Cronbach's alphas were calculated for children's accuracy on addition problems, subtraction problems, and an arithmetic scale that combined addition and subtraction problems within each of the three conditions. As can be seen in Table 2, the internal consistency of addition problems in the three conditions was quite low (.27 to .58), while the scale that combined addition and subtraction problems in each condition raised alpha levels to more acceptable levels (.64 to .78). Therefore, analysis of arithmetic accuracy was based on the proportion of correct responses across addition and subtraction problems in each condition.

Table 2

Cronbach's Alphas for Addition, Subtraction, and Combined Arithmetic Items in Each

Cond	ition
Cona	nion

					Combi	ned
	Addition		Subtraction		Arithmetic	
	α	K	α	K	α	K
Single Math	0.58	4	0.78	4	0.78	8
Dual Visuospatial	0.27	2	0.62	2	0.64	4
Dual Verbal	0.51	2	0.36	2	0.65	4

Arithmetic strategies. Another aspect of children's arithmetic performance that was measured was their problem solving strategies, which included counting, decomposition, retrieval, and other/guess/non-response. Children were considered to use a counting strategy if they reported counting during any portion of the calculation to reach the solution, or if their behaviors indicated such (e.g., counting on fingers, sub-vocal counting). Strategies were coded as decomposition if the child reported breaking the problem apart into a series of smaller problems; the exact steps the child reported using were recorded to validate the child's report. Strategies were coded as retrieval if the child claimed that they just knew the answer, although there were certain limitations to this rule. For example, if the child claimed to "just know" the answer, but had not provided their answer immediately (i.e., within approximately 3 s), then the experimenter prompted them to explain a little more. This usually resulted in a modified report of the strategy they used, such as stating, "Well, I counted part of it", or describing breaking the problem apart and retrieving intermediate solutions from memory (i.e., decomposition).

Similarly, if the child's behavior indicated the obvious use of a counting strategy, then this was reported and the child's strategy was coded as counting. There were, of course, instances when the child's reported strategy was initially unclear and could not be clarified by further prompting. These instances, along with cases where they reported guessing or simply did not respond, were all coded as such, but then collapsed into the category of "other".

Chapter 4: Results

Two sets of research questions were addressed by the analyses reported below. The first set of questions was addressed through associational analyses, which examined predictors of arithmetic performance. These analyses began with an exploration of the relation between two types of working memory (visuospatial and verbal) and children's accuracy in solving arithmetic problems. Next, the cognitive process through which working memory is related to arithmetic accuracy was explored. Specifically, two questions were addressed: (1) Do measures of number sense and problem solving strategies mediate the relation between working memory capacity and arithmetic performance? (2) Are visuospatial and verbal working memory capacities related to arithmetic accuracy through the same cognitive processes, or are there different mediators for each type of working memory?

In the second set of questions, experimental data were analyzed to examine the extent of working memory involvement during children's problem solving on different types of mathematical tasks. Specifically, the degree of visuospatial and verbal working memory involvement in children's performance was compared on number line, number sets, and arithmetic tasks. Further, the question of whether the extent of involvement differs for second and fourth graders, or for high- and low-performing students, was addressed.

Note that in the first set of questions, the focus was on working memory capacity as a predictor of math performance and thus the analysis involved examining the relation between children's performance on single memory tasks (capturing the child's memory capacity) and single math tasks (capturing the child's level of math skill). In the second set of questions, the focus was on the extent to which working memory resources were utilized when children completed different mathematical tasks and thus the analysis involved examining decrements in children's performance in the dual task, compared to the single tasks.

Descriptive Statistics

Before addressing the research questions, descriptive statistics were calculated for measures of children's visuospatial and verbal working memory capacities, number sense, and arithmetic accuracy in the single task conditions (presented in Table 3). Overall, all measures seemed to match the children's level of ability, in that none of the tasks appeared to be too easy or too difficult for children. With regard to arithmetic accuracy, children's performance covered a wide range, from 1 correct problem (13%) to 8 correct problems (100%), and averaged around 62% correct. Children's number line performance, which was represented as the average proportion absolute error (PAE) across 4 trials (8 estimates in total), indicated that while some students were quite accurate in their answers (1% error), others were much less accurate (37% error), with a mean error of 12%. Children's number sets scores represented the average proportion of correctly identified number sets (i.e., hits) on each trial. There were 4 possible hits on trials that had a target of 5, and 3 possible hits on trials that had a target of 9; the proportion of hits on each trial was averaged across all four trials to obtain the average proportion of hits. The minimum score was .13, which reflects 1 out of 4 possible hits on each of the two target 5 trials, but no hits on each of the two target 9 trials

((.25+.25+0+0)/4); the maximum average was 1, which reflects 4 out of 4 hits on target 5 trials and 3 out of 3 hits on target 9 trials ((1 + 1 + 1 + 1)/4). Across all children, the mean score was 0.76, indicating that children generally did well on the task, but still showed some difficulty.

Regarding the memory measures, the ranges observed for visuospatial and verbal working memory were slightly different from each other. Across the 8 visuospatial trials in the single memory condition, the minimum average percentage of recalled stimuli (out of 3 on each trial) was 13%, which represents 3 correct dot locations, out of 24 in total. The maximum score across all 8 trials was 83%, amounting to about 2 to 3 correct dot locations on each trial. On average, children recalled 46% of all possible dot locations, which amounts to about 1 to 2 dots per trial. Across the 8 verbal working memory trials, the minimum average percentage of recalled letters (out of 5 on each trial) was 15%, which represents 6 letters recalled across all trials, or about 1 letter per trial. The maximum score was 100%, and the average was 60%, suggesting that children typically recalled 3 letters per trial. Thus, both measures of working memory capacity seemed to match children's ability level, although the visuospatial task might have been slightly more challenging for them.

Table 3

Means, Standard Deviations, Minimum Values, and Maximum Values for Measures of Number Sense, Arithmetic, and Working Memory Capacity

	Min	Max	M	SD
Arithmetic Accuracy	.13	1.00	.62	.25
Number Line Error	.01	.37	.12	.08
Number Sets Accuracy	.13	1.00	.76	.21
Visuospatial WM Recall	.13	.83	.46	.15
Verbal WM Recall	.15	1.00	.60	.20

Note. WM = Working Memory. Accuracy = Proportion Correct. Error = Proportion Absolute Error. Recall = Proportion Correct.

Table 4 displays the percentage of children who used specific arithmetic strategies at three levels of frequency: none of the problems, some of the problems (between 1 and 7 times), and all of the problems. Counting and retrieval strategies were less common than decomposition. Specifically, while more than half of children never used a counting strategy (58%), and the majority of children never used retrieval (84%), a minority of children never used decomposition (25%), illustrating the popularity of decomposition in this sample. Regarding the use of counting strategies, almost half of the children relied on it for some, but not all of the problems (45%), while a smaller number relied on it for all problems (13%). In other words, most children were capable of implementing memory-based strategies at least some of the time. In contrast, with regard to the use of retrieval, a very small minority of children (3%) reported using this strategy for every problem, while slightly more (13%) reported using it for just some of the problems, suggesting that

most of the problems were too difficult for children to solve by retrieving the solution from memory, which can be expected for multi-digit arithmetic. Children's use of decomposition, however, was more uniformly distributed, with 42% of children using it some of the time and 33% using it on all of the problems. Therefore, this strategy appears to be a good choice for students who have advanced beyond the use of counting strategies, but are unable to retrieve solutions to multi-digit arithmetic problems from memory.

Table 4

Percentage of Students Using Counting, Decomposition, Retrieval, and Other Strategies at Various Frequencies

Frequency	Counting	Decomposition	Retrieval	Other
0	58%	25%	84%	62%
1-7	29%	42%	13%	37%
8	13%	33%	3%	1%

Working Memory Capacity and Arithmetic Performance: Mediating Factors

Correlation analysis. Bi-variate correlations were run to examine interrelations between children's working memory and arithmetic accuracy in the single-task conditions, as well as the measures of number sense and strategy use. The results are shown in Table 5. In examining measures of working memory in relation to measures of math performance and arithmetic strategies, distinct patterns of correlations emerged for visuospatial and verbal working memory. High visuospatial working memory was significantly associated with lower number line PAE scores (a measure of the error in children's estimates), higher arithmetic accuracy, and greater use of the decomposition
strategy. While high verbal working memory was also significantly associated with higher arithmetic accuracy, it was not significantly correlated with use of the decomposition strategy, and was only marginally significantly associated with lower PAE scores (p = .075). Higher verbal working memory was, however, significantly associated with more frequent use of the retrieval strategy.

