Housing markets, business cycles and monetary policy

Author: Margarita Rubio

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MARGARITA RUBIO

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Margarita Rubio

Advisors: Fabio Ghironi, Matteo Iacoviello

This dissertation studies the implications of housing market heterogeneity for the transmission of shocks, welfare and the conduct of monetary policy.

In the first chapter I focus on mortgage contract heterogeneity (fixed vs. variable-rate mortgages). I develop and solve a New Keynesian dynamic stochastic general equilibrium model that features a housing market and a group of constrained individuals who need housing collateral to obtain loans. A given proportion of constrained households borrows at a variable rate, while the rest borrows at a fixed rate. The model predicts that in an economy with mostly variable-rate mortgages, an exogenous interest rate shock has larger effects on borrowers than in a fixed-rate economy. For plausible parametrizations, aggregate differences are muted by wealth effects on labor supply and by the presence of savers. More persistent shocks cause larger aggregate differences. From a normative perspective I find that, in the presence of collateral constraints, the optimal Taylor rule is less aggressive against inflation than in the standard sticky-price model. Furthermore, for given monetary policy, a high proportion of fixed-rate mortgages is welfare enhancing.

Then, I develop a two-country version of the model to study the implications of housing market heterogeneity for a monetary union as well as costs and benefits of being in a monetary union when there are asymmetric shocks. Results show that consumption reacts more strongly to common shocks in countries with high loan-to-value ratios (LTVs), a high proportion of borrowers or variable-rate mortgages. I also find that country-specific housing price shocks increase consumption not only in the country where the shock takes place. Welfare analysis shows that housing-market homogeneization is not beneficial per se, only when it is towards low LTVs or predominantly fixed-rate mortgages. As for costs and benefits of monetary unions, when there is a technology shock in one of the countries and they are symmetric, the monetary union regime is welfare worsening. However, results are dependent on whether or not countries are symmetric and on the source of the asymmetry.

1 Fixed and Variable-Rate Mortgages, Business Cycles and Monetary Policy

Abstract[1]

The aim of this paper is twofold. First, I study how the proportion of fixed and variable-rate mortgages in an economy can affect the way shocks are propagated. Second, I analyze optimal implementable simple monetary policy rules and the welfare implications of this proportion. I develop and solve a New Keynesian dynamic stochastic general equilibrium model that features a housing market and a group of constrained individuals who need housing collateral to obtain loans. A given proportion of constrained households borrows at a variable rate, while the rest borrows at a fixed rate. The model predicts that in an economy with mostly variable-rate mortgages, an exogenous interest rate shock has larger effects on borrowers than in a fixed-rate economy. Aggregate effects are also larger for the variable-rate economy. For plausible parametrizations, differences are muted by wealth effects on labor supply and by the presence of savers. More persistent shocks, such as inflation target and technology shocks, cause larger aggregate differences. From a normative perspective I find that, in the presence of collateral constraints, the optimal Taylor rule is less aggressive against inflation than in the standard sticky-price model. Furthermore, for given monetary policy, a high proportion of fixed-rate mortgages is welfare enhancing.

Keywords: Fixed/Variable-rate mortgages, monetary policy, housing market, collateral constraint

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"[...]*the structure of mortgage contracts may matter for consumption behavior. In countries like the United Kingdom, for example, where most mortgages have adjustable rates, changes in short-term interest rates have an almost immediate effect on household cash flows.* [...] *In an economy where most mortgages carry fixed rates, such as the United States, that channel of effect may be more muted. I do not think we know at this point whether, in the case of households, these effects are quantitatively significant in the aggregate. Certainly, these issues seem worthy of further study".* Ben Bernanke, June 15, 2007.

1.1 Introduction

Mortgage contracts in an economy can be fixed or variable rate. The proportion of variable-rate mortgages varies from country to country. In countries such as the United States, Germany and France, the majority of mortgages are fixed rate. However, the predominant type of mortgages in countries such as the United Kingdom, Australia and Spain is variable.

Mortgage rate changes affect the amount of mortgage interest payments, causing a direct cash-flow effect on consumption. Interest rate changes also affect housing demand and housing prices. If households are using housing as a collateral, the value of this collateral changes, inducing a wealth effect on household behavior and indirectly affecting consumption (ECB (2003), HM Treasury (2003)). Interest rate shocks affect mortgage rates differently depending on whether the mortgage is fixed or variable rate. Variable-rate mortgages are mortgage loans for which the interest rate is adjusted periodically, typically in line with some measured short-term interest rate. Hence, interest rate shocks directly affect variable rates. In contrast, fixed-rate mortgages are mortgage loans for which the interest rate remains constant through the term of the loan. The fixed interest rate is tied to a longer-term interest rate and is less sensitive to changes in the policy rate.

This raises important questions: How does the mortgage rate structure affect the way macroeconomic shocks are propagated? What are the implications in terms of monetary policy and welfare? These questions are of academic and policy interest. To give an illustrative example, the United Kingdom Treasury explicitly mentions the difference in mortgage structures as an important reason not to join the euro area. In the UK, the vast majority of borrowers have variable-rate mortgages, as opposed to the large countries of the euro area. According to the UK Treasury, British households are more exposed to monetary policy changes than, say, German households (HM Treasury (2003), Miles (2004)).

To address these questions, I build a New Keynesian dynamic stochastic general equilibrium model with housing and collateral constraints to explore how shocks are propagated in the presence of mortgage heterogeneity. I introduce fixed and variable-rate mortgages in the model. For the proportion of variable-rate mortgages to matter via the direct, cash-flow effect of mortgage interest payments on consumption, borrowers and savers are needed. Then, the effect of interest rate changes on borrowing does not cancel out by the presence of a representative consumer. For the indirect, wealth effect to appear, one needs non-durable consumption to be related to house prices. The introduction of collateral constraints tied to housing value for one type of consumers solves both problems since it motivates the presence of borrowers and savers and relates housing prices to consumption. In this model, monetary policy has real effects that are comparable with other sticky-price models. Furthermore, since the model is microfounded it allows me to study optimal monetary policy and welfare.[2]

It is not the aim of this paper to explain how the decision between fixed and variable-rate mortgages is made.[3] For simplicity, I hold the proportion of fixed and variable-rate borrowers constant and exogenous. Although this proportion can vary in reality, there is evidence that it fluctuates around a constant mean which is higher or lower depending on the country.[4] We could think of these cross-country differences as due to institutional, historical or cultural factors, out of the scope of this model.[5]

I use the model to compute impulse responses to interest rate, inflation target and

[2] The analysis of optimal monetary policy is restricted to optimization over parameters of a simple implementable Taylor rule.

[3] See Miles (2004) or Campbell and Cocco (2003) for studies that cover this from a microeconomic perspective.

[4] See Appendix 1 for evidence for the UK and the US.

[5] The European Mortgage Federation (EMF) highlights that cultural differences play an important role for the predominant type of mortgage contract in a country. They are linked to real estate law, borrowers' risk aversion, funding system or frequency of house moves.

technology shocks. I consider two extreme cases; one in which the economy is composed by variable-rate borrowers and one where the fixed rate is the predominant type of mortgage.

Results show that interest rate shocks affect more strongly those borrowers that have variable-rate mortgages. Given an increase in the interest rate set by the central bank, variable-rate borrowers reduce their consumption and housing demand by more than fixed-rate borrowers. The intuition is as follows: After a monetary policy shock (increase in the interest rate), fixed and variable-rate consumers differ in the real interest rate they face. Consider the most extreme case in which the variable rate changes one for one with the interest rate set by the central bank and the fixed rate is constant. After the shock, the nominal mortgage rate increases for the variable-rate individuals and inflation decreases. For the fixed-rate borrowers, the nominal mortgage interest rate does not react, but inflation is still decreasing because the economy is contracting. As a result, real rates increase by more if the mortgage is variable rate. In real terms, payments are increasing by more for variable-rate consumers, and their consumption and housing decrease by more (this is a pure cash-flow effect). A second, wealth effect comes through the collateral constraint. Banks are willing to lend as long as debt repayments do not exceed a fixed proportion of the value of the house collateral. For borrowers with variable-rate mortgages the value of their collateral has been reduced by more since they are demanding less houses. These effects make consumption decrease more strongly for variable-rate borrowers.

Aggregate consumption also declines by more after a monetary policy shock when the economy is mainly borrowing at a variable rate. However, aggregate differences are more muted due to the behavior of savers. In equilibrium, borrowing and saving must be equal. If borrowing decreases, saving must also decrease. Savers are the owners of financial intermediaries in the model, so any loss for the borrowers is a gain for the savers. These manage to offset part of the decrease in consumption following a positive interest rate shock. Results for monetary policy shocks are very robust to different model specifications.

Some aggregate differences arise because the borrowers' marginal propensity to consume is larger than the savers'.[6] However, aggregate differences are not large because interest rate shocks are not very persistent. Also, income effects on labor supply are important in this model. With the type of preferences used in standard real business cycle models, labor effort is determined together with the intertemporal consumption choice. When consumption is reduced, individuals tend to work more to compensate and smooth consumption. Using preferences as in Greenwood, Hercowitz and Huffman (1988)(GHH henceforth), this effect is eliminated. In this case, the channels that are important for the purposes of this paper are emphasized and aggregate effects are larger.

In contrast, inflation target shocks generate larger aggregate differences between scenarios. In particular, when the inflation target increases, output responds by more when variable rates are predominant. Real interest rates fall persistently and house prices increase by less than with fixed mortgage rates. Variable-rate borrowers increase their nondurable consumption by more. Since house prices do not increase as much in the variable-rate case, also savers can consume more nondurables.

Finally, I consider technology shocks. A favorable technology shock increases output and lowers prices. Monetary policy responds in a persistent way and real rates increase. Variable-rate borrowers consume less because the real rate increase affects them and dampens the positive effects of the technology shock for them. The increase in real rates does not affect fixed-rate consumers as much and they can consume more. Output increases by more when fixed rates are predominant.

I also study welfare and optimal monetary policy in the context of fixed and variable rate mortgages. In particular, I search over parameters of a simple, implementable interest rate rule so that welfare is maximized. I find that, in the presence of collateral constraints, a social welfare maximizing central bank should respond to inflation less aggressively than in the absence of collateral constraints. Results also show that when the central bank focuses only on the savers'welfare, thus ignoring the collateral constraint, the optimized inflation parameter in the Taylor rule is higher. However, when borrowers are taking into account,

[6] In this model borrowers face collateral constraints and are more impatient than savers. This makes their consumption respond by more to changes in wealth.

the central bank optimally responds less to inflation. The central bank faces a trade-off between the borrowers and savers' welfare because on the one hand, low inflation corrects the sticky-price distortion but, on the other hand, inflation relaxes the collateral constraint and improves borrowers' welfare. Comparing welfare across mortgage rate scenarios for given policy shows that this inflation channel is more effective the higher the proportion of fixed-rate mortgages in the economy. Therefore, borrowers are better off with fixed-rate mortgages although this comes at the cost of lower welfare for savers. For aggregate welfare, I find that predominantly fixed-rate contracts are welfare enhancing.

This paper relates to different strands of literature. First, it contributes to the literature on New Keynesian general equilibrium models with housing and collateral constraints such as Aoki et al. (2004) and Iacoviello (2005), who do not consider heterogeneous mortgage contracts. Second, it is also related to a literature that studies fixed and variable-rate mortgages. Campbell and Cocco (2003) and Miles (2004) study the fixed versus variable rate choice from a partial equilibrium perspective. Graham and Wright (2007) develop a model in which some households face binding credit constraints and debt contracts can be fixed or variable rate. However, they do not include a housing market and thus the constraint is not tied to housing stock and housing prices, eliminating the wealth channel. Calza et al. (2007) study how institutional factors, including mortgage contracts, can affect the monetary transmission mechanism. In my model, I focus on fixed versus variable rate mortgages. My results on monetary policy shocks are comparable to theirs under some parameter specifications. Relative to them, I do not only study the exogenous component of monetary policy but also the systematic response to other shocks. The existent literature is silent about how mortgage heterogeneity affects the way shocks such as changes in inflation target or technology are propagated. Finally the paper contributes to the literature on optimal monetary policy with heterogeneous consumers and collateral constraints. See for instance Monacelli (2006) or Mendicino and Pescatori (2007). However, none of these papers studies optimal monetary policy in the context of different mortgage contracts.

Section II explains the basic model I build for the analysis. Section III shows the results and dynamics and business cycles of the model. Section IV analyzes optimal monetary policy. Section V presents the conclusions. Appendix 1 contains graphs and tables on the empirical evidence mentioned above. Appendix 2 shows model derivations.

1.2 The Baseline Model

I consider an infinite-horizon economy in which households consume, work and demand real estate. There is a representative financial intermediary that provides mortgages and accepts deposits from consumers. Firms set prices subject to Calvo (1983)-Yun (1996) nominal rigidity. The monetary authority sets interest rates endogenously, in response to inflation and output, following a Taylor rule.

1.2.1 The Consumer's Problem

There are three types of consumers: unconstrained consumers, constrained consumers who borrow at a variable rate, and constrained consumers who borrow at a fixed rate. Constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments for both mortgages and loans cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. This assumption ensures that the borrowing constraint is binding, so that constrained individuals do not save and wait until they have the funds to self-finance their consumption. This generates an economy in which households divide into borrowers and savers. Furthermore, borrowers are divided into two groups, those who borrow at a fixed rate and those who borrow at a variable rate. The proportion of each type of borrower is fixed and exogenous. All households derive utility from consumption, housing services assumed proportional to the housing stock and leisure.[7]

[7] I do not allow for renting. This is needed to generate borrowers and savers in the economy. If renting were allowed, borrowers could use renting to save and the wealth effect would disappear. Furthermore, in the US, homeownerships have been quite high in the last years (about 65 percent, according to the US Census

Unconstrained Consumers (Savers) Unconstrained consumers maximize:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{1}$$

where, E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, and C_t^u, H_t^u and L_t^u are consumption at t, the stock of housing and hours worked, respectively; $1/(\eta - 1)$ is the labor supply elasticity, $\eta > 0$ and $j > 0$ represents the weight of housing in the utility function.

The budget constraint is:

$$C_t^u + q_t H_t^u + b_t^u \leq q_t H_{t-1}^u + w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t^v + S_t^v, \tag{2}$$

where q_t is the real housing price and w_t^u is the real wage for unconstrained consumers. These can buy houses or sell them at the current price q_t. I assume zero housing depreciation for simplicity. As we will see, this group will choose not to borrow at all; they are the savers in this economy. b_t^u is the amount they save. They receive interest R_{t-1} for their savings. π_t is inflation in period t. S_t and F_t are lump-sum profits received from the firms and the financial intermediary, respectively. We can think of these consumers as the wealthy agents in the economy, who own the firms and the financial intermediary.

The first-order conditions for this unconstrained group are:

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \tag{3}$$

$$w_t^u = (L_t^u)^{\eta-1} C_t^u, \tag{4}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}. \tag{5}$$

Bureau).

Equation (3) is the Euler equation for consumption, equation (4) is the labor-supply condition, and equation (5) is the Euler equation for housing. This states that the benefits from consuming housing must be equal to the costs at the margin.

Constrained Consumers (Borrowers) Constrained consumers can be of two types: those who borrow at a variable rate and those who do it at a fixed rate. The proportion of variable-rate consumers is fixed and exogenous and equal to $\alpha \in [0,1]$.

Constrained and unconstrained consumers are different in the way they discount the future. Constrained consumers are more impatient than unconstrained ones. I assume that constrained consumers face a limit on the debt they can acquire. The maximum amount they can borrow is proportional to the value of their collateral, in this case the stock of housing. That is, the debt repayment next period cannot exceed a proportion of tomorrow's value of today's stock of housing:

$$E_t \frac{R_t^{ci}}{\pi_{t+1}} b_t^{ci} \leq k E_t q_{t+1} H_t^{ci}, \tag{6}$$

where $i = v$ if the constrained consumer borrows at a variable rate and $i = f$ if he or she borrows at a fixed rate, and $R_t^{ci} = R_t$ if $i = v$, $R_t^{ci} = \overline{R}_t$ if $i = f$.

Constrained consumers maximize their lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max \; E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln C_t^{ci} + j \ln H_t^{ci} - \frac{\left(L_t^{ci}\right)^\eta}{\eta} \right), \tag{7}$$

subject to:

$$C_t^{ci} + q_t H_t^{ci} + \frac{R_{t-1}^{ci} b_{t-1}^{ci}}{\pi_t} \leq q_t H_{t-1}^{ci} + w_t^{ci} L_t^{ci} + b_t^{ci}, \tag{8}$$

and (6).[8]

As noted above, constrained consumers are more impatient than unconstrained ones,

[8] We will see from the firm's problem that $w_t^{cv} = w_t^{cf} = w_t^c$.

so that $\widetilde{\beta} < \beta$. This assumption is crucial for the borrowing constraint to be binding and therefore, for there to be both borrowers and savers in the economy.

The first-order conditions for constrained consumers are:

$$\frac{1}{C_t^{ci}} = \widetilde{\beta} E_t \left(\frac{R_t^{ci}}{\pi_{t+1} C_{t+1}^{ci}} \right) + \lambda_t^{ci} R_t^{ci}, \tag{9}$$

$$w_t^{ci} = \left(L_t^{ci} \right)^{\eta - 1} C_t^{ci}, \tag{10}$$

$$\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{ci}} q_{t+1} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}. \tag{11}$$

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint $\left(\lambda_t^{ci} \right)$ appears in equations (9) and (11). From the Euler equations for consumption of unconstrained consumers, we know that $R = 1/\beta$ in steady state. If we combine this result with the Euler equation for consumption of constrained individuals we have that $\lambda^{ci} = \left(\beta - \widetilde{\beta} \right) / C^{ci} > 0$ in steady state. This means that the borrowing constraint holds with equality in steady state. Since we log-linearize the model around the steady state and assume that uncertainty is low, we can generalize this result to off-steady-state dynamics. Then, the borrowing constraint is always binding, so that constrained individuals are going to borrow the maximum amount they are allowed to and unconstrained consumers are never in debt.

Given the borrowing amount implied by (6) at equality, consumption for constrained individuals can be determined by their flow of funds:

$$C_t^{ci} = w_t^{ci} L_t^{ci} + b_t^{ci} + q_t \left(H_{t-1}^{ci} - H_t^{ci} \right) - \frac{R_{t-1}^{ci} b_{t-1}^{ci}}{\pi_t}, \tag{12}$$

and the first-order condition for housing becomes:

$$\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} \left(q_t - \frac{k E_t q_{t+1} \pi_{t+1}}{R_t^{ci}} \right) - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{ci}} \left(1 - k \right) q_{t+1}. \tag{13}$$

Aggregate Variables Given the fraction α of variable-rate borrowers, we can define aggregates across constrained consumers as $C_t^c \equiv \alpha C_t^{cv} + (1-\alpha) C_t^{cf}, L_t^c \equiv \alpha L_t^{cv} + (1-\alpha) L_t^{cf}, H_t^c \equiv \alpha H_t^{cv} + (1-\alpha) H_t^{cf}, b_t^c \equiv \alpha b_t^{cv} + (1-\alpha) b_t^{cf}$.

Therefore, economy-wide aggregates are: $C_t \equiv C_t^u + C_t^c, L_t \equiv L_t^u + L, H_t \equiv H_t^u + H_t^c$. In this model, aggregate supply of housing is fixed, so that market clearing requires[9]: $H_t = H_t^u + H_t^c = H$.

1.2.2 The Financial Intermediary

The financial intermediary accepts deposits from savers, and extends both fixed and variable-rate loans to borrowers. The profits of the financial intermediary are:

$$F_t = \alpha R_{t-1} b_{t-1}^{cv} + (1-\alpha) \overline{R}_{t-1} b_{t-1}^{cf} - R_{t-1} b_{t-1}^u. \tag{14}$$

To simplify, since the typical time horizon of a mortgage is large, I consider the maturity of mortgages to be infinite, although this assumption is not crucial for the dynamics of the problem.

In equilibrium, aggregate borrowing and saving must be equal, that is,

$$b_t^c = b_t^u. \tag{15}$$

Substituting (15) into (54) ,we obtain,

$$F_t = (1-\alpha) b_{t-1}^{cf} \left(\overline{R}_{t-1} - R_{t-1} \right). \tag{16}$$

I assume that the financial intermediary operates under perfect competition. Therefore, the optimality condition for the financial intermediary implies that at each point in time τ, the intermediary is indifferent between lending at a variable or fixed rate. Hence, the expected discounted profits that the intermediary obtains by lending new debt in a

[9] This assumption provides an easy way to specify the supply of housing and have variable prices. A two-sector model with production of housing does not generate significatively different results (see Appendix 2).

given period at a fixed interest rate must be equal to the expected discounted profits the intermediary would obtain by lending it at variable rate:

$$E_{\tau}\sum_{i=\tau+1}^{\infty}\Lambda_{\tau,i}\overline{R}_{\tau}^{*}\left(b_{\tau}^{cf}-b_{\tau-1}^{cf}\right)=E_{\tau}\sum_{i=\tau+1}^{\infty}\Lambda_{\tau,i}R_{i-1}\left(b_{\tau}^{cf}-b_{\tau-1}^{cf}\right), \tag{17}$$

where $\Lambda_{t,i}=\beta^{i-t}\left(\frac{C_t^u}{C_{t+i}^u}\right)$ is the unconstrained consumer relevant discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary's problem.

We can obtain the optimal value of the fixed rate in period τ from expression (56) :

$$\overline{R}_{\tau}^{*}=\frac{E_{\tau}\sum_{i=\tau+1}^{\infty}\Lambda_{\tau,i}R_{i-1}}{E_{\tau}\sum_{i=\tau+1}^{\infty}\Lambda_{\tau,i}}. \tag{18}$$

Equation (57) states that, for every new debt issued at date τ, there is a different fixed interest rate that has to be equal to a discounted average of future variable interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period τ and the new amount of debt in period τ are fixed for all future periods. However, the fixed interest rate varies with the date the debt was issued, so that in every period there is a new fixed interest rate associated with new debt in this period. If we consider fixed-rate loans to be long-term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed interest rate that is the one the financial intermediary effectively charges every period. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period optimal fixed interest rate and new amount of debt. Therefore, the effective fixed interest rate that the financial intermediary charges for the stock of fixed-rate debt every period is:

$$\overline{R}_t = \frac{\overline{R}_{t-1} b^{cf}_{t-1} + \overline{R}^*_t \left(b^{cf}_t - b^{cf}_{t-1} \right)}{b^{cf}_t}. \tag{19}$$

Equation (58) states that the fixed interest rate that the financial intermediary is actually charging today is an average of what it charged last period for the previous stock of mortgages and what it charges this period for the new amount. Importantly, this assumption is not crucial for results. Both $\overline{R}^*_\tau$ and $\overline{R}_t$ are practically unaffected by interest rate shocks. This assumption is a way to reconcile the model with the fact that fixed-rate loans are not one-period assets but longer term ones.

As noted above, if any, profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and it does not make profits in absence of shocks, if there is a shock at a given point in time, the fact that only the variable interest rate is affected can generate non-zero profits.

1.2.3 Firms

Final Goods Producers There is a continuum of identical final goods producers that aggregate intermediate goods according to the production function

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{20}$$

where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The final good firm chooses $Y_t(z)$ to minimize its costs, resulting in demand of intermediate good z:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t. \tag{21}$$

The price index is then given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{\varepsilon-1}}. \tag{22}$$

Market clearing for the final good requires:

$$Y_t = C_t = C_t^u + C_t^c.$$

Intermediate Goods Producers The intermediate goods market is monopolistically competitive. Intermediate goods are produced according to the production function:

$$Y_t(z) = A_t L_t^u(z)^{\gamma} L_t^c(z)^{(1-\gamma)}, \tag{23}$$

where $\gamma \in [0,1]$ measures the relative size of each group in terms of labor. [10] A_t represents technology and it follows the following autoregressive process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + u_{At}, \tag{24}$$

where ρ_A is the autorregressive coefficient and u_{At} is a normally distributed shock to technology.

Labor demand is determined by:

$$w_t^u = \frac{1}{X_t}\gamma \frac{Y_t}{L_t^u}, \tag{25}$$

$$w_t^c = \frac{1}{X_t}(1-\gamma)\frac{Y_t}{L_t^c}, \tag{26}$$

where X_t is the markup, or the inverse of marginal cost.[11]

The price-setting problem for the intermediate good producers is a standard Calvo-Yun setting. An intermediate good producer sells its good at price $P_t(z)$, and $1-\theta, \in [0,1]$, is the probability of being able to change the sale price in every period. The optimal reset

[10] This Cobb-Douglas production function implies that labor efforts of constrained and unconstrained consumers are not perfect substitutes. This assumption can be justified by the fact that savers are the managers of the firms and their wage is not the same as the one of the borrowers. Experimenting with a production function in which hours are substitutes leads to very similar results (See Appendix 2). The Cobb-Douglas specification is analytically tractable and allows for closed form solutions for the steady state of the model.

[11] Symmetry across firms allows us to write the demands without the index z.

price $P_t^*(z)$ solves:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left\{ \Lambda_{t,k} \left[\frac{P_t^*(z)}{P_{t+k}} - \frac{\varepsilon/(\varepsilon-1)}{X_{t+k}} \right] Y_{t+k}^*(z) \right\} = 0. \tag{27}$$

The aggregate price level is then given by:

$$P_t = \left[\theta P_{t-1}^{\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}. \tag{28}$$

Using (63) and (64), and log-linearizing, we can obtain a standard forward-looking New Keynesian Phillips curve which is presented in the A.

