Commentaries on "Unit Root Testing in Practice: Dealing with Uncertainty over the Trend and Initial Condition," by David I. Harvey, Stephen J. Leybourne, and A.M. Robert Taylor

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I congratulate Harvey, Leybourne, and Taylor on writing a truly interesting and stimulating paper on an important issue in practice: how to conduct unit root tests. The authors provided a simple practical guide that should prove very useful in time series applications. I find myself with really nothing to criticize. I will comment on a few issues related to this research problem.

The Impact of Preliminary Data Analysis

In the last two decades, there has been much discussion on how to improve sampling performance of unit root testing, and many procedures were proposed. Depending on the empirical feature of the data, there are different directions that can potentially improve the performance of unit root testing. A partial list of these directions includes

1. initial condition;
2. trend uncertainty and efficient detrending;
3. information of the innovation distribution (non-Gaussianity);
4. regression model specification;
5. using useful covariates to improve univariate unit root testing (as Hansen, 1995).

Harvey, Leybourne, and Taylor focuses on issues related to trend and initial condition uncertainty. Initial condition is not observable or estimable in practice, and so it is impossible to directly use information about initial condition. However, initial condition uncertainty does affect the selection of a more efficient (say) detrending procedure. For this reason, researchers want to take into account initial condition uncertainty when selecting a detrending method.

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How to choose an appropriate method to improve the empirical performance of unit root testing largely depends on the empirical feature of the time series and availability of additional information. For this reason, it is natural to consider some preliminary data analysis that can give us better knowledge about the empirical feature of the data and, thus, help us choose an appropriate method.

However, such a strategy also entails the choice of an appropriate preliminary data analysis procedure, and the performance of a unit root test coupled with a preliminary data analysis procedure will be affected by the choice of such a preliminary procedure. How to choose a good preliminary data analysis procedure adds another practical issue in addition to the already complicated problem. Also, an even more important issue in using a preliminary data checking procedure is that if, say, a pre-test for trend is used, the asymptotic properties (size and power) of the unit root test will be affected by the size of the pre-test and the magnitude of the trend component. It is well known that preliminary data analysis such as pre-testing may impact the size and power of the second stage inference procedures (for studies on similar issues in other inference problems, see, e.g., Kabaila, 1995; Leeb and Pötscher, 2005; Guggenberger, 2007). From this point of view, an appropriately chosen simple strategy that does not depend on a preliminary test may have some desirable properties in the presence of limited uncertain issues.

In Harvey, Leybourne, and Taylor, the pre-testing based strategy naturally entails the choice of an appropriate test for the trend. Consequently the performance of such unit root tests will depend on the ability of the pre-test in detecting the trend component. In addition, practical issues such as bandwidth selection in the pre-test may also influence the second stage unit root testing. Monte Carlo evidence in Harvey, Leybourne, and Taylor indicates that some pre-test might be better than others in terms of unit root testing. In the presence of a local trend, trend tests have poor power properties because they are inconsistent in this case. Consequently, this inability to consistently detect the trend component will be translated into the second stage unit root testing. As shown by the authors, the asymptotic size and power of the second stage unit root test will be a function of κ. Similarly, the second strategy of Harvey, Leybourne, and Taylor that uses a data-dependent weighted average of unit root tests requires choosing the weights based on a trend test statistic. Again choice of the trend statistic (i.e., construction of the weight) affects the sampling performance of the unit root test. The third test, the union of rejections (UR) test, does not depend on a preliminary checking procedure. The Monte Carlo evidence in Harvey, Leybourne, and Taylor indicates that such a simple strategy has more "robust" performance with respect to trend and initial condition uncertainty.

Despite the additional complication and uncertainty that a preliminary data analysis procedure may bring into unit root testing, a preliminary check on the empirical feature of the data (and other types of information) can still provide very valuable information to researchers and decision makers—it helps the researchers to identify plausible directions that could improve the performance of unit root testing. This is important because in some applications, there is simply not much room for additional improvement along certain directions, and there is no strategy that is better (than others) uniformly over all types of uncertainty.

Strategies that use a preliminary data analysis are sequential tests where a hypothesis about the trend component and the unit root hypothesis are considered sequentially. An alternative way to deal with trend uncertainty in unit root testing might be to consider a test for unit root and trend uncertainty jointly. Of course, a different method may be needed for such joint tests. For example, methods based on model selection (e.g., Phillips and Ploberger, 1994) might be useful for such a purpose.

Dealing with Initial Condition and Trend Uncertainty in Practice

To focus on the issue of trend uncertainty, Harvey, Leybourne, and Taylor first considered strategies of dealing with trend uncertainty under the assumption that the initial condition is negligible. Similarly, their discussion on unit root testing with respect to the initial condition uncertainty is given under the assumption that there is no uncertainty about the trend. This is certainly convenient from the point of view of theoretical analysis and helps the readers to focus on each individual issue.

However, in practice, researchers have to deal with the initial condition and trend uncertainty (and also other types of uncertainty) simultaneously. Because the purpose of this paper is to consider unit root testing in "practice," simultaneously dealing with initial condition and trend uncertainty seems appropriate. From the analysis of Harvey, Leybourne, and Taylor, it seems that the following augmented union of rejections (*UR*) strategy should be used in practice to deal with uncertainty over the trend and initial condition:

> Reject the I(1) null if any of the following tests rejects unit root: $DF\text{-}OLS^{\mu}$, $DF\text{-}QD^{\mu}$, $DF\text{-}OLS^{\tau}$, $DF\text{-}QD^{\tau}$.

Similar to the other *UR* tests proposed in Harvey, Leybourne, and Taylor, the preceding test is not size controlled, and appropriate correction along the lines of Section 3.3.2 of the paper may be needed.

More generally, unit root testing involves a large array of practical issues. For example, other issues such as weak dependence in ε_t (thus lag length selection in the augmented Dickey–Fuller [ADF] regression) has to be handled simultaneously with initial condition and trend uncertainty, and innovation distributional uncertainty is also an important direction. When we face more uncertain issues in practice, it is more difficult to deal with them simultaneously. Appropriately dealing with these issues can help us obtain more efficient statistical inference.

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