Table 5

Zero-order Correlations Between Single-Task Measures of Working Memory, Number Sense, and Arithmetic Performance

		1	2	3	4	5	6	7
Working Memory Capacity								
1.	Visuospatial WM	_						
2.	Verbal WM	.173	_					
Number Sense								
3.	Number Line PAE	327**	191	_				
4.	Number Sets Hits	.156	.136	185 [†]	_			
Arit	hmetic Strategies							
5.	Counting	138	207	.292*	391***	_		
6.	Decomposition	.295**	.150	393**	.304**	750***	_	
7.	Retrieval	184	.229*	075	.234*	154	244*	_
8.	Arithmetic Accuracy	.221*	.340**	378**	.172	317**	.452***	.258*

Note. PAE = Proportion Absolute Error. WM = Working Memory. *p < .05. **p < .01. ***p < .001. Different patterns of associations were also seen when examining the two measures of number sense in relation to arithmetic accuracy and strategies. Lower error in number line estimates (i.e., greater accuracy) was associated with higher arithmetic accuracy, higher use of a counting strategy, and higher use of the decomposition strategy. Conversely, number sets scores were not significantly correlated with arithmetic accuracy, although they were significantly correlated with all three measures of arithmetic problem solving strategies (counting, decomposition, and retrieval).

Last, interrelations among arithmetic strategies and their relation to arithmetic accuracy were examined. All three problem solving strategies were significantly correlated with arithmetic accuracy, with higher accuracy associated with greater use of decomposition and retrieval strategies but lower use of the counting strategy. Greater use of decomposition was also associated with greater use of retrieval, and lower use of counting, while there was no significant relation between counting and retrieval strategies.

Mediation analysis. Having established a relation between working memory and arithmetic accuracy, further analysis was conducted to explore mediators of this relation, focusing specifically on number sense and problem solving strategies. Separate chains of mediation were considered for visuospatial and verbal working memory. Figure 3 displays three hypothetical mediation models that were explored in this analysis for each type of working memory. The specific measures that represented "number sense" and "arithmetic strategies" were selected according to the pattern of correlations described above.



Figure 3. Hypothesized processes through which verbal and visuospatial working memory relate to arithmetic performance.

In examining bi-variate correlations, two separate patterns of associations emerged for visuospatial and verbal working memory, which supported the decision to test separate mediation models for each of these working memory components. With regard to visuospatial working memory, the correlation analysis showed that one measure of number sense (number line PAE) and one type of strategy (decomposition) correlated with both visuospatial working memory and arithmetic accuracy, as well as with each other. This pattern of correlations suggested that all three mediation models shown in Figure 3 (Models 1a, 1b, and 1c) could be tested. However, since Model 1a would be subsumed by Models 1b and 1c (i.e., the specific paths tested in Model 1a represent a subset of paths tested in Models 1b and 1c), only the latter two models were tested. With regard to verbal working memory, the correlation analysis showed that one type of strategy (retrieval) was correlated both with this type of working memory and with arithmetic accuracy. No measure of number sense was correlated with verbal working memory. This pattern of associations supported the decision to test only one chain of two parallel chains shown in Model 2a (Figure 3). Specifically, only the arithmetic strategies piece of this model was tested, since the correlation analysis did not show a correlation between verbal working memory and either measure of number sense.

The mediation analysis was conducted using bias-corrected bootstrapping, which tests the significance of the mediation effect based on estimation of confidence intervals, and is the recommended method in smaller samples (Dearing & Hamilton, 2006; Preacher & Hayes, 2008). To conduct the analysis, the SPSS macro "PROCESS", created by Preacher and Hayes (2004), was used to randomly select 5,000 samples with replacement from the complete data file. Regression coefficients were estimated for each of the bootstrap samples, after which the estimated coefficients were averaged across all samples. This method allowed for detection of direct and indirect effects (i.e., mediation) of visuospatial and verbal working memory on arithmetic accuracy, via number sense and arithmetic strategies. Testing for indirect effects required estimating the regression coefficients for a number of pathways, as outlined by Hayes (2013). When examining possible mediators of the relation between visuospatial working memory and arithmetic accuracy (i.e., Models 1b, and 1c), verbal working memory was included in the models as a covariate; likewise, when examining the mediator of the relation between verbal working memory and arithmetic accuracy (Model 2a), visuospatial working memory was included as a covariate in the model.

To test Models 1b and 1c, seven paths were estimated. The paths that were estimated were the same for the two models, with the exception of the order of the mediating variables; in Model 1b, number line PAE was the first mediator (M_1) and decomposition was the second mediator (M_2), while in Model 1c, decomposition was the first mediator (M_1) and number line PAE was the second mediator (M_2). Otherwise, the estimated paths for both models were as follows: (1) visuospatial working memory (X) to M_1 , controlling for verbal working memory (path a_1), (2) M_1 to arithmetic accuracy (Y), controlling for verbal working memory (path b_1), (3) M_1 to M_2 , controlling for X and verbal working memory (path d), (4) X to M_2 , controlling for M_1 and verbal working memory (path b_2), (6) X to Y, controlling for verbal working memory (path c), and (7) X to Y,

controlling for M_1 , M_2 , and verbal working memory (path *c'*). To test the significance of the indirect effect of visuospatial working memory on arithmetic accuracy via M_1 and M_2 , the product of paths a_1 , d, and b_2 was calculated for each bootstrap sample (i.e., 5,000 times for each indirect effect), generating a distribution of indirect effects. From this distribution, a 95% confidence interval was estimated, which served as a test of significance for the indirect effect. By convention, the indirect effect is concluded to be statistically significant if the confidence interval does not contain the value 0 (Preacher & Hayes, 2008).

Similarly, for Model 2a, the following four paths were estimated: (1) verbal working memory (X) to retrieval (M), controlling for visuospatial working memory (path a), (2) M to arithmetic accuracy (Y), controlling for X and visuospatial working memory (path b) (3) X to Y, controlling for visuospatial working memory (path c) and (4) X to Y, controlling for M and visuospatial working memory (path c'). The indirect effect of verbal working memory on arithmetic accuracy via retrieval was estimated for each of the bootstrap samples as the product of paths a and b. Once again, the 95% confidence interval for the distribution of indirect effects was calculated to test the significance of retrieval as a mediator.

Figure 4 displays the results pertaining to Models 1b and 1c, which yielded evidence supporting mediation effects. The two models were estimated separately, but are reported in parallel here due to their similar structure. Visuospatial working memory was marginally significantly predictive of number line PAE scores, b = -0.11, $s_b = .06$, p= .062, and was significantly predictive of decomposition, b = 0.79, $s_b = .32$, p = .016, after controlling for verbal working memory. After partitioning variance explained by visuospatial and verbal working memory, number line PAE was significantly predictive of decomposition, b = -1.85, $s_b = .61$, p = .003, and decomposition use was significantly predictive of number line PAE, b = -0.06, $s_b = .02$, p = .006. Decomposition significantly predicted arithmetic accuracy after partitioning variance explained by number line PAE , visuospatial working memory, and verbal working memory, b = 0.20, $s_b = .07$, p = .030; likewise, number line PAE significantly predicted arithmetic accuracy when holding both types of working memory and decomposition constant, b = -0.84, $s_b = .38$, p = .005. When controlling for both of the mediators and verbal working memory, the relation between visuospatial working memory and arithmetic accuracy was non-significant, b = 0.08, $s_b = .19$, p = .674, whereas both indirect effects were statistically significant, indicating full mediation for both models. The indirect effects were estimated as $a_1*d*b_2 = 0.04$, 95% CI [0.01, 0.16] for Model 1b, and as $a_1*d*b_2 = 0.04$, 95% CI [0.01, 0.14], for 1c.



Figure 4. Mediation of the relation between visuospatial working memory and arithmetic accuracy. Solid lines indicate significant effects; dashed lines indicate non-significant effects. WM = working memory. [†] p < .10, ^{*} p < .05, ^{**} p < .01.

With regard to Model 2a (see Figure 5), verbal working memory was significantly predictive of use of children's use of the retrieval strategy after partitioning variance explained by visuospatial working memory, b = 0.24, $s_b = .10$, p = .017. Use of the retrieval strategy, in turn, was significantly predictive of arithmetic accuracy when holding visuospatial and verbal working memory constant, b = 0.37, $s_b = .16$, p = .029. In

contrast to the analysis of visuospatial working memory, verbal working memory was directly related to arithmetic accuracy after partitioning variance explained by visuospatial working memory and use of the retrieval strategy, b = 0.32, $s_b = .14$, p =.027. A test of indirect effects showed that this relation was partially mediated by use of the retrieval strategy, a*b = 0.09, 95% CI [0.01, 0.22].



Figure 5. Mediation of the relation between verbal working memory and arithmetic accuracy. Solid lines indicate significant effects; dashed line indicates non-significant effects. WM = working memory. *p < .05.

In summary, the results reported above suggest that visuospatial and verbal working memory capacities are related to arithmetic performance through different cognitive processes. While the relation between visuospatial working memory and arithmetic performance was mediated by children's number sense and use of the decomposition strategy, the relation between verbal working memory and arithmetic performance was partially mediated only by children's use of the retrieval strategy.