1.2.4 Monetary Policy

The model is closed with a Taylor Rule with interest rate smoothing, to describe the conduct of monetary policy by the central bank:[12]

$$R_t = (R_{t-1})^{\rho} \left[\left(\frac{\pi_t}{\pi_t^*} \right)^{(1+\phi_\pi)} R \right]^{1-\rho} \varepsilon_{Rt}, \tag{29}$$

where $0 \leq \rho \leq 1$ is the parameter associated with interest-rate inertia, and $\phi_\pi > 0$ measures the response of interest rates to current inflation. R is the steady-state values of the interest rate. ε_{Rt} is a white noise shock with zero mean and variance σ_ε^2. π_t^* is the inflation target that evolves according to:

$$\log(\pi_t^*) = \rho_\pi \log(\pi_{t-1}^*) + \varepsilon_{\pi t}, \tag{30}$$

where $\varepsilon_{\pi t}$ is normally distributed with variance σ_π^2.

[12] This is a realistic policy benchmark for most of the industrialized countries. A more realistic rule would also include output but it complicates building intuition about the workings of the model. Furthermore, estimations deliver a small response to the output gap in the last two decades (See Clarida, Gali and Gertler (2000)).

1.3 Shock Transmission and Business Cycles

I linearize the equilibrium equations around the steady state. Details are shown in Appendix 2. For calibration, I consider the following parameter values: The discount factor, β, is set to 0.99 so that the annual interest rate is 4% in the steady state. The discount factor for borrowers, $\widetilde{\beta}$, is set to 0.98. Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency. Results are not sensitive to different values within this range. This value of $\widetilde{\beta}$ is low enough to endogenously divide the economy into borrowers and savers. The weight of housing on the utility function, j, is set to 0.1 in order for the ratio of housing wealth to GDP in the steady state to be consistent with the data. This value of j implies a ratio of approximately 1.40, in line with the Flow of Funds data.[13] I set $\eta = 2$, implying a value of the labor supply elasticity of 1.[14] For the loan-to-value ratio, I pick $\kappa = 0.9$, consistent with the evidence that in the last years borrowing constrained consumers borrowed on average more than 90% of the value of their house.[15] The labor income share of unconstrained consumers, γ, is set to 0.64, following the estimate in Iacoviello (2005). I pick a value of 6 for ε, the elasticity of substitution between intermediate goods. This value implies a steady state markup of 1.2. The probability of not changing prices, θ, is set to 0.75, implying that prices change every four quarters. For the Taylor Rule parameters I use $\rho = 0.8$, $\phi_\pi = 0.5$. The first value reflects a realistic degree of interest-rate smoothing.[16] The second one, is consistent with the original parameter proposed by Taylor in 1993. For α, I consider two polar cases for comparison. In the first case, the proportion of variable-rate mortgages in the economy is 0, that is, all constrained consumers in the economy borrow at a fixed rate. In the second case, the proportion of variable-rate mortgages is 1. Table 1 shows a summary of the parameter values.

[13] See Table B.100. In this model, consumption is the only component of GDP. To make the ratio comparable with the data I multiply it by 0.6, which is approximately what nondurable consumption and services account for in the GDP, according to the data in the NIPA tables.

[14] Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.

[15] We can identify constrained consumers with those that borrow more than 80% of their home. In the US, among those borrowers, the average LTV ratio exceeds 90% for the period 1973-2006. See the data from the Federal Housing Finance Board.

[16] See McCallum (2001).

Parameter Values		
β	.99	Discount Factor for Savers
$\widetilde{\beta}$	.98	Discount Factor for Borrowers
j	.1	Weight of Housing in Utility Function
η	2	Parameter associated with labor elasticity
k	.9	Loan-to-value ratio
γ	.64	Labor share for Savers
α	0/1	Proportion of variable-rate borrowers
X	1.2	Steady-state markup
θ	.75	Probability of not changing prices
ρ_π	.975	Inflation target persistence
ρ_A	.9	Technology persistence
ρ	.8	Interest-Rate-Smoothing Parameter in Taylor Rule
ϕ_π	.5	Inflation Parameter in Taylor Rule

Table 1: Parameter Values

1.3.1 Impulse Responses

Monetary Policy Shock Impulse responses to a one standard deviation (0.29 percent) increase of the interest rate are presented in Figure 1.[17] We can see that when the economy is mainly composed by individuals indebted at a variable-rate, the effects of monetary policy on consumption for the borrowers are stronger than in the fixed-rate case. Borrowers' housing demand, initially, also decreases more strongly after a monetary policy shock if the predominant type of mortgages in the economy is variable rate. These findings show that the proportion of variable-rate mortgages matters for the monetary transmission mechanism.

[17] Iacoviello (2005) estimates a Taylor Rule for the US economy and finds a 0.29 percent standard deviation on a quarterly basis. I use this number as an empirically plausible one-standard deviation increase in the interest rate.

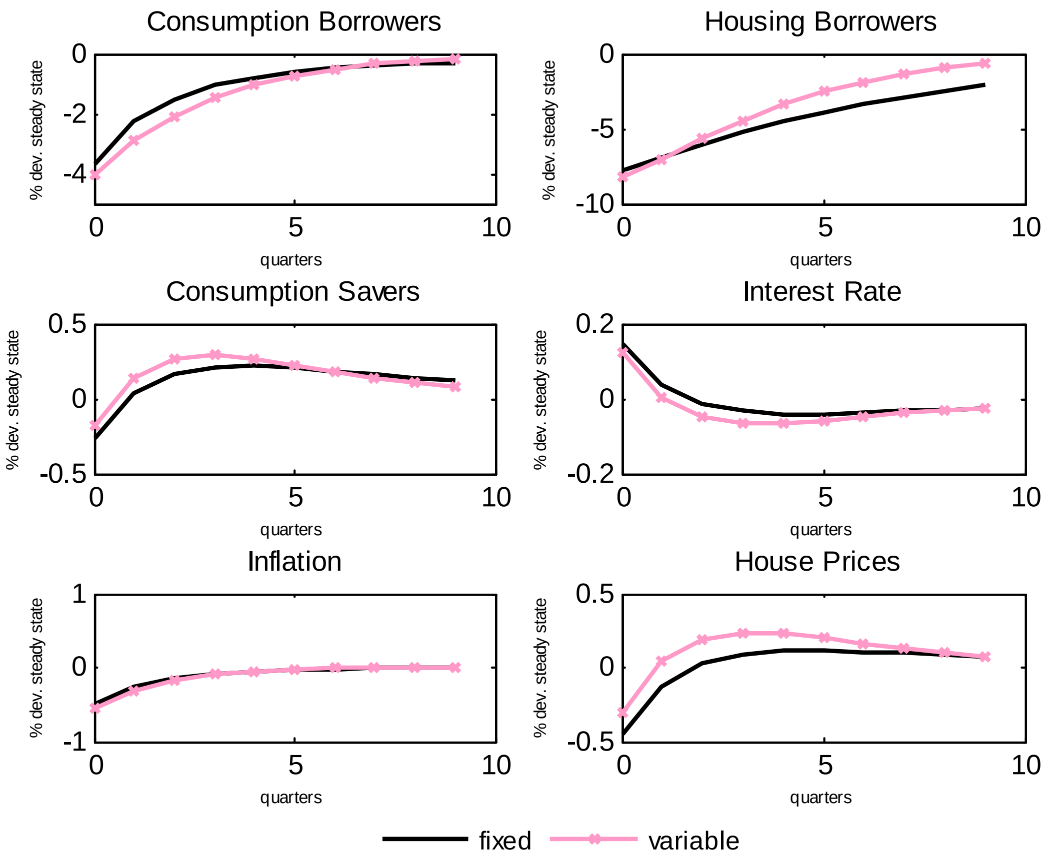

Figure 1: Impulse Responses to a Monetary Policy Shock. Baseline Specification.

When the proportion of variable-rate borrowers is very high, a monetary policy shock affects more strongly those individuals who are constrained and need to borrow.

In the aggregate, output in the variable-rate economy also decreases more strongly (See Figure 2). There is a redistribution between borrowers and savers but we can still find aggregate differences because borrowers are more sensitive to changes in wealth (they are more impatient and use housing wealth as collateral).

Sensitivity Analysis Differences between the two scenarios are not larger because monetary policy shocks are not very persistent and the share of borrowers in the economy is not very large. Results are sensitive to the wage share of unconstrained individuals in the

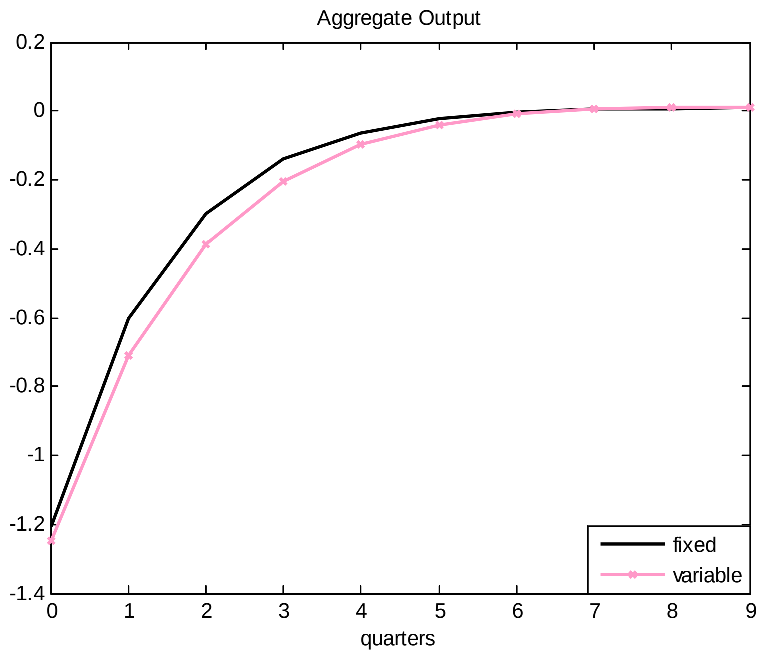

Figure 2: Aggregate Output Response to a Monetary Policy Shock. Baseline Specification.

economy.[18] Figure 3 shows that by decreasing the size of the savers aggregate differences are amplified.

In this model income effects on the labor supply decision are important. In the baseline model preferences are separable in consumption and labor. In this case, the labor supply decision depends on the level of consumption. Given a negative shock to the economy, labor supply moves both in response to a substitution and an income effect. On the one hand, lower wages make consumers want to work less. On the other hand, lower consumption generates an income effect that makes consumers want to work more. Income effects can partly offset aggregate differences. GHH preferences have the property of shutting down the income effect on the labor supply decision. In this preferences, labor and consumption are non-separable. This makes labor effort to be determined independently from the intertemporal consumption-savings choice.[19] There an extensive literature that has also

[18] This parameter represents the relative economic size of each group in the economy.
[19] See Appendix 2 for details on GHH preferences and derivations.

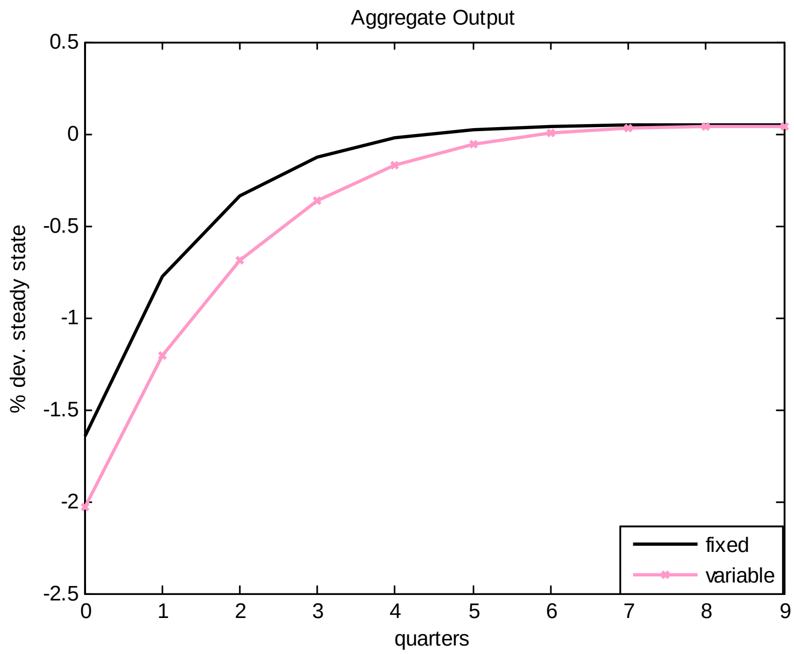

Figure 3: Aggregate Output Response to a Monetary Policy Shock. Increasing the share of borrowers to 60%.

used these preferences to emphasize other channels that are partially offset by this income effects.[20] Impulse responses, in line with other studies that use GHH preferences, show how consumption responses are stronger and aggregate differences are amplified (See Figure 4).

Results for monetary policy shocks with standard preferences are very robust to alternative model specifications. We can introduce capital in the basic model or assume nonseparability between housing and consumption in the utility function. The basic results for the variables of interest are maintained.[21]

Inflation Target Shock Instead of a shock to the interest rate, we can also consider a more persistent monetary policy disturbance such as a shock to the inflation target. Figure 5 shows the responses of the variables of interest to an increase in the inflation target of 0.1

[20] See for example Raffo (2006) and references therein.

[21] The details of the model are presented in Appendix 2. The parameter values used for the calibration are 0.025 for capital depreciation and 10 for capital adjustment costs. The elasticity of substitution between non-durable consumption goods and housing of 0.5. The rest of the parameter values are the same as in the baseline model.

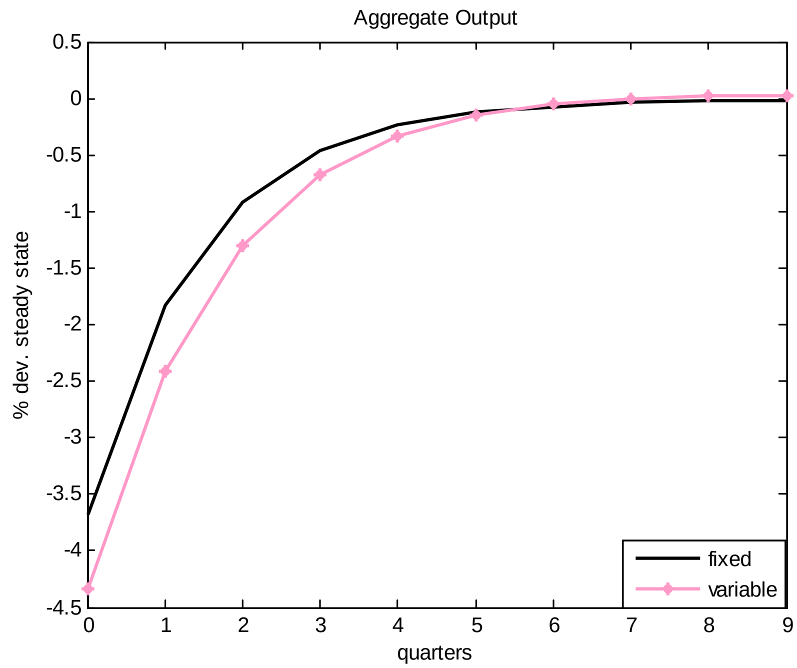

Figure 4: Aggregate Output Response to a Monetary Policy Shock. GHH Preferences.

percent, with 0.975 persistence.[22]

Aggregate differences are amplified with this type of shock. Output increases by more in the variable-rate case. Monetary policy responds systematically to the shock in a very persistent way. Real interest rates fall persistently and house prices increase by less in the variable-rate economy. Variable-rate borrowers increase by more their nondurable consumption because real rates fall. Since house prices do not increase that much in the variable-rate case, also savers can consume more nondurables.

Technology Shock A shock to technology may also have different effects on the economy depending on whether individuals are mainly borrowing at variable or fixed rate. Impulse responses to a 1 percent positive shock to technology with 0.9 persistence are showed in Figure 6.[23] We see that the economy responds more strongly after a technology shock when

[22] In line with Adolfson et al. (2007) or Iacoviello and Neri (2008).

[23] This high value of persistence is consistent with estimates in the literature. See for instance Iacoviello and Neri (2008).

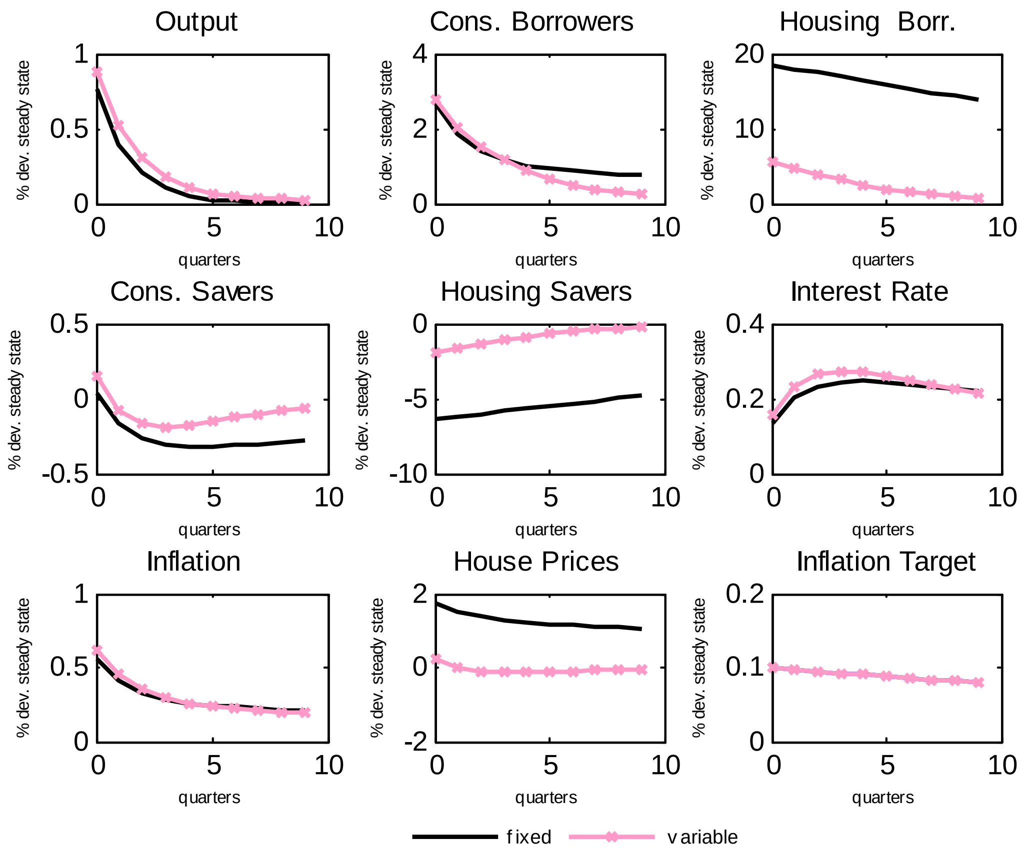

Figure 5: Impulse Responses to an Inflation Target Shock. Baseline Specification.

the majority of its borrowers have a fixed-rate mortgage. A technology shock increases output and lowers prices. As a reaction, real interest rates increase in a very persistent way. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. As a result, output increases by more for fixed-rate consumers.

Monetary policy shocks or inflation target shocks cause the real interest rate to vary countercyclically, which is why flexible-rate mortgages amplify the effects those shocks. Technology shocks, by contrast, cause the real interest rate to vary procyclically: it rises when output rises, which is why flexible-rate mortgages dampen the effects of those shocks.

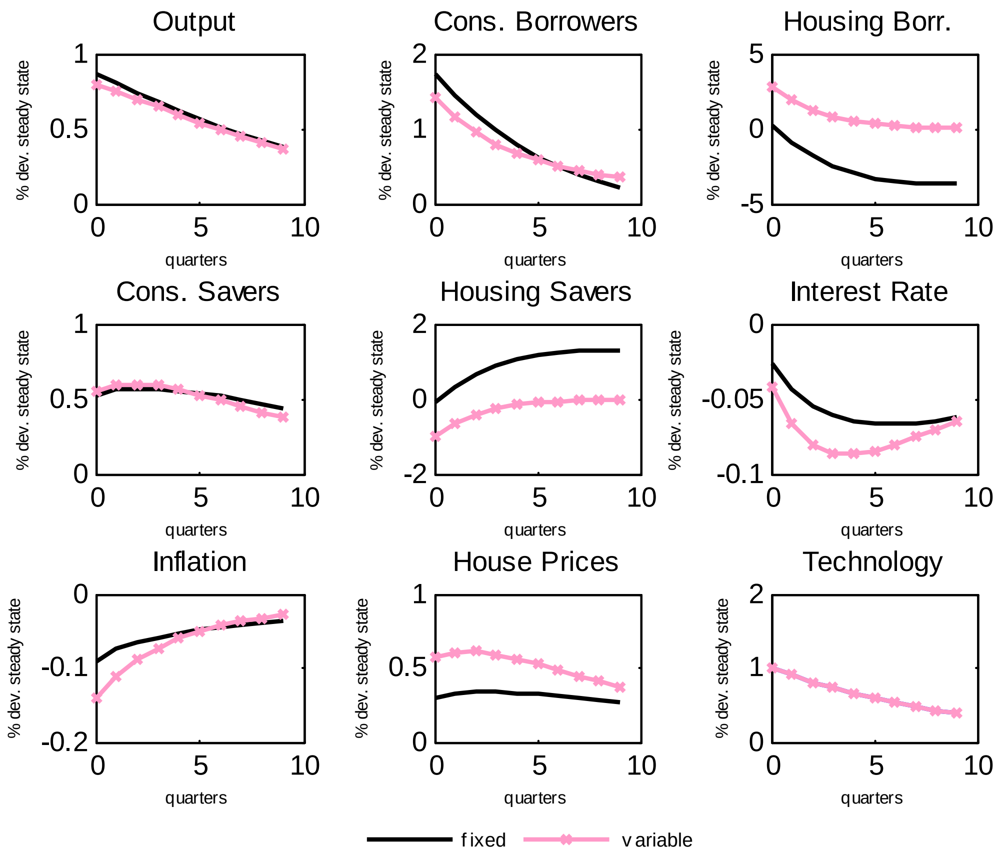

Figure 6: Impulse Responses to a Technology Shock. Baseline Model.

1.3.2 Second Moments

Table 2 shows the standard deviations of the main variable both from the model and the data.[24] The model generates a standard deviation of GDP of 2.0127 for the variable-rate case and 2.126 for the fixed-rate economy. This is slightly smaller but close to the data (2.26), especially for the fixed-rate economy.[25]The volatility of consumption and housing demand is always greater for those individuals that are constrained but smaller in the case of variable rates. The volatility of inflation and house prices is smaller in the model than in the data while the correlation between output and house prices is greater.

[24] Theoretical moments calculated for technology shocks. Standard deviations from the data taken from Davis and Heathcote (2005).

[25] Davis and Heathcote (2005) also find smaller output volatility.

Business Cycle Properties				
% SD Rel. to GDP		**Data**	**Model (Fixed Rates)**	**Model (Variable Rates)**
	$\mathbf{y}$	2.26	2.126	2.013
	$\mathbf{c}^u$		0.931	0.904
	$\mathbf{c}^c$		1.413	1.304
	$\mathbf{h}^u$		2.276	0.646
	$\mathbf{h}^c$		6.525	1.852
	π	0.78	0.094	0.121
	$\mathbf{q}$	1.37	0.552	0.911
Correlations	$\mathbf{y}, \mathbf{q}$	0.65	0.960	0.993

Table 2: Business Cycle Properties.

1.4 Welfare and Optimal Monetary Policy

In this section, I compare different simple monetary policy rules based on welfare evaluations, both for the whole economy and for different types of consumers, in order to provide some normative assessment.

The individual welfare for savers and borrowers respectively is defined as follows:[26]

$$V_{u,t} \equiv \; E_t \sum_{m=0}^{\infty} \beta^m \left(\ln C^u_{t+m} + j \ln H^u_{t+m} - \frac{\left(L^u_{t+m}\right)^{\eta}}{\eta} \right), \tag{31}$$

$$V_{ci,t} \equiv \; E_t \sum_{m=0}^{\infty} \widetilde{\beta}^m \left(\ln C^{ci}_{t+m} + j \ln H^{ci}_{t+m} - \frac{\left(L^{ci}_{t+m}\right)^{\eta}}{\eta} \right), \tag{32}$$

Following Mendicino and Pescatori (2007), I define social welfare as a weighted sum of individual welfare for the different types of households:

$$V_t = (1-\beta)\, V_{u,t} + \left(1-\widetilde{\beta}\right) \left[\alpha V_{cv,t} + (1-\alpha)\, V_{cf,t}\right]. \tag{33}$$

[26] I numerically compute the second order approximation of the utility function as a measure of welfare.

Borrowers and savers' welfare are weighted by $\left(1-\widetilde{\beta}\right)$ and $(1-\beta)$ respectively, so that the two groups receive the same level of utility from a constant consumption stream. As in Mendicino and Pescatori (2007), I take this approach to be able to evaluate the welfare of the three types of agents separately.[27]

To begin, I evaluate the welfare achieved under the ad-hoc Taylor rule used in the baseline model. Results are presented in Table 3:

Ad-hoc Taylor Rule: $\rho = 0.8, \phi_\pi = 0.5$		
	Variable Rate	**Fixed Rate**
Social Welfare	-6.0693	-4.7097
Savers Welfare	21.5748	-822.6597
Borrowers Welfare	-314.2514	175.8456
$\boldsymbol{\sigma}(\pi)$	0.2436	0.1999

Table 3: Welfare comparison. Ad-hoc Taylor Rule.

The economy with fixed-rate mortgages achieves a higher level of welfare than the variable-rate economy. Notice as well that there is a trade-off between savers and borrowers' welfare: Although a larger fraction of fixed-rate borrowers raises aggregate welfare, this comes at the cost of lower welfare for savers.

Figure (7) shows how the welfare level varies with the proportion of variable rate mortgages in the economy.[28] This figure clearly illustrates this trade-off. When mortgages are at a fixed rate, savers, who own the financial intermediary, bear all the risk associated with interest rate changes and therefore their welfare is lower. Borrowers, are however insured against interest-rate risk and their collateral constraint is relaxed when mortgage rates are fixed, and thus their welfare is higher. If we look at the loglinearized collateral constraint (see equation (80) in Appendix 2), we can observe that, at a given level of inflation, in real terms, mortgage payments are lower, the lower the value of α is. As a result of this trade-off

[27] See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.

[28] Welfare is rescaled so that it appears in the positive axis. Additionally, borrowers and savers' welfare is divided by 100.

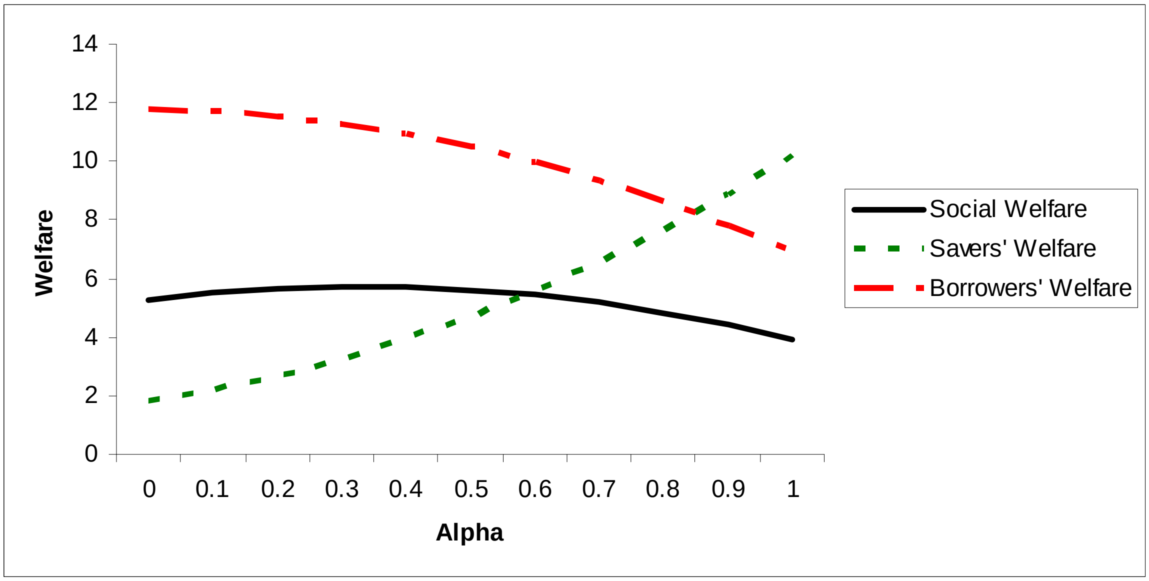

Figure 7: Welfare level for different values of α. Ad-hoc Taylor rule.

between borrowers and savers, the economy achieves the maximum level of social welfare at around the value of $\alpha = 0.3$, that is, when 70 percent of the mortgages are fixed rate.

Next, I study what is the monetary policy that maximizes welfare. The design of optimal monetary policy in the presence of collateral constraints is more complicated than in the standard sticky-price setting. In this case, there are two types of distortions, price rigidities and credit frictions. On the one hand, the central bank should aim at lowering inflation volatility because, given sticky prices, inflation distorts production decisions. On the other hand, inflation relaxes the borrowing constraints and improves the borrowers' welfare. And, as noticed above, this inflation channel is much more effective when fixed-rate mortgages are predominant. The loglinearized collateral constraint shows that mortgage payments decrease with inflation but increase with the interest rate. Inflation relaxes the collateral constraint for borrowers, as long as the interest rate does not react too much to it. Therefore, the inflation channel for borrower welfare is stronger the less the central bank responds to inflation but also the lower the value of α. In the limit, an economy with just fixed-rate mortgages maximizes the favorable effects of inflation on the collateral constraint.

Given a grid of possible parameters for the Taylor rule, I perform a search that maximizes welfare, subject to determinacy requirements. For simplicity, I start by keeping the value of ρ fixed to 0.8 and I search over different values of ϕ_π, the response coefficient to inflation. In this way, I can build intuition about on much the central bank should respond to inflation in different cases for the same degree of interest-rate smoothing. Results are presented in Table 4:

Optimized Taylor Rule (Maximize Social Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 0.1}$	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 0.1}$
Social Welfare	-3.0619	-0.9131
Savers Welfare	-256.1426	-3398.0803
Borrowers Welfare	-25.0259	1653.3866
$\boldsymbol{\sigma}(\pi)$	0.5611	0.5233
Optimized Taylor Rule (Maximize Savers Welfare)		
	Variable Rate	**Fixed Rate**
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 3.85}$	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 20}$
Social Welfare	-10.4962	-8.8568
Savers Welfare	146.6523	-4.4361
Borrowers Welfare	-598.1339	-440.6205
$\boldsymbol{\sigma}(\pi)$	0.0353	0.0058
Optimized Taylor Rule (Standard Sticky Price)		
	$\boldsymbol{\rho = 0.8, \phi_\pi^* = 20}$	
Social Welfare	-51.0065	
$\boldsymbol{\sigma}(\pi)$	0.0182	

Table 4: Welfare Values for Optimized Taylor Rule

For the model with collateral constraints, I consider two cases: a central bank that is a social welfare maximizer and a central bank that neglects the borrowers' welfare. Within each case, mortgage contracts are either fixed or variable rate. Then, I compare the results with a model without collateral constraints. As in Monacelli (2006) and Mendicino and Pescatori (2007), lenders prefer the central bank being aggressive against inflation. However, borrowers obtain welfare gains from a monetary policy that minimizes credit market inefficiencies. The central bank aggressively fights inflation if it considers only the welfare of those not facing credit constraints. However, economies with fixed-rate contracts achieve a higher welfare in all cases because they are less distorted by the collateral constraint. If we compare the results with a model without collateral constraints, we clearly see that the central bank should respond to inflation less aggressively than in the standard sticky-price model, without collateral constraints. In fact, for the model with collateral constraints the optimal value of ϕ_π corresponds to the minimum value allowed in the search while in the absence of collateral constraints it corresponds to the maximum one.

Optimized Taylor Rule (Maximize Social Welfare)		
	Variable Rate	**Fixed Rate**
	$\rho^* = 0.9, \phi_\pi^* = 0.35$	$\rho^* = 0.1, \phi_\pi^* = 0.1$
Social Welfare	-2.9985	20.7278
Savers Welfare	-183.2848	-16768.7875
Borrowers Welfare	-58.2802	9420.7829
$\sigma(\pi)$	0.3490	0.9423

Optimized Taylor Rule (Maximize Savers Welfare)		
	Variable Rate	**Fixed Rate**
	$\rho^* = 0.1, \phi_\pi^* = 0.35$	$\rho^* = 0.9, \phi_\pi^* = 20$
Social Welfare	-13.0488	-8.8798
Savers Welfare	187.7239	-1.2493
Borrowers Welfare	-746.3038	-443.3643
$\sigma(\pi)$	0.3428	0.0077

Optimized Taylor Rule (Standard Sticky Price)	
	$\rho^* = 0.1, \phi_\pi^* = 20$
Social Welfare	-50.8178
$\sigma(\pi)$	0.0126

Table 5: Welfare Values for Optimized Taylor Rule

Table 5 shows results for an optimized Taylor rule in which I search for both the values of ρ and ϕ_π so that welfare is maximized. Again in this case we can clearly see that the optimal response to inflation by the central bank is less aggressive in the presence of collateral constraints.

1.5 Conclusions

In this paper, I have developed a New Keynesian general equilibrium model with housing and collateral constraints to study first, how the proportion of variable-rate mortgages in the economy can affect the transmission of shocks and then, what the welfare implications of mortgage contracts are. There are unconstrained and constrained individuals that correspond to the savers and borrowers of the economy. I explicitly introduce fixed and variable-rate mortgages, that is, constrained individuals can be of two types: those who borrow at a variable rate and those who borrow at a fixed rate.

Model responses are in line with the intuition. A monetary policy shock affects more strongly those individuals who are borrowing in economies in which the predominant type of mortgages is at variable rate. Consumption and housing demand decrease by more after an interest rate increase if constrained consumers are variable rate. In a general equilibrium framework, the partial equilibrium effects are maintained, but muted by a redistribution between borrowers and savers and strong wealth effects in labor supply decisions. GHH preferences generate larger aggregate differences between the two scenarios considered.

Monetary policy shocks are not persistent. More persistent shocks such as technology or inflation target shocks are able to generate much larger differences in the aggregate economy.[29] Monetary policy responds to these shocks in a very persistent way causing large aggregate differences between the fixed and the variable-rate economy. Inflation target shocks have more effect on output in variable-rate economies. On the contrary, technology shocks increase output by more in those economies mainly borrowing at a fixed rate, due to the procyclicality of real interest rates in this case.

From a normative perspective, I find that the optimal interest-rate response to inflation by the Central Bank is weaker when a group of consumers need collateral to obtain loans,

[29] This is also consistent with Krusell and Smith (1998) or Gourinchas (2001). They study the effects of the distribution of income and wealth and the implications of precautionary savings and life cycle for the macroeconomy in a general equilibrium framework with heterogeneous agents. Their results are not very different from what one would obtain in a representative agent model, behaviors of different agents practically offset each other in the aggregate when considering realistic parameter specification. They also find that permanent shocks would generate larger effects on the aggregate economy.

as compared to the standard sticky-price model. Inflation relaxes the collateral constraint and therefore reduces the distortions created by this extra friction. However, this channel is stronger the higher the proportion of fixed-rate mortgages in the economy. A high proportion of fixed-rate contracts is welfare enhancing.

The model presented here can set directions for future research. The proportion of fixed and variable-rate mortgages is kept constant. A natural extension would be to endogeneize it by modelling the mortgage choice. For instance, borrowers could be heterogeneous in their risk aversions or market-powered banks could price mortgages charging a spread on fixed-rate mortgages depending on economic conditions. Furthermore, this model is not able to keep track of the new fixed-rate mortgages issued every period. For tractability I assume that the financial intermediary charges an average of the new fixed interest rate and the old interest rate for fixed-rate mortgages every period. An overlapping generations version could solve this issue. It would also be interesting to study shock transmission and monetary policy in international versions of the model with heterogeneous mortgage structures across countries.

2 Housing Market Heterogeneity in a Monetary Union

Abstract[30]

This paper studies the implications of cross-country housing market heterogeneity for a monetary union as well as costs and benefits of being in a monetary union when there are asymmetric shocks. I develop a two-country New Keynesian general equilibrium model with housing and collateral constraints to explore this issue. Results show that consumption reacts more strongly to common shocks in countries with high loan-to-value ratios (LTVs), a high proportion of borrowers or variable-rate mortgages. As for asymmetric technology shocks, output and house prices increase by more in the country receiving the shock if it can conduct monetary policy independently. I also find that country-specific housing price shocks increase consumption not only in the country where the shock takes place, there is an international transmission. From a normative perspective, I conclude that housing-market homogeneization is not beneficial per se, only when it is towards low LTVs or predominantly fixed-rate mortgages. As for costs and benefits of monetary unions, when there is a technology shock in one of the countries and they are symmetric, the monetary union regime is welfare worsening. However, results are dependent on whether or not countries are symmetric and on the source of the asymmetry.

Keywords: Housing market, collateral constraint, monetary policy, monetary union

[30] I am truly thankful to Fabio Ghironi, Matteo Iacoviello and Peter Ireland for their invaluable advice. A great part of this project has been undertaken during my stays at the Federal Reserve Board and at the Bundesbank. I thank them for their financial support and helpful comments. All errors are mine.

2.1 Introduction

Costs and benefits of monetary unions are a much discussed topic, especially in relation to the Europe's Economic and Monetary Union (EMU). Different national characteristics such as heterogeneous institutions, consumption patterns or financial structures can be a source of different transmission of common shocks in a monetary union. Also, country-specific shocks derived from member heterogeneity can enhance the possible divergence. In this paper, I consider how heterogeneous housing markets across members can contribute to the transmission of shocks in a currency area. I also study the effects of asymmetric shocks for both a monetary union and a flexible exchange rate regime. Finally, I use welfare analysis to evaluate whether homogeneization is beneficial in a monetary union and whether countries with asymmetric shocks should join in a monetary union.

Countries in Europe clearly differ in their housing market characteristics. There is evidence of different loan-to-value ratios (LTVs), different proportion of residential debt relative to GDP across countries and heterogeneous mortgage contracts. Also, house price movements do not show the same pattern in every country.[31] Maclennan et al (1998) point out the importance of such heterogeneity in a monetary union. They conclude that there should be an effort toward institutional homogeneization among European countries to alleviate possible tensions.

LTVs in Europe range from 16% in France[32] to 73% in Germany or 95% in Sweden. European countries also differ in their proportion of borrowers. The residential debt to GDP ratio ranges from values such as 18.7% in Italy to 98.4% in the Netherlands or 100.8% in Denmark. In those countries with a high LTV or a high proportion of indebted consumers, housing collateral effects are stronger. Therefore, shocks that affect the value of the collateral constraint could potentially have amplified effects on aggregate variables. This is a clear example of the financial accelerator mechanism, first modeled by Bernanke et al (1999).

[31] Tables in the Appendix summarize this evidence.

[32] According to the EMF, the average loan-to-value ratio for first-time buyers reached a low 16% in 2004 due to house price inflation and low interest rates.

Differences in mortgage contracts across countries are another important source of heterogeneity in Europe. In countries such as Germany or France, the majority of mortgages are fixed rate. On the contrary, the predominant type of mortgages in countries such as the United Kingdom, Spain or Greece is variable rate. Calza et al (2007) and Rubio (2008) show that the mortgage structure of an economy matters for the transmission of shocks, especially for those shocks that display more persistence.

Asymmetric shocks can also pose a problem for monetary unions. For example, different housing markets can also lead to an asymmetric evolution of house prices. Data on house price movements for European countries in the last years show such asymmetry. There are countries such as Spain, the United Kingdom or France that have experienced large house price increases. However, house prices have been pretty stable and even slightly decreased in Germany. Country-specific house price shocks can create extra divergence across monetary union members. It is important to assess to what extent asymmetric house price movements in a specific region can be transmitted to other areas. House prices increasing in one area increase consumer's wealth and therefore consumption. Since countries are trading also production in other areas can increase. Furthermore, interest rates respond to inflation and creating house price movements in the whole union. Asymmetric technology shocks can also be considered to study costs and benefits of forming a monetary union. If the shock occurs in one of the countries, the interest rate response would be different if the economy can conduct its independent monetary policy or if it is in a monetary union regime. Furthermore, differences in the transmission of shocks when Central Banks have asymmetric reaction functions are also an issue when countries consider joining in a monetary union with a unique monetary policy.

There is an extensive literature discussing differences in the transmission mechanisms between European countries but little focus on the consequences of housing market heterogeneity from a theoretical standpoint. A microfounded general equilibrium model is needed to understand the implications of housing market differences, explore all the interrelations that take place in the economy and do some normative analysis. Calza et al (2007) and

Rubio (2008) use a closed economy framework and thus cannot address housing market heterogeneity in a monetary union. Gilchrist et al (2002) build a two-country model with a financial accelerator and cross-country financial heterogeneity to explore differences between monetary regimes (monetary union vs. non monetary union). Nevertheless, their model is silent about differences in housing markets. Iacoviello and Smets (2006) develop a monetary union model with housing market heterogeneity. However, they do not compare it with a non monetary union framework. Also, they do not focus on the role of mortgage contract heterogeneity. Aspachs and Rabanal (2008) have a two-country model with housing and collateral constraints but just focus on the case of Spain and the EMU. Carré and Collard (2003) also study the implications asymmetric technology shocks both from a positive and a normative perspective. However, their model does not consider a housing market and collateral constraints.

This paper presents a two-country dynamic stochastic general equilibrium (DSGE) model that features a housing market. There is a group of individuals in each country that are credit constrained and need housing collateral to obtain loans. Countries trade goods and savers in each country have access to foreign assets. Across countries, I allow for differences in LTVs, in the proportion of borrowers and in the structure of mortgage contracts (fixed vs. variable rate). I also consider idiosyncratic house price and technology shocks. Finally, I also analyze asymmetries in the monetary policy reaction functions across countries. Under this general setting, I compare the case in which the two countries have independent monetary policy and different currencies with the case of a monetary union.

Results show that in a monetary union, common shocks (monetary policy and technology) have a different impact across countries when there exists housing market heterogeneity. In particular, consumption reacts more strongly after a shock when the LTV is high, the proportion of borrowers is high, or when mortgages are predominantly variable rate. Concerning asymmetric house price shocks, I find that consumption increases in the country where a positive house price shock takes place but also in the other country. House price shocks are transmitted internationally. Results are robust to the monetary regime considered if countries are symmetric in their interest rate reaction functions. Asymmetric technology shocks have different effects on both economies depending on the monetary regime considered because the interest rate response is different. Asymmetries in the monetary policy response across countries also generate different transmission of shocks both across countries and across regimes.

From a normative perspective I find that homogeneity per se is not necessarily beneficial. For instance, total welfare is higher in a situation where LTVs are asymmetric than in a situation where they are equal but very high because in the latter case collateral constraints have a strong distorting effect on the economy. Also, for mortgage contracts, homogeneization is welfare improving only if it is towards fixed-rate mortgages. As for benefits and costs of forming a monetary union when there is an asymmetric shock, the results depend on whether or not countries are symmetric and on the source of asymmetry. In the case of asymmetric house price shocks, differences across regimes in terms of welfare are not significant. My results also show that when countries are different in their monetary policy response to inflation, the country that is more aggressive against inflation is especially better off under a monetary union regime.

The paper is organized as follows. Section 2 presents both the baseline model (two countries with different currencies and independent monetary policies) and the monetary union version. Section 3 presents the dynamics of the model. Section 4 analyzes welfare. Section 5 concludes. Tables, steady-state relationships, the linearized model and some sample replication files are in the Appendix.

2.2 A Two-Country Model with Housing

I develop a two-country general equilibrium model with a housing market. As a starting point I consider the case in which each of the countries implements its own monetary policy, under a flexible exchange rate regime. In each country, the central bank sets the interest rate to respond to domestic output and inflation. I allow for mortgage and housing heterogeneity across countries.

2.2.1 The Model

I consider an infinite-horizon, two-country economy with a flexible exchange rate regime. Households consume, work and demand real estate. There is a financial intermediary in each country that provides mortgages and accepts deposits from consumers. Each country produces one differentiated good but households consume goods from both countries. Housing is a non-traded good. I assume that labor is immobile across countries. Firms follow a standard Calvo problem. In this economy, both final and intermediate goods are produced. Prices are sticky in the intermediate goods sector.

The Consumer's Problem There are three types of consumers in each country: unconstrained consumers, constrained consumers who borrow at a variable rate and constrained consumers who borrow at a fixed rate. The proportion of each type of borrower is fixed and exogenous. Consumers can be constrained or unconstrained, in the sense that constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments for both mortgages and loans next period cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. This assumption ensures that the borrowing constraint is always binding, so that constrained individuals do not save and wait until they have the funds to self-finance their consumption. This generates an economy in which households are separated into the ones that mostly borrow and the ones that mostly save.

COUNTRY A

Unconstrained Consumers Unconstrained consumers maximize an expected lifetime utility function in three arguments: non-durable consumption, housing services and labor/leisure. It is assumed that housing services are proportional to the housing stock.

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j_t \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{34}$$

Here, E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, and C_t^u, H_t^u and L_t^u are consumption at t, the stock of housing and hours worked respectively. j_t represents the weight of housing in the utility function. I assume that $\log(j_t) = \log(j) + u_{Jt}$, where u_{Jt} follows an autorregressive process. Shocks to j_t can be interpreted as shocks to the house price.[33] $1/\eta - 1$ is the aggregate labor-supply elasticity.

Consumption is a bundle of domestically and foreign produced goods. The consumption index is defined as: $C_t^u = (C_{At}^u)^n (C_{Bt}^u)^{1-n}$ where n is the size of Country A.

The budget constraint, in units of Country A's currency, is:

$$\begin{aligned} P_{At}C_{At}^u + P_{Bt}C_{Bt}^u + Q_t H_t^u + R_{At-1}B_{t-1}^u + e_t R_{Bt-1} D_{t-1} + \frac{\psi}{2} e_t D_t^2 \leq Q_t H_{t-1}^u + \\ W_t^u L_t^u + B_t^u + e_t D_t + P_{At} F_t + P_{At} S_t, \end{aligned} \tag{35}$$

where P_{At} and P_{Bt} are the prices of the goods produced in Countries A and B, respectively, Q_t is the housing price in Country A, and W_t^u is the wage for unconstrained consumers. Unconstrained consumers can hold bonds. B_t^u represents domestic bonds denominated in home currency. R_{At} is the nominal interest rate in Country A. Positive bond holdings mean borrowing and negative mean savings. However, as we will see, this group will choose not to borrow at all, they are the savers in this economy. D_t are foreign bond holdings by savers in Country A. R_{Bt} is the nominal rate of foreign bonds, which are denominated in foreign currency. e_t is the exchange rate between currency in Country A and Country B. To ensure stationarity of net foreign assets, I introduce a small quadratic cost of deviating from zero foreign borrowing $\frac{\psi}{2} e_t D_t^2$. They obtain interests for their savings. S_t and F_t are lump-sum

[33] A shock to j_t represents a shock to the marginal utility of housing. These shocks directly affect housing demand and therefore can be interpreted as a proxy for exogenous disturbances to house prices.

profits received from the firms and the financial intermediary in Country A, respectively.

Dividing by P_{At}, we can rewrite the budget constraint in terms of good A:

$$C^u_{At}+\frac{P_{Bt}}{P_{At}}C^u_{Bt}+q_t H^u_t+\frac{R_{At-1}b^u_{t-1}}{\pi_{At}}+\frac{e_t R_{Bt-1}D_{t-1}}{P_{At}}+\frac{\psi}{2P_{At}}e_t D^2_t \leq q_t H^u_{t-1}+w^u_t L^u_t+b^u_t+\frac{e_t D_t}{P_{At}}+F_t+S_t, \tag{36}$$

where π_{At} denotes the inflation rate for the good produced in Country A, defined as P_{At}/P_{At-1}.

Maximizing (88) subject to (90), we obtain the first-order conditions for the unconstrained group:

$$\frac{C^u_{At}}{C^u_{Bt}}=\frac{nP_{Bt}}{(1-n)P_{At}} \tag{37}$$

$$\frac{1}{C^u_{At}}=\beta E_t\left(\frac{R_{At}}{\pi_{At+1}C^u_{At+1}}\right), \tag{38}$$

$$\frac{1-\psi D_t}{C^u_{At}}=\beta E_t\left(\frac{R_{Bt}e_{t+1}}{\pi_{At+1}C^u_{At+1}e_t}\right), \tag{39}$$

$$w^u_t=(L^u_t)^{\eta-1}\frac{C^u_{At}}{n}, \tag{40}$$

$$\frac{j_t}{H^u_t}=\frac{n}{C^u_{At}}q_t-\beta E_t\frac{n}{C^u_{At+1}}q_{t+1}. \tag{41}$$

Equation (37) equates the marginal rate of substitution between goods to the relative price. Equation (38) is the Euler equation for consumption. Equation (91) is the first-order condition for net foreign assets. Equation (92) is the labor-supply condition. These equations are standard. Equation (93) is the Euler equation for housing and states that at the margin the benefits from consuming housing have to be equal to the costs.

Combining (38) and (91) we obtain a non-arbitrage condition between home and foreign bonds:

$$R_{At} = \frac{R_{Bt} E_t e_{t+1}}{(1 - \psi D_t)\, e_t}. \tag{42}$$

Since all consumption goods are traded and there are no barriers to trade, I assume in this paper that the law of one price holds:

$$P_{At} = e_t P^*_{At} \tag{43}$$

Constrained Consumers Constrained consumers are of two types: those who borrow at a variable rate and those who do it at a fixed rate. The proportion of variable-rate consumers is constant and exogenous and equal to α_A. The proportion of fixed-rate consumers is equal to $1 - \alpha_A$.

Constrained consumers face a collateral constraint; the expected debt repayment next period cannot exceed a proportion of the expectation of tomorrow's value of today's stock of housing:

$$E_t \frac{R^c_{At}}{\pi_{At+1}} b^{ci}_{At} \leq k_A E_t q_{t+1} H^{ci}_t, \tag{44}$$

where $i = v$ if the constrained consumer borrows at a variable rate and $i = f$ if she borrows at a fixed rate. R^c_t is the rate at which the constrained consumer borrows and it is equal to R_{At} if the constrained consumer is variable rate and equal to $\overline{R_{At}}$ if fixed rate. k_A is the loan-to-value ratio in Country A.

Constrained consumers maximize their lifetime utility function:

$$\max \; E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln C^{ci}_t + j_t \ln H^{ci}_t - \frac{\left(L^{ci}_t\right)^{\eta}}{\eta} \right), \tag{45}$$

where $C^{ci}_t = \left(C^{ci}_{At}\right)^n \left(C^{ci}_{Bt}\right)^{1-n}$, subject to the budget constraint (in real terms):

$$C_{At}^{ci}+\frac{P_{Bt}}{P_{At}}C_{Bt}^{ci}+q_tH_t^{ci}+\frac{R_{At-1}^{c}b_{At-1}^{ci}}{\pi_{At}}\leq q_tH_{t-1}^{ci}+w_t^{ci}L_t^{ci}+b_t^{ci}, \tag{46}$$

and the collateral constraint.

Constrained consumers are more impatient than unconstrained ones, so that $\widetilde{\beta}<\beta$.