Involvement of Working Memory in Mathematical Performance

The second research question was addressed by analyzing the effects of the

experimental memory manipulation on mathematics performance. To determine to what extent visuospatial and verbal working memory are involved in children's math problem solving, separate analyses were conducted for each type of math task: number line, number sets, and arithmetic. In general, analyses were conducted by comparing math scores in the single-task condition to math scores in the dual-task condition, in which children had to perform a math task under interference of either visuospatial or verbal material (visuospatial-load dual task and verbal-load dual task, respectively). Involvement of visuospatial or verbal working memory in children's problem solving would be indicated by a decrement in performance between items in the single math condition and items in the visuospatial- or verbal-load dual condition. For example, if math performance is significantly worse on visuospatial dual-task items, compared to single-task performance, this decrement would suggest that visuospatial working memory is involved in solving this particular type of math problem. Likewise, if math performance is significantly worse on verbal dual-task items, compared to single-task items, verbal working memory would be implicated in children's problem solving.

To examine whether the extent of working memory involvement in problem solving differs for second and fourth grade students (or for high- and low-performing students), the decrement in math performance caused by a visuospatial or verbal load was compared for these two groups of students (i.e., between second and fourth graders, or between high- and low-performing students); different degrees of decrement in performance as a function of grade or performance level would suggest different extents of working memory involvement. Number line estimation. A 3 (condition: single vs. dual visuospatial vs. dual verbal) x 2 (grade level: second vs. fourth) x 2 (gender) mixed ANOVA was conducted mean PAE scores, in which condition was the within-subjects variable, and between-subjects variables were grade level and gender. The only significant result was the main effect of grade level, F(1, 84) = 18.81, p < .001, $\eta_p^2 = .18$, with fourth graders showing lower PAE scores across all conditions (M = 0.09) than second graders (M = 0.15). The following effects were non-significant: the main effects of gender, F(1, 84) = 0.03, p = .858, $\eta_p^2 < .001$, and condition, F(2, 168) = 1.21, p = .302, $\eta_p^2 = .02$; the 2-way interaction effects between condition and grade, F(2, 168) = 0.92, p = .402, $\eta_p^2 = .01$, condition and gender, F(2, 168) = 0.69, p = .504, $\eta_p^2 = .01$, and gender and grade, F(1, 84) = 2.61, p = .110, $\eta_p^2 = .03$; and the 3-way interaction between condition, grade, and gender, F(2, 168) = 1.11, p = .331, $\eta_p^2 = .01$.

In the present study, children from two grades were included to capture different levels of mathematical skill, so as to explore variation in the involvement of working memory in math performance as a function of skill level. As reported above, there was no interaction between grade level and experimental condition. This, however, does not necessarily suggest that working memory involvement in number line estimation does not differ as a function of skill level. In fact, despite the main effect of grade, some second graders may show performance on the number line task that is comparable to that of fourth graders. To explore this possibility, a median split was performed on children's PAE scores in the single-task condition. Children who scored below the median PAE of 0.11 were categorized as high-performing students (n = 41), and those who scored above

the median were categorized as low-performing students (n = 47). A cross-tabulation of performance group and grade level showed that there were 24 second graders (43%) who fell into the high-performing group, while 9 fourth graders (28%) fell into the low-performing group. The overlap in these distributions is depicted in Figure 6.





Figure 6. Frequency distributions of number line, number sets, and arithmetic scores in second and fourth grade students.

Based on these overlapping distributions, the grade level variable was replaced with the dichotomous measure of number line performance and a 3 (condition) x 2 (gender) x 2 (performance group) ANOVA was conducted. As can be expected, this analysis showed a main effect of performance group, F(1, 84) = 90.59, p < .001, $\eta_p^2 = .52$, indicating that children who performed below the median PAE score in the single math condition had significantly lower error across all conditions (M = 0.08) than those who scored above the single condition median value (M = 0.18). There was also a significant 2-way interaction between performance group and condition, F(2, 168) = 3.12, p = .047, $\eta_p^2 = .04$. The following effects were non-significant: the main effect of condition, F(2, 168) = 1.71, p = .184, $\eta_p^2 = .02$, and gender, F(1, 84) < .001, p = .997, $\eta_p^2 < .01$; the 2-way interaction between gender and performance group, and condition, F(2, 168) = 0.02, p = .977, $\eta_p^2 < .01$.

Simple effects tests were performed to interpret the interaction between performance group and condition. Comparisons using the LSD method were made between conditions within each performance level, which revealed that among low-performing children, there were no significant differences in PAE between the single math condition (M = 0.19) and the dual-task condition, for both visuospatial, p = .618, and verbal dual-task items, p = .770, (M = 0.18 in both cases). In high-performing children, on the other hand, PAE scores on visuospatial dual-task items (M = 0.09) were significantly higher than PAE scores in the single math condition (M = 0.08) were only marginally significantly higher than scores in the single math condition, p = .061. Scores on visuospatial and verbal dual-task items were not significantly different from each other, p = .285. Mean PAE scores in each condition for each performance group are depicted in Figure 7.



Figure 7. Mean number line, number sets, and arithmetic scores in single math, dual visuospatial, and dual verbal conditions.

Taken together, this analysis suggests that working memory involvement in number line estimation differs for high- and low-performing students. High-performing students seem to utilize visuospatial working memory and—to a lesser extent—verbal working memory. Lowperforming students, on the other hand, appear to rely less on visuospatial and verbal working memory resources. That is, among low-performing students, there was no difference in number line performance, whether or not visuospatial and verbal memory resources were demanded by an additional task.

Number sets identification. To determine how working memory is related to children's performance on the number sets task, a 3 (condition: single vs. dual visuospatial vs. dual verbal) x 2 (grade: second vs. fourth) x 2 (gender) mixed ANOVA was conducted on the mean proportion of correct target detections across number sets items in each condition. A main effect of grade level, F(1, 84) = 7.33, p = .008, $\eta_p^2 = .08$, showed that fourth graders correctly identified a greater proportion of number sets (M = 0.77) than second graders (M = 0.68). The main effect of condition was also significant, F(2, 168) = 7.77, p < .001, $\eta_p^2 = .09$. Non-significant effects included: the main effect of gender, F(1, 84) < .01, p = .949, $\eta_p^2 < .01$, and the 2-way interaction effects between gender and grade, F(1, 84) = 0.78, p = .381 $\eta_p^2 = .01$, condition and gender, F(2, 168) = 1.51, p = .224, $\eta_p^2 = .02$, and condition and grade, F(2, 168) = 0.92, p = .396, $\eta_p^2 = .01$; the 3-way interaction effect between condition, gender, and grade level was marginally significant, F(2, 168) = 2.89, p = .058, $\eta_p^2 = .03$

Pairwise comparisons using the LSD method were conducted to follow-up on the main effect of condition. Across genders and grade levels, number sets scores were significantly higher in the single math condition (M = 0.77) than scores on visuospatial items in the dual-task condition (M = 0.70), p = .005, and scores on verbal dual condition items (M = 0.69), p < .001.

There was no significant difference between scores on visuospatial and verbal dual-task items, p = .651.

As was the case with scores on the number line task, there was considerable overlap in the performance distributions of second and fourth grade students. A median split was performed on number sets scores in the single math condition (Mdn = 0.75), which resulted in 44 students in the high-performing group and 44 students in the low-performing group. Cross-tabulation of grade level and number sets performance group showed that 26 second graders (46%) scored above the median, while 14 fourth graders (44%) scored below the median (see Figure 6). Given this substantial overlap, the possibility that working memory involvement differed by performance level, rather than by grade level, was explored next.

The 3 (condition) x 2 (gender) x 2 (performance group) ANOVA was repeated with the performance group variable in lieu of grade level. As can be expected, there was a significant main effect of performance group, F(1, 84) = 84.43, p < .001, $\eta_p^2 = .50$, which showed that students who scored above the median in the single math condition had a higher mean number sets score (M = 0.82) across all conditions than students who scored below the median (M = 0.59). Also significant were the main effect of condition, F(2, 168) = 9.32, p < .001, $\eta_p^2 = .10$, and the 2-way interaction effect between condition and performance group, F(2, 168) = 16.07, p < .001, $\eta_p^2 = .16$. Non-significant effects were observed for gender, F(1, 84) = 1.39, p = .241, $\eta_p^2 = .02$, the two-way interactions between gender and condition, F(2, 168) = 1.43, p = .242, $\eta_p^2 = .02$, and between gender and performance level, F(1, 84) = 0.03, p = .870, $\eta_p^2 < .01$, as well as the three-way interaction between gender, condition, and performance level, F(2, 168) = 0.89, p = .413, $\eta_p^2 = .01$.