The first-order conditions for these consumers are:

$$\frac{C_{At}^{ci}}{C_{Bt}^{ci}}=\frac{nP_{Bt}}{(1-n)P_{At}} \tag{47}$$

$$\frac{n}{C_{At}^{ci}}=\widetilde{\beta}E_t\left(\frac{nR_{At}^{c}}{\pi_{At+1}C_{At+1}^{ci}}\right)+\lambda_{At}^{ci}R_{At}^{c}, \tag{48}$$

$$w_t^{ci}=\left(L_t^{ci}\right)^{\eta-1}\frac{C_{At}^{ci}}{n}, \tag{49}$$

$$\frac{j_t}{H_t^{ci}}=\frac{n}{C_{At}^{ci}}q_t-\widetilde{\beta}E_t\frac{n}{C_{At+1}^{ci}}q_{t+1}-\lambda_{At}^{ci}k_AE_tq_{t+1}\pi_{At+1}. \tag{50}$$

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint $\left(\lambda_t^{ci}\right)$ appears in the equations. As in Iacoviello (2005), the borrowing constraint is always binding, so that constrained individuals borrow the maximum amount they are allowed to and their saving is zero:[34]

$$b_t^{ci}=\frac{k_AE_tq_{t+1}H_t^{ci}\pi_{At+1}}{R_{At}^{c}}. \tag{51}$$

Therefore, consumption for constrained individuals is determined by their flow of funds:

[34] From the Euler equations for consumption of the unconstrained consumers, we know that $R_A=1/\beta$ in steady state. If we combine this result with the Euler equation for consumption for the constrained individual we have that $\lambda^{ci}=n\left(\beta-\widetilde{\beta}\right)/C_A^{ci}>0$ in steady state. This means that the borrowing constraint holds with equality in steady state. Since we log-linearize around the steady state assuming that uncertainty is low, we can generalize this result to off-steady-state dynamics.

$$C^{ci}_{At} + \frac{P_{Bt}}{P_{At}} C^{ci}_{Bt} = w^{ci}_t L^{ci}_t + b^{ci}_t + q_t \left(H^{ci}_{t-1} - H^{ci}_t \right) - \frac{R^c_{At-1} b^{ci}_{t-1}}{\pi_{At}}, \tag{52}$$

and the first-order condition for housing becomes:

$$\frac{j_t}{H^{ci}_t} = \frac{n}{C^{ci}_{At}} \left(q_t - \frac{k_A E_t q_{t+1} \pi_{At+1}}{R^c_{At}} \right) - \widetilde{\beta} E_t \frac{n}{C^{ci}_{At+1}} \left(1 - k_A\right) q_{t+1}. \tag{53}$$

The market clearing conditions for the final good in country A is $nY_{At} = nC_{At} + (1-n) C^*_{At} + n\frac{\psi}{2} d^2_t$ and the world bond market clearing condition is $nd_t + (1-n) \frac{P_{Bt}}{P_{At}} d^*_t = 0$, where d_t denotes the foreign bond in real terms. Variables in Country B are denoted with a star. Everything is similar in Country B.

Within Country Aggregate Variables Given α_A, the fraction of variable-rate borrowers in Country A, we can define aggregates across constrained consumers as the sum of variable-rate consumers aggregates and fixed-rate consumers aggregates, so that $C^c_t \equiv \alpha_A C^{cv}_t + (1-\alpha_A) C^{cf}_t$, $L^c_t \equiv \alpha_A L^{cv}_t + (1-\alpha_A) L^{cf}_t$, $H^c_t \equiv \alpha_A H^{cv}_t + (1-\alpha_A) H^{cf}_t$ and $b^c_t \equiv \alpha_A b^{cv}_t + (1-\alpha_A) b^{cf}_t$.

Therefore, economy-wide aggregates in Country A are $C_t \equiv C^u_t + C^c_t$, $L_t \equiv L^u_t + L^c_t$ and aggregate supply of housing is fixed, so that market clearing requires $H_t \equiv H^u_t + H^c_t = H$.

Aggregates in Country B are constructed symmetrically.

The Financial Intermediary There is a financial intermediary in each country. The financial intermediary accepts deposits from savers, and extends both fixed and variable-rate loans to borrowers.

COUNTRY A The profits of the financial intermediary in Country A are defined as follows:

$$F_t = \alpha_A R_{At-1} b^{cv}_{t-1} + (1-\alpha_A) \overline{R}_{At-1} b^{cf}_{t-1} - R_{At-1} b^u_{t-1}. \tag{54}$$

For simplicity, and given that typically the time horizon of a mortgage is large, I consider the maturity of mortgages to be infinite. This assumption is not crucial for the dynamics of the problem since we are interested in short-term business cycle fluctuations.

In equilibrium, borrowing and savings have to be equal, that is $b_t^c + b_t^u = 0$. Substituting this into (54) we obtain:

$$F_t = (1 - \alpha_A) \overline{R_A}_{t-1} b_{t-1}^{cf} - (1 - \alpha_A) R_{At-1} b_{t-1}^{cf}. \tag{55}$$

Profits from financial intermediation are rebated every period to the unconstrained consumers in Country A.

I assume a competitive framework for the financial intermediary. Therefore, optimality implies that the intermediary is indifferent between lending at a variable or fixed rate at each point in time τ. Hence, the expected discounted profits that the intermediary obtains by lending at a fixed interest rate have to be equal to the expected discounted profits the intermediary would obtain by lending at the variable rate:

$$E_\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i} \overline{R_A}_\tau^{OPT} \left(b_\tau^{cf} - b_{\tau-1}^{cf} \right) = E_\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i} R_{Ai-1} \left(b_\tau^{cf} - b_{\tau-1}^{cf} \right), \tag{56}$$

where $\Lambda_{t,i} = \beta^{i-t} \dfrac{C_{At}^u}{C_{At+i}^u}$ is the unconstrained consumer's discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary's problem. Notice that this is not a condition on the stock of debt, but on the new amount obtained on a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period τ and the new amount of debt in period τ are going to be fixed for all periods. However, the fixed interest rate varies with the date the debt was issued, so that there is a new fixed interest rate associated with new debt in every period.

We can obtain the optimal value of the fixed rate in period τ from expression (56) :

$$\overline{R}_{A\tau}^{OPT} = \frac{E_{\tau}\sum\limits_{i=\tau+1}^{\infty}\beta^{i-\tau}\Lambda_{\tau,i}R_{Ai-1}}{E_{\tau}\sum\limits_{i=\tau+1}^{\infty}\beta^{i-\tau}\Lambda_{\tau,i}}. \tag{57}$$

Equation (57) states that, for every new debt issued at date τ, there is a different fixed interest rate that has to be equal to a discounted average of future interest rates. However, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Thus, we must define a new aggregate fixed interest rate, which is the one that the financial intermediary effectively charges every period. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period optimal fixed interest rate and new debt. Therefore, the fixed interest rate that the financial intermediary effectively charges for the stock of fixed-rate debt every period is defined as:

$$\overline{R}_{At} = \frac{\overline{R}_{At-1}b_{t-1}^{cf} + \overline{R}_{At}^{OPT}\left(b_{t}^{cf} - b_{t-1}^{cf}\right)}{b_{t}^{cf}}. \tag{58}$$

Equation (58) states that the fixed interest rate that the financial intermediary is actually charging today is an average between what it charged last period for the previous stock of mortgages and what it charges this period for the new amount.

The financial intermediary problem for Country B is symmetric.

Firms

Final Goods Producers In Country A, there is a continuum of final goods producers that aggregate intermediate goods according to the production function

$$Y_{1t}^{k} = \left[\int_{0}^{1} Y_{1t}^{k}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{59}$$

where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods.

The total demand of intermediate good z is given by $Y_{At}(z) = \left(\frac{P_{At}(z)}{P_{At}}\right)^{-\varepsilon} Y_{At}$, and the price index is $P_{At} = \left[\int_0^1 P_{At}(z)^{1-\varepsilon} dz\right]^{\frac{1}{\varepsilon-1}}$.

Intermediate Goods Producers The intermediate goods market is monopolistically competitive. Intermediate goods are produced according to the following production function:

$$Y_{At}(z) = Z_t \left(L_t^u(z)\right)^{\gamma_A} \left(L_t^c(z)\right)^{(1-\gamma_A)}, \tag{60}$$

where Z_t represents technology. I assume that $\log Z_t = \rho_Z \log Z_{1t-1} + u_{Zt}$ where ρ_Z is the autorregressive coefficient and u_{Zt} is a normally distributed shock to technology.γ measures the relative size of each group in terms of labor.[35]

The first-order conditions for labor demand are the following:[36]

$$w_t^u = \frac{Z_t}{X_t} \gamma_A \frac{Y_{At}}{L_t^u}, \tag{61}$$

$$w_t^c = \frac{Z_t}{X_t} (1-\gamma_A) \frac{Y_{At}}{L_t^c}, \tag{62}$$

where X_t is the markup, or the inverse of marginal cost.

The price-setting problem for the intermediate goods producers is a standard Calvo-Yun setting. An intermediate good producer sells good at price $P_{At}(z)$, and $1-\theta$ is the probability of being able to change the sale price in every period. The optimal reset price $P_{At}^{OPT}(z)$ solves:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left\{ \Lambda_{t,k} \left[\frac{P_{At}^{OPT}(z)}{P_{At+k}} - \frac{\varepsilon/(\varepsilon-1)}{X_{t+k}} \right] Y_{At+k}^{OPT}(z) \right\} = 0. \tag{63}$$

The aggregate price level is given by:

[35] This Cobb-Douglas production function implies that labor inputs of the two groups are not perfect substitutes. This assumption can be justified by the fact that savers are the managers of the firms and their wage is not the same as that of the borrowers. The Cobb-Douglas specification is analytically tractable and allows for closed form solutions for the steady state of the model.

[36] Symmetry across firms allows to avoid the index z.

$$P_{At} = \left[\theta P^{\varepsilon}_{At-1} + (1-\theta)\left(P^{OPT}_{At}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{64}$$

Using (63) and (64) and log-linearizing, we can obtain the standard forward-looking Phillips Curve (See equation in the Appendix 3).[37]

The firm problem is analogous in Country B.

Monetary Policy The model is closed with a Taylor Rule with interest-rate smoothing for interest-rate setting by each country's central bank.[38] In Country A,

$$R_{At} = (R_{At-1})^{\rho A}\left(\pi_{At}^{(1+\phi_{\pi A})} R_A\right)^{1-\rho A} \varepsilon_{AR,t}, \tag{65}$$

$0 \leq \rho A \leq 1$ is the parameter associated with interest-rate inertia. $(1+\phi_{\pi A})$ measures the sensitivity of interest rates to current inflation. $\varepsilon_{AR,t}$ is a white noise shock process with zero mean and variance σ^2_{ε}. In Country B, R_{Bt} is set similarly.

2.2.2 The Monetary Union Case

Now we can consider the case in which Country A and Country B form a monetary union. The problem for consumers in this case differs from the previous one in that prices are denominated in a common currency and therefore there is no need for the use of the exchange rate. Monetary policy is now conducted by a single central bank that reacts to inflation and output in both countries weighted by its relative size. Equations are presented in the Appendix.

[37] This Phillips curve is consistent with other two-country models with financial accelerator. See for instance Gilchrist et al (2002) or Iacoviello and Smets (2006).

[38] This rule is consistent with the primary objective of the ECB being price stability. This type of rule is also used in other monetary union models. See Iacoviello and Smets (2007) or Aspachs and Rabanal (2008)

2.3 Dynamics

2.3.1 Parameter Values

We can use the model to explore how shocks are transmitted across different experiments. I linearize the equilibrium equations around the steady state. Details are shown in the Appendix. The discount factor for savers, β, is set to 0.99 so that the annual interest rate is 4% in steady state. The discount factor for borrowers, $\widetilde{\beta}$, is set to 0.98.[39] The steady-state weight of housing in the utility function, j, is set to 0.1 in order for the ratio of housing wealth to GDP in steady state to be approximately 1.40.[40] I set $\eta = 2$, implying a value of the labor supply elasticity of 1.[41] For the loan-to-value ratio, I pick $\kappa = 0.8$ for the baseline calibration, consistent with a weighted average of LTVs in 2004 calculated by the European Mortgage Federation (EMF) on European countries.[42]However, one of the experiments I perform consists of testing the sensitivity of results to this parameter. The labor income share of unconstrained consumers, γ, is set to 0.7 in both countries as a reference point. Nonetheless, as for the LTV ratio, experiments with different values of γ will be performed.[43] I pick a value of 6 for ε, the elasticity of substitution between intermediate goods. This value implies a steady-state markup of 1.2. The probability of not changing prices, θ, is set to 0.75, implying that prices change on average every four quarters. For the Taylor Rule parameters I use $\rho = 0.8$, $\phi_\pi = 0.5$. The first value reflects a realistic degree of interest-rate smoothing.[44] ϕ_π is consistent with the original parameters proposed by Taylor in 1993. For the baseline model, I consider α to be equal to 1, that is, all mortgages are variable rate.[45] Results when $\alpha = 0$ will also be checked.

[39] Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency.

[40] This value corresponds to the US. I assume here that the ratio is similar across most industrialized countries, given the lack of housing wealth data for European countries. See Aspachs and Rabanal (2008).

[41] Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.

[42] The countries that are included in the sample are Belgium, Germany, Greece, Spain, France, Italy, Hungary, Poland, Sweden and the United Kingdom.

[43] This value is in the range of the estimates of Iacoviello (2005), Iacoviello and Neri (2008) and Campbell and Mankiw (1991) for the US, Canada, France and Sweden.

[44] See McCallum (2001).

[45] This value makes the model comparable with the standard models.

Monetary policy shocks are represented by a one percent increase of the interest rate. A technology shock will be a one percent positive technology with 0.9 persistence[46]. House price shocks have a 0.8 persistence.[47] I set the size of the shock to the housing demand parameter to 20% so that house prices increase roughly by 1 %.

Table 6 presents a summary with the parameter values:

Parameter Values in Baseline Model		
β	.99	Discount Factor for Savers
$\widetilde{\beta}$	.98	Discount Factor for Borrowers
j	.1	Weight of Housing in Utility Function
η	2	Parameter associated with labor elasticity
k	.9	Loan-to-value ratio
γ	.64	Labor share for Savers
α	1	Proportion of variable-rate borrowers
X	1.2	Steady-state markup
n	.5	Size of Country A
θ	.75	Probability of not changing prices
ρ	.8	Interest-Rate-Smoothing Parameter in Taylor Rule
ϕ_π	.5	Inflation Parameter in Taylor Rule
ψ	.0001	Adjustment Cost Net Foreign Assets

Table 6: Parameter Values

2.3.2 Common Shock with Housing Market Heterogeneity

LTV Asymmetry When countries in a monetary union are asymmetric, a common shock can affect them differently. The first source of asymmetry that I consider is differences in

[46]This high persistence value for technology shocks is consistent with what is commonly used in the literature. Smets and Wouters (2002) estimate a value of 0.822 for this parameter in Europe, Iacoviello and Neri (2008) estimate is 0.93 for the US.

[47]The persistence of the house price shock is consistent with the estimates in Iacoviello (2005) and Iacoviello and Neri (2008).

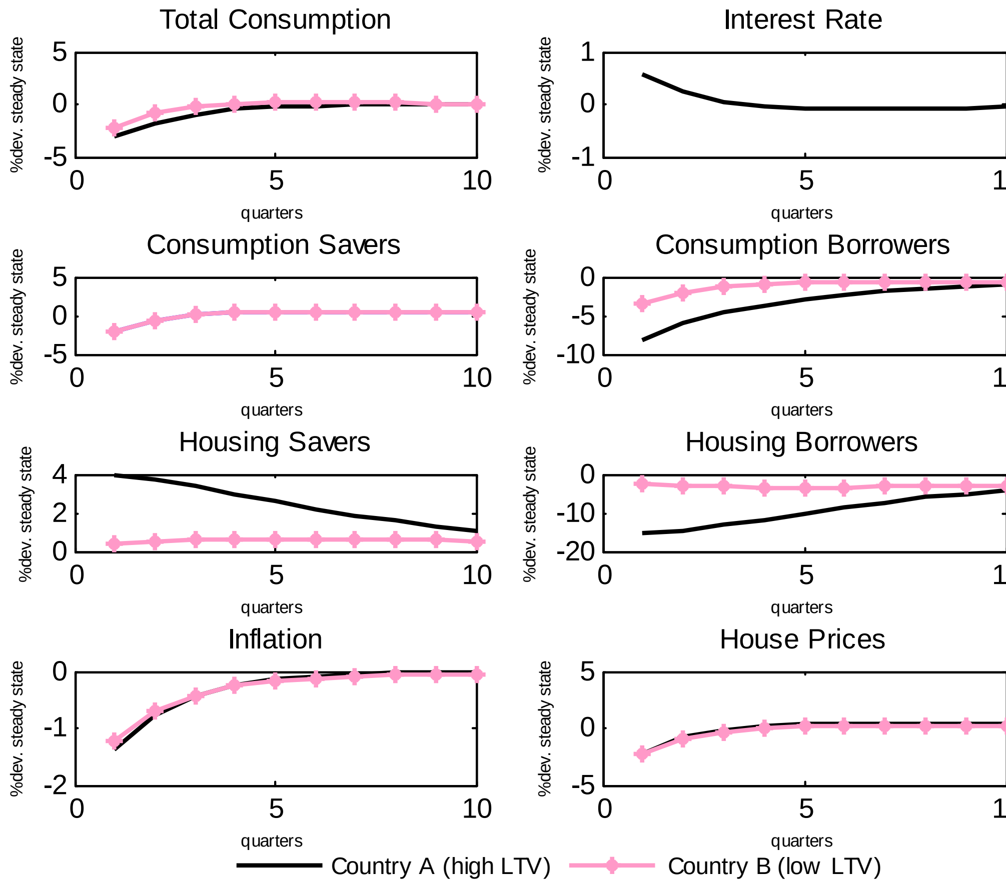

Figure 8: Impulse Responses to a Monetary Policy Shock in a Monetary Union.

LTVs. The loan-to-value ratio is a crucial parameter because it implies the degree of credit accessibility for borrowers and therefore the strength of the financial accelerator. When LTVs are high, shocks that affect the value of the collateral are amplified due to the financial accelerator effect.

Figure 8 shows the effects of a monetary policy shock in a monetary union when countries differ in their LTVs. We consider Country B to be a country with a low LTV of 0.2, as opposed to the rest of the union which has an LTV of 0.8. The size of Country B is set to 0.1. This theoretical experiment could illustrate the case of France in 2004, when LTVs for first-time buyers reached a low 16% due to house price inflation and low interest rates.

An increase in the interest rate contracts the economy. Savers substitute intertemporally and prefer to save today to consume tomorrow. For borrowers, there is both a direct and an indirect effect that make their consumption decrease. First, their mortgage payments increase and therefore they consume less. The second effect comes through the collateral constraint. Since housing prices decrease following the interest rate increase, the value of their collateral decreases. Impatient agents are able to borrow less and hence consume less. This collateral effect, however, is stronger the higher the LTV parameter. We can see that the effects of this shock are amplified if the country has a high LTV, meaning that the financial accelerator is stronger there.

The experiment for a common technology shock would be analogous. Also in this case total consumption would react more in the country that has a high LTV ratio. The interest rate would decrease and housing prices in both countries increase. The collateral effect is greater in that country with the higher LTV and therefore its consumption would increase by more.

Results for common monetary policy shocks are robust to the monetary regime considered if the policy reaction functions are equal across countries and also equal to the monetary union's counterpart. However, an interesting experiment that can be performed is to see how results vary across monetary regimes (monetary union vs. flexible exchange rates and independent monetary policies) when just one of the countries is hit by a technology shock.

Borrowers Proportion Asymmetry The proportion of borrowers is also a source of cross-country asymmetry that matters for the transmission of shocks. This proportion is somehow captured by $1-\gamma$ in the model, the labor income share of borrowers. We consider Country B having a higher proportion of borrowers ($\gamma_B = 0.2$) than the rest of the union (Country A) where $\gamma_A = 0.7$. The reasoning for this is similar to the LTV heterogeneity case. When borrowers are very numerous, collateral constraints are a more pervasive feature of the economy. Therefore, we should expect that in this case, common shocks also affect the variables of interest in a stronger way. Figure 9 confirms this intuition. After a monetary policy shock, consumption reacts more strongly where the proportion of borrowers is higher.

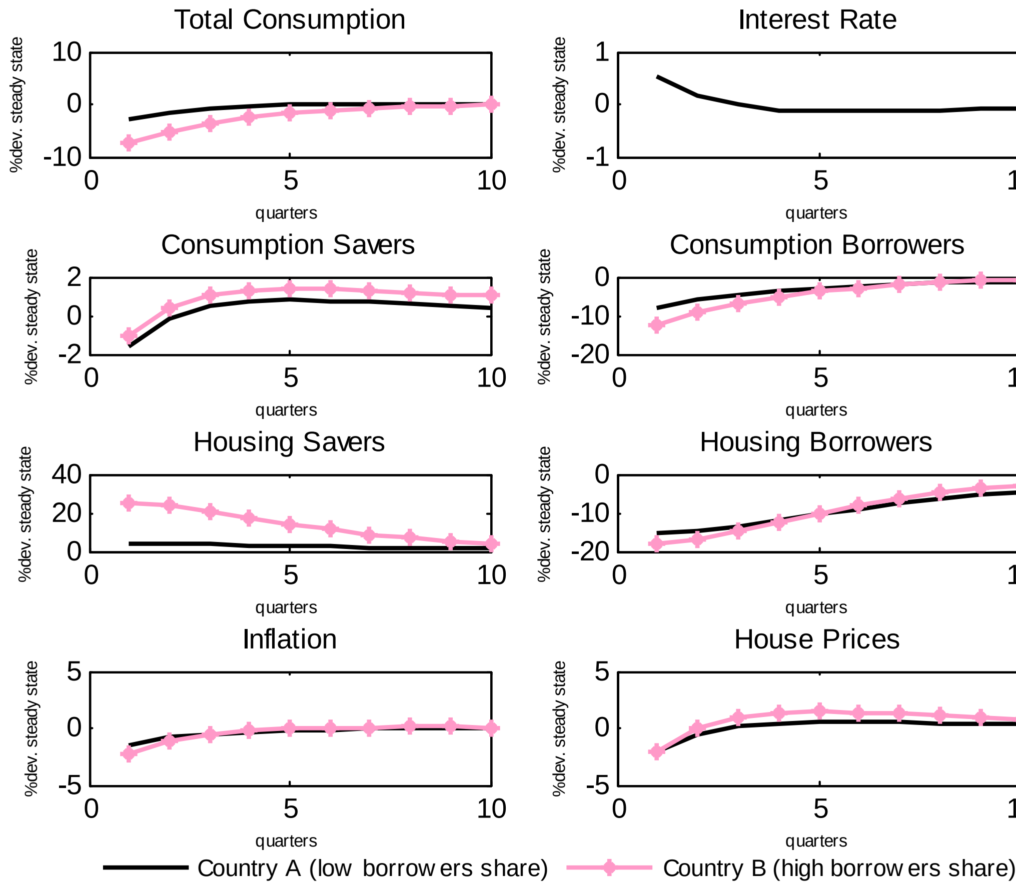

Figure 9: Impulse Responses to a Monetary Policy Shock in a Monetary Union.

Mortgage Contract Asymmetry Another source of heterogeneity in housing markets is the cross-country mortgage structure heterogeneity. Let us analyze now the case in which the structure of mortgage contracts in Country A is fixed rate and variable rate in Country B. This could be seen as Country B being for instance Spain and Country B the rest of EMU. Consider first an interest rate shock in a monetary union. For those consumers with variable-rate mortgages, after a positive interest-rate shock, interest rate payments increase by more than for the fixed-rate case. Also, the value of their collateral decreases by more. Then, the monetary policy shock hits strongly those individuals that are constrained. We can observe in Figure 10 that consumption and housing demand for borrowers decrease more persistently in the country in which consumers borrow at a variable

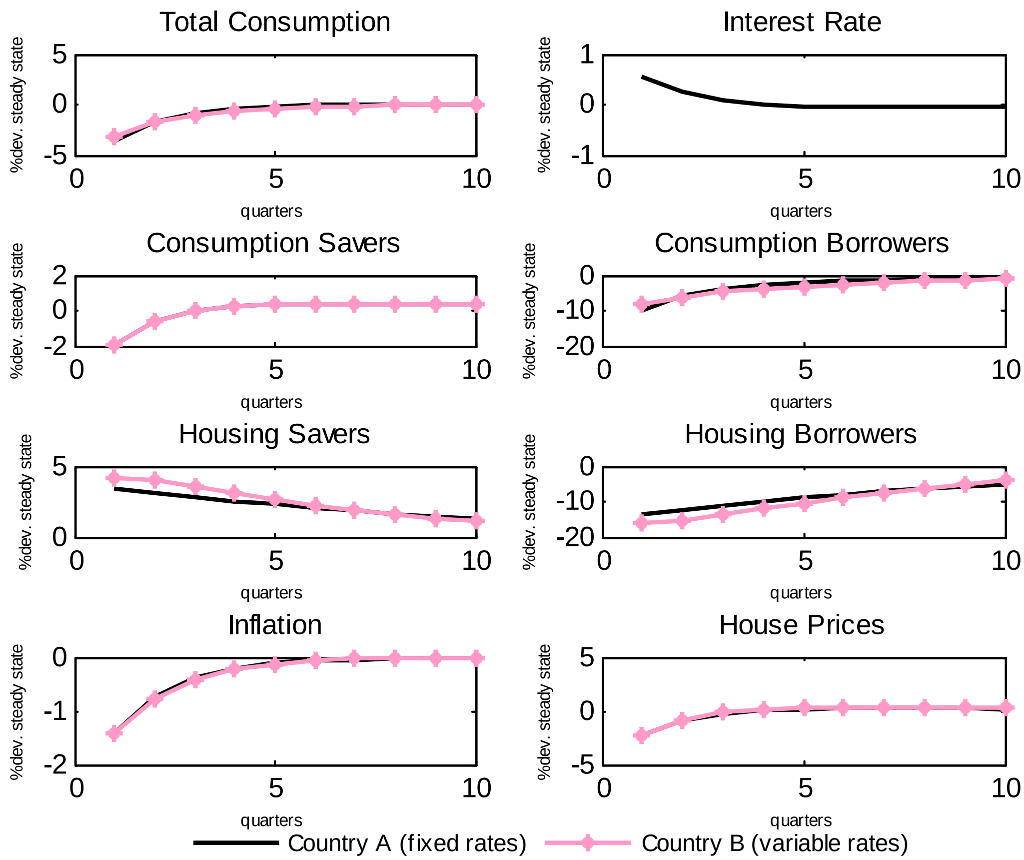

Figure 10: Impulse Responses to a Monetary Policy Shock in a Monetary Union.

rate. However, in the aggregate the effects between the two countries are quantitatively small. General equilibrium effects partially offset aggregate differences: On the one hand, there is a redistribution between borrowers and savers. On the other hand, there are important wealth effects in the labor-supply decision, that is, variable-rate borrowers can simply decide to work more to compensate their consumption loss.[48]

In Figure 11 we see that a more persistent shock, such as a technology shock delivers larger aggregate differences between the two countries when the structure of mortgage contracts differs among them. In particular, we see strong differences in the behaviors of housing demand and house prices across countries. Total consumption reacts by more in

[48] This results are in line with Rubio (2008).

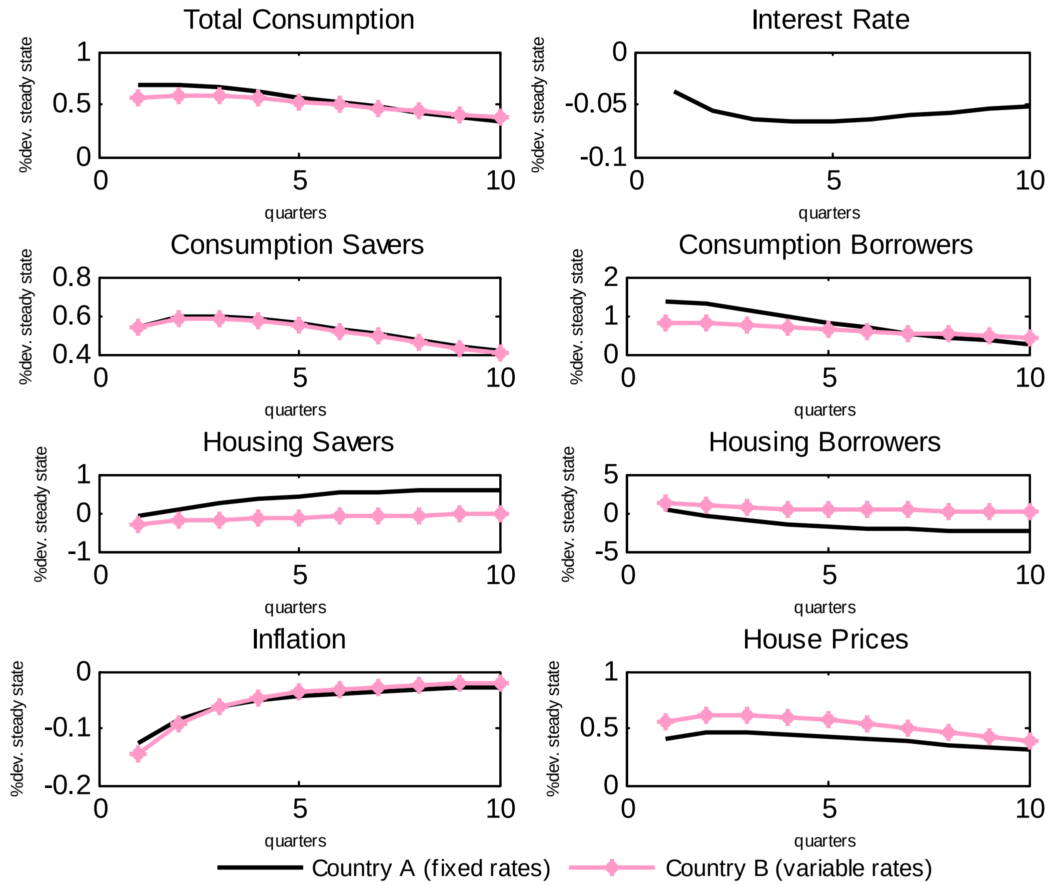

Figure 11: Impulse Responses to a common Technology Shock. Monetary Union.

the fixed-rate case due to the procyclicality of the real interest rate. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. In Rubio (2008) it is also the case that aggregate differences increase with the persistence of the shock.

2.3.3 Asymmetric Technology Shock

Even if countries are symmetric, they can suffer asymmetric shocks. Figure 12 shows impulse responses of a technology shock that occurs only in Country B. I compare the monetary union versus the flexible exchange rates regime. When technology improves in Country B, interest rates react little in a monetary union. However, if Country B conducts monetary

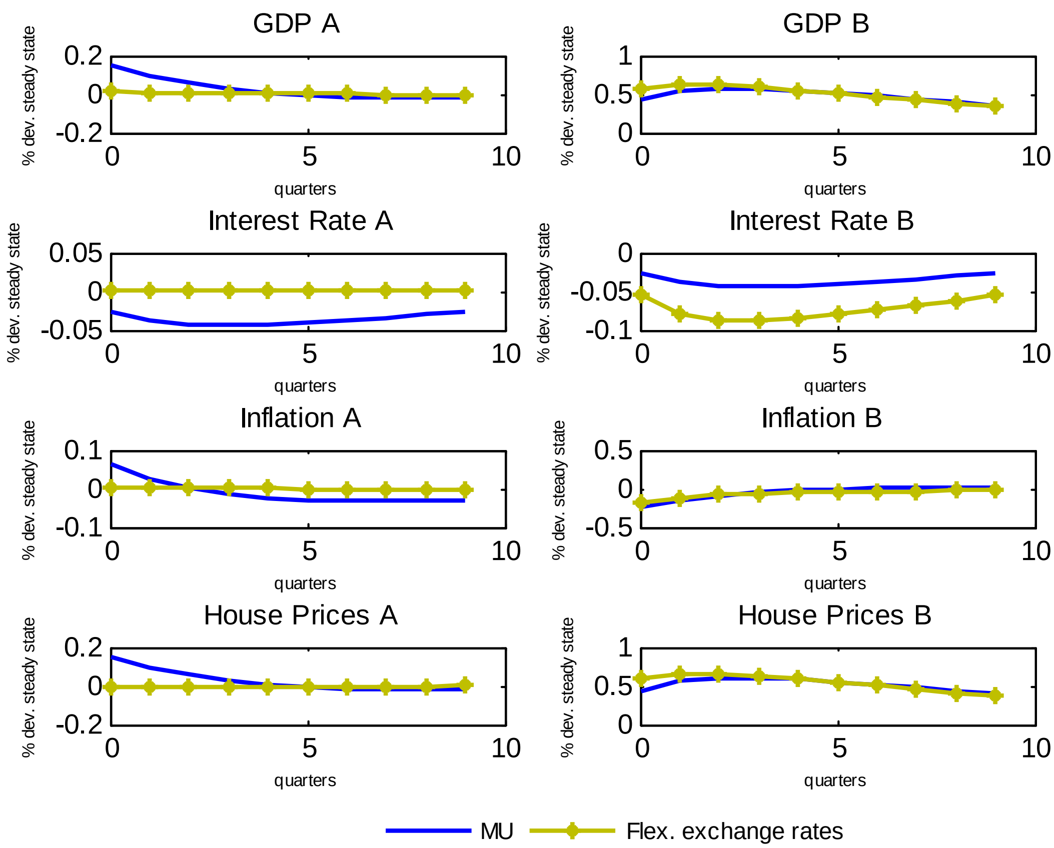

Figure 12: Impulse Responses to a Technology Shock in Country B. Monetary Union versus Flexible Exchange Rate regime.

policy independently, the interest rate in Country B is going to decrease significantly as compared to the monetary union regime. As a consequence, under the flexible exchange rate regime, housing prices in Country B increase and therefore consumption increases by more.

In Country A, when it conducts its own monetary policy, interest rates do not move because the shock happened in Country B and inflation is not changing. However, under the monetary union regime, the common interest rates goes down and that expands Country A' economy. Furthermore, the decrease in interest rates make house prices in Country A increase and this increases consumption and output further due to the positive wealth effects for borrowers.

2.3.4 Idiosyncratic House Price Shock

I can also study in this framework how asymmetric house price shocks are transmitted across countries. In a closed economy, a positive house price shock increases the value of the collateral, and total consumption increases, mainly due to the increase in consumption of borrowers. However, in an open economy, a country-specific house price shock can be transmitted internationally to other countries. If that were the case, the divergence caused by an asymmetric shock would be alleviated. Figure 13 shows the effects of a house price shock in Country A. Consumption in this country increases initially because of wealth effects. Housing demand by borrowers also increases. However, this asymmetric shock is slightly transmitted to Country B where consumption also increases because the countries are trading. Interest rates, especially in the union, decrease and this makes house prices in Country B increase as well.

2.3.5 Monetary Policy Response Asymmetry

So far I have assumed that countries have the same policy parameters independently on their monetary regime. One source of asymmetry that is worth studying is different reaction functions across countries. It is sensible to believe that when countries have independent monetary policies, central bankers have preferences that differ from the ones that a single central banker for a monetary union would have. Figure 14 compares the monetary union case with the flexible exchange rate one for a common technology shock when countries are symmetric in everything but in their Taylor rule. In particular, under flexible exchange rates $\phi_{\pi A} = 0.01$ and $\phi_{\pi B} = 5$, meaning that monetary policy in Country B is more aggressive against inflation. On the contrary, for the monetary union, $\phi_{\pi} = 0.5$. Given the same technology shock, variables react differently across countries and monetary regimes. When the country is very aggressive against inflation, inflation does not decrease so much after a technology shock and also the interest rate goes down by less in response. However,

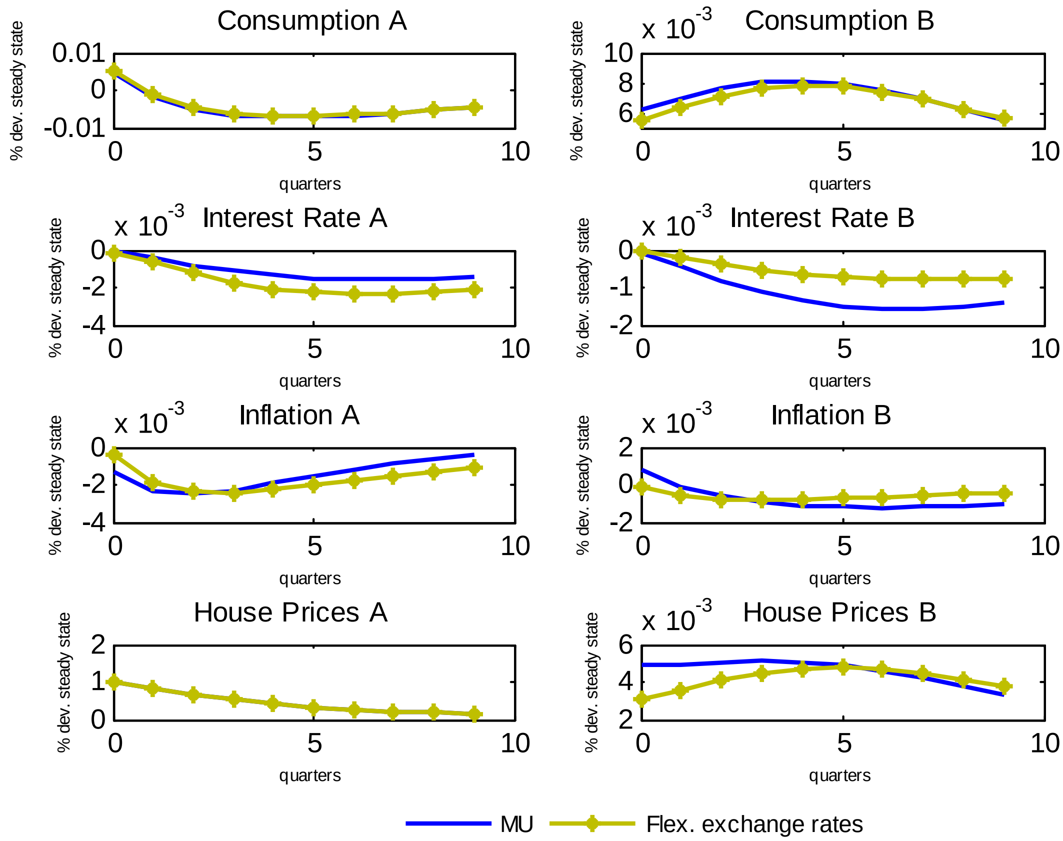

Figure 13: Impulse Responses to a House Price Shock in Country A. Monetary Union versus Flexible Exchange Rates.

house prices increase by more and this amplifies the response of output by the wealth effect channel.

2.4 Welfare Analysis

In this section, I evaluate how cross-country asymmetries affect welfare for a given policy rule. Notice that in this economy there are two types of distortions, price rigidities and credit frictions. The individual welfare for savers and borrowers in Country A respectively is defined as follows:[49]

[49] I numerically compute the second order approximation of the utility function as a measure of welfare. I consider a common technology shock.

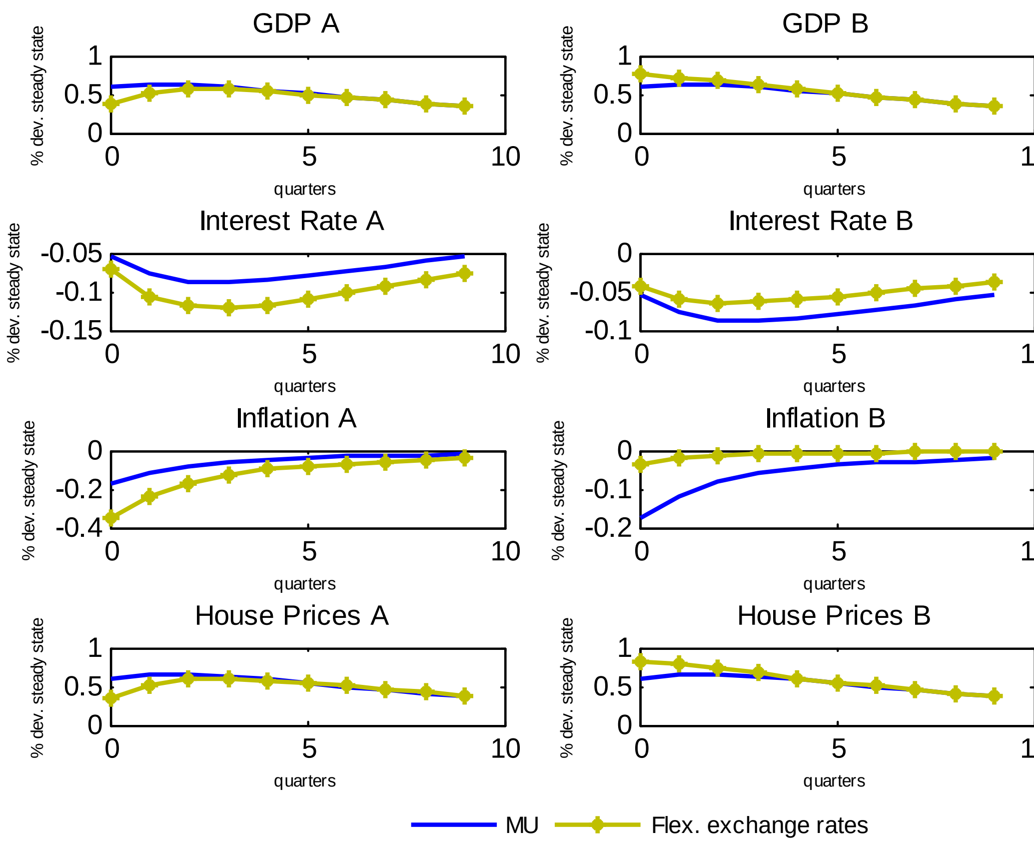

Figure 14: Impulse Responses to a Common Technology Shock. Monetary Union versus Flexible Exchange Rate Regime (Country B more aggressive against inflation).

$$V_{u,t} \equiv E_t \sum_{m=0}^{\infty} \beta^m \left(\ln C^u_{t+m} + j \ln H^u_{t+m} - \frac{\left(L^u_{t+m}\right)^\eta}{\eta} \right), \tag{66}$$

$$V_{ci,t} \equiv E_t \sum_{m=0}^{\infty} \widetilde{\beta}^m \left(\ln C^{ci}_{t+m} + j \ln H^{ci}_{t+m} - \frac{\left(L^{ci}_{t+m}\right)^\eta}{\eta} \right), \tag{67}$$

Following Mendicino and Pescatori (2007), I define social welfare as a weighted sum of individual welfare for the different type of households:

$$V_t = (1-\beta) V_{u,t} + \left(1 - \widetilde{\beta}\right) \left[\alpha V_{cv,t} + (1-\alpha) V_{cf,t}\right]. \tag{68}$$

Borrowers and savers' welfare are weighted by $\left(1-\widetilde{\beta}\right)$ and $(1-\beta)$ respectively, so that the two groups receive the same level of utility from a constant consumption stream. As in Mendicino and Pescatori (2007), I take this approach to be able to evaluate the welfare of the three types of agents separately.[50] Everything is symmetric for Country B.

This analysis allows me to see if there are benefits from homogeneization when countries are heterogeneous in their housing markets and they are hit by a common technology shock. Table 7 presents welfare comparisons under different LTVs. Country B is a small country in a monetary union. We can observe that heterogeneity in LTVs is not necessarily welfare worsening. High LTVs imply stronger collateral effects and that enhances one of the distortions in the economy. A symmetric economy that has high LTVs does worse than an asymmetric economy in which one of the countries has a low LTV ratio. However, if the economy is symmetric and both countries have low LTVs, welfare improves because the collateral effects are less important. Notice that the welfare improvements from moving to a scenario with a lower LTV are mainly driven by the borrowers side, who are the ones that are affected by the collateral constraint. There is a trade-off between savers and borrowers welfare and savers actually lose. However, overall the economy is better off when collateral effects are not so important.

	$\boldsymbol{k_A = k_B = .8}$	$\boldsymbol{k_A = k_B = .2}$	$\boldsymbol{k_A = .8/k_B = .2}$
Social Welfare A	**-3.486**	**-3.087**	**-3.746**
Savers Welfare A	-116.53	-136.66	-144.31
Borrowers Welfare A	-116.05	-86.02	-115.14
Social Welfare B	**-3.486**	**-3.087**	**-0.817**
Savers Welfare B	-116.53	-136.66	117.71
Borrowers Welfare B	-116.05	-86.02	-99.73
Total Welfare	**-3.486**	**-3.087**	**-3.453**

[50] See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.

Table 7: Welfare evaluation. Monetary Union. Common Technology Shock. Different LTVs

Table 8 shows welfare comparisons for a common technology shock under different values of γ, the labor income share of savers. The economy as a whole achieves a higher welfare when the labor income share of savers is small and countries are symmetric. When γ is lower, that is, the economic size of borrowers is larger, borrowers are worse off because collateral effects are more important in this case and the economy is more distorted. However, savers are better off. Overall, the welfare improvement of savers outweighs that of the borrowers and the economy has higher welfare with low values of γ. Nevertheless, in this case heterogeneity delivers the lowest welfare result. Country B especially, the small heterogeneous country improves welfare significantly by homogeneization.

	$\gamma_A = \gamma_B = .7$	$\gamma_A = \gamma_B = .2$	$\gamma_A = .7/\gamma_B = .2$
Social Welfare A	**-3.486**	**-3.208**	**-3.233**
Savers Welfare A	-116.53	-80.29	-80.85
Borrowers Welfare A	-116.05	-120.27	-121.24
Social Welfare B	**-3.486**	**-3.208**	**-9.555**
Savers Welfare B	-116.53	-80.29	-790.72
Borrowers Welfare B	-116.05	-120.27	-82.43
Total Welfare	**-3.486**	**-3.208**	**-3.865**

Table 8: Welfare evaluation. Monetary Union. Common Technology Shock. Different Borrowers Proportions

In Table 9 we see that homogeneization towards fixed rate mortgages is welfare improving. Countries with variable-rate mortgages are worse off than countries with fixed-rate mortgage because the collateral constraint is tighter. The heterogeneity by itself is not welfare worsening, it is the fact that in the variable-rate country the collateral constraint is more distorting. This suggests that countries such as Spain or the United Kingdom should increase the proportion of fixed-rate contracts.

	$\boldsymbol{\alpha_A = \alpha_B = 1}$	$\boldsymbol{\alpha_A = \alpha_B = 0}$	$\boldsymbol{\alpha_A = 0/\alpha_B = 1}$
Social Welfare A	**-3.486**	**-1.476**	**-1.067**
Savers Welfare A	-116.53	-469.86	-447.89
Borrowers Welfare A	-116.05	161.08	170.57
Social Welfare B	**-3.486**	**-1.476**	**-6.547**
Savers Welfare B	-116.53	-469.86	-392.47
Borrowers Welfare B	-116.05	161.08	-131.13
Total Welfare	**-3.486**	**-1.476**	**-1.615**

Table 9: Welfare evaluation. Monetary Union. Common Technology Shock. Different Mortgage Contracts

Welfare comparisons allow me to evaluate also costs and benefits of forming a monetary union. Table 10 shows the welfare implications of an asymmetric technology shock in Country B when both countries are symmetric. We can observe that in this case, it is beneficial for both countries to conduct their independent monetary policies.

Symmetric Countries	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-4.688**	**-3.608**
Savers Welfare A	-162.86	-196.60
Borrowers Welfare A	-152.99	-82.11
Social Welfare B	**-3.465**	**-2.603**
Savers Welfare B	-177.31	-28.91
Borrowers Welfare B	-84.59	-115.71
Total Welfare	**-4.076**	**-3.105**

Table 10: Technology Shock in Country B. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Symmetric Countries.

However, as noticed in Table 11, results depend on the size of the country receiving the shock. For the country that experiments the shock, it is beneficial to be in a monetary union. However, the other country is better off under a flexible exchange rate regime. These results are in line with Carré and Collard (2003). Overall, the economy is better off if countries form a monetary union. When there is a positive technology shock in Country B, production costs go down and output increases under both regimes. However, the benefits obtained by exporting to the big country are enhanced in the monetary union case because there is not exchange rate volatility.

Country B small	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-3.259**	**72.58**
Savers Welfare A	-91.25	11585.57
Borrowers Welfare A	-117.33	-2163.66
Social Welfare B	**-2.960**	**-1373.23**
Savers Welfare B	-188.41	-140947.46
Borrowers Welfare B	-53.83	1811.85
Total Welfare	**-3.229**	**-71.99**

Table 11: Technology Shock in Country B. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Country B Small.

This conclusion are maintained under LTV asymmetry. The small country receiving the shock has now a low LTV. I still find that this economy is better off in the monetary union. However, we see that across regimes Country B is better off with a low LTV because collateral effects are less important.

Country B small and low LTV	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-3.270**	**10.64**
Savers Welfare A	-92.04	2057.31
Borrowers Welfare A	-117.50	-496.57
Social Welfare B	**-2.451**	**-259.29**
Savers Welfare B	-191.17	-26295.32
Borrowers Welfare B	-27.00	183.00
Total Welfare	**-3.188**	**-16.35**

Table 12: Technology Shock in Country B. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Country B Small and low LTV.

However, results are reversed if the asymmetry comes from the proportion of borrowers. In this case, it is beneficial for Country A to be in the union and welfare worsening for Country B.

Country B small and high prop. borrowers	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-3.552**	**-14.73**
Savers Welfare A	-127.59	-1935.32
Borrowers Welfare A	-113.83	230.93
Social Welfare B	**-1.336**	**401.71**
Savers Welfare B	-72.83	40958.57
Borrowers Welfare B	-30.38	-393.74
Total Welfare	**-3.331**	**26.91**

Table 13: Technology Shock in Country B. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Country B Small and high prop. of borrowers.

When there is asymmetry in mortgage contracts. In this case, it is not beneficial to be in a monetary union because Country B is truly worse off under this regime. This is an important result when the UK considers whether or not to enter the EMU.

Country B small and vble .rate mortgages	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-3.244**	**-9.015**
Savers Welfare A	-94.12	-976.52
Borrowers Welfare A	-115.16	37.50
Social Welfare B	**-2.978**	**102.59**
Savers Welfare B	-185.94	10581.42
Borrowers Welfare B	-55.96	-160.72
Total Welfare	**-3.218**	**2.146**

Table 14: Technology Shock in Country B. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Country B Small and vble. rate mortgages.

I can also analyze if it is beneficial to form a monetary union when there are house price shocks in just one of the countries. Table 15 shows that for an asymmetric house price shock there is not much difference in terms of welfare between being in a monetary union or a flexible exchange rate regime.

Symmetric Countries	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-59.93**	**-60.02**
Savers Welfare A	2049.08	2052.47
Borrowers Welfare A	-4021.08	-4027.26
Social Welfare B	**-2.27**	**-2.27**
Savers Welfare B	-40.40	-40.73
Borrowers Welfare B	-93.67	-93.62
Total Welfare	**-31.10**	**-31.15**

Table 15: House Price Shock in Country A. Welfare Comparison. Monetary Union vs. Flexible Exchange Rates. Symmetric Countries.