To follow-up on the main effect of condition, pairwise comparisons using the LSD method were conducted. Consistent with the previous ANOVA, number sets scores were significantly higher in the single math condition (M = 0.76) than scores on the visuospatial (M = 0.69) and verbal dual-task items (M = 0.68), p's < .001, but scores on visuospatial and verbal dual-task items were not significantly different from each other, p = .748. A test of simple effects was performed to follow up on the 2-way interaction effect between condition and performance group, in which comparisons of scores were made (using the LSD method) between the three conditions, within performance groups. Among low-performing students, scores in the single task condition (M = 0.59) were not significantly different from scores on visuospatial dual-task items (M = 0.63), p = .159, or verbal dual-task items (M = 0.57), p = .395. In contrast, high-performing students showed significantly lower scores on dual visuospatial (M = 0.74) and dual verbal (M = 0.80) items, compared to their scores in the single math condition (M = 0.93), p's < .001. The mean score in each condition within performance group is depicted in Figure 7.

The pattern of effects that emerged from the second ANOVA suggests that visuospatial and verbal working memory resources are both involved in children's number sets identification. However, high-performing children, regardless of gender or grade level, show significantly greater involvement of both visuospatial and verbal working memory in identifying number sets than low-performing children, which echoes the pattern observed on the number line task.

Arithmetic problem solving. Parallel to the analysis of number line and number sets performance, a 3 (condition: single vs. dual visuospatial vs. dual verbal) x 2 (grade: second vs. fourth) x 2 (gender) mixed ANOVA was conducted on the proportion of correct responses on arithmetic problems; gender and grade level served as between-subjects variables and condition served as a within-subjects variable. A main effect of grade level, F(1, 84) = 9.79, p = .002, $\eta_p^2 =$

.10, revealed significantly higher performance among fourth graders (M = 0.67) than second graders (M = 0.50), across genders and conditions. There was also a significant main effect of condition, F(2, 168) = 17.86, p < .001, $\eta_p^2 = .18$. Non-significant results were found for the following effects: the main effect of gender, F(1, 84) = 0.96, p = .330, $\eta_p^2 = .01$; the 2-way interaction effects between gender and grade level, F(1, 84) = 0.64, p = .427, $\eta_p^2 = .01$, and grade and condition, F(2, 168) = 0.61, p = .543, $\eta_p^2 = .01$, and condition and gender, F(1, 84) = 2.37, p=.096, $\eta_p^2 = .03$; and the 3-way interaction effect between gender, condition, and grade level, F(2, 168) = 0.17, p = .842, $\eta_p^2 < .01$.

Pairwise LSD comparisons were conducted to interpret the main effect of condition. Across genders and grade levels, scores in the single math condition (M = 0.65) were significantly higher than scores on the dual verbal items (M = 0.48), p < .001, but were not significantly different from scores on dual visuospatial items (M = 0.63), p = .485. Dual visuospatial item scores were significantly higher than dual verbal item scores, p < .001.

Since uneven performance distributions were observed in number sets and number line scores, the possibility that the same was true for arithmetic scores was explored. The performance distributions of second and fourth grade students were examined by conducting a median split on children's arithmetic scores from the single task condition (Mdn = 0.63), resulting in two groups nearly even in size (high-performing = 39, low-performing = 49). Cross-tabulation of performance group and grade level revealed that there were 17 second graders who performed above the median (30%), and 10 fourth graders who performed below the median (31%). Once again, the overlapping performance distributions of second and fourth graders provided the rationale for examining the possibility that visuospatial and verbal working memory involvement in arithmetic problem solving differed for high- and low-performing students.

Performance distributions for each grade are depicted in Figure 6.

The 3 (condition) x 2 (gender) x 2 (performance group) ANOVA was repeated with performance level as a replacement for grade level. Reflecting similar patterns as those observed when examining number line and number sets performance, results showed significant main effect of performance level, F(1, 84) = 89.87, p < .001, $\eta_p^2 = .52$, whereby high-performing students (M = 0.77) scored significantly higher than low-performing students (M = 0.40) across conditions. There was also a significant main effect of condition, F(2, 168) = 18.79, p < .001, η_p^2 = .18, as well as a significant two-way interaction effect between condition and performance level, F(2, 168) = 3.93, p = .021, $\eta_p^2 = .05$. The main effect of gender, F(1, 84) = 3.67, p = .059, $\eta_p^2 = .04$, the two-way interactions between gender and condition, F(2, 168) = 1.91, p = .151, $\eta_p^2 =$.02, and between gender and performance level, F(1, 84) = 0.89, p = .348, $\eta_p^2 = .01$, as well as the three-way interaction between gender, condition, and performance level, F(2, 168) = 2.20, p =.114, $\eta_p^2 = .03$, were all non-significant.

To follow-up on the main effect of condition, pairwise comparisons using the LSD method were conducted. Scores in the single task condition (M = 0.64) and in the dual visuospatial condition (M = 0.62) were significantly higher than scores in the dual verbal condition (M = 0.48), p's < .001. However, scores in the single task condition were not significantly different from scores in the dual visuospatial condition, p = .444. The 2-way interaction between condition and performance group was further analyzed through simple effects tests, in which scores in each condition were compared within each performance group, using the LSD method. Within high-performing students, scores in the single task condition (M = 0.76), p = .017, and the dual verbal condition (M = 0.69), p < .001, but scores in the latter two were only

marginally significantly different from each other, p = .075. Within low-performing students, on the other hand, single task scores (M = 0.43), were significantly higher than scores in the dual verbal condition (M = 0.29), p < .001, but were not significantly different from scores in the dual visuospatial condition (M = .48), p = .109; scores in the dual visuospatial condition were significantly higher than scores in the dual verbal condition, p < .001. Figure 7 shows the mean arithmetic score in each condition for each performance group.

Taken together, these results suggest that verbal working memory plays a larger role in arithmetic problem solving than visuospatial working memory, among second and fourth grade children, regardless of their baseline arithmetic performance. However, of note, reliance on visuospatial working memory differed for high- and low-performing children: while children showing low arithmetic performance appeared to depend minimally on visuospatial working memory, children showing high arithmetic performance appeared to require a substantial amount of visuospatial working memory resources to solve arithmetic problems.

In observing differences in the extent of working memory involvement between highand low-performing students, the point could be raised that these differences were due to a floor effect in the low-performing group (i.e., this group did not have as much room to drop as the high-performing group). However, a close look at the level of performance of children in the low-performing group showed that they scored highly enough on the number sets and arithmetic tasks, and showed low enough error on the number line task, to have shown a significant decrement in their performance. Specifically, with regard to the number sets task, highperforming children showed a decrement of 0.19 points between single-task items and visuospatial dual-task items, while low-performing children showed a mean single-task number sets score of 0.59—much more than 0.19 points away from the floor. On the arithmetic task, high-performing children showed a decrement of 10 percentage points from single-task items to visuospatial dual-task items, and the mean single-task score among low-performing children was 43%, well above 10 percentage points from the floor. Last, on the number line task, high-performing children showed an increase of 0.03 points in their mean PAE from single-task items to visuospatial dual-task items, while the mean single-task PAE among children in the low-performing group was 0.19, over 0.81 points away from the worst possible error score of 0.99. Thus, the individual differences in working memory involvement detected in these analyses could not be explained by a floor effect.

Analysis of visuospatial and verbal working memory recall. The results pertaining to each math task, reported above, showed that children who performed above the median score on the single number line, number sets, and arithmetic tasks showed a greater decrement in math scores than children who scored below the median. This pattern of results suggests that highperforming children utilized working memory resources to a greater extent than low-performing children. However, an alternative explanation is possible: that high-performing children dedicated more cognitive effort to remembering visuospatial or verbal stimuli in the dual-task condition than to solving the math problems, at the expense of their math performance. If this were the case, high-performing children would show a smaller decrement in recall scores than low-performing children. For example, as reported above, visuospatial interference caused a greater decrement in high-performing children's arithmetic scores than low-performing children's scores. If visuospatial interference caused a greater decrement in high-performing children because they were more focused on the visuospatial memory component of the dual task than the arithmetic component, then they should show a smaller decrement, when compared to low-performing children, in visuospatial recall scores as a result of arithmetic interference.

To test this possibility, analyses parallel to those reported above were conducted, in which children's visuospatial and verbal working memory recall scores were analyzed, rather than their scores on the three math tasks. Specifically, children's visuospatial and verbal recall scores in the single memory condition were compared to their visuospatial and verbal recall scores in the dual-task condition. To make this comparison, three separate analyses were conducted, which differed according to the dual-task items that served as the comparison. First, dual-task items that involved the number line task were analyzed, followed by dual-task items that involved the number sets task, then by dual-task items that involved the arithmetic task. In each of these analyses, the between-subjects variable was the corresponding dichotomous measure of math performance group (i.e., number line performance, number sets performance, and arithmetic performance groups). Including the relevant performance group variable in each of the three analyses made it possible to test the possibility of a smaller decrement in recall scores in the high-performing group than the low-performing group, by examining 2-way interaction effects between performance group and condition (single vs. dual), or 3-way interaction effects between performance group, condition, and stimulus (visuospatial vs. verbal).

The results of the analyses conducted for each math task are reported below. Note that any significant interaction effects were interpreted by conducting simple effects tests, in which mean comparisons were made using the LSD method. Table 6 displays estimated marginal means and standard errors for each analysis.

Table 6

Estimated Marginal Means and Standard Errors for Analyses of Visuospatial and Verbal Recall Scores in Single- and Dual-Task Conditions

	Number Line		Numb	Number Sets		Arithmetic	
	М	SE	М	SE	М	SE	
High-performing							
Visuospatial							
Single	0.42	0.02	0.43	0.02	0.43	0.02	
Dual	0.33	0.04	0.28	0.04	0.33	0.02	
Verbal							
Single	0.54	0.03	0.55	0.03	0.55	0.03	
Dual	0.26	0.03	0.28	0.04	0.23	0.03	
Low-performing							
Visuospatial							
Single	0.48	0.02	0.47	0.02	0.48	0.02	
Dual	0.34	0.03	0.34	0.04	0.36	0.03	
Verbal							
Single	0.64	0.03	0.63	0.03	0.65	0.03	
Dual	0.39	0.03	0.38	0.04	0.29	0.03	

Number Line Task. A mixed 2 (condition: single vs. dual number line) X 2 (stimulus: visuospatial vs. verbal) X 2 (number line performance group: high vs. low) was conducted on children's memory stimulus recall scores. Significant main effects of stimulus, F(1, 86) = 8.97, p = .004, $\eta_p^2 = .09$, and condition, F(1, 86) = 113.70, p < .001, $\eta_p^2 = .57$, showed that children recalled a greater proportion of visuospatial stimuli than verbal stimuli and that they recalled a greater proportion of stimuli on items in the single memory condition than on items with a number line component in the dual-task condition. In addition, a main effect of number line performance group, F(1, 86) = 7.66, p = .007, $\eta_p^2 = .08$, showed that children who scored below the median PAE score in the single math condition had significantly higher recall than children

who scored below the median PAE score. A stimulus by condition interaction effect, F(1, 86) = 113.70, p < .001, $\eta_p^2 = .57$, revealed that there was a greater decrement in verbal recall scores than visuospatial recall scores. Non-significant interaction effects between condition and performance group, F(1, 86) = 0.04, p = .845, $\eta_p^2 < .01$, and between condition, performance group, and stimulus, F(1, 86) = 1.96, p = .165, $\eta_p^2 = .02$, showed that the decrement to recall scores caused by arithmetic interference was comparable across the high- and low-performing groups. Thus, it appears as through children in both groups focused on stimulus recall to the same extent.

Number Sets Task. A mixed 2 (condition: single visuospatial. dual number sets) X 2 (stimulus: visuospatial vs. verbal) X 2 (number sets performance group: high vs. low) was conducted on children's memory stimulus recall scores. Significant main effects of stimulus, F(1, 86) = 8.37, p = .005, $\eta_p^2 = .09$, condition, F(1, 86) = 128.23 p < .001, $\eta_p^2 = .60$, and number sets performance group, F(1, 86) = 6.30, p = .014, $\eta_p^2 = .07$, as well as the interaction effect between stimulus and condition, F(1, 86) = 14.64, p < .001, $\eta_p^2 = .15$, revealed the same pattern of mean differences reported with respect to the number line task (see Table 6 for estimated marginal means). Again, non-significant interaction effects between condition and performance group, F(1, 86) = 0.42, p = .520, $\eta_p^2 = .01$, and between condition, performance group, and stimulus, F(1, 86) = 0.02, p = .878, $\eta_p^2 < .01$, revealed that the interference effect cause by the number sets task resulted in similar levels of decrement in that high- and low-performing children.

Arithmetic Task. A mixed 2 (condition: single vs. dual arithmetic) X 2 (stimulus: visuospatial vs. verbal) X 2 (arithmetic performance group: high vs. low) was conducted on children's memory stimulus recall scores. Once again, there were significant main effects of condition, F(1, 86) = 213.13, p < .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = 213.13, p < .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = 213.13, p < .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = 213.13, p < .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = 213.13, p < .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = .001, $\eta_p^2 = .71$, and arithmetic performance group, F(1, 86) = .001, $\eta_p^2 = .001$, $\eta_p^2 =$

5.44, p = .022, $\eta_p^2 = .06$, as well as a significant interaction effect between stimulus and condition, F(1, 86) = 71.99, p < .001, $\eta_p^2 = .46$, all of which echoed the mean differences reported with respect to the number line and number sets tasks (see Table 6). The main effect of stimulus was non-significant in this analysis, F(1, 86) = 2.31, p = .132, $\eta_p^2 = .03$. The interaction effects between condition and performance group, F(1, 86) = 0.42, p = .520, $\eta_p^2 = .01$, and between condition, performance group, and stimulus, F(1, 86) = 0.02, p = .878, $\eta_p^2 < .01$, were also nonsignificant, again suggesting that high- and low-performing children focused on stimulus recall to the same extent.

Chapter 5: Discussion

Performance in mathematics during childhood is strongly predictive of long-term mathematical achievement (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Tyson, Lee, Borman, & Hanson, 2007). By adulthood, mathematical competence increases the chance of full-time employment after completing school, over and above a number of competing factors, such as reading literacy and level of education (Rivera-Batiz, 1992). Given these longterm implications, this dissertation aimed to address limitations in existing literature that is centered on understanding the cognitive underpinnings of children's mathematical performance. Specifically, the nature of relations between children's working memory and two mathematical abilities that provide a foundation for children's mathematical development—namely, number sense and arithmetic strategies—were examined.

Two sets of research questions guided this investigation. The first set of questions pertains to predictors of arithmetic performance. Research has suggested that working memory may support the development of number sense and procedural knowledge of arithmetic, both of which can facilitate arithmetic calculations (De Rammelaere, Stuyven, & Vandierendonck, 1999; Geary et al, 2004, 2007, 2012; Fuchs et al., 2010b), but this mechanism has not been explicitly tested in previous investigations. The present study addressed this gap in the literature by examining children's number sense and arithmetic strategies as mediators of the relation between working memory and arithmetic performance.

The second set of questions examined the involvement of working memory in children's number sense and arithmetic performance. While a large and growing body of experimental investigations have reported the involvement of specific working memory components in adults' arithmetic problem solving (DeStefano & LeFevre, 2004), there is currently a dearth of such

investigations conducted with children. Existing research on the involvement of working memory in children's math problem solving have examined only one aspect of children's mathematical performance, their calculation of single-digit arithmetic problems. To extend this literature to other important areas of children's mathematical performance, the present research investigated the involvement of the visuospatial sketchpad and the phonological loop in children's number sense, including number line estimation and number sets identification, and their multi-digit arithmetic problem solving.

Mediators of the Working Memory Capacity and Arithmetic Performance Relation

The relation between working memory and arithmetic performance is complex. Research conducted with adults and children has documented different relations between performance on span measures of specific working memory components and performance on different mathematical tasks, such as arithmetic, algebraic, and word problem solving (Raghubar et al., 2010). The present study examined the mechanisms underlying relations between arithmetic performance and two components of working memory—the visuospatial sketchpad and the phonological loop. The literature suggests several underlying mechanisms for each type of working memory, but none have ever been tested in a complete mediation model. The findings of this study indicated that the relations of the visuospatial sketchpad and phonological loop to arithmetic performance and use of the decomposition strategy mediated the relation, while for the phonological loop, only children's use of the retrieval strategy mediated the relation. Below, the findings are discussed in the context of the extant literature.

Visuospatial working memory and arithmetic performance. There is a long history of research that has documented relations between spatial skills—such as the ability to visualize,

mentally transform, and mentally rotate objects in space—and performance in various mathematical domains (de Hevia, Vallar, & Girelli, 2008; Gunderson, Ramirez, Beilock, & Levine, 2012; Mix & Cheng, 2012; Hegarty & Kozhevnikov, 1999). Therefore, it is not surprising that the ability to encode and retain visuospatial information in working memory has emerged as an important predictor of various measures of mathematical performance, especially on more difficult math problems (Alloway & Passolunghi, 2010; Bull, Johnston, & Roy, 1999; Holmes & Adams, 2006; Holmes, Adams, & Hamilton, 2008) and among high-achieving students (Geary et al., 2009b). Indeed, in the present study, children's visuospatial working memory was found to be positively correlated with their arithmetic performance. To better understand the processes involved in this relation, visuospatial working memory was examined in relation to a number of other factors that have emerged in the literature as potential mediators.