I can also study if it is beneficial for countries with different Central Bank preferences to form a monetary union with a unique Central Bank. Table 16 shows welfare comparisons for the two monetary regimes when countries are asymmetric in their interest rate response to inflation. We see that the country that fights inflation more aggressively (Country B) gains from joining a monetary union in which the level of aggressiveness is intermediate. The Country with a small inflation coefficient in the Taylor Rule also gains slightly. Overall, the economy benefits from joining in a monetary union.

Asymmetric TR	**Monetary Union**	**Flex. Exchange Rates**
Social Welfare A	**-3.486**	**-3.677**
Savers Welfare A	-116.53	-161.23
Borrowers Welfare A	-116.05	-103.27
Social Welfare B	**-3.486**	**-4.614**
Savers Welfare B	-116.53	31.09
Borrowers Welfare B	-116.05	-246.26
Total Welfare	**-3.486**	**-4.146**

Table 16: Welfare Comparisons. Asymmetric Policy Responses, Common Technology Shock. Monetary Union vs. Flexible Exchange Rates

2.5 Concluding Remarks

This paper explores first how cross-country housing market heterogeneity affects the transmission of shocks in a monetary union. Since there is evidence of such heterogeneity across countries in Europe, it is relevant to evaluate to what extent this is important. Then, I also study the effects of asymmetric shocks and asymmetric monetary policy responses across members. Results are presented both from a positive and a normative perspective.

For this purpose, I build a two-country dynamic stochastic general equilibrium (DSGE) model that features a housing market. A group of individuals in each country are credit constrained and need housing collateral to obtain loans. I consider two monetary regimes: the two countries conducting its own monetary policy under a flexible exchange rate system and a monetary union between the two countries.

I find that after a common monetary policy or technology shock, variables respond more strongly if the country has a high LTV, a high proportion of borrowers or mainly variable rate mortgages. As for country-specific shocks, I find that the effects of a house price shock in one country are transmitted internationally to the other country and that the effects of asymmetric technology shocks depend on the monetary regime. I also study how asymmetries in the monetary policy reaction functions lead to different transmission of common shocks across monetary regimes.

From a normative perspective, I find that housing market homogeneization per se is not necessarily beneficial. Since low LTVs imply that collateral effects are less important and therefore the collateral constraint less distorting, homogeneization towards high LTVs decreases welfare. However, in the case of the borrowers proportion, homogeneization would increase welfare. As for mortgage contracts, result suggest that countries with predominantly variable-rate contracts should move towards fixed-rate contracts. Fixed-rate mortgages reduce the distorting effects of the collateral constraint. In terms of analyzing costs and benefits of forming a monetary union, I find that, when countries are symmetric and there is an asymmetric technology shock, a monetary union regime decreases welfare. However, this result may change once asymmetries are introduced. When the country that receives the shock is small I find that, as in Carré and Collard (2003), this country is better off under a monetary union. Nevertheless, the other country is better off under a flexible exchange rates regime. This result is still maintained if the small country has a low LTV. However, if the small country has a high proportion of borrowers or variable-rate mortgages, as opposed to the rest of countries in the union, the monetary union regime is welfare worsening. On the contrary, for an asymmetric house price shock, the monetary regime does not make any significant difference for welfare. When countries are asymmetric in their aggressiveness towards inflation, I find that overall a monetary union with an intermediate response to inflation is beneficial.

For future research, it would be interesting to study what is the optimal monetary policy under the different sources of asymmetry for the monetary union as a whole and for the two countries separately.

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Appendix 1: Tables and Figures

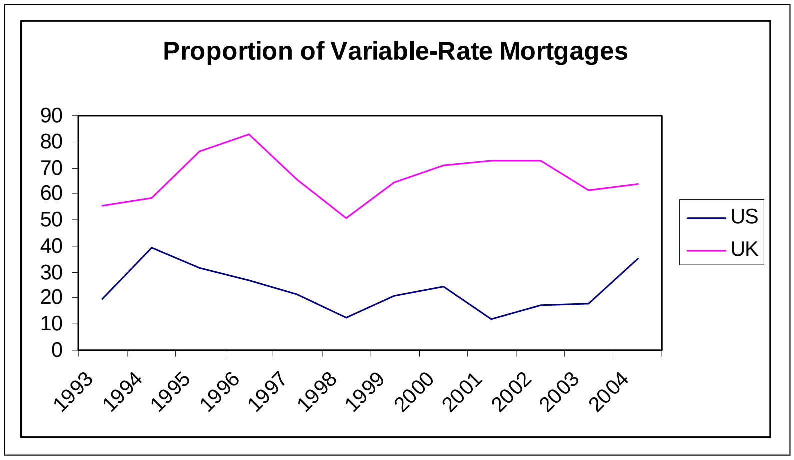

Figure 15: Proportion of Variable-Rate Mortgages in the US and UK. Source: Federal Housing Finance Board and Council of Mortgage Lenders

Loan-to-value ratios in European Countries (2006)	
Germany	73%
France	16%
Sweden	up to 95%
Denmark	50%
Spain	66.5%
Italy	60% and higher
United Kingdom	78%
Source: European Mortgage Federation (Factsheets)	

Table 17[51]

[51] Average LTV for all buyers. For France, first-time buyers in 2004.

Predominant Type of Mortgage Interest Rate			
Australia	Vble (mostly)	**Italy**	Mixed (72%)
Austria	Fixed (75%)	**Japan**	Mixed /Vble (64%)
France	Fixed (86%)	**Spain**	Vble (75%)
Germany	Fixed (mostly)	**United Kingdom**	Vble (72%)
Greece	Vble (80%)	**United States**	Fixed (85%)
Source: ECB (2003), Debelle (2004), Calza et al. (2006)			

Table 18

Residential Debt to GDP Ratio (2006)			
Belgium	36.3%	**Italy**	18.7%
Denmark	100.8%	**Netherlands**	98.4%
Germany	51.3%	**Austria**	23.5%
Greece	29.3%	**United Kingdom**	83.1%
Spain	58.6%	**Sweden**	56.7%
France	32.2%	**EU 27**	49.0%
Source: European Mortgage Federation			

Table 19

House Price % Change in European Countries						
	2001	**2002**	**2003**	**2004**	**2005**	**2006**
Belgium	5.1	7.1	7.2	5.5	16.3	12.1
Denmark	7.9	5.3	5.2	11.7	17.0	16.2
Germany	0.0	-1.6	-0.8	-0.8	-1.7	-0.9
Greece	14.6	13.0	5.7	5.2	13.1	9.0
Spain	9.9	15.7	17.6	17.4	13.9	10.4
France	8.1	9.0	11.5	17.6	14.7	9.9
Ireland	8.0	3.6	14.1	11.2	10.6	13.6
Italy	7.9	10.0	10.7	n/a	n/a	n/a
Latvia	n/a	14.0	17.5	4.9	48.6	n/a
Hungary	8.6	-1.1	n/a	n/a	n/a	n/a
Poland	10.0	-4.2	-6.9	n/a	n/a	n/a
Sweden	8.0	6.3	6.6	9.6	9.6	11.4
UK	8.4	17.0	15.7	11.8	5.5	6.3
Source: European Mortgage Federation						

Table 20

Appendix 2: Model Derivations and Alternative Specifications for "Fixed and Variable-Rate Mortgages, Business Cycles and Monetary Policy"

Steady-State Relationships

Using (3) in the steady state we obtain $R = 1/\beta$. From (57) and (58) we have that $\overline{R}^{*} = \overline{R} = R = 1/\beta$.

From the first order conditions for housing we can obtain the steady-state consumption-to-housing ratio for both constrained and unconstrained consumers:

$$\frac{C^u}{qH^u} = \frac{1}{j}(1-\beta), \tag{69}$$

$$\frac{C^c}{qH^c} = \frac{1}{j}\left(1-\widetilde{\beta}-k\left(\beta-\widetilde{\beta}\right)\right) = \frac{q}{j}\Phi, \tag{70}$$

where $\Phi \equiv \left(1-\widetilde{\beta}-k\left(\beta-\widetilde{\beta}\right)\right)$. From (52) and (62)we obtain the constrained and unconstrained consumption-to-output ratio in the steady state:

$$\frac{C^c}{Y} = \frac{1-\gamma}{X}\left(\frac{\Phi}{\Phi+jk(1-\beta)}\right), \tag{71}$$

$$\frac{C^u}{Y} = 1-\frac{C^c}{Y}, \tag{72}$$

where $X = \varepsilon/(\varepsilon-1)$

The housing-to-output ratio for constrained and unconstrained consumers:

$$\frac{qH^c}{Y} = \frac{(1-\gamma)j}{X}\left(\frac{1}{\Phi+jk(1-\beta)}\right), \tag{73}$$

$$\frac{qH^u}{Y} = \frac{Xj\left(\Phi + jk\left(1-\beta\right)\right) - j\left(1-\gamma\right)\Phi}{X\left(\Phi + jk\left(1-\beta\right)\right)\left(1-\beta\right)}. \tag{74}$$

Log-Linearized Model

The model can be reduced to the following linearized system in which all lower-case variables with a hat denote percent changes from the steady state and steady-state levels are denoted by dropping the time index:

Financial intermediary

$$\widehat{\overline{r}}_{\tau}^{*} = \frac{(1-\beta)}{\beta} E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \widehat{r}_{i-1}, \tag{75}$$

$$\widehat{\overline{r}}_t = \widehat{\overline{r}}_{t-1} \Rightarrow \widehat{\overline{r}}_t = \widehat{\overline{r}} = 0. \tag{76}$$

Equation (75) is the log-linearized fixed interest rate in each period τ. Using this result we can obtain the log-linearized aggregate fixed interest rate, which is zero in deviations from the steady state (equation (76)), given the initial condition of being at the steady state in the absence of shocks.

Aggregate Demand

$$\widehat{y}_t = \frac{C^u}{Y}\widehat{c}_t^u + \frac{C^c}{Y}\widehat{c}_t^c, \tag{77}$$

$$\widehat{c}_t^u = E_t\widehat{c}_{t+1}^u - \left(\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right), \tag{78}$$

$$\begin{aligned}\widehat{c}_t^c = {} & \left(\frac{\Phi + jk\left(1-\beta\right)}{\Phi}\right)\left(\widehat{y}_t - \widehat{x}_t\right) - \frac{j}{\Phi}\left(\widehat{h}_t^c - \widehat{h}_{t-1}^c\right) \\ & + \frac{kj}{\Phi}\left(\beta\widehat{b}_t^c - \widehat{b}_{t-1}^c\right) - kj\left(\alpha\widehat{r}_{t-1} - \widehat{\pi}_t\right),\end{aligned} \tag{79}$$

$$\widehat{b}_t^c = E_t \widehat{q}_{t+1} + \widehat{h}_t^c - (\alpha \widehat{r}_t - E_t \widehat{\pi}_{t+1}) \,. \tag{80}$$

Equation (77) is the log-linearized goods market clearing condition. Equation (172) is the Euler equation for unconstrained consumption. Equation (79) is the budget constraint for constrained individuals, which determines constrained consumption. Equation (80) is the log-linearized collateral constraint.

Housing Equations

$$\frac{H^u}{Y} \widehat{h}_t^u + \frac{H^c}{Y} \widehat{h}_t^c = 0, \tag{81}$$

$$\widehat{h}_t^u = \frac{1}{1-\beta} \left(\widehat{c}_t^u - \widehat{q}_t \right) - \frac{\beta}{1-\beta} E_t \left(\widehat{c}_{t+1}^u - \widehat{q}_{t+1} \right), \tag{82}$$

$$\widehat{h}_t^c = \frac{1-k\beta}{\Phi} \widehat{c}_t^c - \frac{1}{\Phi} \widehat{q}_t - \frac{k\beta}{\Phi} \left(\alpha \widehat{r}_t - E_t \widehat{\pi}_{t+1} \right) + \frac{\widetilde{\beta}}{\Phi} \widehat{q}_{t+1} - \frac{\widetilde{\beta}\,(1-k)}{\Phi} E_t \widehat{c}_{t+1}^c. \tag{83}$$

Equation (81) is the log-linearized market clearing condition for housing. Equation (82) is the housing margin for unconstrained consumers. Equation (181) is the analogous expression for constrained consumers.

Aggregate Supply

$$\widehat{y}_t = -\frac{1}{\eta - 1} \left(\gamma \widehat{c}_t^u + (1-\gamma)\, \widehat{c}_t^c + \widehat{x}_t \right), \tag{84}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} - \widetilde{k} \widehat{x}_t + u_{\pi t}. \tag{85}$$

Equation (183) is the production function combined with labor market clearing. Equation (85) is the New Keynesian Phillips curve that relates inflation positively to future inflation and negatively to the markup ($\widetilde{k} \equiv (1-\theta)(1-\beta\theta)/\theta$). $u_{\pi t}$ is a normally distributed cost-push shock.

Monetary Policy

$$\widehat{r}_t = \rho\widehat{r}_{t-1+}\left(1-\rho\right)\left[\left(1+\phi_\pi\right)\left(\widehat{\pi}_t-\widehat{\pi}_t^*\right)+\phi_y\widehat{y}_t\right]+e_t. \tag{86}$$

Alternative Model Specifications

The Model with GHH Preferences

Under GHH preferences, savers maximize:

$$E_0\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t^u-\frac{\left(L_t^u\right)^\eta}{\eta}\right)+j\ln H_t^u\right], \tag{87}$$

The first-order conditions are:

$$\frac{1}{C_t^u-\frac{\left(L_t^u\right)^\eta}{\eta}}=\beta E_t\left[\frac{R_t}{\pi_{t+1}\left(C_{t+1}^u-\frac{\left(L_{t+1}^u\right)^\eta}{\eta}\right)}\right], \tag{88}$$

$$w_t^u=\left(L_t^u\right)^{\eta-1}, \tag{89}$$

$$\frac{j}{H_t^u}=\frac{1}{C_t^u-\frac{\left(L_t^u\right)^\eta}{\eta}}q_t-\beta E_t\frac{1}{C_{t+1}^u-\frac{\left(L_{t+1}^u\right)^\eta}{\eta}}q_{t+1}. \tag{90}$$

Note that consumption no longer appears in the labor-supply decision (equation (89)).

Similarly, we can obtain the first-order conditions for borrowers:

$$\frac{1}{C_t^{ci}-\frac{\left(L_t^{ci}\right)^\eta}{\eta}}=\widetilde{\beta}E_t\left[\frac{R_t^{ci}}{\pi_{t+1}\left(C_{t+1}^{ci}-\frac{\left(L_{t+1}^{ci}\right)^\eta}{\eta}\right)}\right]+\lambda_t^{ci}R_t^{ci}, \tag{91}$$

$$w_t^{ci}=\left(L_t^{ci}\right)^{\eta-1}, \tag{92}$$

$$\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci} - \frac{\left(L_t^{ci}\right)^{\eta}}{\eta}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{ci} - \frac{\left(L_{t+1}^{ci}\right)^{\eta}}{\eta}} q_{t+1} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}. \tag{93}$$

The Model with Capital

We can add capital to the model so that unconstrained consumers have more saving choices. Since borrowers would not hold capital, the only part of the model that changes is the one of the unconstrained consumers:

Unconstrained consumers maximize their expected lifetime utility function:

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^{\eta}}{\eta} \right), \tag{94}$$

subject to the budget constraint which includes capital:

$$C_t^u + q_t H_t^u + K_t - (1-\delta) K_{t-1} + \frac{\phi}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right)^2 + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t H_{t-1}^u \tag{95}$$
$$+ z_t K_{t-1} + w_t^u L_t^u + b_t^u + F_t^v + S_t^v,$$

So, in this case, savers can buy houses or sell them at the current price q_t and hold bonds. They can also hold capital K_t, whose price is normalized to unity, which they rent to firms at rental price z_t. δ is the depreciation rate of capital. Consumers also have to pay quadratic adjustment costs for capital.

Maximizing (94) subject to (95), we obtain the first-order conditions:

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \tag{96}$$

$$w_t^u = (L_t^u)^{\eta - 1} C_t^u, \tag{97}$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}. \tag{98}$$

$$\frac{1}{C_t^u}\left[1+\phi\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)\right] = \beta E_t \frac{1}{C_{t+1}^u}\left(z_t + (1-\delta) + \phi \frac{K_{t+1}}{K_t^2}\left(\frac{K_{t+1} - K_t}{K_t}\right)\right), \tag{99}$$

Now, we have a fourth first order condition, equation (99), which is the first order condition with respect to capital.

Unconstrained individuals are not going to hold capital in equilibrium so their problem remains unchanged.

Intermediate goods are going to be produced according to the following production function:

$$Y_t = (L_t^u)^{\mu\gamma} (L_t^c)^{\mu(1-\gamma)} K_{t-1}^{1-\mu}, \tag{100}$$

where γ measures the relative size of each group in terms of labor and μ is the labor share.

Firms choose employment and capital to

$$\min w_t^u L_t^u + w_t^c L_t^c + z_t K_{t-1},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor and capital demand are the following:

$$w_t^u = \frac{1}{X_t} \gamma\mu \frac{Y_t}{L_t^u}, \tag{101}$$

$$w_t^c = \frac{1}{X_t}(1-\gamma)\mu \frac{Y_t}{L_t^c}, \tag{102}$$

$$z_t = \frac{1}{X_t}(1-\mu)\frac{Y_t}{K_{t-1}}, \tag{103}$$

Non Separability between Housing and Non-Durable Consumption in the Utility Function

Unconstrained consumers consume an index of non-durable goods and housing defined as:

$$I_t^u = \left[(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \tag{104}$$

where ν is the share of housing in the composite consumption index and μ is the elasticity of substitution between non-durable consumption goods and housing.

Unconstrained consumers maximize an expected lifetime utility function with two arguments; the consumption index and labor/leisure.

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln I_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{105}$$

Subject to the budget constraint:

$$C_t^u + q_t H_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t H_{t-1}^u + w_t^u L_t^u + b_t^u + F_t^v + S_t^v, \tag{106}$$

Maximizing (105) subject to (106), we obtain the first-order conditions:

$$\frac{\left(C_t^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{\mu-1}{\mu}}} = \beta E_t \left[\left(\frac{R_t}{\pi_{t+1}} \right) \left(\frac{\left(C_{t+1}^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_{t+1}^u\right)^{\frac{\mu-1}{\mu}}} \right) \right], \tag{107}$$

$$w_t^u \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{\mu-1}{\mu}}} = \left(L_t^u\right)^{\eta-1}, \tag{108}$$

$$\begin{aligned} \frac{\nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{\mu-1}{\mu}}} &= \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^u\right)^{\frac{\mu-1}{\mu}}} q_t \\ &- \beta E_t \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^u\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^u\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_{t+1}^u\right)^{\frac{\mu-1}{\mu}}} q_{t+1} \end{aligned} \tag{109}$$

In the same way, we have the problem of the **constrained consumers**.

They also consume a consumption index that aggregates non-durable goods and housing:

$$I_t^{ci} = \left[(1-\nu)^{\frac{1}{\mu}} \left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H^{ci}\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{110}$$

Constrained consumers maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max \ E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln I_t^{ci} - \frac{\left(L_t^{ci}\right)^{\eta}}{\eta}\right), \tag{111}$$

subject to:

$$C_t^{ci} + q_t H_t^{ci} + \frac{R_{t-1}^c b_{t-1}^{ci}}{\pi_t} \leq q_t H_{t-1}^{ci} + w_t^{ci} L_t^{ci} + b_t^{ci}, \tag{112}$$

$$E_t \frac{R_t^c}{\pi_{t+1}} b_t^{ci} \leq k E_t q_{t+1} H_t^{ci}. \tag{113}$$

The first-order conditions for the consumers are:

$$\frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}} = \widetilde{\beta} E_t \left[\left(\frac{R_t^c}{\pi_{t+1}}\right) \left(\frac{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}}}\right)\right] \tag{114}$$

$$+\lambda_t^{ci} R_t^c,$$

$$w_t^{ci} \frac{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}} \left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}} + \nu^{\frac{1}{\mu}} \left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}} = \left(L_t^{ci}\right)^{\eta-1}, \tag{115}$$

$$
\begin{aligned}
\frac{\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}} &= \frac{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}} q_t \\
&-\widetilde{\beta} E_t \frac{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}}} q_{t+1}-\lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}.
\end{aligned} \tag{116}
$$

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint $\left(\lambda_t^{ci}\right)$ appears in the equations. From the Euler equations for consumption of the unconstrained consumers, we know that $R = 1/\beta$ in steady state. If we combine this result with the Euler equation for consumption for the constrained individual we have that $\lambda^{ci} = \frac{\left(\beta-\widetilde{\beta}\right)(1-\nu)^{\frac{1}{\mu}}\left(C^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H^{ci}\right)^{\frac{\mu-1}{\mu}}} > 0$ in steady state, given that $\widetilde{\beta} < \beta$. This means that the borrowing constraint holds with equality in steady state. As in Iacoviello (2005), since we log-linearize around the steady state and assuming that uncertainty is low, we can generalize this steady-state result. Then, the borrowing constraint is always binding, so that constrained individuals are going to borrow the maximum amount they are allowed to and their savings are going to be zero:

$$
b_t^{ci} = \frac{k E_t q_{t+1} H_t^{ci} \pi_{t+1}}{R_t^c}. \tag{117}
$$

Therefore, consumption for constrained individuals is determined by their flow of funds:

$$
C_t^{ci} = w_t^{ci} L_t^{ci} + b_t^{ci} + q_t\left(H_{t-1}^{ci} - H_t^{ci}\right) - \frac{R_{t-1}^c b_{t-1}^{ci}}{\pi_t}, \tag{118}
$$

And the first-order condition for housing becomes:

$$\frac{\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}}=\frac{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_t^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_t^{ci}\right)^{\frac{\mu-1}{\mu}}}\left(q_t-\frac{kE_t q_{t+1}\pi_{t+1}}{R_t^c}\right) \tag{119}$$

$$-\widetilde{\beta}E_t\frac{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^{ci}\right)^{\frac{-1}{\mu}}}{(1-\nu)^{\frac{1}{\mu}}\left(C_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}}+\nu^{\frac{1}{\mu}}\left(H_{t+1}^{ci}\right)^{\frac{\mu-1}{\mu}}}(1-k)\,q_{t+1}.$$

The problem of the financial intermediary, the firms and the monetary policy is identical to the baseline model.

A Two-Sector Model

We can relax the assumption that the housing supply is fixed and consider a two-sector model in which consumers can supply labor to the housing sector and the consumption sector.

Unconstrained consumers:

$$\max\ E_0\sum_{t=0}^{\infty}\beta^t\left(\ln C_t^u+j\ln H_t^u-\left(\left(L_{ct}^u\right)^{1-\nu}+\left(L_{ht}^u\right)^{1-\nu}\right)^{\frac{1+\eta}{1-\nu}}\right), \tag{120}$$

subject to the budget constraint:

$$C_t^u+q_tH_t^u+\frac{R_{t-1}b_{t-1}^u}{\pi_t}\leq q_t(1-\delta)H_{t-1}^u+w_{ct}^uL_{ct}^u+w_{ht}^uL_{ht}^u+b_t^u+F_t^v+S_t^v, \tag{121}$$

Maximizing (120) subject to (121), we obtain the first-order conditions:

$$\frac{1}{C_t^u}=\beta E_t\left(\frac{R_t}{\pi_{t+1}C_{t+1}^u}\right), \tag{122}$$

$$w_{ct}^u=(1+\eta)\left(\left(L_{ct}^u\right)^{1-\nu}+\left(L_{ht}^u\right)^{1-\nu}\right)^{\frac{\nu+\eta}{1-\nu}}\left(L_{ct}^u\right)^{-\nu}C_t^u, \tag{123}$$

$$w_{ht}^{u}=(1+\eta)\left(\left(L_{ct}^{u}\right)^{1-\nu}+\left(L_{ht}^{u}\right)^{1-\nu}\right)^{\frac{\nu+\eta}{1-\nu}}\left(L_{ht}^{u}\right)^{-\nu}C_{t}^{u}, \tag{124}$$

$$\frac{j}{H_{t}^{u}}=\frac{1}{C_{t}^{u}}q_{t}-\beta E_{t}\frac{1}{C_{t+1}^{u}}\left(1-\delta\right)q_{t+1}. \tag{125}$$

Equations (122) is the consumption Euler equation. Equations (123) and (124) are the labor-supply condition for the consumption and the housing sector, respectively. Equation (125) is the Euler equation for housing and states that the benefits from consuming housing have to be equal to the costs.

Constrained consumers maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

$$\max\ E_{0}\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\left(\ln C_{t}^{ci}+j\ln H_{t}^{ci}-\left(\left(L_{ct}^{ci}\right)^{1-\nu}+\left(L_{ht}^{ci}\right)^{1-\nu}\right)^{\frac{1+\eta}{1-\nu}}\right), \tag{126}$$

subject to:

$$C_{t}^{ci}+q_{t}H_{t}^{ci}+\frac{R_{t-1}^{c}b_{t-1}^{ci}}{\pi_{t}}\leq q_{t}\left(1-\delta\right)H_{t-1}^{ci}+w_{ct}^{ci}L_{ct}^{ci}+w_{ht}^{ci}L_{ht}^{ci}+b_{t}^{ci}, \tag{127}$$

$$E_{t}\frac{R_{t}^{c}}{\pi_{t+1}}b_{t}^{ci}\leq kE_{t}q_{t+1}H_{t}^{ci}. \tag{128}$$

The first-order conditions for the consumers are:

$$\frac{1}{C_{t}^{ci}}=\widetilde{\beta}E_{t}\left(\frac{R_{t}^{c}}{\pi_{t+1}C_{t+1}^{ci}}\right)+\lambda_{t}^{ci}R_{t}^{c}, \tag{129}$$

$$w_{ct}^{ci}=(1+\eta)\left(\left(L_{ct}^{ci}\right)^{1-\nu}+\left(L_{ht}^{ci}\right)^{1-\nu}\right)^{\frac{\nu+\eta}{1-\nu}}\left(L_{ct}^{ci}\right)^{-\nu}C_{t}^{ci}, \tag{130}$$

$$w_{ht}^{ci}=(1+\eta)\left(\left(L_{ct}^{ci}\right)^{1-\nu}+\left(L_{ht}^{ci}\right)^{1-\nu}\right)^{\frac{\nu+\eta}{1-\nu}}\left(L_{ht}^{ci}\right)^{-\nu}C_{t}^{ci}, \tag{131}$$

$$\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{ci}} (1-\delta) q_{t+1} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}. \tag{132}$$

Using the fact that the collateral constraint is binding, consumption for constrained individuals is determined by their flow of funds:

$$C_t^{ci} = w_{ct}^{ci} L_{ct}^{ci} + w_{ht}^{ci} L_{ht}^{ci} + b_t^{ci} + q_t \left((1-\delta) H_{t-1}^{ci} - H_t^{ci} \right) - \frac{R_{t-1}^c b_{t-1}^{ci}}{\pi_t}, \tag{133}$$

And the first-order condition for housing becomes:

$$\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} \left(q_t - \frac{k E_t q_{t+1} \pi_{t+1}}{R_t^c} \right) - \widetilde{\beta} E_t \frac{1}{C_{t+1}^{ci}} (1-k)(1-\delta) q_{t+1} \tag{134}$$

The problem for the financial intermediary and the final good producer is identical to the baseline model. The problem for the intermediate good producer is slightly changed. Intermediate goods are produced according to the following production function:

$$Y_{ct} = \left(L_{ct}^u\right)^{\gamma} \left(L_{ct}^c\right)^{(1-\gamma)}, \tag{135}$$

where γ measures the relative size of each group in terms of labor.

Analogously, the production function for the housing sector is the following:

$$Y_{ht} = \left(L_{ht}^u\right)^{\gamma} \left(L_{ht}^c\right)^{(1-\gamma)}, \tag{136}$$

Firms choose employment to

$$\min w_{ct}^u L_{ct}^u + w_{ct}^c L_{ct}^c + w_{ht}^u L_{ht}^u + w_{ht}^c L_{ht}^c,$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

$$w_{ct}^u = \frac{1}{X_t} \gamma \frac{Y_{ct}}{L_{ct}^u}, \tag{137}$$

$$w_{ct}^{c} = \frac{1}{X_t}\left(1-\gamma\right)\frac{Y_{ct}}{L_{ct}^{c}}, \tag{138}$$

$$w_{ht}^{u} = q_t\gamma\frac{Y_{ht}}{L_{ht}^{u}}, \tag{139}$$

$$w_{ht}^{c} = q_t\left(1-\gamma\right)\frac{Y_{ht}}{L_{ht}^{c}}, \tag{140}$$

Production function in which labor for savers and labor for borrowers are substitutes

The consumers' and financial intermediary's problem remains unchanged. However, the intermediate goods firm's production function turns into the following one:

$$Y_t = \omega L_t^{u} + \left(1-\omega\right)\left[\alpha L_t^{cv} + \left(1-\alpha\right)L_t^{cf}\right] = \omega L_t^{u} + \left(1-\omega\right)L_t^{c}, \tag{141}$$

where ω is the size of the unconstrained group.

Firms choose employment to

$$\min \omega w_t^{u} L_t^{u} + \left(1-\omega\right) w_t^{c} L_t^{c},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

$$w_t^{u} = w_t^{c} = \frac{1}{X_t}. \tag{142}$$

We see that in this case, the wage paid to each group is the same.

Aggregate variables are defined as follows:

$$C_t \equiv \omega C_t^{u} + \left(1-\omega\right)C_t^{c}. \tag{143}$$

$$L_t \equiv \omega L_t^u + (1-\omega) L_t^c. \tag{144}$$

$$H_t \equiv \omega H_t^u + (1-\omega) H_t^c. \tag{145}$$

Appendix 3: Model Derivations and Codes for " Housing Market Heterogeneity in a Monetary Union"

The Monetary Union Case

Unconstrained consumers in Country A:

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j_t \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{146}$$

subject to:

$$C_{At}^u + \frac{P_{Bt}}{P_{At}} C_{Bt}^u + q_t H_t^u + \frac{R_{At-1} b_{t-1}^u}{\pi_{At}} + \frac{R_{t-1} d_{t-1}}{\pi_{At}} + \frac{\psi}{2} d_t^2 \leq q_t H_{t-1}^u + w_t^u L_t^u + b_t^u + d_t + F_t + S_t, \tag{147}$$

where R_t is an international interest rate. The non-arbitrage condition between home and foreign bonds implies now that

$$R_{At} = \frac{R_t}{(1-\psi d_t)}, \tag{148}$$

The equations for consumers in Country B are symmetric. The problem for the firms and the financial intermediary in each country is identical to the non-monetary union case.

The Taylor Rule becomes:

$$R_t = (R_{t-1})^\rho \left(\left[(\pi_{At})^n (\pi_{Bt})^{(1-n)} \right]^{(1+\phi_\pi)} \left[\left(\frac{Y_{At}/Y_{At-1}}{Y_A} \right)^n \left(\frac{Y_{Bt}/Y_{Bt-1}}{Y_B} \right)^{1-n} \right]^{\phi_y} R \right)^{1-\rho} \varepsilon_{R,t}, \tag{149}$$

Steady-State Relationships

Relative prices in the steady state are derived from equations (37), (47)and their counterparts for Country B:

$$\frac{n}{1-n}\frac{P_B}{P_A} = \frac{C_A^u}{C_B^u} = \frac{C_A^{ci}}{C_B^{ci}} = \frac{C_A^{u*}}{C_B^{u*}} = \frac{C_A^{ci*}}{C_B^{ci*}} \tag{150}$$

Interest rates:

$$R_A = R = R_B = \overline{R} = \overline{R}^* = 1/\beta \tag{151}$$

We can find the consumption to housing ratio for savers and borrowers in Country A by using the first order conditions for housing:

$$\frac{C_A^u}{qH^u} = \frac{n}{j}(1-\beta) \tag{152}$$

$$\frac{C_A^c}{qH^c} = \frac{n}{j}\left[\left(1-\widetilde{\beta}\right) - k_A\left(\beta-\widetilde{\beta}\right)\right] = \frac{n}{j}\zeta \tag{153}$$

Similarly, for Country B:

$$\frac{C_B^{u*}}{q^*H^{u*}} = \frac{(1-n)}{j^*}(1-\beta) \tag{154}$$

$$\frac{C_B^{c*}}{q^*H^{c*}} = \frac{(1-n)}{j^*}\left[\left(1-\widetilde{\beta}\right) - k_B\left(\beta-\widetilde{\beta}\right)\right] = \frac{(1-n)}{j^*}\zeta^* \tag{155}$$

Borrowing in the steady state:

$$b^{ci} = \beta k_A q H^{ci}. \tag{156}$$

$$b^u + b^c = 0$$

$$b^{ci*} = \beta k_B q^* H^{ci*}. \tag{157}$$

$$b^{u*} + b^{c*} = 0$$

From the problem of the firm we have that in the steady state:

$$w^u = \frac{1}{X} \gamma \frac{Y_A}{L^u}, \tag{158}$$

$$w^c = \frac{1}{X} (1 - \gamma) \frac{Y_A}{L^c}, \tag{159}$$

$$w^{u*} = \frac{1}{X^*} \gamma \frac{Y_B}{L^{u*}}, \tag{160}$$

$$w^{c*} = \frac{1}{X^*} (1 - \gamma) \frac{Y_B}{L^{c*}}, \tag{161}$$

where $X = X^* = \dfrac{\varepsilon - 1}{\varepsilon}$.

Combining the steady-state budget constraint for the unconstrained consumers in Country A with(152) and (158) we obtain:

$$\frac{C_A^u}{Y_A} = \frac{n (\gamma + X - 1)}{X (1 - jk_A)} \tag{162}$$

Similarly, for constrained consumers:

$$\frac{C_A^c}{Y_A} = \frac{1 - \gamma}{X} \frac{\zeta n}{\zeta + jk_A (1 - \beta)} \tag{163}$$

The market clearing condition for the good produced in Country A implies:

$$\frac{C_A^*}{Y_A} = \frac{n}{1 - n} \left(1 - \frac{C_A^u}{Y_A} - \frac{C_A^c}{Y_A} \right)$$

Using (152) and (162) we can find the housing to output ratio for the savers in Country A:

$$\frac{H^u}{Y_A} = \frac{j\left(\gamma + X - 1\right)}{Xq\left(1 - jk_A\right)\left(1 - \beta\right)} \tag{164}$$

Analogously, using (153) and (163) we can find the housing to output ratio for the constrained consumers in Country A:

$$\frac{H^c}{Y_A} = \frac{\left(1 - \gamma\right)j}{Xq}\frac{n}{\zeta + jk_A\left(1 - \beta\right)} \tag{165}$$

Similarly, for Country B:

$$\frac{C_B^{u*}}{Y_B} = \frac{\left(1 - n\right)\left(\gamma + X^* - 1\right)}{X^*\left(1 - j^*k_B\right)} \tag{166}$$

$$\frac{C_B^{c*}}{Y_B} = \frac{1 - \gamma}{X^*}\frac{\zeta\left(1 - n\right)}{\zeta^* + j^*k_B\left(1 - \beta\right)} \tag{167}$$

$$\frac{H^{u*}}{Y_B} = \frac{j^*\left(\gamma + X^* - 1\right)}{X^*q^*\left(1 - j^*k_B\right)\left(1 - \beta\right)} \tag{168}$$

$$\frac{H^{c*}}{Y_B} = \frac{\left(1 - \gamma\right)j^*}{X^*q^*}\frac{\left(1 - n\right)}{\zeta^* + j^*k_B\left(1 - \beta\right)} \tag{169}$$

Log-linearized Equations

Variables in deviations from the steady state are expressed in lower-case and with a hat.

Interest Rates

$$\hat{r}_{At} = \hat{r}_{Bt} + E_t\left(\hat{e}_{t+1} - \hat{e}_t\right) + \psi, \tag{170}$$

$$\widehat{\overline{r}}_{At} = \widehat{\overline{r}}_{Bt} = 0. \tag{171}$$

Aggregate Demand

$$\widehat{c}^{u}_{At} = E_t \widehat{c}^{u}_{At+1} - \left(\widehat{r}_{At} - E_t \widehat{\pi}_{At+1}\right), \tag{172}$$

$$\widehat{c}^{u*}_{Bt} = E_t \widehat{c}^{u*}_{Bt+1} - \left(\widehat{r}_{Bt} - E_t \widehat{\pi}_{Bt+1}\right), \tag{173}$$

$$\begin{aligned}\hat{c}^{c}_{At} = &\left(\frac{\zeta + jk_A\left(1-\beta\right)}{\zeta}\right)\left(\hat{y}_{At} + \widehat{z}_t - \hat{x}_t\right) - \frac{j}{\zeta}\left(\hat{h}^{c}_{t} - \hat{h}^{c}_{t-1}\right) \\ &+ \frac{k_A j}{\zeta}\left(\beta \hat{b}^{c}_{t} - \hat{b}^{c}_{t-1}\right) - k_A j\left(\alpha_A \hat{r}_{At-1} - \hat{\pi}_{At}\right),\end{aligned} \tag{174}$$

$$\hat{b}^{c}_{t} = E_t \hat{q}_{t+1} + \hat{h}^{c}_{t} - \left(\alpha_A \hat{r}_{At} - E_t \hat{\pi}_{At+1}\right), \tag{175}$$

$$\begin{aligned}\hat{c}^{c*}_{Bt} = &\left(\frac{\zeta^* + j^* k_B\left(1-\beta\right)}{\zeta^*}\right)\left(\hat{y}_{Bt} + \widehat{z}^{*}_{t} - \hat{x}_t\right) - \frac{j^*}{\zeta^*}\left(\hat{h}^{c*}_{t} - \hat{h}^{c*}_{t-1}\right) \\ &+ \frac{k_B j^*}{\zeta^*}\left(\beta \hat{b}^{c*}_{t} - \hat{b}^{c*}_{t-1}\right) - k_B j^*\left(\alpha_B \hat{r}_{Bt-1} - \hat{\pi}_{Bt}\right),\end{aligned} \tag{176}$$

$$\hat{b}^{c*}_{t} = E_t \hat{q}^{*}_{t+1} + \hat{h}^{c*}_{t} - \left(\alpha_B \hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}\right), \tag{177}$$

$$\hat{c}_{At} - \hat{c}_{Bt} = \hat{c}^{*}_{At} - \hat{c}^{*}_{Bt} \tag{178}$$

Housing Equations

$$\widehat{h}^{u}_{t} = \frac{1}{1-\beta}\left(\widehat{c}^{u}_{At} - \widehat{q}_t\right) - \frac{\beta}{1-\beta} E_t\left(\widehat{c}^{u}_{At+1} - \widehat{q}_{t+1}\right), \tag{179}$$

$$\widehat{h}^{u*}_{t} = \frac{1}{1-\beta}\left(\widehat{c}^{u*}_{Bt} - \widehat{q}^{*}_t\right) - \frac{\beta}{1-\beta} E_t\left(\widehat{c}^{u*}_{Bt+1} - \widehat{q}^{*}_{t+1}\right), \tag{180}$$

$$\hat{h}_t^c = \frac{1-k_A\beta}{\zeta}\hat{c}_t^c - \frac{1}{\zeta}\hat{q}_t - \frac{k_A\beta}{\zeta}\left(\alpha_A \hat{r}_{At} - E_t\hat{\pi}_{At+1}\right) + \frac{\widetilde{\beta}}{\zeta}E_t\hat{q}_{t+1} - \frac{\widetilde{\beta}\left(1-k_A\right)}{\zeta}E_t\hat{c}_{t+1}^c. \quad (181)$$

$$\hat{h}_t^{c*} = \frac{1-k_B\beta}{\zeta^*}\hat{c}_t^{c*} - \frac{1}{\zeta}\hat{q}_t^* - \frac{k_B\beta}{\zeta^*}\left(\alpha_B \hat{r}_{Bt} - E_t\hat{\pi}_{Bt+1}\right) + \frac{\widetilde{\beta}}{\zeta^*}E_t\hat{q}_{t+1}^* - \frac{\widetilde{\beta}\left(1-k_B\right)}{\zeta^*}E_t\hat{c}_{t+1}^{c*}. \quad (182)$$

Aggregate Supply

$$\hat{y}_{At} = \frac{\eta+1}{\eta-1}\widehat{z}_t - \frac{1}{\eta-1}\left(\gamma\hat{c}_{At}^u + (1-\gamma)\,\hat{c}_{At}^c + \hat{x}_t\right), \quad (183)$$

$$\hat{y}_{At} = \left(\frac{C_A^u}{Y_A} + \frac{C_A^c}{Y_A}\right)\hat{c}_{At} + \left(1 - \frac{C_A^u}{Y_A} - \frac{C_A^c}{Y_A}\right)\hat{c}_{At}^* \quad (184)$$

$$\hat{y}_{Bt} = \frac{\eta+1}{\eta-1}\widehat{z}_t^* - \frac{1}{\eta-1}\left(\gamma\hat{c}_{Bt}^{u*} + (1-\gamma)\,\hat{c}_{Bt}^{c*} + \hat{x}_t^*\right), \quad (185)$$

$$\hat{y}_{Bt} = \left(\frac{C_B^{u*}}{Y_B} + \frac{C_B^{c*}}{Y_B}\right)\hat{c}_{Bt}^* + \left(1 - \frac{C_B^{u*}}{Y_B} - \frac{C_B^{c*}}{Y_B}\right)\hat{c}_{Bt}, \quad (186)$$

$$\hat{\pi}_{At} = \beta\hat{\pi}_{At+1} - \widetilde{k}\hat{x}_t + u_{At}, \quad (187)$$

$$\hat{\pi}_{Bt}^* = \beta\hat{\pi}_{Bt+1}^* - \widetilde{k}\hat{x}_t^* + u_{Bt}, \quad (188)$$

where $\widetilde{k} = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and u_{At} and u_{Bt} are cost-push shocks.

Monetary Policy

$$\hat{r}_{At} = \rho_A\hat{r}_{At-1+}\left(1-\rho_A\right)\left[\left(1+\phi_{A\pi}\right)\hat{\pi}_{At} + \phi_{Ay}\hat{y}_{At}\right] + \epsilon_{AR,t}, \quad (189)$$

$$\hat{r}_{Bt} = \rho_B\hat{r}_{Bt-1+}\left(1-\rho_B\right)\left[\left(1+\phi_{B\pi}\right)\hat{\pi}_{Bt}^* + \phi_{By}\hat{y}_{Bt}\right] + \epsilon_{BR,t}, \quad (190)$$

Notice that under the monetary union regime (189) and (190) become:

$$\hat{r}_t = \rho\hat{r}_{t-1+}\left(1-\rho\right)\left\{\left(1+\phi_\pi\right)\left[n\hat{\pi}_{At}+\left(1-n\right)\hat{\pi}_{Bt}\right]+\phi_y\left[n_{At}\hat{y}_{At}+\left(1-n\right)\hat{y}_{Bt}\right]\right\}+\epsilon_{R,t} \quad (191)$$

Matlab Code for Welfare Analysis (Monetary Union Case)

These matlab files replicate the results on pages 61, 62 and 63 of this dissertation. It requires the use of Dynare (http://www.cepremap.cnrs.fr/~michel/dynare/). The "mod" file needs to be saved with the extension "mod" as "name.mod" and it is the main file that has to be run. It also requires the use of the auxiliary "m" file containing the steady states of the model, which has to be saved with the name "name_steadystate.m". The output of the file reports the welfare calculations on the first columns of Tables 7, 8 and 9 of this dissertation for the case of a monetary union. The results of the rest of the columns can be obtained by modifying the value of the parameters accordingly. Files for the flexible exchange rate case are available upon request.