One potential mediator that has been implicated is children's number sense, which was supported by the present findings. Consistent with previous investigations (Fuchs et al., 2010b; Geary et al., 2007), a relation was detected between children's visuospatial working memory and their performance on the number line task. However, visuospatial working memory was not found to be related to number sets task performance, despite the fact that this relation was found previously. The reason for this inconsistency is not completely clear, but one possibility pertains to differences in the measurement of visuospatial working memory. Whereas previous work (Fuchs et al., 2010b; Geary et al., 2007) has relied on a standardized assessment of visuospatial working memory known as the WMTB-C (Pickering & Gathercole, 2001), which involves a Block Recall task and a Memory for Mazes task, the measurement of visuospatial working memory in the present study was an adaptation of the Visual Patterns Task (Della Sala et al., 1997). While the former tasks capture more dynamic spatial elements of visuospatial working

memory, the latter task captures more static visual elements. Thus, it is possible that the more abstract spatial processing elements of visuospatial working memory are related to the apprehension of symbolic representations of quantities, and the measure used in the present study might not have been as sensitive to these elements. Therefore, a relation might have been detected in the present study had a different measure of visuospatial working memory been used.

Another possible mechanism is implied by the notion that visuospatial working memory facilitates the development of basic and fundamental arithmetic skills and knowledge. For example, visuospatial working memory may be related to the facility with which children are able to align addends in columns when implementing written arithmetic algorithms, and to their development of a conceptual understanding of place value (Geary, 2003; Rourke & Finlayson, 1978). While these aspects of arithmetic problem solving were not directly assessed in the present study, they are implicit in children's ability to execute certain problem solving strategies (Geary, 2006). Indeed, visuospatial working memory was correlated with the frequency with which children used the decomposition strategy in the present study. This is consistent with previous investigations, which have documented relations between visuospatial processing and the execution of certain arithmetic problem solving strategies, including decomposition and mental visualization of arithmetic algorithms (Geary et al., 2007; Imbo & LeFevre, 2010; Laski, Casey, Yu, Dulaney, Heyman, & Dearing, 2013; Trbovich & LeFevre, 2003).

Thus, there may be multiple mechanisms through which visuospatial working memory is related to arithmetic problem solving. Given the shared relations found between children's visuospatial working memory, number line task performance, use of the decomposition strategy, and arithmetic performance, two mediation models were analyzed in this research. First, number line estimation and use of the decomposition strategy were tested as serial mediators of the relation between the visuospatial working memory capacity and arithmetic accuracy, respectively. Second, both factors were tested as serial mediators again, but in the opposite order, whereby decomposition was the first mediator and number line estimation was the second mediator. The findings with respect to both of these models are discussed next.

Number line estimation and decomposition as serial mediators. Children's number line estimation performance and use of the decomposition strategy mediated the relation between visuospatial working memory and arithmetic accuracy. The finding of a mediation effect of children's number line estimation is not only consistent with previous findings of bi-variate relations between these variables (Fuchs et al., 2010b; Geary et al., 2007), but it also resonates with the perspective that one way individuals interpret numerical magnitude is through visuospatial codes that resemble a mental number line or continuum (Dehaene, 1992, 1997; Dehaene, Bossini, & Giraux, 1991; Dehaene & Mehler, 1992), which has been found to facilitate arithmetic learning and performance (Booth & Siegler, 2008; Griffin et al., 1994). An explanation for the mediation effect of the decomposition strategy is not as clear, but it is possible that strong visuospatial working memory may support children's learning of the decomposition strategy, possibly by alleviating demands on the central executive and phonological components of working memory, as executing the decomposition strategy is thought to be particularly demanding of these resources (Imbo & Vaniderendonck, 2007, 2010). This process, as suggested by the present findings, may result in greater success in arriving at correct problem solutions. Furthermore, children who use the decomposition strategy gain additional practice in retrieving simple arithmetic facts when executing this strategy, and must exercise arithmetic procedures that improve their understanding of arithmetic concepts, such as the property of commutativity (Canobi et al., 1998; Cowan & Renton, 1996; Geary, 2006). Thus,

these children already seem to have a solid foundation on which to perform multi-digit calculations.

Importantly, a serial mediation effect was detected for these two mediators, in both orders. In other words, strong visuospatial working memory was associated with better number line estimation skills, which, in turn, were related to greater use of the decomposition strategy, and finally, to higher arithmetic accuracy. Likewise, strong visuospatial working memory predicted more frequent use of the decomposition strategy, which was associated with better number line estimation skills, and again, higher arithmetic accuracy. The fact that both serial mediation models were significant suggests not only that children's capacity for storing and manipulating visuospatial information supports both the development of spatial numerical representations and the acquisition of skills needed to execute the decomposition strategy, but also, that there may be a bi-directional relation between these two factors. That is, an understanding of numerical magnitudes might facilitate children's use of the decomposition strategy, while skilled use of the decomposition strategy could clarify their representation of number magnitudes.

Understanding the possible bi-directional relation between children's decomposition and number line estimation requires further research. Children's arithmetic strategies have not often been linked to their number line estimation skills. Therefore, the mechanism underlying this relation is not immediately clear. In considering how use of the decomposition strategy might improve the accuracy of number line estimates, it is useful to consider factors that have been found to improve children's performance on the number line task. For example, Laski and Siegler (2007) provided children with feedback on their categorization of number magnitudes as "really small", "small", "medium", "big", and "really big" and found that this feedback

improved the accuracy not only of their categorizations but also of their estimates on the number line task. If experiential factors such as number categorization can improve children's numerical magnitude representations, then it is plausible that experience in executing certain arithmetic strategies, such as decomposition, could have a similar effect. Executing the decomposition strategy requires children to practice making judgments about the relative magnitudes of problem addends, which may help them to categorize numerical magnitudes and ultimately improve the precision of their number line estimates. While the relation found between decomposition and number line estimation raises this possibility, this mechanism should be tested in future research.

With regard to the way in which number line estimation ability might facilitate use of the decomposition strategy, findings on children's memory for numbers become relevant. Specifically, there is evidence to suggest that children showing more accurate internal representations of numerical magnitude—as measured by several tasks, including number line estimation—were also better at remembering numbers introduced to them in a story, while other types of numerical knowledge, such as knowledge of counting, were unrelated to their recall of the numbers presented (Thompson & Siegler, 2010). Since decomposition is a strategy that involves temporary storage of intermediate results, any mechanism that might support their ability to do so is particularly relevant to the present finding; it appears as though skill in making spatial-numerical relations, as measured by number line task, may indeed be one such mechanism. This possibility should be tested empirically in future studies.

Verbal working memory and arithmetic performance. Also related to arithmetic performance is verbal working memory. It has been argued that one of the primary roles of verbal working memory in mental arithmetic is to assist in the storage of problem information. This idea has been supported by the finding of a minimal role for verbal working memory when

problem information is visually available during problem solving. For example, when Fürst and Hitch (2000) displayed problem information continuously during adults' arithmetic problem solving, taxing their verbal working memory in a dual-task condition had no effect on their performance. In the present study, however, problem information was visually available for the complete duration of children's problem solving and yet, verbal working memory was still found to be correlated with and involved in arithmetic performance. This finding implicates a role for verbal working memory in arithmetic performance, despite the visuospatial availability of problem information.

There are a couple of explanations for the contrasting findings between the present study and those of Fürst and Hitch. First, it is important to note the ages of participants in both studies; in the present study, participants were of elementary school age, while in the Fürst and Hitch study, participants were college-aged students. Therefore, the inconsistency in the two sets of findings might partially reflect developmental differences in working memory or arithmetic skill. Second, verbal working memory may be useful for other aspects of arithmetic problem solving. Indeed, it has been argued that another role of verbal working memory in mental arithmetic is that of encoding and storing verbal information used for counting (Healy & Nairne, 1985; Nairne & Healy, 1983). This idea may provide some explanation for the link between verbal working memory capacity and arithmetic performance in the present study, as a small correlation was found between verbal working memory capacity and use of counting strategies, although this was only marginally significant. However, children's use of another strategy, retrieval, was significantly related to their verbal working memory capacity. This finding is consistent with literature that has suggested a relation between verbal working memory and the early acquisition of number facts (Geary et al., 2004), which are thought to be encoded and retrieved verbally

(Dehaene & Cohen, 1995). In line with this argument, the present study was the first to test children's use of the retrieval strategy as a mediator of the relation between verbal working memory and arithmetic performance.

Retrieval as a mediator. One of the key findings in the present study is that the retrieval strategy partially mediates the relation between verbal working memory and arithmetic performance. In other words, children with strong verbal working memory showed higher accuracy in their arithmetic problem solving, and this apparent advantage was explained, in part, by their more frequent use of the retrieval strategy. This finding suggests that the capacity to temporarily store and process verbal information may provide children with an important advantage in performing arithmetic calculations by supporting their learning of arithmetic facts that would be accessed through the retrieval strategy.

It is not surprising that children's use of retrieval during the strategy assessment was positively related to their arithmetic accuracy in the single-math condition. After all, use of the retrieval strategy typically represents the most advanced level of skill in arithmetic problem solving (Ashcraft & Stazyk, 1981; Geary, 2011; Geary et al., 2004; Shrager & Siegler, 1998; Siegler 1988). Compared to decomposition and counting, retrieval is the most efficient method of solving an arithmetic problem, as it requires the least amount of time to execute and is the least demanding of central executive working memory resources (Imbo & Vandierendonck, 2006, 2007). Children with mathematical difficulties are much less likely to use the retrieval strategy than their typically-achieving peers (Bull & Johnston, 1997), as they struggle to learn the arithmetic facts that would support their use of this strategy (Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1995; Geary, 2004; Jordan, Hanich, & Kaplan, 2003). The present findings suggest that this may be due, in part, to a lack of available resources from verbal working

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memory. While this mechanism has been suggested by previous findings, this study was the first to directly test this mediation model.

Lack of a mediation effect of number sense. Unlike visuospatial working memory, which was related to children's number line estimation, children's verbal working memory was not significantly related to either measure of number sense. This is consistent with a previous finding, which showed that among the three primary components of working memory, verbal working memory was the only one that bared no relation to children's number line or number sets task performance (Geary et al., 2007). Thus, while verbal working memory has been implicated in a number of mathematical skills (Bull, Espy, & Wiebe, 2008; Fürst & Hitch, 2000; Hecht, 2002; Imbo & LeFevre, 2010; Lee & Kang, 2002; Logie et al., 1994; Trbovich & LeFevre, 2003), including those assessed in the present study, children's number line estimation and number sets identification do not appear to be mechanisms underlying this relation.

Involvement of Working Memory in Number Sense and Arithmetic Performance

Despite widely documented relations of visuospatial and verbal working memory capacity to children's arithmetic performance, the present investigation is one of the few that have experimentally investigated the involvement of working memory resources in children's math performance using dual-task methodology. The advantage of this approach is that it involves experimentally taxing specific working memory components during mathematical tasks. This manipulation allows for a comparison of the extent to which the assessed components of working memory are involved in completing math tasks, and whether this involvement varies as a function of the types of mathematical performance that are measured. This differs from examining relations between measures of working memory capacity and measures of arithmetic performance in that it yields evidence of causal relations. Furthermore, finding that working memory is related to arithmetic performance is not the same as finding how it is actually involved in children's problem solving. The latter type of finding can provide information about which aspects of children's performance are most constrained by limitations in specific types of working memory, which is useful in identifying specific areas of difficulty for children, especially when individual differences are considered.

One major contribution of the present research is that it is the first to use a dual-task design to examine visuospatial and verbal working memory involvement in children's number sense and multi-digit arithmetic problem solving. In particular, there is no previous research that has used a dual-task procedure to study working memory involvement in children's or adults' number line and number sets task performance. The number line and number sets tasks capture children's spatial and symbolic representations of number magnitudes (Dehaene, 1992; Geary et al., 2009a; Siegler & Opfer, 2003); therefore, examining the extent to which children use visuospatial and verbal working memory resources during these tasks provides meaningful information about the visuospatial and verbal processing involved in accessing internal representations of numbers. With regard to arithmetic, previous dual-task investigations have only measured children's single-digit arithmetic performance. Performance on multi-digit arithmetic can represent a wider range of arithmetic competencies, including an understanding of place value, knowledge of single-digit arithmetic facts, and the ability to adaptively execute a range of problem solving strategies, since children are less likely to use retrieval on these types of problems. Thus, multi-digit arithmetic places a greater load on elementary school children's working memory than single-digit arithmetic (Adams & Hitch, 1997).

Evidence of visuospatial or verbal working memory involvement in children's performance on a math task is found in a decrement in children's math performance between the

single- and dual-task conditions. However, in interpreting decrements in number line, number sets, and arithmetic performance caused by dual-task interference, it is important to note that a lack of a significant effect of visuospatial or verbal interference does not necessarily imply a lack of visuospatial or verbal working memory involvement in children's performance. Rather, it is likely to suggest that working memory resources required for the task are minimal. That is, any given math task may require some amount of resources from visuospatial and verbal working memory, but if the amount is minimal, then the combined load of the math task and retaining visuospatial or verbal stimuli does not exceed working memory resources, resulting in no decrement to math performance.

In the present study, the findings showed variable levels of involvement of both visuospatial and verbal components of working memory in children's number line estimation, number sets identification, and arithmetic calculations. Importantly, significant involvement of specific working memory components was not always found for all students, but instead was detected only in students who demonstrated a high level of skill on the particular math task being assessed. These results are discussed in greater detail below.

Working memory involvement in number sense. According to the present findings, visuospatial and verbal working memory components appear to be differentially involved in the two measured aspects of children's numerical representations. In examining performance on the number line task, at first glance there appears to be minimal visuospatial and verbal processing involved in generating spatial representations of number magnitude. However, a look at differences between high- and low-skill students indicates a varied pattern of working memory involvement. Specifically, high-skill children showed a highly significant involvement of visuospatial working memory and a marginally significant involvement of verbal working

memory. Low-skill children, on the other hand, show minimal involvement of both types of working memory. In examining performance on the number sets task, on the other hand, both the visuospatial sketchpad and the phonological loop were highly involved, but again, only for highskill students. Therefore, low-skill students appear to involve minimal resources from both types of working memory in both aspects of number sense.

The finding that the visuospatial sketchpad was significantly involved in high-performing students' number line estimation is consistent with literature suggesting that number line estimation is a visuospatial process (Dehaene, 1992, 1997; Siegler & Opfer, 2003). It also echoes the correlation between visuospatial working memory capacity and the accuracy of children's number line estimates that was detected in the mediation analysis. Thus, the accuracy of children's number line estimates is related to their visuospatial working memory capacity and the extent to which they involve visuospatial working memory resources.

The finding that high-performing children relied heavily on both types of working memory when completing the number sets task is consistent with the argument, made by those that developed the task, that completing the task involves multiple domain-general and domainspecific cognitive processes (Geary et al., 2009a), including those required for processing approximate and exact visuospatial and verbal codes of quantity (Dehaene, 1992). However, as was discussed previously and in contrast to the number line task, performance on the number sets task was not significantly related to either type of working memory. Thus, the use of resources from visuospatial and verbal working memory is related to greater accuracy in identifying number sets, but large capacities in both working memory components are not associated with higher performance.

Working memory involvement in arithmetic problem solving. It is clear from the

present and previous findings that verbal working memory plays a large role in arithmetic calculations, particularly those presented in a horizontal format (Trbovich & LeFevre, 2003). In fact, the present findings indicate a larger role for verbal working memory in arithmetic calculations than for visuospatial working memory. However, a comparison between students with high versus low levels of arithmetic skill revealed differential involvement of visuospatial working memory. Specifically, highly skilled children showed a significant involvement of visuospatial working memory and verbal working memory, while children with a lower level of arithmetic skill primarily showed involvement of verbal working memory.

One possible explanation for this finding is that children who can delegate various components of arithmetic calculations to the visuospatial sketchpad and the phonological loop, thus involving both storage systems in their problem solving, have higher success in arriving at the correct solution to the problem. The role of the visuospatial sketchpad is variegated, in that it can be used to encode and retain aspects of arithmetic calculations of a more static or abstract spatial nature (Knops, Thirion, Hubbard, Vincent, Dehaene, 2009; Zorzi, Priftis, & Umilta, 2002). Therefore, it is possible that some children may temporarily store intermediate results in the visuospatial sketchpad by visually encoding numeric symbols or spatial representations of numbers when executing arithmetic calculations, as has been suggested previously (Logie et al., 2004), while assigning other components of the task, such as fact retrieval, to the phonological loop.

The delegation of different aspects of arithmetic calculation to different working memory components can be illustrated by the following example. To solve the problem 52 + 29, a high-performing child might first add 9 and 2 to obtain the value 11, but will need to temporarily retain this value in memory while they execute the next steps, which might be to add 50 and 20

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to obtain 70, and finally to add 70 and 11 to obtain 81. Executing all three of these steps would require considerable phonological and central executive resources. Therefore, the child might have more success in executing these steps by temporarily encoding intermediate results visually (i.e., visualizing the number 11 in a mental space), rather than verbally (i.e., sub-vocally repeating the number 11). Delegating the first intermediate result of 11 to the visuospatial sketchpad could make it easier for them to sub-vocally process the next step, e.g., "... 2 and 9 have been added... 5 plus 2 is 7, then add a 0 to make it 70". They might then recruit the visuospatial code of 11 from the visuospatial sketchpad to sub-vocally complete the final step of adding both intermediate results—e.g., "11 plus 70 is 81—to arrive at the solution. As illustrated here, the potential negotiation of resources between these two storage systems may facilitate calculation and thus increase the likelihood of achieving the correct answer. While the present study did not explicitly test this mechanism, it provides some preliminary support for this possibility.