Mod File

```
//%----------------------------------------------------------------
    //% Declare endogenous and exogenous variables
    //%----------------------------------------------------------------
    var
    b bf bu c ca ca1 ca1v ca1f c1 cb cb1 cb1v cb1f c1v c1f c1v_star c1f_star
    dpa h h1 h1v h1f lmf lmv nc nc1 nc1f ncc q uca uca1 uca1f ucb ucb1 ucb1f
    X Y b_star bf_star c_star ca_star ca1_star ca1v_star ca1f_star c1_star
    cb_star cb1_star cb1v_star cb1f_star dpb h_star h1_star h1v_star h1f_star
    lmf_star lmv_star nc_star nc1_star nc1f_star ncc_star q_star uca_star
    uca1_star uca1f_star ucb_star ucb1_star ucb1f_star uu welfu ucv welfcv
    ucf welfcf welf uu_star welfu_star ucv_star welfcv_star ucf_star welfcf_star
    welf_star welfunion X_star Y_star d r ra rb rbar
    data_CCA data_CCB data_DPE data_QQA data_QQB data_RRE GDPA GDPB
    a_a a_c a_j1 a_j2 a_z1 a_z2 ;
    //% shocks
```

```
varexo eps_a eps_j1 eps_j2 eps_e eps_c eps_e eps_a1 eps_j2 eps_p eps_z1 eps_z2 ;
//%———————————————————————
//% Declare model parameters
//%———————————————————————
parameters BETA BETA1 JEI MUC MUH DKC DKH DH ETA EC EC_STAR FIK;
parameters FIK_STAR FIH M M_STAR GAMMA GAMMA_STAR NU NA HB ;
parameters TETA TAYLOR_R TAYLOR_P X_SS LAGP RHO_AA RHO_AC ;
parameters RHO_AJ1 RHO_AJ2 RHO_AK1 RHO_AK2 RHO_AZ1 RHO_AZ2 ;
parameters CC_SS CC_SS_star IK_SS IK_SS_star QQ_SS QQ_SS_star ;
parameters ALPHA ALPHA_STAR;
X_SS = 1.2 ;
BETA = 0.99 ;
BETA1 = 0.98 ;
JEI=0.1 ;
MUC=0 ;
MUH=0 ;
DKC=0 ;
DKH=0 ;
DH=0 ;
ETA=1 ;
NU=0 ;
LAGP = 0 ;
TETA = 0.75 ;
TAYLOR_R = 0.8 ;
TAYLOR_P = 0.5 ;
FIH=0 ;
FIK=0 ;
FIK_STAR=0 ;
EC=0;
EC_STAR=0;
GAMMA=0.7 ;
GAMMA_STAR=0.7;
M=0.8;
M_STAR=0.8 ;
NA = .9 ;
HB = 0 ;
ALPHA=1;
ALPHA_STAR=1;
//%Shocks parameters
RHO_AA = 0.9 ;
RHO_AC = 0.8 ;
RHO_AJ1 = 0.8 ;
RHO_AJ2 = 0.8 ;
RHO_AK1 = 0.8 ;
RHO_AK2 = 0.8 ;
```

```
RHO_AZ1 = 0.8 ;
RHO_AZ2 = 0.8 ;
STDERR_AA = 0.01 ;
STDERR_AC = 0.01 ;
STDERR_AP = 0.01 ;
STDERR_AE = 0.01 ;
STDERR_AJ1 = 0.2 ;
STDERR_AJ2 = 0.2 ;
STDERR_AK1 = 0.05 ;
STDERR_AK2 = 0.05 ;
STDERR_AZ1 = 0.03 ;
STDERR_AZ2 = 0.03 ;
//%———————————————————————
//% Model equations
//%———————————————————————
model ;
//% Welfare
uu=c+JEI*h-(exp(nc)^(1+ETA))/(1+ETA);
welfu=uu+BETA*welfu(+1);
ucv=c1v+JEI*h1v-(exp(nc1)^(1+ETA))/(1+ETA);
welfcv=ucv+BETA1*welfcv(+1);
ucf=c1f+JEI*h1f-(exp(nc1f)^(1+ETA))/(1+ETA);
welfcf=ucf+BETA1*welfcf(+1);
welf=(1-BETA)*welfu+(1-BETA1)*(ALPHA*welfcv+(1-ALPHA)*welfcf);
uu_star=c_star+JEI*h_star-(exp(nc_star)^(1+ETA))/(1+ETA);
welfu_star=uu_star+BETA*welfu_star(+1);
ucv_star=c1v_star+JEI*h1v_star-(exp(nc1_star)^(1+ETA))/(1+ETA);
welfcv_star=ucv_star+BETA1*welfcv_star(+1);
ucf_star=c1f_star+JEI*h1f_star-(exp(nc1f_star)^(1+ETA))/(1+ETA);
welfcf_star=ucf_star+BETA1*welfcf_star(+1);
welf_star=(1-BETA)*welfu_star+(1-BETA1)*(ALPHA_STAR*welfcv_star
+(1-ALPHA_STAR)*welfcf_star);
welfunion=NA*welf+(1-NA)*welf_star;
//%——————————————————————————————-
//%
//% COUNTRY A
//%
//%——————————————————————————————-
//% PATIENT HOUSEHOLDS
//% real budget constraint
exp(ca) + exp(ucb-uca)*exp(cb) + exp(q+h) + exp(bu) = (1-DH)*exp(q+h(-1))
+ exp(log(1-MUC)+log(GAMMA)+Y-X) + (1-1/exp(X))*exp(Y)
+ exp(ra(-1)-dpa+bu(-1)) + d - exp(a_c)*exp(r(-1)-dpa)*d(-1) - 0.0001*d^2/2
+ (1-ALPHA)*exp(bf(-1))*(exp(rbar(-1)-dpa)-exp(ra(-1)-dpa)) ;
//% Saving=borrowing
```

```
exp(bu)=ALPHA*exp(b)+(1-ALPHA)*exp(bf);
//% housing
exp(q+uca) =exp(log(JEI)+a_j1-h) + BETA*(exp(q(+1)+uca(+1))) ;
//% euler
exp(uca) = BETA*exp(ra-dpa(+1)+uca(+1)) ;
//% nfa
exp(uca)*(1-0.0001*d) = BETA*exp(r-dpa(+1)+uca(+1)+a_c) ;
//% labor supply
( exp(nc)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc)^(-NU)
= exp( log(1-MUC)+log(GAMMA)+Y-X-nc +uca) ;
//% IMPATIENT HOUSEHOLDS
//VARIABLE
//% budget constraint
exp(ca1v) + exp(ucb-uca)*exp(cb1v) + exp(q+h1v) - (1-DH)*exp(q+h1v(-1)) =
exp(log(1-MUC)+log(1-GAMMA)+Y-X) + exp(b) - exp(ra(-1)-dpa+b(-1)) ;
//% collateral constraint
b = log(M) + q(+1) + h1v - ra + dpa(+1) ;
//% relative marginal utilities
exp(ucb1) = exp(uca1)*exp(ucb-uca) ;
//% housing
exp(q+uca1) = exp(log(JEI)+a_j1-h1v) +BETA1*exp(q(+1)+uca1(+1)) +
M*exp(lmv+q(+1)-ra+dpa(+1));
//% euler
exp(uca1) = BETA1*exp(ra-dpa(+1)+uca1(+1)) + exp(lmv) ;
//% labor supply
( exp(nc1)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc1)^(-NU)
= exp(log(1-MUC)+log(1-GAMMA)+Y-X-nc1 +uca1) ;
//FIXED
//% budget constraint
exp(ca1f) + exp(ucb-uca)*exp(cb1f) + exp(q+h1f) - (1-DH)*exp(q+h1f(-1)) =
exp(log(1-MUC)+log(1-GAMMA)+Y-X) + exp(bf) - exp(rbar(-1)-dpa+bf(-1)) ;
//% collateral constraint
bf = log(M) + q(+1) + h1f - rbar + dpa(+1) ;
//% relative marginal utilities
exp(ucb1f) = exp(uca1f)*exp(ucb-uca) ;
//% housing
exp(q+uca1f) = exp(log(JEI)+a_j1-h1f) +BETA1*exp(q(+1)+uca1f(+1)) +
M*exp(lmf+q(+1)-rbar+dpa(+1));
//% euler
exp(uca1f) = BETA1*exp(rbar-dpa(+1)+uca1f(+1)) + exp(lmf) ;
//% labor supply
( exp(nc1f)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc1f)^(-NU)
= exp(log(1-MUC)+log(1-GAMMA)+Y-X-nc1f +uca1f) ;
//% FIRMS
//% Production function
```

```
Y = (1-MUC)*a_a + (1-MUC)*GAMMA*nc + (1-MUC)*(1-GAMMA)*ncc ;
//% Phillips curve
dpa - LAGP*dpa(-1) = BETA*(dpa(+1) - LAGP*dpa)
- ((1-TETA)*(1-BETA*TETA)/TETA)*(X-log(X_SS)) + eps_p ;
//% MARKET CLEARING
//% goods
(NA)*(exp(ca)+exp(ca1)) + (1-NA)*(exp(ca_star)+exp(ca1_star))
= (NA)*exp(Y) - NA*0.0001*d^2/2 ;
//% housing
exp(h) + exp(h1) = 1 ;
//% VARIOUS DEFINITIONS
exp(ca1)=ALPHA*exp(ca1v)+(1-ALPHA)*exp(ca1f);
exp(cb1)=ALPHA*exp(cb1v)+(1-ALPHA)*exp(cb1f);
exp(h1)=ALPHA*exp(h1v)+(1-ALPHA)*exp(h1f);
exp(ncc)=ALPHA*exp(nc1)+(1-ALPHA)*exp(nc1f);
//%
c = (NA+HB)*ca + (1-NA-HB)*cb ;
c1 = (NA+HB)*ca1 + (1-NA-HB)*cb1 ;
c1v = (NA+HB)*ca1v + (1-NA-HB)*cb1v ;
c1f = (NA+HB)*ca1f + (1-NA-HB)*cb1f ;
//%
exp(uca) = exp(a_z1) * ((1-EC)/(1-BETA*EC)) * ( 1 / ( exp(c) - EC * exp (c(-1)) )
- BETA * EC / ( exp(c(+1)) - EC*exp(c) ) ) * (NA+HB) * exp(c)/exp(ca) ;
exp(uca1) = exp(a_z1) * ((1-EC)/(1-BETA1*EC)) *
( 1 / ( exp(c1) - EC * exp (c1(-1)) )
- BETA1 * EC / ( exp(c1(+1)) - EC*exp(c1) ) ) * (NA+HB) * exp(c1)/exp(ca1v) ;
exp(uca1f) = exp(a_z1) * ((1-EC)/(1-BETA1*EC)) * ( 1 / ( exp(c1)
- EC * exp (c1(-1)) ) - BETA1 * EC / ( exp(c1(+1)) - EC*exp(c1) ) )
* (NA+HB) * exp(c1)/exp(ca1f) ;
exp(ucb) = exp(a_z1) * ((1-EC)/(1-BETA*EC)) * ( 1 / ( exp(c) - EC * exp (c(-1)) )
- BETA * EC / ( exp(c(+1)) - EC*exp(c) ) ) * (1-NA-HB) * exp(c)/exp(cb) ;
exp(ucb1) = exp(a_z1) * ((1-EC)/(1-BETA1*EC)) *
( 1 / ( exp(c1) - EC * exp (c1(-1)) )
- BETA1 * EC / ( exp(c1(+1)) - EC*exp(c1) ) ) * (1-NA-HB) * exp(c1)/exp(cb1v) ;
exp(ucb1f) = exp(a_z1) * ((1-EC)/(1-BETA1*EC)) *
( 1 / ( exp(c1) - EC * exp (c1(-1)) )
- BETA1 * EC / ( exp(c1(+1)) - EC*exp(c1) ) ) * (1-NA-HB) * exp(c1)/exp(cb1f) ;
//%---------------------------------
//%
//% COUNTRY B
//%
//%---------------------------------
//% STAR_PATIENT HOUSEHOLDS
//% budget constraint
//% DROP THIS EQUATION AND IMPOSE MARKET CLEARING ;
```

```
//% relative marginal utilities
exp(ucb_star) = exp(uca_star)*exp(ucb-uca) ;
//% housing
exp(q_star+ucb_star) =exp(log(JEI)+a_j2-h_star) + BETA*exp(q_star(+1)+ucb_star(+1));
//% euler
exp(ucb_star) = BETA*exp(rb-dpb(+1)+ucb_star(+1)) ;
//% nfa
exp(ucb_star)*(1+0.0001*d) = BETA*exp(r-dpb(+1)+ucb_star(+1)+a_c) ;
//% labor supply
( exp(nc_star)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc_star)^(-NU)
= exp(log(1-MUC)+log(GAMMA_STAR)+Y_star-X_star-nc_star+ucb_star) ;
//% definition fixed rate
exp(rbar)=1/BETA;
//% STAR_IMPATIENT HOUSEHOLDS
//VARIABLE
//% budget constraint
exp(ca1v_star)/exp(ucb-uca) + exp(cb1v_star) + exp(q_star+h1v_star)
- (1-DH)*exp(q_star+h1v_star(-1)) = exp(log(1-MUC)
+log(1-GAMMA_STAR)+Y_star-X_star) + exp(b_star) - exp(rb(-1)
-dpb+b_star(-1)) ;
//% collateral constraint
b_star = log(M_STAR) + q_star(+1) + h1v_star - rb + dpb(+1) ;
//% relative marginal utilities
exp(ucb1_star) = exp(uca1_star)*exp(ucb-uca) ;
//% housing
exp(q_star+ucb1_star) = exp(log(JEI)+a_j2-h1v_star) +
BETA1*exp(q_star(+1)+ucb1_star(+1)) +
M_STAR*exp(lmv_star+q_star(+1)-rb+dpb(+1));
//% euler
exp(ucb1_star) = BETA1*exp(rb-dpb(+1)+ucb1_star(+1)) + exp(lmv_star) ;
//% labor supply
( exp(nc1_star)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc1_star)^(-NU)
= exp(log(1-MUC)+log(1-GAMMA_STAR)+Y_star-X_star-nc1_star+ucb1_star) ;
//FIXED
//% STAR_10
exp(ca1f_star)/exp(ucb-uca) + exp(cb1f_star) + exp(q_star+h1f_star)
- (1-DH)*exp(q_star+h1f_star(-1)) = exp(log(1-MUC)+log(1-GAMMA_STAR)
+Y_star-X_star) + exp(bf_star) - exp(rbar(-1)-dpb+bf_star(-1)) ;
//% collateral constraint
bf_star = log(M_STAR) + q_star(+1) + h1f_star - rbar + dpb(+1) ;
//% relative marginal utilities
exp(ucb1f_star) = exp(uca1f_star)*exp(ucb-uca) ;
//% housing
exp(q_star+ucb1f_star) = exp(log(JEI)+a_j2-h1f_star) +
BETA1*exp(q_star(+1)+ucb1f_star(+1)) +
```

```
M_STAR*exp(lmf_star+q_star(+1)-rbar+dpb(+1));
//% euler
exp(ucb1f_star) = BETA1*exp(rbar-dpb(+1)+ucb1f_star(+1)) + exp(lmf_star) ;
//% labor supply
( exp(nc1f_star)^(1-NU) )^((ETA+NU)/(1-NU)) * exp(nc1f_star)^(-NU)
= exp(log(1-MUC)+log(1-GAMMA_STAR)+Y_star-X_star-nc1f_star+ucb1f_star) ;
//% STAR_FIRMS
//% Production function
Y_star = (1-MUC)*a_a + (1-MUC)*GAMMA_STAR*nc_star
+ (1-MUC)*(1-GAMMA_STAR)*ncc_star ;
//% Phillips curve
dpb - LAGP*dpb(-1) = BETA*(dpb(+1) - LAGP*dpb)
- ((1-TETA)*(1-BETA*TETA)/TETA)*(X_star-log(X_SS)) + eps_p ;
//% STAR_MARKET CLEARING
//% goods
(1-NA)*( exp(cb_star)+exp(cb1_star))
+(NA)*(exp(cb)+exp(cb1)) = (1-NA)*exp(Y_star) - 0.0001*NA*(d/exp(ucb-uca))^2/2;
//% housing
exp(h_star) + exp(h1_star) = 1 ;
//% STAR_VARIOUS DEFINITIONS
exp(ca1_star)=ALPHA_STAR*exp(ca1v_star)+(1-ALPHA_STAR)*exp(ca1f_star);
exp(cb1_star)=ALPHA_STAR*exp(cb1v_star)+(1-ALPHA_STAR)*exp(cb1f_star);
exp(h1_star)=ALPHA_STAR*exp(h1v_star)+(1-ALPHA_STAR)*exp(h1f_star);
exp(ncc_star)=ALPHA_STAR*exp(nc1_star)+(1-ALPHA_STAR)*exp(nc1f_star);
//%
c_star = (NA-HB)*ca_star + (1-NA+HB)*cb_star ;
c1_star = (NA-HB)*ca1_star + (1-NA+HB)*cb1_star ;
c1v_star = (NA-HB)*ca1v_star + (1-NA+HB)*cb1v_star ;
c1f_star = (NA-HB)*ca1f_star + (1-NA+HB)*cb1f_star ;
//%
exp(uca_star) = exp(a_z2) * ((1-EC)/(1-BETA*EC)) *
( 1 / ( exp(c_star) - EC * exp (c_star(-1)) ) -
BETA * EC / ( exp(c_star(+1)) - EC*exp(c_star) ) ) * (NA-HB)
* exp(c_star)/exp(ca_star) ;
exp(uca1_star) = exp(a_z2) * ((1-EC_STAR)/(1-BETA1*EC_STAR)) *
( 1 / ( exp(c1_star) - EC_STAR * exp (c1_star(-1)) ) -
BETA1 * EC_STAR / ( exp(c1_star(+1)) - EC_STAR*exp(c1_star) ) )
* (NA-HB) * exp(c1_star)/exp(ca1v_star) ;
exp(uca1f_star) = exp(a_z2) * ((1-EC_STAR)/(1-BETA1*EC_STAR)) *
( 1 / ( exp(c1_star) - EC_STAR * exp (c1_star(-1)) ) -
BETA1 * EC_STAR / ( exp(c1_star(+1)) - EC_STAR*exp(c1_star) ) )
* (NA-HB) * exp(c1_star)/exp(ca1f_star) ;
exp(ucb_star) = exp(a_z2) * ((1-EC_STAR)/(1-BETA*EC_STAR)) *
( 1 / ( exp(c_star) - EC_STAR * exp (c_star(-1)) ) -
BETA * EC_STAR / ( exp(c_star(+1)) - EC_STAR*exp(c_star) ) )
```

```
* (1-NA+HB) * exp(c_star)/exp(cb_star) ;
exp(ucb1_star) = exp(a_z2) * ((1-EC_STAR)/(1-BETA1*EC_STAR)) *
( 1 / ( exp(c1_star) - EC_STAR * exp (c1_star(-1)) )
- BETA1 * EC_STAR / ( exp(c1_star(+1)) - EC_STAR*exp(c1_star) ) )
* (1-NA+HB) * exp(c1_star)/exp(cb1v_star) ;
exp(ucb1f_star) = exp(a_z2) * ((1-EC_STAR)/(1-BETA1*EC_STAR)) *
( 1 / ( exp(c1_star) - EC_STAR * exp (c1_star(-1)) )
- BETA1 * EC_STAR / ( exp(c1_star(+1)) - EC_STAR*exp(c1_star) ) )
* (1-NA+HB) * exp(c1_star)/exp(cb1f_star) ;
//%-------------------------------------------------
//%
//% AREA-WIDE EQUATIONS
//%
//%-------------------------------------------------
//% Taylor rule
r = TAYLOR_R*r(-1) + (1-TAYLOR_R)*(1+TAYLOR_P)*(NA*dpa+(1-NA)*dpb)
+ (1-TAYLOR_R)*log(1/BETA) + eps_e ;
//% Definition of relative price
dpb = ucb - uca - ucb(-1) + uca(-1) + dpa ;
//% Definitions
data_CCA = (exp(ca) + exp(ca1) + exp(cb) + exp(cb1)) ;
data_CCB = (exp(cb_star) + exp(cb1_star) + exp(ca_star) + exp(ca1_star)) ;
data_DPE = NA*dpa+(1-NA)*dpb ;
data_QQA = q - QQ_SS ;
data_QQB = q_star - QQ_SS_star ;
data_RRE = r - log(1/BETA) ;
GDPA=exp(ca) + exp(ca1) + exp(ca_star) + exp(ca1_star) ;
GDPB=exp(cb) + exp(cb1) + exp(cb_star) + exp(cb1_star) ;
//% STOCHASTIC PROCESSES FOR THE SHOCKS
a_a = RHO_AA * a_a(-1) + eps_a ;
a_c = RHO_AC * a_c(-1) + eps_c ;
a_j1 = RHO_AJ1 * a_j1(-1) + eps_j1 ;
a_j2 = RHO_AJ2 * a_j2(-1) + eps_j2 ;
a_z1 = RHO_AZ1 * a_z1(-1) + eps_z1 ;
a_z2 = RHO_AZ2 * a_z2(-1) + eps_z2 ;
end ;
//%-----------------------------------
//% Call steady state
//%-----------------------------------
steady(solve_algo=0) ;
//%-----------------------------------
//% Declare shocks
//%-----------------------------------
shocks;
var eps_a ; stderr 100*STDERR_AA ;
```

```
//var eps_c ; stderr STDERR_AC ;
//var eps_e ; stderr 100*STDERR_AE ;
//var eps_j1 ; stderr 100*STDERR_AJ1 ;
//var eps_j2 ; stderr 100*STDERR_AJ2 ;
//var eps_p ; stderr STDERR_AP ;
//var eps_z1 ; stderr STDERR_AZ1 ;
//var eps_z2 ; stderr STDERR_AZ2 ;
end;
stoch_simul(dr_algo=0,order=2,irf=0) welfu welfu_star welfcv welfcf welfcv_star welfcf_star welfunion welf welf_star ;
```

Steady-State File

```
function [ys,check]=name_steadystate(junk,ys)
    global NU X_SS MUC MUH M DH DKH DKC BETA BETA1 ys
    global JEI logxxx ZETA0 ZETA1 ZETA2 ZETA3
    global GAMMA r ALPHA ALPHA_STAR
    global CC_SS RR_SS QQ_SS R_SS
    global ETA
    global q h h1 c c1 CC b uh uh1 CY q_star h_star h1_star
    global CC_star
    global CHI1 CHI2 CHI3 CHI4 CHI5 CHI6 R1
    global wh wh1 wc wc1 xxx
    global H1_SS H_SS NA
    global QQ_SS_star H1_SS_star
    global CC_SS_star QI QI_star
    global GDP GDP_star DEBT DEBT_star HOUSING HOUSING_star
    global M_STAR GAMMA_STAR HB
    % Nominal, real rates and inflation
    R_SS = 1/BETA ;
    R1 = R_SS-1;
    dpa = 1 ;
    dpb = 1 ;
    r = 1 / BETA ;
    ra = r ;
    rb = r ;
    rbar=r;
    d = 1 ;
    check=0;
    ZETA0 = BETA*MUC/(1-BETA*(1-DKC))/X_SS ;
    ZETA1 = BETA*MUH/(1-BETA*(1-DKH));
    ZETA2 = JEI/(1-BETA*(1-DH)) ;
    ZETA3 = JEI/(1-BETA1*(1-DH)-(BETA-BETA1)*M) ;
    ZETA0_STAR = BETA*MUC/(1-BETA*(1-DKC))/X_SS ;
    ZETA1_STAR = BETA*MUH/(1-BETA*(1-DKH));
    ZETA2_STAR = JEI/(1-BETA*(1-DH)) ;
```

```
ZETA3_STAR = JEI/(1-BETA1*(1-DH)-(BETA-BETA1)*M_STAR) ;
CHI1 = 1+DH*ZETA2*(1-R1*ZETA1-GAMMA*(1-MUH)) ;
CHI2 = (R1*ZETA1+GAMMA*(1-MUH))*DH*ZETA3+R1*BETA*M*ZETA3 ;
CHI3 = (X_SS-1+R1*ZETA0*X_SS+GAMMA*(1-MUC))/X_SS ;
CHI4 = 1+DH*ZETA3*(1-(1-GAMMA)*(1-MUH))+R1*BETA*M*ZETA3 ;
CHI5 = (1-GAMMA)*(1-MUH)*DH*ZETA2 ;
CHI6 = (1-GAMMA)*(1-MUC)/X_SS ;
CHI1_STAR = 1+DH*ZETA2_STAR*(1-R1*ZETA1_STAR-GAMMA_STAR*(1-MUH));
CHI2_STAR = (R1*ZETA1_STAR+GAMMA_STAR*(1-MUH))*DH*ZETA3_STAR+R1*BETA*M_STAR*ZETA3_STAR ;
CHI3_STAR = (X_SS-1+R1*ZETA0_STAR*X_SS+GAMMA_STAR*(1-MUC))/X_SS ;
CHI4_STAR = 1+DH*ZETA3_STAR*(1-(1-GAMMA_STAR)*(1-MUH))+R1*BETA*M_STAR*ZETA3_STAR ;
CHI5_STAR = (1-GAMMA_STAR)*(1-MUH)*DH*ZETA2_STAR ;
CHI6_STAR = (1-GAMMA_STAR)*(1-MUC)/X_SS ;
CY = (CHI3*CHI4+CHI2*CHI6)/(CHI1*CHI4-CHI2*CHI5) ;
CY1 = (CHI1*CHI6+CHI3*CHI5)/(CHI1*CHI4-CHI2*CHI5) ;
CY_STAR = (CHI3_STAR*CHI4_STAR+CHI2_STAR*CHI6_STAR)/(CHI1_STAR*CHI4_STAR-CHI2_STAR*CHI5_STAR);
CY1_STAR = (CHI1_STAR*CHI6_STAR+CHI3_STAR*CHI5_STAR)/(CHI1_STAR*CHI4_STAR-CHI2_STAR*CHI5_STAR);
nc = ((1-MUC)*GAMMA/CY/X_SS)^(1/(1+ETA)) ;
nc1 = ((1-MUC)*(1-GAMMA)/CY1/X_SS)^(1/(1+ETA)) ;
nc1f=nc1;
ncc=nc1;
nc_star = ((1-MUC)*GAMMA_STAR/CY_STAR/X_SS)^(1/(1+ETA)) ;
nc1_star = ((1-MUC)*(1-GAMMA_STAR)/CY1_STAR/X_SS)^(1/(1+ETA)) ;
nc1f_star=nc1_star;
ncc_star=nc1_star;
Y = (nc^GAMMA)*(ncc^(1-GAMMA));
q=1;
Y_star = (nc_star^GAMMA_STAR)*(ncc_star^(1-GAMMA_STAR));
q_star=1;
num_rpb = (CY+CY1)*Y*NA*(1-NA-HB) ;
den_rpb = (CY_STAR+CY1_STAR)*Y_star*(1-NA)*(NA-HB) ;
rpb = num_rpb / den_rpb ;
ca = (NA+HB)*CY*Y ;
cb = (1-NA-HB)*CY*Y/rpb ;
c = ca^(NA+HB)*cb^(1-NA-HB) ;
ca1v = (NA+HB)*CY1*Y ;
ca1f=ca1v;
ca1=ca1v;
cb1v = (1-NA-HB)*CY1*Y/rpb ;
cb1f=cb1v;
cb1=cb1v;
c1 = ca1^(NA+HB)*cb1^(1-NA-HB) ;
c1v = ca1v^(NA+HB)*cb1v^(1-NA-HB) ;
c1f = ca1f^(NA+HB)*cb1f^(1-NA-HB) ;
ca_star = (NA-HB)*CY_STAR*Y_star*rpb ;
```

```
cb_star = (1-NA+HB)*CY_STAR*Y_star ;
c_star = ca_star^(NA-HB)*cb_star^(1-NA+HB) ;
ca1v_star = (NA-HB)*CY1_STAR*Y_star*rpb ;
ca1f_star=ca1v_star;
ca1_star=ca1v_star;
cb1v_star = (1-NA+HB)*CY1_STAR*Y_star ;
cb1f_star=cb1v_star;
cb1_star=cb1v_star;
c1_star = ca1_star^(NA-HB)*cb1_star^(1-NA+HB) ;
c1v_star = ca1v_star^(NA+HB)*cb1v_star^(1-NA-HB) ;
c1f_star = ca1f_star^(NA+HB)*cb1f_star^(1-NA-HB) ;
h = ZETA2*ca/q/(NA+HB) ;
h1v = ZETA3*ca1/q/(NA+HB) ;
h1f=h1v;
h1=h1v;
h_star = ZETA2_STAR*cb_star/q_star/(1-NA+HB) ;
h1v_star = ZETA3_STAR*cb1_star/q_star/(1-NA+HB) ;
h1f_star=h1v_star;
h1_star=h1v_star;
b = BETA*M*q*h1 ;
bf=b;
bu=b;
b_star = BETA*M_STAR*q_star*h1_star ;
bf_star=b_star;
lmv = (1-BETA1/BETA)/ca1*(NA+HB) ;
lmv_star = (1-BETA1/BETA)/cb1_star*(1-NA+HB) ;
lmf=lmv;
lmf_star=lmv_star;
uh = JEI/h;
uh1 = JEI/h1;
uh_star = JEI/h_star;
uh1_star = JEI/h1_star;
uca = (NA+HB)/ca;
uca1 = (NA+HB)/ca1;
uca1f=uca1;
ucb = (1-NA-HB)/cb;
ucb1 = (1-NA-HB)/cb1;
ucb1f=ucb1;
uca_star = (NA-HB)/ca_star;
uca1_star = (NA-HB)/ca1_star;
uca1f_star=uca1_star;
ucb_star = (1-NA+HB)/cb_star;
ucb1_star = (1-NA+HB)/cb1_star;
ucb1f_star=ucb1_star;
CC = ca + ca1 + cb + cb1 ;
```

```
CC_star = cb_star + cb1_star + ca_star + ca1_star ;
GDP = CC ;
GDP_star = CC_star;
HOUSING = q*(h+h1) ;
HOUSING_star = q_star*(h_star+h1_star) ;
DEBT = b ;
DEBT_star = b_star ;
GDPA=ca+ca1+ca_star+ca1_star;
GDPB=cb+cb1+cb_star+cb1_star;
H_SS = log(h) ;
H1_SS = log(h1) ;
H_SS_star = log(h_star) ;
H1_SS_star = log(h1_star) ;
X = X_SS ;
X_star = X_SS ;
uu=exp(log(c)+JEI*log(h)-((nc)^(1+ETA))/(1+ETA));
welfu= exp(log(uu)/(1-BETA));
ucv=exp(log(c1v)+JEI*log(h1v)-((nc1)^(1+ETA))/(1+ETA));
welfcv=exp(log(ucv)/(1-BETA1));
ucf=exp(log(c1f)+JEI*log(h1f)-((nc1f)^(1+ETA))/(1+ETA));
welfcf=exp(log(ucf)/(1-BETA1));
welf=exp((1-BETA)*log(welfu)+(1-BETA1)*(ALPHA*log(welfcv)+(1-ALPHA)*log(welfcf)));
uu_star=exp(log(c_star)+JEI*log(h_star)-((nc_star)^(1+ETA))/(1+ETA));
welfu_star=exp(log(uu_star)/(1-BETA));
ucv_star=exp(log(c1v_star)+JEI*log(h1v_star)-((nc1_star)^(1+ETA))/(1+ETA));
welfcv_star=exp(log(ucv_star)/(1-BETA1));
ucf_star=exp(log(c1f_star)+JEI*log(h1f_star)-((nc1f_star)^(1+ETA))/(1+ETA));
welfcf_star=exp(log(ucf_star)/(1-BETA1));
welf_star=exp((1-BETA)*log(welfu_star)+(1-BETA1)*(ALPHA*log(welfcv_star)+(1-ALPHA)*log(welfcf_star)));
welfunion=exp(NA*log(welf)+(1-NA)*log(welf_star));
% Shocks
shocks = 1 ;
% Log of some variables in steady state
CC_SS = log(CC) ;
CC_SS_star = log(CC_star);
QQ_SS = log(q) ;
QQ_SS_star = log(q_star);
RR_SS = log(1/BETA) ;
xxx = [ ...
shocks
shocks
shocks
shocks
shocks
shocks
```

b

b_star

bf

bf_star

bu

c

c1

c1_star

c1f

c1f_star

c1v

c1v_star

c_star

ca

ca1

ca1_star

ca1v

ca1v_star

ca1f

ca1f_star

ca_star

cb

cb1

cb1_star

cb1v

cb1v_star

cb1f

cb1f_star

cb_star

d

CC/CC

CC_star/CC_star

1

q/q

q_star/q_star

1

dpa

dpb

GDPA

GDPB

h

h1

h1_star

h1v

h1v_star

```
h1f
h1f_star
h_star
lmv
lmv_star
lmf
lmf_star
nc
nc1
nc1f
ncc
nc1_star
nc_star
nc1f_star
ncc_star
q
q_star
r
ra
rb
rbar
uca
uca1
uca1_star
uca1f
uca1f_star
uca_star
ucb
ucb1
ucb1_star
ucb1f
ucb1f_star
ucb_star
ucf
ucf_star
ucv
ucv_star
uu
uu_star
welf
welf_star
welfcf
welfcf_star
welfcv
welfcv_star
```

```
welfu
welfu_star
welfunion
X
X_star
Y
Y_star ] ;
logxxx = log(xxx) ;
ys = logxxx ;
```