Working Memory Capacity versus Working Memory Involvement

This research examined the nature of the relation between working memory and math performance in two different ways. First, associations between measures of children's visuospatial and verbal working memory capacities and measures of their number sense, arithmetic strategies, and arithmetic performance were examined. Second, visuospatial and verbal working memory components were experimentally manipulated to examine their involvement in children's completion of mathematical tasks. Thus, the former investigation examined correlations between working memory capacity and mathematical performance, while the latter examined the extent to which children involve resources from working memory when executing mathematical tasks. These two investigations have different theoretical and practical implications. On one hand, an understanding of the mechanisms underlying relations between working memory capacity and mathematical outcomes may help educators to identify children who are at risk for poor math achievement and focus on helping them improve areas that may be particularly difficult for them as a result of their limited working memory capacities. On the other hand, understanding which working memory resources children recruit when performing mathematical tasks demonstrates how they encode and process numerical content and operations, regardless of their working memory capacity. This information can inform the ways in which mathematical concepts and procedures are taught and highlight the most efficient ways for children to approach mathematical tasks, including problem solving strategies. This has promising implications for children who show limitations in their working memory capacities because it suggests that it may be possible to train children to use even the limited resources they have more efficiently in order to improve their performance. This possibility should be investigated in future research.

Limitations and Future Directions

In addition to the suggestion that future research should consider possible links or lack thereof between the educational implications of working memory capacity versus working memory involvement, additional recommendations for future research are made here, based on limitations of the present research. The first limitation pertains to the measurement of children's arithmetic strategies. While the relation between children's working memory capacity and their use of various problem solving strategies was investigated, the involvement of working memory in children's execution of arithmetic strategies was not investigated, as strategies were not assessed in the dual-task condition. Therefore, it was impossible determine whether children's use of any particular strategy called upon resources from the visuospatial sketchpad or the phonological loop. This limitation was primarily due to time constraints, as the procedures involved in executing the present study required children to be out of their classrooms for a considerable amount of time. In order to measure working memory involvement in children's arithmetic strategies, a separate experimental session would have been required, as half of the present experimental session was dedicated to measuring children's number sense, rather than their arithmetic performance. Thus, future investigations should include an assessment of arithmetic strategies using a dual-task procedure, as an understanding of working memory involvement in children's arithmetic strategies of the mechanisms underlying working memory involvement in children's arithmetic performance.

The second limitation involves the specific components of working memory that were assessed and manipulated in the present study. Only children's visuospatial and verbal working memory capacities were examined, but it is important to understand how central executive resources might be related to and involved in children's number sense and their multi-digit arithmetic strategies and performance. This limitation is, again, due to time constraints, as the length of the experiment and the number of experimental items would need to have been increased by one third in order to measure the central executive component of working memory. However, this is an important next step for future studies, as it has been posited that while the contributions of the visuospatial sketchpad and phonological loop to children's performance are significant, the role of the central executive is also substantial (Barrouillet, Fayol, & Lathuliére, 1997; Geary, Hamson, & Hoard, 2000; Geary et al., 2007; Hubber et al., 2013).

Third, while the present research extended the work of previous dual-task investigations by assessing various types of math performance, it was difficult to examine variations in working memory involvement within each type of math task, due to the limited number of items on each measure. In particular, to conduct an in-depth investigation of variations in visuospatial and verbal working memory involvement in children's multi-digit arithmetic performance, a larger number of addition and subtraction items should be sampled. This would allow for a comparison of visuospatial and verbal working memory involvement in performing addition versus subtraction, as well as an examination of differences in working memory involvement due to problem size or carry and borrow operations. After all, previous research has shown that working memory involvement is in large part specific to the particular arithmetic operation being executed (DeSefano & Lefevre, 2004; Lee & Kang, 2002).

Fourth, while it was possible to assess working memory involvement in three measures of children's mathematical performance in the present study, a study of working memory involvement in other areas of mathematics could yield significant implications for educators. For example, there could be a very different pattern of working memory involvement in children's performance on more spatially based types of mathematics. In fact, national assessments show that more spatially oriented mathematics, such as measurement of volume, are areas of particular difficulty for children (Lubienski, 2003; National Center for Education Statistics, 2011).

Last, it is important to note that the mediation analyses were based on associational components of the study and thus, conclusions of causal relations cannot be derived from this study. However, the associational findings obtained in this research provide a starting point for future dual-task investigations of working memory involvement in children's mathematical performance.

Implications

The present research shows that arithmetic problem solving strategies and visuospatial

representations of numbers represent mechanisms through which children's working memory capacity is related to their arithmetic performance. However, regardless of working memory capacity, children are more successful in their arithmetic calculations when they call upon resources from the visuospatial sketchpad during problem solving. These findings have several implications for interventions aimed at children with deficits in working memory capacity or who demonstrate mathematical difficulties.

The finding that children's working memory capacity is related to their arithmetic performance through their underlying numerical representations and arithmetic strategies highlights the importance of early identification of children with deficits in visuospatial and verbal working memory capacities, in order to intervene before these deficits interfere with their acquisition of critical mathematical skills and knowledge. Specifically, the finding that children's visuospatial working memory capacity is related to their arithmetic performance through their spatial representations of numerical magnitudes and their use of the decomposition strategy uncovers two targets for early interventions; focusing on these mediating factors may improve these children's chances for mastering arithmetic calculations, one of the most important skills they are expected to develop during their educational careers. This finding also underscores the importance of aiming to develop both of these factors, as the relation between them may be bidirectional. Therefore, improving both of these skills may elicit a higher amount of learning overall.

Additionally, limited verbal working memory capacity may represent a barrier to children's ability to commit arithmetic facts to long-term memory or subsequently retrieve them, which can directly impact their execution of arithmetic calculations. Finding methods that make arithmetic facts more salient for children, such as associating facts with visuospatial stimuli or

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presenting facts in novel and creative ways, may improve their ability to encode and recall this information from long-term memory. Indeed, evidence from neurobiological investigations suggests that greater salience of stimuli is associated with higher levels of encoding in critical brain regions (Lisman & Grace, 2005; Lisman & Otmakhova, 2001). Moreover, there is evidence to suggest that children's arithmetic fact learning can be significantly improved when the problem terms are spatially represented on a number line during the learning process (Booth & Siegler, 2004).

It is also possible that interventions aimed at improving children's working memory capacity (Holmes, Gathercole, & Dunning, 2009; Klingberg, Forssberg, & Westerberg, 2002; Thorell, Lindquist, Nutley, Gunilla, & Klingberg, 2009) have the potential to facilitate their acquisition of spatial representations of numerical magnitude and arithmetic problem solving strategies, which would improve their overall mathematical performance. These interventions may also increase the effectiveness of interventions designed to improve children's number sense (Griffin et al., 2004; Laski & Siegler, 2007, 2013) or to increase their use of memory-based strategies. Last, the finding that children who take advantage of resources from visuospatial working memory have higher arithmetic performance, regardless of their working memory capacity, suggests that training children to visualize elements of their arithmetic calculations may improve the efficiency of their problem solving strategies. Thus, children demonstrating difficulties with mathematics might be helped by learning to solve problems more efficiently. For example, children should not just be taught to use the decomposition strategy, but should also be taught how to use the decomposition strategy more efficiently by encoding intermediate solutions in a visuospatial modality.

While further investigation is warranted to test the applications of these findings and the

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validity of these implications, this research contributes substantially to our understanding of how working memory is related to math performance in children. Though limitations in working memory capacity pose serious challenges for children in the course of their mathematical development, awareness of the mediating factors identified in this research can guide the development of interventions that could have real and substantial impacts. Moreover, this research serves as a catalyst for future investigations that can further advance our understanding of children's mathematical thinking, which ultimately serves to advance their chances of success in mathematics.

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Appendix



a. Visuospatial working memory items

b. Verbal working memory items

i. Set A	ii. Set B	iii. Practice item
NFTVD	MHZRG	KA
YCLFK	BMSWH	
PGRSQ	QKVLJ	
SDGBP	XRCDL	
KZFYN	WPXBR	
JQDNB	CLQTV	
TJHMF	HXNGT	
ZWPJC	VYKCX	

c. Arithmetic items	
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i. Set A	ii. Set B
34 + 7	45 + 6
18 + 6	26 + 7
29 + 5	16 + 8
59 + 9	67 + 5
83 - 6	31 - 8
22 - 9	86 - 7
45 - 7	31 - 8
26 - 8	53 - 6

d. Number line items and display

i. Set A ii. S		et B	iii. Display	
6	965	16	655	
47	327	23	504	
481	28	271	34	
773	13	901	8	

e. Number sets stimuli and displays (targets are either 5 or 9)

Å	5 *	• • • • *** •	7	••	
0 8		2 +	5 0	2	
· · · 1		3 5	••3	• * • *	▲ ▲ <mark>* * *</mark> * * *
4 1	3 2	4 •	* 2 * 2	* * * 1	***
	• * *	1 4	3	2 1	* *
3 6	•	3 + 5	▲ ★ ★ ★ ★ ★ ★ ★ ★ ★	7 2	
	2 7	6 •	***	8 *	

i. Stimuli

ii. Displays

